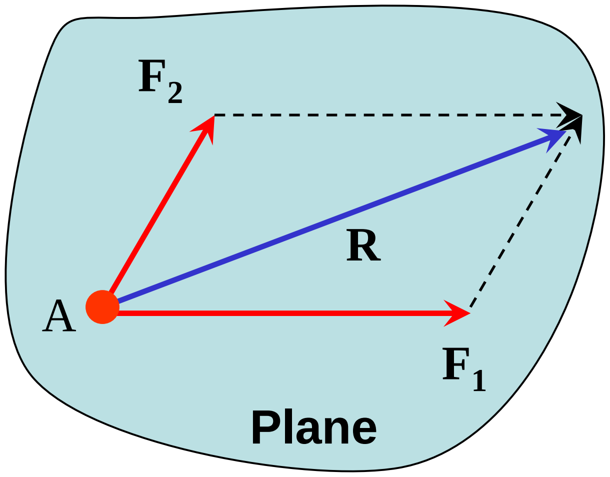
# ME101: Engineering Mechanics

2019-20 (II Semester)

# LECTURE: 2

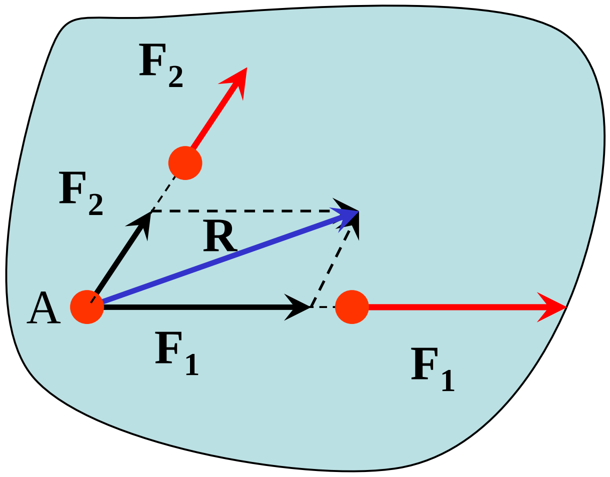
# Force Systems

- **Concurrent forces**
  - Lines of action intersect at a point



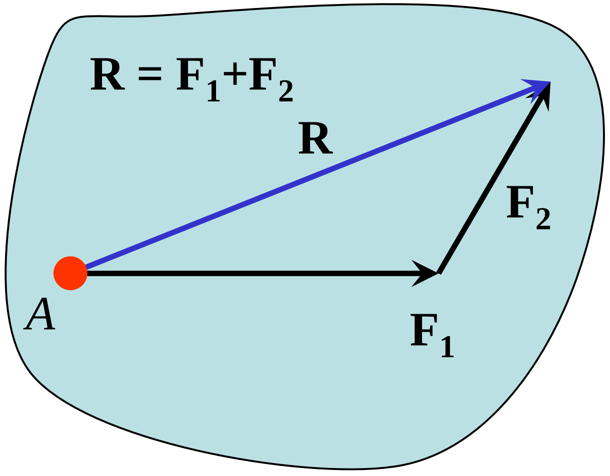

Concurrent Forces

**F₁** and **F₂**




Principle of

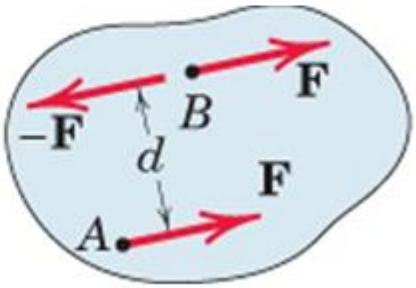
Transmissibility

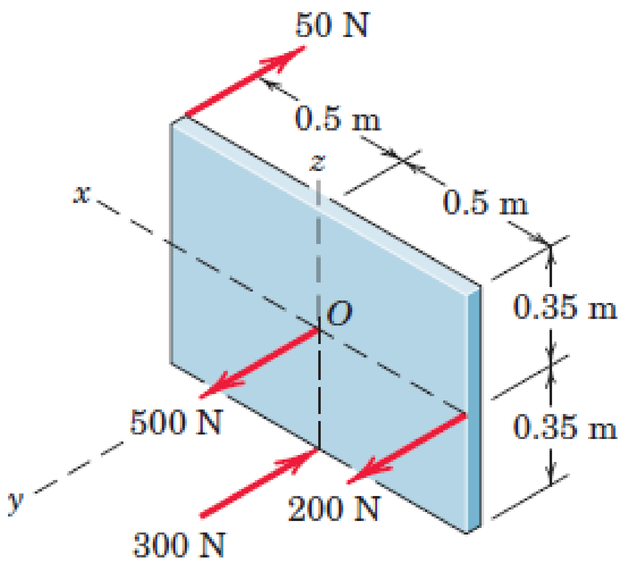


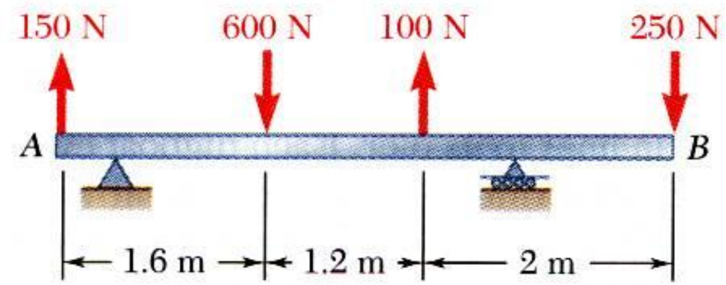

$\mathbf{R} = \mathbf{F_1} + \mathbf{F_2}$

# Force Systems

- **Parallel Forces**
  - Lines of action do not intersect at a point

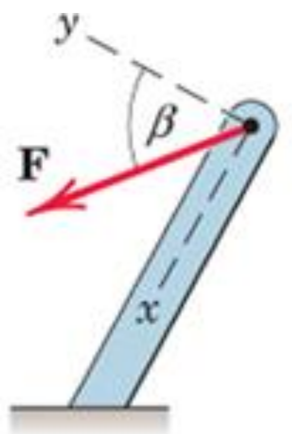


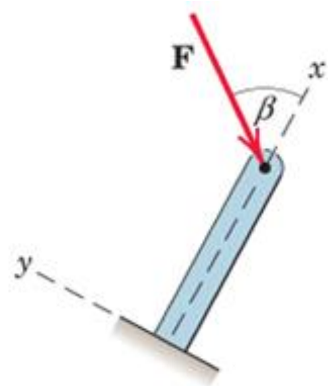



# Components of a Force

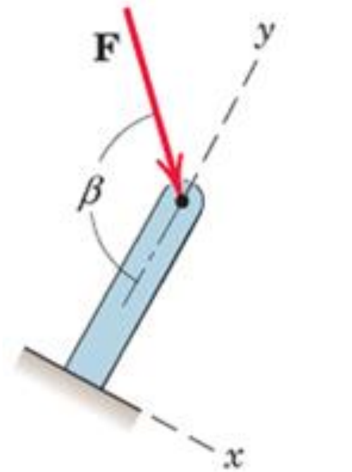
- Examples



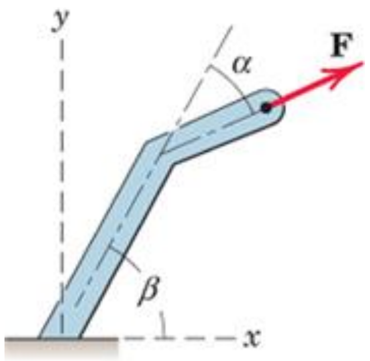

$$F_x = F \sin\beta$$
$$F_y = F \cos\beta$$




$$F_x = -F \cos\beta$$
$$F_y = -F \sin\beta$$




$$F_x = F \sin(\pi - \beta)$$
$$F_y = -F \cos(\pi - \beta)$$

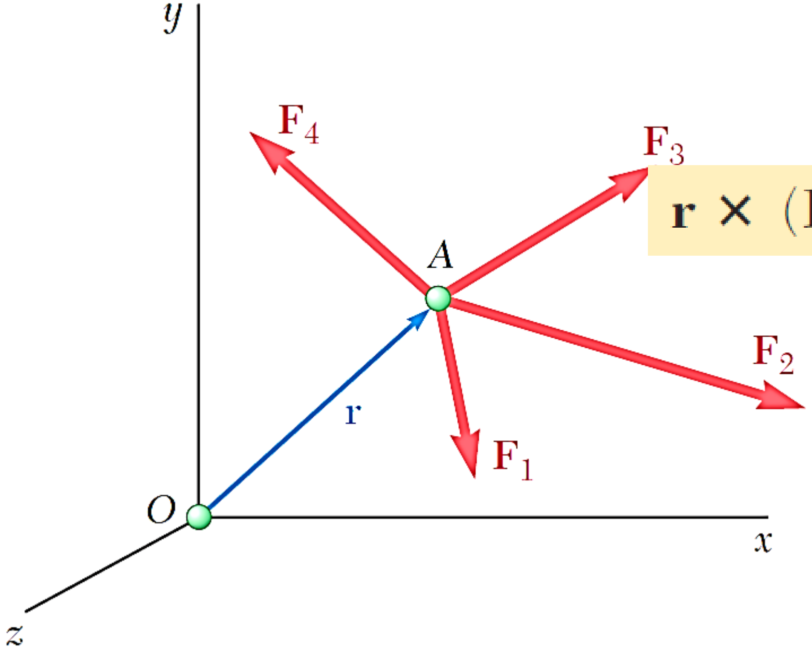



$$F_x = F \cos(\beta - \alpha)$$
$$F_y = F \sin(\beta - \alpha)$$

# Moment of a System of Concurrent Forces

## Varignon's Theorem

– **Moment** of the **resultant** of a system of **concurrent forces** about **a point** is equal to the **sum of the moments of the individual forces** about the **same point**




$$\mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \cdots) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \cdots$$

**Vector cross product**

- **Anti commutative**
- **Not associative**
- **Distributive**

# Rectangular Components of Moment

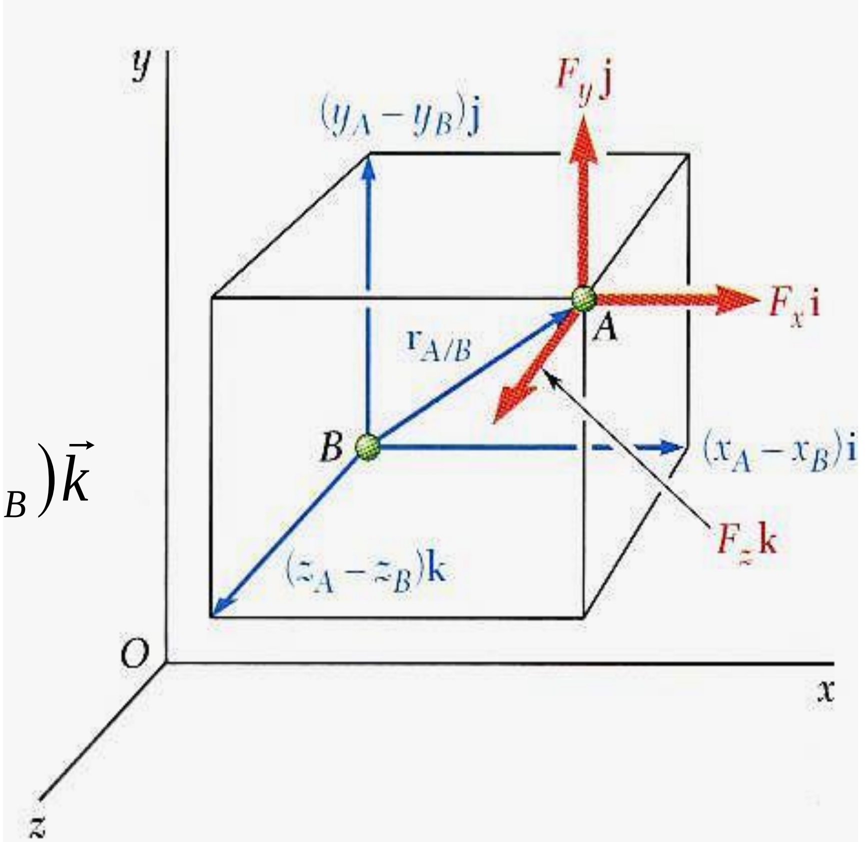
The moment of **F** about *B*,

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

$$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$$

$$= (x_A - x_B)\vec{i} + (y_A - y_B)\vec{j} + (z_A - z_B)\vec{k}$$

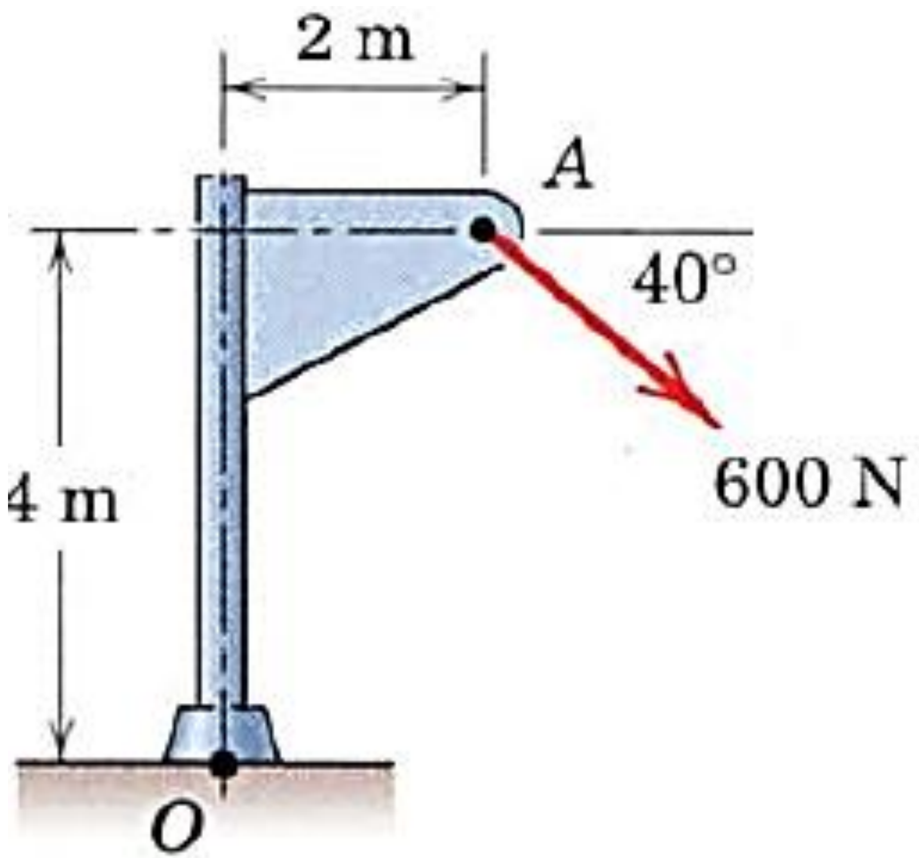
$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$




$$\vec{M}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}$$

# Moment: Example

Calculate the magnitude of the moment about the base point $O$ of the 600-N force in five different ways.



# Moment: Example

**Solution.** (*I*) The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N}\cdot\text{m}$$

(*II*) Replace the force by its rectangular components at $A$

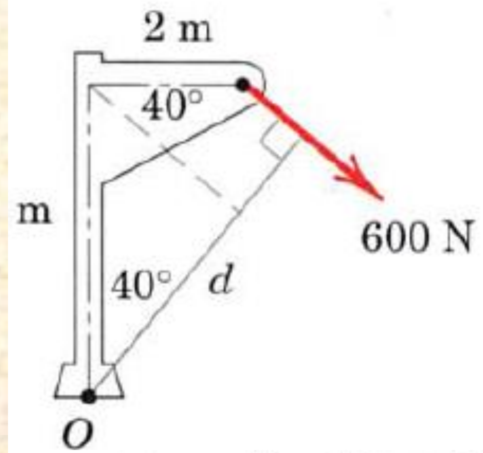
$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

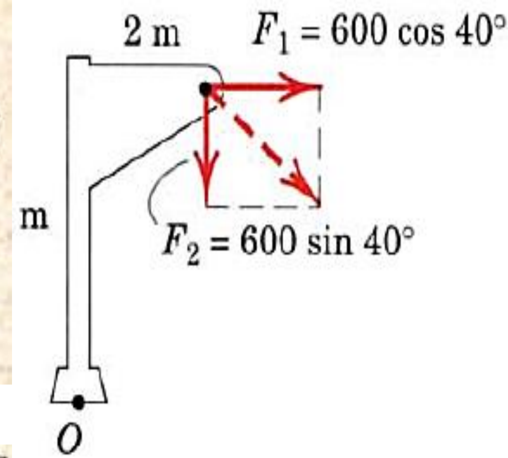
By Varignon's theorem, the moment becomes

$$M_O = 460(4) + 386(2) = 2610 \text{ N}\cdot\text{m}$$

(*III*) By the principle of transmissibility, move the 600-N force along its line of action to point $B$, which eliminates the moment of the component $F_2$. The moment arm of $F_1$ becomes

$$d_1 = 4 + 2 \tan 40° = 5.68 \text{ m}$$



# Moment: Example

and the moment is

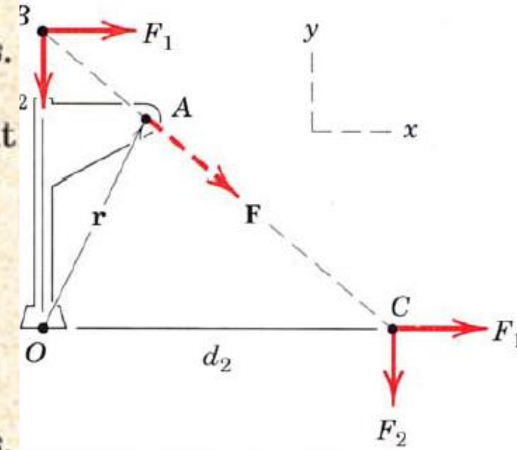
$$M_O = 460(5.68) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

**(IV)** Moving the force to point $C$ eliminates the moment of the component $F_1$. The moment arm of $F_2$ becomes

$$d_2 = 2 + 4 \cot 40° = 6.77 \text{ m}$$

and the moment is

$$M_O = 386(6.77) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$




**(V)** By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have

$$\begin{aligned} \mathbf{M}_O &= \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i} \cos 40° - \mathbf{j} \sin 40°) \\ &= -2610\mathbf{k} \text{ N}\cdot\text{m} \end{aligned}$$

The minus sign indicates that the vector is in the negative $z$-direction. The magnitude of the vector expression is

$$M_O = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

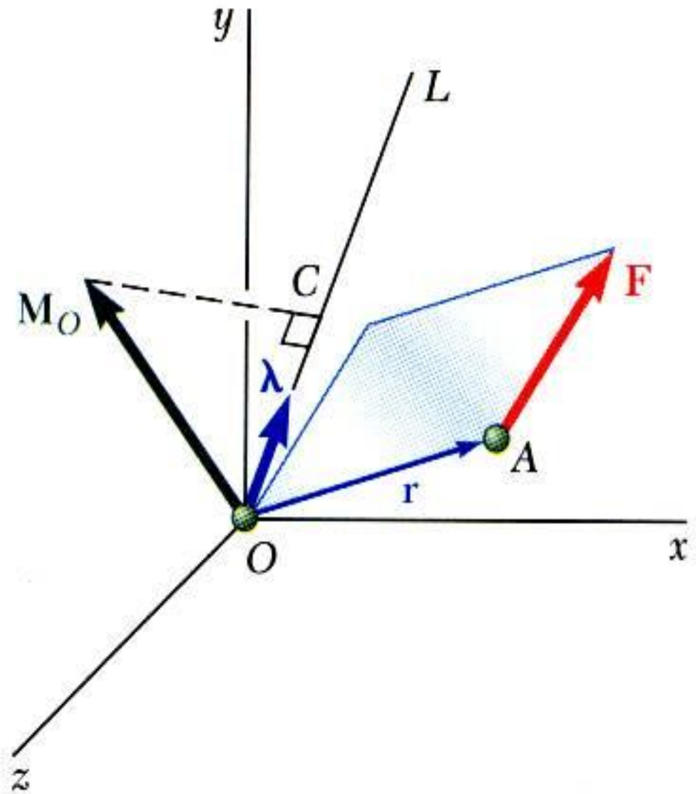
# Moment of a Force About a Given Axis

- Moment $\mathbf{M_O}$ of a force $\mathbf{F}$ applied at the point $\mathbf{A}$ about a point $\mathbf{O}$,

$$\vec{M}_O = \vec{r} \times \vec{F}$$

- **Scalar moment** $M_{OL}$ about an axis OL is the **projection** of the **moment vector** $\mathbf{M_O}$ onto the **axis**,

$$M_{OL} = \vec{\lambda} \bullet \vec{M}_O = \vec{\lambda} \bullet \left(\vec{r} \times \vec{F}\right)$$




**Application** of **Scalar Triple Product**

# Moment of a Force About a Given Axis

## Significance of $M_{OL}$

– $M_{OL}$ is a measure of the **tendency** of the $\mathbf{F}$ to impart a **rigid body rotation** about the axis OL

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

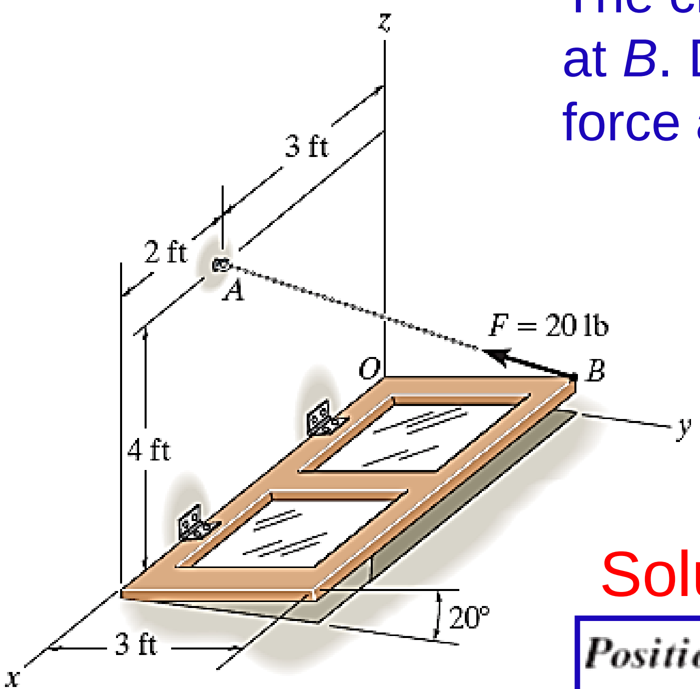
- Moments of $\mathbf{F}$ about the coordinate axes

$$M_x = yF_z - zF_y$$

$$M_y = zF_x - xF_z$$

$$M_z = xF_y - yF_x$$

# Moment: Example




The chain *AB* exerts a force of 20 lb on the door at *B*. Determine the magnitude of the moment of this force along the hinged axis *x* of the door.

Solution:

***Position Vector and Force Vector:***

$$\mathbf{r}_{OA} = \{(3-0)\mathbf{i} + (4-0)\mathbf{k}\}\ \text{ft} = \{3\mathbf{i} + 4\mathbf{k}\}\ \text{ft}$$

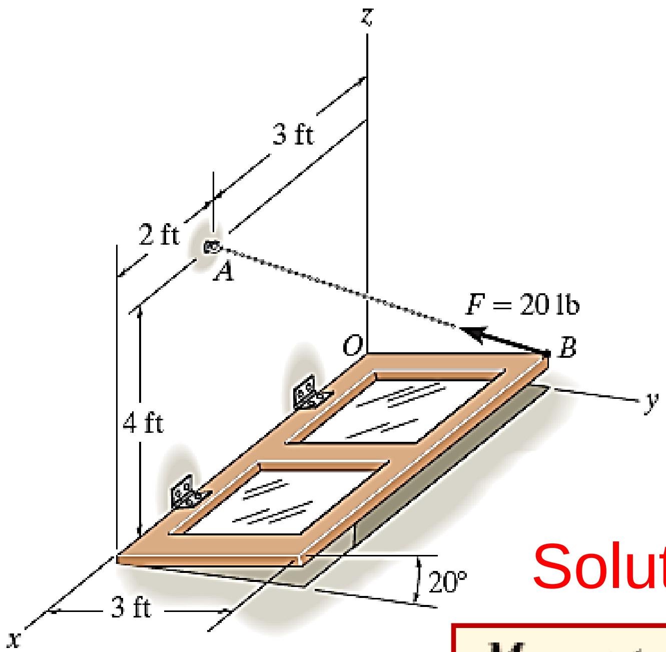
$$\mathbf{r}_{OB} = \{(0-0)\mathbf{i} + (3\cos 20^\circ - 0)\mathbf{j} + (3\sin 20^\circ - 0)\mathbf{k}\}\ \text{ft}$$

$$= \{2.8191\mathbf{j} + 1.0261\mathbf{k}\}\ \text{ft}$$

$$\mathbf{F} = 20\left(\frac{(3-0)\mathbf{i} + (0 - 3\cos 20^\circ)\mathbf{j} + (4 - 3\sin 20^\circ)\mathbf{k}}{\sqrt{(3-0)^2 + (0 - 3\cos 20^\circ)^2 + (4 - 3\sin 20^\circ)^2}}\right)\ \text{lb}$$

$$= \{11.814\mathbf{i} - 11.102\mathbf{j} + 11.712\mathbf{k}\}\ \text{lb}$$

# Moment: Example




Solution:

***Moment of Force* F *About x Axis:*** The unit vector along the $x$ axis is $\mathbf{i}$.

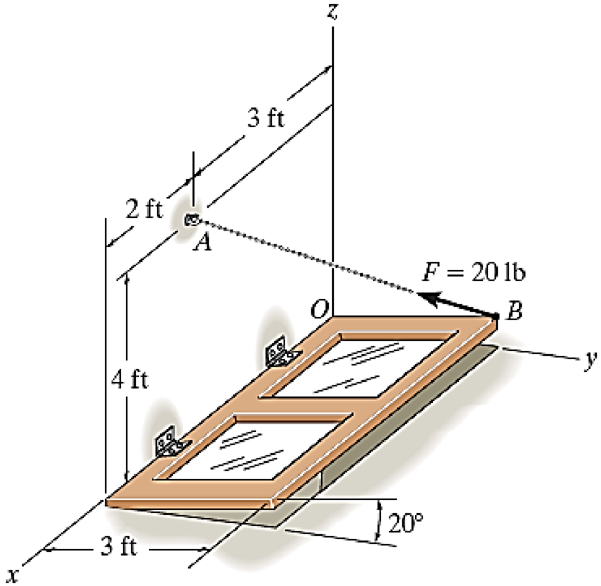
$$M_x = \mathbf{i} \cdot (\mathbf{r}_{OA} \times \mathbf{F})$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 3 & 0 & 4 \\ 11.814 & -11.102 & 11.712 \end{vmatrix}$$

$$= 1[0(11.712) - (-11.102)(4)] - 0 + 0$$

$$= 44.4 \text{ lb} \cdot \text{ft}$$

# Moment: Example




Alternatively

Solution:

$$M_x = \mathbf{i} \cdot (\mathbf{r}_{OB} \times \mathbf{F})$$

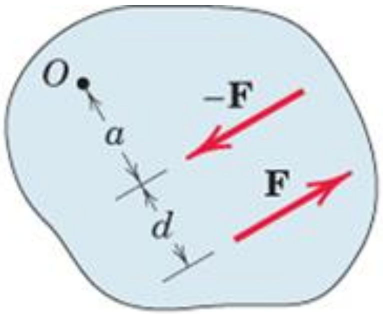
$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2.8191 & 1.0261 \\ 11.814 & -11.102 & 11.712 \end{vmatrix}$$

$$= 1[2.8191(11.712) - (-11.102)(1.0261)] - 0 + 0$$

$$= 44.4 \text{ lb} \cdot \text{ft}$$

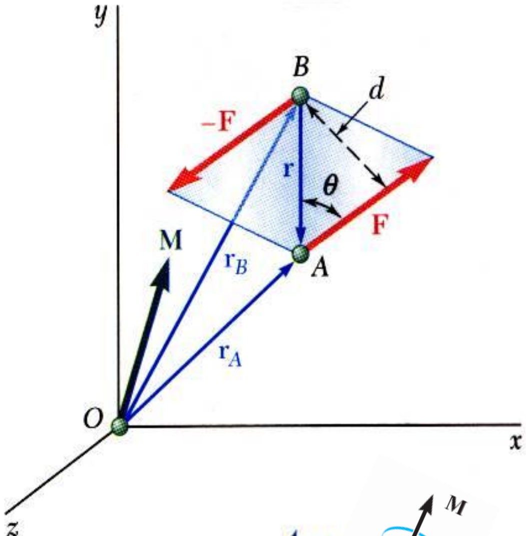
# Moment of a Couple

**Moment** produced by two equal, opposite and noncollinear forces is called a ***couple***.

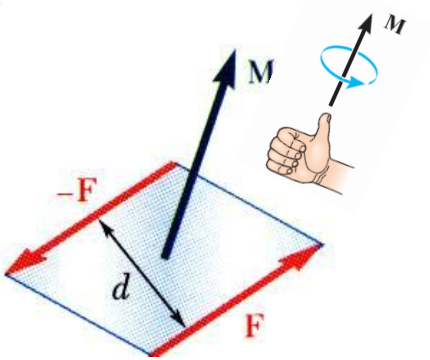



Vector Algebra Method:
Moment of the couple about point O:

$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times \left(-\vec{F}\right)$$
$$= \left(\vec{r}_A - \vec{r}_B\right) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$
$$M = rF\sin\theta = Fd$$



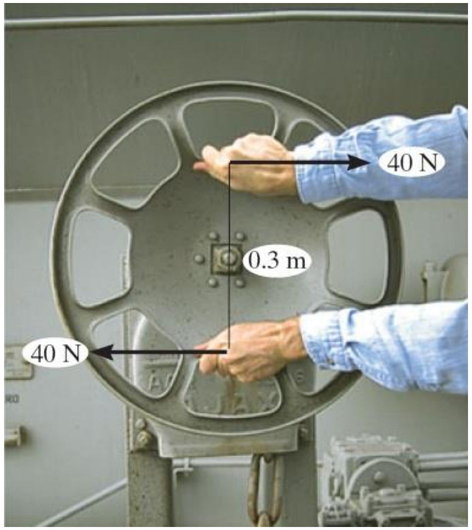

The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.

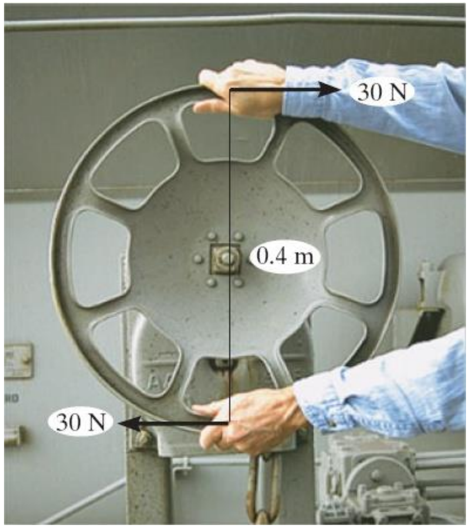


# Couple: Example

Moment reqd to turn the shaft connected at center of the wheel = 12 Nm

- First case: Couple Moment produced by 40 N forces = 12 Nm
- Second case: Couple Moment produced by 30 N forces = 12 Nm




Same couple moment will be produced even if the shaft is not connected at the center of the wheel

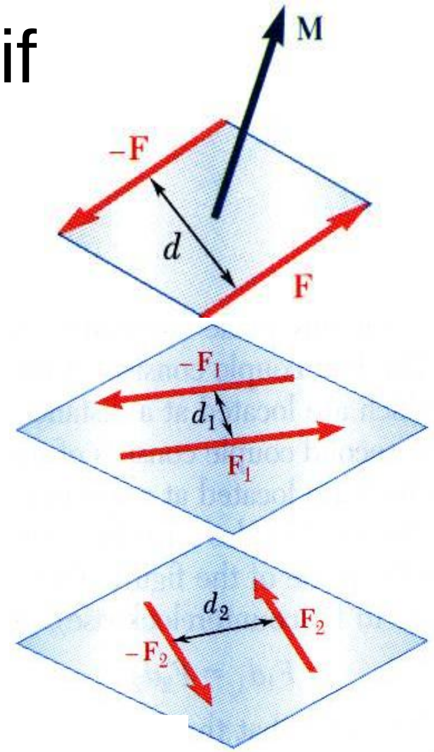
→ Couple Moment is a Free Vector

# Moment of a Couple

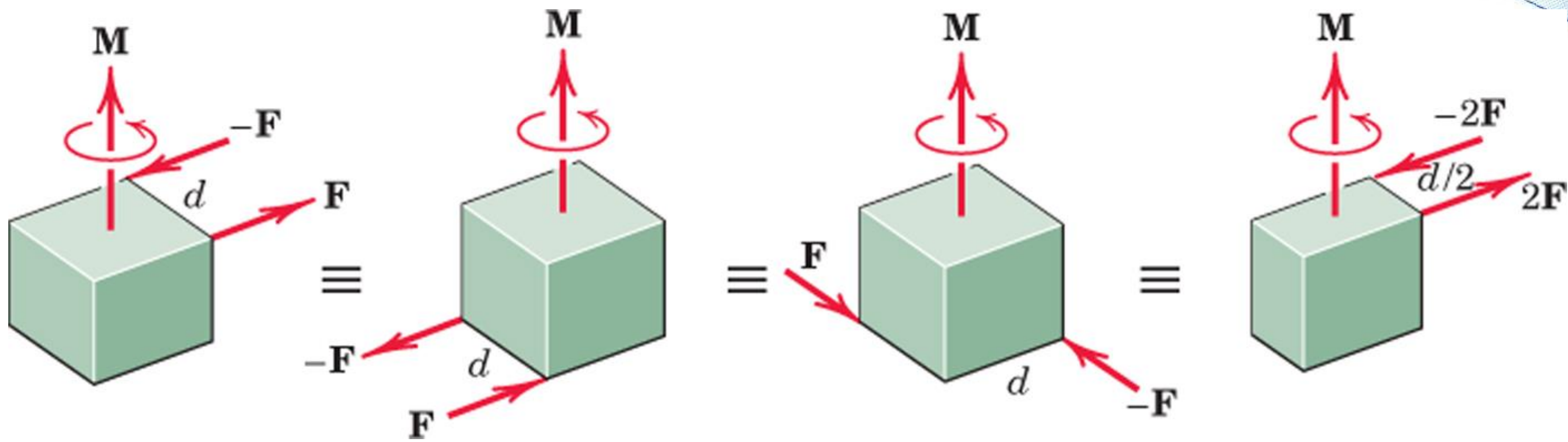
**Two couples** will have **equal moments** if

$$F_1 d_1 = F_2 d_2$$

- the two couples lie in parallel planes, and
- the two couples have the same sense or the tendency to cause rotation in the same direction.




Examples:



# Addition of Couples

- Consider two intersecting planes $P_1$ and $P_2$ with each containing a couple

$$\vec{M}_1 = \vec{r} \times \vec{F}_1 \text{ in plane } P_1$$

$$\vec{M}_2 = \vec{r} \times \vec{F}_2 \text{ in plane } P_2$$

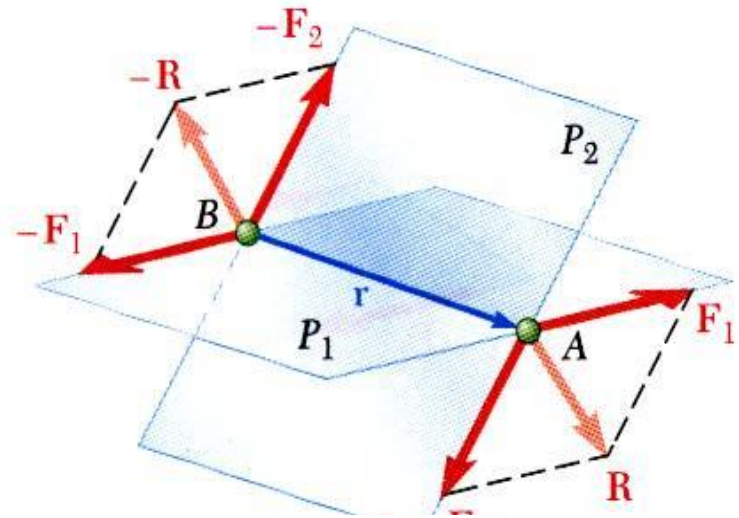
- Resultants of the vectors also form a couple

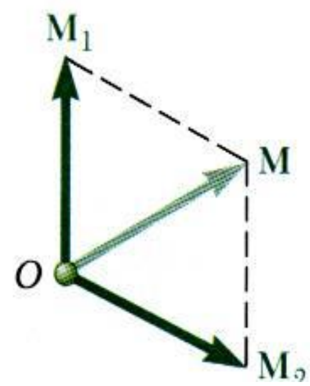
$$\vec{M} = \vec{r} \times \vec{R} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$$

- By Varignon's theorem

$$\vec{M} = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2$$

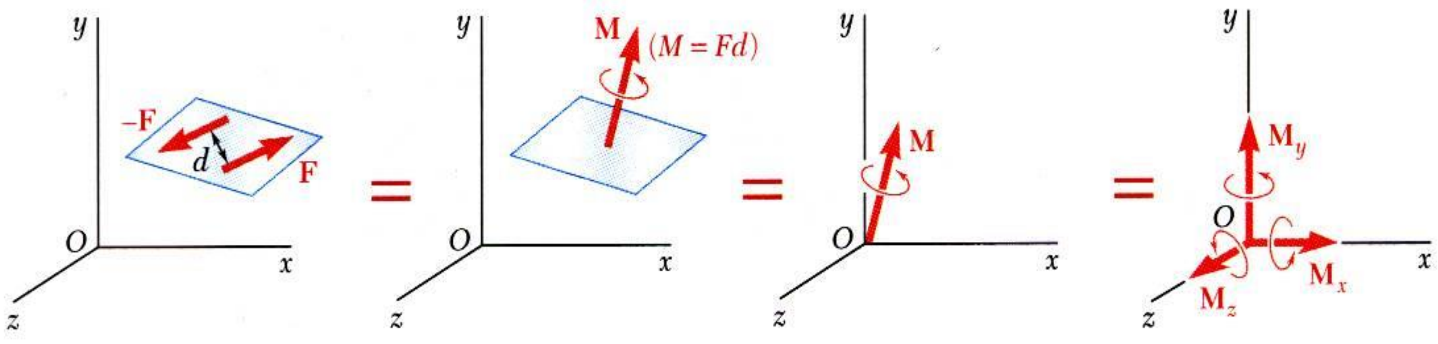
$$= \vec{M}_1 + \vec{M}_2$$




- **Sum** of **two couples** is also a couple that is equal to the **vector sum of the two couples**

# Representation of Couples by Vectors

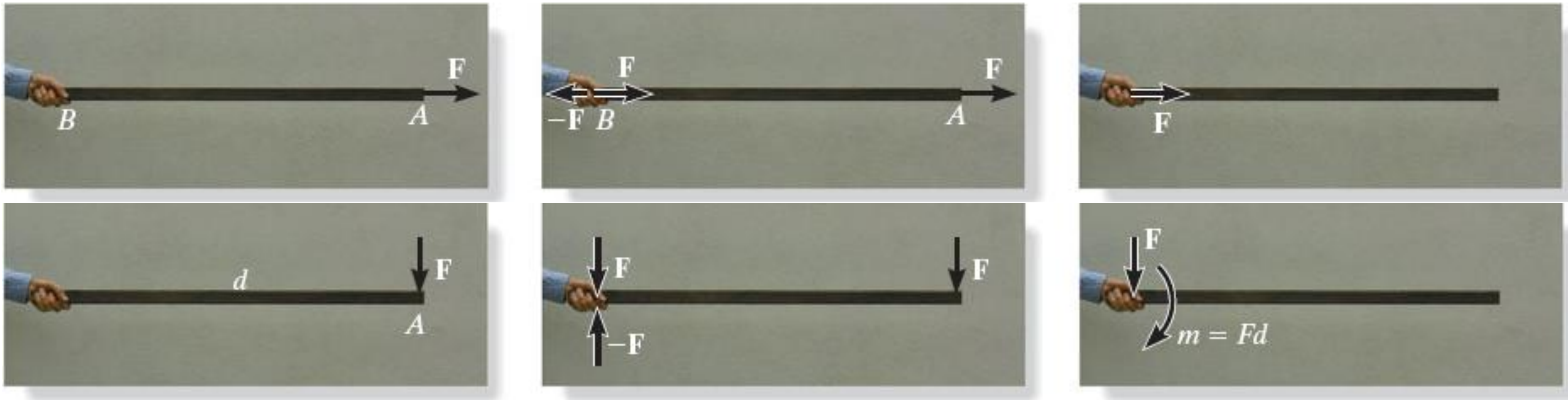


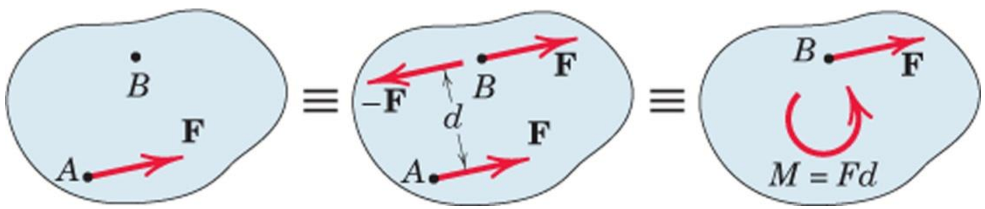

- A couple can be represented by a vector with magnitude and direction equal to the moment of the couple.

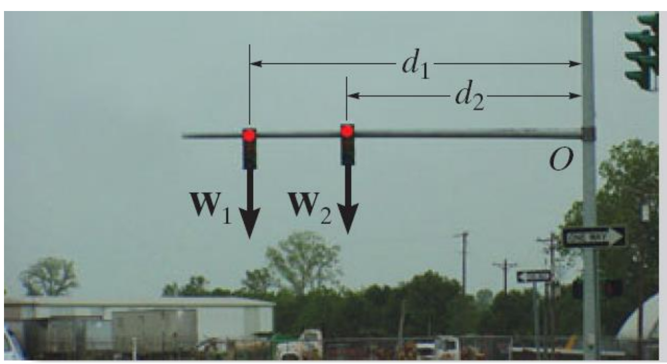
- *Couple vectors* obey the law of addition of vectors.

- Couple vectors are **free vectors**, i.e., the point of application is not significant.
- Couple vectors may be resolved into component vectors.

# Equivalent Systems (Force-Couple Systems)





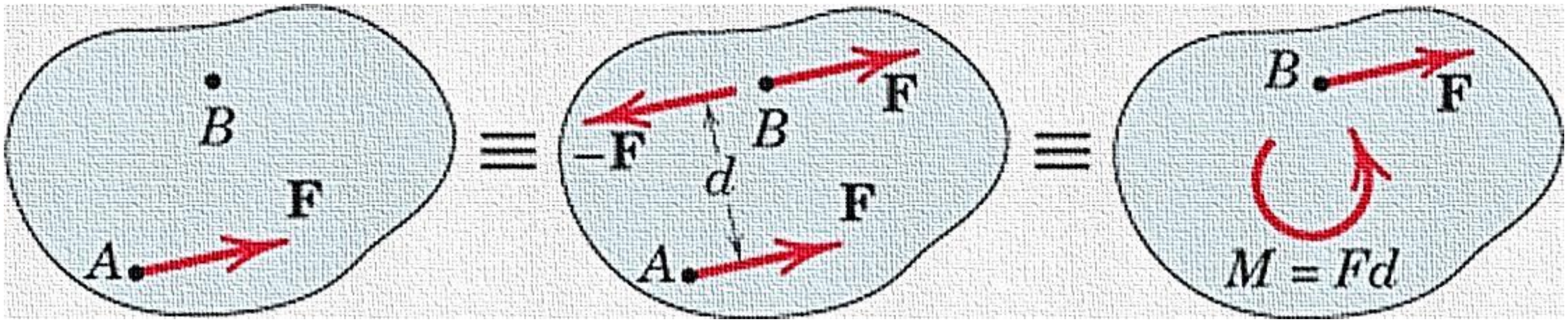
At support O:

$W_R = W_1 + W_2$

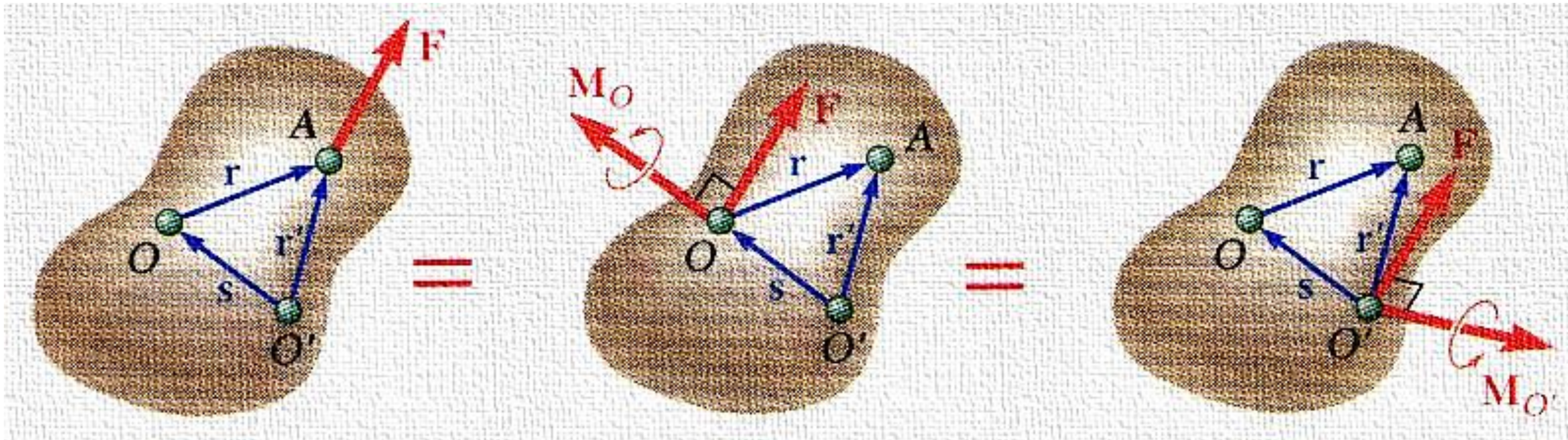
$(M_R)_O = W_1 d_1 + W_2 d_2$

# Equivalent Force and Couple

- Two dimensional plane

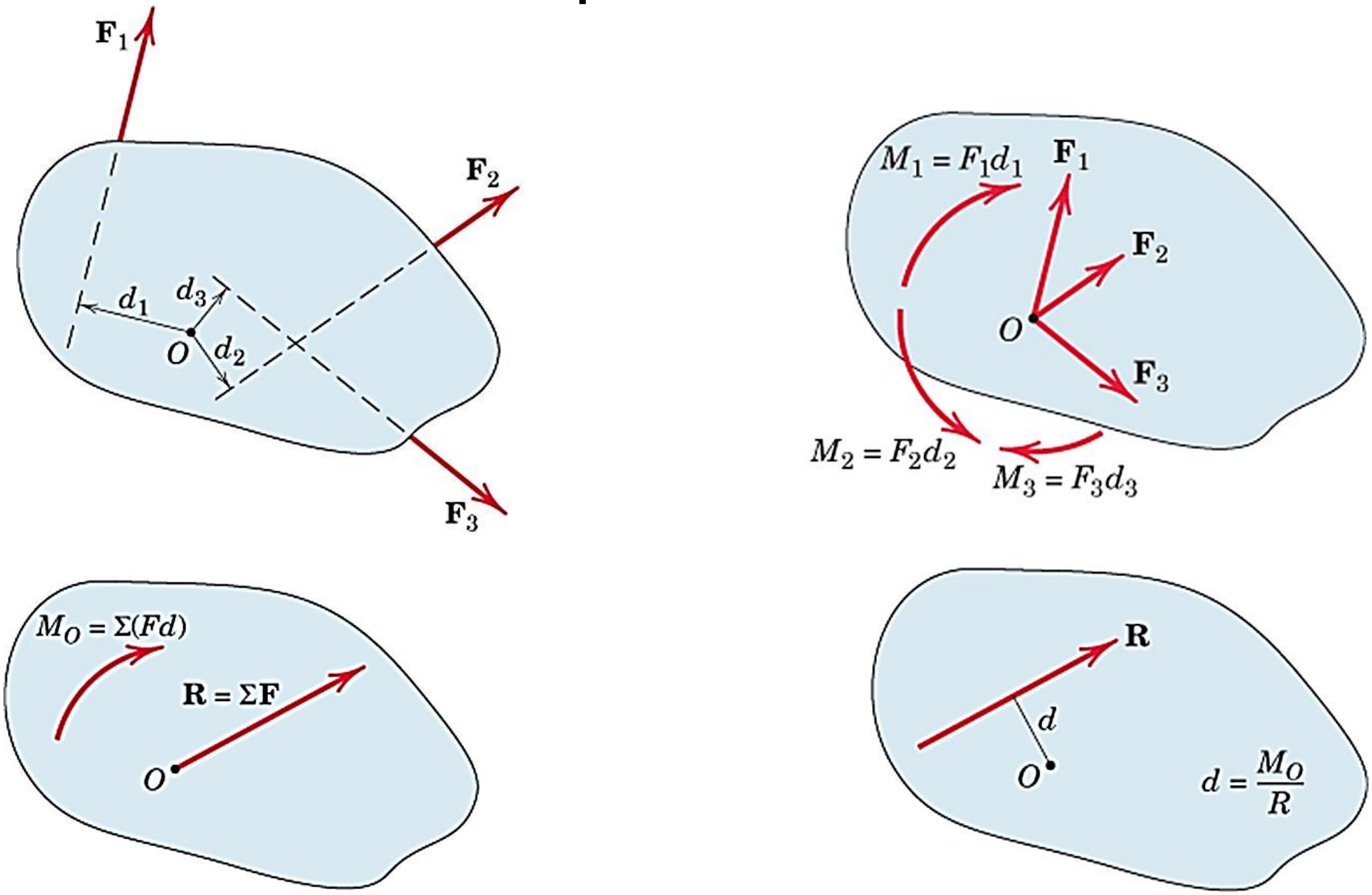



- Three dimensional space

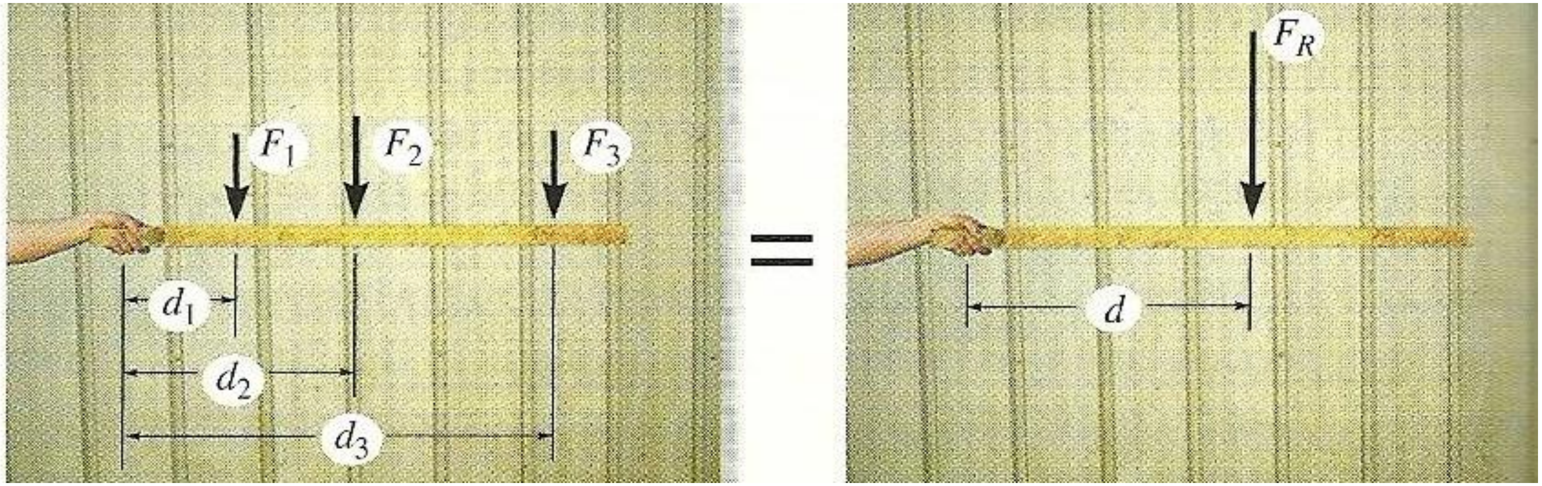


# Resultant of Concurrent Forces

- Two dimensional plane



# Equivalent Systems: Resultants




$F_R = F_1 + F_2 + F_3$

How to find d?

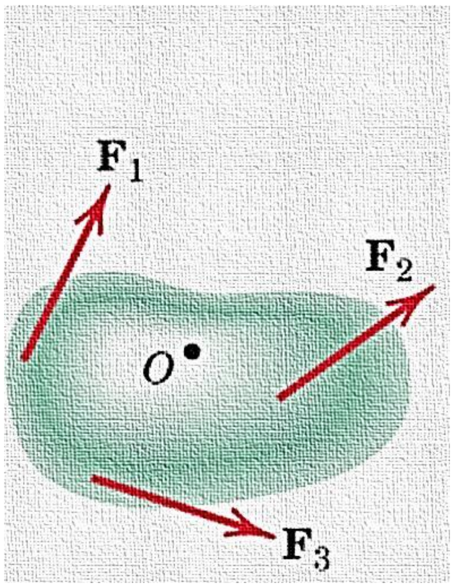
Moment of the Resultant force about the grip must be equal to the moment of the forces about the grip

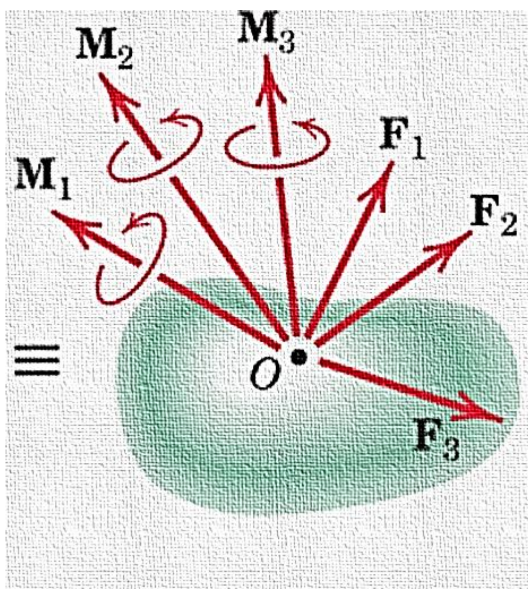
$F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3$

→ Equilibrium Conditions

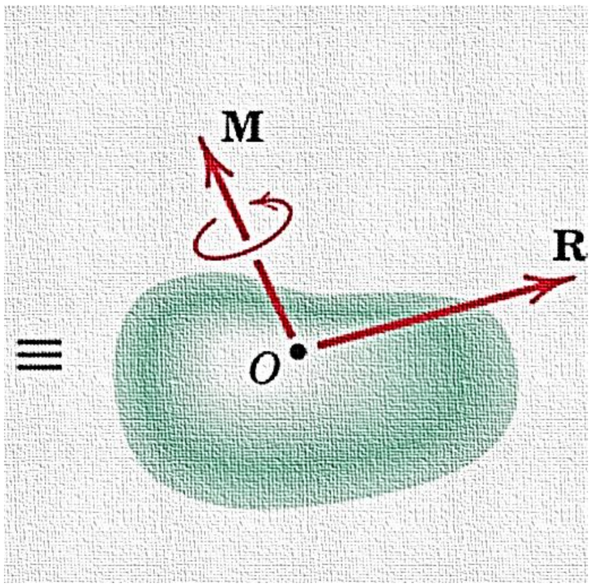
# Resultant of Force System:: 3D

- Three dimensional space



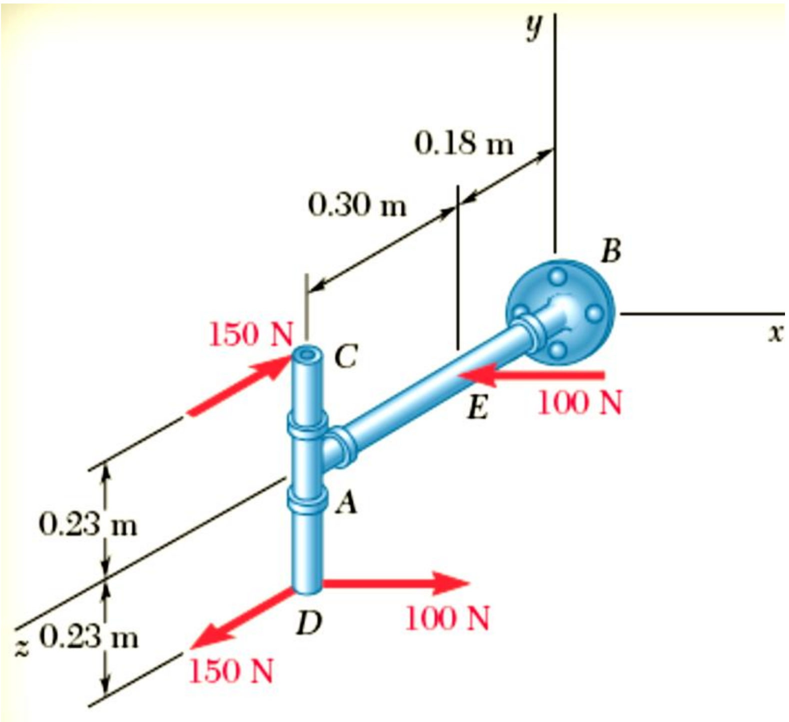




Equivalent force-couple systems for each force




Resultant equivalent force-couple system

# Sample Problem

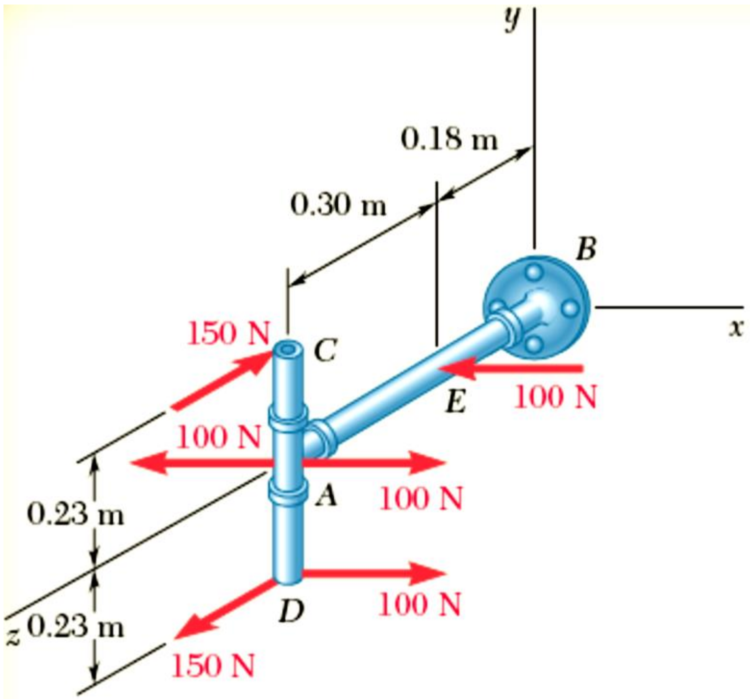



Determine the components of the single couple equivalent to the couples shown.

SOLUTION:

- Attach equal and opposite 100 N forces in the $\pm x$ direction at *A*, thereby producing 3 couples for which the moment components are easily computed.

- Alternatively, compute the sum of the moments of the four forces about an arbitrary single point. The point *D* is a good choice as only two of the forces will produce non-zero moment contributions.

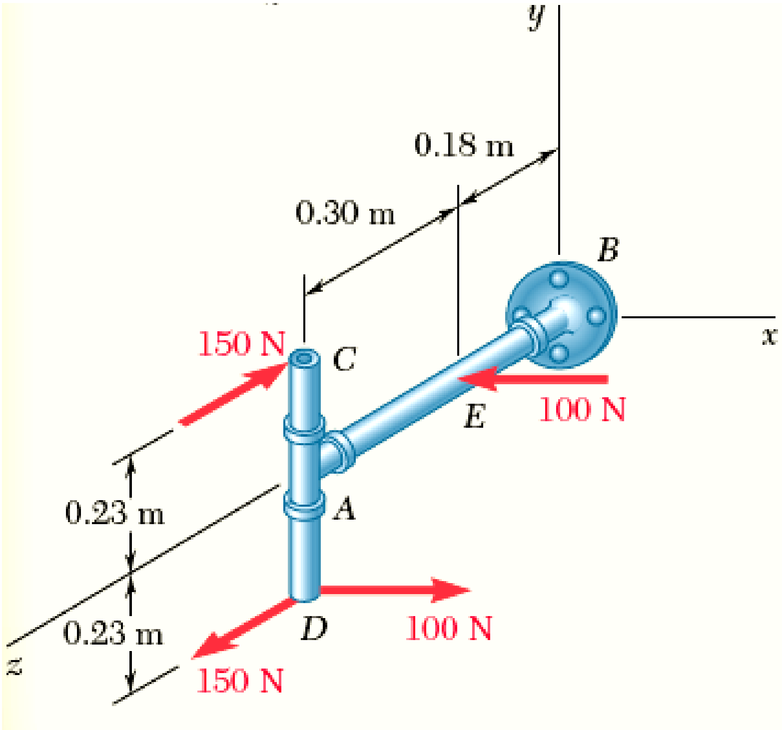



- Attach equal and opposite 100 N forces in the $\pm x$ direction at $A$

- The three couples may be represented by three couple vectors,

$$M_x = -(150\ \text{N})(0.46\ \text{m}) = -69\ \text{N}\cdot\text{m}$$
$$M_y = +(100\ \text{N})(0.3\ \text{m}) = +30\ \text{N}\cdot\text{m}$$
$$M_z = +(100\ \text{N})(0.23\ \text{m}) = +23\ \text{N}\cdot\text{m}$$

$$\boxed{\begin{aligned}\mathbf{M} = &-(69\ \text{N}\cdot\text{m})\mathbf{i} + (30\ \text{N}\cdot\text{m})\mathbf{j}\\ &+ (23\ \text{N}\cdot\text{m})\mathbf{k}\end{aligned}}$$




- Alternatively, compute the sum of the moments of the four forces about *D*.

- Only the forces at *C* and *E* contribute to the moment about *D*.

$$\mathbf{M} = \mathbf{M}_D = (0.46\ \text{m})\mathbf{j} \times (-150\ \text{N})\mathbf{k} + [(0.23\ \text{m})\mathbf{j} - (0.3\ \text{m})\mathbf{k}] \times (-100\ \text{N})\mathbf{i}$$

$$\boxed{\mathbf{M} = -(69\ \text{N}\cdot\text{m})\mathbf{i} + (30\ \text{N}\cdot\text{m})\mathbf{j} + (23\ \text{N}\cdot\text{m})\mathbf{k}}$$