Quantum Mechanics (PH101)
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Tutorial-10
due on Wednesday, 20th of November, 2019 (8:00Hrs IST)

1. Consider a particle's wave function at $t = 0$ as $\Psi(x, 0) = c_1\psi_1(x) + c_2\psi_2(x)$. Consider c_1 and c_2 to be real, where, $\psi_1(x)$ and $\psi_2(x)$ are the stationary eigen states. What will the wave function $\Psi(x, t)$ at a later time $t > 0$? Find the probability density and comment over its behaviour with time. Do you find that the $\psi(x, t)$ is also the stationary state?

2. A particle of mass m is confined to a one dimensional infinite well in the region $0 \leq x \leq a$. At $t = 0$ its normalized wave function is

$$\Psi(x,\ t = 0) = \sqrt{\frac{8}{5a}}\left[1 + cos\left(\frac{\pi x}{a}\right)\right]\ sin\ \left(\frac{\pi x}{a}\right)$$

   (a) What is the wave function at a later time $t = t_0$?
   (b) What is the average energy of the system at $t = 0$ and at $t = t_0$?
   (c) What is the probability that the particle is found in the left half of the box (i.e., in the region $0 \leq x \leq a/2$) at $t = t_0$?

3. Using the uncertainty relation $\Delta p \Delta x \geq \hbar/2$, estimate the ground state energy of the harmonic oscillator.

4. Consider that at $t = 0$ the particle is in the state

$$\psi(x) = \frac{1}{\sqrt{2}}[\phi_0(x) + \phi_1(x)]$$

where, $\phi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-m\omega x^2/2\hbar}$ and $\phi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \sqrt{\frac{2m\omega}{\hbar}} x e^{-m\omega x^2/2\hbar}$ are the stationary state eigenfunction corresponding to the ground and first excited state of the one-dimensional Harmonic oscillator respectively. Compute the $< x >$ at time $t > 0$.