ME101: Engineering Mechanics (3 1 0 8)

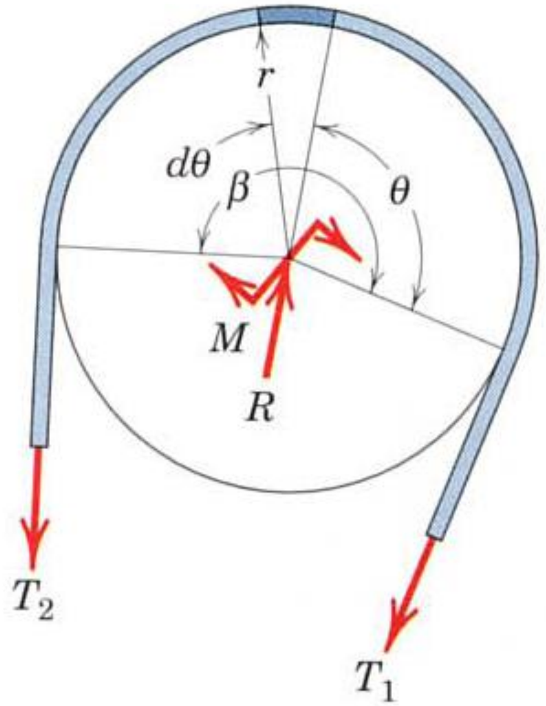
2019-20 (II Semester)

LECTURE: 12

Belt Friction
(Relation between belt tensions)

❖ Impending slippage of flexible cables, belts, ropes over sheaves, wheels, drums

It is necessary to estimate the frictional forces developed between the belt and its contacting surface.



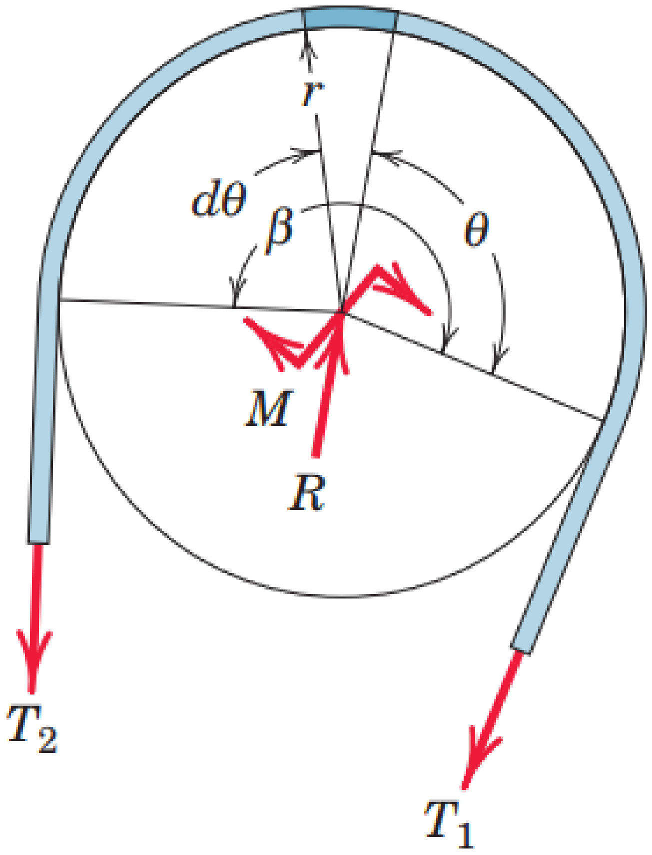
❖ Consider a drum subjected to two belt tensions (*T1* and *T2*).

- ✓ *M* is the torque necessary to prevent rotation of the drum.
- ✓ *R* is the bearing reaction.
- ✓ *r* is the radius of the drum.
- ✓ β is the total contact angle between belt and surface (in radians).
- ✓ *T2* > *T1* since *M* is clockwise.

Introduction

- Concept of Belt friction is extensively used in Belt Drives.
- Just one turn of rope around pulley, can increase tension by large magnitude.
- Hence, Tensions in the ropes are analysed by the concept of Belt friction
- Designer has to make sure that Belt tension does not exceed the Permissible limits to avoid system failure.

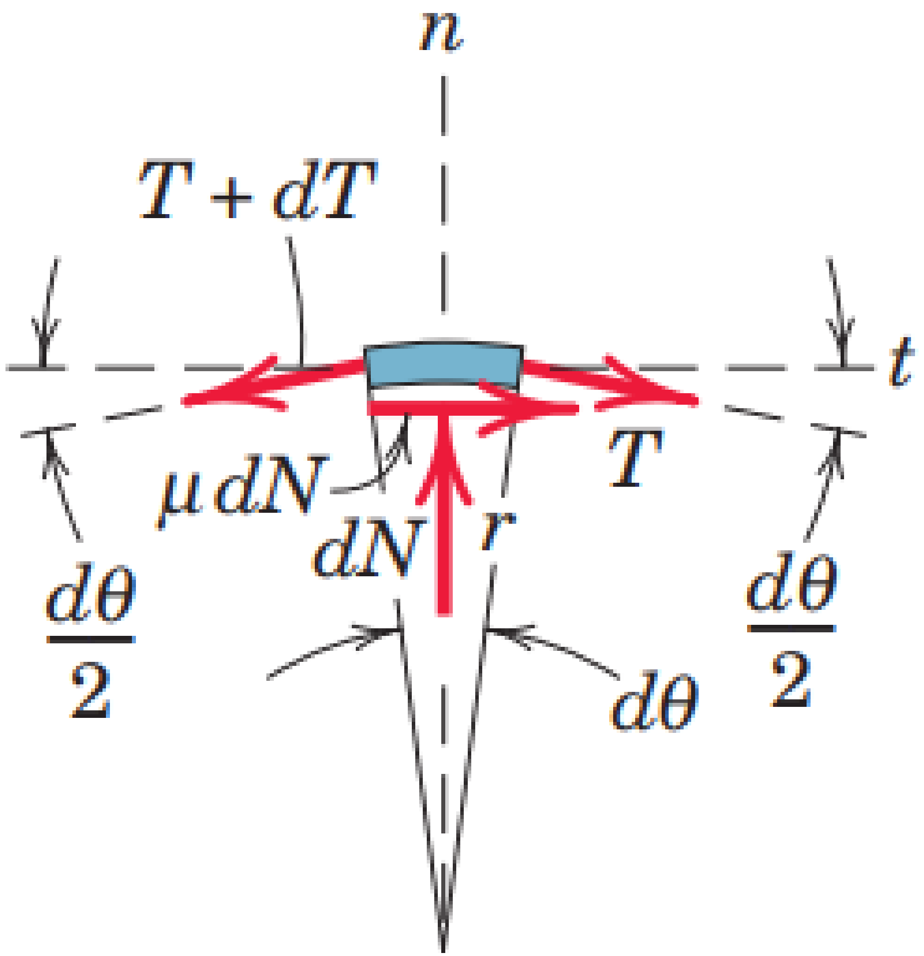
Analysis of Belt Friction



(a)

- A DRUM is wrapped with a Belt as shown
- It is subjected to two Belt Tensions $\mathbf{T_1}$ and $\mathbf{T_2}$
- Torque $\boldsymbol{M}$ necessary to prevent rotation
- Logically, with $\boldsymbol{M}$ in the CW direction shown, $\mathbf{T_2}$ is greater than $\mathbf{T_1}$.
- The tension increases from $\boldsymbol{T}$ at the angle $\boldsymbol{\theta}$ to $\boldsymbol{T + dT}$ at the angle $\boldsymbol{\theta + d\theta}$ in element

Analysis of Belt Friction



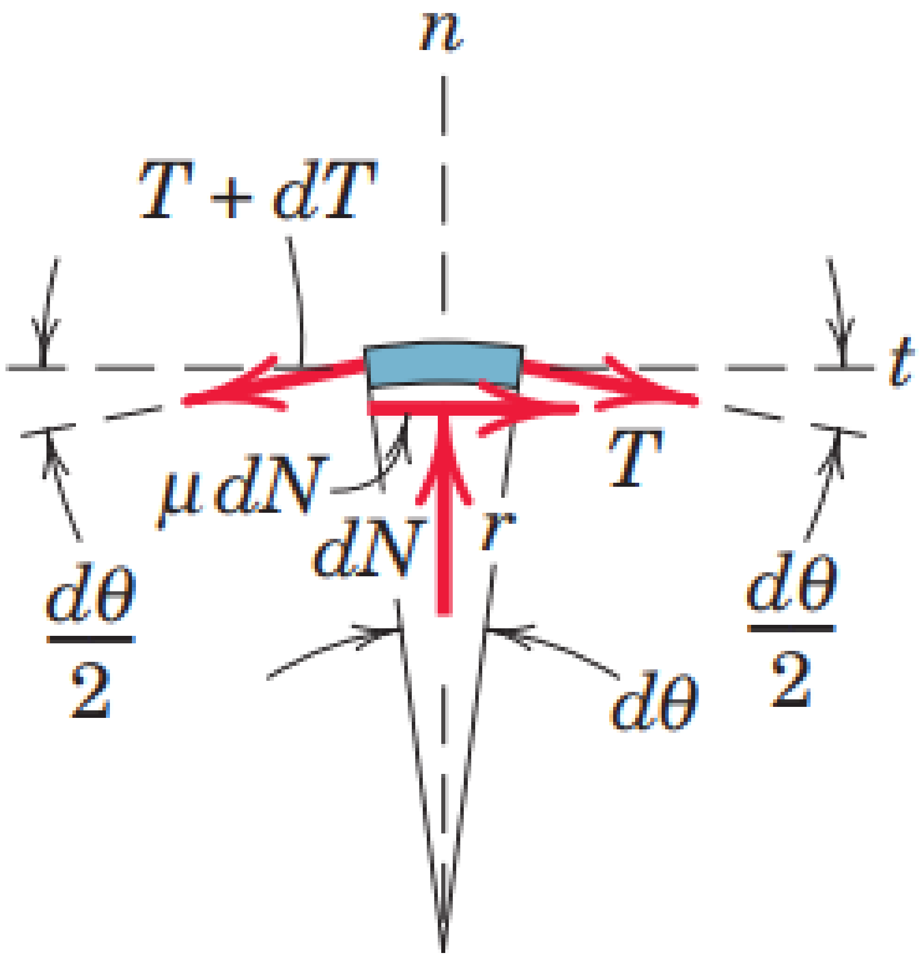
- The tension increases from $\boldsymbol{T}$ at the angle $\boldsymbol{\theta}$ to $\boldsymbol{T + dT}$ at the angle $\boldsymbol{\theta + d\theta}$ in element

- The freebody diagram of an element of the belt of length "$\boldsymbol{r d\theta}$" is shown

- The normal force is a differential dN, since it acts on a differential element of area

- Friction force, which must act on the belt in a direction to oppose slipping, is a differential and is $\boldsymbol{\mu dN}$

Analysis of Belt Friction



- Force balance in t-direction is

$$\boldsymbol{T\cos\frac{d\theta}{2} + \mu dN = (T + dT)\cos\frac{d\theta}{2}}$$

$$\boldsymbol{\mu dN = dT} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

- Force Balance in n-direction

$$\boldsymbol{dN = (T + dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2}}$$

$$\boldsymbol{dN = Td\theta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

- Combining equation (1) and (2),

$$\boldsymbol{\frac{dT}{T} = \mu d\theta}$$

Analysis of Belt Friction

$$\frac{dT}{T} = \mu d\theta$$

Integrating between corresponding limits yields

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu d\theta$$

$$\ln \frac{T_2}{T_1} = \mu\beta$$

$$T_2 = T_1 e^{\mu\beta}$$

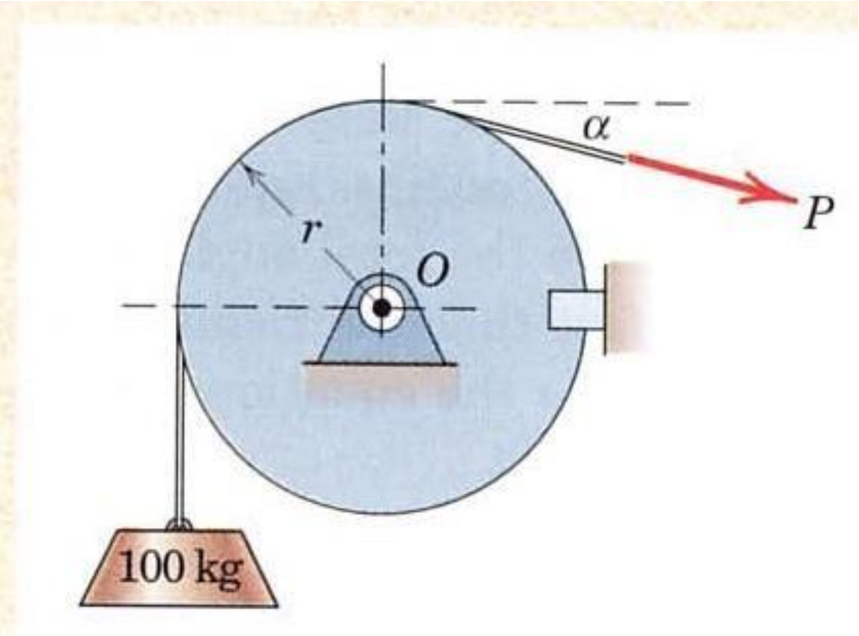
$$T_2 = T_1 e^{\mu\beta}$$

- Here, "β" is called as Angle of Wrap or Total angle of Belt contact.

- For "n" rotations of belt over pulley, angle of contact will be $2n\pi+\beta$

Sample Problem

A flexible cable which supports the 100-kg load is passed over a fixed circular drum and subjected to a force P to maintain equilibrium. The coefficient of static friction μ between the cable and the fixed drum is 0.30. (*a*) For $\alpha = 0$, determine the maximum and minimum values which P may have in order not to raise or lower the load. (*b*) For $P = 500$ N, determine the minimum value which the angle α may have before the load begins to slip.



Solution on next page

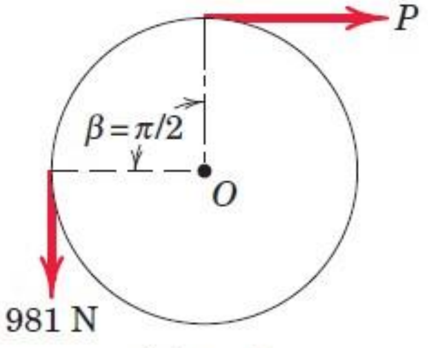
Impending slippage of the cable over the fixed drum is given by: $T_2 = T_1 e^{\mu\beta}$

1 **(a)** With $\alpha = 0$ the angle of contact is $\beta = \pi/2$ rad. For impending upward motion of the load, $T_2 = P_{max}$, $T_1 = 981$ N, and we have

2 $$P_{max}/981 = e^{0.30(\pi/2)} \qquad P_{max} = 981(1.602) = 1572 \text{ N} \qquad \textit{Ans.}$$

For impending downward motion of the load, $T_2 = 981$ N and $T_1 = P_{min}$. Thus,

$$981/P_{min} = e^{0.30(\pi/2)} \qquad P_{min} = 981/1.602 = 612 \text{ N} \qquad \textit{Ans.}$$



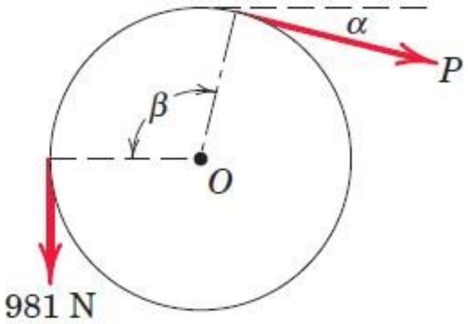
(a) α = 0

(b) With $T_2 = 981$ N and $T_1 = P = 500$ N, Eq. 6/7 gives us

$$981/500 = e^{0.30\beta} \qquad 0.30\beta = \ln(981/500) = 0.674$$

$$\beta = 2.25 \text{ rad} \qquad \text{or} \qquad \beta = 2.25\left(\frac{360}{2\pi}\right) = 128.7°$$

$$\alpha = 128.7° - 90° = 38.7°$$



(b) P = 500 N

FBD

Wheel Friction or Rolling Resistance

- Resistance of a wheel to roll over a surface is caused by deformation between two materials of contact.

- ✓ This resistance is not due to tangential frictional forces
- ✓ Entirely different phenomenon from that of dry friction

Steel is very stiff
Low Rolling Resistance

Significant Rolling Resistance between rubber tyre and tar road

Large Rolling Resistance due to wet field

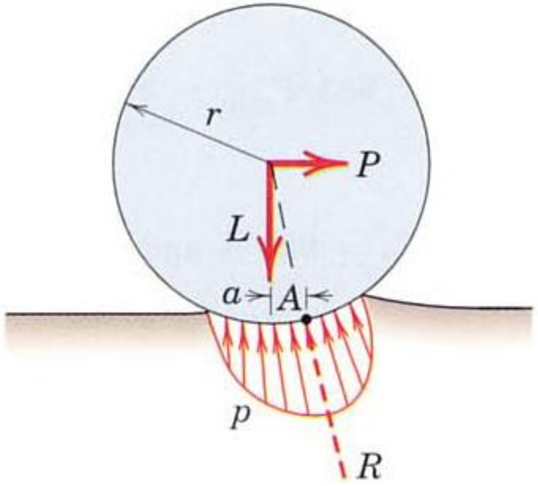
Deformations due to Rolling Resistance

- ❖ Actually materials are not rigid and deformation occurs at the point of contact.
- ✓ Reaction of surface on the cylinder consists of a distribution of normal pressure.
- ❑ Consider a wheel under action of a load L on axle and a force P applied at its center to produce rolling;
- ✓ Deformation of wheel and supporting surface as shown in **Fig(f)**
- ✓ Resultant R of the distribution of normal pressure must pass through wheel center for the wheel to be in equilibrium (i.e., rolling at a constant speed).
- ✓ R acts at point A on right of wheel center for rightwards motion.
- ✓ Force P required to maintain rolling at constant speed can be apprx. estimated as:

$$\sum M_A = 0 \Rightarrow L\ a = P\ r \cancelto{1}{\cos\theta}$$

$$\Rightarrow\ P = \frac{a}{r}L = \mu_r L$$

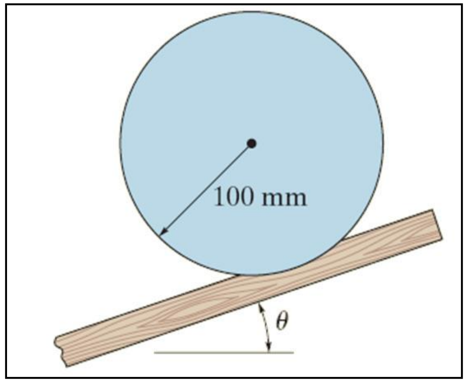
- μ_r is the ratio of resisting force to the normal force analogous to μ_s *or* μ_k and called **Coefficient of Rolling Resistance**
- No slippage or impending slippage in interpretation of μ_r



Fig(f)

Problem on Rolling Resistance

A 10 kg steel wheel (radius = 100 mm) rests on an inclined plane made of wood. At θ =1.2º, the wheel begins to roll-down the incline with constant velocity. Determine the coefficient of rolling resistance in Fig(g)?



Fig(g)

Solution:

- When the wheel has impending motion, the normal reaction *N* acts at point *A* defined by the dimension *a*. Draw the FBD for the wheel: *r* = 100 mm, 10 kg = 98.1 N
- Using simplified equation directly: $P = \frac{a}{r} L = \mu_r L$

Here *P* = 98.1(sin1.2) = 2.05 N

L = 98.1(cos1.2) = 98.08 N, Coefficient of Rolling Resistance, μ_r= 0.0209

❖ Alternatively, $\sum M_A = 0$

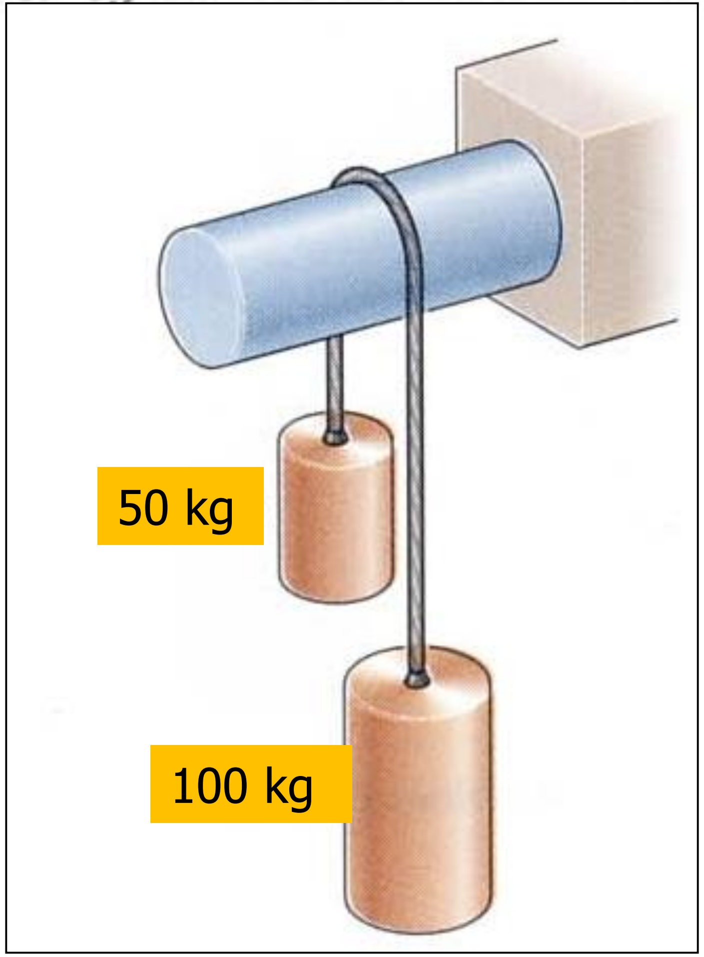
98.1(*sin*1.2)(*r* appx) = 98.1(*cos*1.2)*a*

(since *r*cos1.2 = *r*x0.9998 *r*)

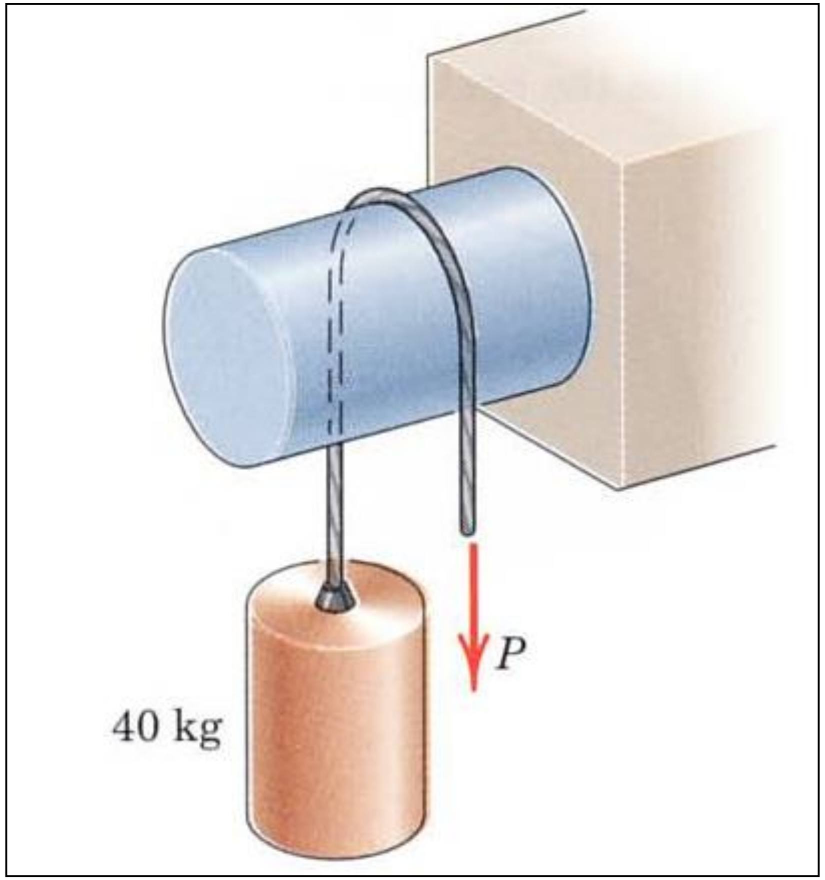
$a/r = \mu_r = 0.0209$

Assignments

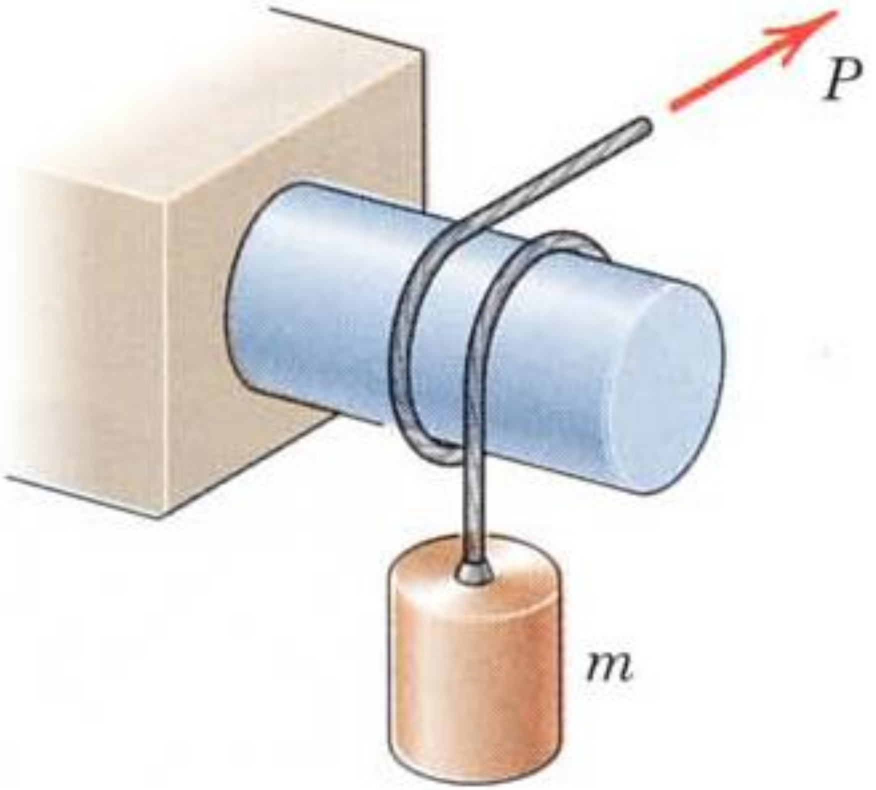
What is the minimum coefficient of friction μ between the rope and the fixed shaft which will prevent the unbalanced cylinders from moving?



Determine the force P required to (*a*) raise and (*b*) lower the 40-kg cylinder at a slow steady speed. The coefficient of friction between the cord and its supporting surface is 0.30.



6/90 A force $P = mg/6$ is required to lower the cylinder at a constant slow speed with the cord making $1\frac{1}{4}$ turns around the fixed shaft. Calculate the coefficient of friction μ between the cord and the shaft.

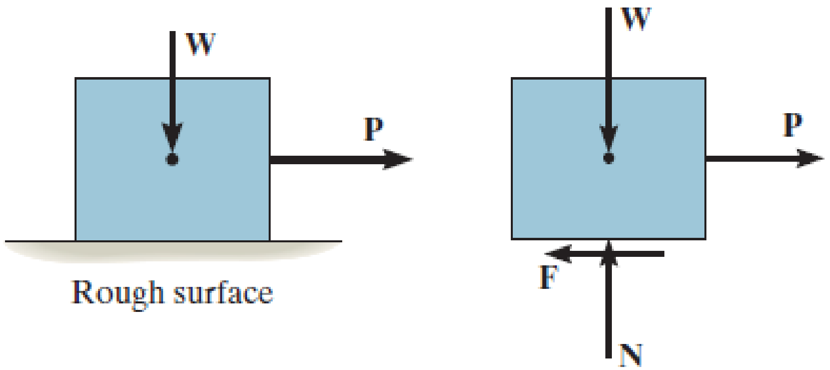


Problem 6/90

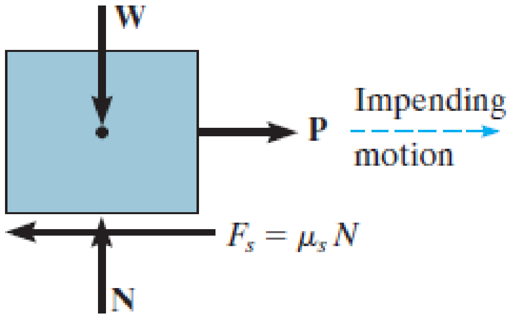
Chapter review

Dry Friction

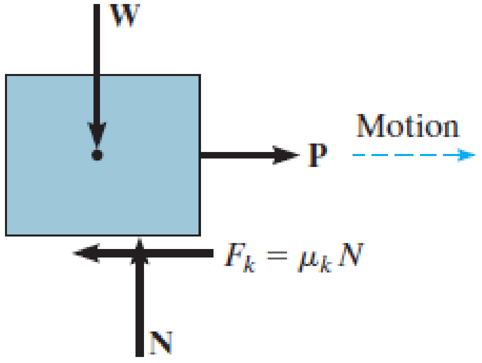
Frictional forces exist between two rough surfaces of contact. These forces act on a body so as to oppose its motion or tendency of motion.



A static frictional force approaches a maximum value of $F_s = \mu_s N$, where μ_s is the *coefficient of static friction*. In this case, motion between the contacting surfaces is *impending*.



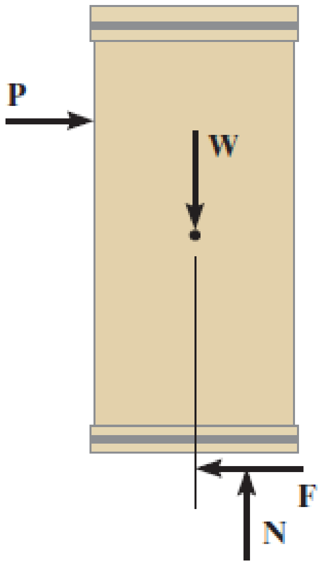
If slipping occurs, then the friction force remains essentially constant and equal to $F_k = \mu_k N$. Here μ_k is the *coefficient of kinetic friction*.



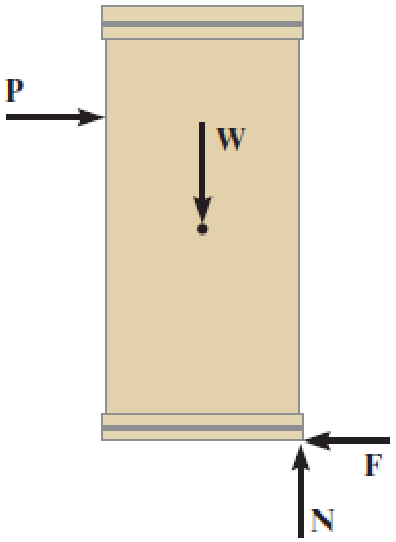
Chapter review *(contd.)*

The solution of a problem involving friction requires first drawing the free-body diagram of the body. If the unknowns cannot be determined strictly from the equations of equilibrium, and the possibility of slipping occurs, then the friction equation should be applied at the appropriate points of contact in order to complete the solution.

It may also be possible for slender objects, like crates, to tip over, and this situation should also be investigated.



Impending slipping
$F = \mu_s N$



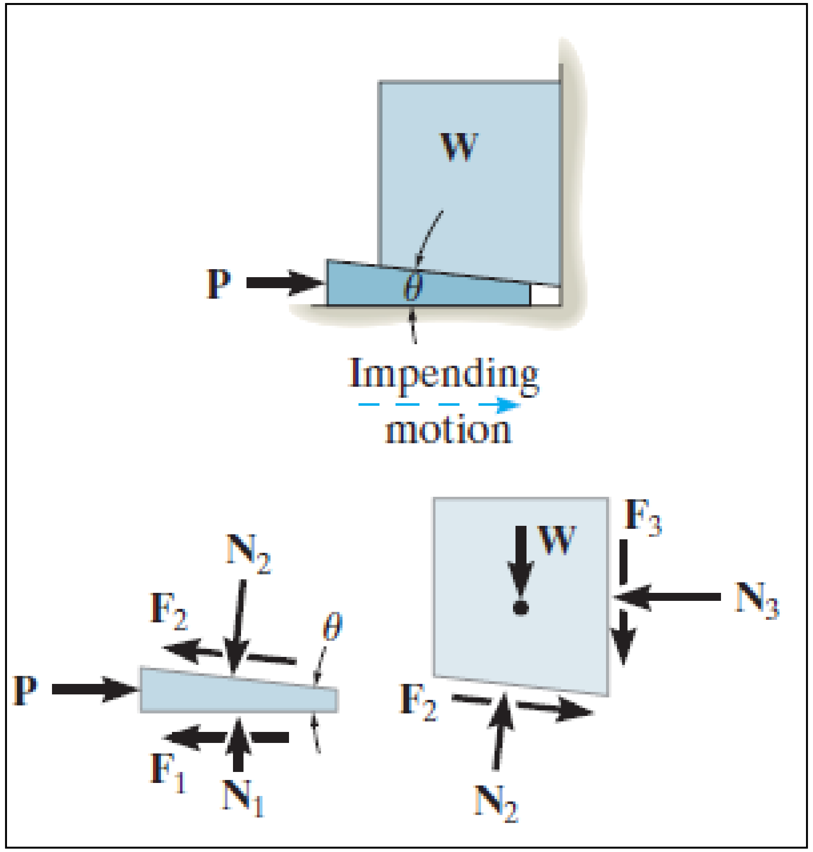
Tipping

Wedges

Wedges are inclined planes used to increase the application of a force. The two force equilibrium equations are used to relate the forces acting on the wedge.

An applied force **P** must push on the wedge to move it to the right.

If the coefficients of friction between the surfaces are large enough, then **P** can be removed, and the wedge will be self-locking and remain in place.



$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

Screws

Square-threaded screws are used to move heavy loads. They represent an inclined plane, wrapped around a cylinder.

The moment needed to turn a screw depends upon the coefficient of friction and the screw's lead angle θ.

If the coefficient of friction between the surfaces is large enough, then the screw will support the load without tending to turn, i.e., it will be self-locking.

$$M = rW \tan(\theta + \phi_s)$$

Upward Impending Screw Motio

$$M' = rW \tan(\theta - \phi_s)$$

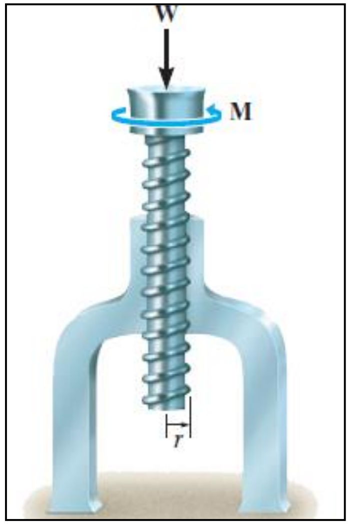
Downward Impending Screw Motion

$$\theta > \phi_s$$

$$M'' = rW \tan(\phi_s - \theta)$$

Downward Screw Motion

$$\phi_s > \theta$$

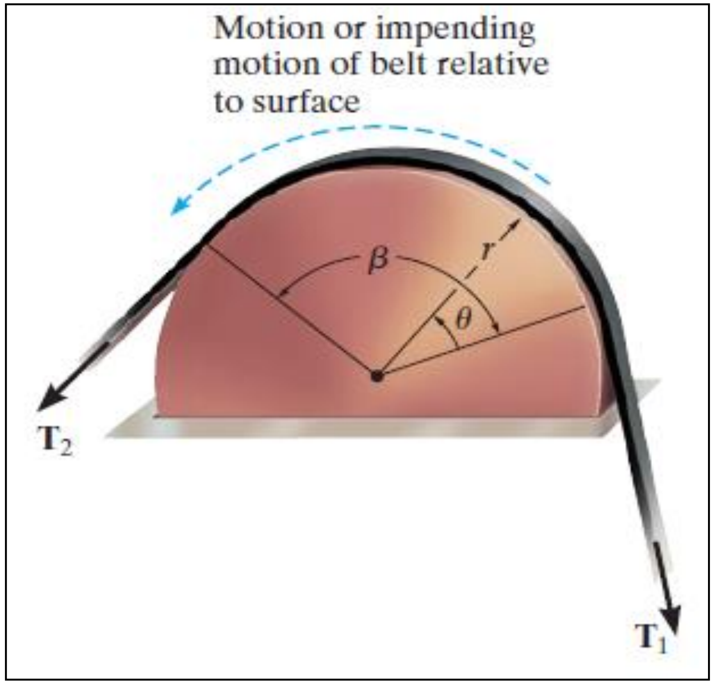


Flat Belts

The force needed to move a flat belt over a rough curved surface depends only on the angle of belt contact, β, and the coefficient of friction.

$$T_2 = T_1 e^{\mu\beta}$$

$$T_2 > T_1$$

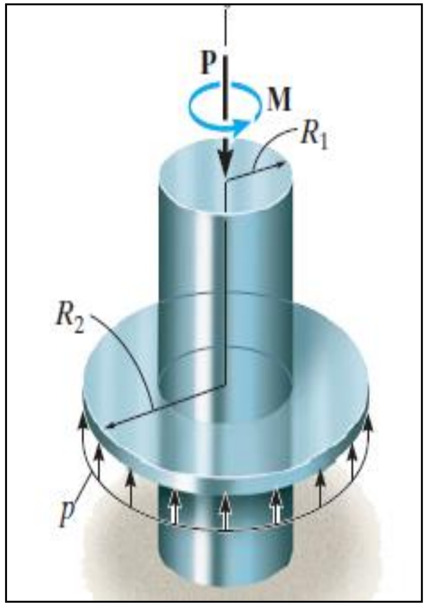


Collar Bearings and Disks

The frictional analysis of a collar bearing or disk requires looking at a differential element of the contact area. The normal force acting on this element is determined from force equilibrium along the shaft, and the moment needed to turn the shaft at a constant rate is determined from moment equilibrium about the shaft's axis.

If the pressure on the surface of a collar bearing is uniform, then integration gives the result shown.

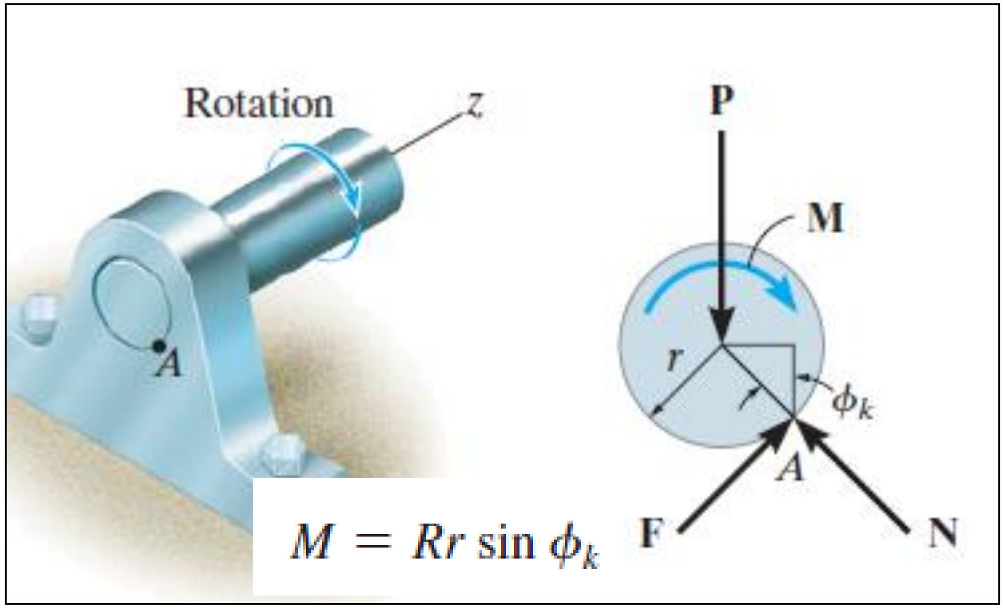
$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$



Chapter review *(contd.)*

Journal Bearings

When a moment is applied to a shaft in a nonlubricated or partially lubricated journal bearing, the shaft will tend to roll up the side of the bearing until slipping occurs. This defines the radius of a friction circle, and from it the moment needed to turn the shaft can be determined.



$$M = Rr \sin \phi_k$$

Rolling Resistance

The resistance of a wheel to rolling over a surface is caused by localized *deformation* of the two materials in contact. This causes the resultant normal force acting on the rolling body to be inclined so that it provides a component that acts in the opposite direction of the applied force **P** causing the motion. This effect is characterized using the *coefficient of rolling resistance*, *a*, which is determined from experiment.

$$P \approx \frac{Wa}{r}$$