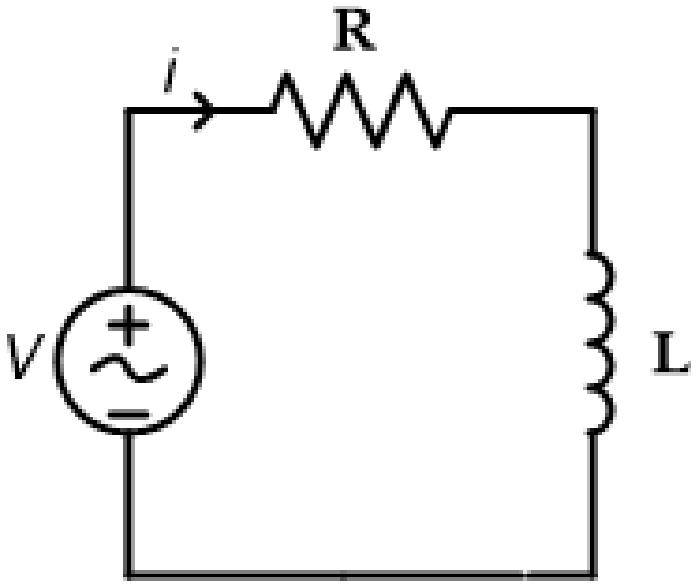
Lecture 9

Instantaneous Power, Average and Complex Power, Apparent Power and Power Factor

Instantaneous Power



Let $v = V_m \sin(\omega t + \theta)$

Then $i = I_m \sin(\omega t + \phi)$

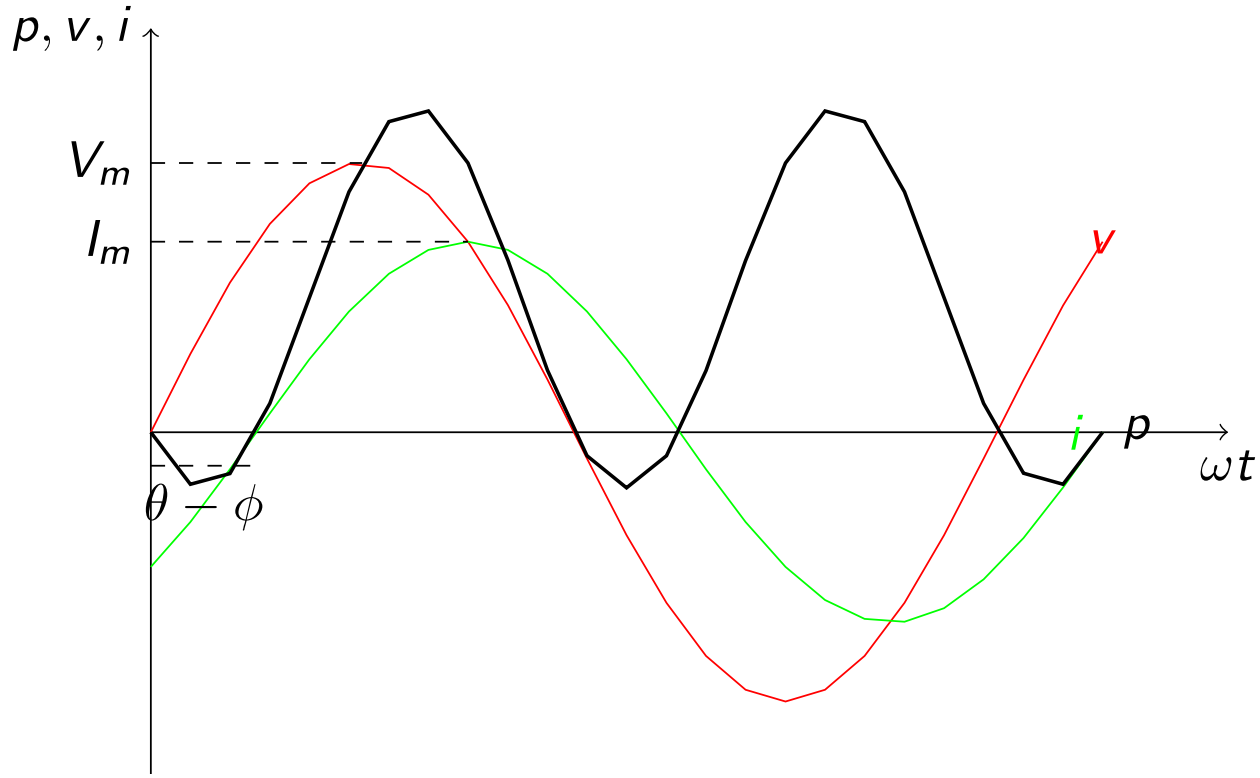
$$p(t) = v(t)i(t)$$

$$p(t) = V_m \sin(\omega t + \theta) I_m \sin(\omega t + \phi)$$

$$p(t) = \frac{V_m I_m}{2}(\cos(\theta - \phi) - \cos(2\omega t + \theta + \phi))$$

$$p(t) = \frac{V_m I_m}{2}\cos(\theta - \phi) - \frac{V_m I_m}{2}\cos(2\omega t + \theta + \phi)$$

Average Power



$$P = \frac{1}{T}\int_0^T p(t)dt$$

$$P = \frac{1}{2}V_m I_m \cos(\theta - \phi)$$

Average Power

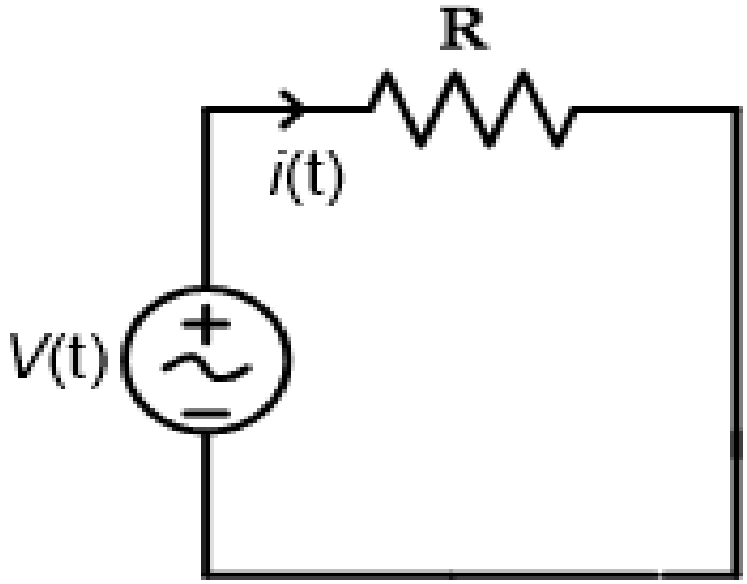
Resistor: V_m and I_m are in phase.

$$P = \frac{1}{2} V_m I_m \cos(0) = \frac{1}{2} V_m I_m$$

Ideal Inductor : V_m leads I_m by 90°.

$$P = \frac{1}{2} V_m I_m \cos(90^\circ) = 0$$

RMS current and voltage



$$V = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

$$I = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

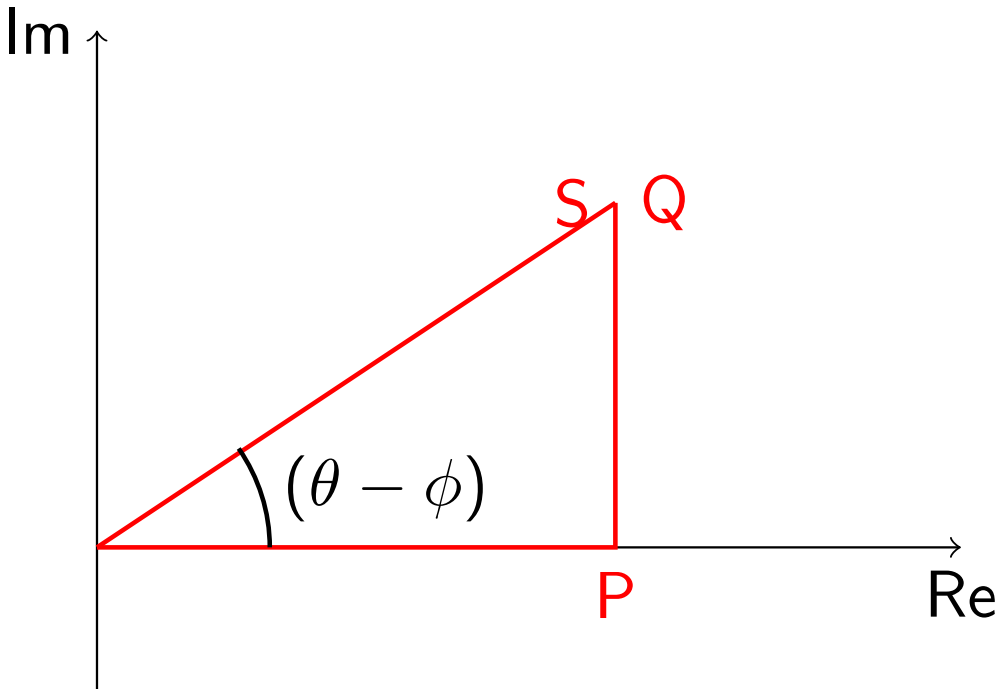
Average or Real Power

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = VI \cos(\theta - \phi)$$

where

$$V = \frac{V_m}{\sqrt{2}}, \quad I = \frac{I_m}{\sqrt{2}}$$

Power Triangle



$$\mathbf{S} = VI\cos(\theta - \phi) + \jmath VI\sin(\theta - \phi)$$

P = Real power in W, Q=Reactive power in VAR

Complex Power

$$\mathbf{S} = VI\cos(\theta - \phi) + \jmath VI\sin(\theta - \phi)$$
$$= VI\angle(\theta - \phi) = V\angle\theta\ I\angle -\phi$$

Phasor voltage and current are

$$\mathbf{V} = V\angle\theta, \quad \mathbf{I} = I\angle\phi$$

Then the complex power is

$$\mathbf{S} = \mathbf{V}\mathbf{I}^*$$

Apparent Power

Magnitude of complex power is called the apparent power

$$|S| = VI$$

Power factor

pf is the ratio of real power to the apparent power

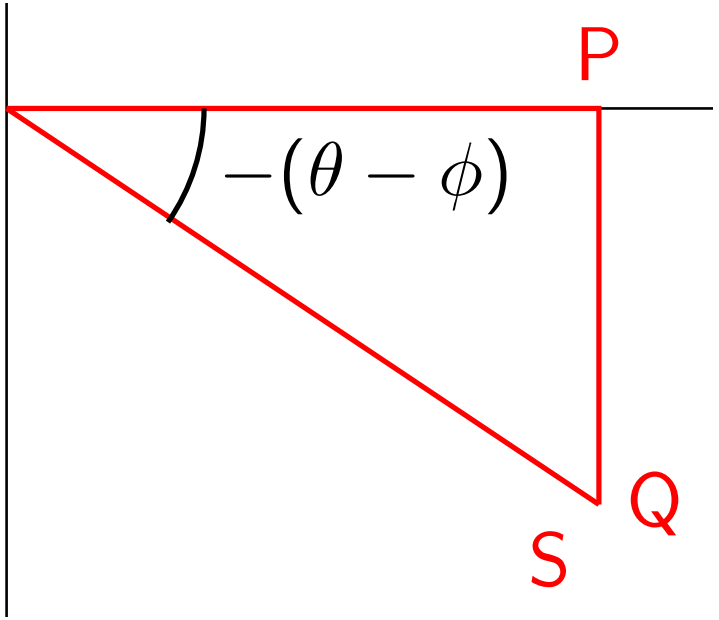
$$\text{pf} = \frac{VI\cos(\theta - \phi)}{VI} = \cos(\theta - \phi)$$

Maximum value of pf is 1

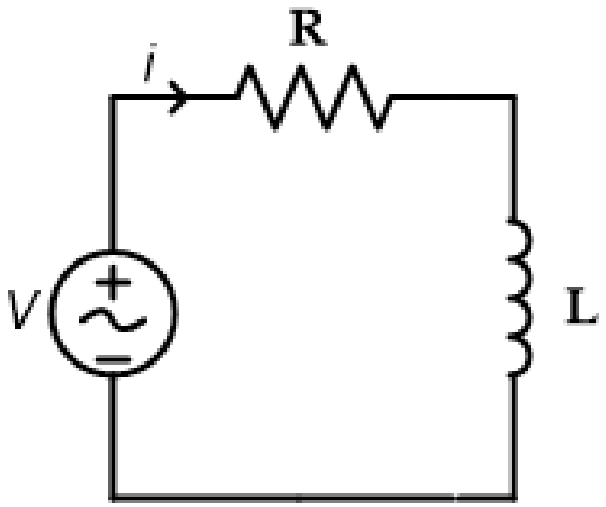
$(\boldsymbol{\theta - \phi})$ < 0 power factor is LEADING

Test

- Draw the power triangle of a RC load connected to a sinusoidal source



Estimate average, reactive powers and pf



Given, $v=230\sqrt{2}\sin(1000t)$, R = 4Ω and L = 3mH

Answers: **8.5 kW**, 6.3 kVAR, 0.8 lagging