PH102, Electromagnetism
Mid-semester Exam

March 3, 2018
2 - 4 PM

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Question 1: Contains three parts; answer them in the space provided.

a) Three static charges q_1, q_2, q_3 are situated at $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ respectively. Find the divergence of electric field produced by these charges at a point $\vec{r}$ and therefore find the volume charge density (ρ) at that point. [1.5+0.5]

Answer:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1(\vec{r}-\vec{r}_1)}{|(\vec{r}-\vec{r}_1)|^3} + \frac{q_2(\vec{r}-\vec{r}_2)}{|(\vec{r}-\vec{r}_2)|^3} + \frac{q_3(\vec{r}-\vec{r}_3)}{|(\vec{r}-\vec{r}_3)|^3}\right]$$

$$\therefore \vec{\nabla}\cdot\vec{E} = \frac{1}{4\pi\epsilon_0}\left[q_1\vec{\nabla}\cdot\frac{(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} + q_2\vec{\nabla}\cdot\frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|^3} + q_3\vec{\nabla}\cdot\frac{\vec{r}-\vec{r}_3}{|\vec{r}-\vec{r}_3|^3}\right].$$

Using $\vec{\nabla}\cdot\frac{(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} = 4\pi\delta^3(\vec{r}-\vec{r}_1)$,

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\epsilon_0}\left[q_1\delta^3(\vec{r}-\vec{r}_1) + q_2\delta^3(\vec{r}-\vec{r}_2) + q_3\delta^3(\vec{r}-\vec{r}_3)\right].$$

Using Gauss' law: $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$. where ρ is volume charge density.

$$\therefore \boxed{\rho = q_1\delta^3(\vec{r}-\vec{r}_1) + q_2\delta^3(\vec{r}-\vec{r}_2) + q_3\delta^3(\vec{r}-\vec{r}_3)}$$

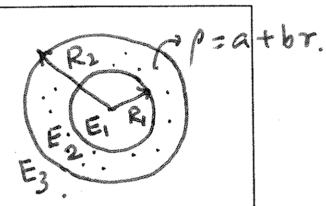
so that $\int_{\text{all space}} \rho\, d\tau = q_1+q_2+q_3 \rightarrow$ total charge given in the problem.

b) A static electric charge is distributed in a spherical shell of inner radius R_1 and outer radius R_2. The electric charge density is given by $\rho = a + br$, where r is the distance from the centre and a, b are constants.

(i) Find electric field in $r < R_1$, $R_1 < r < R_2$ and $r > R_2$.

(ii) Find out the electrostatic energy stored at $r < R_1$. [1.5+0.5]

Answer (continues to backside):



Following Gauss' law $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$.

- $E_1\ (r<R_1) = 0$ as $Q_{enc} = 0$.

- For $R_1<r<R_2$: $E_2 \cdot 4\pi r^2 = \frac{1}{\epsilon_0}\int_{R_1}^{r}(a+br)\,4\pi r^2 dr$

$$= \frac{1}{\epsilon_0}4\pi\cdot\left[\frac{a}{3}(r^3-R_1^3) + \frac{b}{4}(r^4-R_1^4)\right].$$

$$\therefore \vec{E}_2 = \frac{1}{\epsilon_0 r^2}\left[\frac{a}{3}(r^3-R_1^3) + b/4\,(r^4-R_1^4)\right]\hat{r}.$$

- For $r > R_2$: $E_3 \cdot 4\pi r^2 = \frac{1}{\epsilon_0}\int_{R_1}^{R_2}(a+br)\,4\pi r^2 dr$

Answer to Question 1 (b) continued :

$$\therefore \bar{E}_3 = \frac{1}{\epsilon_0 r^2}\left[\frac{a}{3}\left(R_2^3 - R_1^3\right) + b/4\left(R_2^4 - R_1^4\right)\right]\hat{r}.$$

Electrostatic energy in free space

$$W = \frac{1}{2}\epsilon_0 \int E^2 d\tau$$

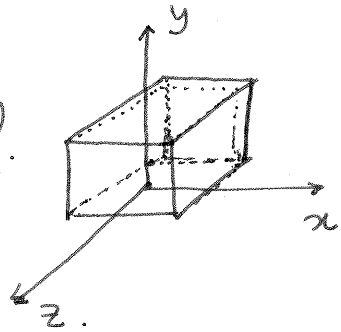
$\therefore$ For $r < R_1$: $E_1 = 0$.

$\therefore \boxed{W = 0}$

c) A dielectric cube of side s centred at origin, carries a "frozen in" polarisation $\vec{P} = k\vec{r}$, where k is a constant. Find all the bound charges and check that they add up to zero. [1.5+0.5]

Answer:

Polarization: $\bar{P} = k\bar{r}$
$= k(x\hat{x} + y\hat{y} + z\hat{z})$.



- Bound surface charge density:

$\boxed{\sigma_b = \vec{P}.\hat{n}}$ where $\hat{n}$ points $\perp^r$ above the surface.

For six different surfaces of the cube:

$\hat{n}$: $\hat{x}, -\hat{x}, \hat{y}, -\hat{y}, \hat{z}$ & $-\hat{z}$.

$\therefore \sigma_b = \{kx, -kx, ky, -ky, kz, -kz\}$ respectively.

$\therefore$ The total surface charge ~~density~~ at $x = s/2$ surface :

$$Q_b\Big|_{x=s/2} = \int \sigma_b da = \int kx\, dy\, dz = k\int \frac{s}{2} dy\, dz = \frac{ks}{2}s^2.$$

Similarly $Q_b\Big|_{x=-s/2} = -\int kx\, dy\, dz = -k\int(-s/2)dy\, dz$
$= \frac{ks}{2}s^2.$

$\therefore$ Total Bound surface charge $= 6.\frac{ks}{2}s^2 = \boxed{3ks^3.} = Q_b^A\Big|_{Tot}$

- Bound volume charge density : $\boxed{\rho_b = -\vec{\nabla}.\vec{P} = -3k.}$

$\therefore$ Total bound volume charge $Q_b^V = \int \rho\, d\tau = -3ks^3$.

$$\therefore Q_b^V + Q_b^A\Big|_{Tot} = 3ks^3 - 3ks^3 = 0.$$

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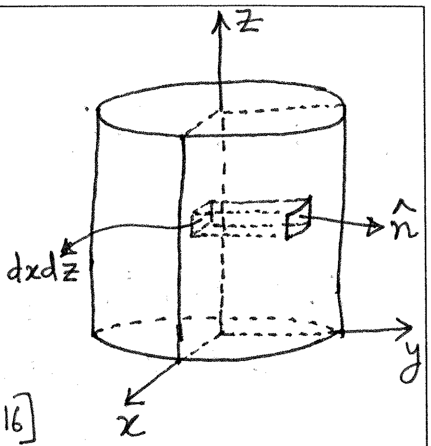
Question 2: Contains two parts; answer them in the space provided.

a) Evaluate the flux of electric field $\vec{E} = yz\,\hat{x} + zx\,\hat{y} + xy\,\hat{z}$ through the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. [3]

Answer:

$$\iint_S \vec{E}\cdot\hat{n}\, ds = \iint \vec{E}\cdot\hat{n}\,\frac{dx\,dz}{|\hat{n}\cdot\hat{y}|}$$



Normal to the surface $x^2+y^2=16$ is

$$\vec{\nabla}(x^2+y^2) = 2x\,\hat{x} + 2y\,\hat{y}$$

$\therefore$ Unit normal : $\hat{n} = \dfrac{2x\hat{x}+2y\hat{y}}{\sqrt{4x^2+4y^2}}$

$$= \frac{x\hat{x}+y\hat{y}}{4} \; ; \; [\text{since } x^2+y^2=16]$$

$$\therefore\ \vec{E}\cdot\hat{n} = (yz\,\hat{x} + zx\,\hat{y} + xy\,\hat{z})\cdot\frac{(x\hat{x}+y\hat{y})}{4} = \frac{xyz}{4} + \frac{xyz}{4} = \frac{1}{2}xyz$$

and $\hat{n}\cdot\hat{y} = \dfrac{x\hat{x}+y\hat{y}}{4}\cdot\hat{y} = \dfrac{y}{4}$

Hence $\displaystyle \iint \vec{E}\cdot\hat{n}\,\frac{dx\,dz}{|\hat{n}\cdot\hat{y}|} = \iint \frac{xyz}{2}\,\frac{dx\,dz}{|\frac{y}{4}|} = \int_{z=0}^{5}\int_{x=0}^{4} 2\,\frac{xyz}{y}\,dx\,dz$

$$= 2\int_0^5\int_0^4 xz\,dx\,dz$$

$$= \cancel{2}\cdot\frac{x^2}{\cancel{2}}\Big|_0^4\ \frac{z^2}{2}\Big|_0^5$$

$$= 16 \times \frac{25}{2}$$

$$= 200.$$

Hence the flux of the electric field will be 200 units through the surface mentioned in the problem.

b) The parabolic cylindrical coordinate system (u, v, z) is described by the following transformations with respect to Cartesian coordinates (x, y, z) as:

$$x = \frac{1}{2}(u^2 - v^2),\ y = uv,\ z = z; \text{ where } -\infty < u < \infty,\ v \geq 0,\ -\infty < z < \infty.$$

(i) Find the unit vectors in this coordinate system in terms of those in Cartesian coordinates.

(ii) A vector field in this coordinate is given by : $\vec{F} = \frac{1}{2}u\sqrt{u^2+v^2}\,\hat{u} + \frac{1}{2}v\sqrt{u^2+v^2}\,\hat{v} + z\hat{z}$. Show explicitly whether $\vec{F}$ can be a valid electric field. [1.5+1.5]

Answer:

(i) $\vec{r} = \frac{1}{2}(u^2 - v^2)\hat{x} + uv\hat{y} + z\hat{z}$ and $d\vec{r} = h_1 du_1 \hat{e}_1 + h_2 du_2 \hat{e}_2 + h_3 du_3 \hat{e}_3$

Where $h_i = \left|\frac{\partial \vec{r}}{\partial u_i}\right|$; $\hat{e}_i = \frac{\partial \vec{r}/\partial u_i}{|\partial \vec{r}/\partial u_i|}$

$\therefore\ h_1 = h_u = \left|\frac{\partial \vec{r}}{\partial u}\right| = \sqrt{u^2+v^2}$

$h_2 = h_v = \left|\frac{\partial \vec{r}}{\partial v}\right| = \sqrt{u^2+v^2}$ and $h_3 = h_z = \left|\frac{\partial \vec{r}}{\partial z}\right| = 1.$

$\hat{e}_1 = \hat{u} = \frac{\partial \vec{r}/\partial u}{|\partial \vec{r}/\partial u|} = \frac{u\hat{x} + v\hat{y}}{\sqrt{u^2+v^2}}$; $\hat{e}_2 = \hat{v} = \frac{\partial \vec{r}/\partial v}{|\partial \vec{r}/\partial v|} = \frac{-v\hat{x} + u\hat{y}}{\sqrt{u^2+v^2}}$; $\hat{e}_3 = \hat{z} = \hat{z}$

(ii) Hence : $\vec{\nabla} \times \vec{F} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 E_1 & h_2 E_2 & h_3 E_3 \end{vmatrix}$

$$= \frac{1}{(u^2+v^2)}\begin{vmatrix} (u^2+v^2)^{1/2}\hat{u} & (u^2+v^2)^{1/2}\hat{v} & \hat{z} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial z} \\ (u^2+v^2)\frac{u}{2} & (u^2+v^2)\frac{v}{2} & z \end{vmatrix}$$

$$= \frac{1}{(u^2+v^2)}\left[\hat{u}.0 + \hat{v}.0 + \hat{z}\left(\frac{v}{2}.2u - \frac{u}{2}.2v\right)\right]$$

$= 0.$

Therefore $\vec{F}$ is a valid electric field.

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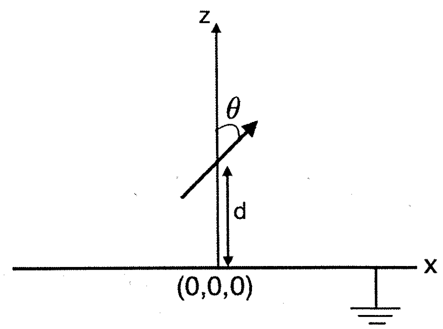
Question 3: A dipole is kept on the z axis at a distance d from the origin. The dipole moment $\vec{p} = p_x\hat{x} + p_z\hat{z}$ is in the xz plane making an angle θ with the z axis as shown in the figure. A conducting grounded sheet lies in the xy plane.

a) What is the dipole moment of the image and what is its location?

b) Write down the potential at a general point (x, y, z) in the region $z > 0$. Show explicitly that the potential at a point on the conducting plane, due to the dipole and its image, is zero. You may use cartesian coordinates.

c) Find the induced surface charge density on the conducting plane.

d) Find the torque on the dipole. For which angles θ, the magnitude of the torque is maximum?



[1+(2+0.5)+1+(1+0.5)]

Answer (Use backside too):

(a) Dipole moment of the image: $\vec{p}\,' = -p_x\hat{x} + p_z\hat{z}$ and the image will be located at $(-d)\hat{z}$.

(b) $$V(x,y,z) = \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{p}\cdot(\vec{r}-d\hat{z})}{(x^2+y^2+(z-d)^2)^{3/2}} + \frac{\vec{p}\,'\cdot(\vec{r}+d\hat{z})}{(x^2+y^2+(z+d)^2)^{3/2}}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{p_x x + p_z(z-d)}{(x^2+y^2+(z-d)^2)^{3/2}} + \frac{-p_x x + p_z(z+d)}{(x^2+y^2+(z+d)^2)^{3/2}}\right]$$

It is evident from the expression of $V(x,y,z)$ that $\boxed{V(x,y,z=0)=0}$.

(c) $$\sigma(x,y) = \epsilon_0 E_z\Big|_{z=0} = -\epsilon_0\frac{\partial V}{\partial z}\Big|_{z=0}$$

$$= -\frac{[3p_x d x + p_z(x^2+y^2-2d^2)]}{2\pi(d^2+x^2+y^2)^{5/2}}$$

(d) Electric field due to the image dipole at the point of the original dipole at $z>0$ is:

$$\vec{E} = \frac{1}{4\pi\epsilon_0(2d)^3}\left[3(\vec{p}\,'\cdot\hat{z})\hat{z} - \vec{p}\,'\right] \text{ with } \vec{p}\,' = -p_x\hat{x} + p_z\hat{z}$$

Answers to question 3 continued:

a, $\vec{E} = \frac{1}{4\pi\epsilon_0 (2d)^3}\left(p_x \hat{x} + 2p_z \hat{z}\right)$

Therefore the torque is: $\vec{N} = \vec{p} \times \vec{E}$

$$\boxed{\vec{N} = -\frac{p_x p_z \hat{y}}{4\pi\epsilon_0 (2d)^3}}$$

Now, note that; $\sin\alpha = \frac{p_z}{|\vec{p}|}$; $\cos\alpha = \frac{p_x}{|\vec{p}|}$ & $\tan\alpha = \frac{p_z}{p_x}$

Hence $|\vec{N}| = -\frac{|p|^2}{4\pi\epsilon_0}\frac{\sin 2\alpha}{2}\left(\frac{1}{2d}\right)^3$

Therefore the torque is maximum when $\sin 2\alpha = 1$

$\Rightarrow 2\alpha = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$$\therefore \boxed{\alpha = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}}$$

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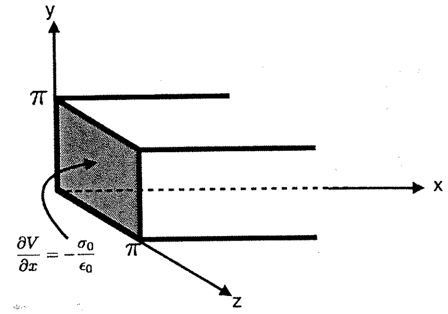
Question 4: An infinitely long metal pipe (with sides π) is grounded. One end, at $x = 0$, is closed by a conducting plate (insulated from the other metal surfaces) carrying constant surface charge density σ_0 so that $\frac{\partial V}{\partial x}|_{x=0} = -\frac{\sigma_0}{\epsilon_0}$ as shown by the shaded region in figure. Find the potential inside the pipe following the steps mentioned below:

a) Write down appropriate boundary conditions.
b) Use method of separation of variables to find the potential at any point inside the pipe in a product form.
c) Applying the boundary conditions write down the potential in a series form.



[1+2+3]

Answer (Use backside too): As there is no free or bound charge in the prob: potential satisfies Laplace's Equ^n: $\boxed{\nabla^2 V = 0}$

There exists no translational symmetry, so that
$V = V(x, y, z)$

$$\therefore \boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.}$$

- Boundary Conditions:

$$\boxed{\begin{aligned} &V(x, 0, z) = 0. \\ &V(x, \pi, z) = 0. \\ &V(x, y, 0) = 0. \\ &V(x, y, \pi) = 0. \\ &V \to 0 \text{ as } x \to \infty. \\ &\frac{\partial V}{\partial x} = -\sigma_0/\epsilon_0 \text{ at } x = 0. \end{aligned}}$$

- Separation of variables: $V(x, y, z) = X(x)\, Y(y)\, Z(z)$.

$$\therefore \nabla^2 V = 0 \rightarrow \frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0.$$

This is only possible, if $\boxed{\frac{1}{X}\frac{d^2X}{dx^2} = C_1}$, $\boxed{\frac{1}{Y}\frac{d^2Y}{dy^2} = C_2}$ and $\boxed{\frac{1}{Z}\frac{d^2Z}{dz^2} = C_3}$, all are constants, with $\boxed{C_1 + C_2 + C_3 = 0}$

- Boundary condition suggests that solution along z & y will be Harmonic.

$\therefore$ We can choose: $C_2 = -K^2$, $C_3 = -l^2$ with K, l ~~integers~~, real numbers.

$\therefore C_3 = K^2 + l^2$.

Answer to question 4 continued :

$$\therefore \frac{d^2Y}{dy^2}+k^2Y=0 \rightarrow Y = C\sin ky + D\cos ky \rightarrow \boxed{D=0} \text{ as } V=0 \text{ at } y=0.$$

$\cdot$ $V=0$ at $y=\pi$ yields k : integer.

$$\frac{d^2Z}{dz^2}+l^2Z=0 \rightarrow Z = E\cos lz + F\sin lz \rightarrow \boxed{E=0} \text{ as } V=0 \text{ at } z=0$$

$V=0$ at $z=\pi$ yields l : integer.

$$\therefore \frac{d^2X}{dx^2}-(k^2+l^2)X=0.$$

$$\Rightarrow X(x) = Ae^{\sqrt{k^2+l^2}\,x} + Be^{-\sqrt{k^2+l^2}\,x}. \rightarrow \boxed{A=0} \text{ as } V=0 \text{ for } x\to\infty.$$

$$\therefore V(x,y,z) = X(x)\,Y(y)\,Z(z) = Ce^{-\sqrt{k^2+l^2}\,x}\sin ky \sin lz.$$ All const. are dumped in C.

Now, (i) Above solutions are infinite possible with $k=1,2\ldots,\ l=1,2\ldots$

(ii) None of the specific solutions will be able to match last boundary condition;

We will take a linear combination of all solutions & Apply Fourier's trick to find the unknown coeff. C by matching with boundary cond. at $x=0$.

$$\therefore V(x,y,z) = \sum_{k=1,2..}\ \sum_{l=1,2..} C_{kl}\, e^{-\sqrt{k^2+l^2}\,x} \sin ky \sin lz.$$

$$\text{Now, } -\frac{\partial V}{\partial x}\Big|_{x=0} = \sum_k\sum_l \sqrt{k^2+l^2}\, C_{kl} \sin ky \sin lz = \frac{\sigma_0}{\epsilon_0}.$$

$$\therefore \sum_k\sum_l C_{kl}\sqrt{k^2+l^2}\, \underbrace{\int_0^\pi \sin ky \sin ny\, dy}_{\frac{\pi}{2}\delta_{kn}}\ \underbrace{\int_0^\pi \sin lz \sin mz\, dz}_{\frac{\pi}{2}\delta_{lm}} = \frac{\sigma_0}{\epsilon_0}\ \underbrace{\int_0^\pi \sin ny\, dy}_{(2/n)_{n:\text{odd}}}\ \underbrace{\int_0^\pi \sin mz\, dz}_{(2/m)_{m\ \text{odd}}}.$$

$$\rightarrow C_{nm}\sqrt{n^2+m^2}\cdot\left(\frac{\pi}{2}\right)^2 = \frac{\sigma_0}{\epsilon_0}\cdot\left(\frac{2}{n}\right)_{n:\text{odd}}\left(\frac{2}{m}\right)_{m:\text{odd}}.$$

$$\therefore C_{nm} = \frac{16\,\sigma_0}{\pi^2\epsilon_0\, nm\sqrt{n^2+m^2}}. \quad (n,m:\text{odd}) \text{ or } C_{nm}=0.$$

$$\therefore V(x,y,z) = \frac{16\,\sigma_0}{\pi^2\epsilon_0} \sum_{n=1,3,5}\ \sum_{m=1,3,5\ldots} \frac{e^{-\sqrt{n^2+m^2}\,x}\sin ny \sin mz}{nm\sqrt{n^2+m^2}}$$

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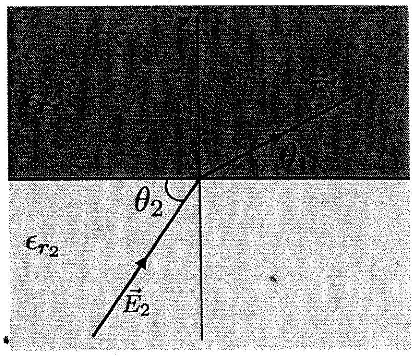
Question 5: Two extensive homogeneous, isotropic dielectric media meet on plane $z = 0$. The dielectric constants of the medium situated at $z > 0$ is $\epsilon_{r_1} = 4$ and the same for $z < 0$ is $\epsilon_{r_2} = 3$.

(a) Write down appropriate boundary conditions for electric field ($\vec{E}$) and electric displacement ($\vec{D}$).
(b) Now consider, a uniform electric field $\vec{E}_1 = 5\hat{x} - 2\hat{y} + 3\hat{z}$ kV/m exists for $z \geq 0$. Find $\vec{E}_2$ for $z \leq 0$ using the boundary conditions (*Hint: Divide* $\vec{E}_1$ *into* $E_1^{||}$ *and* $E_1^{\perp}$).
(c) Find the angles θ_1 and θ_2 that $\vec{E}_1$ and $\vec{E}_2$ makes with the interface respectively.
(d) Find energy densities in both media in Joule/m^3.



[1+2+2+1]

Answer (Use backside too):

(a) Boundary conditions : * For electric displacement :

(i) $D_1^{\perp} - D_2^{\perp} = \sigma_f$ with $\sigma_f = 0$ in this problem

$\Rightarrow D_1^{\perp} = D_2^{\perp}$

(ii) $D_1^{||} - D_2^{||} = P_1^{||} - P_2^{||}$

* For electric field :

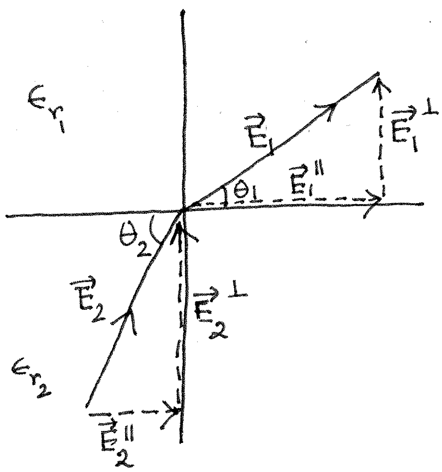
(i) $E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\epsilon_0} = \frac{\sigma_b}{\epsilon_0}$ (in absence of σ_f)

(ii) $E_1^{||} = E_2^{||}$

(b) Given: $\vec{E}_1 = 5\hat{x} - 2\hat{y} + 3\hat{z}$; $\Rightarrow E_1^{\perp} = \vec{E}_1 \cdot \hat{z} = 3 \Rightarrow \vec{E}_1^{\perp} = 3\hat{z}$ kV/m

Now, $\vec{E}_1 = \vec{E}_1^{\perp} + \vec{E}_1^{||}$

Hence, $\vec{E}_1^{||} = \vec{E}_1 - \vec{E}_1^{\perp} = 5\hat{x} - 2\hat{y}$ kV/m



Using the boundary conditions :

$\vec{E}_1^{||} = \vec{E}_2^{||}$,

we find $\vec{E}_2^{||} = 5\hat{x} - 2\hat{y}$ kV/m

The other boundary condition $\vec{D}_1^{\perp} = \vec{D}_2^{\perp}$

implies : $\epsilon_{r_1} \vec{E}_1^{\perp} = \epsilon_{r_2} \vec{E}_2^{\perp}$

$$\Rightarrow \vec{E}_2^{\perp} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}} \vec{E}_1^{\perp} = \frac{4}{3} \cdot 3\hat{z} = 4\hat{z} \text{ kV/m}$$

Hence $\vec{E}_2 = \vec{E}_2^{\perp} + \vec{E}_2^{||} = 5\hat{x} - 2\hat{y} + 4\hat{z}$ kV/m

Answer to question 5 continued :

(c) $\tan\theta_1 = \dfrac{3}{\sqrt{5^2+2^2}} = \dfrac{3}{\sqrt{29}} \Rightarrow \theta_1 = \tan^{-1}\left(\dfrac{3}{\sqrt{29}}\right) = 29.1^\circ$

$$\tan\theta_2 = \frac{E_2^{\perp}}{E_2^{\parallel}} = \frac{4}{\sqrt{29}} \Rightarrow \theta_2 = \tan^{-1}\left(\frac{4}{\sqrt{29}}\right) = 36.6^\circ$$

(d) $W_1 = \frac{1}{2}\epsilon_1 |\vec{E}_1|^2 = \frac{1}{2}\epsilon_0 \epsilon_{r_1} |\vec{E}_1|^2$

$$= \frac{1}{2}\cdot 4 \cdot \left(\frac{10^{-9}}{36\pi}\right)\frac{C}{V.m}\left(\sqrt{25+4+9}\right)^2 \times 10^6 \frac{V^2}{m^2}$$

$$= \left(\frac{38}{18\pi}\times 10^{-3}\right)\frac{CV}{m^3}$$

$$= 671.9 \times 10^{-6}\ J/m^3 = 671.9\ \mu J/m^3$$

$W_2 = \frac{1}{2}\epsilon_2 |\vec{E}_2|^2 = \frac{1}{2}\epsilon_0 \epsilon_{r_2} |\vec{E}_2|^2$

$$= \frac{1}{2}\cdot 3 \cdot \left(\frac{10^{-9}}{36\pi}\right)\frac{C}{V.m}\left(\sqrt{25+4+16}\right)^2 \times 10^6 \frac{V^2}{m^2}$$

$$= \left(\frac{45}{24\pi}\times 10^{-3}\right)\frac{CV}{m^3}$$

$$= 596.8 \times 10^{-6}\ J/m^3 = 596.8\ \mu J/m^3$$