Quantum Mechanics (PH101)
Course Instructors: Pankaj Mishra and Tapan Mishra
Tutorial-8
due on Wednesday, 6th of November, 2019 (8:00Hrs IST)

1. The lifetime of a given atom in an excited state is 10^{-8} s. It comes to the ground state by emitting a photon of wavelength 5800 $\mathring{A}$. Find the energy uncertainty and wavelength uncertainty of the photon. Use the minimum time-Energy uncertainty principle $\Delta E \Delta t = \hbar/2$.

2. Find the uncertainty in the velocity of a particle if the uncertainty in its position is equal to its (a) de Broglie wavelength (b) Compton wavelength. Use the minimum position and momentum uncertainty relation.

3. Check if $\Psi = Ae^{i(kx-\omega t)}$ and $\Psi = Asin(kx - \omega t)$ are acceptable solutions of the time-dependent Schroedinger's equation. The time-dependent Schroedinger'e equation is given by

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial^2 x} + U\Psi$$

4. The normalized wave function of the ground state of the Quantum harmonic oscillator is given by $\psi(x) = C_0 e^{-\alpha x^2}$, where $C_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$ and $\alpha = \frac{m\omega}{2\hbar}$. m is the mass and ω is the angular frequency of the oscillator.

   Compute the $\Delta x \Delta p$ for this state, where Δx and Δp are the uncertainties in the position x and momentum p, respectively. Please comment over the result whether it is consistent with the uncertainty principle. **Use the Gaussian integral** $\int_{-\infty}^{\infty} e^{\alpha x^2 + \beta x} dx = \sqrt{\frac{\pi}{\alpha}} e^{\beta^2/4\alpha}$.

5. An electron is described by the wave function

$$\psi(x) = \begin{cases} 0, & \text{for } x \leq 0 \\ Ce^{-x}(1 - e^{-x}), & \text{for } x > 0, \end{cases}$$

   where x is in nm and C is a constant.

   (a) Determine the value of C that normalizes $\psi(x)$.

   (b) Where is the electron most likely to be found?

   (c) Calculate the average position or expectation value of the position $< x >$ for the electron. Compare this with the most likely position, and comment on the difference.

6. A particle is represented by the wavefunction at time $t = 0$ by

$\Psi(x) = A(a^2 - x^2)$ if $-a \leq x \leq a$ and zero at all other places. Here A and a are constant.

(a) Determine the normalization constant A.
(b) What is the expectation value of x at $t = 0$?
(c) What is the expectation value of p at $t = 0$?
(d) Evaluate $\langle x^2 \rangle$ and $\langle p^2 \rangle$ at $t = 0$.
(e) Obtain the uncertainty relation $(\Delta x \Delta p)$ and comment over your result whether you are getting minimum uncertainty relation or not.