# PH 101: Physics I

Module 3: Introduction to Quantum Mechanics

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# Contents

**Quantum Mechanics:** **Two-slit experiment. De Broglie’s hypothesis. Uncertainty Principle, wave function and wave packets, phase and group velocities. Schrödinger Equation. Probabilities and Normalization. Expectation values. Eigenvalues and eigenfunctions.**

**Applications in one dimension: Particle in a box, Finite Potential well, Harmonic oscillator.**

**Text / References:**


1. **R. Eisberg and R. Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles,* 2nd Ed., John-Wiley, 1985.**
2. **E. W. Wichman, *Quantum Physics*, Berkeley Physics Course Vol. 4, Tata McGraw Hill (Indian Edition), 2008**
3. **D. J. Griffiths, *Introduction to Quantum Mechanics,* Pearson, 2005.**
4. **R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics, Vol. III*, Pearson, 2006.**
5. **A. Beiser, *Concepts of Modern Physics,* Tata McGraw-Hill, New Delhi, 1995.**
6. **S. Gasiorowicz, *Quantum Physics,* John Wiley (Asia), 2000.**

# Classical Mechanics


**Newton, Sir Isaac, PRS, (1643 – 1727), English physicist and mathematician**

$$\vec{F} = m\frac{d^2\vec{x}}{dt^2}$$

$$\vec{F} = -G\frac{m_1 m_2}{|\vec{x}-\vec{x}'|^3}(\vec{x}-\vec{x}')$$

$$G = 6.6726 \times 10^{-11}\mathrm{m}^3\cdot\mathrm{s}^{-2}\cdot\mathrm{kg}^{-1}$$

$$g = 9.8067\mathrm{m}\cdot\mathrm{s}^{-2}$$

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{f, H\}$$

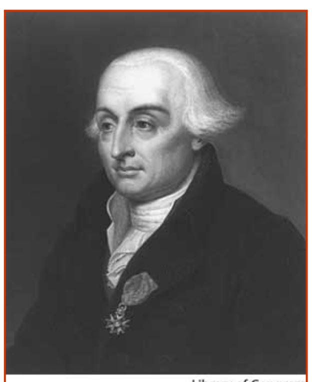
$$H(q,p;t) = \sum_i p_i\dot{q}_i - L(q,\dot{q};t)$$

$$p = \frac{\partial L}{\partial \dot{q}}$$

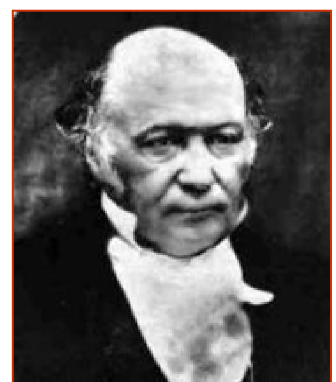
$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

**Euler, Leonhard (1707 -- 1783), Swiss mathematician.**




**Lagrange, Joseph Louis (1736 -- 1813), Italian-French mathematician, astronomer and physicist.**



**Hamilton, William Rowan (1805 -- 1865), Irish mathematician and astronomer.**

# Classical Electrodynamics

Coulomb, Charles Augustin (1736 – 1806), French physicist


Biot, Jean Baptiste (1774 --1862), French Physicist;
Savart, Félix (1791 -- 1841),
French Physicist

b / André Marie Ampère (1775-1836).

Ampere, Andre Marie (1775 -- 1836), French Physicist

h / Michael Faraday (1791-1867), the son of a poor blacksmith, discovered induction experimentally.

Faraday, Michael (1791 -- 1867), English Physicist

**Hendrik Antoon Lorentz**

1/2 of the prize

the Netherlands

Leiden University
Leiden, the Netherlands

b. 1853
d. 1928

Lorentz, Hendrik Antoon (1853 -- 1928), Dutch Physicist


$$\nabla \cdot \vec{E}(\vec{x},t) = 4\pi\rho(\vec{x},t),$$

$$\nabla \cdot \vec{B}(\vec{x},t) = 0,$$

$$\nabla \times \vec{E}(\vec{x},t) = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}(\vec{x},t),$$

$$\nabla \times \vec{B}(\vec{x},t) = \frac{4\pi}{c}\vec{j}(\vec{x},t) + \frac{1}{c}\frac{\partial}{\partial t}\vec{E}(\vec{x},t),$$

Maxwell, James Clerk (1831 – 1879), **Scottish** physicist

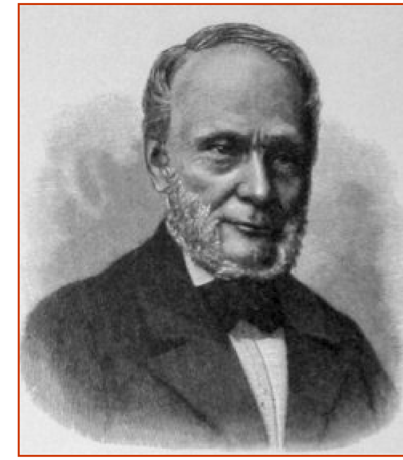
# Classical Physics of collection of large number of particles

Dalton, John (1766 -- 1844), British chemist and physicist.

Carnot, Nicolas Léonard Sadi (1796 -- 1832), French physicist.

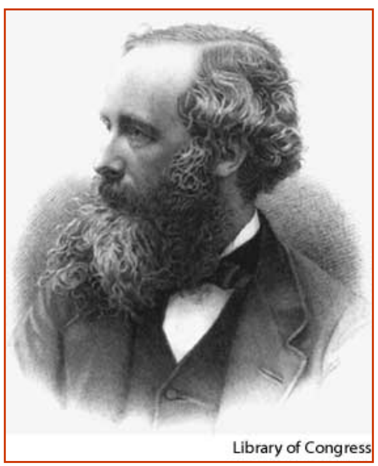
Joule, James Prescott (1818 -- 1889), British physicist.

Helmholtz, Hermann Ludwig Ferdinand von (1821 -- 1894), German physicist and physician.



Clausius, Rudolf Julius Emanuel (1822 -- 1888), German mathematical physicist.


Thomson, William (Baron Kelvin) (1824 - 1907), British physicist and mathematician.

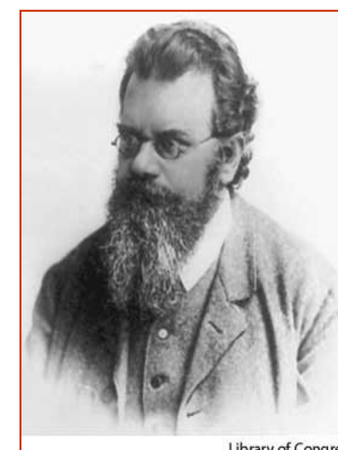



Maxwell, James Clerk (1831 – 1879), Scottish physicist

Lord and Lady Kelvin at the coronation of King Edward VII in 1902.

Boltzmann, Ludwig, (1844 – 1906),

Austrian physicist.



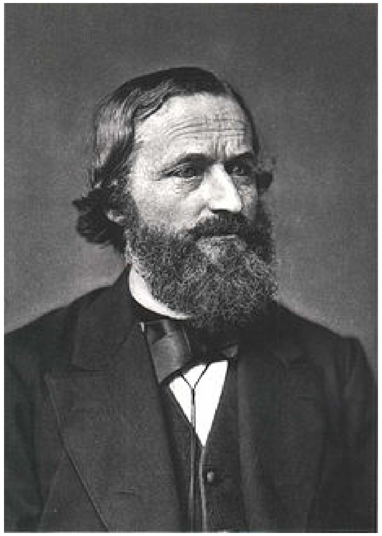
"At this stage, there is nothing new to be discovered in
Physics now, because all the
Dynamical properties of matter and field can be understood using the
Classical Physics.
It was understood that if we know the initial conditions the future state of
a system can be precisely predicted.

All that remains is more and more precise measurement."

# Two revolutionary experiments in the late 19th century

Michelson, Albert

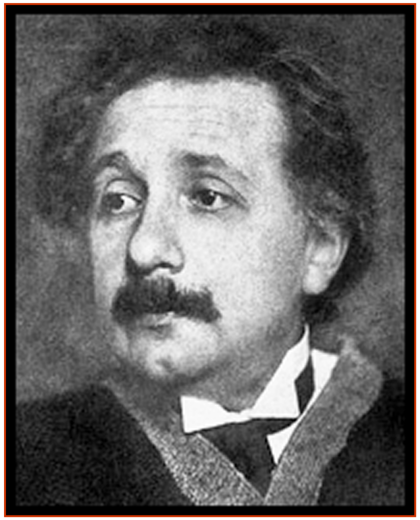
Morley, Edward



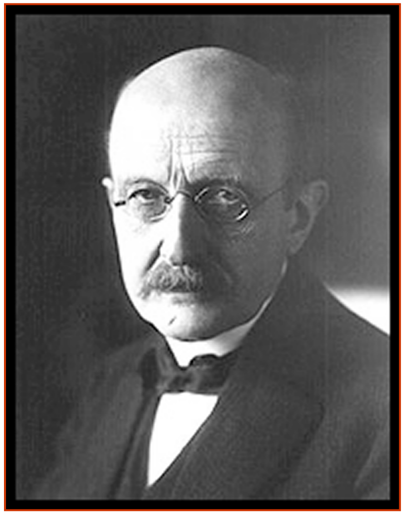
Gustav Kirchhoff

**Michelson-Morley Experiment (1887)**

**Ultraviolet catastrophy in blackbody radiation (before October, 1900)**



Einstein, Albert



Planck, Max

# Necessity of a new Theory

At this stage, several experimental observations could not be explained using the already existing classical theory.

Black body radiation

Photo electric effect

Atomic line spectra

# Blackbody Radiation

All material objects(bodies) absorb and emit radiations simultaneously.

At thermal equilibrium the body must absorb and emit thermal energy at the same rate.

A blackbody is a material which abosorbs all the radiation falling on it and reflects none.

A perfect black body is one which also radiate all the radiation abosrbed.

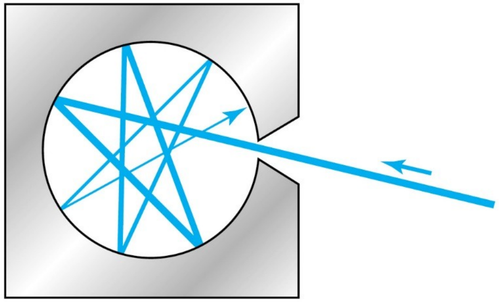
The interesting properties of blackbody radiation is that it is independent of materials.

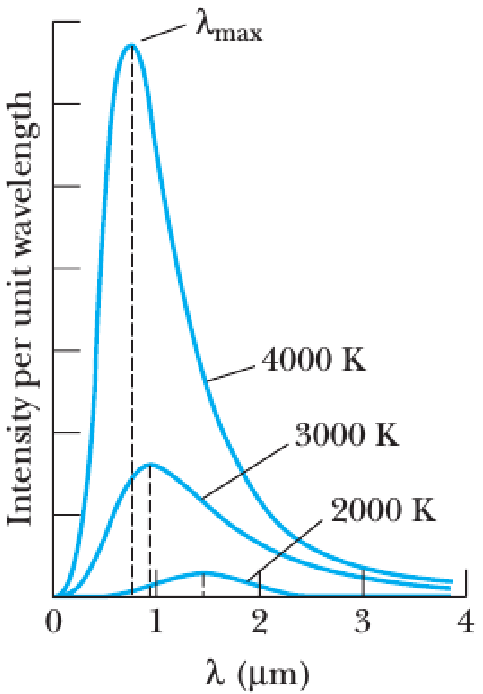
Continuous spectrum.

Can be assumed as a cavity with a hole.

The radiation which enters though the whole gets reflected several times inside the cavity and finally gets absorbed. A small fraction of that may get reemitted through the hole.

At thermal equilibrium this should also be an perfect emitter of radiation.



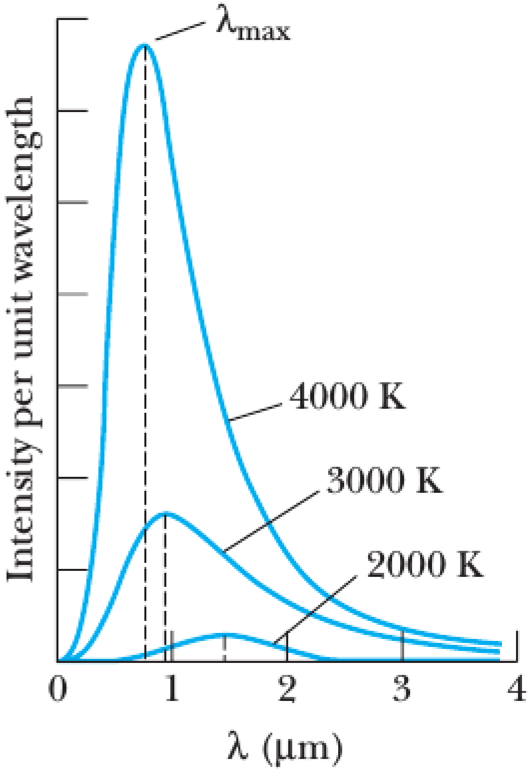



Intensity vs Wavelength
(Spectral distribution)
Intensity=Total power radiated per unit area per unit wavelength at a given temperature.

# Blackbody Radiation

**Observations:**

--The maximum of the distribution shifts to smaller wavelengths as the temperature is increased.
--The total power radiated increases with the temperature.




Wien's displacement law:

The wave length $\lambda_{max}$ at the maximum power varies inversely with temperature (T).

$$\lambda_{max}T = 2.898 \times 10^{-3} m.K$$

The proper formula for the power per unit area at temperature T was given by Stefan and Boltzmann as:

$$R(T) \propto \sigma T^4$$

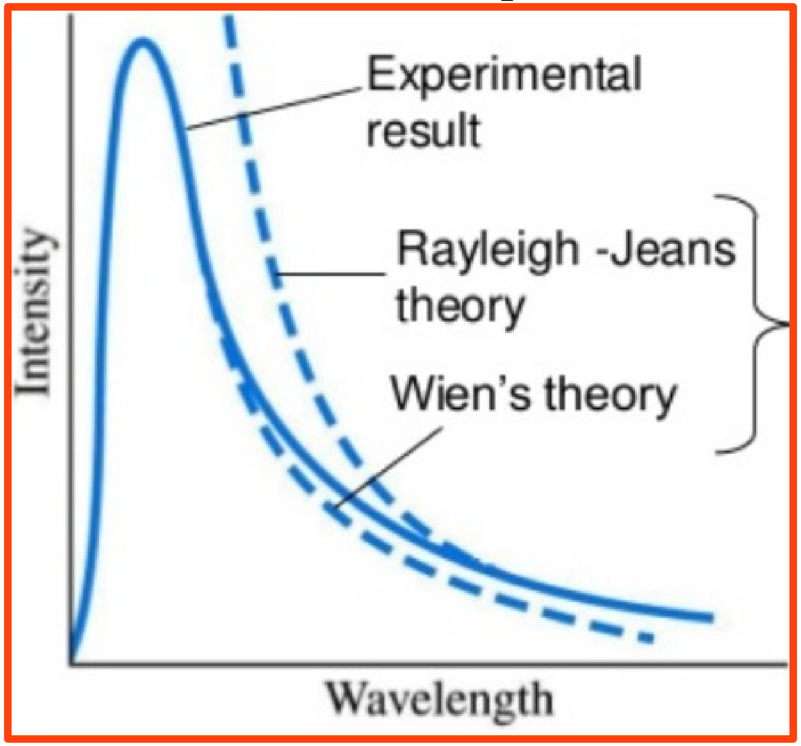
Stefan-Boltzmann constant

# Blackbody Radiation

Lord Rayleigh and James Jeans used the classical theories of electromagnetism and thermodynamics to show that the blackbody spectral distribution can be represented as

$$I(\lambda,\ T) = \frac{2\pi ckT}{\lambda^4}$$

Rayleigh-Jeans formula




Classical Result

This explains the experimental data only in the limit of large wavelength.

When $\lambda \longrightarrow 0$ the above formula diverges.

This is called the ultraviolet catastrophe

Wien's proposed the spectral distribution as

$$I(\lambda, T) \propto \frac{\exp\left(-\frac{hc}{kT\lambda}\right)}{\lambda^3}$$

This is consistent with experimentalo data for small wavelength.

Rayleigh-Jeans and Wien's formula for spectral distribution were purely based on the Classical Physics and able to explain the experimental observation in the extreme limits of the wavelegth.

Around 1900, this failure indicated that the electromagnetic radiation may not be just a wave!

A correct spectral distribution was necessary to explain the blackbody radiation....

# Blackbody Radiation: Planck's distribution formula

Max Planck had the best possible experimental data for the blackbody radiation over a broad range of wavelengths and used the method of curve fitting to obtain the following formula that fitted the best.

$$I(\lambda,\ T) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} \qquad h = 6.6261 \times 10^{-34}\,\mathrm{J\cdot s}$$

To provide a theoretical explanation to this he assumed that the radiation in the cavity was emitted by some sort of oscillators that are contained in the wall of the cavity.

The oscillators can have certain discrete energies determined by *E=nhf*, where n is an integer and h is the called the Plank's constant.

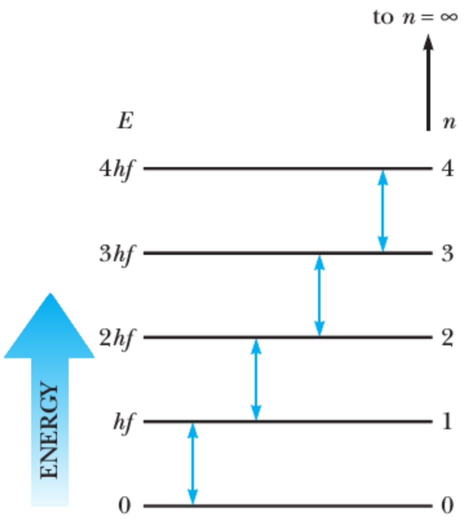
The oscillators can absorb or emit only in discrete multiples of the quanta of energy given by :

$$E = hf$$

Allowed energy levels according to Planck's hypothesis for an oscillator with frequency "f" .

Allowed transitions are indicated by the double headed arrows.

*At this point energy can be zero, which will we see is incorrect!*



# Blackbody Radiation: Planks distribution formula

$$I(\lambda,\ T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

$$\frac{1}{e^{hc/\lambda kT} - 1} = \frac{1}{\left[1 + \frac{hc}{\lambda kT} + \left(\frac{hc}{\lambda kT}\right)^2 \frac{1}{2} + \cdots\right] - 1} \rightarrow \frac{\lambda kT}{hc}$$

for large $\lambda$

$$I(\lambda,\ T) = \frac{2\pi c^2 h}{\lambda^5} \frac{\lambda kT}{hc} = \frac{2\pi ckT}{\lambda^4}$$

Rayleigh-Jean's Formula

For small wavelength $\lambda \to 0$

$$I(\lambda, T) \propto \frac{\exp\left(-\frac{hc}{kT\lambda}\right)}{\lambda^5}$$

Wien's formula

# Photoelectric Effect

Photoelectric effect: Incident light (electromagnetic radiation) shining on the material transfers energy to the electrons, allowing them to escape.

Electromagnetic radiation interacts with electrons within metals and gives the electrons increased kinetic energy. Light can give electrons enough extra kinetic energy to allow them to escape. We call the ejected electrons **photoelectrons**.

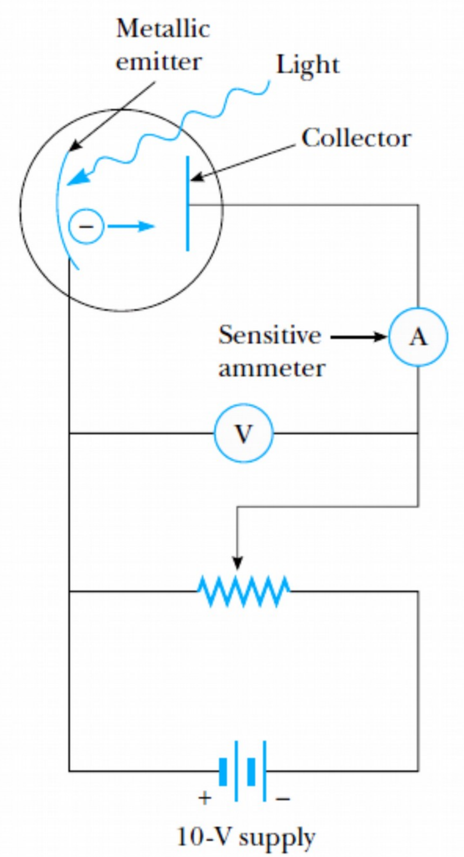



**Figure 3.14** Photoelectric effect apparatus.

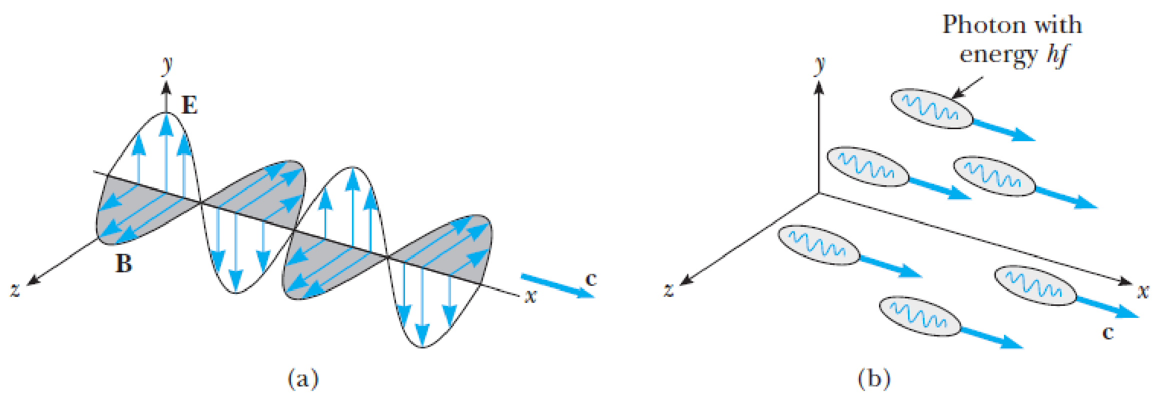



**Figure 3.16** (a) A classical view of a traveling light wave. (b) Einstein's photon picture of "a traveling light wave."

**Table 3.1 Work Functions of Selected Metals**

| Metal | Work Function, $\phi$, (in eV) |
|---|---|
| Na | 2.28 |
| Al | 4.08 |
| Cu | 4.70 |
| Zn | 4.31 |
| Ag | 4.73 |
| Pt | 6.35 |
| Pb | 4.14 |
| Fe | 4.50 |

The energy needed to get electron out of a metal is known as work function.

http://www.walter-fendt.de/ph14e/photoeffect.htm

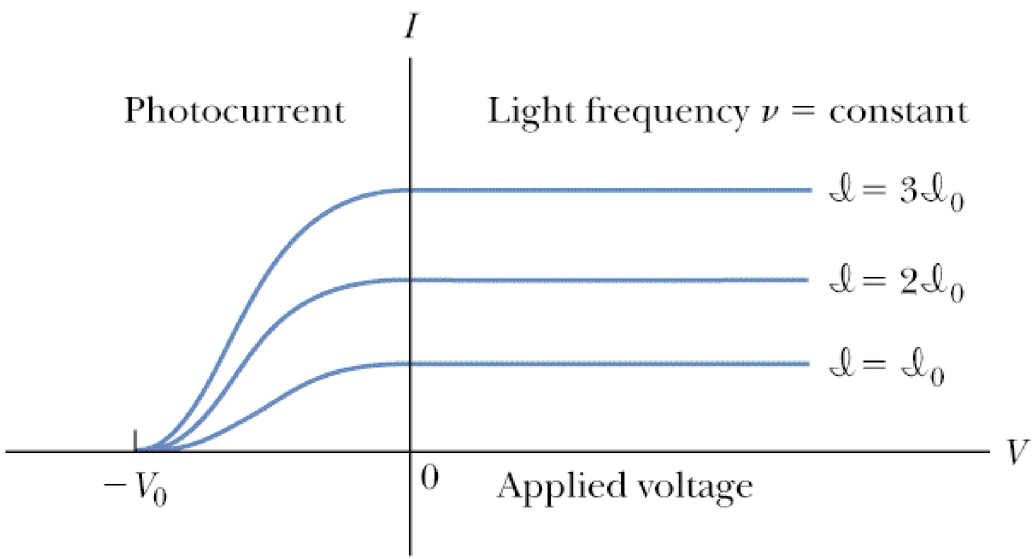
# Photoelectric Effect

**Experimental observations:**

1) The kinetic energies of the photoelectrons are independent of the light intensity.
2) Only sufficiently energetic light makes an effect, there can be an enormous intensity of light that is not sufficiently energetic and no effect is observed.
3) The maximum kinetic energy of the photoelectrons, for a given emitting material, depends only on the frequency of the light.
4) The smaller the work function ($\varphi$) of the emitter material, the smaller is the threshold frequency of the light that can eject photoelectrons.
5) However, the number of photoelectrons are produced is proportional to the intensity of light.
6) The photoelectrons are emitted almost instantly following illumination of the emitter, independent of the intensity of the light.

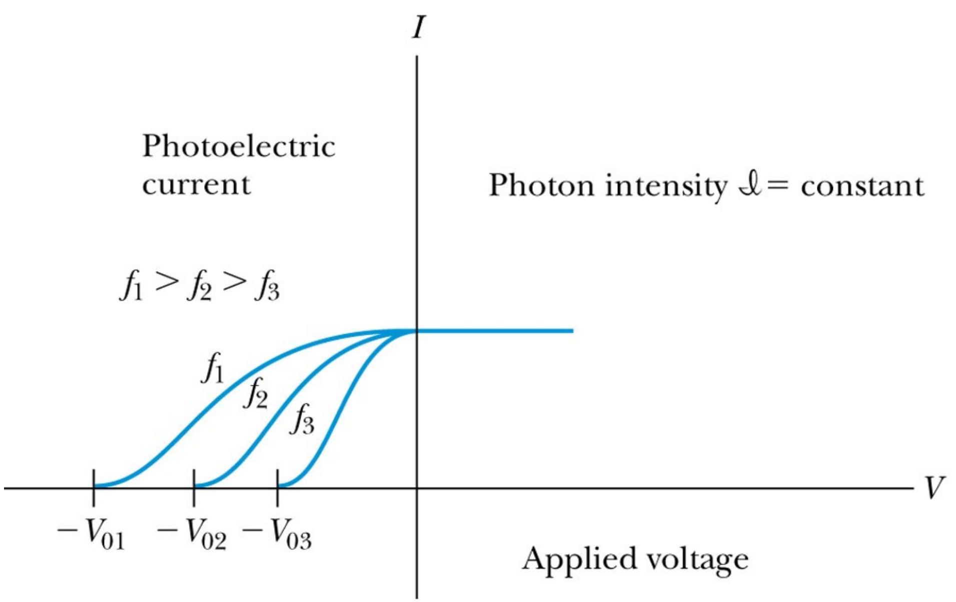
# Photoelectric Effect

Thornton/Rex, Modern Physics for Scientists and Engineers, 2/e
Figure 3.11





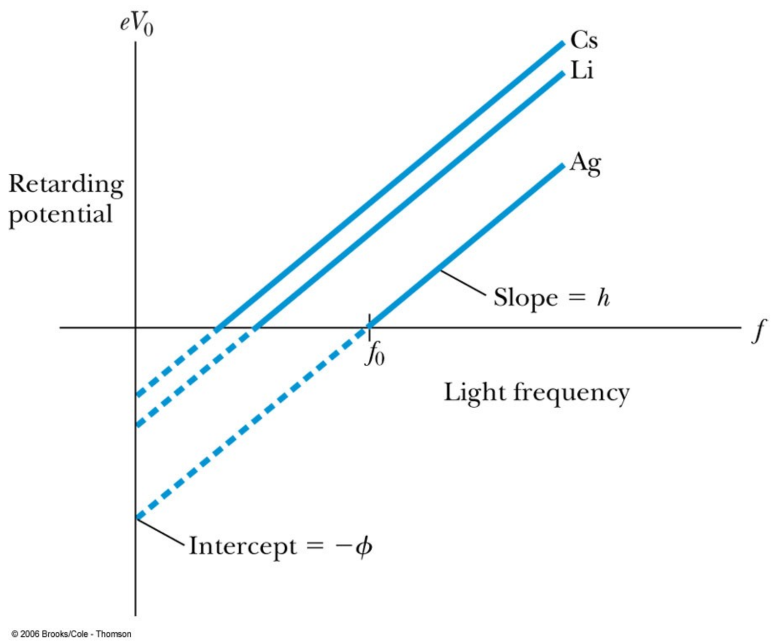

Same emitter material ($V_0$), same kind of light (f or ν) with 1, 2, and 3 fold intensity, **photocurrent intensity depends on incoming light intensity if sufficiently energetic light has been used.**





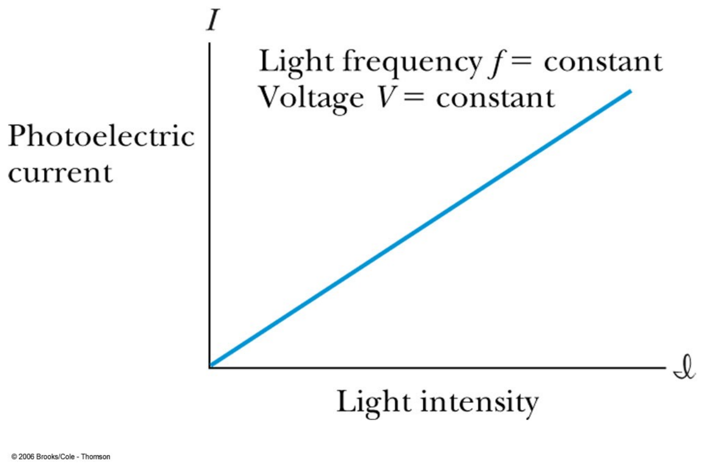

Three different emitter materials, three different kinds of light (*f*) are required to get a photocurrent

# Photoelectric Effect




$$\frac{1}{2}mv_{max}^2 = eV_0 = hf - \phi$$

**Different materials have different work functions**, i.e. amounts of energy required to allow an electron to escape a metal block, note that the slope of all of these curves is just h




Photocurrent is linear function of light intensity if it is sufficiently energetic to overcome the work function.

# Photoelectric Effect

1) The existence of a threshold frequency is completely inexplicable in classical theory.
2) Classical theory predicts that the total amount of energy in a light wave increases as the light intensity increases.
3) Classical theory would predict that for extremely low light intensities, a long time would elapse before any one electron could obtain sufficient energy to escape. We observe, however, that the photoelectrons are ejected almost immediately.
4) Conclusion: light can not be simple a wave, must be something else, a stream of particles (as thought by Newton in deriving geometric (ray-) optic).

Einstein suggested that the electromagnetic radiation field is quantized into particles called **photons**. Each photon possesses the quantum of total energy (which is all kinetic as they do not have rest mass ):

$$E = hf = \frac{hc}{\lambda}$$

h: Planck's constant and $\nu$ is the frequency of the light wave.

Energy before (photon) = energy after (electron)

$$hf = \phi + \text{K.E. (electron)}$$

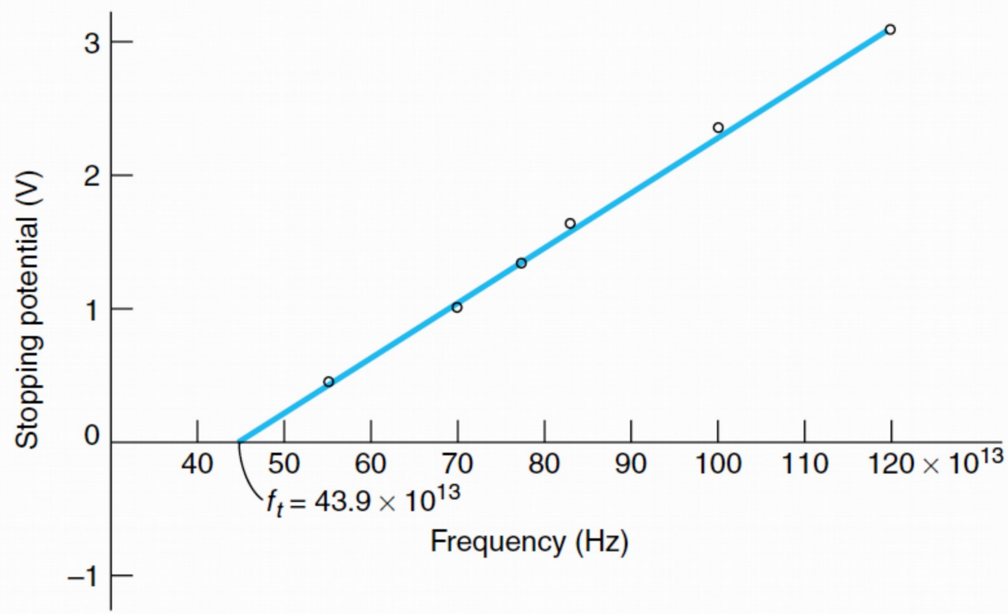
$$hf = \phi + \frac{1}{2}mv_{\max}^2 \qquad eV_0 = \frac{1}{2}mv_{\max}^2$$

# Photoelectric Effect

Einstein in 1905 predicted that the stopping potential was linearly proportional to the light frequency, with a slope *h*, the same constant found by Planck.

$$eV_0 = \frac{1}{2} m v_{\max}^2 = hf - hf_0 = h(f - f_0)$$

Millikan's data for stopping potential versus frequency for the photoelectric effect. The data fall on a straight line with slope $h/e$, as predicted by Einstein a decade before the experiment. The intercept on the stopping potential axis is $-\phi/e$. [*R. A. Millikan, Physical Review, 7, 362 (1915).*]




This suggests the light wave can be envisaged to be made of small packets of energies knows as light quanta (Photon)!

Nobel Prize for A. Einstein, 1921 (none for his other great achievements as people had difficulty with special relativity for a long time). Also Nobel Prize for Milikan in 1923 for experiment.

# Photoelectric Effect: Summary

1. In **1839**, ***Alexandre Edmond Becquerel*** observed the photoelectric effect via an electrode in a conductive solution exposed to light.

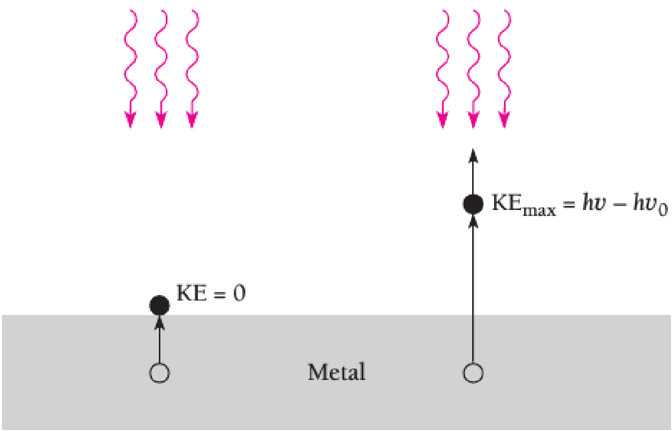
2. In **1873**, ***Willoughby Smith*** found that selenium is photoconductive.

3. In **1887**, ***Heinrich Hertz*** made observations of the photoelectric effect and of the production and reception of electromagnetic (EM) waves.

4. In **1899**, ***Joseph John Thomson* (N)** investigated ultraviolet light in Crookes tubes.

5. In **1901**, ***Nikola Tesla*** received the U.S. Patent 685957 (Apparatus for the Utilization of Radiant Energy) that describes radiation charging and discharging conductors by "radiant energy".

6. In **1902**, ***Philipp von Lenard* (N)** observed the variation in electron energy with light frequency.



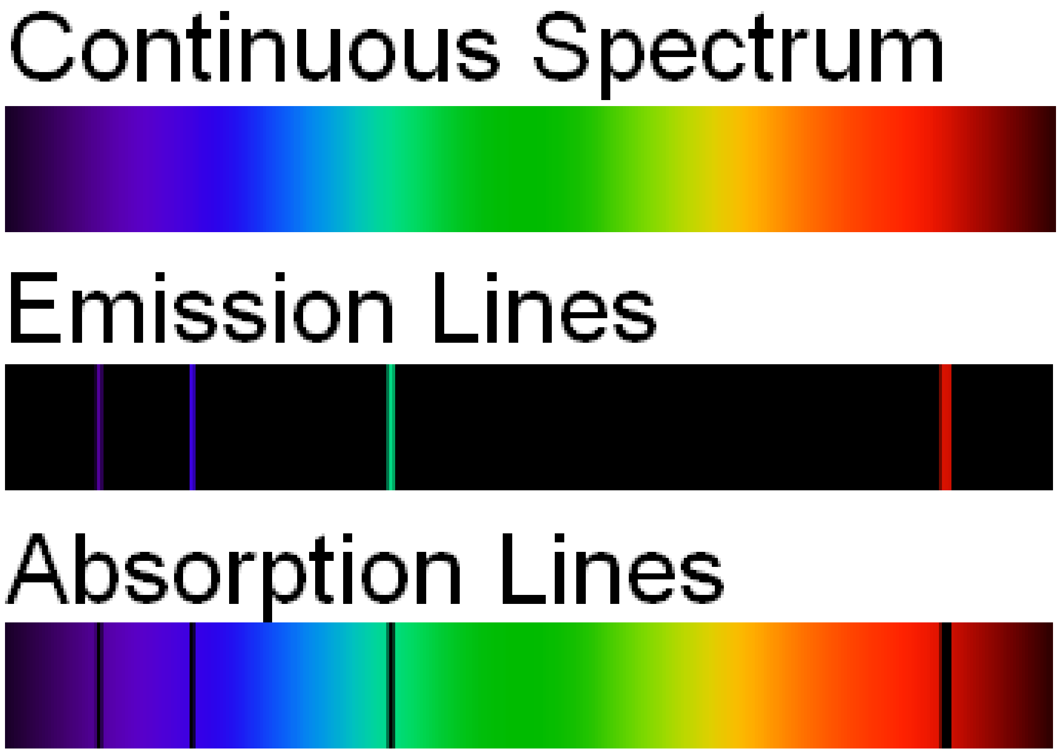

$$eV_0 = \frac{1}{2}mv_{max}^2 = hf - hf_0 = h(f - f_0)$$

In **1905**, ***Albert Einstein* (N)** proposed the well-known Einstein's equation for photoelectric effect.

In **1916**, ***Robert Andrews Millikan* (N)** finished a decade-long experiment to confirm Einstein's explanation of photoelectric effect.

# Atomic Line Spectra

Chemical elements were observed to produce unique wavelengths of light when burned or excited in an electrical discharge.




**dense material**, i.e. hot "black body", big- bang background radiation, sun if one does not look too carefully.

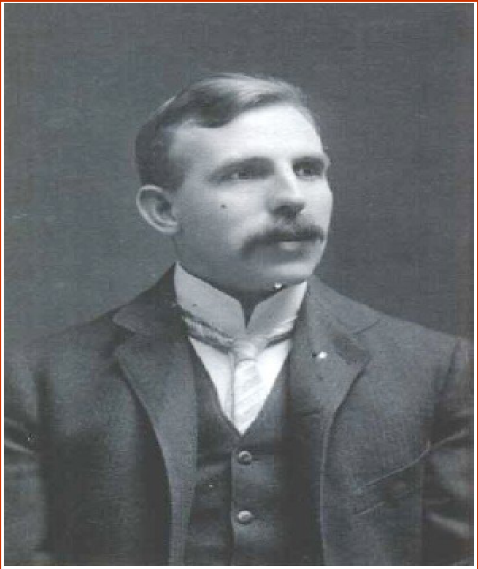
As the white light (with continuous spectrum) passes through an absorbing layer of atomic elements and analyzed with spectrograph a discrete set of emission lines are observed.

For every absorption line, there is an emission line, but **typically** not the other way around.

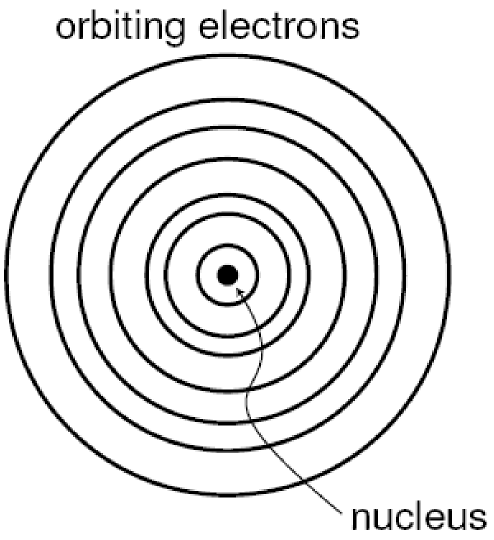
This shows the discrete nature of the atomic transition, i.e., an atom can absorb or emit the EM radiation in the selective manner!

# Explannation of Line Spectra using Classical picture of atom

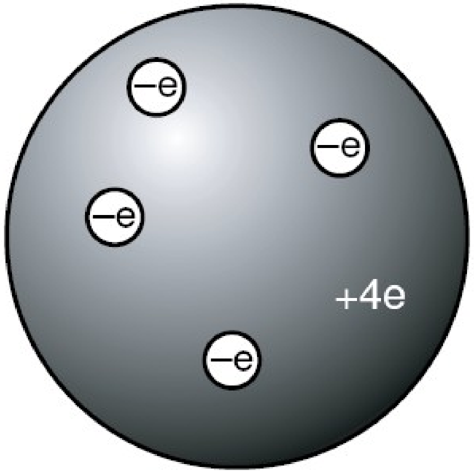
j / J.J. Thomson in the lab.



e / Ernest Rutherford (1871-1937).

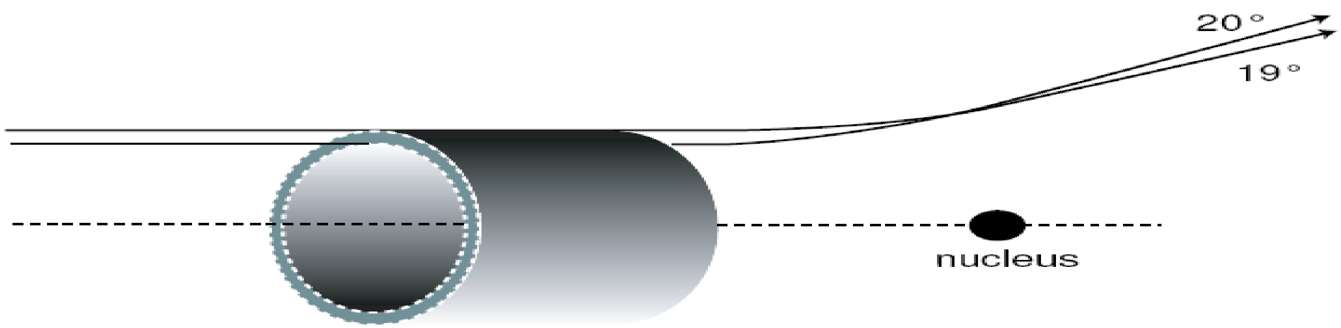



h / The planetary model of the atom.



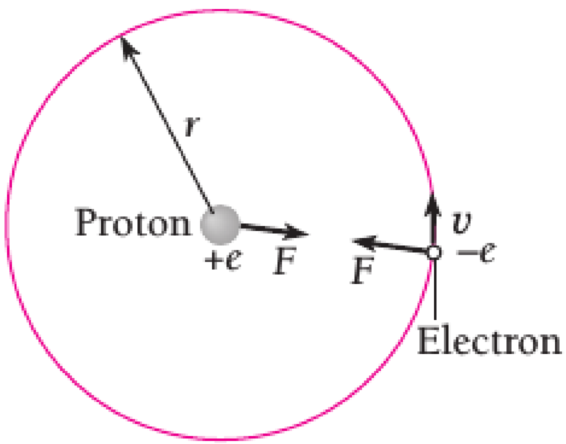

l / The raisin cookie model of the atom with four units of charge, which we now know to be beryllium.

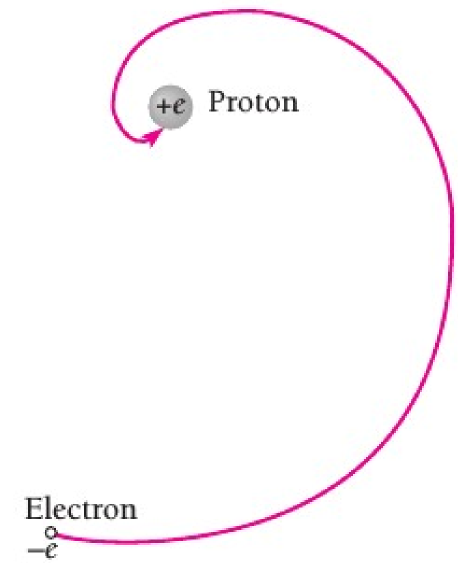
***Nuclear atom model*** **(1911):** ***Ernest Rutherford***



# Explannation of Line Spectra using Classical picture of atom

## Classical physics: atoms should collapse!

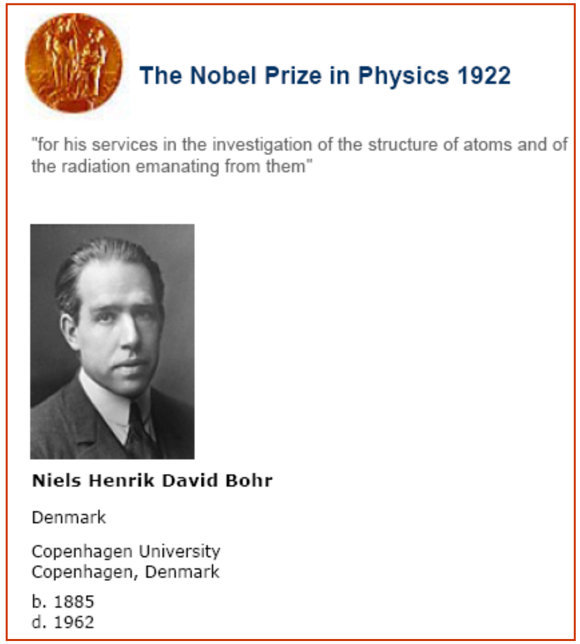



**<u>Classical Electrodynamics:</u> charged particles radiate EM energy (photons) when their velocity vector changes (e.g. they accelerate).**

**This means an electron should fall into the nucleus.**

New mechanics is needed!




**Bohr, Niels Henrik David (1885 -- 1962), Danish physicist.**

**_Bohr's model_ of atomic structure, 1913**

**The electron's _orbital angular momentum_ is _quantized_**

$$\mathbf{L} = n \cdot \hbar = n \cdot \frac{h}{2\pi}$$

$$E_n = \frac{-13.6 \text{ eV}}{n^2} = \frac{-m_e q_e^4}{8h^2\epsilon_0^2}\frac{1}{n^2}$$

$$E = E_i - E_f = \frac{m_e e^4}{8h^2\epsilon_0^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = \frac{m_e e^4}{8ch^3\epsilon_0^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

**The theory predicts that _electrons travel in discrete orbits around the atom's nucleus_, with the chemical properties of the element being largely determined by the number of electrons in each of the outer orbits**

**The idea that an electron could _drop_ from a _higher-energy orbit to a lower one_, _emitting a photon_ (light quantum) of discrete energy (this became the basis for quantum theory).**

We shall continue in the next class