PH 102 : Electromagnetism **Department of Physics**
Tutorial 3 **Indian Institute of Technology Guwahati**
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1. A uniformly charged solid sphere of radius R carries a total charge Q, and is set spinning with angular velocity ω about the z axis.
(a) What is the magnetic dipole moment of the sphere?
(b) Find the magnetic field at a point (r, θ) inside the sphere.
(c) Using the results of (b) find the average magnetic field within the sphere. Hint: Average magnetic field is defined as

$$\vec{B}_{\text{avg}} = \frac{1}{\frac{4}{3}\pi R^3}\int \vec{B} d\tau$$

Compare this result with the result of (a) and show that the average magnetic field is related to the magnetic dipole moment as

$$\vec{B}_{\text{avg}} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{R^3}$$

2. Suppose the field inside a large piece of magnetic material is $\vec{B}_0$, so that $\vec{H}_0 = \vec{B}_0/\mu_0 - \vec{M}$.
(a) Now a small spherical cavity is hollowed out of the material (as shown in figure 1). Find the field at the centre of the cavity, in terms of $\vec{B}_0, \vec{M}$. Also find $\vec{H}$ at the centre of the cavity in terms of $\vec{H}_0, \vec{M}$.
(b) Do the same for a long needle-shaped cavity running parallel to $\vec{M}$.
(c) Do the same for a thin wafer-shaped cavity perpendicular to $\vec{M}$.

3. Given that $\vec{H}_1 = -2\hat{i} + 6\hat{j} + 4\hat{k}$ A/m in the region $y - x - 2 \leq 0$, where $\mu_1 = 5\mu_0$. Calculate
(a) $\vec{M}_1$ and $\vec{B}_1$.
(b) $\vec{M}_2$ and $\vec{B}_2$ in the region $y - x - 2 \geq 0$, where $\mu_2 = 2\mu_0$.

4. A short circular cylinder of radius a and length L carries a "frozen-in" uniform magnetisation $\vec{M}$ parallel to its axis. Find the bound current and sketch the magnetic field of the cylinder: one for $L \gg a$, one for $L \langle a$ and one for $L \approx a$.

5. (a) Find the magnetic dipole moment of a spherical shell, of radius R, carrying a uniform surface charge σ which is set to spin at angular velocity $\vec{\omega}$. **2**
(b) Consider a charge of $3pC$ being distributed over a sphere of radius $1cm$ and having a uniform surface charge density σ. If this sphere is rotated about its diameter with

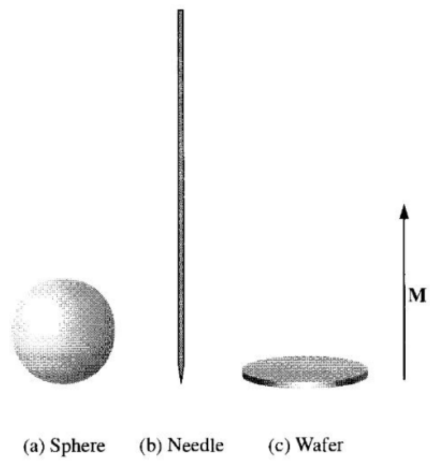

Figure 1: Figure for take home problem 2.

angular velocity $\omega = 10^6$ radians per second, find the magnetic dipole moment of the sphere.

6. A circular loop of radius a is at a distance D above a tiny magnetic dipole of infinitesimal area dS carrying a current I_1, as shown in figure 2. Assume current through the circular loop $I_2 = 0$, for the time being. Also, the distance D and loop radius a are related as $D = \sqrt{3}a$. Write your final answers only in terms of I_1, dS, a and fundamental constants.

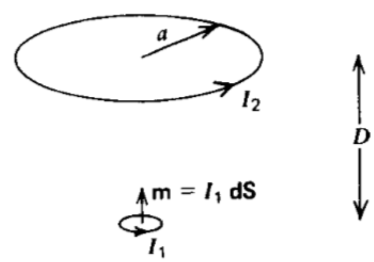

Figure 2: Figure for problem 6

(i) What is the vector potential due to the dipole at all points on the circular loop.

(ii) Consider the loop to be carrying a current $I_2 \neq 0$. The relation between D and a remains same as before $D = \sqrt{3}a$. What is the magnetic field due to I_2 at the position of the tiny dipole? What is the force on the magnetic dipole? What is the torque on the magnetic dipole?