

**Instruction:** The task can be carried out individually or in groups of two (recommended), and it should be presented orally (code demonstration) along with submission of code and a compact report.

**Attention!** Plagiarism check (including usage of AI tools) will be performed on all submissions.

In this assignment, you are going to study the diffusion equation for temperature  $T$  (i.e., the heat conduction equation), which can be written as

$$0 = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + S \quad (1)$$

Discretize this equation using FVM (Eq. 4.57 in Versteeg & Malalasekara [1]) and solve using Gauss-Seidel iteration. MATLAB is recommended for programming.

The computational domain and boundary conditions are as shown in figure 1 and table 1, respectively (pick one Case and register your team). Discuss the results from both a physical and a numerical point of view. Present the results as contour plots (wherever applicable). The submission must include the following:

1. Use different meshes to solve the problem (i.e.,  $10 \times 10$ ,  $20 \times 20$ ,  $40 \times 40$ , etc) and show mesh independence. Stretch and refine the mesh in regions where you expect large gradients and discuss the solution outcome.
2. Plot residual error ( $R$ ) vs number of iterations ( $N$ ). Increase or decrease the error tolerance ( $\epsilon$ ) and discuss the outcome.
3. What happens if you change the boundary conditions? e.g. change Neumann to Dirichlet or Dirichlet to Neumann on one side. Discuss the results.
4. In order to illustrate the heat flow, plot the heat flux vector as a vector plot and discuss.

$$\dot{q}_x = -k \frac{\partial T}{\partial x}, \dot{q}_y = -k \frac{\partial T}{\partial y} \quad (2)$$

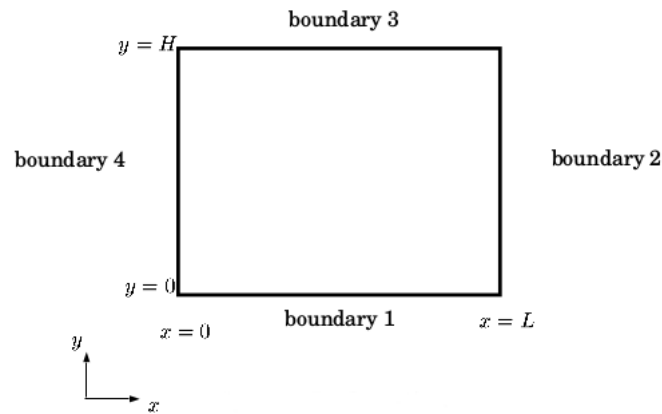


Figure 1: Computational domain.

Case	T1	T2	T3	T4	S	k
1	15	$5(1 - y/H) + 15\sin(\pi y/H)$	10	$dT/dx = 0$	$4 - 5T^3$	$16(y/H + 1)$
2	15	$5(1 - y/H) + 15\sin(\pi y/H)$	10	$dT/dx = 0$	$500000 - 30000 * T$	$16(y/H + 1)$
3	15	$q = +5000$	10	$dT/dx = 0$	-1.5	$16(y/H + 1)$
4	15	$5(1 - y/H) + 15\sin(\pi y/H)$	10	$q = -5000$	-1.5	$16(y/H + 1)$
5	15	$5(1 - y/H) + 15\sin(\pi y/H)$	10	$dT/dx = 0$	-1.5	$16((x/L) + 1)((y/H) + 1)$
6	15	$5(1 - y/H) + 15\sin(\pi y/H)$	10	$dT/dx = 0$	-1.5	$16((x/L) + 1)$

Table 1:  $L = 1$  and  $H = 0.5$ ;  $S$  is source term (per area);  $k$  is coefficient of conductivity.

## References

- [1] H. Versteeg and W. Malalasekera. *An Introduction to Computational Fluid Dynamics - The Finite Volume Method*. Longman Scientific & Technical, Harlow, England, 1st edition, 1995.