Lens Tolerancing

Lens Design OPTI 517

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Lens Tolerancing goals

- Modeling the “as built” performance of a lens system
- We want to know the associated statistics and fabrication yield
- Assign tolerances to the constructional parameters

Design process

Application Specs

Literature search

Finding solutions

Tolerancing

Evaluation Review

Alignment
Assembly
Testing
Verification plans

Optics shop
Opto-mechanics
Electronics

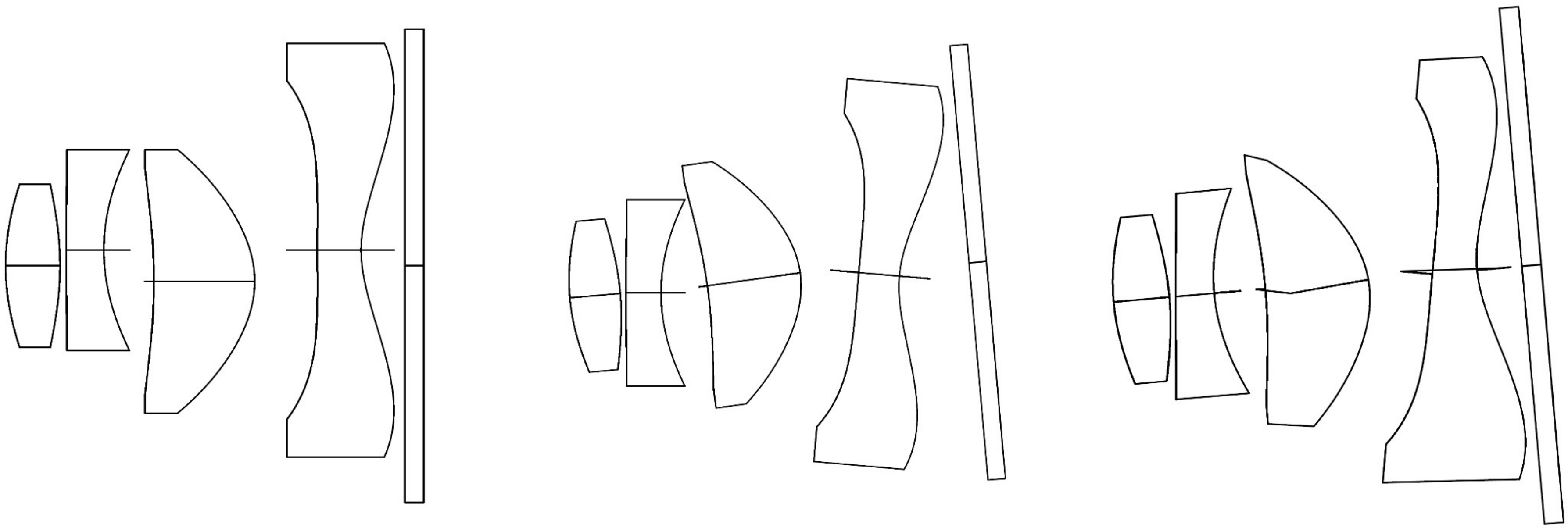
Checks

Fabrication

Tolerancing I

- Since lenses can not be perfectly manufactured some tolerancing must be specified
- Errors are associated with: radius, figure, index, wedge, thickness, spacing, opto-mechanics, assembling, etc.
- These errors decrease the design merit function and affect image quality.
- Tolerancing is a science and an art.
- Test plate fit, index fit, thickness fit
- Compensators: image plane distance; line of sight, aberrations, other
- Tolerances and cost
- Shop tendencies and communication

Lens element decenter, tilt, wedge



Tolerancing II

- Set criteria for lens performance such as merit function; assume small changes.
- Distribution of errors.
- Sensitivity
- Inverse sensitivity
- Worst case
- Standard deviation
- Montecarlo simulation

Some references

- Shannon's Chapter 6 and his chapter in the OSA Handbook of optics
- Warren Smith, Modern Lens Design, chapter 23
- Warren Smith, Fundamentals of the optical tolerance budget. SPIE paper.
- Papers; i.e. in SPIE proceedings

Parameter	Commercial	Precision	High precision
Thickness	0.1 mm	0.01 mm	0.001 mm
Radius	1%	0.1%	0.001%
Index	0.001	0.0001	0.00001
V-number	1%	0.1%	0.01%
Decenter	0.1 mm	0.01 mm	0.001 mm
Tilt	1 arc min	10 arc sec	1 arc sec
Irregularity	1 ring	0.25 ring	<0.1 ring
Sphericity	2 rings	1 ring	0.25 rings
Wavefront residual	0.25 wave rms	0.1 wave rms	<0.07 wave rms

From R. Shannon

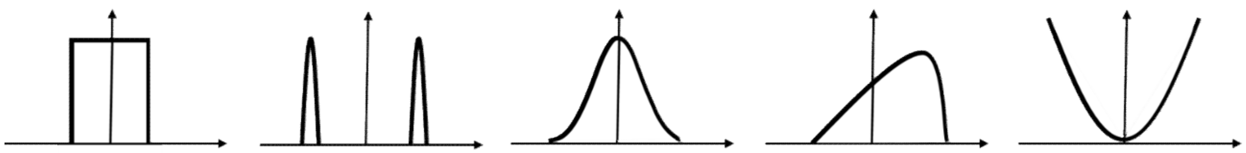
	Surface quality	Diameter, mm	Thickness, mm	Radius	Irregularity	Linear dimension, mm	Angular dimensions
Low cost	120-80	+/- 0.2	0.5	Gage	Gage	0.5	Degrees
Commercial	80-50	+/- 0.07	.25	10 Fr.	3 Fr.	0.25	15 arc-min
Precision	60-40	+/- 0.02	0.1	5 Fr.	1 Fr.	0.1	5-10 arc-sec
Extra-precise	60-40	+/- 0.01	0.02	1 Fr.	1/5 Fr.	0.01	Seconds
Plastic	80-50			10 Fr.	5 Fr.	0.02	

From Warren Smith

ATTRIBUTE	COMMERCIAL QUALITY	PRECISION QUALITY	“MAXIMUM” QUALITY
DIAMETER (mm)	+0.00/-0.10	+0.000/-0.05	+0.000/-0.025
CENTER THICKNESS (mm)	0.150	0.050	0.025
RADIUS (POWER)	0.2% (8 rings)	0.1% (4 rings)	0.05% (2 rings)
IRREGULARITY (Waves@633nm)	1	0.25	0.1
WEDGE (mm)	0.05	0.005	.0025
DECENTER (arc min)	0.05	0.01	0.005
SCRATCH - DIG	80 - 50	60 - 40	20 -10
AR COATING (R avg)	< 1.5%	< 0.5%	< 0.25%

From Special Optics

Parameter statistics



Parameter error distributions. From left to right, uniform, end limited, truncated normal, skewed, parabolic

Tolerancing Analysis

Sensitivity

Surface	Item	Design value	Specified tolerance	Merit function change
2	radius	50.3	5 rings	0.005
3	thickness	13	0.1 mm	0.001
4	radius	24.34	0.2 mm	0.007

Inverse Sensitivity

Surface	Item	Design value	Specified tolerance	Merit function change
2	radius	50.3	2 rings	0.001
3	thickness	13	0.01 mm	0.001
4	radius	24.34	0.03 mm	0.001

Worst case

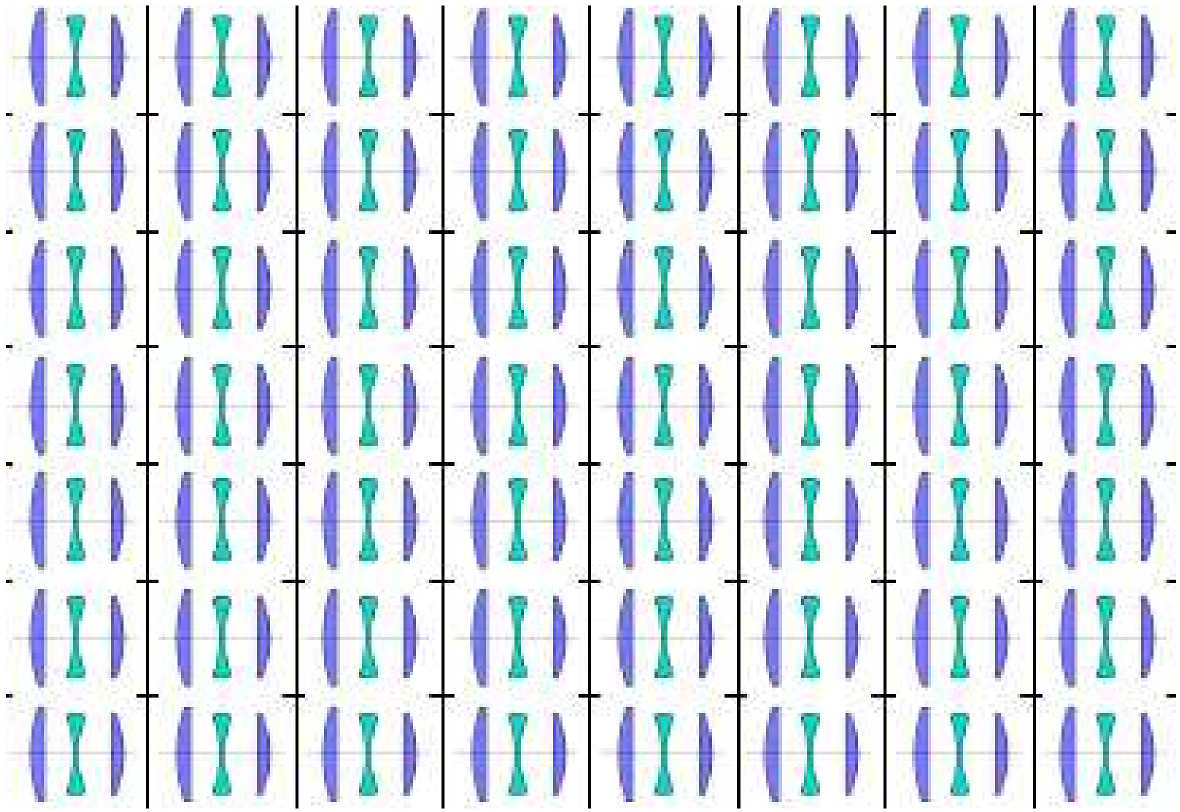
1) **Absolute**: This involves evaluating the system in every possible situation and finding the worst case.
This procedure is not practical due the large number of possibilities.

2) **Statistical**: Use a statistical worse case approach form sensitivity data by summing the absolute values of the individual performance change for each constructional parameter.
This approach is pessimistic.

The statistical nature of tolerancing

- Cannot predict perfectly the final performance
- Must use common sense and statistics
- <u>We are after the statistics</u>

Experience shows that there is a distribution in the performance of lens systems



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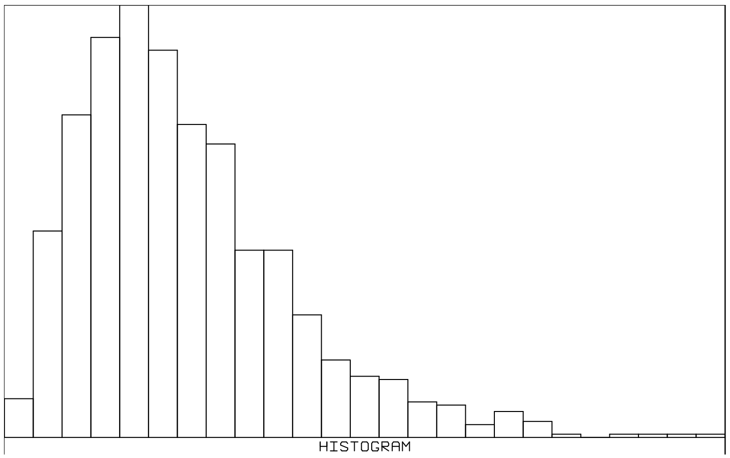
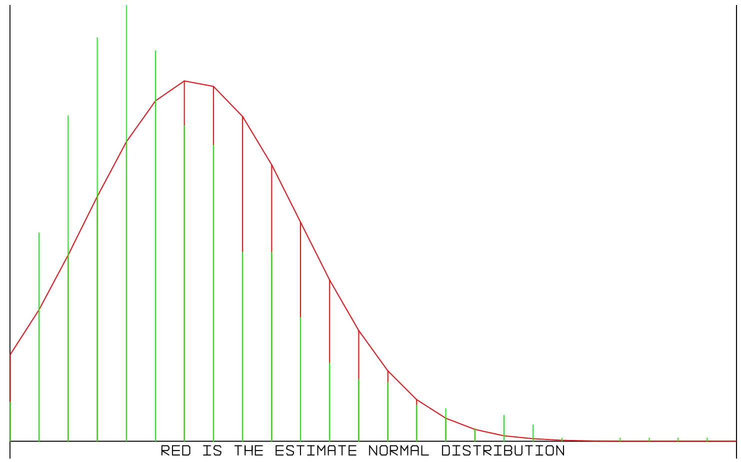
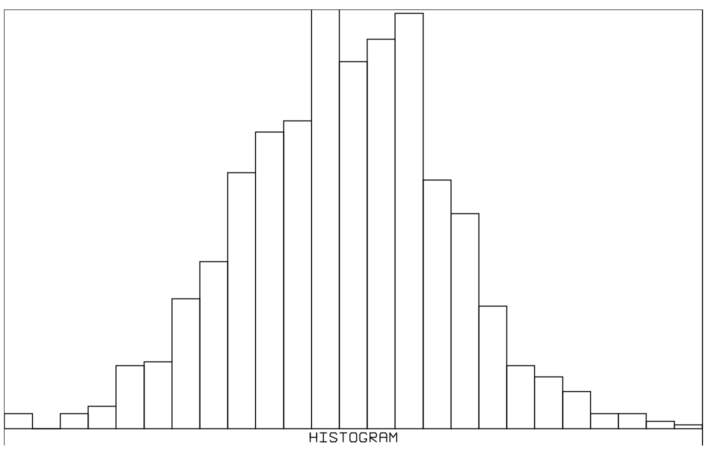
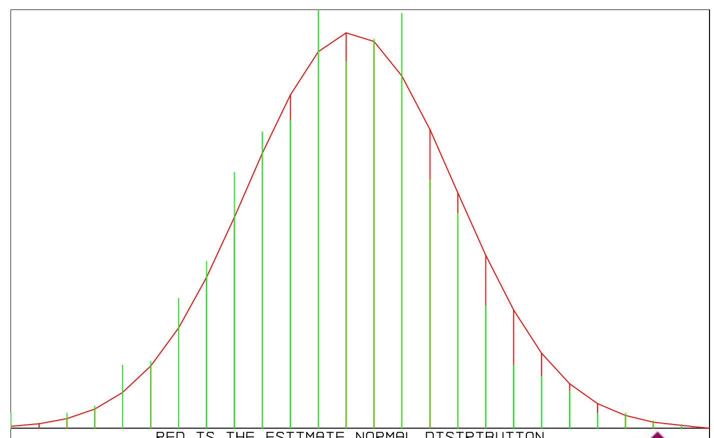
Performance distribution









Statistical theory I

- Let So be the nominal system performance:
- So = S(r0, k0, f0, n0, t0, …)
- S_i is the change in system performance when the i-th system parameter changes from x0 to xi.
- The change in system performance is: $\delta S_i = S_i - S0$
- Consider small changes and assume system is linear so that:
- $\delta Si = \alpha i \; xi$ and therefore: $\delta S = \Sigma \delta Si = \Sigma \alpha i \; xi$.

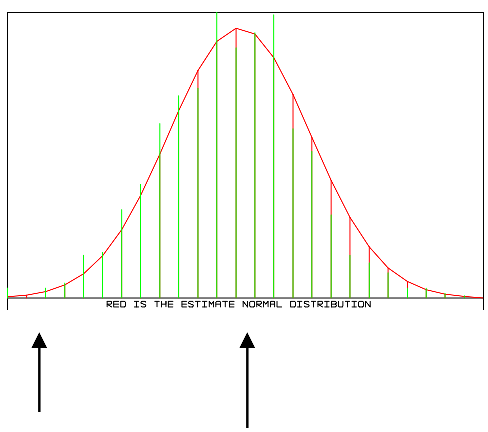
Statistical theory II

- Note that each system parameter has its own probability distribution function: Uniform, normal, end limited, Poisson, etc. Shops for example tend to have lens thickness over the positive side.
- How do we relate these individual probability density functions to the overall probability function for the figure of merit ?
- We make use of the central limit theorem: For a set of n independent, random variables, y1, y2, y3,…. yn, the probability density function for: $z = \Sigma$ yi approaches a Gaussian density function as $i \rightarrow \infty$ for just about any set of probability density functions associated with the {yi} that are encountered in practice.

Statistical theory III

In our case:

$$p(S)=\frac{1}{\sigma_S\sqrt{2\pi}}\exp\left\{\frac{-\left(S-\langle S\rangle\right)^2}{2\sigma_S{}^2}\right\}$$



So <S>

Where: σ_S is the standard variation.

Statistical theory IV

- Now the mean <S> is given by (Frieden p81):
  $\langle S \rangle = S_o + \Sigma \langle \delta S_i \rangle$
- $\Sigma \langle \delta S_i \rangle$ would be zero if the system would be linear
- After assuming statistical independence the variance is given by: $\sigma^2 = \Sigma[\alpha_i \, \sigma_{xi}]^2$
- If we assume $\sigma_{xi} = \Delta xi$, then we obtain the famous Root Sum Squares (RSS) rule:

$$\sigma_S = \sqrt{\left\{ \sum_i \alpha_i^2 \Delta x_i^2 \right\}} = \sqrt{\sum_i (\delta S_i)^2}$$

Statistical theory V

$$\sigma_{\delta S} = \sqrt{\left\{ \sum_i \alpha_i^2 \Delta x_i^2 \right\}} = \sqrt{\sum_i (\delta S_i)^2}$$

Note:

- For $\alpha_i \Delta X_i = 1$ then worst case performance change is: i; compare with standard deviation which gives: $\sqrt{i}$
- "It is the big-ones-that-dominate-effect" Assume that there are ten tolerances effects of +/- 1 and one of +/- 10. The RSS rule gives +/- 10.49 for all of them vs. +/- 10 for the big one.
- We have assumed some linearity and independence in the merit function and random variables.

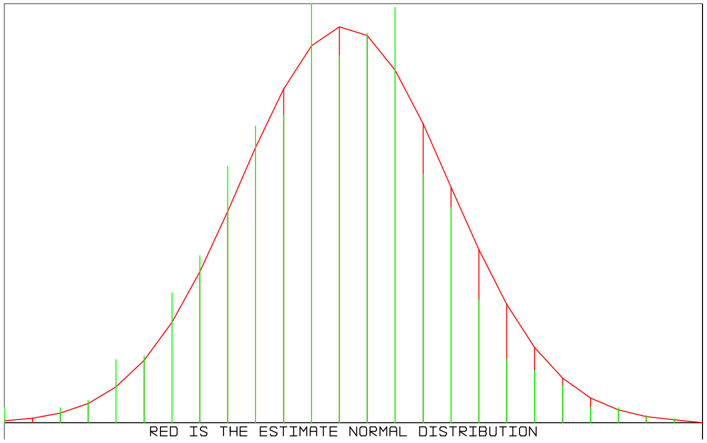
Statistical theory VI

- By integrating the probability density function we can compute the probability of success or estimate how many systems will meet a given performance.

$\delta S_{maximum/\sigma S}$	Probability of success
0.67	0.50
0.80	0.58
1.00	0.68
1.50	0.87
2.00	0.95
2.50	0.99

Monte Carlo Simulation

Trial	Criteria	Change
1	0.011641912	-0.000416137
2	0.011852301	-0.000205748
3	0.012500180	0.000442130
4	0.013553553	0.001495504
5	0.013302508	0.001244459
6	0.012657815	0.000599766
7	0.012147368	8.9319E-005
8	0.012476468	0.000418418
9	0.012603767	0.000545718
10	0.013268314	0.001210265
11	0.012484824	0.000426775
12	0.012649567	0.000591518
13	0.012606634	0.000548585
14	0.012213631	0.000155581
15	0.012496208	0.000438159
16	0.012499526	0.000441477
17	0.013030449	0.000972400
18	0.012641473	0.000583423
19	0.013554178	0.001496128
20	0.012582269	0.000524220



- Nominal 0.012058049
- Best 0.011641912
- Worst 0.013554178
- Mean 0.012638147
- Std Dev 0.000490635

90% <= 0.013302508
50% <= 0.012582269
10% <= 0.011852301

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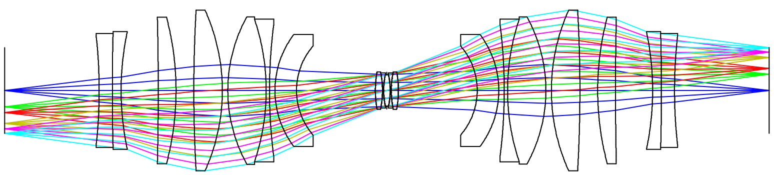
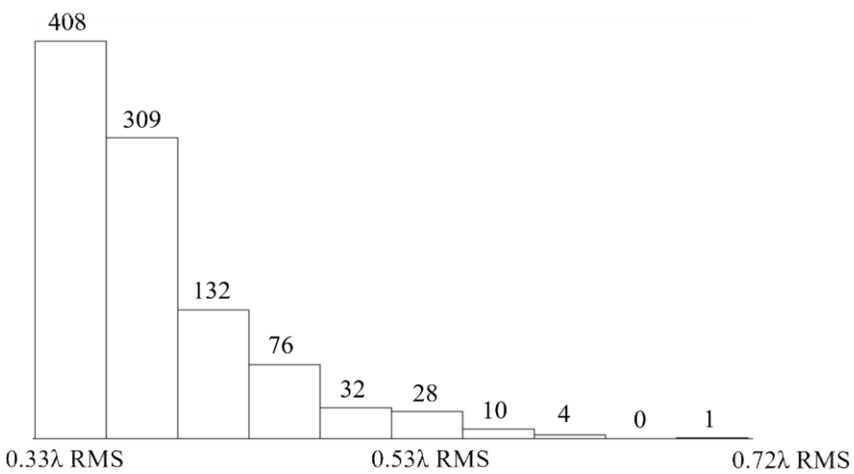
Example I

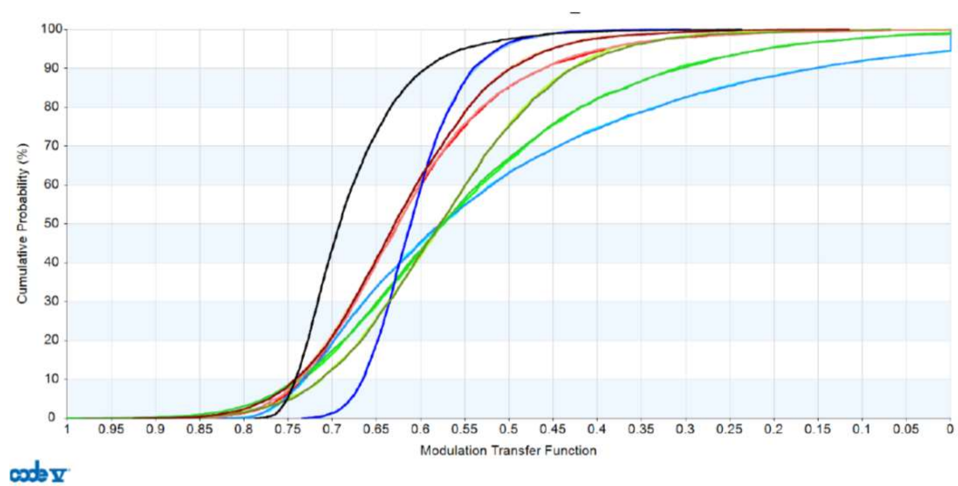
- 10 micrometers in thickness
- 20 micrometers in radius
- 20 arc-seconds in surface tilt
- 0.0001 in index
- 0.1 in Abbe number



- 500 Monte Carlo runs,
  no compensators except for focus
- Nominal 0.000478525
- Best 0.000563064
- Worst 0.003506513
- Mean 0.001304656
- Std Dev 0.000487365

Histogram
Cumulative probability
Yield





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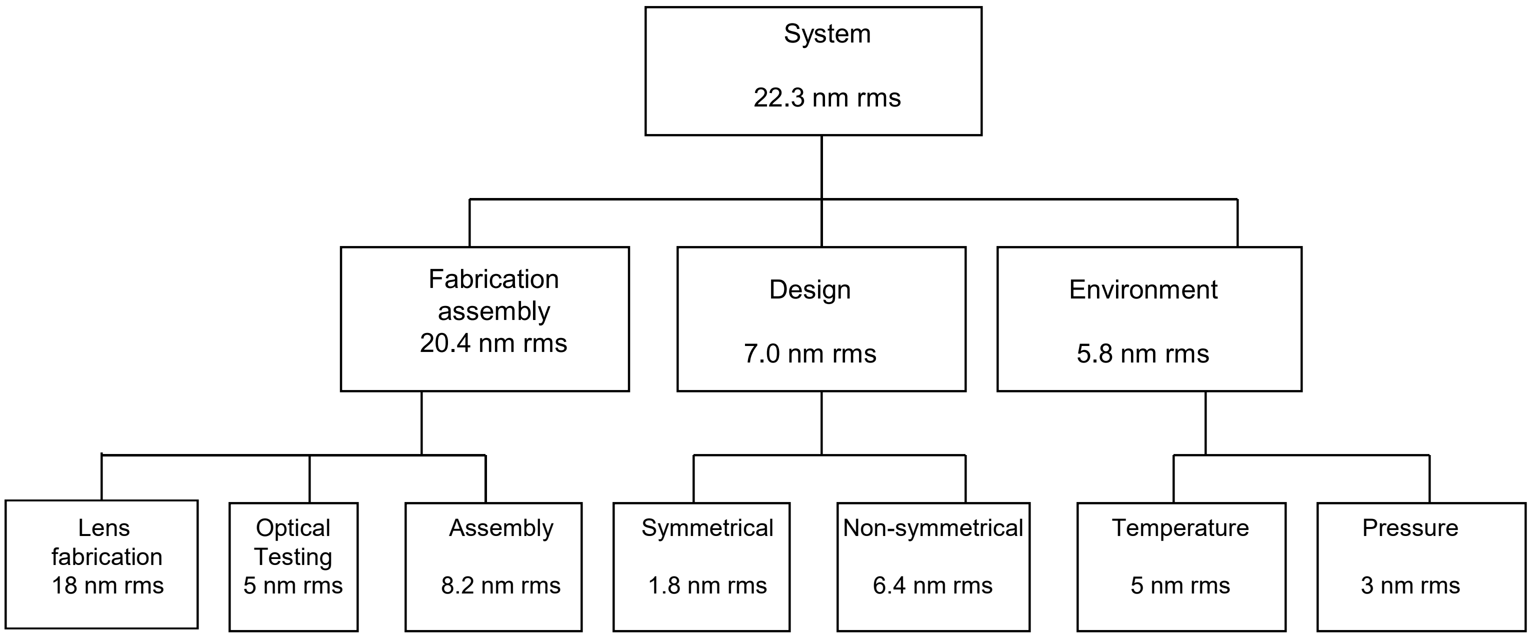
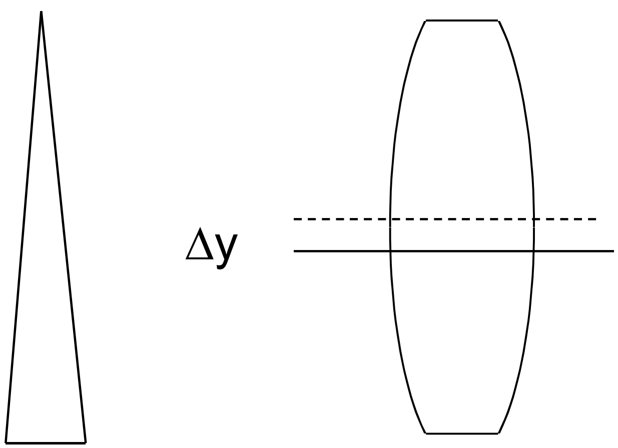
Error Tree



System
22.3 nm rms
Fabrication assembly
20.4 nm rms
Design
7.0 nm rms
Environment
5.8 nm rms
Lens fabrication
18 nm rms
Optical Testing
5 nm rms
Assembly
8.2 nm rms
Symmetrical
1.8 nm rms
Non-symmetrical
6.4 nm rms
Temperature
5 nm rms
Pressure
3 nm rms

Other approaches to tolerancing

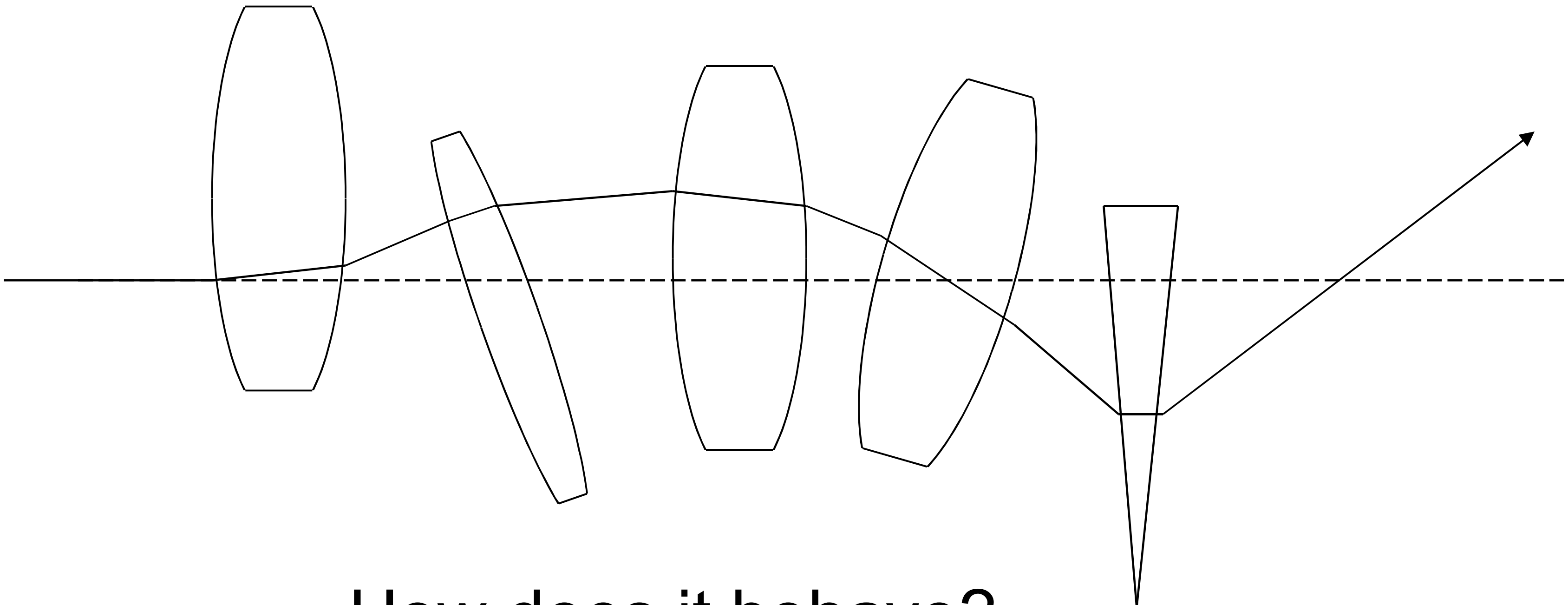
- Efficient tolerancing requires insight into what is happening
- Treat system as plane symmetric
- Parameters that relate the axial symmetry: r, t, n
- Parameters that relate to plane symmetry: surface tilt
- Element decenter is treated as thickness change and surface tilt



Surface Tilt = $\Delta y/R$

Thickness change = $(\Delta y)^2/(2R)$=Tilt X $\Delta y/2$

Plane symmetric system



How does it behave?

Aberrations of a Plane symmetric system

Vector form.		Scalar form.		Name.
First group.				
W_{00000}		W_{00000}		Constant piston
Second group.				
W_{01001}	$\mathbf{i}\cdot\boldsymbol{\rho}$	W_{01001}	$\rho\cos(\beta)$	Field Displacement
W_{10010}	$\mathbf{i}\cdot\mathbf{H}$	W_{10010}	$H\cos(\alpha)$	Linear piston
W_{02000}	$\boldsymbol{\rho}\cdot\boldsymbol{\rho}$	W_{02000}	ρ^2	Defocus
W_{11100}	$\mathbf{H}\cdot\boldsymbol{\rho}$	W_{11100}	$H\rho\cos(\phi)$	Linear distortion
W_{20000}	$\mathbf{H}\cdot\mathbf{H}$	W_{20000}	H^2	Quadratic piston
Third group.				
W_{02002}	$(\mathbf{i}\cdot\boldsymbol{\rho})^2$	W_{02002}	$\rho^2\cos^2(\beta)$	Constant astigmatism
W_{11011}	$(\mathbf{i}\cdot\mathbf{H})(\mathbf{i}\cdot\boldsymbol{\rho})$	W_{11011}	$H\rho\cos(\alpha)\cos(\beta)$	Anamorphism
W_{20020}	$(\mathbf{i}\cdot\mathbf{H})^2$	W_{20020}	$H^2\cos^2(\alpha)$	Quadratic piston
W_{03001}	$(\mathbf{i}\cdot\boldsymbol{\rho})(\boldsymbol{\rho}\cdot\boldsymbol{\rho})$	W_{03001}	$\rho^3\cos(\beta)$	Constant coma
W_{12101}	$(\mathbf{i}\cdot\boldsymbol{\rho})(\mathbf{H}\cdot\boldsymbol{\rho})$	W_{12101}	$H\rho^2\cos(\phi)\cos(\beta)$	Linear astigmatism
W_{12010}	$(\mathbf{i}\cdot\mathbf{H})(\boldsymbol{\rho}\cdot\boldsymbol{\rho})$	W_{12010}	$H\rho^2\cos(\alpha)$	Field tilt
W_{21001}	$(\mathbf{i}\cdot\boldsymbol{\rho})(\mathbf{H}\cdot\mathbf{H})$	W_{21001}	$H^2\rho\cos(\beta)$	Quadratic distortion I
W_{21110}	$(\mathbf{i}\cdot\mathbf{H})(\mathbf{H}\cdot\boldsymbol{\rho})$	W_{21110}	$H^2\rho\cos(\phi)\cos(\alpha)$	Quadratic distortion II
W_{30010}	$(\mathbf{i}\cdot\mathbf{H})(\mathbf{H}\cdot\mathbf{H})$	W_{30010}	$H^3\cos(\alpha)$	Cubic piston
W_{04000}	$(\boldsymbol{\rho}\cdot\boldsymbol{\rho})^2$	W_{04000}	ρ^4	Spherical Aberration
W_{13100}	$(\mathbf{H}\cdot\boldsymbol{\rho})(\boldsymbol{\rho}\cdot\boldsymbol{\rho})$	W_{13100}	$H\rho^3\cos(\phi)$	Linear coma
W_{22200}	$(\mathbf{H}\cdot\boldsymbol{\rho})^2$	W_{22200}	$H^2\rho^2\cos^2(\phi)$	Quadratic astigmatism
W_{22000}	$(\mathbf{H}\cdot\mathbf{H})(\boldsymbol{\rho}\cdot\boldsymbol{\rho})$	W_{22000}	$H^2\rho^2$	Field curvature
W_{31100}	$(\mathbf{H}\cdot\mathbf{H})(\mathbf{H}\cdot\boldsymbol{\rho})$	W_{31100}	$H^3\rho\cos(\phi)$	Cubic distortion
W_{40000}	$(\mathbf{H}\cdot\mathbf{H})^2$	W_{40000}	H^4	Quartic piston

Plane symmetric aberration coefficients

$$J_I = -\frac{1}{2} n^2 \sin^2(I) \Delta\left(\frac{u}{n}\right) x$$

$$J_{II} = -\frac{1}{2} n \sin(I) A \Delta\left(\frac{u}{n}\right) x$$

$$J_{III} = -n \sin(I) \Psi \Delta\left(\frac{u}{n}\right) x$$

$$J_{IV} = -\frac{1}{2} \frac{n \sin(I)}{R} \Psi \Delta\left(\frac{1}{n}\right) x$$

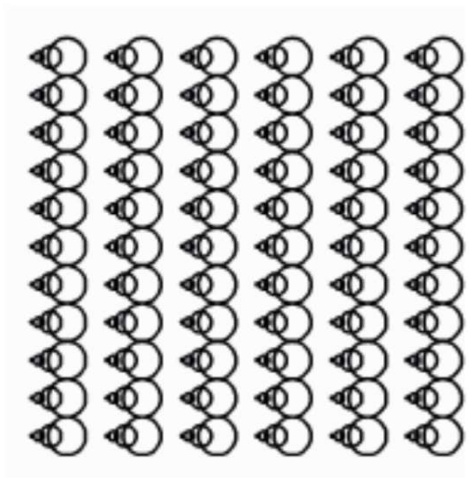
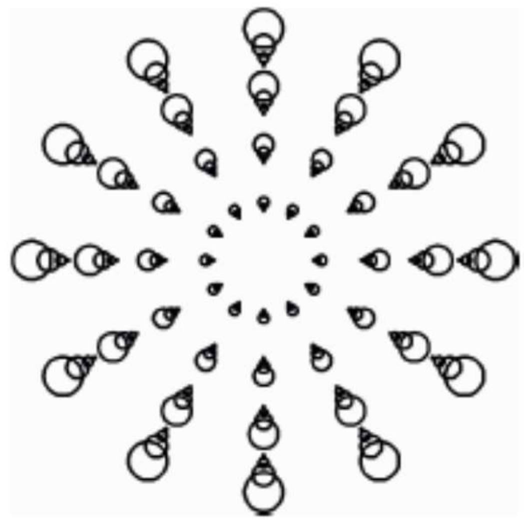
$$J_V = -\frac{1}{2} n \sin(I) \Psi^2 \Delta\left(\frac{1}{n^2}\right) \frac{1}{x}$$

$$A = ni$$

$$\Psi = \bar{A} x - A \bar{x} = n \bar{u} x - n u \bar{x}$$

$W_{02002} = \sum_{i=1}^{j} \{J_I\}_i$	Constant Astigmatism
$W_{11011} = \sum_{i=1}^{j} \left\{2\left(\frac{\bar{x}}{x}\right) J_I\right\}_i$	Anamorphism
$W_{20020} = \sum_{i=1}^{j} \left\{\left(\frac{\bar{x}}{x}\right)^2 J_I\right\}_i$	Quadratic Piston
$W_{03001} = \sum_{i=1}^{j} \{J_{II}\}_i$	Constant Coma
$W_{12101} = \sum_{i=1}^{j} \left\{2\left(\frac{\bar{x}}{x}\right) J_{II} + J_{III}\right\}_i$	Linear Astigmatism
$W_{12010} = \sum_{i=1}^{j} \left\{\left(\frac{\bar{x}}{x}\right) J_{II} + J_{IV}\right\}_i$	Field Tilt
$W_{21001} = \sum_{i=1}^{j} \left\{\left(\frac{\bar{x}}{x}\right)^2 J_{II} + \frac{\bar{x}}{x} J_{III} + J_V\right\}_i$	Quadratic Distortion I
$W_{21110} = \sum_{i=1}^{j} \left\{2\left(\frac{\bar{x}}{x}\right)^2 J_{II} + \frac{\bar{x}}{x}\left(J_{III} + 2J_{IV}\right)\right\}_i$	Quadratic Distortion II
$W_{30010} = \sum_{i=1}^{j} \left\{\left(\frac{\bar{x}}{x}\right)^3 J_{II} + \left(\frac{\bar{x}}{x}\right)^2 \left(J_{III} + J_{IV}\right) + \frac{\bar{x}}{x} J_V\right\}_i$	Piston

Uniform and linear coma

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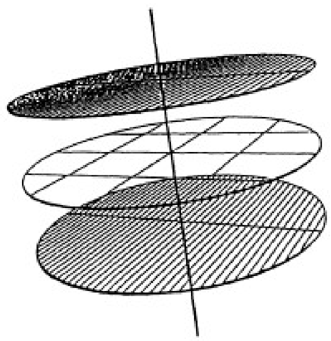
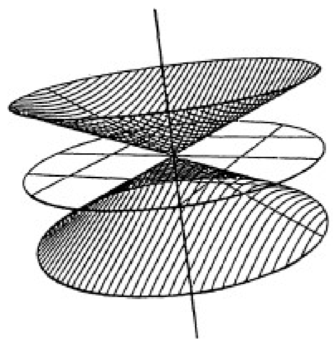
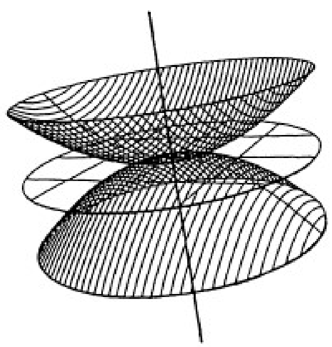
Astigmatism



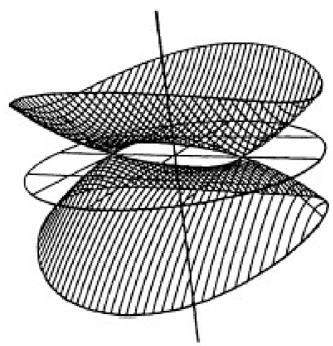
Constant astigmatism



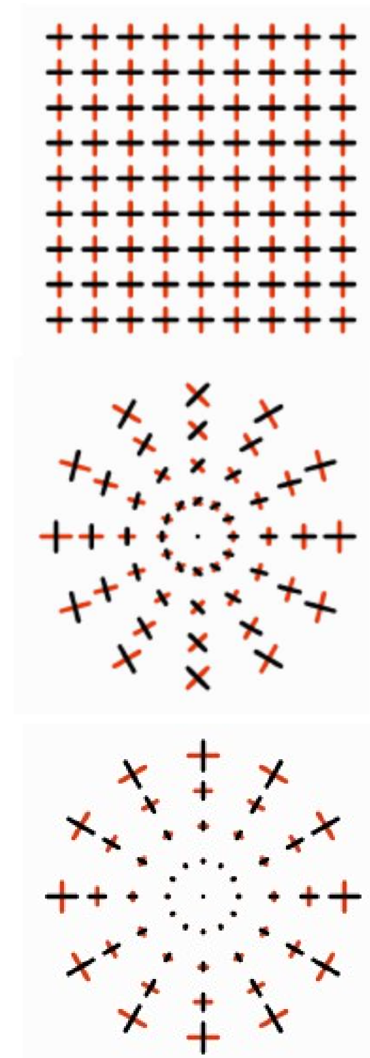
Linear astigmatism



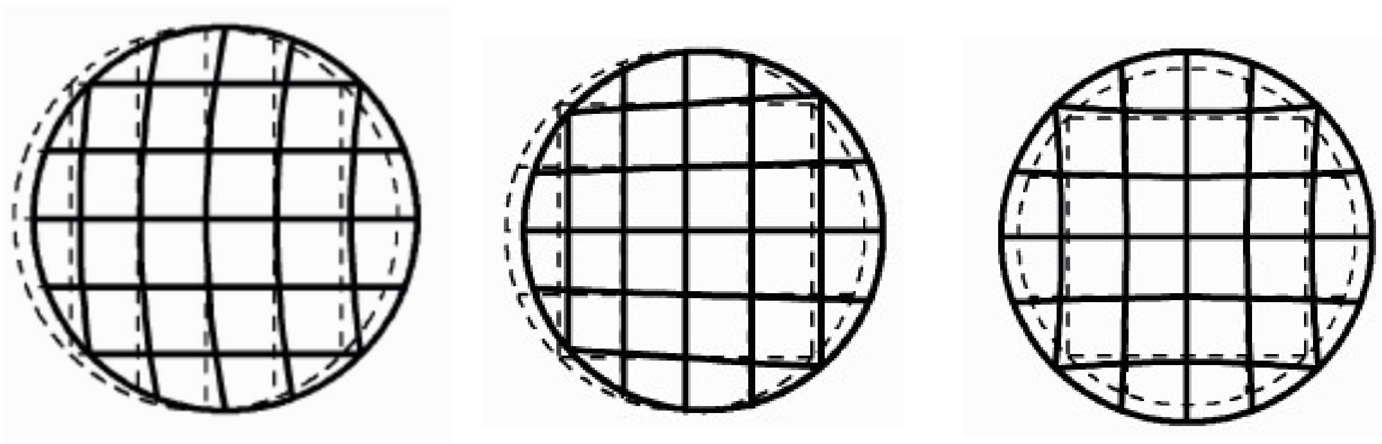
Quadratic astigmatism



Binodal astigmatism



Distortion



Possible distortion under surface tilts

Tolerancing using third-order theory

USA PATENT 2503751 LITLEW

sur	sph	coma	ast	petz	dist	focal
Seidel	1.093	-0.657	-0.255	-1.006	-0.00440	1.002556

Radius tolerancing

1	-0.676	-0.621	0.907	0.273	-0.00137	1.084722
2	0.031	-0.073	0.053	-0.011	0.00015	0.997192
3	-1.728	3.098	-1.049	0.305	-0.00183	1.075346
4	1.685	1.336	-0.776	-0.583	0.00110	0.880381
5	0.000	0.000	0.000	0.000	0.00000	1.000000
6	-0.236	-0.112	-0.139	0.077	0.00014	1.030200
7	5.398	-0.608	-0.578	-0.538	-0.00137	0.804431

Thickness tolerancing

1	0.070	0.537	-0.514	0.000	0.00179	1.030493
2	0.649	-1.600	2.724	0.000	0.01485	1.045549
3	-0.585	2.863	-2.636	0.000	-0.00217	1.036535
4	1.919	5.453	-0.429	0.000	-0.00751	0.919094
5	0.000	0.000	0.000	0.000	0.00000	1.000000
6	0.057	2.563	-1.048	0.000	-0.00190	0.997565
7	0.000	0.000	0.000	0.000	0.00000	1.000000

Index tolerancing

1	0.003	0.003	0.001	-0.002	0.00003	0.999073
2	0.000	0.000	0.000	0.000	0.00000	1.000000
3	-0.016	0.009	0.000	0.004	-0.00002	1.001634
4	0.000	0.000	0.000	0.000	0.00000	1.000000
5	0.000	0.000	0.000	0.000	0.00000	1.000000
6	0.019	-0.012	0.004	-0.003	-0.00001	0.997992
7	0.000	0.000	0.000	0.000	0.00000	1.000000

Tilt tolerancing

sur	ast	coma	last	tilt	anaI	distI	distII
1	0.001	-0.147	-0.066	-0.148	0.00000	0.00019	-0.00049
2	0.012	0.259	-0.524	-0.035	0.00002	-0.00077	-0.00040
3	-0.005	-0.446	0.608	0.144	-0.00001	0.00060	0.00024
4	-0.007	0.305	0.177	0.113	-0.00001	-0.00023	0.00040
5	0.000	0.000	0.000	0.000	0.00000	0.00000	0.00000
6	0.003	-0.219	-0.116	-0.052	0.00000	0.00053	-0.00006
7	0.014	0.577	-0.297	-0.018	-0.00002	-0.00034	0.00049

sur	anaII	focal	shift	OAR angle	Image tilt
1	1.00000	1.000001	0.007510	0.203025	-0.247975
2	1.00009	1.000004	-0.006197	-0.230103	-0.335562
3	0.99997	1.000013	0.004687	0.209677	0.646533
4	1.00007	1.000047	-0.004568	-0.218521	0.125974
5	1.00000	1.000000	0.000000	0.000000	0.000000
6	0.99997	0.999983	0.005379	0.338287	0.179453
7	1.00009	0.999912	-0.005404	-0.401050	-0.456490

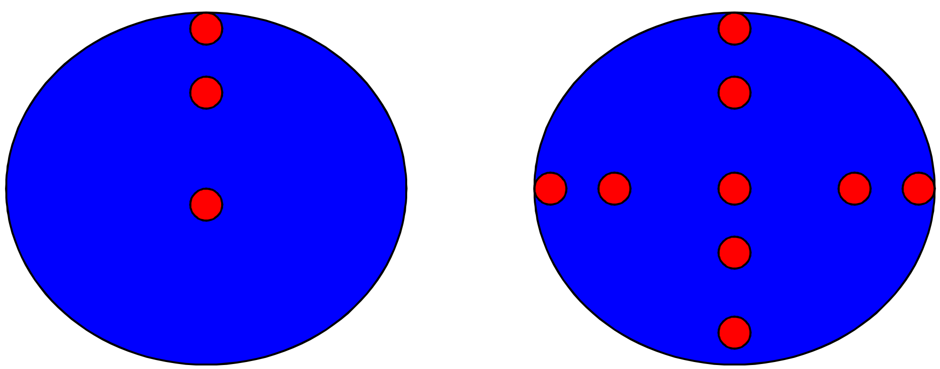
RESULTS

	Axis	off-axis	distortion	focal
Radius				
Abs	9.753	20.890	0.005954	1.508263
rss	5.956	10.704	0.002886	1.257547
Thickness				
Abs	3.280	23.648	0.028211	1.195918
rss	2.110	11.718	0.016983	1.104360
Index				
Abs	0.038	0.075	0.000065	1.004569
rss	0.025	0.048	0.000041	1.002750
Tilt				
Abs	1.996	3.784	0.004778	1.000170
rss	0.892	1.715	0.001426	1.000102
Totals				
Abs	15.067	48.396	0.039008	1.708920
rss	6.382	15.963	0.017285	1.277901

CODE V> GO

Field sampling

- With surface tilts there is no axial symmetry and then one most sample the field at several positions all over the field of view.



Design and tolerance approaches

- Statistical theory
- Monte Carlo simulation
- Aberration theory

- Relaxing the lens (several approaches)
- Global search and then sorting
- Optimization accounting for tolerances
- Accounting for uniform coma and linear astigmatism or distortion
- Using a multi-configuration setting that includes perturbed systems

Summary

- In tolerancing we are after the statistics
- Statistical approach
- Monte Carlo runs
- Aberration theory approach
- Other approaches
- Tolerance error tree and budget