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Mid-Semester Examination

PHYSICS II: PH102

March 2, 2020

Indian Institute of Technology Guwahati

Guwahati, India - 781031

Please read the instructions carefully

a) **Write your name and roll number on each page**. There is a total of **three sheets** in this booklet.

b) All questions are compulsory.

c) You **must** write all parts of the answers in the space provided for the given questions.

d) Write **only the final answer** in this booklet. There is **no partial credit** for intermediate steps leading to the final answer.

e) You **must** write the final answer only in the space provided for the given question. Answers written outside the box will not be evaluated. It is *advised* that you solve the problems in the rough book (usual answer book) and then copy the final answer in the space provided for that problem.

Useful Formulas

Divergence in spherical coordinates: $\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$.

Divergence in cylindrical coordinates: $\vec{\nabla}\cdot\vec{V} = \frac{1}{s}\frac{\partial}{\partial s}(sV_s) + \frac{1}{s}\frac{\partial V_\phi}{\partial\phi} + \frac{\partial V_z}{\partial z}$.

Hyperbolic function: $\sinh x = \frac{1}{2}(e^x - e^{-x})$, $\cosh x = \frac{1}{2}(e^x + e^{-x})$.

Examiner's Signature	Student's Signature	Invigilator's Signature

PH102. Electromagnetism
Mid-semester Exam

March 2, 2020
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Question 1: This question contains **two** parts; Write only the final answers of them in the corresponding space provided.

(a) Find the values of x where the Dirac-delta function $\delta(\sin x)$ is non-vanishing. [1]

Answer: $x = n\pi$, $n \in Z$ (integers)

(b) Using the formula $\delta(y(x)) = \sum_i \frac{\delta(x - x_i)}{|\frac{dy}{dx}|_{x=x_i}}$, where x_i is the i^{th} root of $y(x) = 0$, evaluate

$$\int_{-\infty}^{+\infty} dx\ \delta(\sin x)\ e^{-|x|}\ .$$

[2]

Answer: $\frac{1+e^{-\pi}}{1-e^{-\pi}}$

Question 2: Consider an electric displacement vector $\vec{D} = zs\cos^2\phi\hat{z}$ units in cylindrical coordinates in some region of space. Determine the following:

(a) The free volume charge density at any point. [1]

Answer: $s\cos^2\phi$

(b) The total free volume charge enclosed within the cylinder of radius 1 unit with $-2 \leq z \leq 2$ units. [2]

Answer: $\frac{4\pi}{3}$

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Question 3: This question consists of **three** parts. Write only the final answers at the respective spaces.

a) Consider a uniformly charged sphere of radius R with total charge Q. Find the electric field at a point $\vec{r}$ inside the sphere. [1]

Answer: $\frac{Q\vec{r}}{4\pi\varepsilon_0 R^3}$ 1

b) Determine the z-component of the force on a volume element $d\tau$ at any position $\vec{r}$ in the northern hemisphere. [2]

Answer: $\frac{3Q^2 d\tau}{16\pi^2 R^6 \varepsilon_0}(\vec{r}\cdot\hat{k})$ 2

c) Using the above results find the net force that the southern hemisphere exerts on the northern hemisphere. [2]

Answer: $\frac{3Q^2}{64\pi R^2 \varepsilon_0}\hat{k}$ 2

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Question 4: This question consists of **two** parts. Write only the final answers at the respective spaces.

a) Find the potential in the vicinity of an infinite straight line charge with uniform line charge density λ. [1]

Answer:

$V(r) = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r}{e}\right)$, r is distance from line charge

e is ref. point.

b) The above infinite straight line charge is placed at a distance d above an infinite grounded conducting plane. Let's say that the wire runs parallel to x-axis, directly above the grounded conducting plane situated at $z = 0$.

(i) What is/are the boundary surface/s for the region $z > 0$? Also specify the boundary conditions necessary for solving the Poisson's equation. [0.5 + 0.5]

Answer: $z = 0$ plane , upper half hemisphere cylinder $S_{+\infty}$.

$V(z=0) = 0$

(ii) Find the potential in the region above the plane. [2]

Answer:

$$V(x,y,z) = \frac{-\lambda}{4\pi\epsilon_0} \ln\left(\frac{y^2+(z+d)^2}{y^2+(z-d)^2}\right)$$

2 ; $z > 0$

(iii) Find the charge density σ induced at any point on the conducting plane. [2]

Answer:

$$\sigma(x,y) = \frac{-\lambda d}{\pi(y^2+d^2)}$$

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Question 5: This question consists of **three** parts. Write only the final answers at the respective spaces.

Consider an uncharged polarized dielectric sphere of radius R with "frozen in" polarization

$$\vec{P} = -\frac{\rho_0}{3}\left(1 - \frac{3r}{4R}\right)\vec{r}\,,$$

where ρ_0 is a positive constant. Find the following:

a) Surface and volume bound charge densities. [1 + 1]

Answer: $\sigma_b = -\frac{\rho_0 R}{12}$, $\rho_b = \frac{\rho_0}{3}\left(3 - \frac{3r}{R}\right)$

$\rho_b = \rho_0\left(1 - \frac{r}{R}\right)$ 2

b) Magnitude of the electric fields as a function of distance from the center, both inside and outside the polarized sphere. [1 + 2]

Answer: $\vec{E}_{out} = 0$, $\vec{E}_{in} = \frac{\rho_0}{3\varepsilon_0}\left(1 - \frac{3r}{4R}\right)\vec{r}$ 3

c) The maximum value of the electric field E_{max} and the corresponding distance r_{max} from the center. [1 + 1]

Answer: $E_{max} = \frac{\rho_0}{3\varepsilon_0}$, $r_{max} = 0$

$E_{max} = \frac{\rho_0 R}{9\varepsilon_0}$, $r_{max} = \frac{2R}{3}$ 2

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Question 6: This question consists of **four** parts. Write only the final answers at the respective spaces.

Consider a vector field $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$, and the part of the paraboloid, $z = \frac{x^2}{4} + \frac{y^2}{9}$, such that $z \leq 1$.

a) Find the unit vector on the curved surface of the paraboloid corresponding to traversing its contour at $z = 1$ in the positive (counter clockwise) direction. [1]

Answer: $\left(\frac{-\frac{2y}{3}\hat{i} + \frac{3x}{2}\hat{j} + \hat{k}}{\sqrt{\frac{4y^2}{9} + \frac{9x^2}{4} + 1}}\right)$ where $\frac{x^2}{4} + \frac{y^2}{9} = 1$

b) Determine the curl of the vector field $\vec{F}$. [0.5]

Answer: $\hat{i} + \hat{j} + \hat{k}$

1/2

c) The positive flux of $\vec{F}$ through the above part of the paraboloid may be expressed as the double integral

$$\Phi = \iint_{\mathcal{R}} g(x, y)\, dR,$$

where $\mathcal{R}$ is the projection of the paraboloid on the xy-plane.

Determine (i) the equation of the region $\mathcal{R}$, (ii) the limits of the above flux integral, and (iii) the function $g(x, y)$. [1 + 1 + 1]

Answer: i) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ ii) $-2 \leq x \leq 2$, $-\frac{3}{2}\sqrt{4 - x^2} \leq y \leq \frac{3}{2}\sqrt{4 - x^2}$

iii) $-\frac{x^3}{8} + \frac{xy^2}{18} + \frac{2xy}{9} + y$

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d) Determine the flux Φ. [1.5]

Answer: 0

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