PH 102 : Electromagnetism **Department of Physics**
Tutorial 6 **Indian Institute of Technology Guwahati**
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1. Consider the propagation of en electromagnetic wave in a conducting medium where $\vec{J} = \sigma \vec{E}$ with σ representing the conductivity of the medium. Show that $\vec{E}$ would now satisfy the equation

$$\nabla^2 \vec{E} - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

2. Consider a plane electromagnetic wave of frequency ω, travelling in the z direction, polarised in the x direction, incident at the interface ($z = 0$) separating medium 1 and medium 2. In the derivation for reflected and transmitted amplitudes discussed in the class, the reflected and transmitted waves were assumed to have the same polarisation as the incident wave (along x direction). Prove that this has to be true always. Hint: Let the polarisation vectors of the transmitted and reflected waves be $\hat{n}_T = \cos\theta_T \hat{x} + \sin\theta_T \hat{y}, \hat{n}_R = \cos\theta_R \hat{x} + \sin\theta_R \hat{y}$, and then prove from the electromagnetic boundary conditions that $\theta_T = \theta_R = 0$.

3. Consider a plane wave (with its electric field to be along the y-axis) incident normally on a dielectric film of thickness d. Calculate the reflectivity of the film.

4. Fermat's principle of least time states that light, when reflected or refracted off an interface, will pick the path of least time to propagate between two points. A beam of light from point A is incident upon a dielectric interface at angle θ_i from the normal and is reflected through the point B at angle θ_r (see figure 1).
(a) In terms of $\theta_i, \theta_r, h_1, h_2$ and speed of light c_1, how long does it take for light to travel from A to B along this path? What other relation is there between $\theta_i, \theta_r, L_{AB}, h_1, h_2$?
(b) Find θ_i that satisfies Fermat's principle. What is θ_r?
(c) In terms of $\theta_i, \theta_r, h_1, h_2$ and the light speeds c_1, c_2 in each medium, how long does it take for light to travel from A to C?
(d) Find the relationship between θ_i and θ_t that satisfies Fermat's principle.

5. In many cases the permeability of dielectric media equals that of free space. In this limit (use the general expressions derived in class) show that the reflection and transmission coefficients for waves obliquely incident upon dielectric media are

$$R = \frac{\sin^2(\theta_I - \theta_T)}{\sin^2(\theta_I + \theta_T)}, \; T = \frac{\sin 2\theta_I \sin 2\theta_T}{\sin^2(\theta_I + \theta_T)} \quad (\vec{E} \text{ parallel to the interface})$$

$$R = \frac{\tan^2(\theta_T - \theta_I)}{\tan^2(\theta_T + \theta_I)}, \; T = \frac{\sin 2\theta_I \sin 2\theta_T}{\sin^2(\theta_T + \theta_I)\cos^2(\theta_T - \theta_I)} \quad (\vec{B} \text{ parallel to the interface})$$

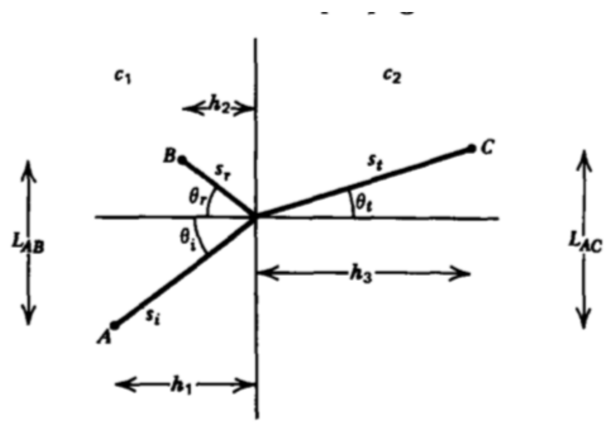

Figure 1: Figure for problem 4.