Magnetically Coupled Circuits

In case of an inductor, the voltage is proportional to the time rate of change of current. It is given as

$$v_1 = L\frac{di}{dt}$$

L is called as self inductance. A current flowing in one coil can produce a flux which can link a second coil present nearby. If the current is time varying, then the flux produced by this current will also be time varying. This time varying flux will induce a voltage in the second coil. The induced voltage or emf in the second coil can be given as

$$v_2 = M\frac{di_1}{dt}$$

where V_2 is the voltage induced in the second coil by the current i_1 in the first coil. The constant ***M*** is the mutual inductance between the two coils.

Dot Convention

The polarity of the induced voltage is decided by the dot convention. According to this convention if the current enters into the dotted terminal of the first coil then the voltage induced by this current will have its positive polarity at the corresponding dotted terminal of the second coil. If the current leaves the dotted terminal, then the corresponding dotted terminal will be marked with the negative polarity of the induced voltage. Fig. 10 shows the signs of the induced voltages in the second coil with the dotted terminals.

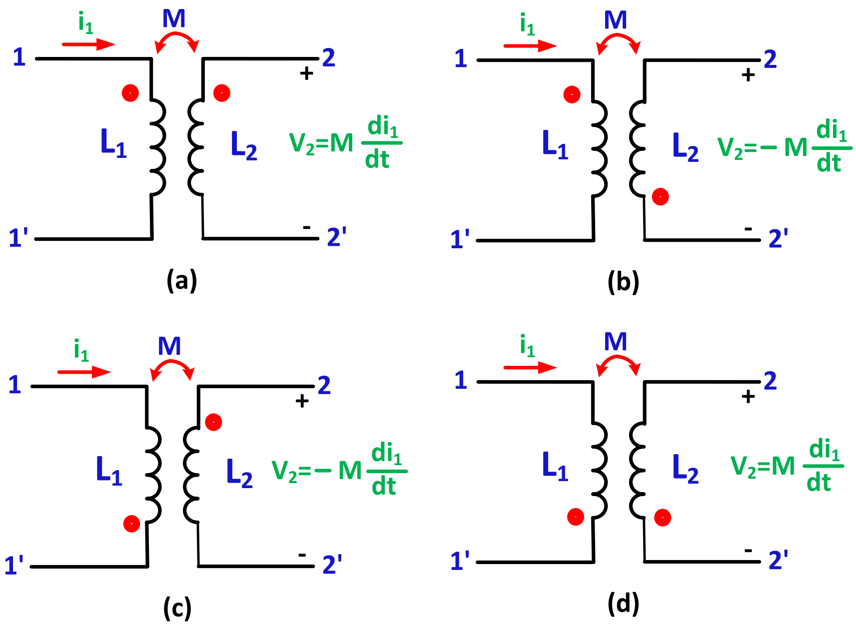

Fig. 10

V_2 is the voltage at the terminal ***22'***. In Fig. 10(a), the current i_1 enters the dotted terminal. The voltage induced by this current in the second coil will have its positive polarity at the corresponding dotted terminal. Hence the terminal voltage $v_2 = M\frac{di_1}{dt}$. In Fig. 10 (b) the dotted terminal in the second coil will be positive. So the terminal voltage $v_2 = -M\frac{di_1}{dt}$. In Fig. 10(c), the current i_1, leaves the dotted terminal. Hence, the corresponding dotted terminal in the second coil, i.e. the terminal 2, will have the negative polarity of the induced voltage. So the terminal voltage will be $v_2 = -M\frac{di_1}{dt}$. With similar analysis, the terminal voltage for the circuit in Fig. 10(d) can be determined as $v_2 = M\frac{di_1}{dt}$.

Analysis of mutually coupled circuits

Normal circuit analysis tools can be used for mutually coupled circuits. The induced voltage in a coil due to current in another coil is the additional term in this case. The important point in such an analysis is the polarity of the induced voltage. For the circuit shown in Fig. 11(a) the KVL equations will be

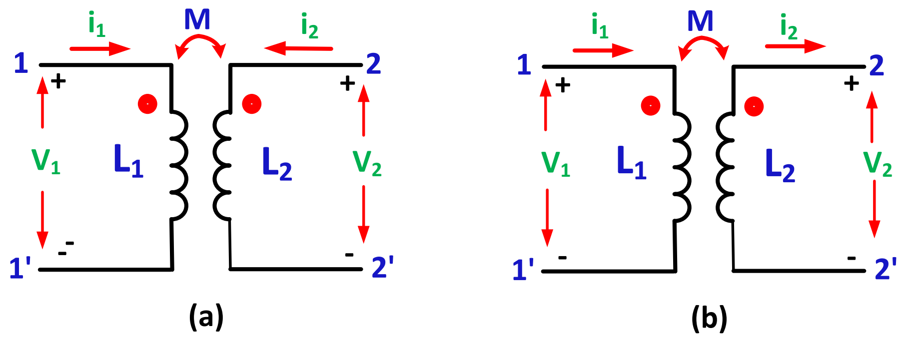

Fig. 11

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$v_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

The induced voltage in coil ***11'*** will be $M\frac{di_2}{dt}$. As the current i_2 enters into the dotted terminal in coil ***22'***, the polarity of the induced voltage in coil ***11'*** will be positive in its dotted terminal. For the circuit shown in Fig. 11(b), the current i_2 leaves the dotted terminal. So the polarity of the induced voltage will be negative in the first coil. The KVL equations for the circuit in Fig. 11(b) are

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$
$$v_2 = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

For phasor analysis of these circuits, the self inductances can be replaced with an impedance term of the form of ***jwL*** and the mutual inductance term will be replaced with ***jwM***. The KVL equations for the circuit in Fig. 11(b) will be

$$V_1 = j\omega L_1 I_1 - j\omega M I_2$$
$$V_2 = -j\omega L_2 I_2 + j\omega M I_1$$

Placement of dots

Dot convention enables us to supress the physical construction of the coils by placing a dot at one terminal of each coil. The placement of the dots are decided by the direction of the flux and the polarity of the induced voltage. Fig. 12(a) shows two coils wound around a magnetic core. If terminal ***1*** in coil ***11'*** is dotted, then we need to find the corresponding dotted terminal of coil ***22***'. This can be decided by the steps given below.

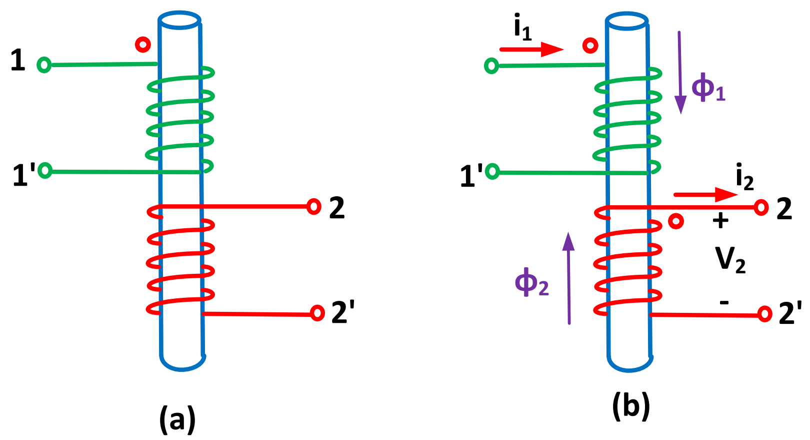

Fig. 12

- Assume that a current i_1 in ***11'***.
- Based on the direction of the current i_1, flux Φ_1 will be produced. The direction of this flux can be found using right hand thumb rule or right hand screw rule. In Fig. 12(b), the direction of this flux is downward.
- Assuming the flux as time varying, this flux will induce a voltage in the coil ***22'*** This induced voltage will oppose its cause, i.e. the flux responsible for its

generation. It should produce a flux (Φ_2) in the opposite direction. This will require the current in coil ***22'*** to flow out of terminal ***2*** as shown in Fig. 12(b).

- For this direction of the current i_2, the induced voltage (V_2) responsible for this current must have its positive polarity at terminal ***2*** as shown in Fig. 12(b).
- The current i_1 is entering its dotted terminal. The corresponding dotted terminal must be positive. Hence the positive polarity terminal in the second coil will be the corresponding dotted terminal.

This can be extended to more than two coils.