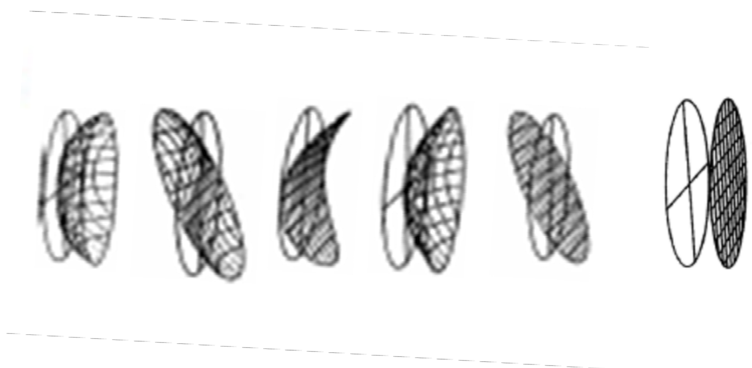
Lens Design OPTI 517

Seidel aberration coefficients



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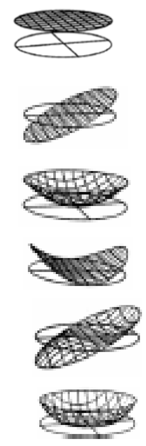
Fourth-order terms

$$W\left(\vec{H},\vec{\rho}\right)=W_{040}(\vec{\rho}\cdot\vec{\rho})^2+W_{131}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2$$
$$+W_{220}\left(\vec{H}\cdot\vec{H}\right)(\vec{\rho}\cdot\vec{\rho})+W_{311}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+W_{400}\left(\vec{H}\cdot\vec{H}\right)^2$$

Spherical aberration
Coma
Astigmatism (cylindrical aberration!)
Field curvature
Distortion
Piston



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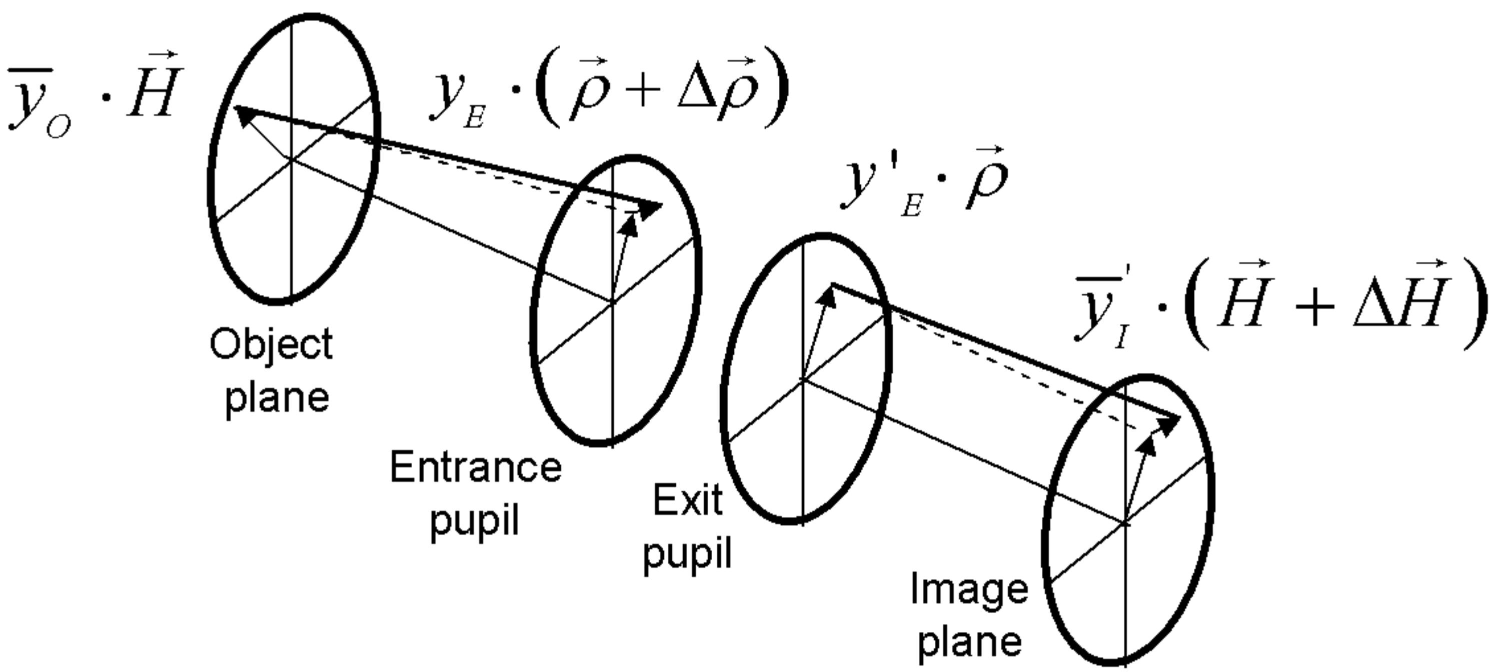
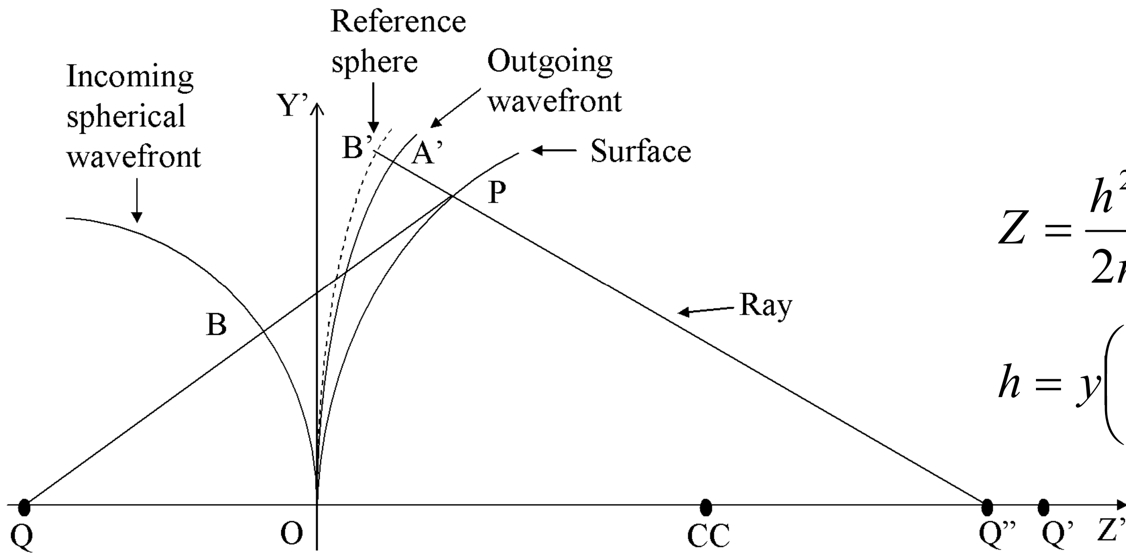
Coordinate system



Spherical aberration



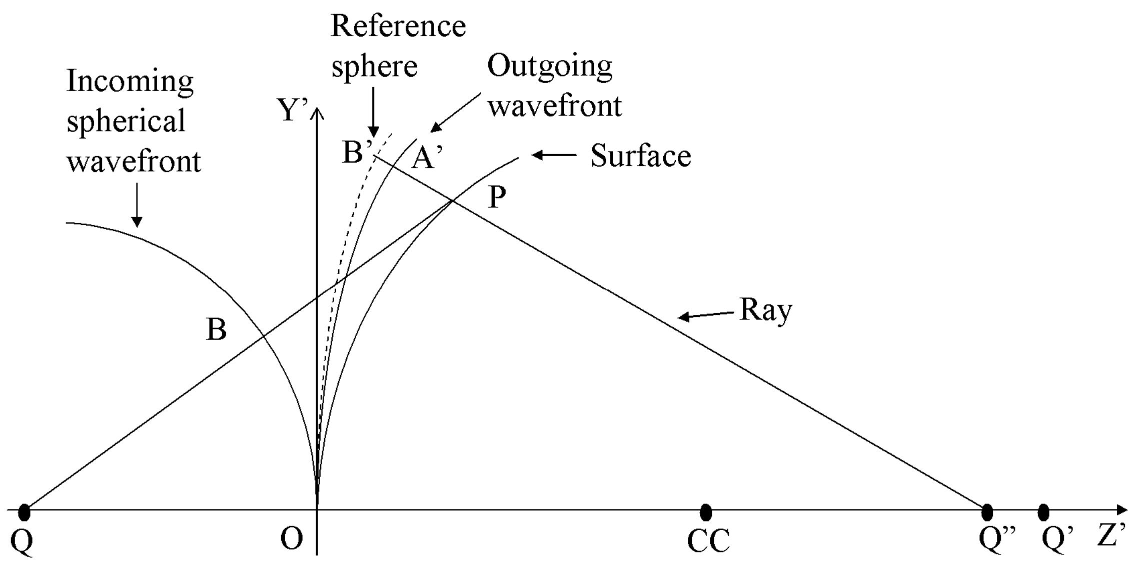
$$Z = \frac{h^2}{2r} + \frac{h^4}{8r^3}$$

$$h = y\left(1 + \frac{u}{2r}y\right)$$

$$W = n'[PB'] - n'[PA'] = n'[PB'] - n[PB]$$

We have a spherical surface of radius of curvature r, a ray intersecting the surface at point P, intersecting the reference sphere at B', intersecting the wavefront in object space at B and in image space at A', and passing in image space by the point Q'' in the optical axis. The reference sphere in object space is centered at Q and in image space is centered at Q'

Question: how do we draw the first order marginal ray in image space?



$$[PQ]^2 = (s - Z)^2 + h^2 = s^2 - 2sZ + Z^2 + h^2$$

$$= s^2 \left\{ 1 + \frac{h^2 - 2s\left[\frac{h^2}{2r} + \frac{h^4}{8r^3}\right] + \left[\frac{h^4}{4r^2}\right]}{s^2} \right\}$$

$$= s^2 \left\{ 1 + \frac{h^2}{s^2}\left[1 - \frac{s}{r}\right] + \frac{h^4}{4r^2 s^2}\left[1 - \frac{s}{r}\right] \right\}$$

$$[PB] = [OQ] - [PQ]$$

$$= -\frac{h^2}{2}\left[\frac{1}{s} - \frac{1}{r}\right] - \frac{h^4}{8r^2}\left[\frac{1}{s} - \frac{1}{r}\right] + \frac{h^4}{8s}\left[\frac{1}{s} - \frac{1}{r}\right]^2$$

$$h = y\left(1 + \frac{u}{2r} y\right)$$

Spherical aberration

$$[PB]=[OQ]-[PQ]$$
$$=-\frac{y^2}{2}\left(1+\frac{u}{2r}y\right)^2\left[\frac{1}{s}-\frac{1}{r}\right]-\frac{y^4}{8r^2}\left[\frac{1}{s}-\frac{1}{r}\right]$$
$$+\frac{y^4}{8s}\left[\frac{1}{s}-\frac{1}{r}\right]^2$$

$$[PB']=[OQ']-[PQ']$$
$$=-\frac{y^2}{2}\left(1+\frac{u}{2r}y\right)^2\left[\frac{1}{s'}-\frac{1}{r}\right]-\frac{y^4}{8r^2}\left[\frac{1}{s'}-\frac{1}{r}\right]$$
$$+\frac{y^4}{8s'}\left[\frac{1}{s'}-\frac{1}{r}\right]^2$$

$$W=n'[PB']-n[PB]=$$
$$=-\frac{y^2}{2}\left(1+\frac{u}{r}y\right)\left\{n'\left[\frac{1}{s'}-\frac{1}{r}\right]-n\left[\frac{1}{s}-\frac{1}{r}\right]\right\}$$
$$-\frac{y^4}{8r^2}\left\{n'\left[\frac{1}{s'}-\frac{1}{r}\right]-n\left[\frac{1}{s}-\frac{1}{r}\right]\right\}$$
$$+\frac{y^4}{8}\left\{\frac{n'}{s'}\left[\frac{1}{s'}-\frac{1}{r}\right]^2-\frac{n}{s}\left[\frac{1}{s}-\frac{1}{r}\right]^2\right\}$$

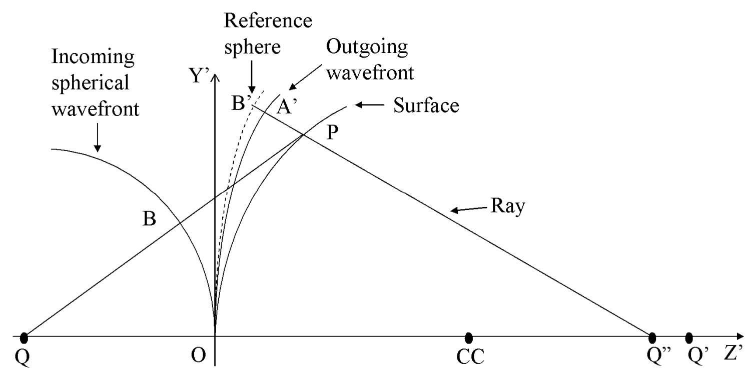
Spherical aberration

$$u = -y/s$$

$$u' = -y/s'$$

$$\Delta\{A\} = 0$$

$$A = ni = -n\left(y/s - y/r\right)$$



$$W_{040} = -\frac{1}{8}A^2 y\Delta\left\{\frac{u}{n}\right\}$$

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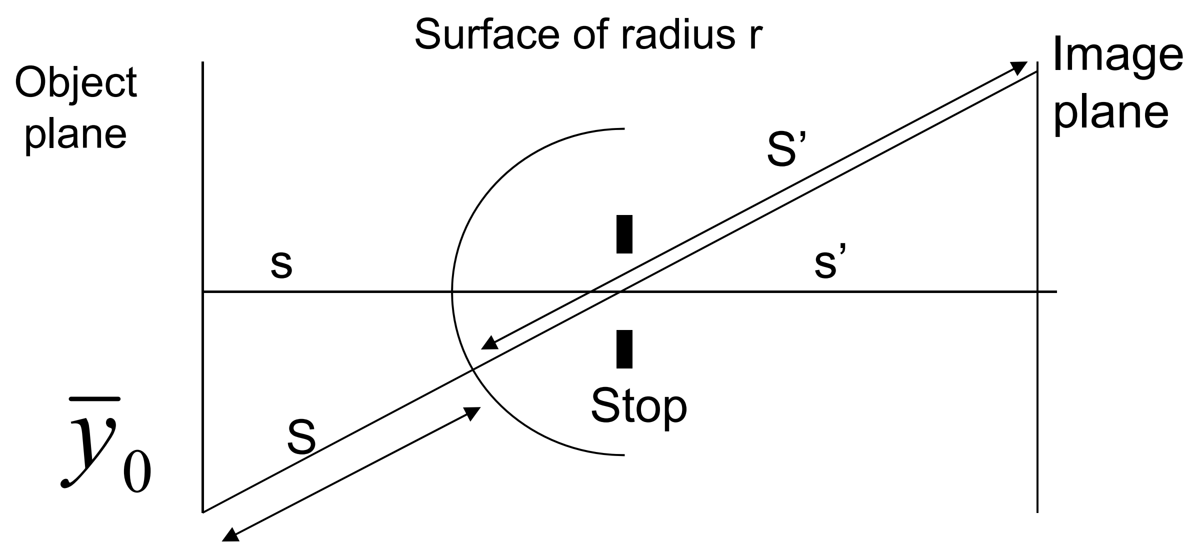
Petzval field curvature W_{220P}



We locate the aperture stop at the center of curvature of the spherical surface. With being the object height, then the inverse of the distance along the chief ray from the off-axis object point to the surface is:

Petzval field curvature

$$\frac{1}{-S}=\frac{1}{-r+\sqrt{(r-s)^2+\bar{y}_0^2}}=\frac{1}{-r+(r-s)\sqrt{1+\frac{\bar{y}_0^2}{(r-s)^2}}}$$

$$\cong\frac{1}{-r+(r-s)\left(1+\frac{1}{2}\frac{\bar{y}_0^2}{(r-s)^2}\right)}=\frac{1}{-s\left(1-\frac{1}{2}\frac{\bar{y}_0^2}{(r-s)s}\right)}$$

$$=-\frac{1}{s}\left(1+\frac{1}{2}\frac{\bar{y}_0^2}{(r-s)s}\right)=-\frac{1}{s}\left(1+\frac{1}{2}\frac{\bar{y}_0^2}{\left(\frac{1}{s}-\frac{1}{r}\right)rs^2}\right)=$$

$$=-\frac{1}{s}\left(1+\frac{u}{2}\frac{\bar{y}_0^2}{irs}\right)=-\frac{1}{s}-\frac{u}{y^2}\frac{1}{2}\frac{\text{Ж}^2}{n^2ri}$$

$$\frac{1}{-S'}\cong-\frac{1}{s'}-\frac{u'}{y^2}\frac{1}{2}\frac{\text{Ж}^2}{n'^2ri'}$$

Petzval field curvature

By inserting the 1/s in the quadratic term of W

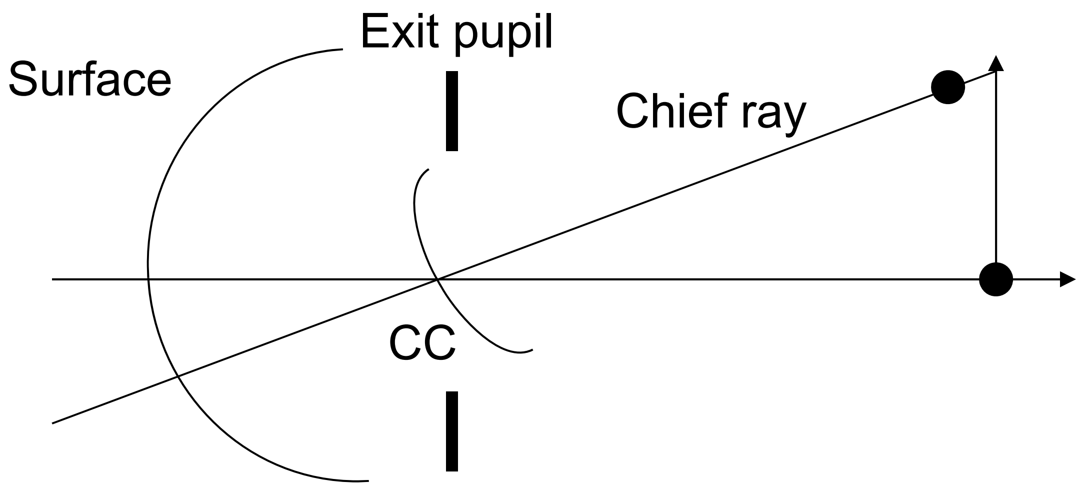
$$W = n^{'}[PB'] - n[PB] =$$
$$= -\frac{y^2}{2}\left(1+\frac{u}{r}y\right)\left\{n^{'}\left[\frac{1}{s^{'}}-\frac{1}{r}\right]-n\left[\frac{1}{s}-\frac{1}{r}\right]\right\}$$
$$-\frac{y^4}{8r^2}\left\{n^{'}\left[\frac{1}{s^{'}}-\frac{1}{r}\right]-n\left[\frac{1}{s}-\frac{1}{r}\right]\right\}$$
$$+\frac{y^4}{8}\left\{\frac{n^{'}}{s^{'}}\left[\frac{1}{s^{'}}-\frac{1}{r}\right]^2-\frac{n}{s}\left[\frac{1}{s}-\frac{1}{r}\right]^2\right\}$$

$\longrightarrow$

$$W = -\frac{y^2}{2}\left\{n^{'}\left[\frac{1}{S^{'}}-\frac{1}{r}\right]-n\left[\frac{1}{S}-\frac{1}{r}\right]\right\}=$$
$$=\left\{n^{'}u^{'}\left(-\frac{1}{4}\frac{Ж^2}{n^{'2}ri^{'}}\right)-nu\left(-\frac{1}{4}\frac{Ж^2}{n^2 ri}\right)\right\}=$$
$$=\left\{-\frac{1}{4}\frac{Ж^2}{Ar}\left(u^{'}-u\right)\right\}=$$
$$=\left\{-\frac{1}{4}\frac{Ж^2}{r}\Delta\left\{\frac{1}{n}\right\}\right\}$$
$$=W_{220P}+O^{(6)}$$

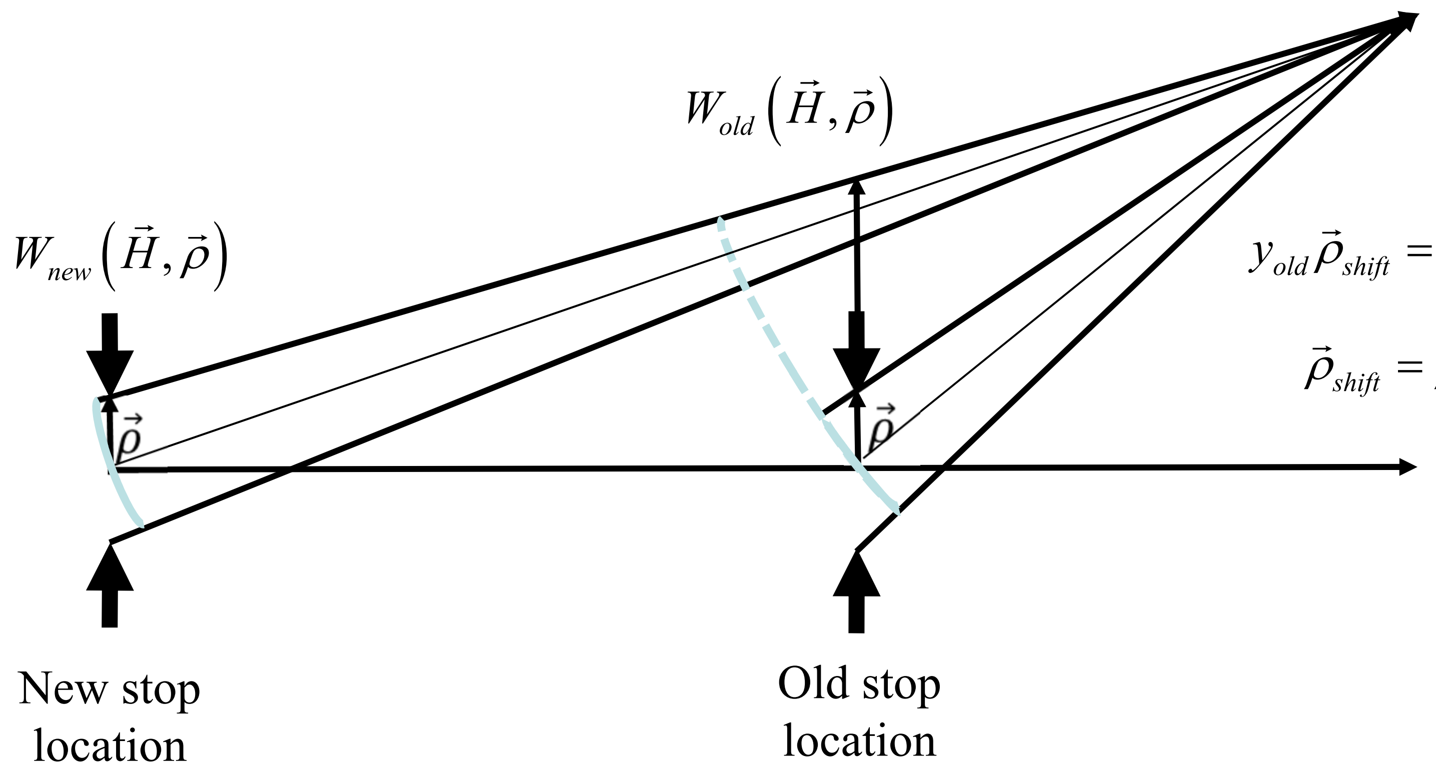
$$\Delta\{A\}=0 \longrightarrow W_{220P} = -\frac{1}{4}\frac{Ж^2}{r}\Delta\left(\frac{1}{n}\right)$$

Aberration function at CC



$$W(\vec{H},\vec{\rho}) = W_{040}(\vec{\rho}\cdot\vec{\rho})^2 + W_{220P}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})$$

New aberration function upon stop shifting



$$y_{old}\vec{\rho}_{shift} = y_{old}\vec{\rho} + \bar{y}_{old}\vec{H}$$

$$\vec{\rho}_{shift} = \vec{\rho} + \frac{\bar{y}_{old}}{y_{old}}\vec{H}$$

$$W_{new}\left(\vec{H},\vec{\rho}\right) = W_{old}\left(\vec{H},\vec{\rho}_{shift}\right) = W_{old}\left(\vec{H},\vec{\rho} + \frac{\bar{y}_{old}}{y_{old}}\vec{H}\right)$$

Expansion about the new chief ray height

$$W\left(\vec{H},\vec{\rho}\right)=W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^{2}+W_{220P}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)\Bigg|_{\vec{\rho}_{shift}=\vec{\rho}+\frac{\bar{y}_E}{y_E}\vec{H}=\vec{\rho}+\frac{\bar{A}}{A}\vec{H}}$$

Quadratic term

$$\vec{\rho}_{shift} \cdot \vec{\rho}_{shift} = \left(\vec{\rho} + \frac{\bar{A}}{A} \vec{H} \right) \cdot \left(\vec{\rho} + \frac{\bar{A}}{A} \vec{H} \right) =$$

$$= \vec{\rho} \cdot \vec{\rho} + 2 \frac{\bar{A}}{A} \vec{H} \cdot \vec{\rho} + \left(\frac{\bar{A}}{A} \right)^2 \vec{H} \cdot \vec{H}$$

$$W_{220P} \left(\vec{H} \cdot \vec{H} \right) \left(\vec{\rho} \cdot \vec{\rho} \right) \rightarrow W_{220P} \left(\vec{H} \cdot \vec{H} \right) \left(\vec{\rho} \cdot \vec{\rho} \right) +$$

$$+ 2 \frac{\bar{A}}{A} W_{220P} \left(\vec{H} \cdot \vec{H} \right) \left(\vec{H} \cdot \vec{\rho} \right) + \left(\frac{\bar{A}}{A} \right)^2 W_{220P} \left(\vec{H} \cdot \vec{H} \right)^2$$

Quartic term

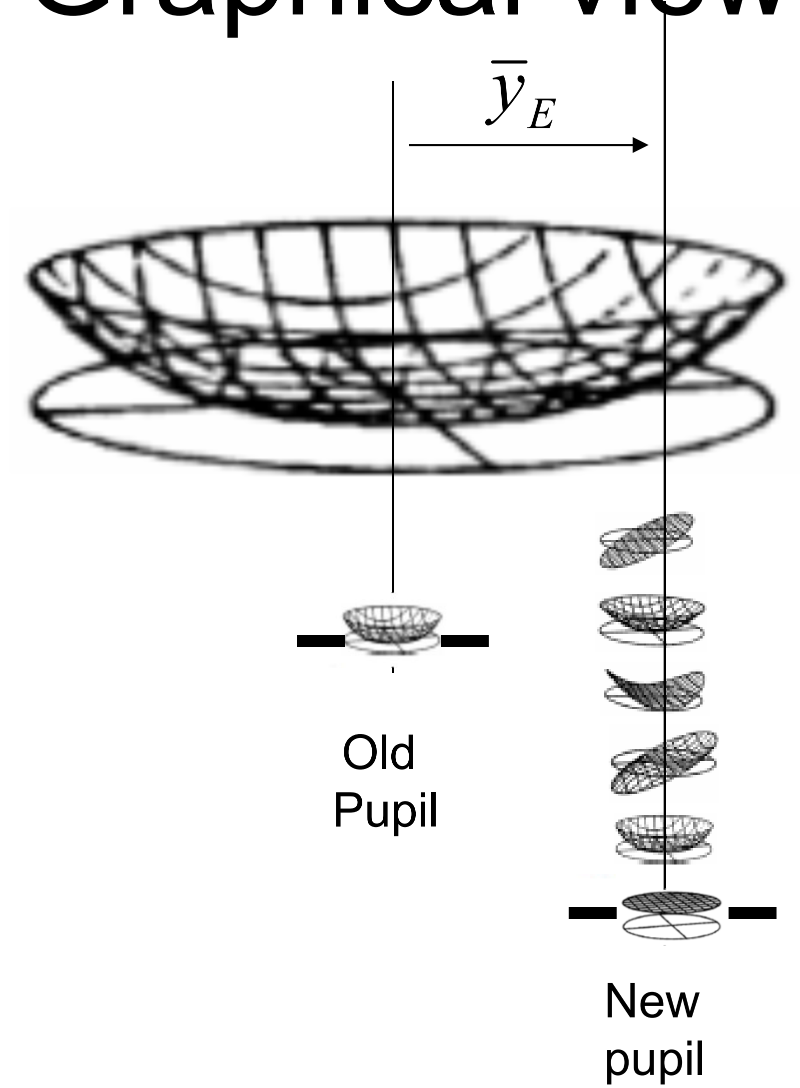
$$\left(\vec{\rho}_{shift}\cdot\vec{\rho}_{shift}\right)^2=\left[\vec{\rho}\cdot\vec{\rho}+2\frac{\bar{A}}{A}\vec{H}\cdot\vec{\rho}+\left(\frac{\bar{A}}{A}\right)^2\vec{H}\cdot\vec{H}\right]\times$$

$$\left[\vec{\rho}\cdot\vec{\rho}+2\frac{\bar{A}}{A}\vec{H}\cdot\vec{\rho}+\left(\frac{\bar{A}}{A}\right)^2\vec{H}\cdot\vec{H}\right]=$$

$$=\left(\vec{\rho}\cdot\vec{\rho}\right)^2+4\frac{\bar{A}}{A}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+4\frac{\bar{A}}{A}\left(\vec{H}\cdot\vec{\rho}\right)^2$$

$$+2\frac{\bar{A}}{A}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+4\left(\frac{\bar{A}}{A}\right)^3\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\left(\frac{\bar{A}}{A}\right)^4\left(\vec{H}\cdot\vec{H}\right)^2$$

Graphical view



All terms

$$W\left(\vec{H},\vec{\rho}\right)=W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^{2}+W_{131}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{222}\left(\vec{H}\cdot\vec{\rho}\right)^{2}$$
$$+W_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{311}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+W_{400}\left(\vec{H}\cdot\vec{H}\right)^{2}$$

$$W_{040}=W_{040}$$

Spherical aberration

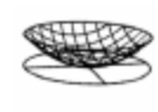
$$W_{131}=4\frac{\bar{A}}{A}W_{040}$$

Coma

$$W_{222}=4\left(\frac{\bar{A}}{A}\right)^{2}W_{040}$$

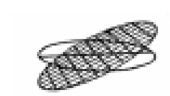
Astigmatism

$$W_{220}=2\left(\frac{\bar{A}}{A}\right)^{2}W_{040}+W_{220P}$$



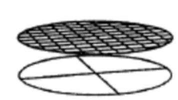
Field curvature
Focus

$$W_{311}=4\left(\frac{\bar{A}}{A}\right)^{3}W_{040}+2\frac{\bar{A}}{A}W_{220P}$$



Distortion

$$W_{400}=\left(\frac{\bar{A}}{A}\right)^{4}W_{040}+\left(\frac{\bar{A}}{A}\right)^{2}W_{220P}$$



Piston

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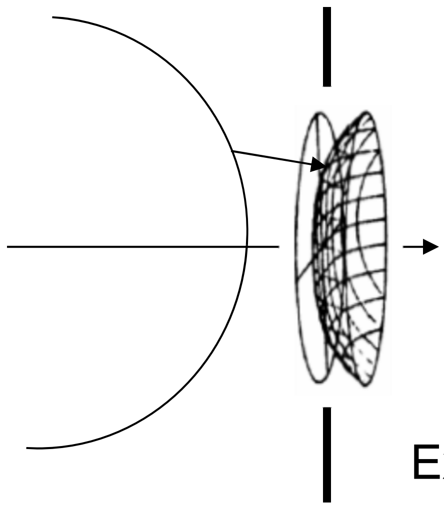
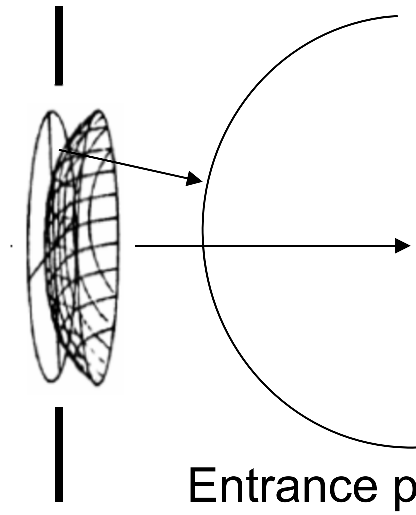
For as system of two surfaces
Exit pupil becomes entrance pupil for next surface.



Exit pupil for surface J



Entrance pupil for surface J+1

Fourth-order contributions

For a given system we add the OPD contributed by each surface.
An issue is that because of pupil aberrations we do not know the pupil coordinates of the ray at previous exit pupils.
However, the error in knowing the ray pupil coordinates leads to six-order aberrations.

To fourth-order we do not have other fourth-order terms to account for.

We are assuming we do not have second-order aberrations. Otherwise these will generate other fourth-order terms.

Order of Error

- We know the ray heights to first-order
- There is an error on the ray heights y and y-bar of third order
- If the third order error is accounted for, it leads to sixth-order terms

$$y \cong y + \alpha y^3$$

$$y^4 \to y^4 + \beta y^6$$

$$W_{040}\left(y^4\right) \to W_{040}\left(y^4\right) + W_{040}\left(y^6\right)$$

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Conclusion

Assume no second order terms in the aberration functions of each surface

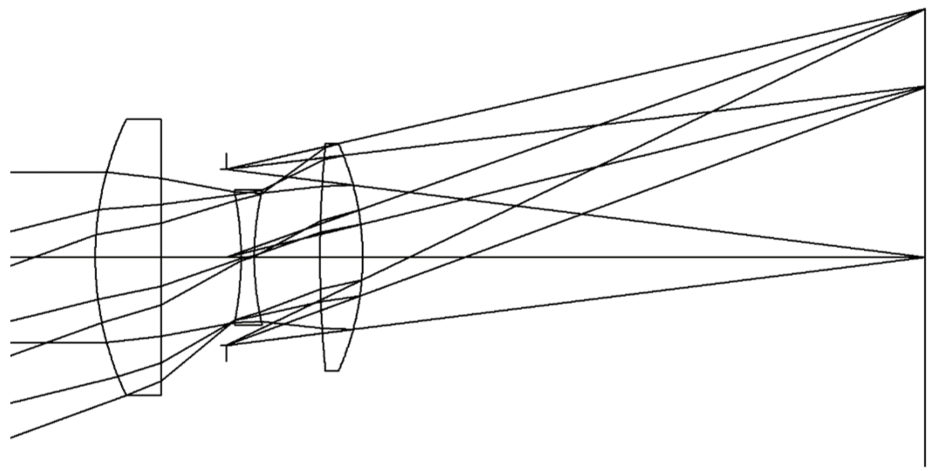
Then for a system of surfaces the fourth-order coefficients are the sum of the coefficients contributed by each surface

There are no fourth-order extrinsic terms from previous aberration in the system

Aberration Coefficients in terms of Seidel sums	
Coefficient	Seidel sum
$W_{040} = \frac{1}{8} S_I$	$S_I = -\sum_{i=1}^{j} \left(A^2 y \Delta\left(\frac{u}{n}\right)\right)_i$
$W_{131} = \frac{1}{2} S_{II}$	$S_{II} = -\sum_{i=1}^{j} \left(A\bar{A} y \Delta\left(\frac{u}{n}\right)\right)_i$
$W_{222} = \frac{1}{2} S_{III}$	$S_{III} = -\sum_{i=1}^{j} \left(\bar{A}^2 y \Delta\left(\frac{u}{n}\right)\right)_i$
$W_{220} = \frac{1}{4} (S_{IV} + S_{III})$	$S_{IV} = -Ж^2 \sum_{i=1}^{j} P_i$
$W_{311} = \frac{1}{2} S_V$	$S_V = -\sum_{i=1}^{j} \left(\frac{\bar{A}}{A}\left[Ж^2 P + \bar{A}^2 y \Delta\left(\frac{u}{n}\right)\right]\right)_i$
$W_{311} = \frac{1}{2} S_V$	$S_V = -\sum_{i=1}^{j} \left(\bar{A}\left[\bar{A}^2 \Delta\left(\frac{1}{n^2}\right) y - \left(Ж + \bar{A} y\right) \bar{y} P\right]\right)_i$
$\delta_\lambda W_{020} = \frac{1}{2} C_L$	$C_L = \sum_{i=1}^{j} \left(A y \Delta\left(\frac{\delta n}{n}\right)\right)_i$
$\delta_\lambda W_{111} = C_T$	$C_T = \sum_{i=1}^{j} \left(\bar{A} y \Delta\left(\frac{\delta n}{n}\right)\right)_i$

Quantities derived from first-order ray data used in computing the aberration coefficients				
Refraction invariant marginal ray	Refraction invariant chief ray	Lagrange invariant	Surface curvature	Petzval sum term
$A = ni = nu + nyc$	$\bar{A} = n\bar{i} = n\bar{u} + n\bar{y}c$	$\mathcal{K} = n\bar{u}y - nu\bar{y} = \bar{A}y - A\bar{y}$	$c = \frac{1}{r}$	$P = c \cdot \Delta\left(\frac{1}{n}\right)$

Example : Cooke triplet lens



First-order ray trace for Cooke triplet						
	Marginal ray y, u, ni			Chief ray y, u, ni		
1.0000	6.2500	-0.1077	0.2636	-4.2509	0.2885	0.1847
2.0000	5.7297	-0.1816	-0.1808	-2.8572	0.4876	0.4872
3.0000	4.6656	-0.0318	-0.3724	-0.0000	0.2915	0.4876
4.0000	4.6345	0.0891	0.3008	0.2842	0.4963	0.5093
5.0000	5.0643	0.0289	0.1475	2.6774	0.2809	0.5272
6.0000	5.1546	-0.1250	-0.3765	3.5557	0.3551	0.1816
Exit pupil	6.4063	-0.1250	-0.1250	-0.0000	0.3551	0.3551
Image	0.0000	-0.1250	-0.1250	18.1985	0.3551	0.3551

Wave coefficients

In waves at 0.000587 mm

	W040	W131	W222	W220	W311	W020	W111
1.0000	5.8831	16.4912	11.5569	37.9424	61.2785	-10.4705	-14.6753
2.0000	4.6978	-50.6336	136.4338	-0.1227	-366.9639	-6.5847	35.4854
3.0000	-22.3709	117.1708	-153.4247	-36.2070	295.7159	18.4586	-48.3398
4.0000	-9.6490	-65.3484	-110.6438	-40.4402	-324.2767	14.8117	50.1564
5.0000	1.6894	24.1504	86.3110	10.3704	382.5921	-4.7481	-33.9387
6.0000	22.0849	-42.6064	20.5492	43.9016	-52.2587	-12.3359	11.8993
Totals	2.3352	-0.7760	-9.2175	15.4444	-3.9128	-0.8690	0.5872

Prescription
F/4, f=50 mm; FOV +/- 20 degrees
Stop at surface 3

SURFACE DATA SUMMARY:

Surf	Type	Radius	Thickness	Glass	Diameter	Conic	Comment
OBJ	STANDARD	Infinity	Infinity		0	0	
1	STANDARD	23.713	4.831	LAK9	20.26679	0	
2	STANDARD	7331.288	5.86		18.17704	0	
3	STANDARD	-24.456	0.975	SF5	9.598584	0	
4	STANDARD	21.896	4.822		9.909458	0	
5	STANDARD	86.759	3.127	LAK9	16.07715	0	
6	STANDARD	-20.4942	-10.0135		16.69285	0	
STO	STANDARD	Infinity	51.25		12.90717	0	
IMA	STANDARD	Infinity			36.46158	0	

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Summary

- Spherical aberration
- Stop shifting
- Off-axis aberrations
- Entrance/exit pupil concatenation
- Seidel sums
- Actual coefficients computation
- Information acquired