PH 102, Electromagnetism,

Post Mid Semester

Lecture 14

Summary Lecture

D. J. Griffiths: Chapters 5, 6, 7, 8 and 9.

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Magnetostatics:

Lorentz force law

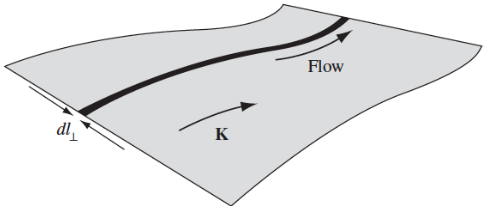
$$\mathbf{F} = Q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})].$$

Magnetic forces do no work.

$$dW_{\text{mag}} = \mathbf{F}_{\text{mag}} \cdot d\mathbf{l} = Q(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v}\, dt = 0.$$

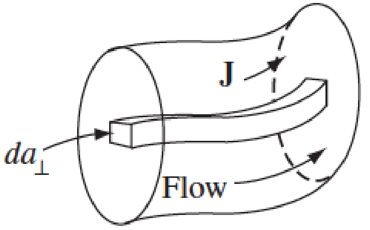
Current

$$\mathbf{I} = \lambda \mathbf{v}. \qquad \mathbf{K} = \sigma \mathbf{v}. \qquad \mathbf{J} = \rho \mathbf{v}.$$



Continuity Equation

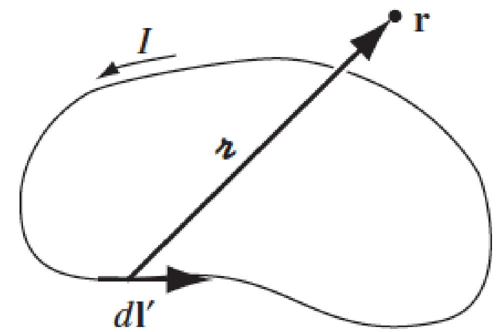
$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} = 0.$$



Magnetostatics:

Biot-Savart Law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \, dl' = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}.$$



Ampere's Law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}, \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}.$$

Ampere's Law is useful, only when symmetry enables pulling ***B*** outside integral

Magnetic vector potential :

Also, $\mathbf{B} = \nabla \times \mathbf{A}.$ doesn't specify Div. A,

Thus one can choose, Div. A = 0

$$\begin{cases} \nabla \cdot \mathbf{B} = 0, & \text{(no name);} \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J}, & \text{(Ampère's law).} \end{cases}$$

$$\nabla \times \mathbf{B} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu_0 \mathbf{J}.$$

Poisson's eq.

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}. \qquad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}} \, d\tau'.$$

Assuming ***J*** goes to zero at infinity

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I}}{\mathscr{r}} \, dl' = \frac{\mu_0 I}{4\pi} \int \frac{1}{\mathscr{r}} \, d\mathbf{l}'; \quad \mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}}{\mathscr{r}} \, da'.$$

Assuming current zero at infinity

Magnetostatic Boundary Conditions

$$\mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0 (\mathbf{K} \times \hat{\mathbf{n}}), \qquad \frac{\partial \mathbf{A}_{\text{above}}}{\partial n} - \frac{\partial \mathbf{A}_{\text{below}}}{\partial n} = -\mu_0 \mathbf{K}.$$

Multipole expansion of vector potential

Magnetic Dipole : In absence of monopoles dipole is the dominant term

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0 I}{4\pi r^2}\oint r'\cos\alpha\, d\mathbf{l}' = \frac{\mu_0 I}{4\pi r^2}\oint (\hat{\mathbf{r}}\cdot\mathbf{r}')\, d\mathbf{l}'.$$

$$= \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2},$$

$$\oint(\vec{c}\cdot\vec{r})d\vec{l} = \vec{a}\times\vec{c}$$

$$\vec{a} = \frac{1}{2}\oint \vec{r}\times d\vec{l}$$

with $\mathbf{c} = \hat{\mathbf{r}}$

$$\oint(\hat{\mathbf{r}}\cdot\mathbf{r}')\, d\mathbf{l}' = -\hat{\mathbf{r}}\times\int d\mathbf{a}'.$$

$$\hat{r}\times\oint(\vec{r}'\times d\vec{r}') = \oint\vec{r}'(\hat{r}\cdot d\vec{r}') - \oint d\vec{r}'(\hat{r}\cdot\vec{r}')$$

$$= \oint d\left[(\vec{r}'(\hat{r}\cdot\vec{r}')\right] - \oint(\hat{r}\cdot\vec{r}')d\vec{r}' - \oint d\vec{r}'(\hat{r}\cdot\vec{r}')$$

$$= -\mathbf{2}\oint(\hat{r}\cdot\vec{r}')d\vec{r}'$$

Here $\boldsymbol{m}$ is the magnetic dipole moment

$$\mathbf{m} \equiv I\int d\mathbf{a} = I\mathbf{a}.$$

Magnetic dipole moment is independent of choice of origin!
No Magnetic monopole

Remember,

Electric dipole moment was independent of origin only
when the total charge vanished

Torque and forces on magnetic dipole

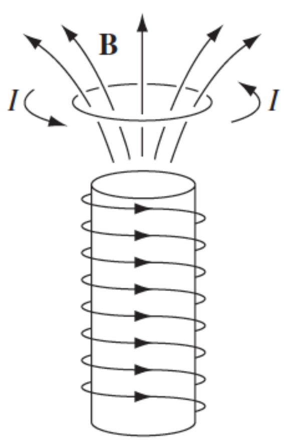
In the uniform field the net force on a current loop is zero!

$$\mathbf{F} = I \oint (d\mathbf{l} \times \mathbf{B}) = I \left(\oint d\mathbf{l} \right) \times \mathbf{B} = \mathbf{0};$$

B being constant comes outside integral and the net displacement around a closed loop also vanishes,

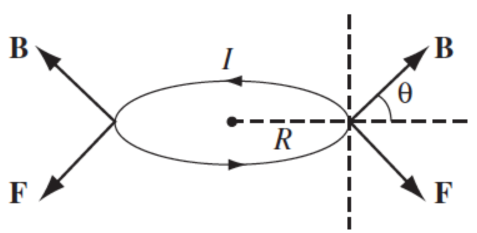
Not true for non uniform field,

Circular wire (***R***) carrying current ***I*** is suspended over a short solenoid over the fringing region.



Here B has a radial component, hence net downward force on loop.

$$\boxed{F = 2\pi I R B \cos\theta.}$$



Post Mid Semester

Lecture 4.

Multipole expansion of vector potential

$$\boxed{\nabla \cdot \mathbf{B} = 0.}$$

No magnetic Monopole

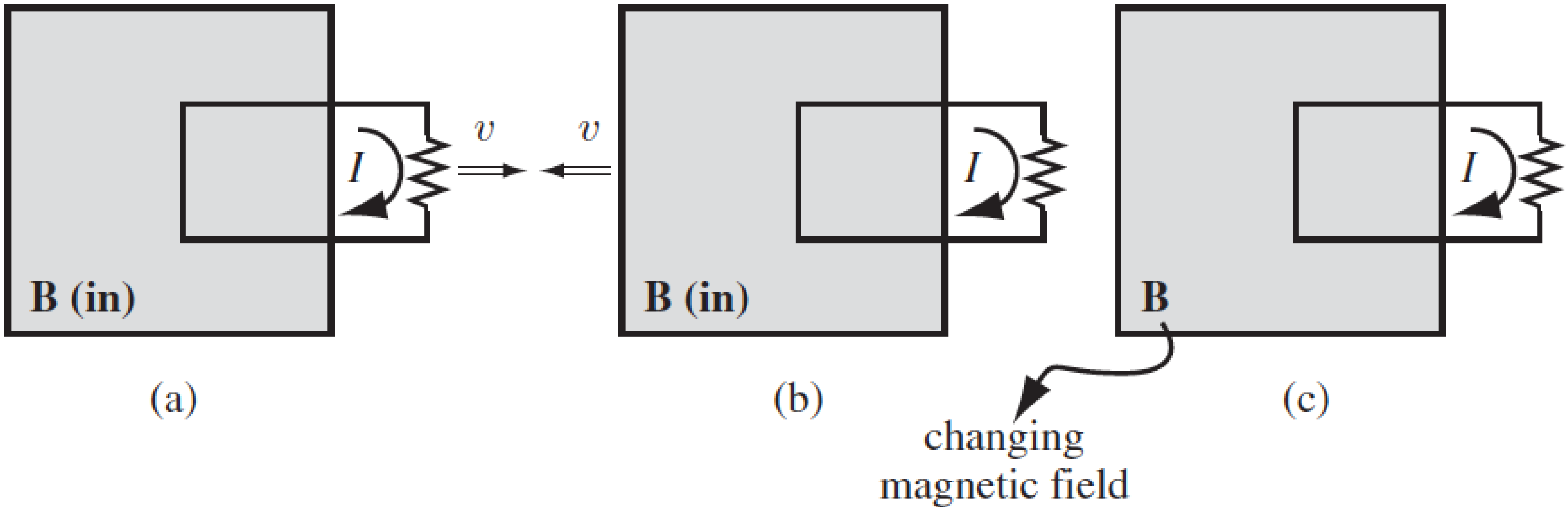
$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2}, \qquad \mathbf{m} \equiv I \int d\mathbf{a} = I\mathbf{a}.$$

Torque and forces on magnetic dipole

$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B}) \qquad \overline{T} = \overline{m} \times \overline{B}.$$

Look at the sample problem from this chapter

Faraday's Law



(a) Case of motional emf;

(b) 2nd Experiment has the same emf, relative motion of loop and the magnet.

(c) 3rd scenario also an electric field gets generated and gives the same emf $\mathcal{E} = -\frac{d\Phi}{dt}$

Changing magnetic field generates an electric field

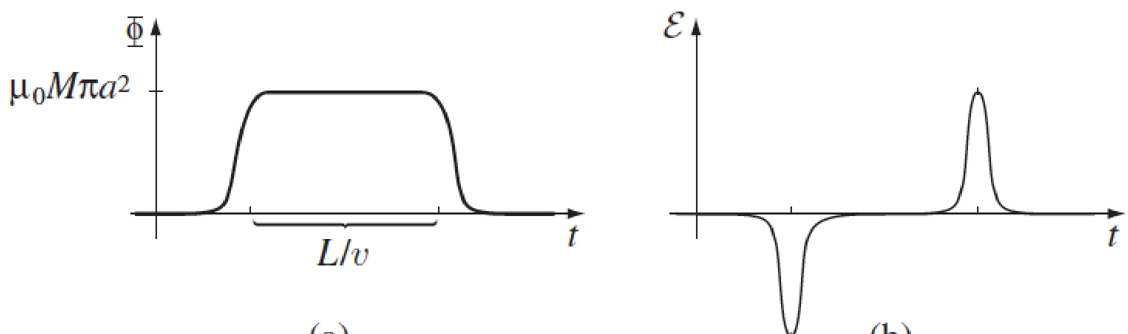
Note: In (a), $\mathbf{v} \times \mathbf{B}$ drives the current ***I*** not ***E***

In (b) & (c), induced ***E*** drives the current ***I***

Lenz's Law

The induced current flows in such a direction that the flux it produces tends to cancel the change. We can not quantify the current but can get the directions right.

Nature abhors a change in flux.



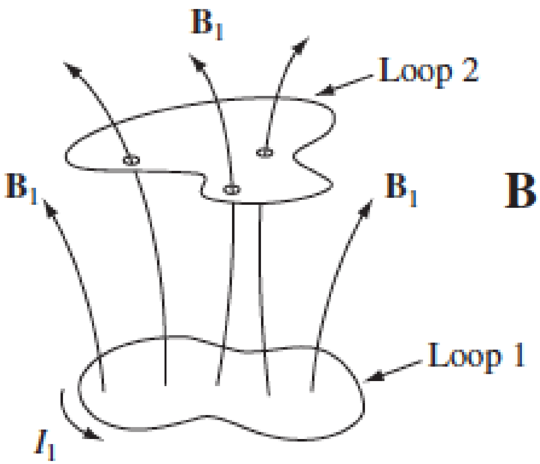
(a) (b)

The magnet enters the ring, flux increases. The current is clockwise to generate field to the right.

Change in the flux is prevented, not flux

The magnet exits ring, flux drops, counterclockwise current to restore the field.

Inductance



$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\mathbf{l}_1 \times \hat{\boldsymbol{\imath}}}{\imath^2}$$

$\mathbf{B}_1$ is proportional to $\mathbf{I}_1$

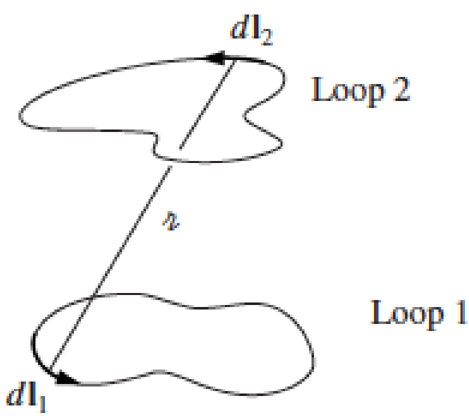
hence flux through loop2

is also proportional to $\mathbf{I}_1$

$$\Phi_2 = \int \mathbf{B}_1 \cdot d\mathbf{a}_2.$$

$$\Phi_2 = M_{21} I_1,$$

here $\mathbf{M}_{21}$ is the Mutual Inductance



$$\Phi_2 = \int \mathbf{B}_1 \cdot d\mathbf{a}_2 = \int (\nabla \times \mathbf{A}_1) \cdot d\mathbf{a}_2 = \oint \mathbf{A}_1 \cdot d\mathbf{l}_2.$$

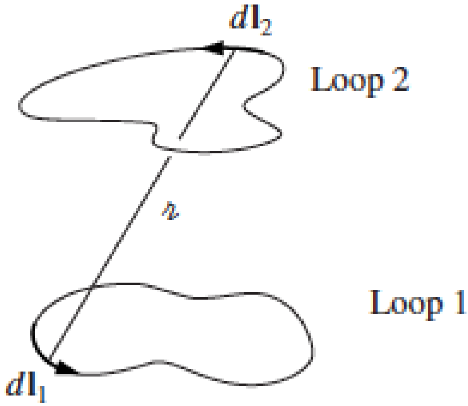
$$= \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\mathbf{l}_1}{\imath} \right) \cdot d\mathbf{l}_2.$$

$$\mathbf{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\mathbf{l}_1}{\imath},$$

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{\imath}.$$

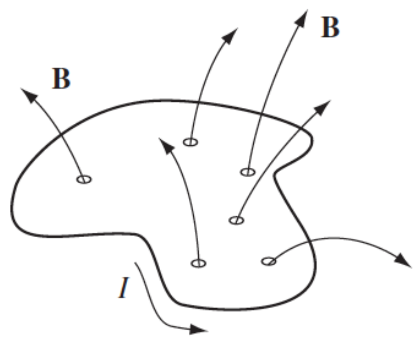
Neumann formula
for mutual inductance

Inductance



emf in loop2 due to change of current in loop1

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M\frac{dI_1}{dt}.$$



Changing current will induce an emf in the source loop as well

$$\Phi = LI.$$

self inductance L also depends on the geometry of the loop!

unit is henries (H), volt-second per ampere

$$\mathcal{E} = -L\frac{dI}{dt}.$$

Back emf, Lenz's Law

Energy in Magnetic field

$$W = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 \, d\tau .$$

In view of this result, we say the energy is "stored in the magnetic field," in the amount ($B_2/2\mu_o$) per unit volume.

In the light of this, it is extraordinary how similar the magnetic energy formulas are to their electrostatic counterparts:

$$W_{\text{elec}} = \frac{1}{2} \int (V\rho) \, d\tau = \frac{\epsilon_0}{2} \int E^2 \, d\tau ,$$

$$W_{\text{mag}} = \frac{1}{2} \int (\mathbf{A} \cdot \mathbf{J}) \, d\tau = \frac{1}{2\mu_0} \int B^2 \, d\tau .$$

Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$+ \quad \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$, can be derived from the above equations.

E and ***B*** are not due to change in ***B*** and ***E*,** rather fields are attributable to ρ and ***J***.

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0},$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} - \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

$$+ \quad \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Fields affect charges!

Maxwell's Equations

Maxwell's Equations in Matter :

$$\nabla \cdot \mathbf{D} = \rho_f, \ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}.$$

For linear media, $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E},$ and $\mathbf{M} = \chi_m \mathbf{H},$

$$\mathbf{D} = \epsilon \mathbf{E}, \ \text{and} \ \mathbf{H} = \frac{1}{\mu}\mathbf{B}$$

where,

$$\epsilon \equiv \epsilon_0(1 + \chi_e)$$

$$\mu \equiv \mu_0(1 + \chi_m)$$

Here, $\mathbf{D}$ is the electric displacement and $\mathbf{J}_d$ is the displacement current

$$\mathbf{J}_d \equiv \frac{\partial \mathbf{D}}{\partial t}$$

Poynting's Theorem

$$\frac{dW}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) d\tau - \frac{1}{\mu_0}\oint_{\mathcal{S}} (\mathbf{E}\times\mathbf{B})\cdot d\mathbf{a},$$

Total energy
in the fields

Rate of energy
transported out of *V, across S*

Work-Energy theorem of electrodynamics.

The 1st term (integral): total energy stored in the fields U_{em}

The 2nd term: rate of energy being transported out of *V*, across *S*, by the EM fields.

Poynting's theorem: *Work done on the charges by the EM force is equal to the decrease in energy stored in the fields, less the energy that flowed out through the surface .*

Poynting vector : Energy per unit time, per unit area, transported by the fields

$$\mathbf{S} \equiv \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}).$$

S . da ~ energy per unit time crossing the infinitesimal surface ***da***: Energy Flux

i.e ***S*** is the energy flux density.

1) Consider the two oppositely traveling electric-field waves,

$$\mathbf{E}_1 = \hat{\mathbf{x}} E_0 \cos(kz - \omega t) \quad \text{and} \quad \mathbf{E}_2 = \hat{\mathbf{x}} E_0 \cos(kz + \omega t).$$

The sum of these two waves is the standing wave, $2\hat{\mathbf{x}} E_0 \cos kz \cos \omega t$.

(a) Find the magnetic field associated with this standing electric wave by finding the **B** fields associated with each of the above traveling **E** fields, and then adding them.

(b) Find the magnetic field by instead using Maxwell's equations to find the **B** field associated with the standing electric wave, $2\hat{\mathbf{x}} E_0 \cos kz \cos \omega t$.

Solution:

(a) The traveling **B** fields must point in the $\pm\hat{\mathbf{y}}$ directions because they must be perpendicular to both the associated **E** field and the direction of propagation, which is $\pm\hat{\mathbf{z}}$. The magnitudes of the **B** fields are E_0/c. The signs are determined by the fact that $\mathbf{E} \times \mathbf{B}$ points in the direction of propagation. The two magnetic waves are therefore

$$\mathbf{B}_1 = \hat{\mathbf{y}}(E_0/c) \cos(kz - \omega t),$$
$$\mathbf{B}_2 = -\hat{\mathbf{y}}(E_0/c) \cos(kz + \omega t).$$

The sum of these waves is $\mathbf{B} = \hat{\mathbf{y}}(2E_0/c) \sin kz \sin \omega t$.

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Solution:

(b) We use the Maxwell equation $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ to find $\mathbf{B}$. The curl of $\mathbf{E} = 2\hat{\mathbf{x}} E_0 \cos kz \cos \omega t$ is

$$\nabla \times \mathbf{E} = -\hat{\mathbf{y}} 2kE_0 \sin kz \cos \omega t. \qquad \mathbf{B} = \hat{\mathbf{y}} (2kE_0/\omega) \sin kz \sin \omega t.$$

$$\omega/k = c,$$

$$\mathbf{B} = \hat{\mathbf{y}} (2E_0/c) \sin kz \sin \omega t.$$

Same as part (a)

2)

Here is a particular electromagnetic field in free space:

$$E_x = 0, \quad E_y = E_0 \sin(kx + \omega t), \quad E_z = 0;$$

$$B_x = 0, \quad B_y = 0, \quad B_z = -(E_0/c)\sin(kx + \omega t).$$

(a) Show that this field can satisfy Maxwell's equations if ω and k are related in a certain way.

(b) Suppose $\omega = 10^{10}\,\mathrm{s}^{-1}$ and $E_0 = 1$ kV/m. What is the wavelength? What is the energy density in joules per cubic meter, averaged over a large region? From this calculate the power density, the energy flow in joules per square meter per second.

Solution:

(a) The fields are

$$\mathbf{E} = \hat{\mathbf{y}} E_0 \sin(kx + \omega t), \qquad \text{and} \qquad \mathbf{B} = -\hat{\mathbf{z}}(E_0/c)\sin(kx + \omega t).$$

(i) & (ii) $\nabla \cdot \mathbf{E} = 0$ & $\nabla \cdot \mathbf{B} = 0$ [In E (B), no y (z)dependence in y (z) component]

Now check, $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = (1/c^2)\partial \mathbf{E}/\partial t$

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$$\nabla \times \mathbf{E} = \hat{\mathbf{z}} k E_0 \cos(kx + \omega t), \qquad \frac{\partial \mathbf{E}}{\partial t} = \hat{\mathbf{y}} \omega E_0 \cos(kx + \omega t),$$

$$\nabla \times \mathbf{B} = \hat{\mathbf{y}} k (E_0/c)\cos(kx + \omega t), \qquad \frac{\partial \mathbf{B}}{\partial t} = -\hat{\mathbf{z}} \omega (E_0/c)\cos(kx + \omega t).$$

To satisfy, $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ & $\nabla \times \mathbf{B} = (1/c^2)\partial \mathbf{E}/\partial t$

The required relation is, $\boxed{k = \omega/c.}$

2)

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Solution:

(b) The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega} = \frac{2\pi(3\cdot 10^8\,\mathrm{m/s})}{10^{10}\,\mathrm{s}^{-1}} = 0.19\,\mathrm{m}.$$

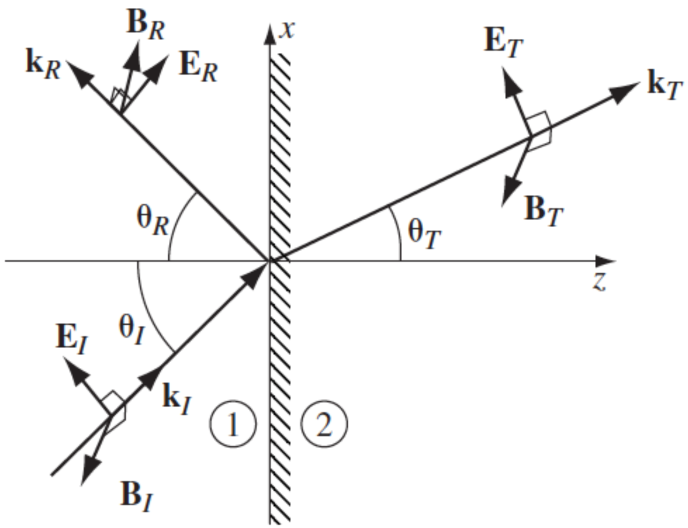
The average energy density,

$$\frac{1}{2}\epsilon_0 E_0^2 = \frac{1}{2}\left(8.85\cdot 10^{-12}\,\frac{\mathrm{s^2\,C^2}}{\mathrm{kg\,m^3}}\right)(10^3\,\mathrm{V/m})^2 = 4.4\cdot 10^{-6}\,\mathrm{J/m^3}.$$

The power density,

$$S = \frac{1}{2}\epsilon_0 E_0^2 c = (4.4\cdot 10^{-6}\,\mathrm{J/m^3})(3\cdot 10^8\,\mathrm{m/s}) = 1300\,\mathrm{J/(m^2 s)}.$$

Electromagnetic Waves



$$\tilde{E}_{0_I} - \tilde{E}_{0_R} = \beta \tilde{E}_{0_T} \qquad \tilde{E}_{0_I} + \tilde{E}_{0_R} = \alpha \tilde{E}_{0_T}$$

$$\boxed{\tilde{E}_{0_R} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right) \tilde{E}_{0_I}, \quad \tilde{E}_{0_T} = \left(\frac{2}{\alpha + \beta}\right) \tilde{E}_{0_I}.}$$

Fresnel's equations, for the case of polarization in the plane of incidence.

- Transmitted wave is in phase with the incident one.
- Reflected wave is either in phase, $\alpha > \beta$ or 180° out of phase, if $\alpha < \beta$.
- Amplitudes of the transmitted & reflected waves depend on the θ_I, as $\alpha \equiv \dfrac{\cos\theta_T}{\cos\theta_I}$.

$$\alpha = \frac{\sqrt{1 - \sin^2\theta_T}}{\cos\theta_I} = \frac{\sqrt{1 - [(n_1/n_2)\sin\theta_I]^2}}{\cos\theta_I}.$$

Normal incidence ($\theta_I = 0$), $\alpha = 1$.
Grazing incidence ($\theta_I = 90^\circ$), α diverges, the wave is totally reflected.

Electromagnetic Waves

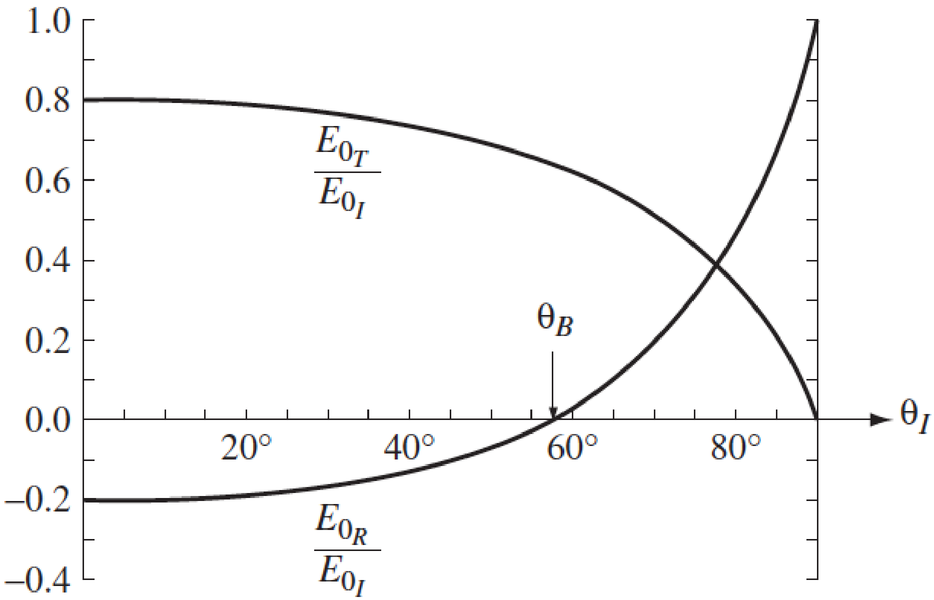
Brewster's Angle:

$$\tilde{E}_{0_R} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)\tilde{E}_{0_I}$$

Intermediate angle, θ_B (Brewster's angle),

Reflected wave is completely extinguished, $\alpha = \beta$, or $\sin^2\theta_B = \dfrac{1-\beta^2}{(n_1/n_2)^2 - \beta^2}$.

For $\mu_1 \cong \mu_2$, so $\beta \cong n_2/n_1$, $\sin^2\theta_B \cong \beta^2/(1+\beta^2)$, $\tan\theta_B \cong \dfrac{n_2}{n_1}$



Transmitted and reflected amplitudes as functions of θ_I, for light incident on glass (n_2=1. 5) from air (n_1=1)

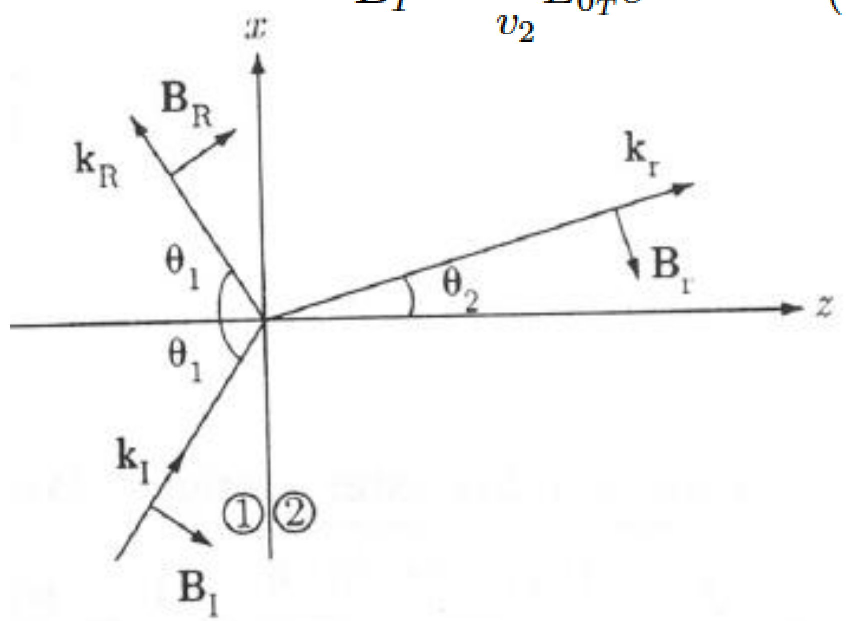
Electromagnetic Waves

$$\text{(i)}\ \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}, \quad \text{(iii)}\ \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel},$$

$$\text{(ii)}\ B_1^{\perp} = B_2^{\perp}, \qquad \text{(iv)}\ \tfrac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \tfrac{1}{\mu_2}\mathbf{B}_2^{\parallel}.$$

$$\begin{aligned} \tilde{\mathbf{E}}_R &= \tilde{E}_{0_R} e^{i(\mathbf{k}_R\cdot\mathbf{r}-\omega t)}\,\hat{\mathbf{y}}, \\ \tilde{\mathbf{B}}_R &= \frac{1}{v_1}\tilde{E}_{0_R} e^{i(\mathbf{k}_R\cdot\mathbf{r}-\omega t)}(\cos\theta_1\,\hat{\mathbf{x}} + \sin\theta_1\,\hat{\mathbf{z}}); \end{aligned}$$

$$\begin{aligned} \tilde{\mathbf{E}}_T &= \tilde{E}_{0_T} e^{i(\mathbf{k}_T\cdot\mathbf{r}-\omega t)}\,\hat{\mathbf{y}}, \\ \tilde{\mathbf{B}}_T &= \frac{1}{v_2}\tilde{E}_{0_T} e^{i(\mathbf{k}_T\cdot\mathbf{r}-\omega t)}(-\cos\theta_2\,\hat{\mathbf{x}} + \sin\theta_2\,\hat{\mathbf{z}}) \end{aligned}$$



$$\begin{aligned} \tilde{\mathbf{E}}_I &= \tilde{E}_{0_I} e^{i(\mathbf{k}_I\cdot\mathbf{r}-\omega t)}\,\hat{\mathbf{y}}, \\ \tilde{\mathbf{B}}_I &= \frac{1}{v_1}\tilde{E}_{0_I} e^{i(\mathbf{k}_I\cdot\mathbf{r}-\omega t)}(-\cos\theta_1\,\hat{\mathbf{x}} + \sin\theta_1\,\hat{\mathbf{z}}); \end{aligned}$$

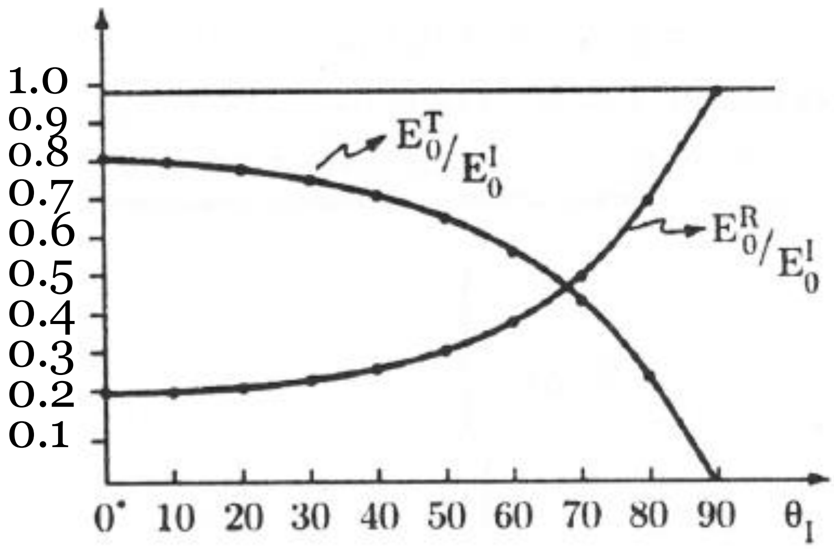
$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}. \ \text{[Note: } \mathbf{k}_I\cdot\mathbf{r} - \omega t = \mathbf{k}_R\cdot\mathbf{r} - \omega t = \mathbf{k}_T\cdot\mathbf{r} - \omega t, \text{ at } z = 0]$$

Electromagnetic Waves

Now,

$$\alpha\beta = \beta\frac{\sqrt{1-\sin^2\theta/\beta^2}}{\cos\theta} = \frac{\sqrt{\beta^2-\sin^2\theta}}{\cos\theta}$$, θ is the angle of incidence!

$$\beta = 1.5,\ \alpha\beta = \frac{\sqrt{2.25-\sin^2\theta}}{\cos\theta}$$



Electromagnetic Waves

Fresnel's equations:

$$E_{0_T} = \left(\frac{2}{1+\alpha\beta}\right) E_{0_I} \quad E_{0_R} = \left|\frac{1-\alpha\beta}{1+\alpha\beta}\right| E_{0_I}$$

At normal incidence, $\alpha = 1$,

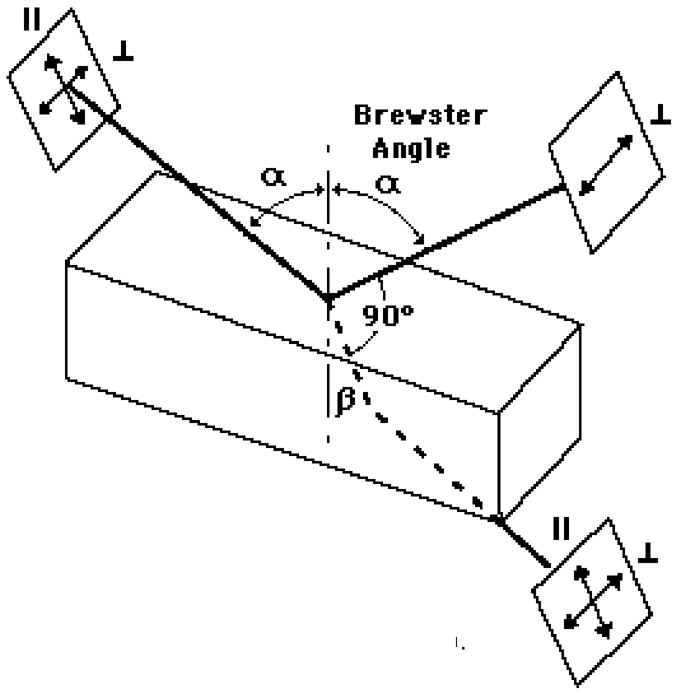
$$E_{0_T} = \left(\frac{2}{1+\beta}\right) E_{0_I} \qquad E_{0_R} = \left|\frac{1-\beta}{1+\beta}\right| E_{0_I}$$

The reflection and transmission coefficients:

$$R = \left(\frac{E_{0_R}}{E_{0_I}}\right)^2 = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2$$

$$T = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\alpha\left(\frac{E_{0_T}}{E_{0_I}}\right)^2 = \alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2$$

$\boxed{R + T = 1}$


II
⊥
Brewster
Angle
α
α
⊥
90°
β
II
⊥

Problem 9.18 The index of refraction of diamond is 2.42. Construct the graph analogous to Fig. 9.16 for the air/diamond interface. (Assume $\mu_1 = \mu_2 = \mu_0$.) In particular, calculate (a) the amplitudes at normal incidence, (b) Brewster's angle, and (c) the "crossover" angle, at which the reflected and transmitted amplitudes are equal.

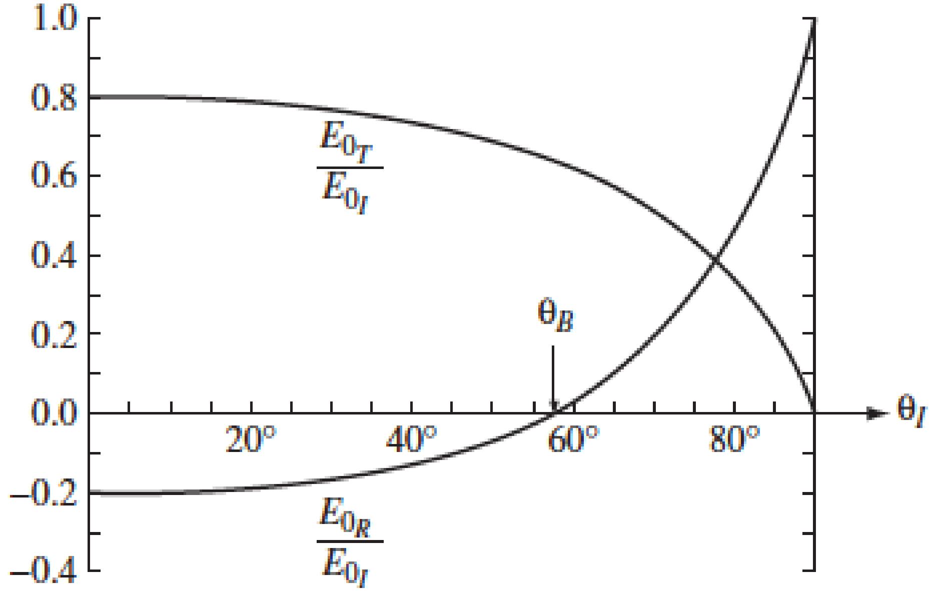
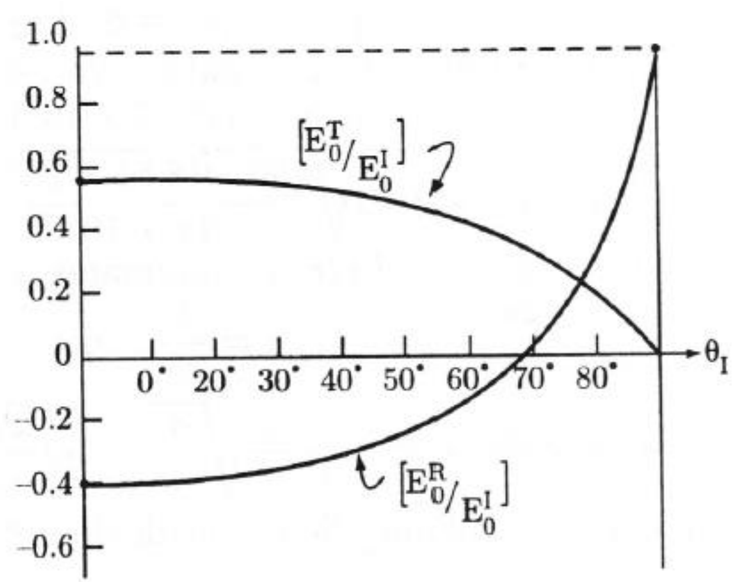


FIGURE 9.16

Problem 9.18 The index of refraction of diamond is 2.42. Construct the graph analogous to Fig. 9.16 for the air/diamond interface. (Assume $\mu_1 = \mu_2 = \mu_0$.) In particular, calculate (a) the amplitudes at normal incidence, (b) Brewster's angle, and (c) the "crossover" angle, at which the reflected and transmitted amplitudes are equal.

Solution: $$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1} = 2.42 \qquad \alpha = \frac{\sqrt{1-\sin^2\theta_T}}{\cos\theta_I} = \frac{\sqrt{1-[(n_1/n_2)\sin\theta_I]^2}}{\cos\theta_I} = \frac{\sqrt{1-(\sin\theta/2.42)^2}}{\cos\theta}.$$

Also,
$$\tilde{E}_{0_R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0_I}, \quad \tilde{E}_{0_T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0_I}.$$



Problem 9.18 The index of refraction of diamond is 2.42. Construct the graph analogous to Fig. 9.16 for the air/diamond interface. (Assume $\mu_1 = \mu_2 = \mu_0$.) In particular, calculate (a) the amplitudes at normal incidence, (b) Brewster's angle, and (c) the "crossover" angle, at which the reflected and transmitted amplitudes are equal.

Solution:
$$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1} = 2.42 \qquad \alpha = \frac{\sqrt{1-\sin^2\theta_T}}{\cos\theta_I} = \frac{\sqrt{1-[(n_1/n_2)\sin\theta_I]^2}}{\cos\theta_I}$$
$$= \frac{\sqrt{1-(\sin\theta/2.42)^2}}{\cos\theta}.$$

Also,
$$\tilde{E}_{0_R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0_I}, \quad \tilde{E}_{0_T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0_I}.$$

(a) $\theta = 0 \Rightarrow \alpha = 1.$
$$\left(\frac{E_{0_R}}{E_{0_I}}\right) = \frac{\alpha-\beta}{\alpha+\beta} = \frac{1-2.42}{1+2.42} = -\frac{1.42}{3.42} = \boxed{-0.415;}$$
$$\left(\frac{E_{0_T}}{E_{0_I}}\right) = \frac{2}{\alpha+\beta} = \frac{2}{3.42} = \boxed{0.585.}$$

(b) $$\tan\theta_B \cong \frac{n_2}{n_1} \qquad \theta_B = \tan^{-1}(2.42) = \boxed{67.5^\circ.}$$

(c) $E_{0_R} = E_{0_T} \Rightarrow \alpha - \beta = 2 \Rightarrow \alpha = \beta + 2 = 4.42;$
$$\sin\theta = 0.979; \ \boxed{\theta = 78.3^\circ.}$$

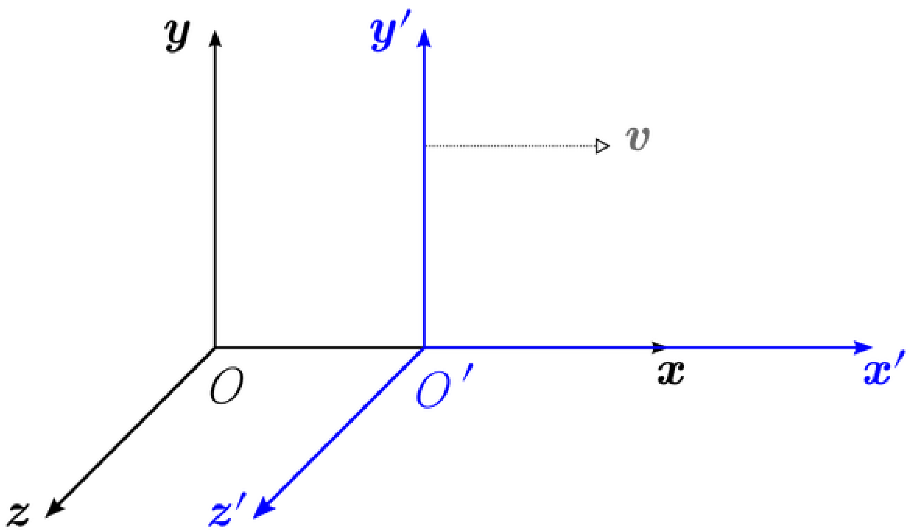
Special Theory of Relativity: Recap

Einstien proposed,

1. The principle of relativity: The laws of physics apply in all inertial reference systems.

2. The universal speed of light: speed of light in vacuum is the same for all inertial observers, regardless of the motion of the source.

Lorentz Transformation:



$$x' = \gamma\left(x - Vt\right)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{V}{c^2}x\right),$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}}\ .$$

Special Theory of Relativity: Recap

Transformation of Electromagnetic Fields

$$\bar{E}_x = E_x, \qquad \bar{E}_y = \gamma(E_y - vB_z), \qquad \bar{E}_z = \gamma(E_z + vB_y),$$

$$\bar{B}_x = B_x, \qquad \bar{B}_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right), \qquad \bar{B}_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right).$$

Fields written in components parallel & perpendicular to the velocity of the moving frame,

$$\mathbf{E} = \mathbf{E}_{\parallel} + \mathbf{E}_{\perp}, \qquad \mathbf{E}' = \mathbf{E}'_{\parallel} + \mathbf{E}'_{\perp},$$

$$\mathbf{B} = \mathbf{B}_{\parallel} + \mathbf{B}_{\perp}, \qquad \mathbf{B}' = \mathbf{B}'_{\parallel} + \mathbf{B}'_{\perp}.$$

The transformation of the parallel & perpendicular components,

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel} \qquad \mathbf{E}'_{\perp} = \gamma\left(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}_{\perp}\right)$$

$$\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel} \qquad \mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp} - (\mathbf{v}/c^2) \times \mathbf{E}_{\perp}\right)$$

Problem 12.44 In system $\mathcal{S}_0$, a static uniform line charge λ coincides with the z axis.

(a) Write the electric field $\mathbf{E}_0$ in *Cartesian* coordinates, for the point (x_0, y_0, z_0).

(b) Use Eq. 12.109 to find the electric in $\mathcal{S}$, which moves with speed v in the x direction with respect to $\mathcal{S}_0$. The field is still in terms of (x_0, y_0, z_0); express it instead in terms of the coordinates (x, y, z) in $\mathcal{S}$.

Solution:

(a) $\mathbf{E} = \dfrac{\lambda}{2\pi\epsilon_0}\dfrac{\hat{\mathbf{s}}}{s} = \boxed{\dfrac{\lambda}{2\pi\epsilon_0}\dfrac{x_0\,\hat{\mathbf{x}} + y_0\,\hat{\mathbf{y}}}{(x_0^2+y_0^2)}}.$

(b) $\bar{E}_x = E_x = \dfrac{\lambda}{2\pi\epsilon_0}\dfrac{x_0}{(x_0^2+y_0^2)}$, $\bar{E}_y = \gamma E_y = \gamma\dfrac{\lambda}{2\pi\epsilon_0}\dfrac{y_0}{(x_0^2+y_0^2)}$, $\bar{E}_z = \gamma E_z = 0$, $\bar{\mathbf{E}} = \boxed{\dfrac{\lambda}{2\pi\epsilon_0}\dfrac{(x_0\,\hat{\mathbf{x}} + \gamma y_0\,\hat{\mathbf{y}})}{(x_0^2+y_0^2)}}.$

$$\bar{E}_x = E_x, \qquad \bar{E}_y = \gamma(E_y - vB_z), \qquad \bar{E}_z = \gamma(E_z + vB_y),$$

$$\bar{B}_x = B_x, \qquad \bar{B}_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right), \qquad \bar{B}_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right).$$