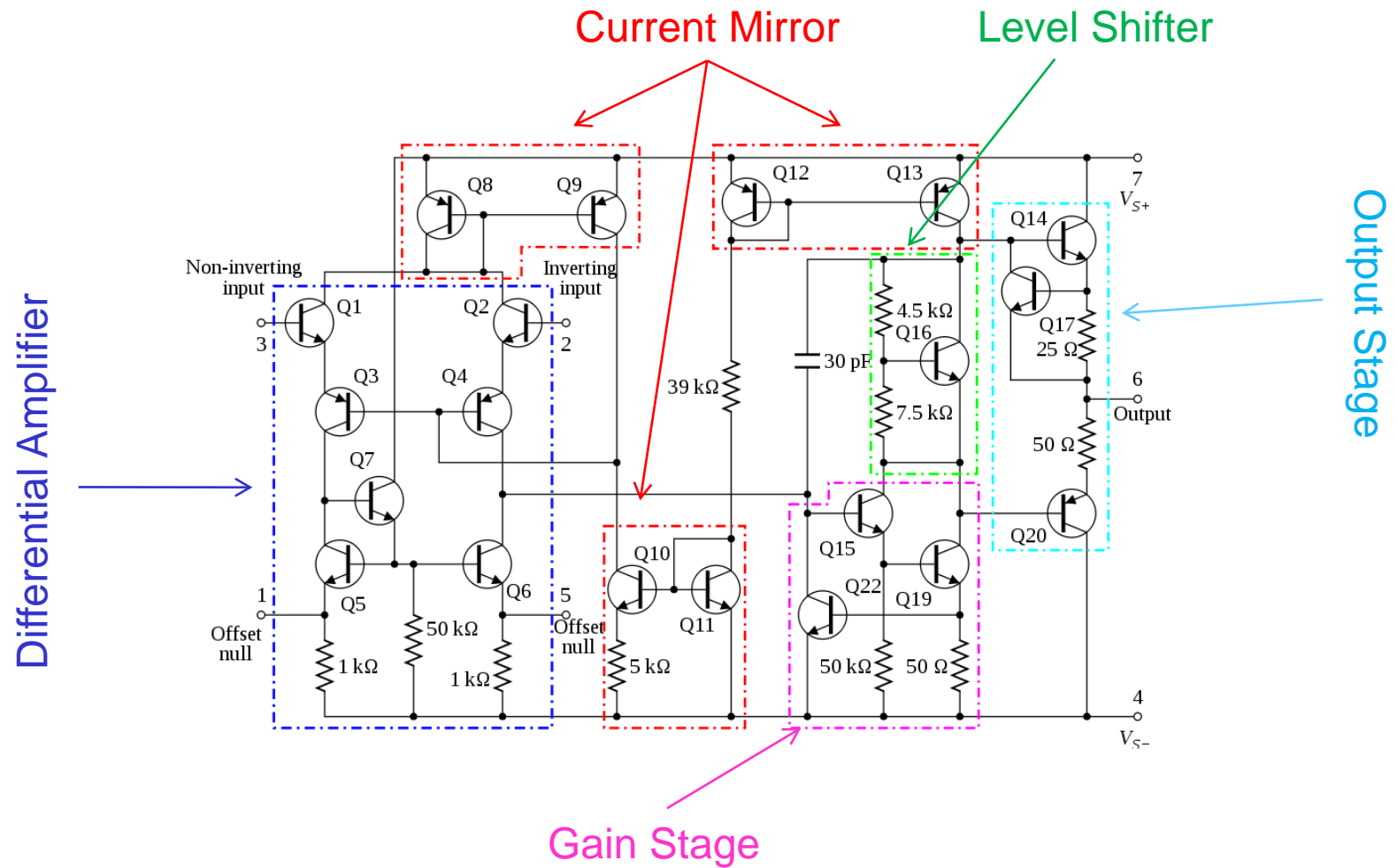


Operational Amplifier (Op-Amp)

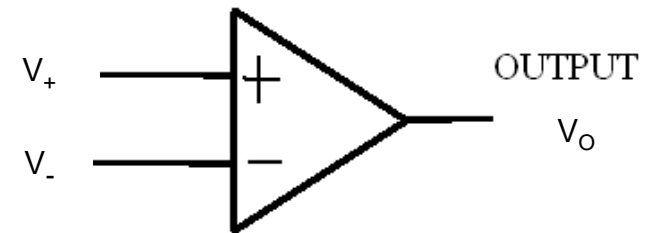
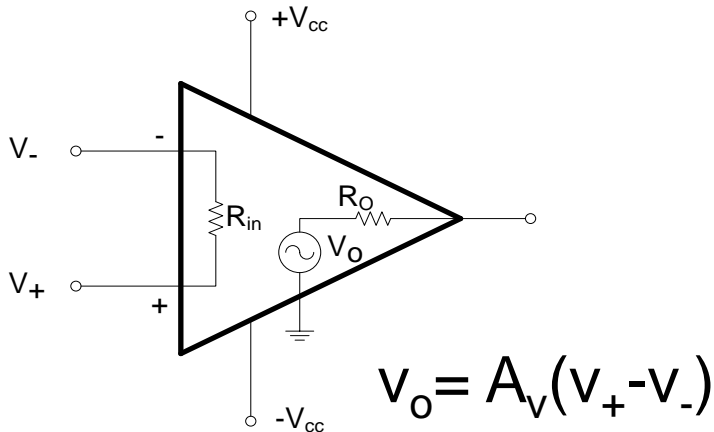
Circuit Diagram of an Op-Amp (IC 741)



Operational Amplifier (Op-Amp)

- An Op-Amp is a very high gain amplifier having a number of differential amplifier stages
- It has high input impedance (typically a few Mega ohm)
- It has a low output impedance (less than 100Ω)

Op-Amp Model



Symbol

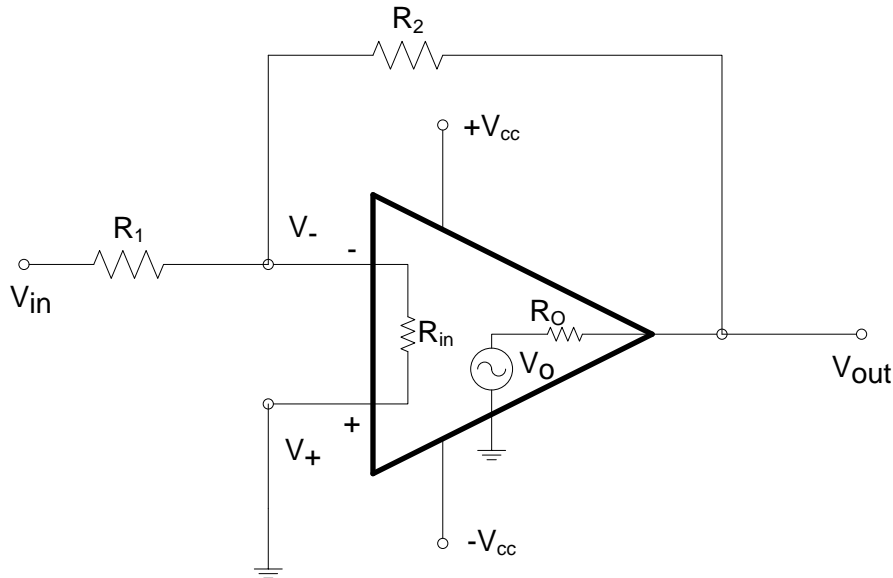
In a good Op-Amp

$$A_v \rightarrow \infty \quad R_{in} \rightarrow \infty \quad R_o \rightarrow 0$$

If v_+ is even slightly higher than v_- , $v_o = +V_{cc}$
 If v_+ is even slightly lower than v_- , $v_o = -V_{cc}$

*Cannot be used
as an amplifier
by itself!*

Consider the circuit shown below



$$v_+ = 0 \quad v_o = -A_v v_-$$

$$\frac{v_{in} - v_-}{R_1} + \frac{v_o - v_-}{R_2 + R_o} = \frac{v_-}{R_{in}}$$

$$v_- \left[\frac{1}{R_2 + R_o} + \frac{1}{R_1} + \frac{1}{R_{in}} \right] - \frac{v_o}{R_2 + R_o} = \frac{1}{R_1} v_{in}$$

$$-\frac{v_o}{A_v} \left[\frac{1}{R_2 + R_o} + \frac{1}{R_1} + \frac{1}{R_{in}} \right] - \frac{v_o}{R_2 + R_o} = \frac{1}{R_1} v_{in}$$

For $A_v \rightarrow \infty$, we get $v_o \left[-\frac{1}{R_2 + R_o} \right] = \frac{v_{in}}{R_1}$

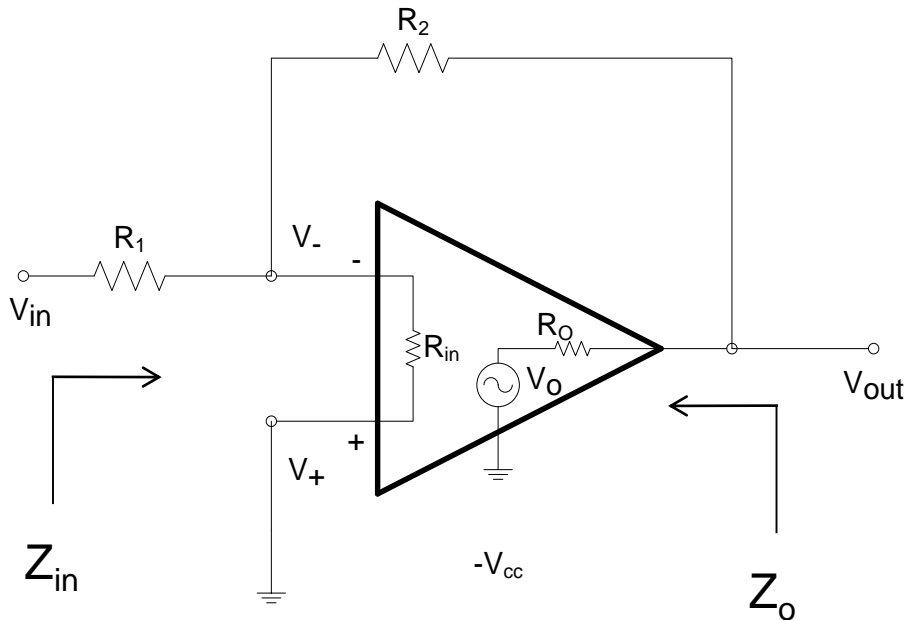
and $v_- = -\frac{v_o}{A_v} \rightarrow 0$

$$\left[\frac{v_o}{R_o + R_2} \right] R_2 = v_{out}$$

Gain $\frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}$

The (-) terminal is effectively at ground. This is referred to as **“Virtual Ground”**

Interesting Points



With $A_v \rightarrow \infty$,

Gain = $-R_2/R_1$
(does not depend on A_v)

(+) is at Ground
(-) is at Virtual Ground

$$Z_{in} = R_1$$
$$Z_{out} = R_o \parallel R_2 \approx R_o$$

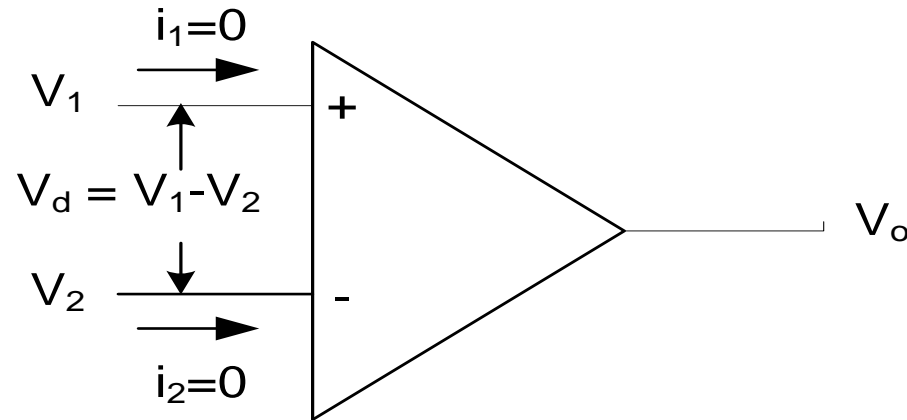
Concept of Virtual Ground

1. $V_- \approx 0 \text{ V}$ (-) terminal is at ground potential
 2. No Current Enters the (-) terminal of the Op-Amp
- Valid only when there is feedback connection between the output and the (-) terminal.

Characteristics of an ideal OP-Amp

- Input Resistance $R_i = \infty$
- Output Resistance $R_o = 0$
- Voltage Gain $A_v = \infty$
- Bandwidth = ∞ (i.e. can work over a wide range of frequencies)
- Perfect balance i.e $v_o = 0$ when $v_1 = v_2$
- Characteristics do not drift with temperature

Ideal Op-Amp analysis



i) $R_i = \infty$,

No current enters into op-amp

Voltage Gain $A_v = \infty$ or, $v_o / v_d = \infty$

or, $v_d = v_o / \infty = 0$ [since v_o is finite]

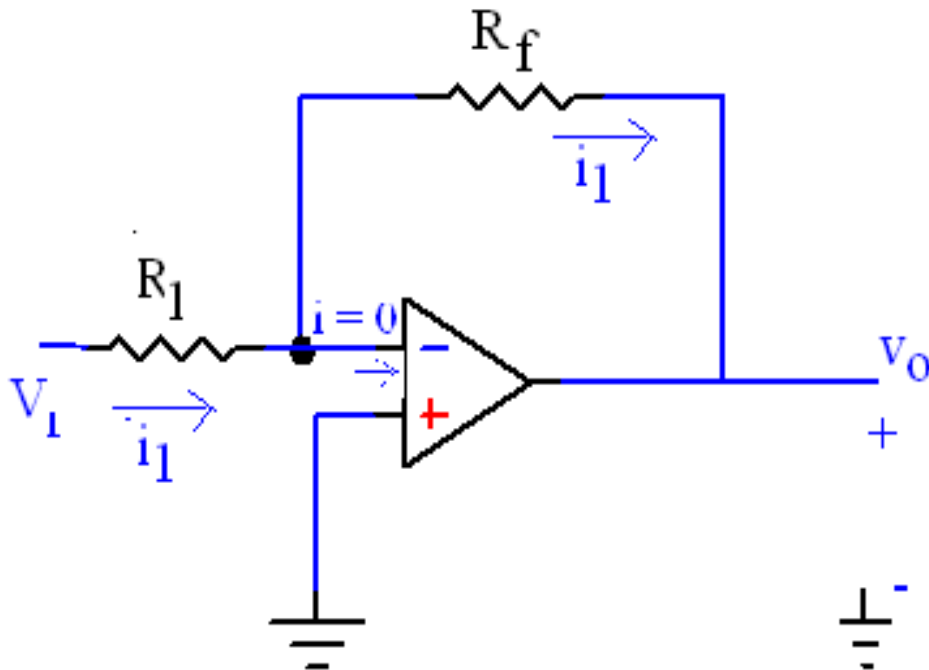
Therefore, $v_1 - v_2 = 0$

or,

ii) $\mathbf{v_1 = v_2}$

Simple OP-AMP Circuits

1. Inverting Amplifier



Using KVL,

$$v_1 - i_1 R_1 = 0$$

$$\Rightarrow i_1 = v_1 / R_1$$

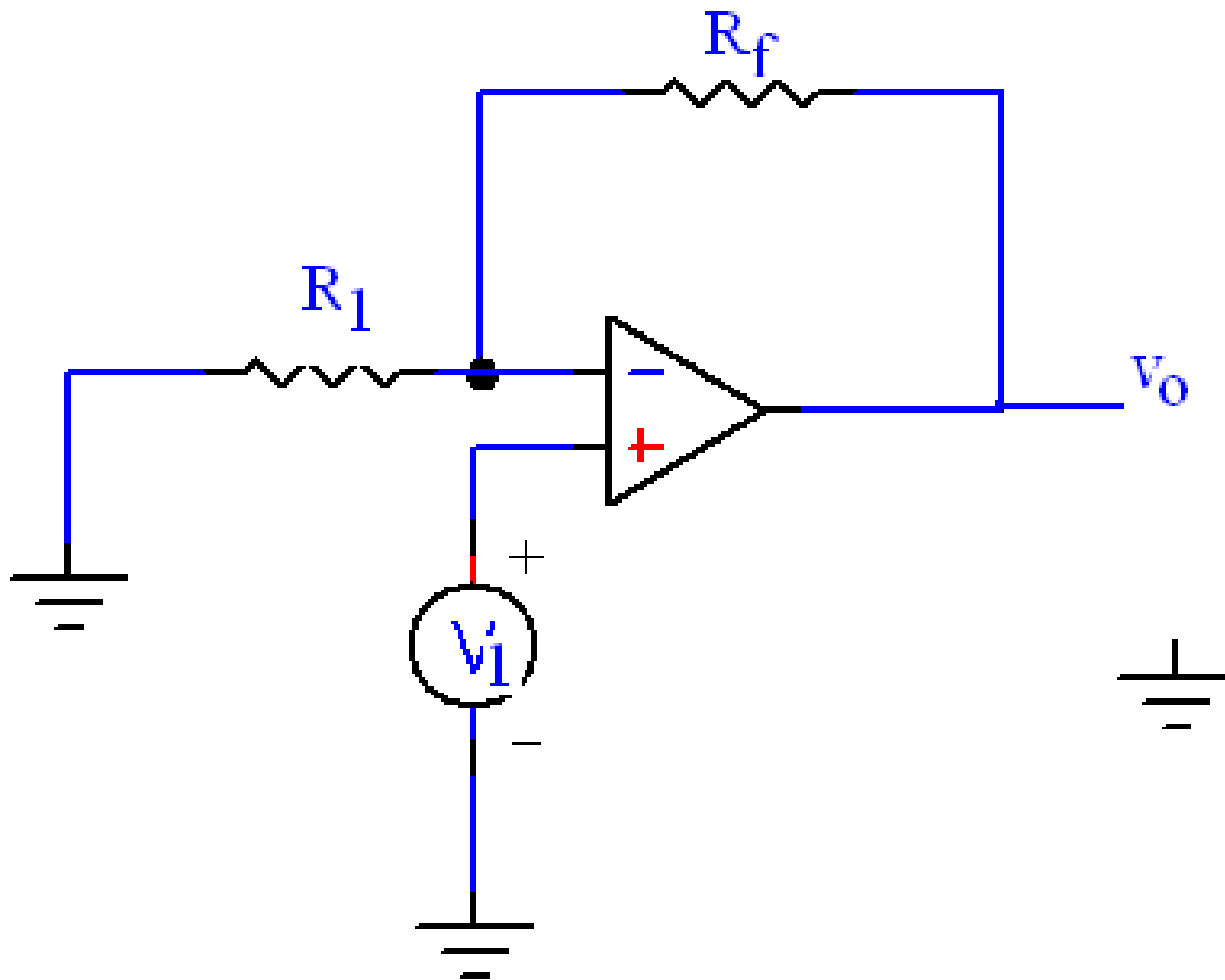
&

$$0 - i_1 R_f - v_o = 0$$

$$\text{or, } v_o = -i_1 R_f = -v_1 R_f / R_1$$

$$v_o / v_1 = -R_f / R_1$$

2. Non Inverting Amplifier



$$v_- = v_+ = v_1$$

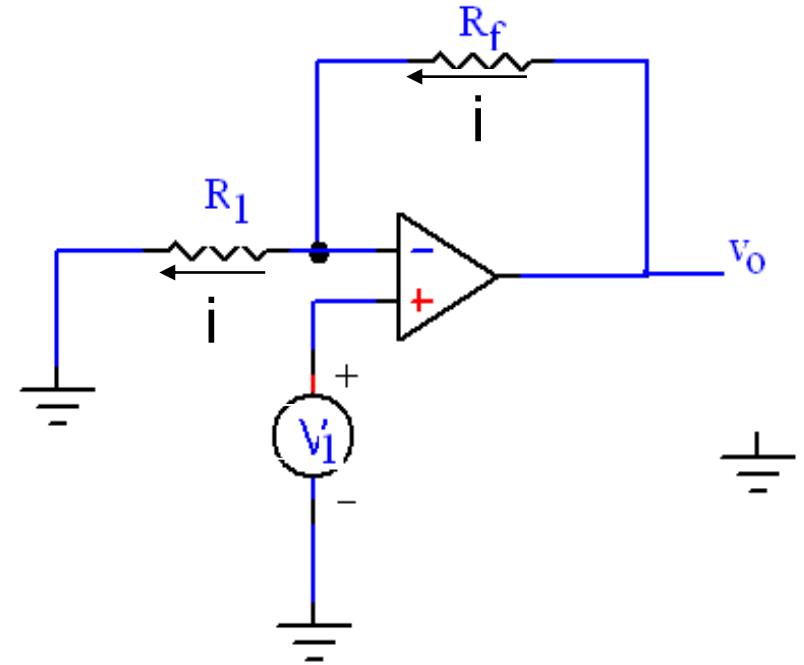
$$i = \frac{v_1}{R_1} = \frac{v_o - v_1}{R_f}$$

$$v_o = R_f \left[\frac{1}{R_1} + \frac{1}{R_f} \right] v_1$$

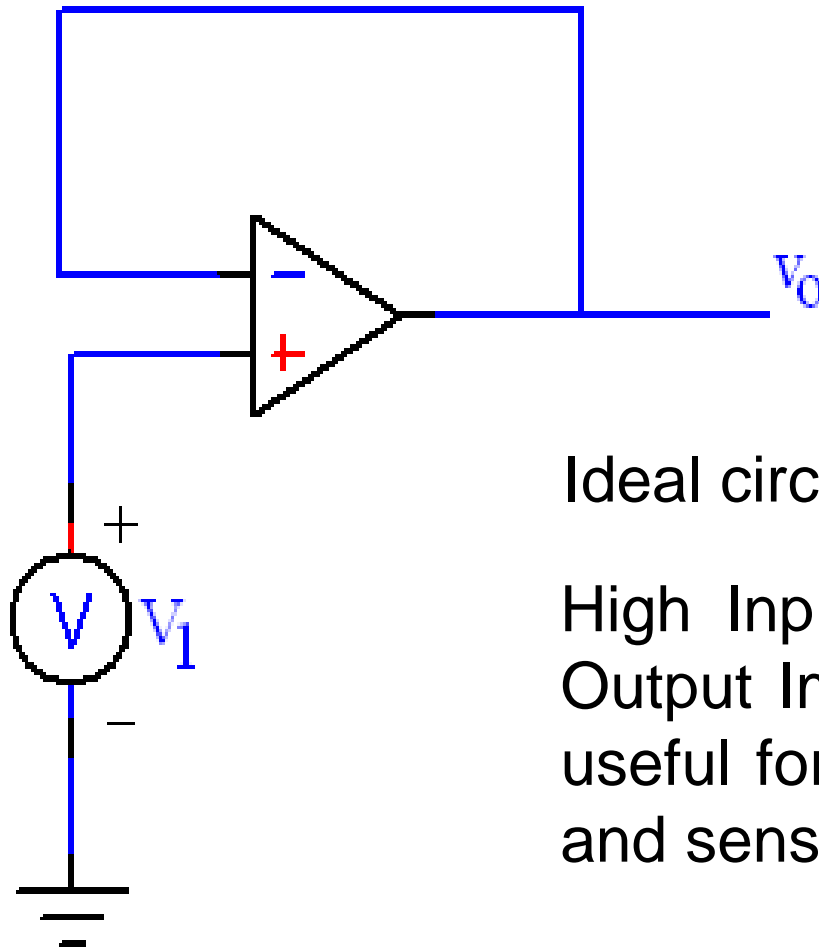
Gain $\frac{v_o}{v_1} = \left(1 + \frac{R_f}{R_1} \right)$

Input Impedance = ∞

Output Impedance = R_o



3. Voltage Follower



$$V_0 = V_1$$

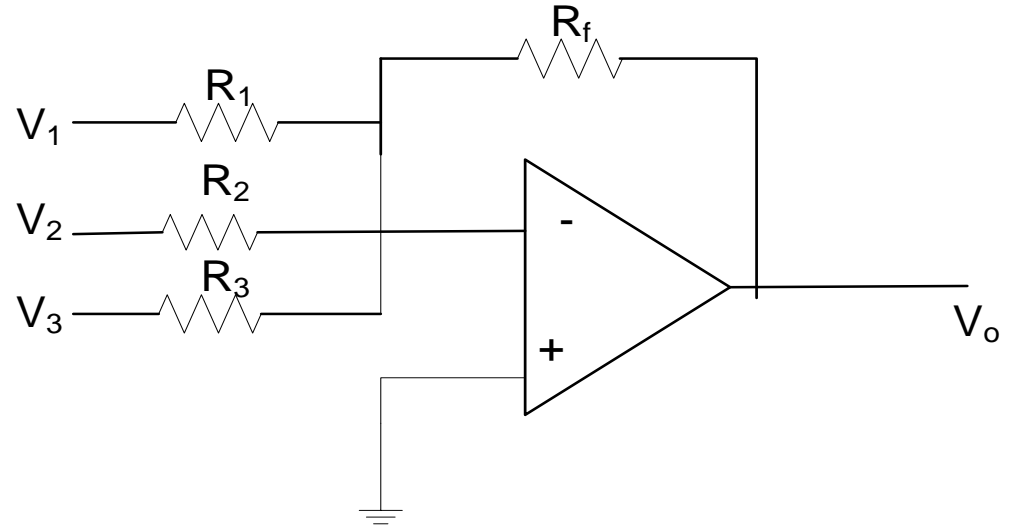
Ideal circuit to test Op-Amp

High Input Impedance and Low Output Impedance also makes it useful for interfacing transducers and sensors to other circuits

4. Summing Amplifier

Use superposition
(i.e. consider one
source at a time and
add their respective
outputs)

Can also be done
directly

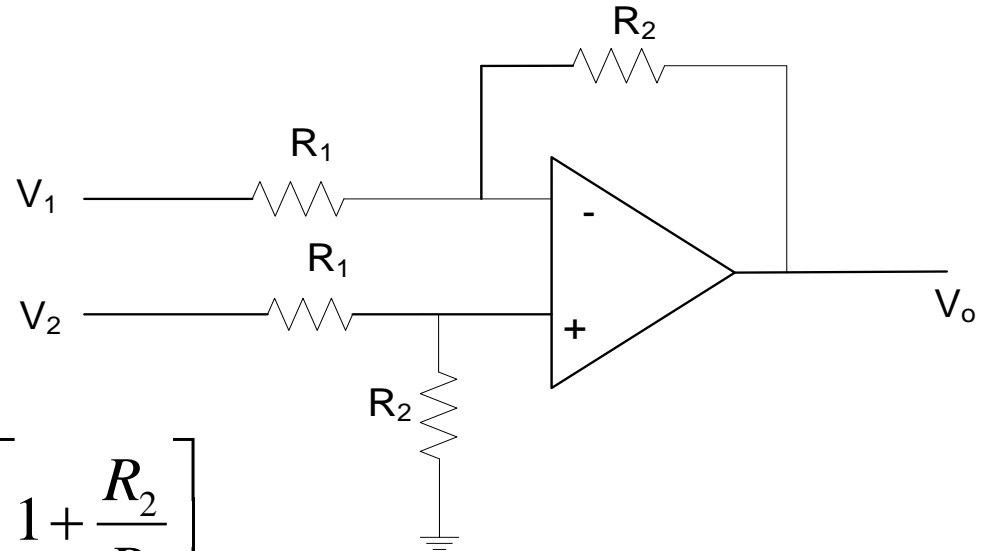


$$v_o = - \left[\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right]$$

Difference Amplifier (or Voltage Subtractor)

Use Superposition

(can also be done directly)



$$\text{If } v_2=0, \quad v_{o1} = -\frac{R_2}{R_1} v_1$$

$$\begin{aligned} \text{If } v_1=0, \quad v_{o2} &= v_2 \left[\frac{R_2}{R_1 + R_2} \right] \left[1 + \frac{R_2}{R_1} \right] \\ &= v_2 \frac{R_2}{R_1} \end{aligned}$$

$$\text{Therefore,} \quad v_o = v_{o1} + v_{o2} = \frac{R_2}{R_1} (v_2 - v_1)$$

Difference Gain = R_2/R_1

Instrumentation Amplifier

$$R_1=10K \quad R_2=2K$$

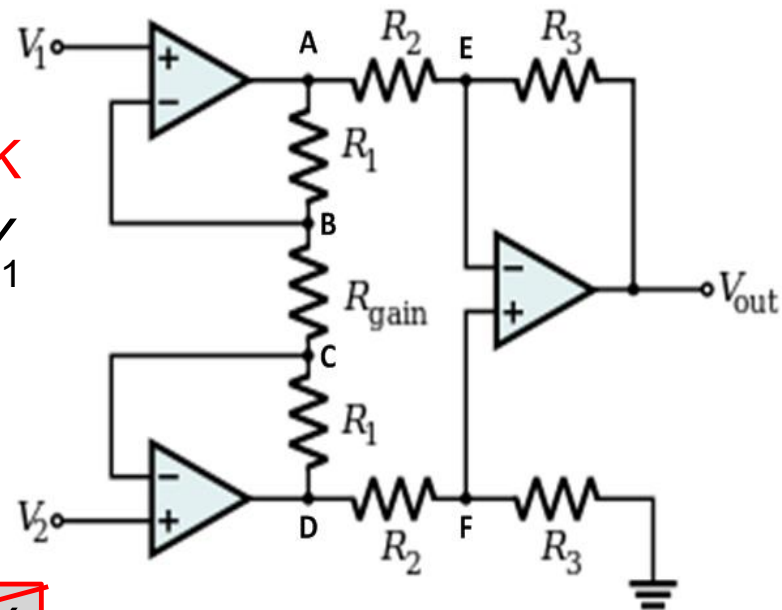
$$R_3=100K \quad R_{gain}=20K$$

$$V_B = V_1 \quad V_C = V_2 \quad V_D = 1.5V_2 - 0.5V_1$$

$$V_A = 1.5V_1 - 0.5V_2$$

~~$$V_E = V_F = 1.4706V_2 - 0.4902V_1$$~~

~~$$V_{out} = 75.006V_2 - 25.002V_1 - 75V_1 + 25V_2$$~~
~~$$= 100.006V_2 - 100.002V_1$$~~
~~$$= 100.004V_2 - 100.002V_1 + V_1$$~~



Mistakes because of Round-Off Errors

$$\frac{V_{out}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{gain}} \right) \frac{R_3}{R_2}$$

Gain can be varied by just changing one resistor, R_{gain}

Circuit also has high input impedance and very low CMRR

Instrumentation Amplifier

$$V_B = V_1 \quad V_C = V_2$$

$$\frac{V_D - V_2}{R_1} = \frac{V_2 - V_1}{R_{gain}} \Rightarrow V_D = V_2 + \frac{R_1}{R_{gain}}(V_2 - V_1)$$

$$\text{Similarly, } V_A = V_1 - \frac{R_1}{R_{gain}}(V_2 - V_1)$$

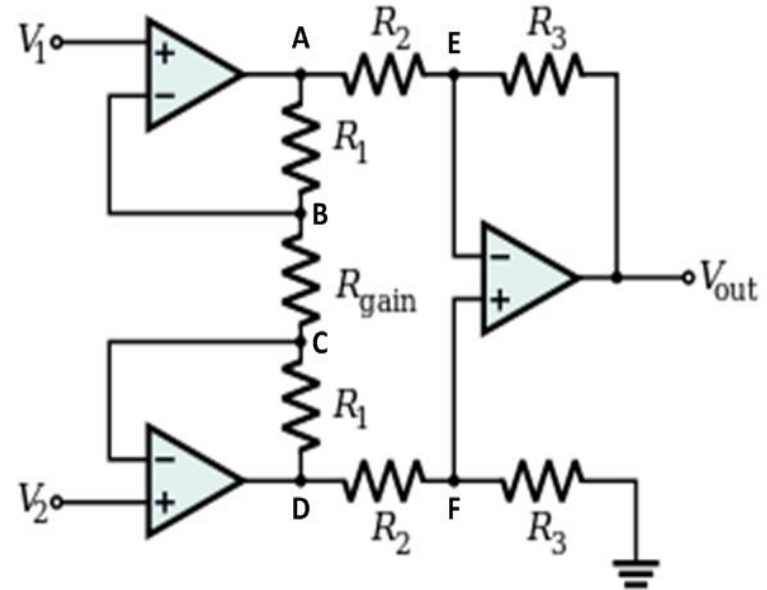
$$V_F = \left(\frac{R_3}{R_2 + R_3} \right) V_D = V_E$$

$$\frac{V_{out} - V_E}{R_3} = \frac{V_E - V_A}{R_2} \Rightarrow V_{out} = \left(1 + \frac{R_3}{R_2} \right) V_E - \frac{R_3}{R_2} V_A$$

$$V_{out} = \frac{R_3}{R_2} (V_D - V_A) = \left(1 + \frac{2R_1}{R_{gain}} \right) \left(\frac{R_3}{R_2} \right) (V_2 - V_1)$$

$$\frac{V_{out}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{gain}} \right) \frac{R_3}{R_2}$$

Gain can be varied by just changing one resistor, R_{gain}



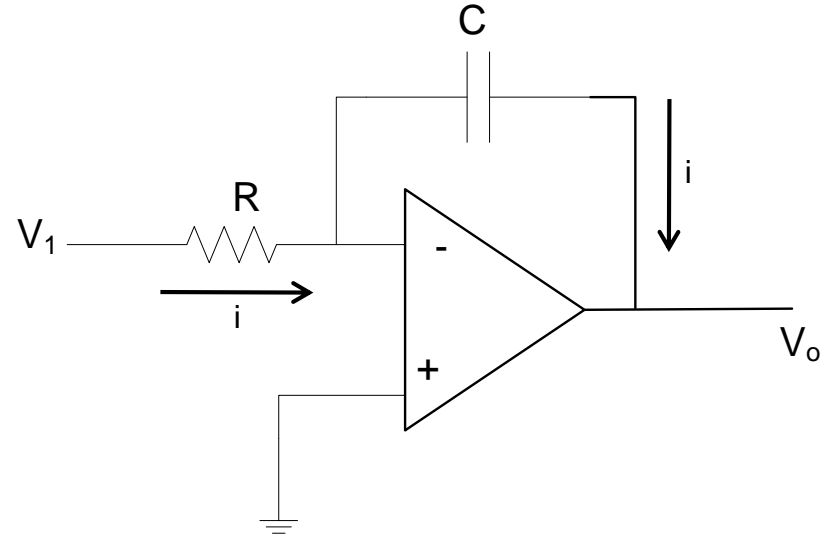
Circuit also has high input impedance and very low CMRR

Integrator

$$i = \frac{v_1}{R} \quad v_o = -\frac{1}{C} \int_0^t i dt \quad \text{with } v_o(0) = 0$$

Therefore,

$$v_o = -\frac{1}{RC} \int_0^t v_1 dt \quad \text{with } v_o(0) = 0$$



v_o

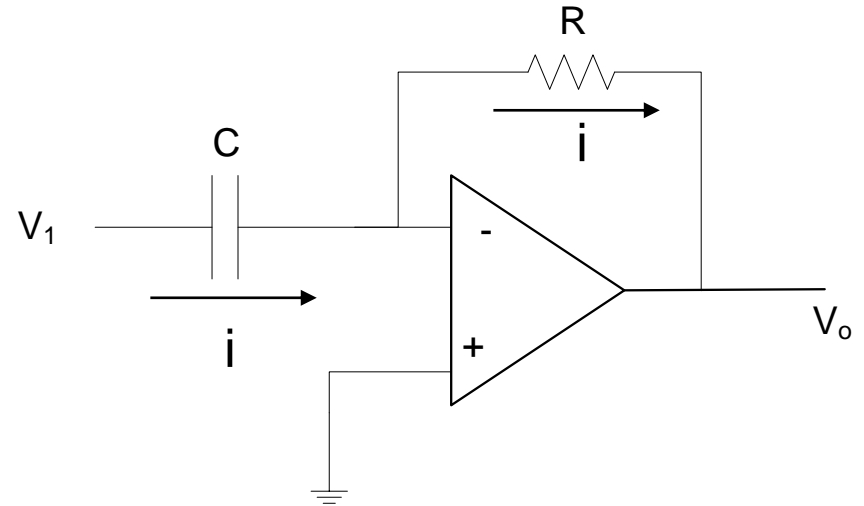
Differentiator

$$v_1 = \frac{1}{C} \int_0^t i dt \quad \text{with } v_1(0) = 0$$

$$v_o = -iR$$

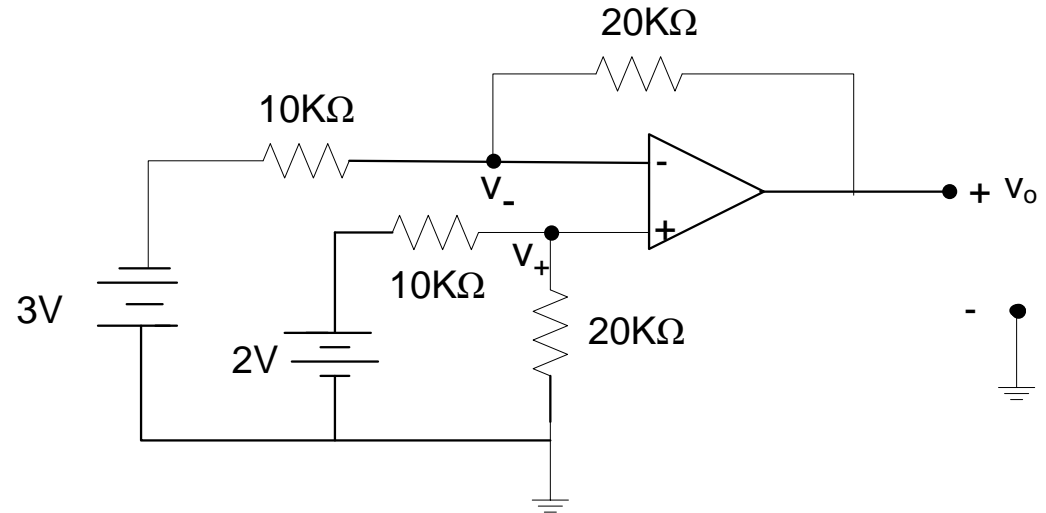
Therefore,
$$v_1 = -\frac{1}{RC} \int_0^t v_o dt$$

or
$$v_o = -RC \frac{dv_1}{dt}$$



Example

Find v_o in the given circuit.



$$v_- = v_+ = 2 \times \frac{20}{30} = \frac{4}{3}$$

$$\frac{v_o - v_-}{20} = \frac{v_- - (-3)}{10}$$

$$\frac{v_o}{20} = v_- \left(\frac{1}{10} + \frac{1}{20} \right) + \frac{3}{10}$$

$$v_o = 3v_- + 6 = 10 \text{ V}$$

Example

Find the gain v_o/v_i in the given circuit.

$v_- = v_+ = 0$ Virtual Ground at (-)

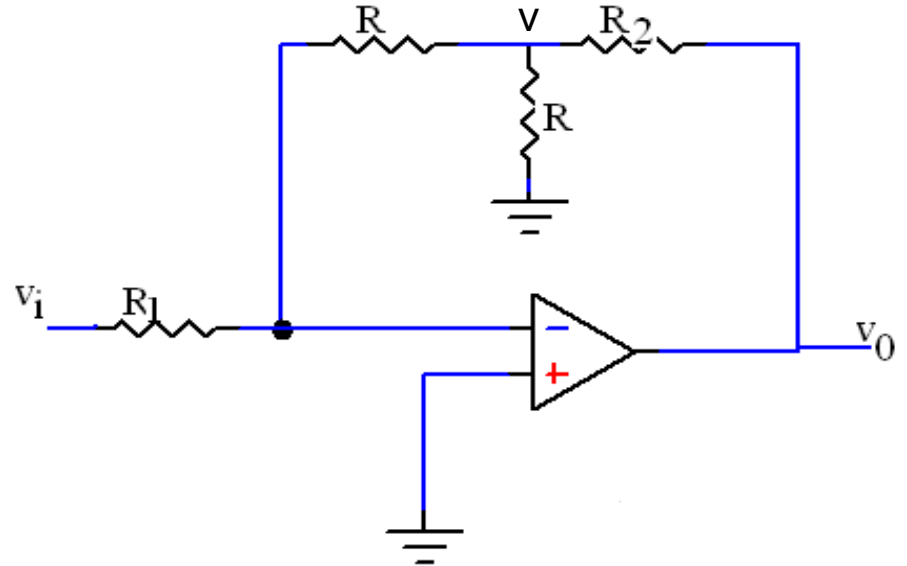
$$\frac{v_i}{R_1} = -\frac{v}{R} \Rightarrow v = -\frac{R}{R_1} v_i$$

$$\frac{v_o - v}{R_2} = \frac{v}{R} + \frac{v}{R} = \frac{2v}{R}$$

$$v_o = v \left[1 + \frac{2R_2}{R} \right]$$

$$= -\left[\frac{R}{R_1} \right] \left[1 + \frac{2R_2}{R} \right] v_i$$

$$= -\left(\frac{R + 2R_2}{R_1} \right) v_i$$



Gain

$$A_v = \frac{v_o}{v_i} = -\left(\frac{R + 2R_2}{R_1} \right)$$

Voltage Controlled Current Source

(with grounded load)

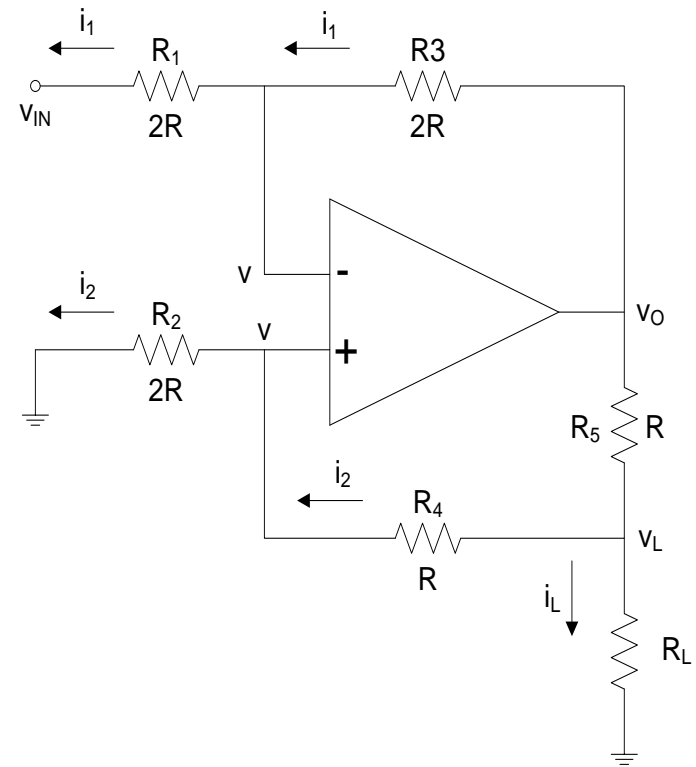
$$i_1 = \frac{v - v_{IN}}{2R} = \frac{v_O - v}{2R} \Rightarrow 2v - v_O = v_{IN}$$

$$i_2 = \frac{v}{2R} = \frac{v_L - v}{R} \Rightarrow v_L = \frac{3}{2}v$$

$$i_L = \frac{v_O - v_L}{R} - \frac{v_L - v}{R} = \frac{v_O - 3v + v}{R} = \frac{v_O - 2v}{R}$$

Therefore, $i_L = -\frac{v_{IN}}{R}$

Note that i_L does not depend on R_L , implying that we have got a current source!

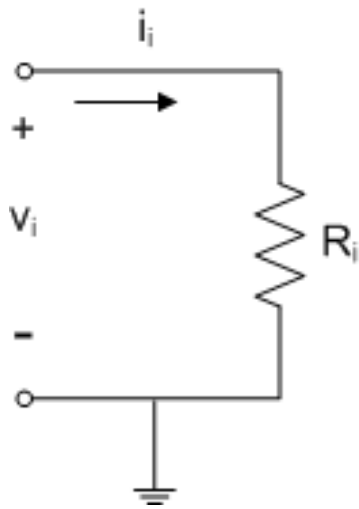


General Condition:

$$R_1 = R_2 \quad \& \quad R_3 = R_4 + R_5$$

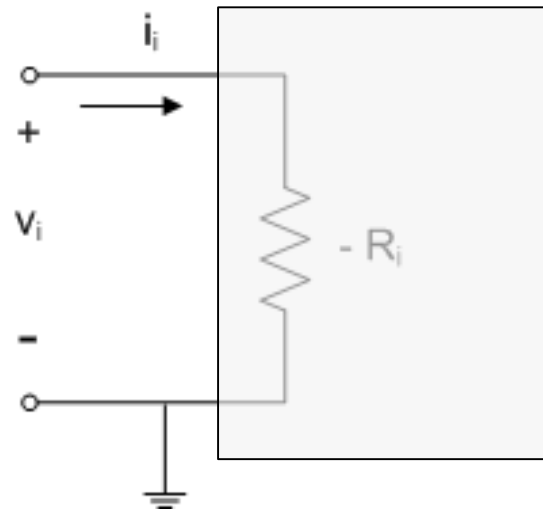
Creating an effectively “Negative Resistance”

What is “Negative Resistance”?



Normal
Resistance

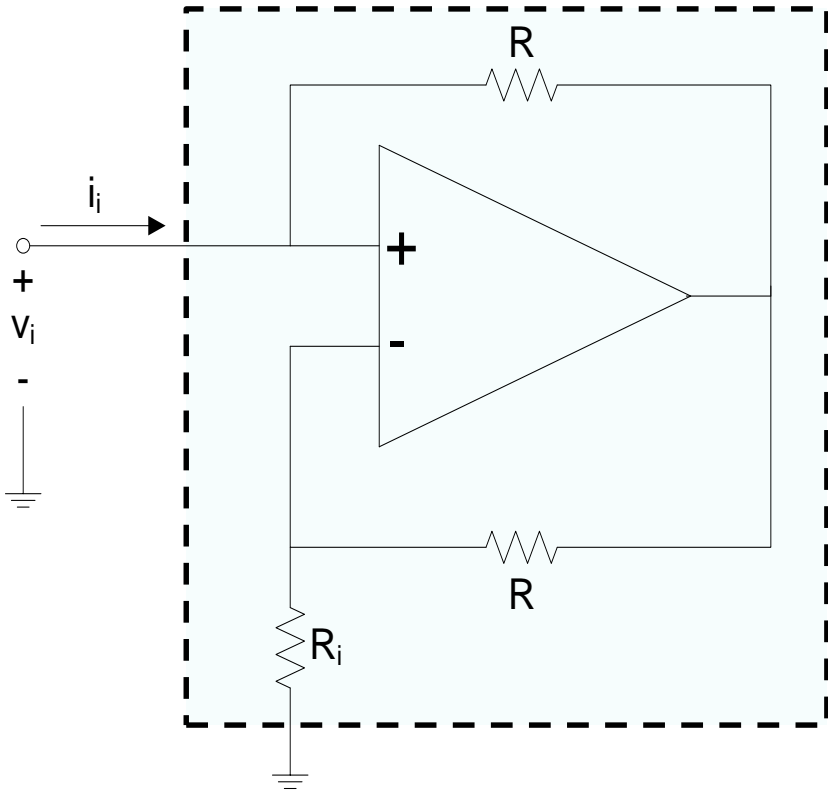
$$\frac{v_i}{i_i} = R_i$$



Negative
Resistance

$$\frac{v_i}{i_i} = -R_i$$

Creating an effectively “Negative Resistance”



Let v_o = opamp output voltage
and $v_+ = v_- = v_i$

$$\frac{v_i}{R_i} = \frac{v_o - v_i}{R} \Rightarrow v_o = v_i \left(1 + \frac{R}{R_i} \right)$$

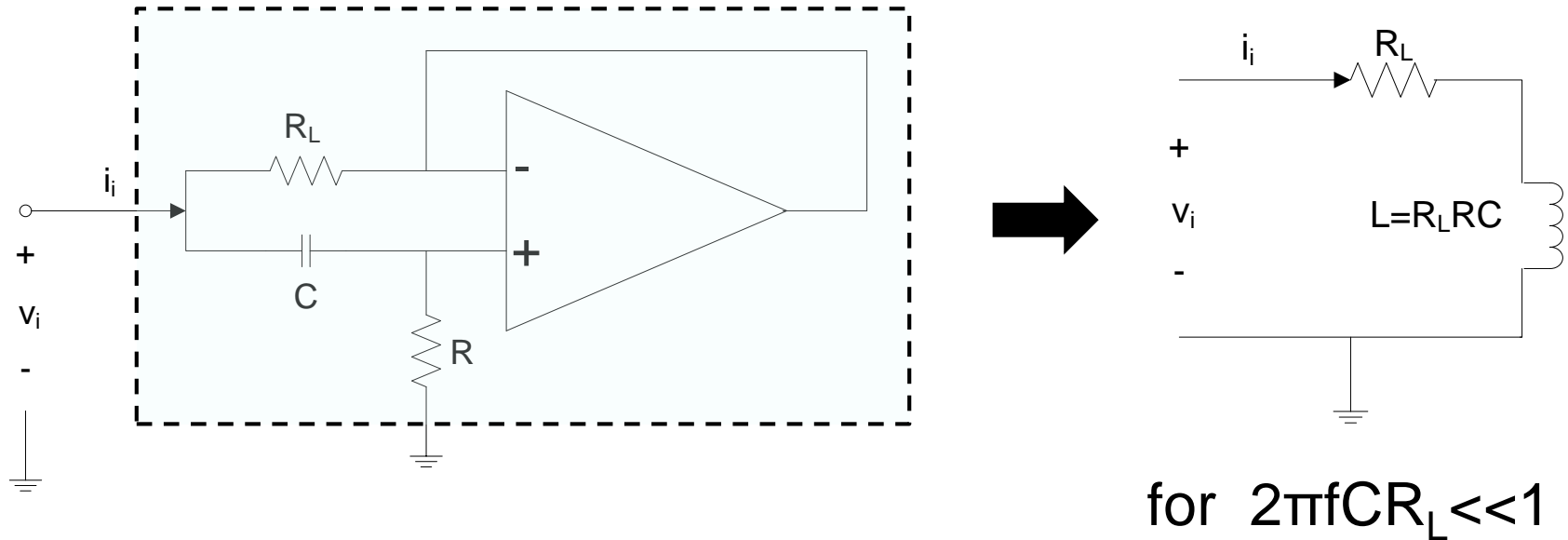
$$i_i = \frac{v_i - v_o}{R} = -\frac{v_i}{R_i}$$

Therefore, $\frac{v_i}{i_i} = -R_i$

$$\frac{v_i}{i_i} = -R_i \quad \text{Negative Resistance}$$

Using Capacitors to make Inductors

(practical to do only small values of inductances)



Using phasors, show that

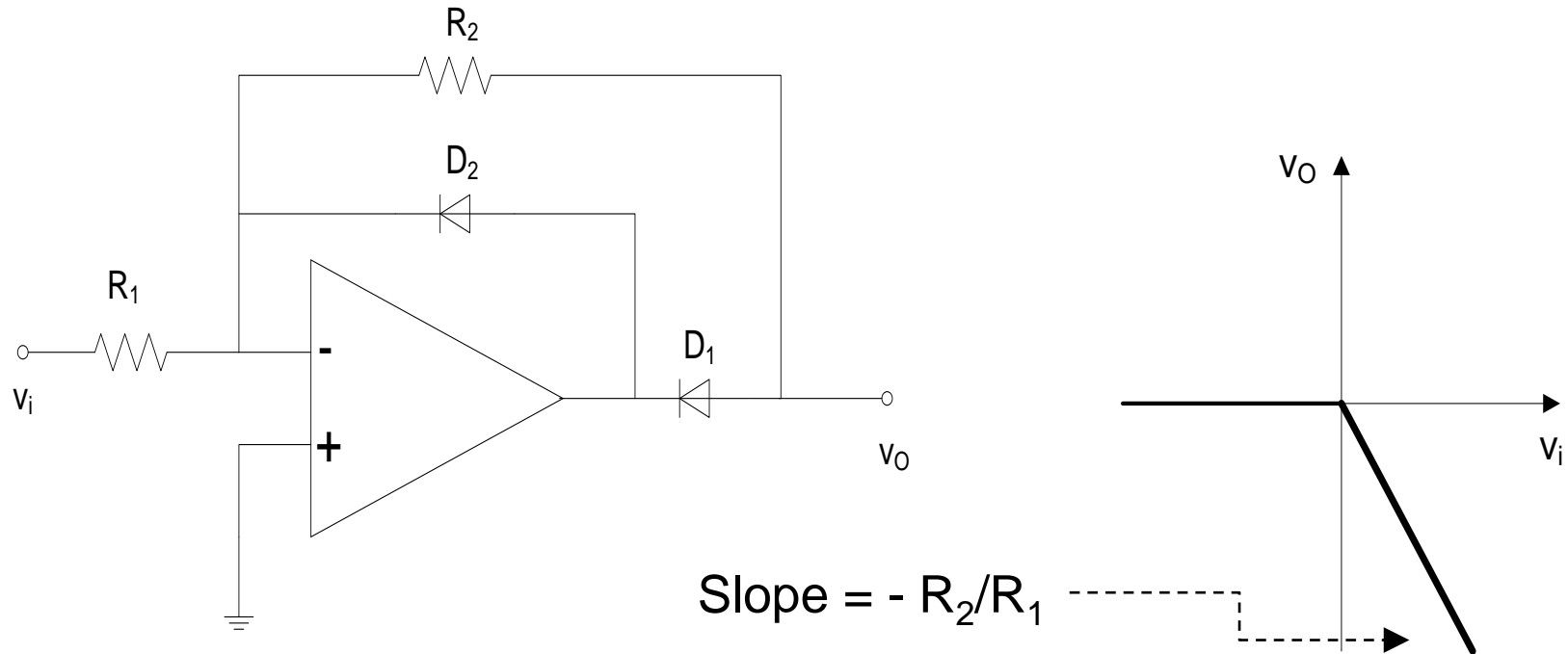
$$\frac{v_i}{i_i} = \frac{R_L + j\omega R C R_L}{(1 + j\omega C R_L)} = R_L + j\omega R C R_L$$

Precision Rectifier

The simple half-wave and full-wave rectifiers we saw earlier have one big drawback – **They do not work for small voltages (say a few millivolts)**. The input voltage must cross the threshold which forward biases the diode for rectification to occur.

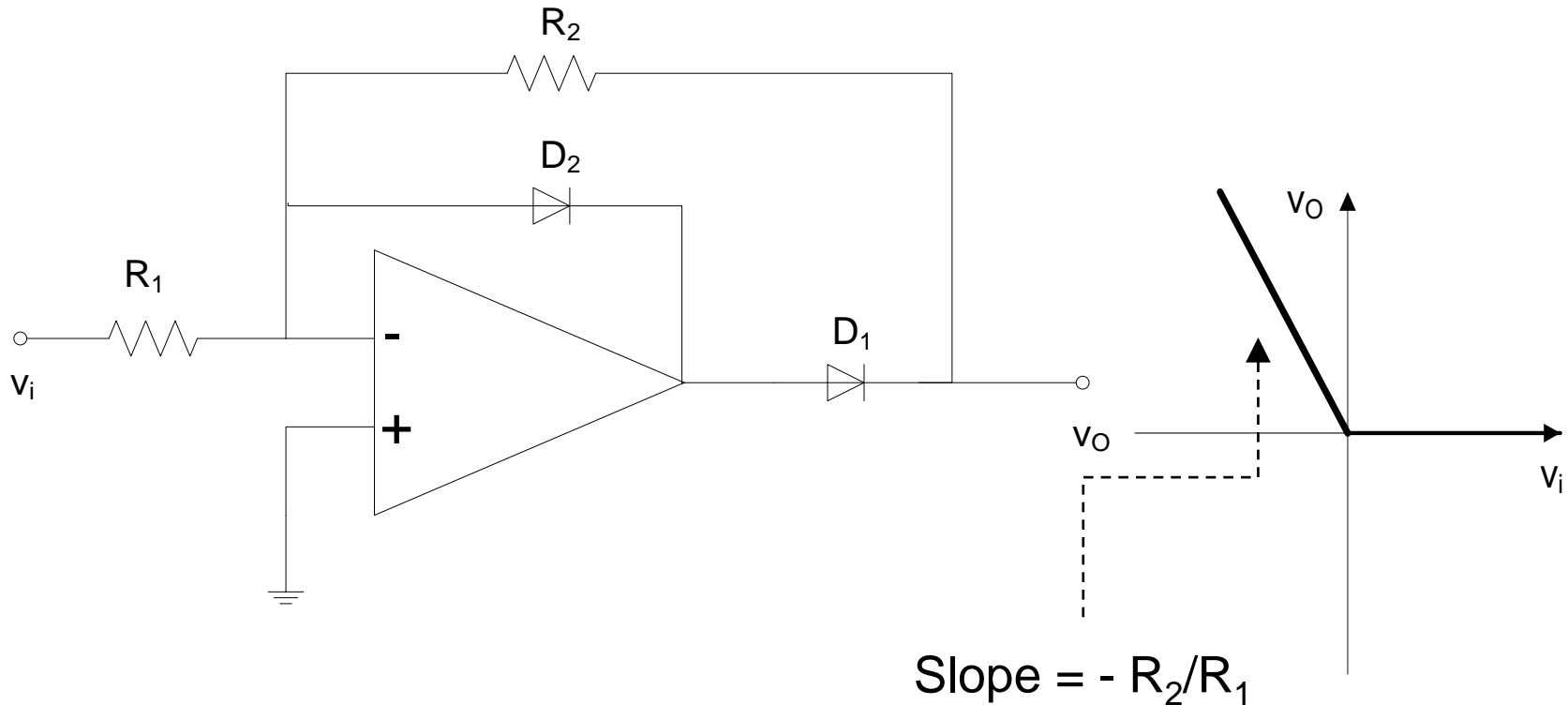
Using Op-Amps, we can design rectifiers which do not have this disadvantage.

Half-Wave Precision Rectifier



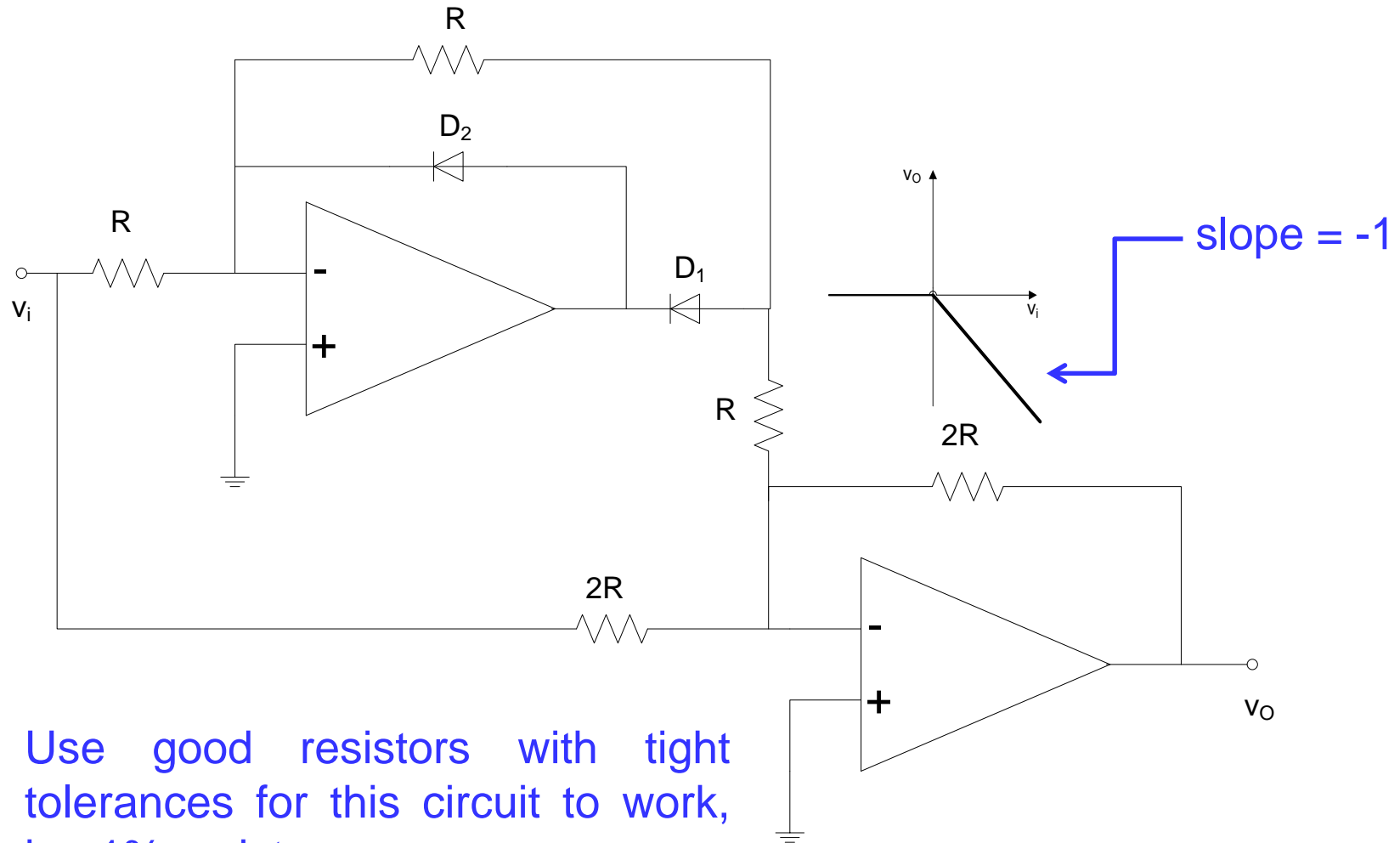
You will be making this circuit in EC102 Lab next semester

What happens when you reverse the diodes?



Build a full wave rectifier using these two half-wave rectifiers and a difference amplifier – *needs three opamps!*

Full-Wave Precision Rectifier



Use good resistors with tight tolerances for this circuit to work, i.e. 1% resistors,