ME101: Engineering Mechanics (3 1 0 8)

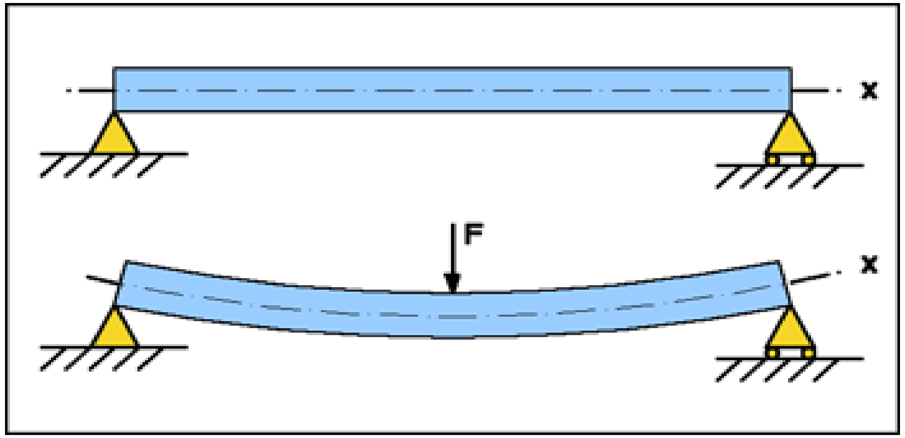
2019-20 (II Semester)

LECTURE: 9

Beams

What is a beam?

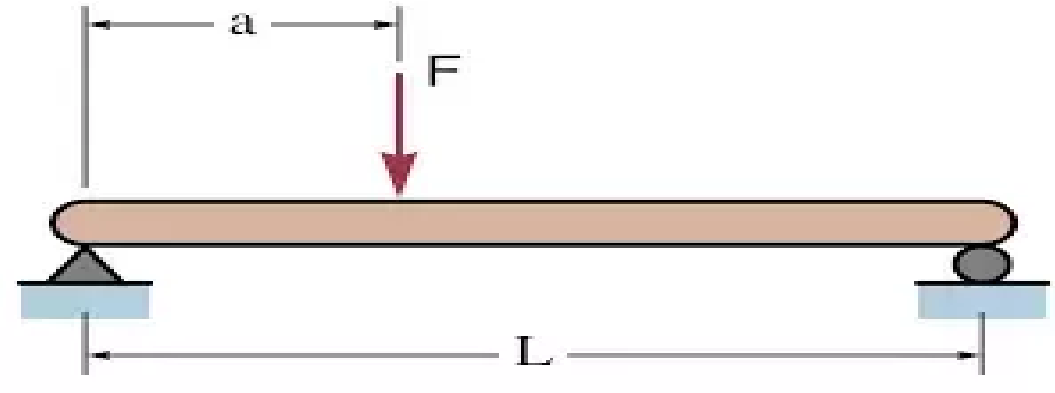
- A beam is a structural element that is subjected to transverse loads or moments which mainly cause it to bend in the axial plane (i.e., the plane containing the beam axis).
- Beams are suitably supported so as to be in equilibrium under the action of external loads or moments.



Types of beams

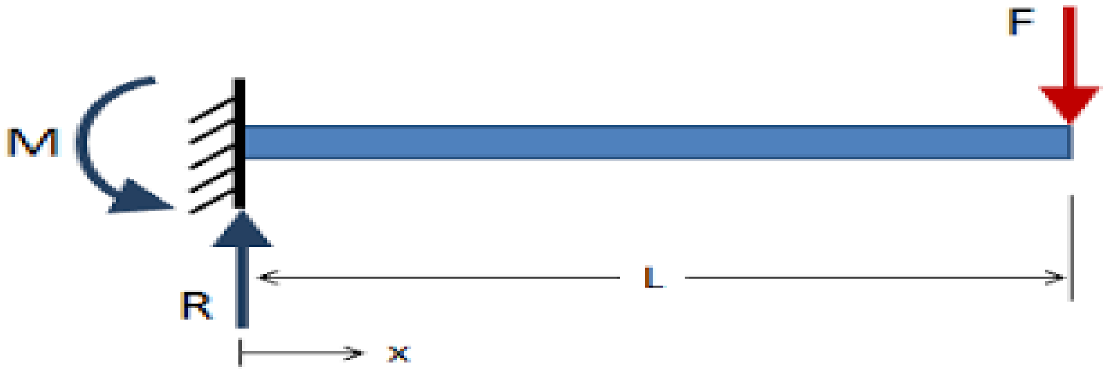
- **Simply supported Beam**

One end of the beam is hinged or pinned and other end of the beam is roller supported.



- **Cantilever beam**

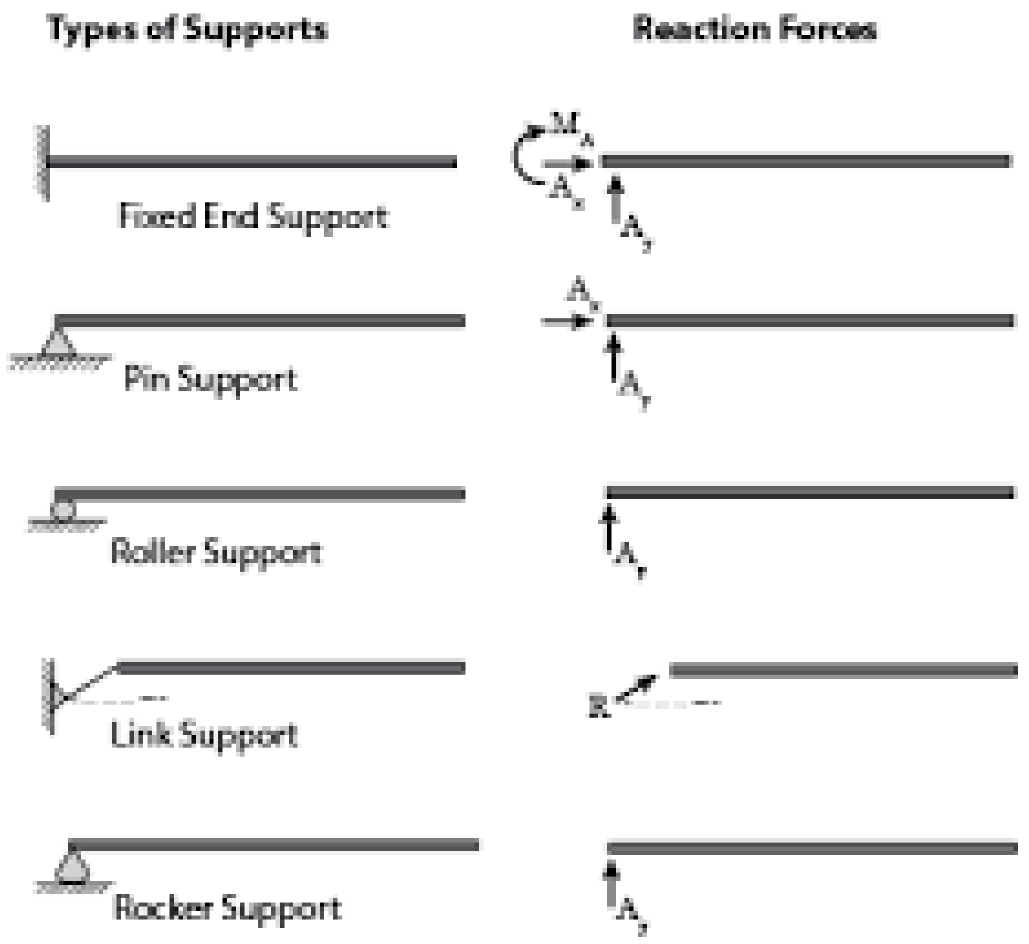
One end of the beam is fixed to a support and other end of the beam is free.



Types of supports

In this section, different types of supports and reaction forces on the beam has been shown.

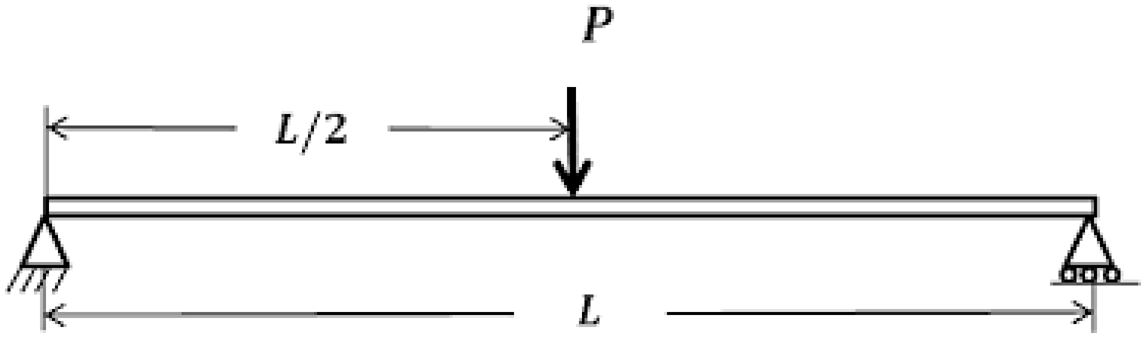
- **Fixed support:** Both horizontal and vertical reaction forces and also reaction moment occur.
- **Pin or Hinge support:** Only horizontal and vertical reaction forces occur but no reaction moment is present.
- **Roller support:** One and only vertical reaction force is present.



Types of Loading

Different types of loading that can act on a beam has been presented below.

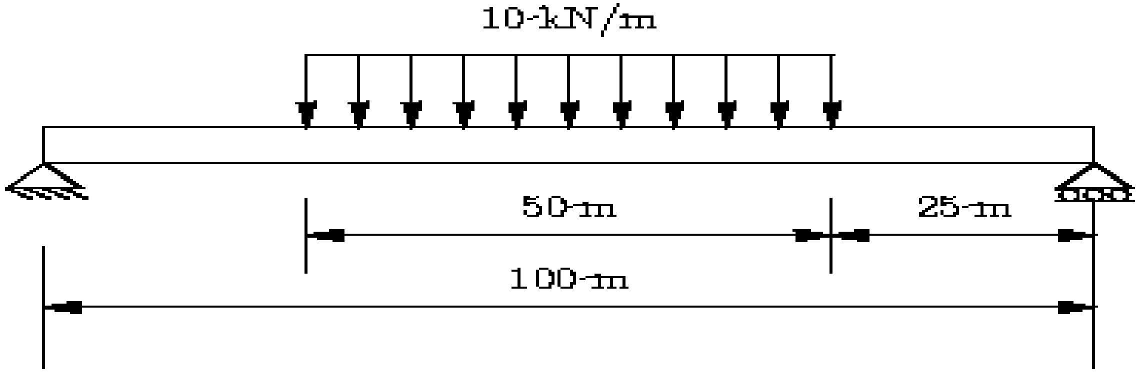
- **Concentrated Load:** When the load acts on a specific points of the beam, then it is known as concentrated load or point load.



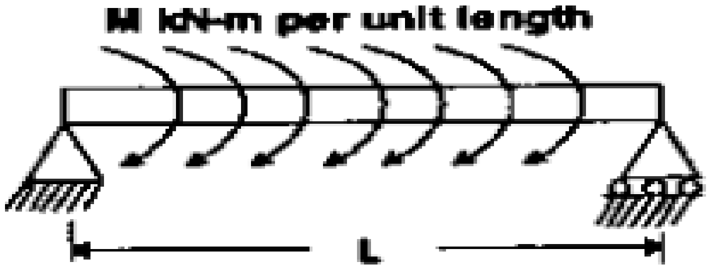
Here the load P on the beam is a concentrated load. Similarly if a moment acts on a specific point of the beam then it is **concentrated moment.**

Types of loading

- **Uniformly distributed load:** When the load acts on a region or better to say on a specific length of the beam and the magnitude of the loading is same or uniform over the entire region of action then it is known as **uniformly distributed load.**



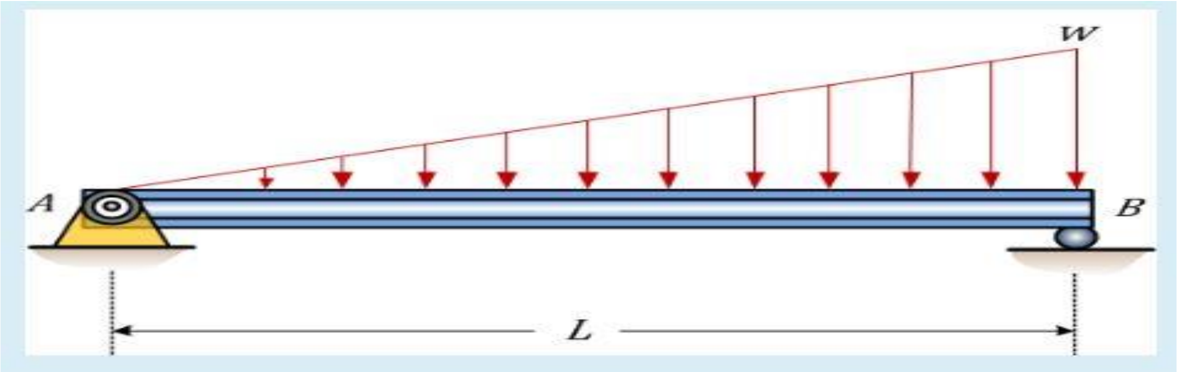
Similarly, **uniformly distributed moment** can act on the beam.



Types of loading

- **Uniformly varying load:** When the load acts on a region or better to say on a specific length of the beam and the magnitude of the loading is varying uniformly over the entire region of action then it is known as **uniformly distributed load.** As example,

➢ **Triangularly varying load:** The load profile varies triangularly over the region of action.



➢ **Trapezoidal varying Load:**

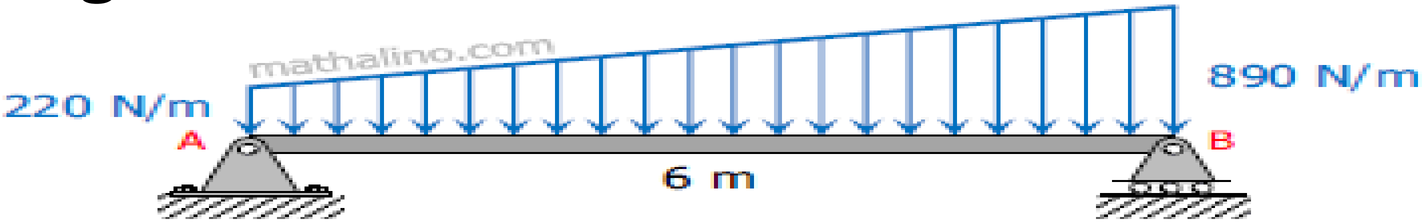
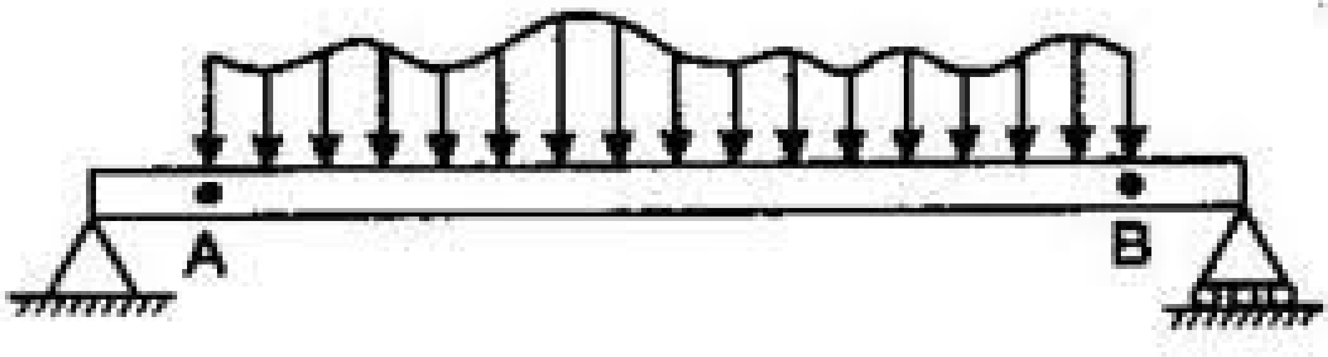


Figure P-238

Types of loading

- **Non-uniformly varying load:** The load profiles varies non-uniformly over the beam length.

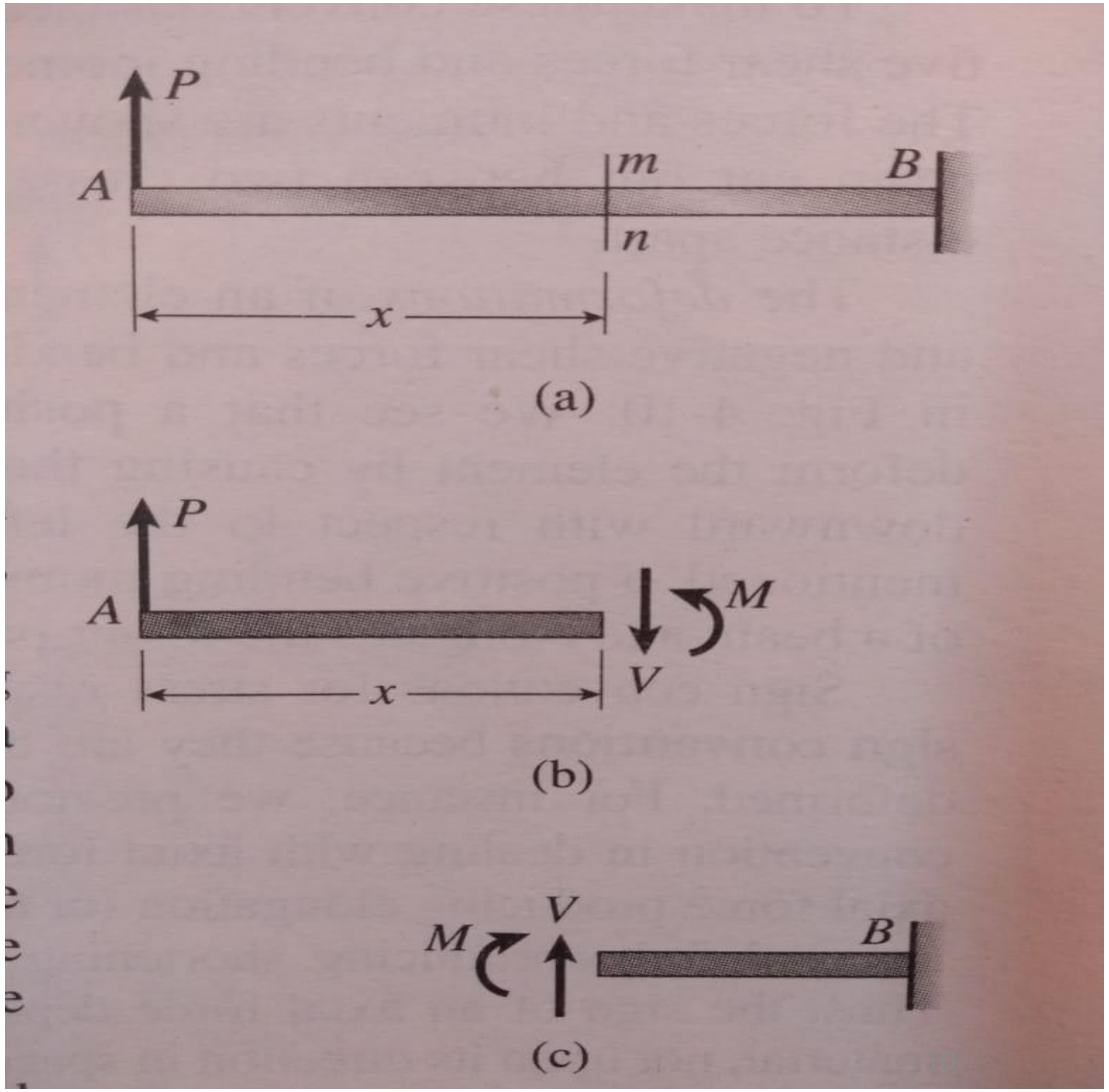


Shear Forces and Bending Moments

When a beam is loaded by forces or couples, stresses and strains are developed throughout the interior of the beam.

The cantilever beam is subjected to a force **P.** We cut through the beam at a c-s *mn* located at a distance *x* from the free end. Isolate the left hand part of the beam as free body and the free body is held in equilibrium by force **P** and the internal stress and moment.

From statics, we know that the resultant of the stresses acting on the cross-section can be reduced to a **shear force (V)** and a **bending moment (M).**



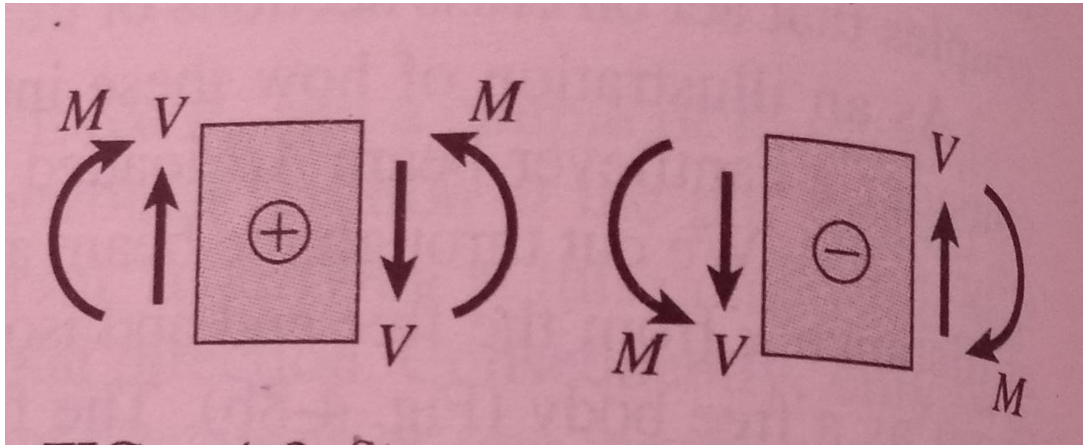
(a)
(b)
(c)

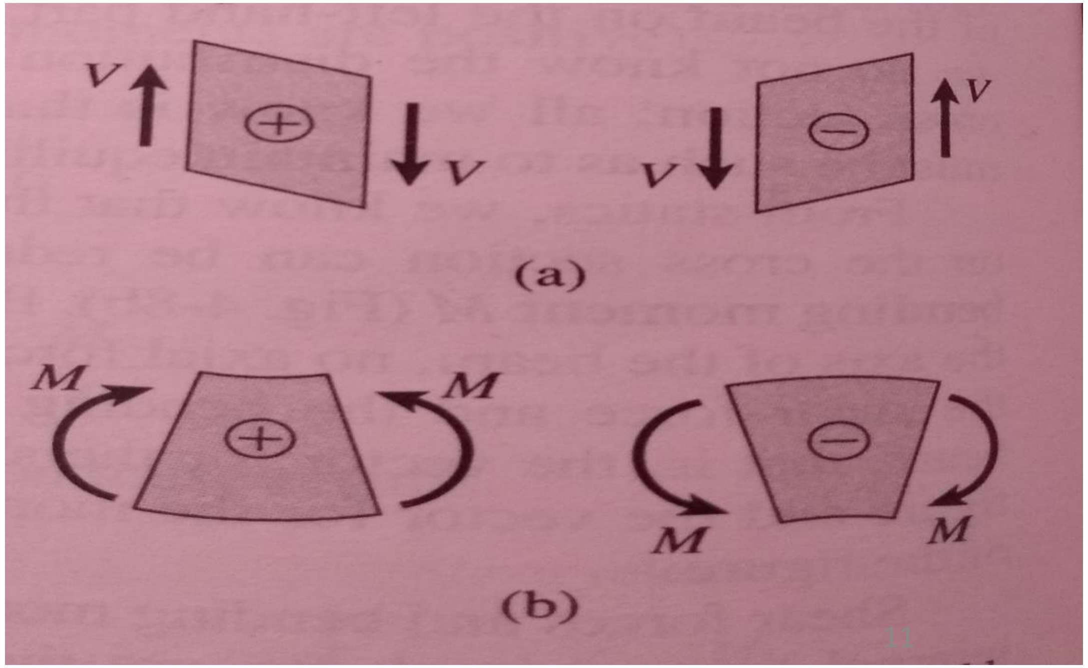
Sign Convention of Shear Force and Bending Moment

In case of a beam, a **positive shear force** acts clockwise against the material and a **negative shear force** acts counter-clockwise against the material.

A **positive bending moment** compresses the upper part of the beam above neutral axis and elongates the lower part of the beam below the neutral axis. A **negative bending moment** compresses the lower part of the beam below the neutral axis and elongates the upper part of the beam above neutral axis.

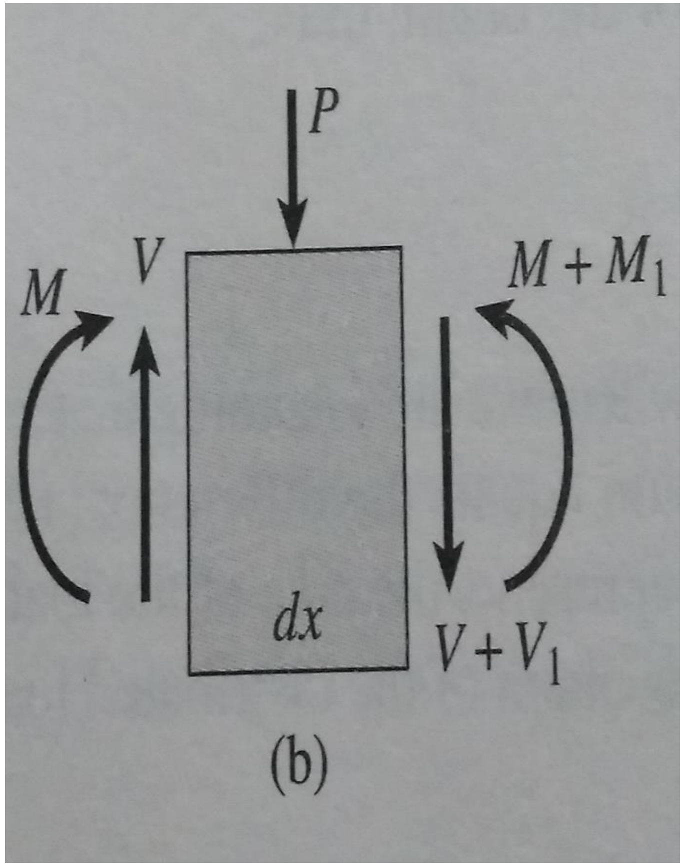
Sign convention for stress resultants are called **deformation sign convention**.





Relationship between Loads, Shear Force and Bending Moment

❑For Concentrated Loads



Let us consider a concentrated load **P** acting on the beam element. From equilibrium of forces in the vertical direction, we get

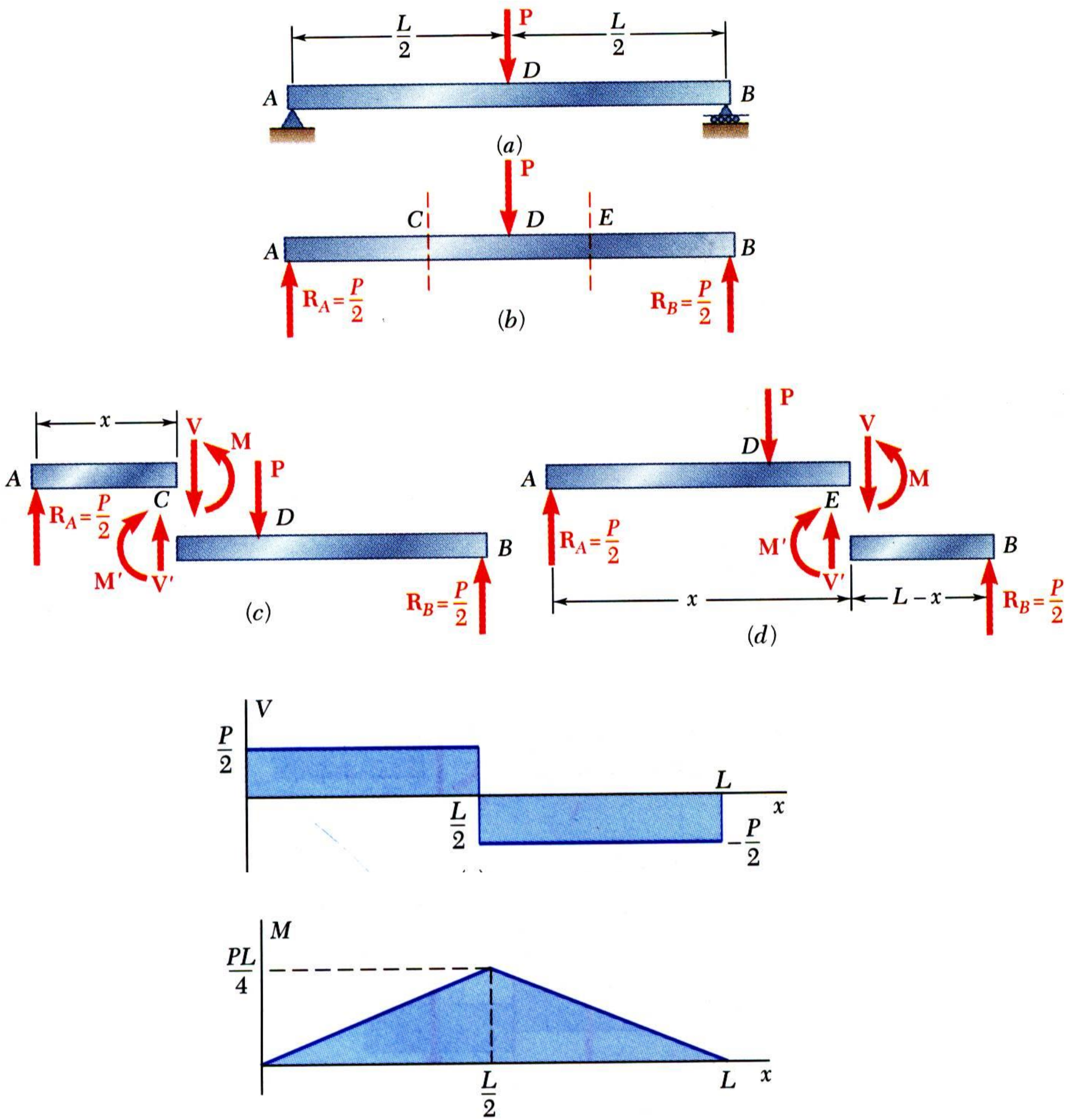
$$\boldsymbol{V - P - (V + V_1) = 0} \quad \text{or,} \quad \boxed{\boldsymbol{V_1 = -P}}$$

As we pass from left to right through the point of load application, the shear force decreases by an equal amount of **P**.

From equilibrium of moments about the left-hand face of the element, we get

$$\boldsymbol{-M - P\left(\frac{dx}{2}\right) - (V + V_1)dx + M + M_1 = 0} \quad \text{or,} \quad \boxed{\boldsymbol{M_1 = P\left(\frac{dx}{2}\right) + Vdx + V_1dx}}$$

Shear Force and Bending Moment Diagrams



- Variation of shear and bending moment along beam may be plotted.
- Determine reactions at supports.
- Cut beam at *C* and consider member *AC*,

$$V = +P/2 \quad M = +Px/2$$

- Cut beam at *E* and consider member *EB*,

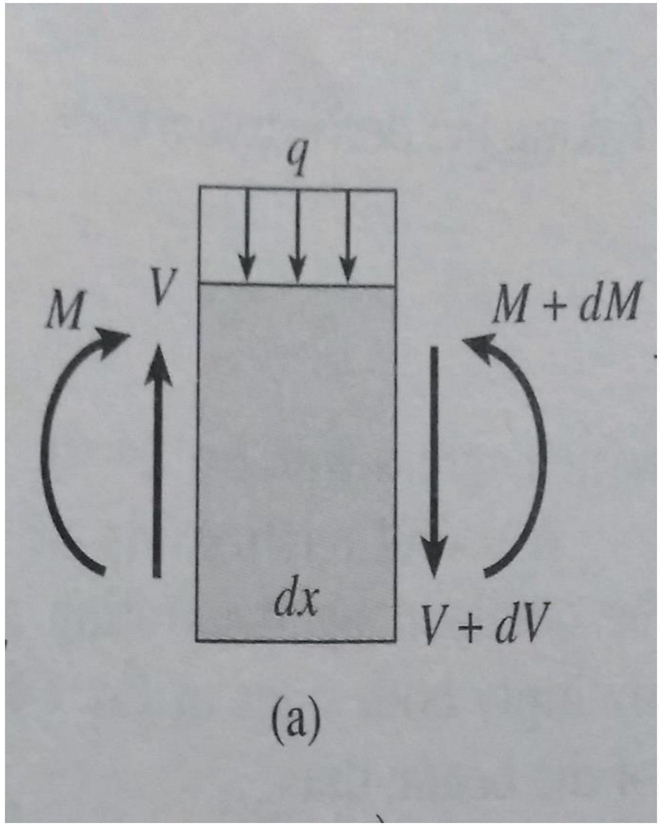
$$V = -P/2 \quad M = +P(L-x)/2$$

- For a beam subjected to concentrated loads, shear is constant between loading points and moment varies linearly.

Relationships between Loads, Shear force and Bending Moments

❑For Distributed Loads

- The rate of change of the shear force at any point on the axis of the beam is equal to the negative of the intensity of the distributed load at that same point (*Note:* If the sign convention for $\boldsymbol{q}$ is reversed, then the minus sign is omitted in the preceding equation).



$$\sum \boldsymbol{F_{vertical}} = \mathbf{0}$$

$$\boldsymbol{V} - \boldsymbol{q}\boldsymbol{dx} - (\boldsymbol{V} + \boldsymbol{dV}) = \mathbf{0}$$

$$\boxed{\frac{\boldsymbol{dV}}{\boldsymbol{dx}} = -\boldsymbol{q}}$$

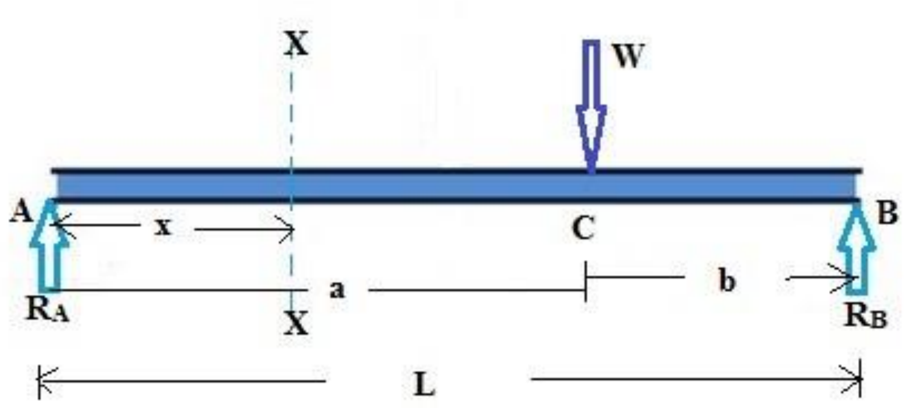
- The rate of change of bending moment at any point on the axis of the beam is equal to the shear force at that same point.

$$\sum \boldsymbol{M} = \mathbf{0} \quad \Rightarrow -\boldsymbol{M} - \boldsymbol{q}\boldsymbol{dx}\left(\frac{\boldsymbol{dx}}{\mathbf{2}}\right) - (\boldsymbol{V} + \boldsymbol{dV})\boldsymbol{dx} + \boldsymbol{M} + \boldsymbol{dM} = \mathbf{0}$$

$$\boxed{\frac{\boldsymbol{dM}}{\boldsymbol{dx}} = \boldsymbol{V}}$$

Shear Force and Bending Moment Diagrams

❑Simply Supported Beam with a Single Concentrated Load

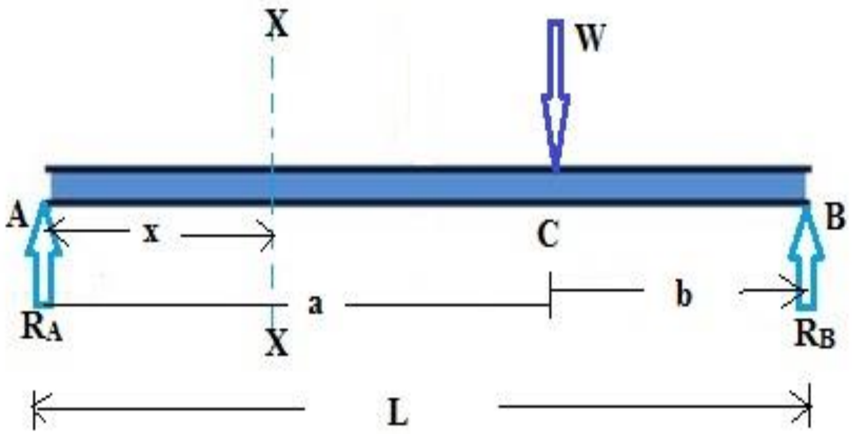


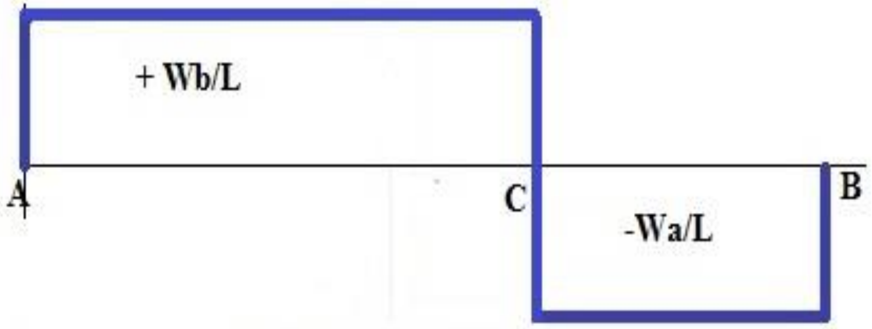
- Figure shows a single point load **W** acting at a distance **a** from the left support of the simply supported beam.
- Two vertical reactions R_A and R_B can be determined from two equilibrium equations.

$$\sum F_{vertical} = 0 \quad => \quad R_A + R_B = W$$

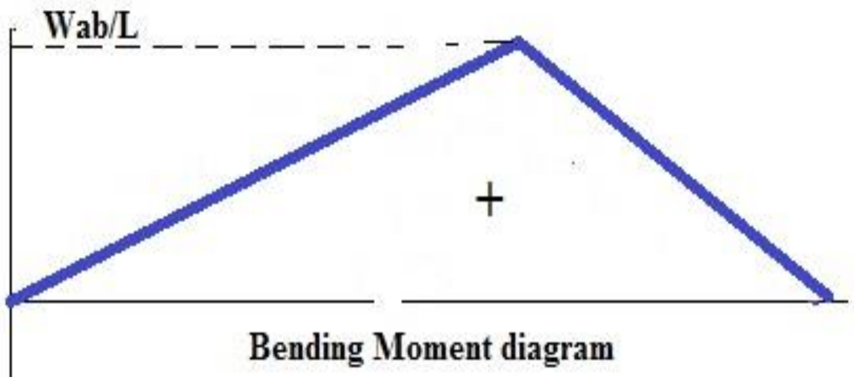
Taking moments of the forces about B and equating to zero i.e., $\sum M_B = 0 \quad => R_a L - Wb = 0$

$$\boxed{R_A = \frac{Wb}{L}} \quad \text{and} \quad \boxed{R_B = \frac{Wa}{L}}$$





Shear force diagram



Bending Moment diagram

➢ For Shear Force Diagram

Consider a section x from point A.

I. Between A and C i.e., $0 \leq x < a$ $\quad \boldsymbol{V_x = R_A = \frac{Wb}{L}}$

II. At C, shear force changes from $\boldsymbol{\frac{Wb}{L}}$ to $\boldsymbol{\frac{Wa}{L}}$

III. Between C and B i.e., $a < x \leq L$ $\quad \boldsymbol{V_x = R_A - W = \frac{Wa}{L}}$

➢ For Bending Moment Diagram

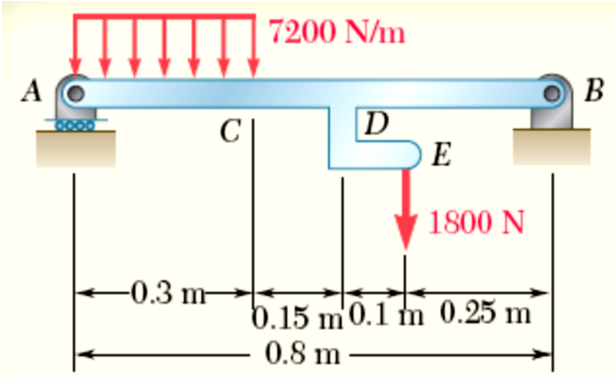
I. Between A and C $(0 \leq x < a)$ $\quad \boldsymbol{M_x = R_A x = \frac{Wb}{L} x}$

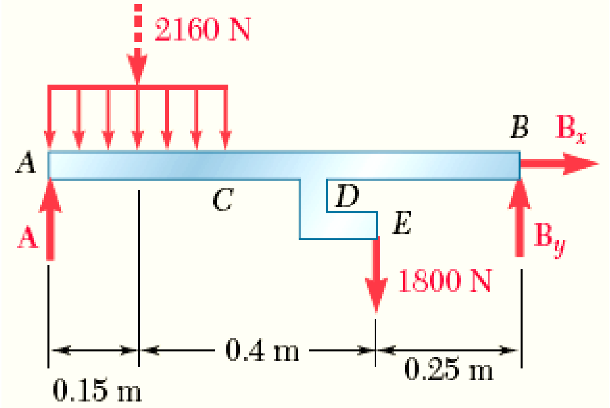
II. At C, $\quad \boldsymbol{M_{C^x} = \frac{Wb}{L} a}$

III. Between B and C $a < x \leq L$

$$\boldsymbol{M_x = R_A x - W(x - a)}$$

Sample Problem





SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *A* and *B*.

$$\sum M_A = 0:$$

$$B_y(0.8\,\mathrm{m}) - (2160\ \mathrm{N})(0.15\,\mathrm{m}) - (1800\ \mathrm{N})(0.55\,\mathrm{m}) = 0$$

$$\boxed{B_y = 1642.5\,\mathrm{N}}$$

$$\sum M_B = 0:$$

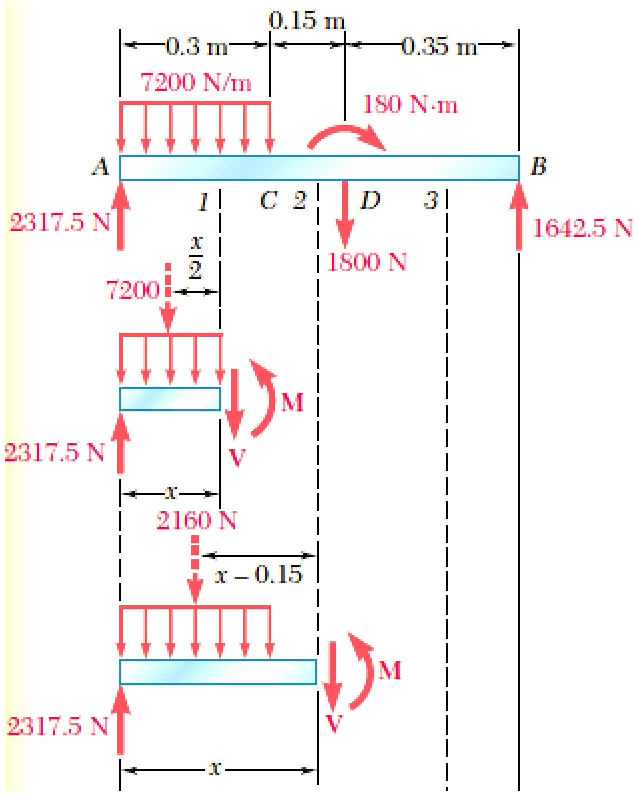
$$(2160\mathrm{N})(0.65\mathrm{m}) + (1800\mathrm{N})(0.25\mathrm{N}) - A(0.8\mathrm{m}) = 0$$

$$\boxed{A = 2317.5\,\mathrm{N}}$$

$$\sum F_x = 0: \qquad \boxed{B_x = 0}$$

- Note: The 1800-N load at *E* may be replaced by a 1800-N force and 180-N . m couple at *D*.

Sample Problem



- Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From *A* to *C*:

$$\sum F_y = 0: \quad 2317.5 - 7200\,x - V = 0$$

$$\boxed{V = (2317.5 - 7200\,x)\,\mathrm{N}}$$

$$\sum M_1 = 0: -2317.5x + 7200\,x\left(\tfrac{1}{2}\,x\right) + M = 0$$

$$\boxed{M = (2317.5x - 3600\,x^2)\,\mathrm{N.m}}$$

From *C* to *D*:

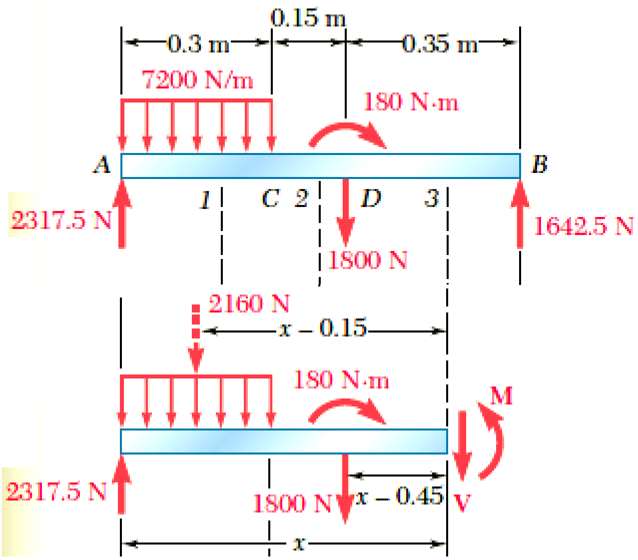
$$\sum F_y = 0: \quad 2317.5 - 2160 - V = 0$$

$$\boxed{V = 157.5\,\mathrm{N}}$$

$$\sum M_2 = 0: -2317.5x + 2160(x - 0.15) + M = 0$$

$$\boxed{M = (324 + 157.5x)\,\mathrm{N \cdot m}}$$

Sample Problem



From *D* to *B*:

$$\sum F_y = 0:$$

$$2317.5 - 2160 - 1800 - V = 0$$

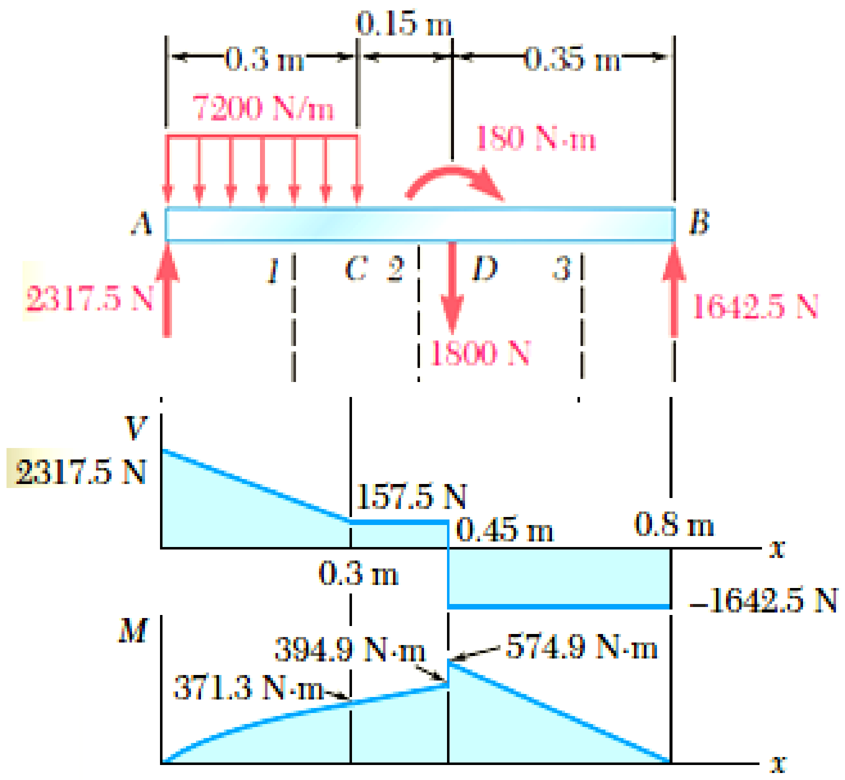
$$\boxed{V = -1642.5\ \mathrm{N}}$$

$$\sum M_3 = 0:$$

$$-2317.5x + 2160(x - 0.15) - 180$$
$$+1800(x - 0.45) + M = 0$$

$$\boxed{M = (1314 - 1642.5x)\ \mathrm{N \cdot m}}$$

Sample Problem



- Plot results.

From A to *C*:

$$\boxed{V = (2317.5 - 7200\,x)\,\mathrm{N}}$$

$$\boxed{M = (2317.5x - 3600\,x^2)\,\mathrm{N\,.\,m}}$$

From *C* to *D*:

$$\boxed{V = 157.5\,\mathrm{N}}$$

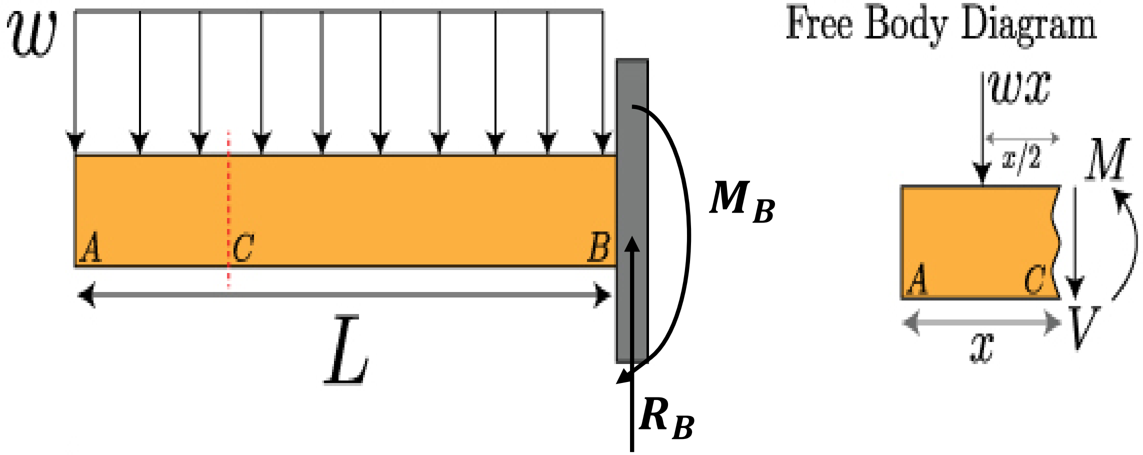
$$\boxed{M = (324 + 157.5x)\,\mathrm{N \cdot m}}$$

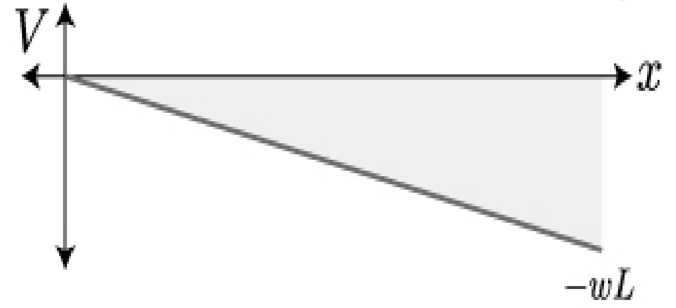
From *D* to *B*:

$$\boxed{V = -1642.5\,\mathrm{N}}$$

$$\boxed{M = (1314 - 1642.5x)\,\mathrm{N \cdot m}}$$

❑ Cantilever Beam with Uniformly Distributed Load

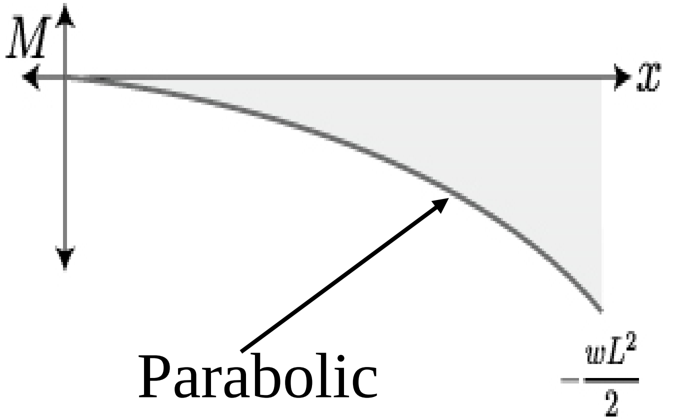




$$+\uparrow \sum F_y = 0$$

$$-wx - V = 0$$

$$\boxed{V = -wx}$$



$$+\circlearrowleft \sum M_C = 0$$

$$wx\left(\frac{x}{2}\right) + M = 0$$

$$\boxed{M = -\frac{wx^2}{2}}$$

➢ Reactions

The reactions at A (fixed support) are obtained from equations of equilibrium.

$$\boldsymbol{R_B = wL} \qquad \boldsymbol{M_B = \frac{wL^2}{2}}$$

➢ Shear Force

Consider a section x-x at a distance x from free end (A).

For $0 \le x \le L$, $\boldsymbol{V_x = -wx}$

Maximum shear force is at fixed end (B).

$$\boldsymbol{V_{\max} = -wL}$$

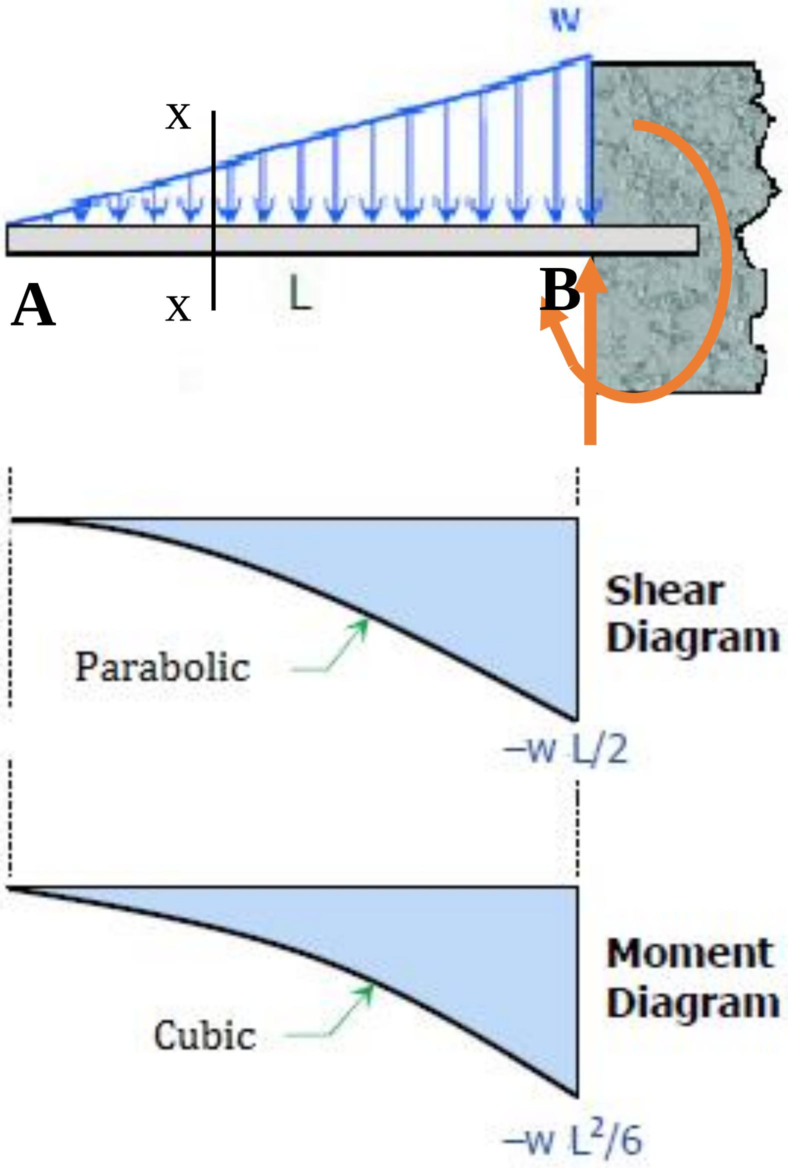
➢ Bending Moment

$$\boldsymbol{M_x = -\frac{wx^2}{2}}$$

Maximum bending moment is at fixed end (B).

$$\boldsymbol{M_{\max} = -\frac{wL^2}{2}}$$

❑Cantilever Beam Carrying a Varying Load



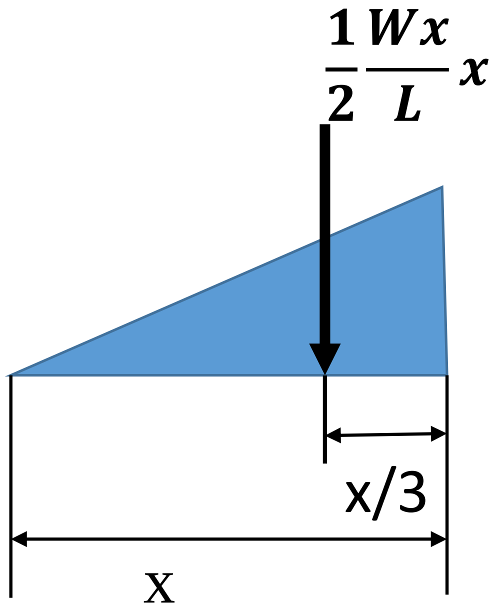
Load is zero at free end (A) and **W** at fixed end (B).

- ➢ Reactions

$$R_B = -\frac{WL}{2}$$

- ➢ Shear Force

$$V_x = -\frac{1}{2}\frac{Wx}{L}x$$



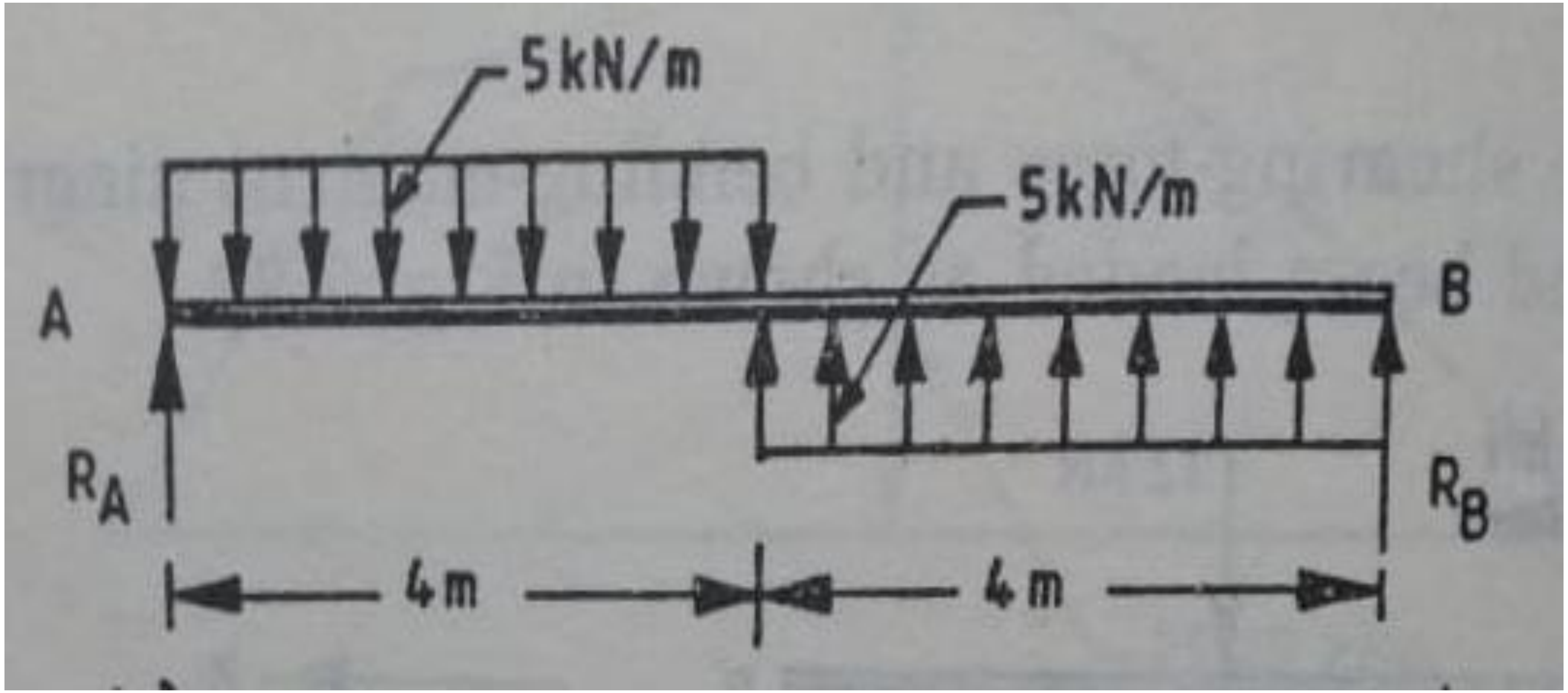
- ➢ Bending Moment

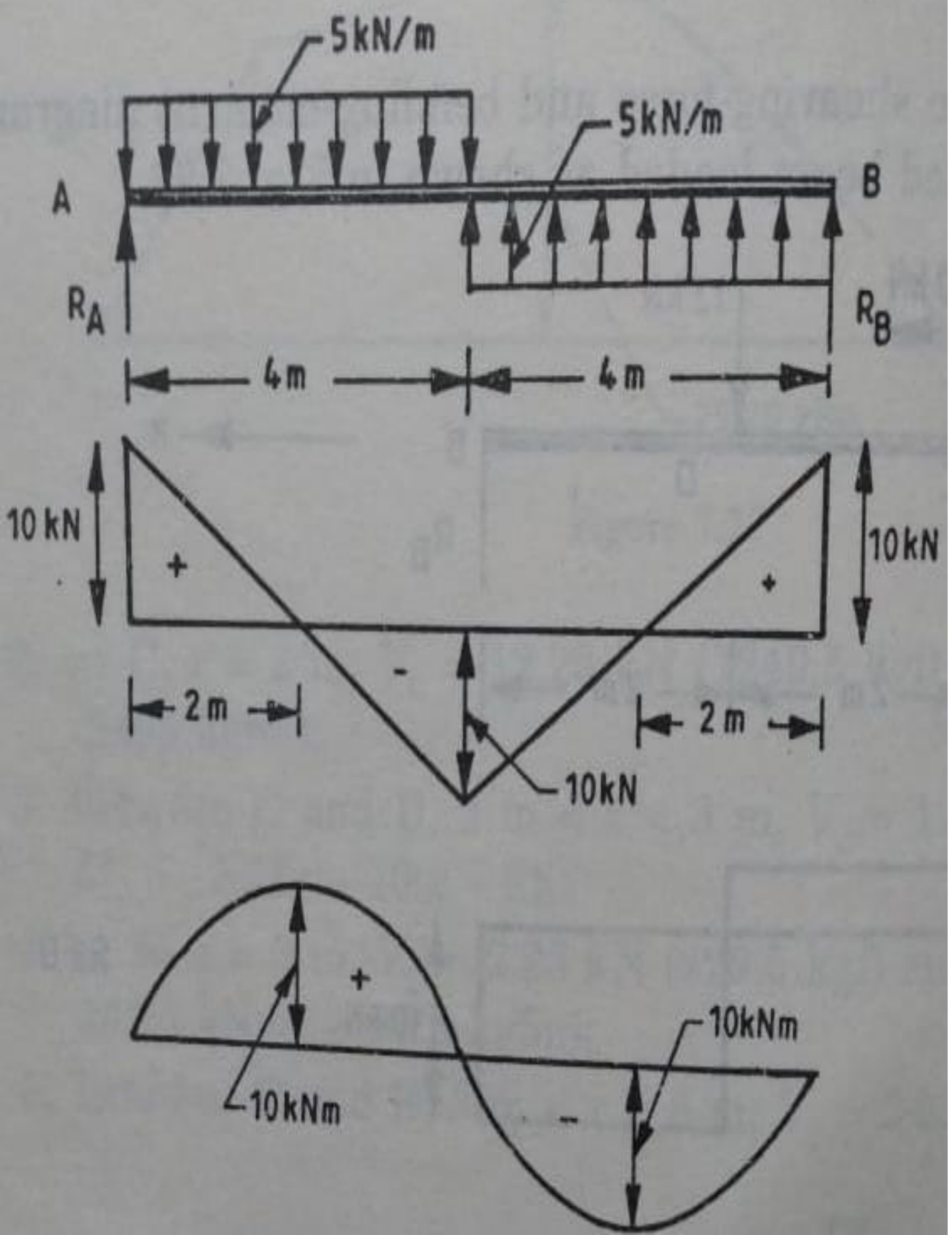
I. Replace the triangular loading up to x by average of it i.e., $\frac{1}{2}\left(0 + \frac{Wx}{L}\right)x$

II. This average load acts through the centroid of the triangle.

III. Moment about x-x $\quad M_x = -\left[\frac{1}{2}\left(\frac{Wx}{L}\right)x\right]\frac{x}{3}$

- Draw SFD and BMD of the simply supported beam AB subjected to downward and upward distributed load.





➢Solution

- Taking moment of all the forces about B, we get

$R_A 8 - 4(5)6 + 4(5)2 = 0 \quad \Rightarrow R_A = 10KN$

- From equilibrium of forces in vertical direction, we can obtain $R_B = -10KN$
- Between A and C, $0 < x < 4m$, $V_x = 10 - 5x$ and $M_x = 10x - \frac{5}{2}x^2$ (parabolic)
- At C, $x = 4m$, $V_c = 10 - 20 = -10KN$ and

$M_c = 10(4) - [\frac{5(4^2)}{2}] = 0$

- Between B and C, $4m < x < 8m$, $V_x = 10 - 20 + 5(x - 4)$ and $M_x = 10x - 20(x - 2) + [\frac{5(x-4)^2}{2}]$
- At B, $x = 8m$, $V_B = \pm 10KN$ and $M_B = 0$.

Some Important Information

- **Notes:**
    I. Where shear force changes sign in SFD, at that point bending moment is maximum or minimum in BMD.
    II. If bending moment changes sign at a section, then curvature will also change sign at that section. Such a point is called **Point of Contraflexure**.
    III. The slope of shear force curve at any section is equal to negative of the loading rate at that section.
    IV. Slope of bending moment curve at any section is magnitude of shear force at that section.