Lecture 8

Sinusoidal Forcing Function,

Phasor Relationship for R, L and C,

Impedance and Admittance,

Phasor Diagram

Why not only ac?

Storage, transmission loss, waste of power

Why not only dc?

Generation, dc/ac/dc conversion

Hybrid systems using ac+dc

Projector, audio system

- Why RMS of ac?

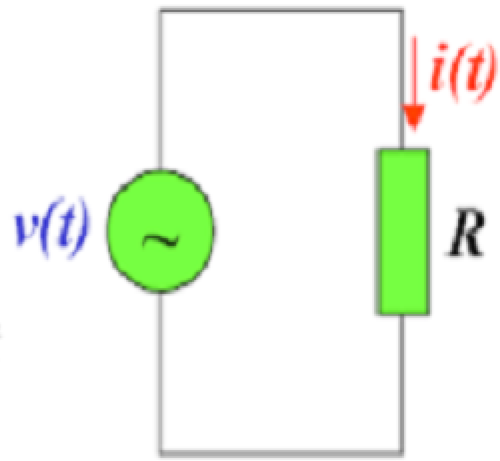
Average captures no information

Peak captures limited information

Sinusoidal Response of a Resistor

$v(t) = V_m \sin\omega_0 t$

$i(t) = I_m \sin\omega_0 t$



$$I_m = \frac{V_m}{R}$$

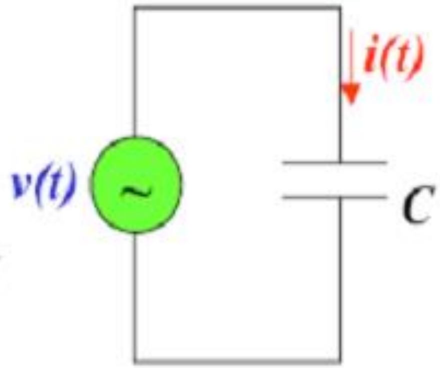
Current is also sinusoidal.

Has the same angular frequency, w_o

Current and voltage of a resistor is in phase $\longrightarrow$ $\phi = 0$

Current magnitude is related to voltage magnitude by $I_m = \dfrac{V_m}{R}$

Sinusoidal Response of a Capacitor



$$i(t) = I_m \sin(\omega_o t + \phi)$$

$$v(t) = V_m \sin \omega_o t$$

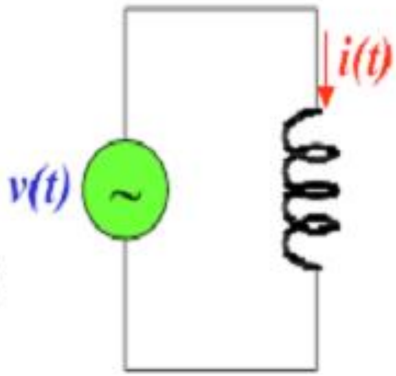
$$i(t) = C\frac{dv(t)}{dt}$$

$$i(t) = \omega_o C V_m \cos \omega_o t$$

$$i(t) = \omega_o C V_m \sin(\omega_o t + 90)$$

$$I_m = \omega_o C V_m$$

Sinusoidal Response of an Inductor



$$i(t) = I_m \sin(\omega_o t + \phi)$$

$$v(t) = V_m \sin \omega_o t$$

$$i(t) = \frac{1}{L}\int v(t)dt = -\frac{1}{\omega_o L} V_m \cos \omega_o t$$

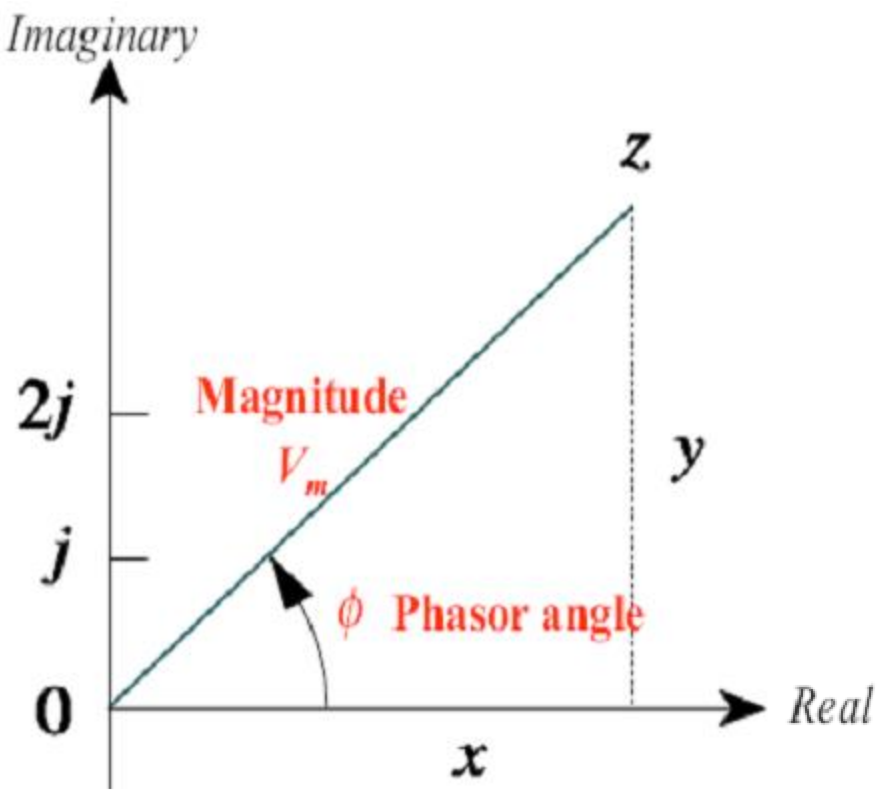
$$i(t) = -\frac{1}{\omega_o L} V_m \sin(\omega_o t + 90)$$

$$i(t) = \frac{1}{\omega_o L} V_m \sin(\omega_o t - 90)$$

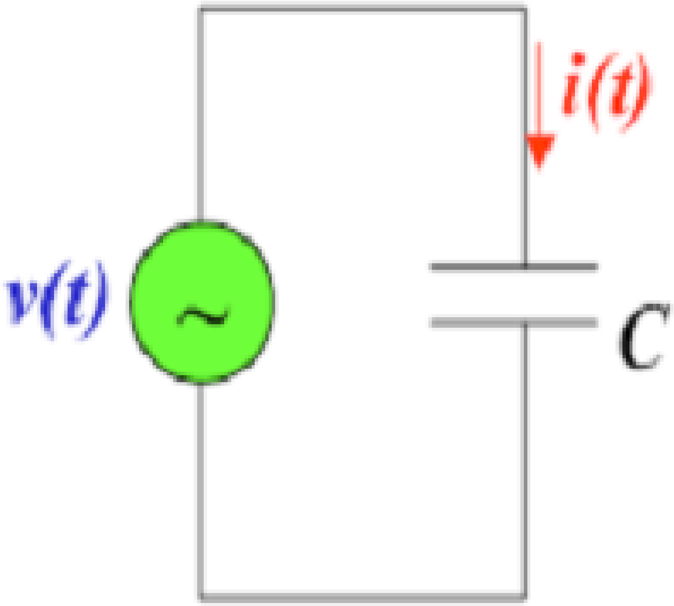
Phasor Plot

$$v(t) = V_m \cos(\omega t + \phi) = \mathrm{Re}\{V_m e^{j(\omega t+\phi)}\} = \mathrm{Re}\{\mathrm{V} e^{j\omega t}\}$$

$$\boxed{\mathrm{V} = V_m e^{j\phi} = V_m \angle \phi}$$



Phasor Response of a Capacitor



If $v(t) = V_m e^{j\omega_o t}$

$$v(t) = V_m \angle 0$$

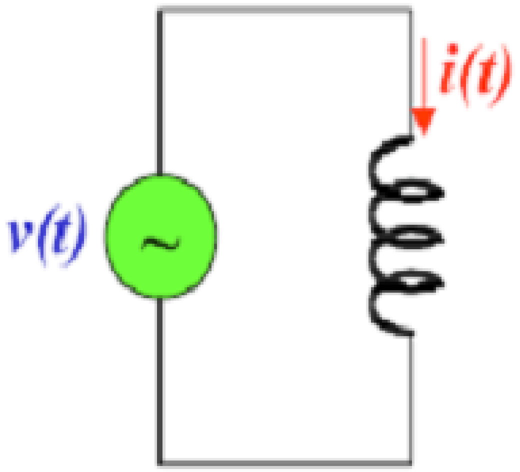
then $i(t) = C\dfrac{dv(t)}{dt}$

$$i(t) = j\omega_o C V_m e^{j\omega_o t}$$

$$= I_m \angle 90^0$$

$$I_m = \omega_0 C V_m$$

Phasor Response of an Inductor



If $v(t) = V_m e^{j\omega_o t}$

$$v(t) = V_m \angle 0$$

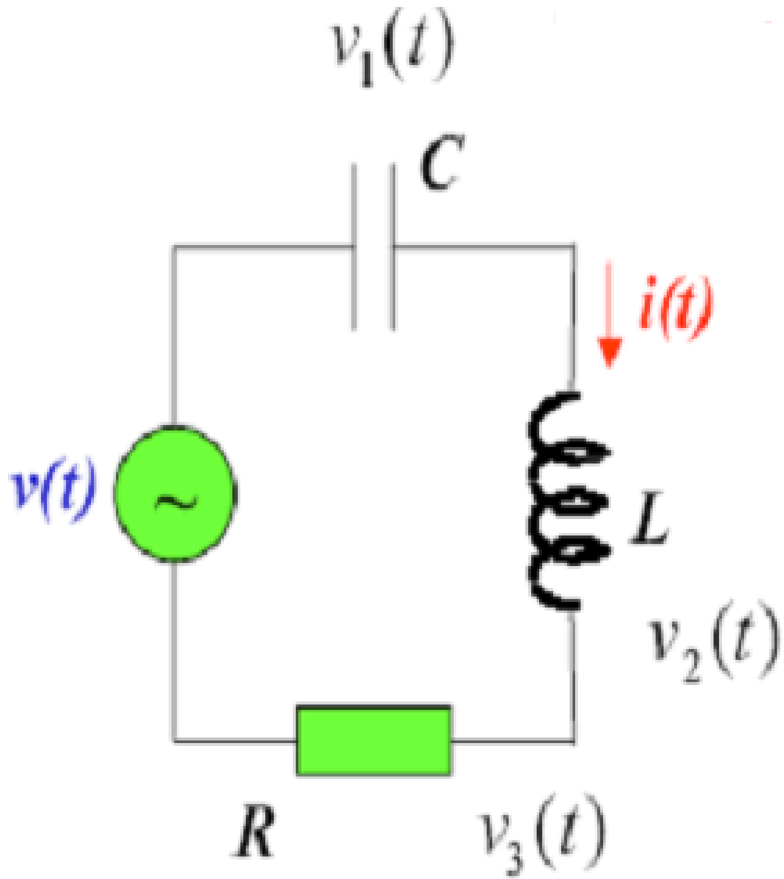
then

$$i(t) = \frac{1}{L}\int v(t)\,dt$$

$$I_m = \frac{V_m}{\omega_0 L}$$

$$i(t) = \frac{1}{j\omega_o L} V_m e^{j\omega_o t} = I_m \angle -90^0$$

Time domain



$$v(t) = V_m \cos \omega_o t$$

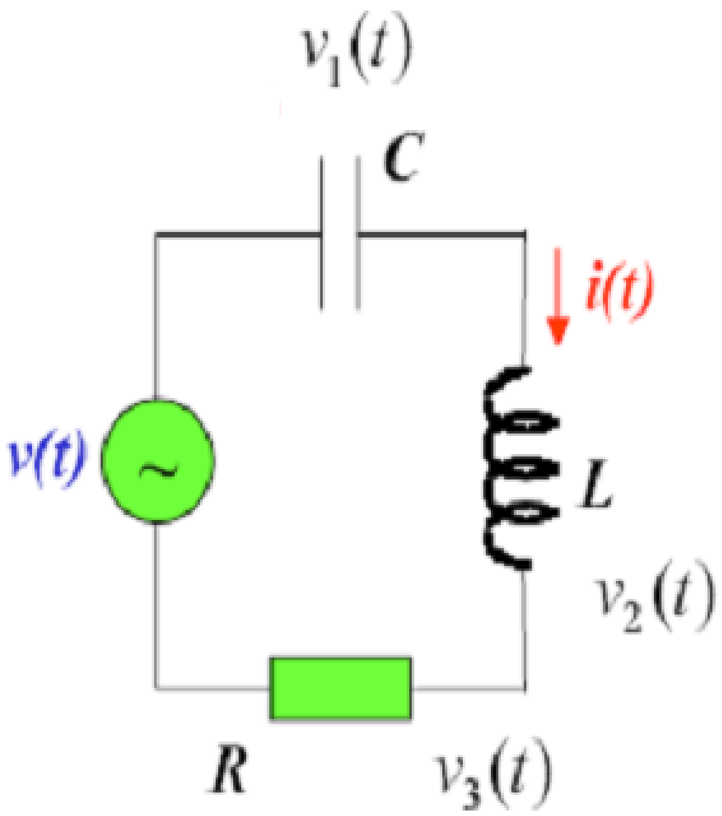
$$i(t) = I_m \cos(\omega_o t + \phi)$$

$$v_1(t) = V_1 \cos(\omega_o t + \phi_1)$$

$$v_2(t) = V_2 \cos(\omega_o t + \phi_2)$$

Phasor Transformation

Phasor domain



$$v(t) = V_m e^{j\omega_o t}$$

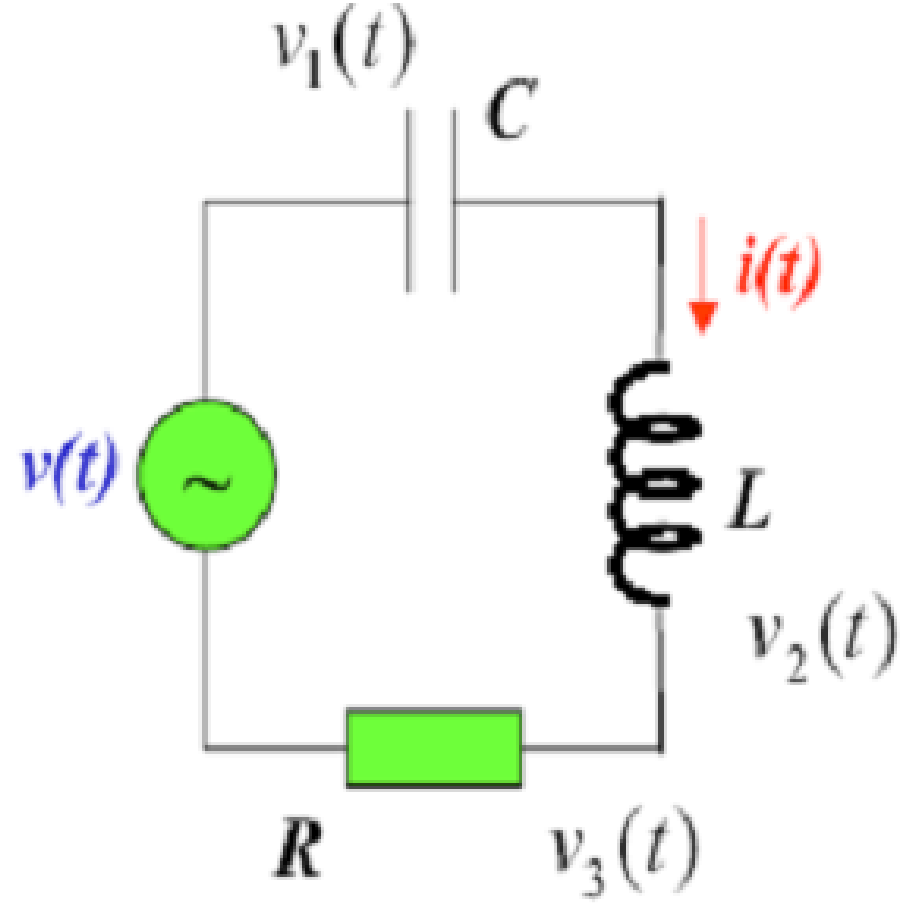
$$i(t) = I_m e^{j(\omega_o t + \phi)}$$

$$v_1(t) = V_1 e^{j(\omega_o t + \phi_1)}$$

$$v_2(t) = V_2 e^{j(\omega_o t + \phi_2)}$$

Example

- Consider the AC circuit



- Let $i(t) \Longleftrightarrow I$
- Then $I = I_m e^{j\omega_o t}$

  $V = V_m e^{j(\omega_o t+\phi)}$
- Then

$$V_1 = \frac{1}{j\omega_o C} I$$

$$V_2 = j\omega_o L I$$

$$V_3 = RI$$

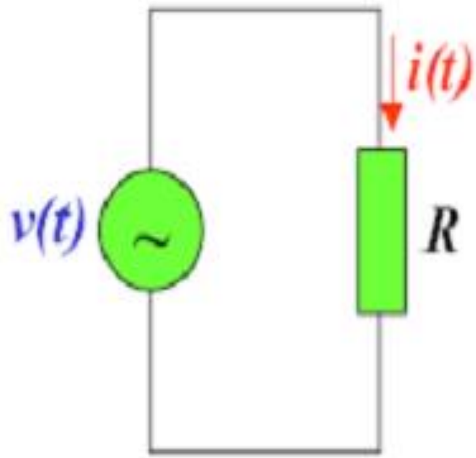
$$V = (R + j\omega_o L + \frac{1}{j\omega_o C})I$$

$$V = [R + j(\omega_o L - \frac{1}{\omega_o C})]I$$

- So given V_m, we can determine I_m
- Or given I_m, we can determine V_m

and we can determine the phase angle between I and V

Impedance of a Resistor



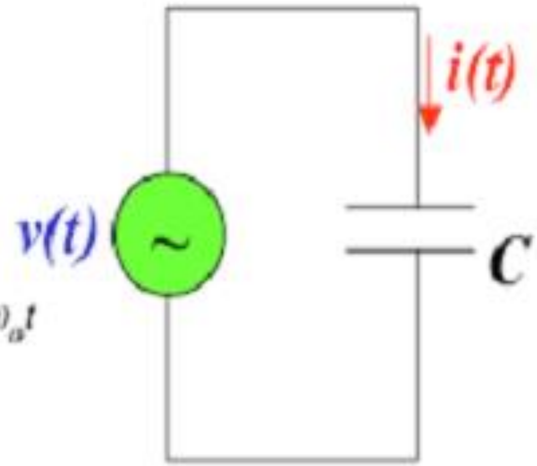
$$v(t) = V_m e^{j(\omega_o t + \phi)}$$

$$i(t) = \frac{V_m}{R} e^{j(\omega_o t + \phi)}$$

$$\frac{v(t)}{i(t)} = R$$

Impedance $= Z$

Impedance of a Capacitor



$$v(t) = V_m e^{j\omega_o t}$$

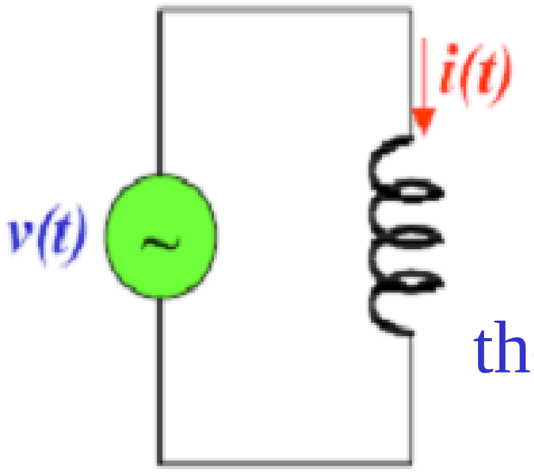
$$i(t) = C\frac{dv(t)}{dt}$$

$$i(t) = j\omega_o C V_m e^{j\omega_o t}$$

Impedance of a capacitor = Z = $\dfrac{v(t)}{i(t)} = \dfrac{1}{j\omega_o C}$

Impedance of an inductor = Z = $\dfrac{v(t)}{i(t)} = j\omega_o L$

Impedance of an inductor



then

If $v(t) = V_m e^{j\omega_o t}$

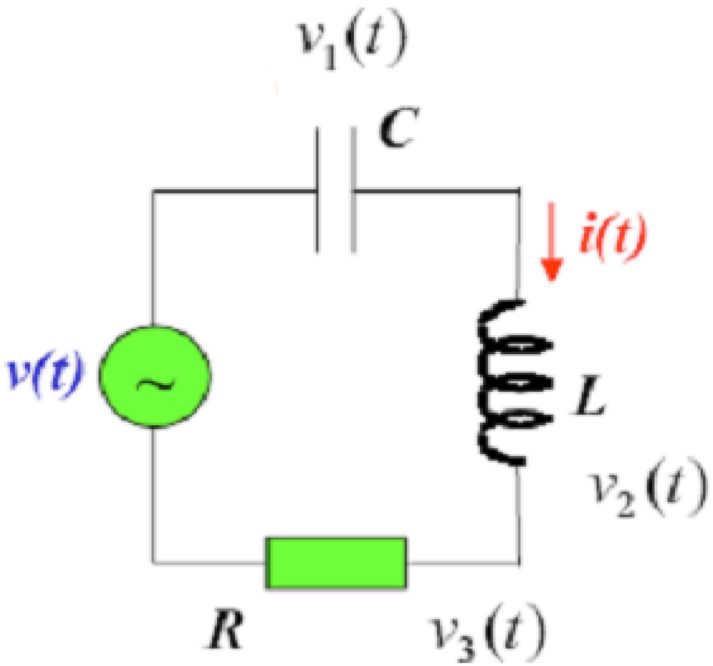
$$i(t) = \frac{1}{L}\int v(t)dt$$

$$i(t) = \frac{1}{j\omega_o L} V_m e^{j\omega_o t}$$

Impedance of an inductor = Z = $\dfrac{v(t)}{i(t)} = j\omega_o L$

Example

- $R = 15\Omega; C = 800\mu F; L = 0.2H$
- $\omega_o = 50 rad/s$ Find *i(t)* if *v(t)* = 15 cosω_0t



- Then $Z_R = 15\Omega$

$$Z_C = \frac{1}{j\omega_o C} = -j25\Omega$$

$$Z_L = j\omega_o L = j10\Omega$$

$$Z = 15 - j15$$

- If $v(t) = 15\cos\omega_o t$
- Then $i(t) = \frac{1}{\sqrt{2}}\cos(\omega_o t + \frac{\pi}{4})$