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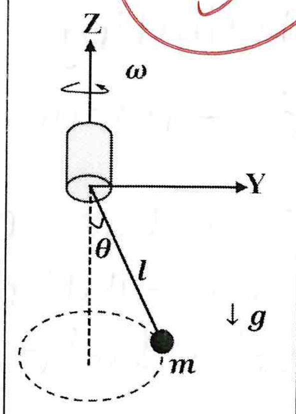
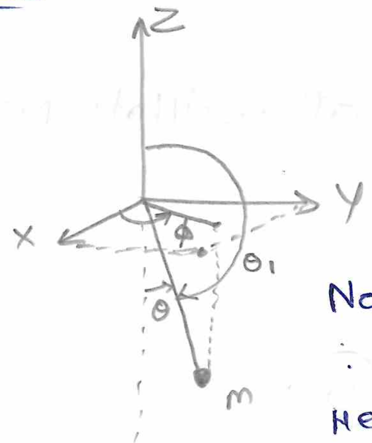
Q2	**[5 marks]** In a simple pendulum, a bob of mass m is hanged using an inextensible and massless string of length l. The pendulum oscillates in YZ plane. Further, the motion of the pendulum is controlled by a motor in such a way that the plane of pendulum oscillation rotates at a constant angular speed ω about Z-axis. Acceleration due to gravity (g) acts downwards. Choose appropriate generalized coordinates from the variables marked in the figure. (i) Express the kinetic and potential energies in terms of generalized coordinates and generalized velocities. (ii) Write down the Lagrangian of the system. (iii) Calculate the equation of motion of mass m. (iv) Considering small θ ($i.e., \cos\theta \approx 1, \sin\theta \approx \theta$), find the frequency of oscillation.	

<u>Soln</u>:



Let ϕ be the angle made by the projection of string on XY plane with +ve x axis.

$\therefore \phi = \omega t$

We use spherical polar coordinate logic to solve the question.

Here, $l = R$, $\theta_1 = 180^\circ - \theta$, $\phi = \omega t$

No. of generalized coordinates $= 1$

$\therefore$ Let θ be the generalised coordinate

Here, $\dot{R} = 0$ as $l = \text{const}$, $\dot{\theta}_1 = -\dot{\theta}$, $\dot{\phi} = \omega$

$\therefore$ Velocity of mass 'm' in spherical polar coordinate is

$$v = \sqrt{\dot{R}^2 + R^2\dot{\theta}_1^2 + R^2\sin^2\theta_1\,\dot{\phi}^2}$$

$$\therefore v^2 = l^2\dot{\theta}^2 + l^2\sin^2\theta\,\omega^2$$

$$\therefore T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(l^2\dot{\theta}^2 + l^2\sin^2\theta\,\omega^2\right) \quad \text{--- (1)}$$

Let $U = 0$ at origin,

Now $z = l\cos\theta_1 = l\cos(180^\circ - \theta) = -l\cos\theta$

$$U = mgz = -mgl\cos\theta \quad \text{--- (2)}$$

$$\therefore L = T - U = \frac{1}{2}m\left(l^2\dot{\theta}^2 + l^2\sin^2\theta\,\omega^2\right) + mgl\cos\theta \quad \text{--- (3)}$$

$\therefore$ From the Euler-Lagrange equation, we have

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\therefore \frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta} \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = ml^2\omega^2 \sin\theta\cos\theta - mgl\sin\theta$$

$\therefore$ Substituting these values in Lagrange's eq^n, we get

$$ml^2\ddot{\theta} - ml^2\omega^2\sin\theta\cos\theta + mgl\sin\theta = 0$$

$$\therefore l\ddot{\theta} - l\omega^2\sin\theta\cos\theta + g\sin\theta = 0 \quad \text{--- (4)}$$

Now, for oscillations, $\sin\theta \approx \theta$, $\cos\theta \approx 1$ as θ is small

$$\therefore l\ddot{\theta} - l\omega^2\theta + g\theta = 0$$

$$\Rightarrow l\ddot{\theta} = -\theta(g - l\omega^2)$$

$$\therefore \ddot{\theta} = -\theta\left(\frac{g}{l} - \omega^2\right)$$

If ω_0 be the angular frequency of oscillations.

$$\therefore \omega_0 = \sqrt{\frac{g}{l} - \omega^2} = 2\pi f$$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{g}{l} - \omega^2} \text{ Hz} \quad \text{--- (5)}$$

f is frequency.