Lectures 9-11 (Engineering Mechanics) - Distributed Forces and Center of Gravity

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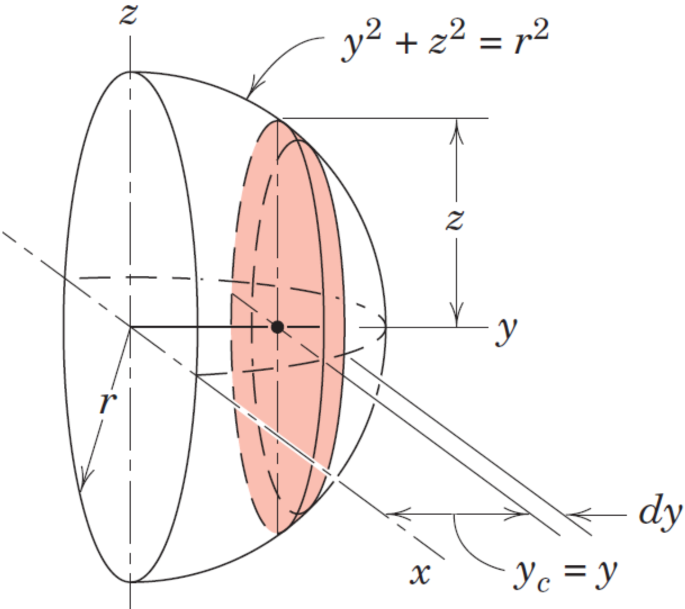
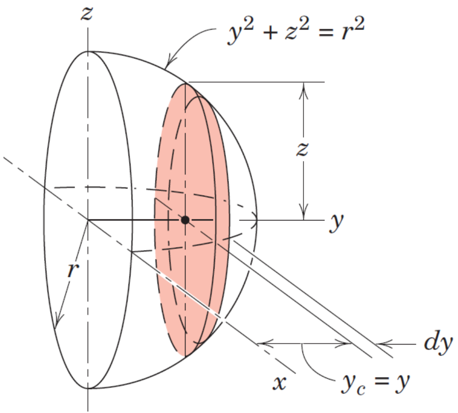
Problem 1 - Locate the centroid of the volume of a hemisphere of radius r with respect to its base.



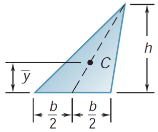
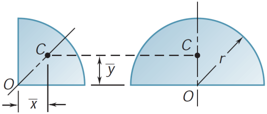
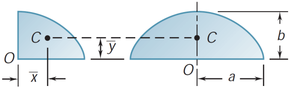
With the axes chosen as shown in the figure, $\bar{x} = \bar{z} = 0$ by symmetry. The most convenient element is a circular slice of thickness dy parallel to the $x - z$ plane. Since the hemisphere intersects the $y - z$ plane in the circle $y^2 + z^2 = r^2$, the radius of the circular slice is $z = +\sqrt{r^2 - y^2}$. The volume of the elemental slice becomes $dV = \pi(r^2 - y^2)dy$



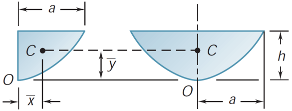
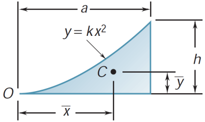
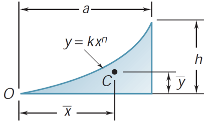
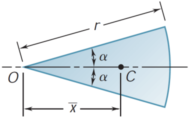
$$\bar{y}V = \int y_c dV, \Rightarrow \bar{y}\int_0^r \pi(r^2 - y^2)dy = \int_0^r y\pi(r^2 - y^2)dy.$$

Integrating gives, $\dfrac{2}{3}\pi r^3 \bar{y} = \dfrac{\pi r^4}{4}, \Rightarrow \bar{y} = \dfrac{3}{8}r$

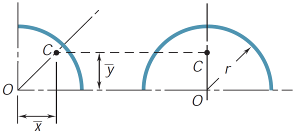
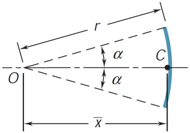
Centroids of common shapes of areas

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

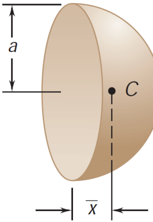
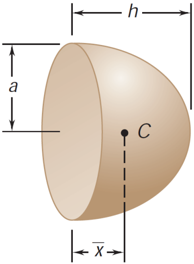
Centroids of common shapes of areas

Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

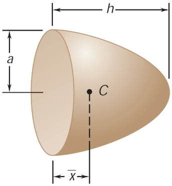
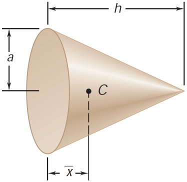
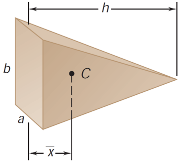
Centroids of common shapes of lines

Shape		$\overline{x}$	$\overline{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Centroids of common 3D shapes

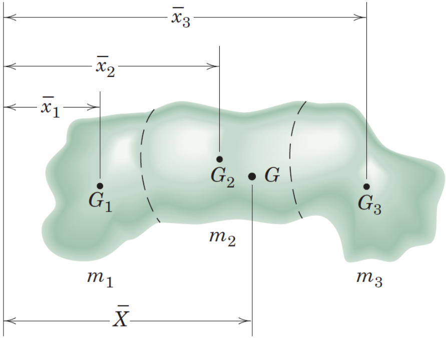
Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$

Centroids of common 3D shapes

Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$

Composite Bodies and Figures

When a body or figure can be conveniently divided into several parts whose mass centers are easily determined, we use the principle of moments and treat each part as a finite element of the whole. Its parts have masses m_1, m_2, m_3 with the respective mass-center coordinates $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$ in the x-direction.

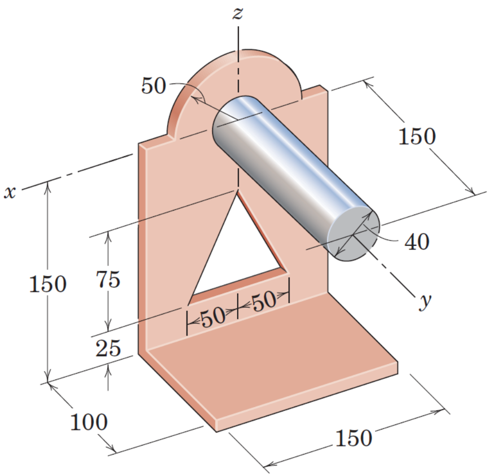


The moment principle gives $(m_1 + m_2 + m_3)\bar{X} = m_1\bar{x}_1 + m_2\bar{x}_2 + m_3\bar{x}_3$

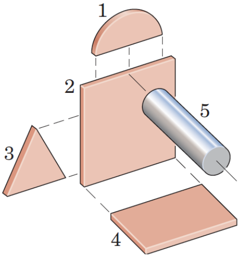
Generalization - $\bar{X} = \dfrac{\sum m\bar{x}}{\sum m}$, $\bar{Y} = \dfrac{\sum m\bar{y}}{\sum m}$, $\bar{Z} = \dfrac{\sum m\bar{z}}{\sum m}$

Analogous relations hold for composite lines, areas, and volumes, where the $m's$ are replaced by $L's$, $A's$, and $V's$, respectively.

Problem 2 - Locate the center of mass of the bracket-and-shaft combination. The vertical face is made from sheet metal which has a mass/unit area of 25 kg/m^2. The material of the horizontal base has a mass/unit area of 40 kg/m^2, and the steel shaft has a density of 7.83 Mg/m^3.



The composite body may be considered to be composed of the five elements. The triangular part will be taken as a negative mass. **For the reference axes indicated it is clear by symmetry that the x-coordinate of the center of mass is zero $\Rightarrow$ IS IT TRUE ???**



PART	m (kg)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{z}$ (mm)	$m\bar{x}$ (kg.mm)	$m\bar{y}$ (kg.mm)	$m\bar{z}$ (kg.mm)
1	0.098	0	0	21.2	0	0	2.08
2	0.562	0	0	-75	0	0	-42.19
3	-0.094	0	0	-100	0	0	9.38
4	0.6	0	50	-150	0	30	-90
5	1.476	0	75	0	0	110.7	0
TOTAL	2.642				0	140.7	-120.73

$$\overline{Y} = \frac{\sum m\bar{y}}{\sum m} = 53.3 \text{ mm}, \ \overline{Z} = \frac{\sum m\bar{z}}{\sum m} = -45.7 \text{ mm}$$

Theorems of Pappus-Guldinus

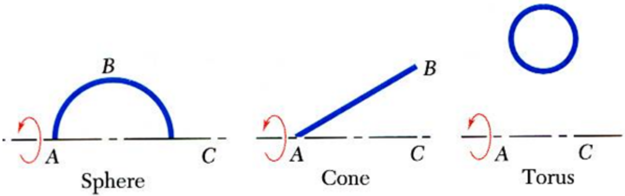


Figure: Surface of revolution is generated by rotating plane curve about fixed axis.

Theorems of Pappus-Guldinus

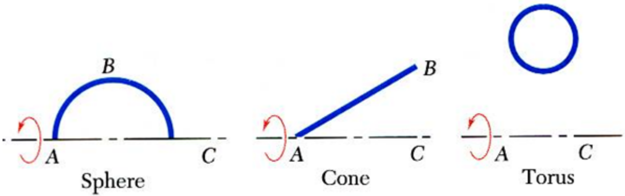
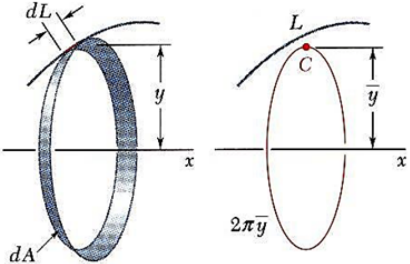


Figure: Surface of revolution is generated by rotating plane curve about fixed axis.

THEOREM I - Area of a surface of revolution is equal to the length of the generating curve times the distance traveled by the centroid through the rotation. $A = 2\pi\bar{y}L$



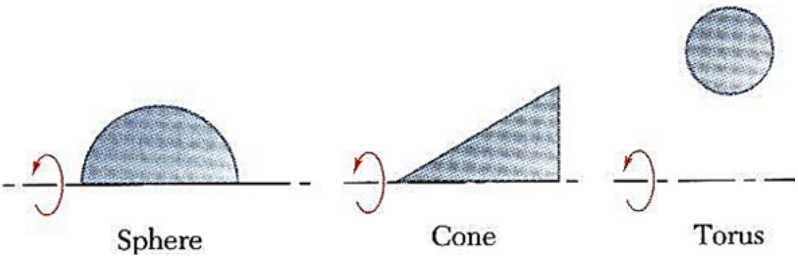


Figure: Body of revolution is generated by rotating plane area about fixed axis.

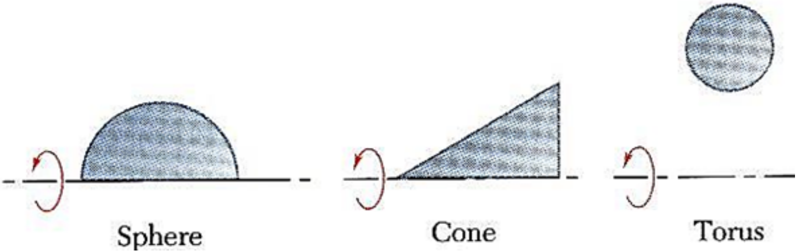
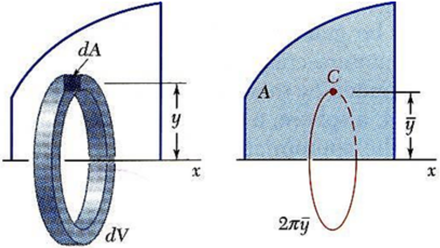
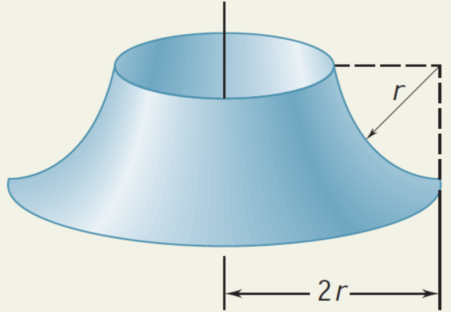


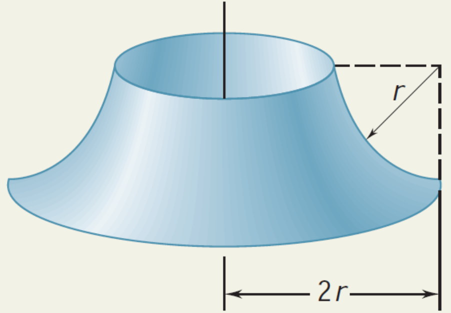
Figure: Body of revolution is generated by rotating plane area about fixed axis.

THEOREM 2 - Volume of a body of revolution is equal to the generating area times the distance traveled by the centroid through the rotation.
$V = 2\pi\bar{y}A$

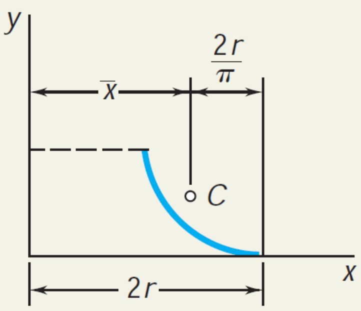




Problem 3 -Determine the area of the surface of revolution shown, which is obtained by rotating a quarter-circular arc about a vertical axis.



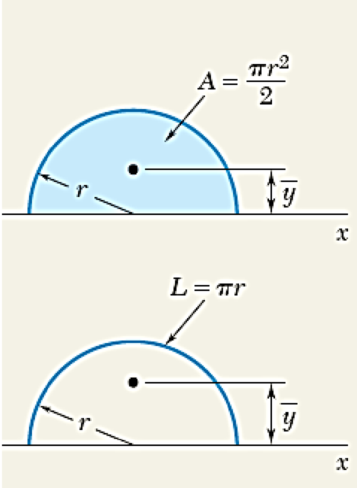
Problem 3 -Determine the area of the surface of revolution shown, which is obtained by rotating a quarter-circular arc about a vertical axis.



According to Theorem I of Pappus-Guldinus, the area generated is equal to the product of the length of the arc and the distance traveled by its centroid. Hence, $\bar{x} = 2r - \dfrac{2r}{\pi}$ and area $A = 2\pi\bar{x}L =$
$$2\pi\left(2r - \frac{2r}{\pi}\right)\left(\frac{\pi r}{2}\right) = 2\pi r^2(\pi - 1)$$

Problem 4 - Using Theorems of Pappus-Guldinus, determine (a) the centroid of a semicircular area, (b) centroid of a semicircular arc. Recall that the volume and surface area of a sphere are $(4/3)\pi r^3$ and $4\pi r^2$ respectively.

Problem 4 - Using Theorems of Pappus-Guldinus, determine (a) the centroid of a semicircular area, (b) centroid of a semicircular arc. Recall that the volume and surface area of a sphere are $(4/3)\pi r^3$ and $4\pi r^2$ respectively.



Volume of a sphere is the product of the area of a semicircle and distance travelled by centroid in one revolution about x-axis.

Hence, $V = 2\pi \bar{y} A$,

$$\Rightarrow \ \frac{4}{3}\pi r^3 = 2\pi \bar{y}(\pi r^2/2) \ \Rightarrow \ \bar{y} = \frac{4r}{3\pi}$$

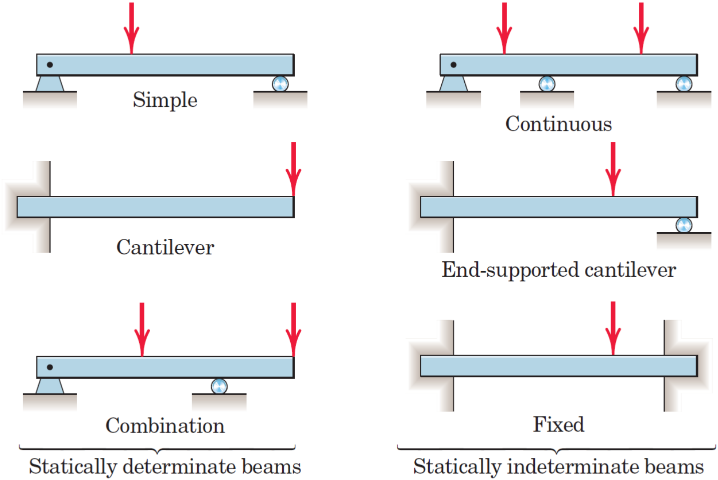
Area of a sphere is the product of the length of a semicircle and distance travelled by centroid in one revolution about x-axis.

Hence, $A = 2\pi \bar{y} L$,

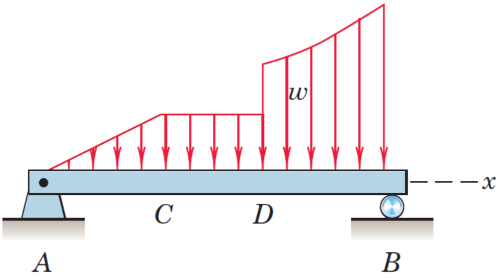
$$\Rightarrow \ 4\pi r^2 = 2\pi \bar{y}(\pi r) \ \Rightarrow \ \bar{y} = \frac{2r}{\pi}$$

BEAMS

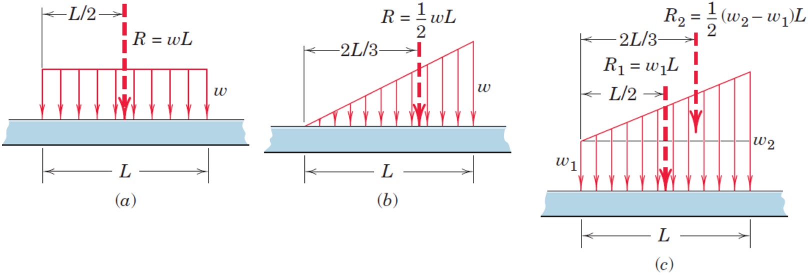
Beams are structural members which offer resistance to bending due to applied loads. Most beams are long prismatic bars, and the loads are usually applied normal to the axes of the bars



Distributed Load on a Beam



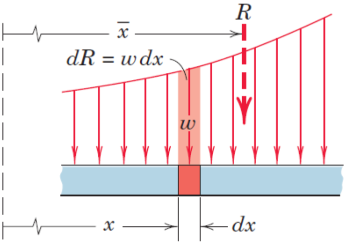
- The intensity w of a distributed load may be expressed as force per unit length of beam.
- The intensity may be constant or variable, continuous or discontinuous
- The intensity of the loading is constant from C to D and variable from A to C and from D to B.
- Intensity is discontinuous at D, where it changes magnitude abruptly.
- Although the intensity itself is not discontinuous at C, the rate of change of intensity dw/dx is discontinuous



- In cases (a) and (b), resultant load R is represented by area formed by the intensity w and the length L over which the force is distributed. The resultant passes through the centroid of this area.
- In part (c), the trapezoidal area is broken into a rectangular and a triangular area, and the corresponding resultants R_1 and R_2 of these subareas are determined separately.
- Note that a single resultant could be determined by using the composite technique for finding centroids. Usually, however, the determination of a single resultant is unnecessary.

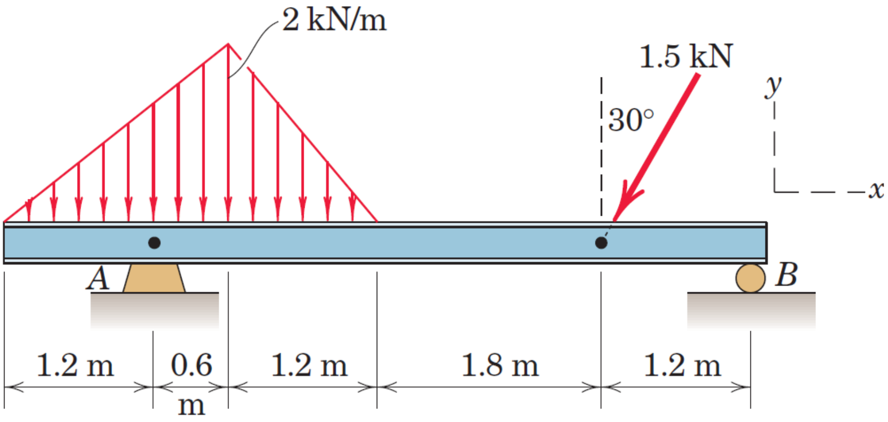
General Load Distribution

- Start with a differential increment of force $dR = wdx$.
- The total load R is then the sum of the differential forces, $R = \int wdx$



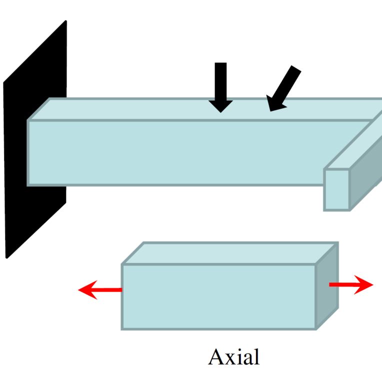
- Resultant R is located at the centroid of the area under consideration.
- The x-coordinate of this centroid is found by the principle of moments
  $R\bar{x} = \int xwdx, \Rightarrow \ \bar{x} = \dfrac{\int xwdx}{R}$
- For the distribution shown, the vertical coordinate of the centroid need not be found
- Once the distributed loads have been reduced to their equivalent concentrated loads, the external reactions acting on the beam may be found by a straightforward static analysis

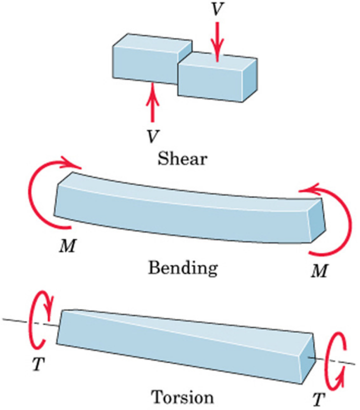
Problem 5 - Determine the reactions at A and B for the beam subjected to a combination of distributed and point loads.



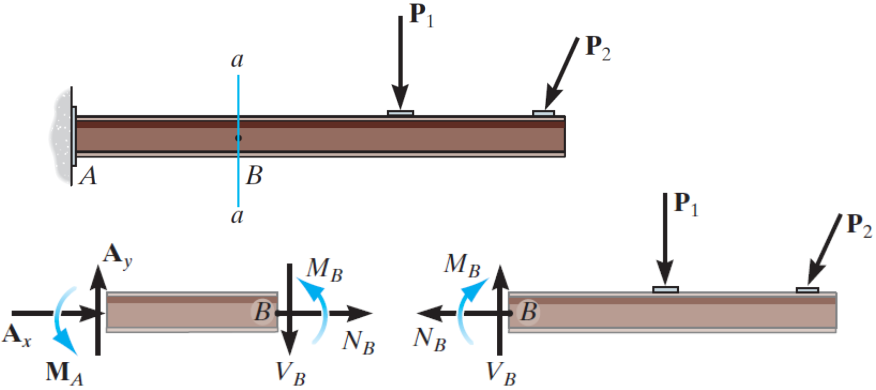
Beams - Internal Effects

- Internal Force Resultants
- Axial force, shear force, bending moment, torsional moment

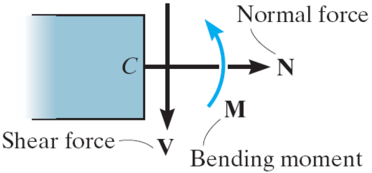




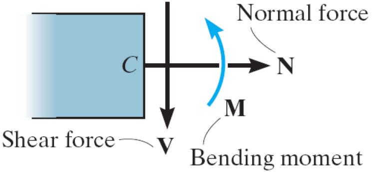
Method of Sections - Internal Force Resultants at B $\rightarrow$ section $a-a$ at B and use equilibrium equations in both cut parts



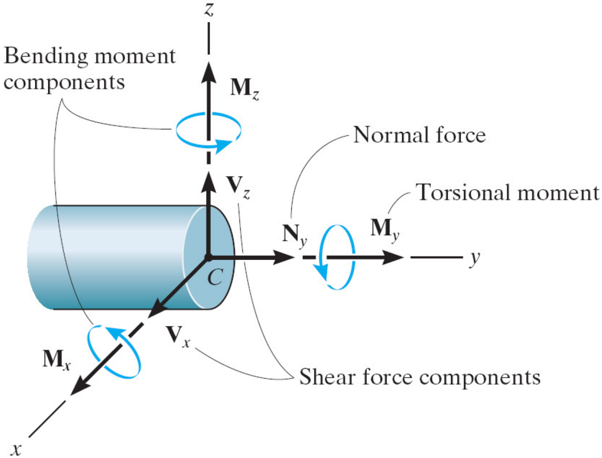
2D BEAM



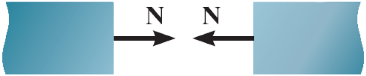
2D BEAM



3D BEAM - Force Resultants act at centroid of the section's cross-sectional area



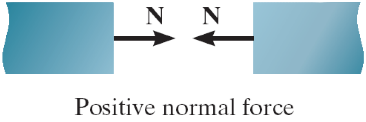
Positive axial force creates Tension



Positive normal force

Sign Convention

Positive axial force creates Tension



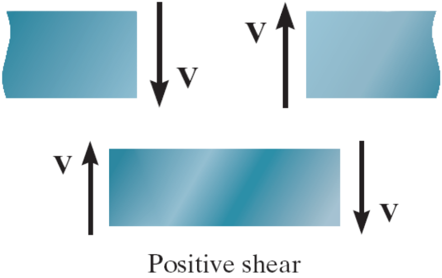
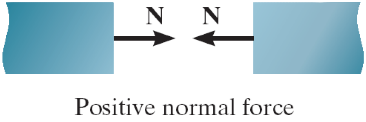


Figure: Positive shear force will cause the Beam segment on which it acts to rotate clockwise

Sign Convention

Positive axial force creates Tension



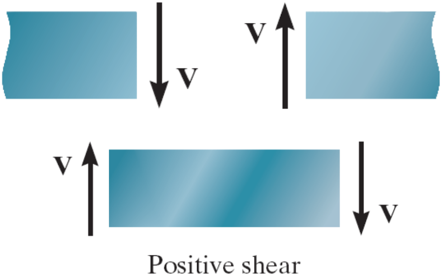


Figure: Positive shear force will cause the Beam segment on which it acts to rotate clockwise

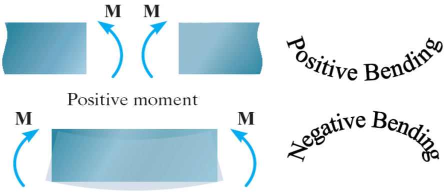
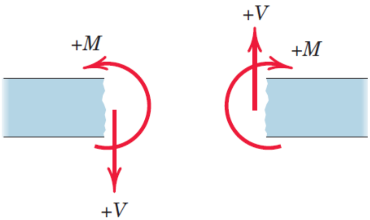
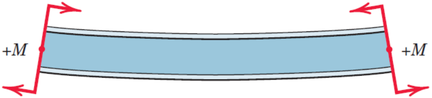


Figure: Positive bending moment will tend to bend the segment on which it acts in a concave upward manner (compression on top of section).

Sign Convention in a single plane

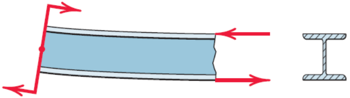


H-section Beam bent by two equal and opposite positive moments applied at ends



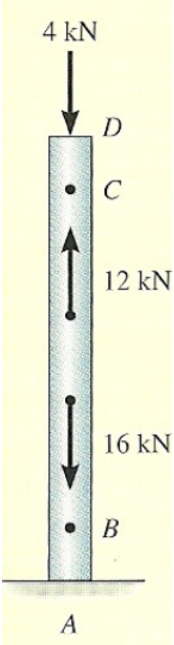
Neglecting resistance offered by web

Compression at top; Tension at bottom

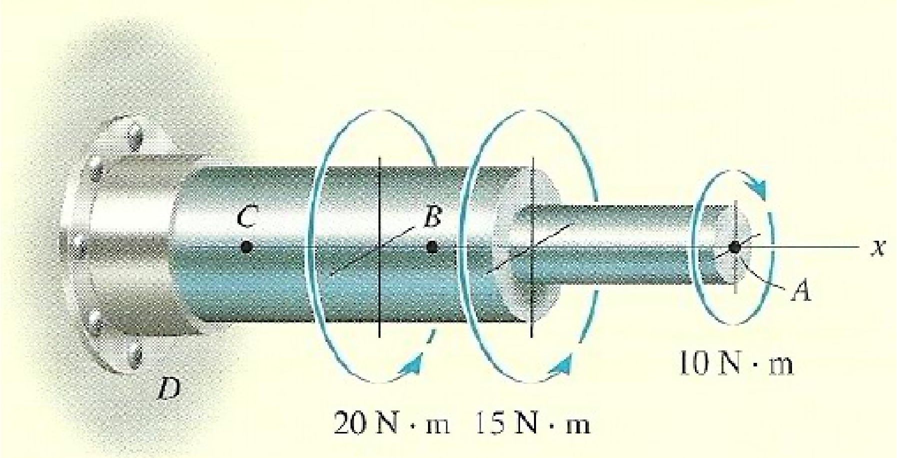


Resultant of these two forces (one tensile and other compressive) acting on any section is a Couple and has the value of the Bending Moment acting on the section.

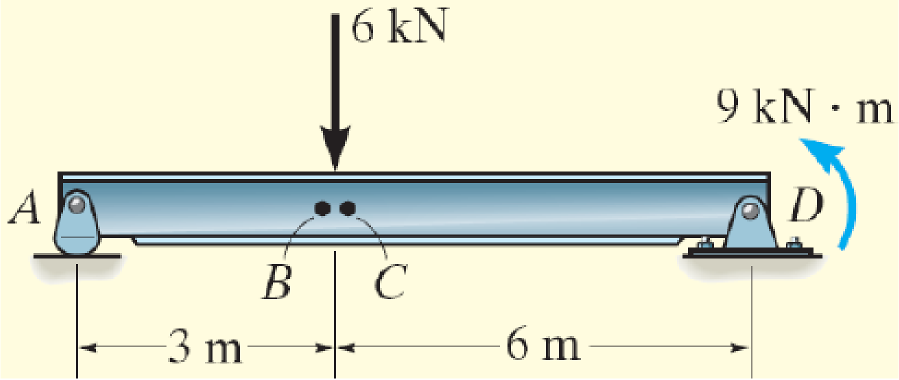
Problem 6 - Find the axial force in the fixed bar at points B and C



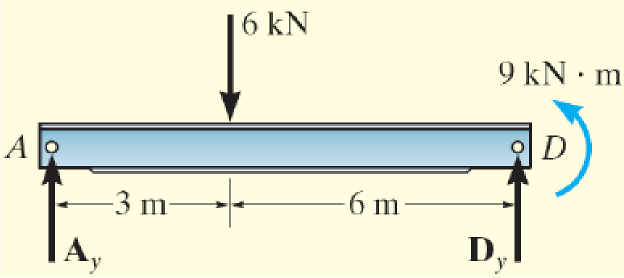
Problem 7 - Find the internal torques at points B and C of the circular shaft subjected to three concentrated torques



Problem 8 - Find the AF, SF, and BM at point B (just to the left of 6 kN) and at point C (just to the right of 6 kN)



Solution - Draw FBD of Entire Beam

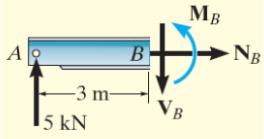


$$\circlearrowleft + \Sigma M_D = 0;\ 9\ \text{kN}\cdot\text{m} + (6\ \text{kN})(6\ \text{m}) - A_y(9\ \text{m}) = 0$$

$$A_y = 5\ \text{kN}$$

Figure: D_y need not be determined if only left part of the beam is analyzed

Draw FBD of segments AB and AC and use equilibrium equations

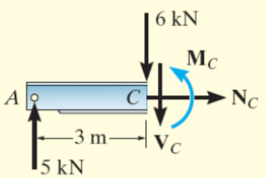


Segment AB

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_B = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 5\text{ kN} - V_B = 0 \qquad V_B = 5\text{ kN}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad -(5\text{ kN})(3\text{ m}) + M_B = 0 \qquad M_B = 15\text{ kN}\cdot\text{m}$$



Segment AC

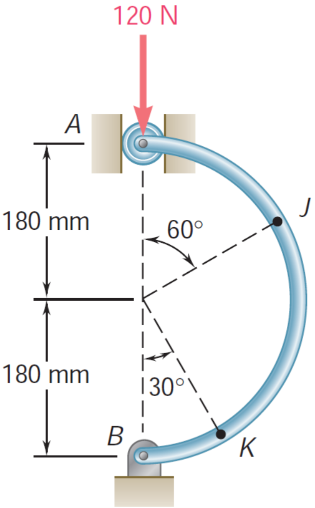
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_C = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 5\text{ kN} - 6\text{ kN} - V_C = 0 \qquad V_C = -1\text{ kN}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -(5\text{ kN})(3\text{ m}) + M_C = 0 \qquad M_C = 15\text{ kN}\cdot\text{m}$$

TRY ON YOUR OWN

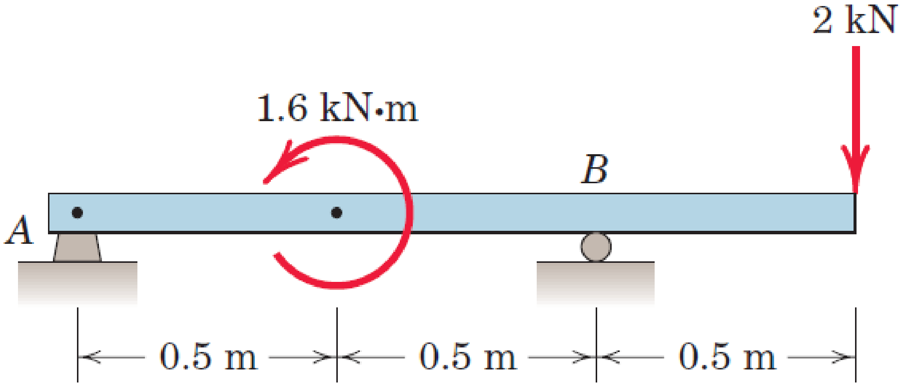
Problem 9 - A semicircular rod is loaded as shown. Determine the internal forces at point J and point K.



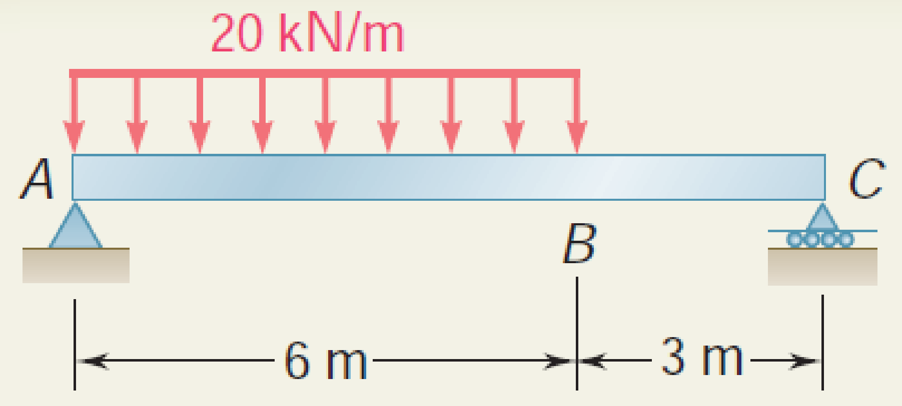
Shear Force and Bending Moment Diagrams

1. Variation of SF and BM over the length of the beam - SFD and BMD
2. Maximum magnitude of BM and SF and their locations is prime consideration in beam design
3. SFDs and BMDs are plotted using method of section
   - Equilibrium of FBD of entire Beam $\rightarrow$ External reactions
   - Equilibrium of a cut part of beam $\rightarrow$ Expressions for SF and BM at the cut section
4. **Use the positive sign convention consistently**
5. The part of the beam which involves the smaller number of forces, either to the right or to the left of the arbitrary section, usually yields the simpler solution
6. We should avoid using a transverse section which coincides with the location of a concentrated load or couple, as such a position represents a point of discontinuity in the variation of shear or bending moment

Problem 10 - Construct the shear and moment diagrams for the beam loaded by the 2-kN force and the 1.6-kN m couple. State the value of the bending moment at point B

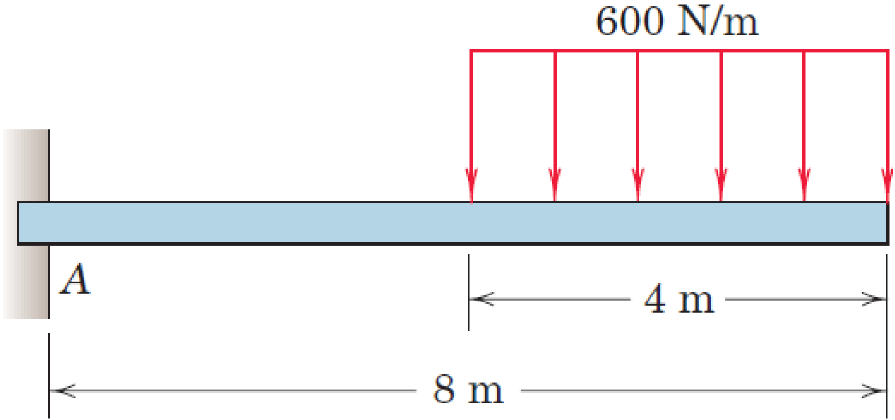


Problem 11 - Draw the shear and bending-moment diagrams for the beam and loading shown and determine the location and magnitude of the maximum bending moment

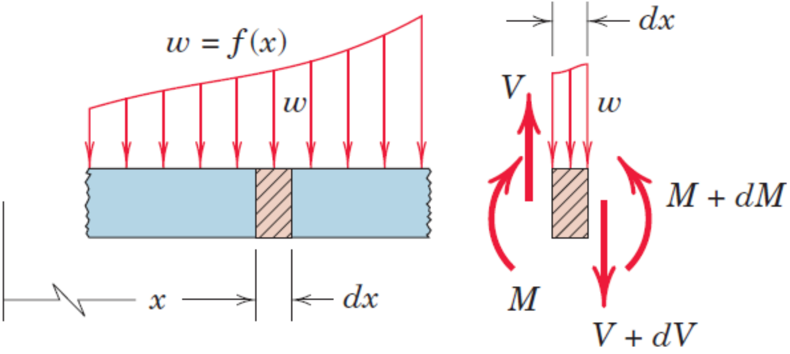


TRY ON YOUR OWN

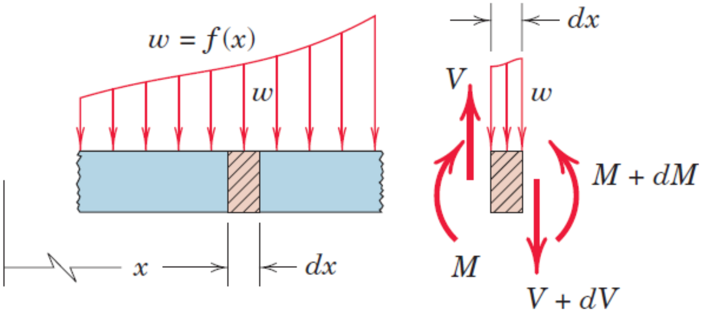
Determine the shear and moment diagrams for the loaded cantilever beam. Specify the shear V and moment M at the middle section of the beam



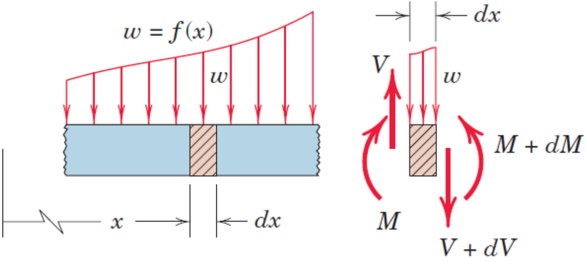
General Loading, Shear, and Moment Relationships



- At the location x the shear V and moment M acting on the element are drawn in their positive directions
- On the opposite side of the element where the coordinate is $x + dx$, these quantities are also shown in their positive directions



- Equilibrium - $V - wdx - (V + dV) = 0 \;\Rightarrow\; w = -\dfrac{dV}{dx}$
- Slope of the shear diagram must everywhere be equal to the negative of the value of the applied loading
- **holds on either side of a concentrated load but not at the concentrated load because of the discontinuity produced by the abrupt change in shear**
- $\int_{V_0}^{V} dV = -\int_{x_0}^{x} wdx, \Rightarrow V = V_0 +$ (the negative of the area under the loading curve from x_0 to x)

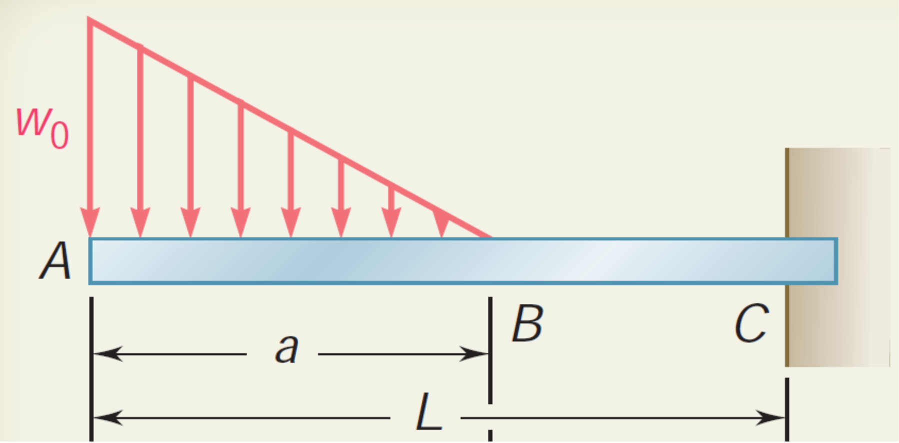


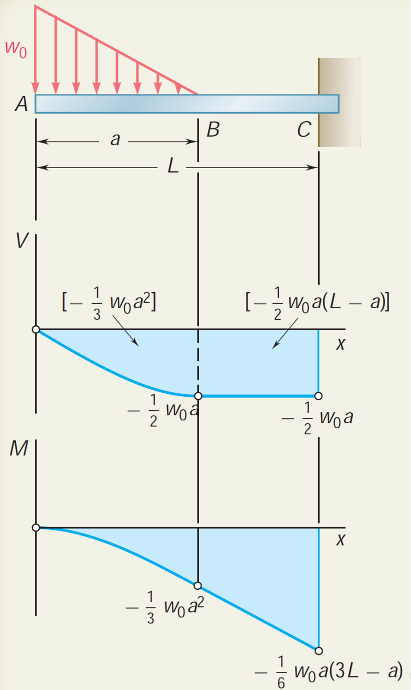
- Equilibrium also requires moment sum to be zero-
  $$M + wdx\left(\frac{dx}{2}\right) + (V + dV)dx - (M + dM) = 0$$
- This gives $V = \dfrac{dM}{dx}$ - which expresses the fact that the shear everywhere is equal to the slope of the moment curve
- **holds on either side of a concentrated couple but not at the concentrated couple because of the discontinuity caused by the abrupt change in moment**
- $\int_{M_0}^{M} dM = \int_{x_0}^{x} Vdx, \Rightarrow M = M_0 +$ (area under the shear diagram from x_0 to x)

- $\int_{M_0}^{M} dM = \int_{x_0}^{x} Vdx, \Rightarrow M = M_0 +$ (area under SFD from x_0 to x)
- M_0 is BM at x_0 and M is BM at x. For beams where there is no externally applied moment M_0 at $x_0 = 0$, the total moment at any section equals the area under the shear diagram up to that section.
- When V passes through zero and is a continuous function of x with $dV/dx \neq 0$, the bending moment M will be a maximum or a minimum, since $dM/dx = 0$ at such a point
- Critical values of M also occur when V crosses the zero axis discontinuously, which occurs for beams under concentrated loads
- **Degree of V in x is one higher than that of w. Also M is of one higher degree in x than is V.** Thus for a beam loaded by $w = kx$, which is of the first degree in x, the shear V is of the second degree in x and the bending moment M is of the third degree in x
- $\dfrac{d^2M}{dx^2} = -w$, (M can be obtained by two integrations provided that the limits of integration are properly evaluated each time). Method only usable ONLY if w is a continuous function of x.
- When bending in a beam occurs in more than a single plane - do separate analysis in each plane and combine results vectorially

TRY ON YOUR OWN

Problem 13 - Sketch the shear and bending-moment diagrams for the cantilever beam shown



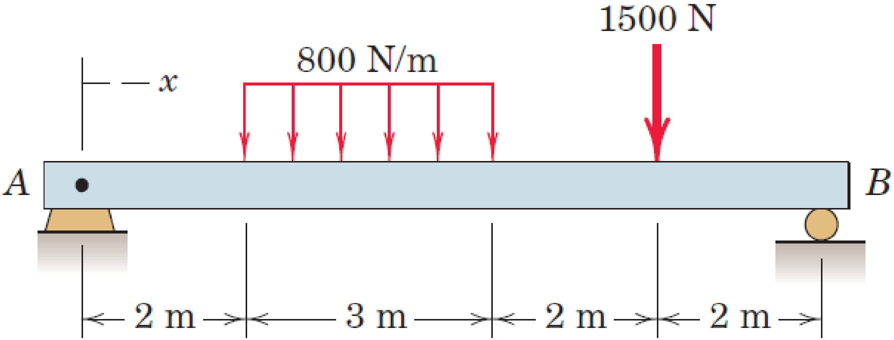


w_0
A
B
C
a
L
V
$[-\frac{1}{3}w_0a^2]$
$[-\frac{1}{2}w_0a(L-a)]$
x
$-\frac{1}{2}w_0a$
$-\frac{1}{2}w_0a$
M
x
$-\frac{1}{3}w_0a^2$
$-\frac{1}{6}w_0a(3L-a)$

Important Information

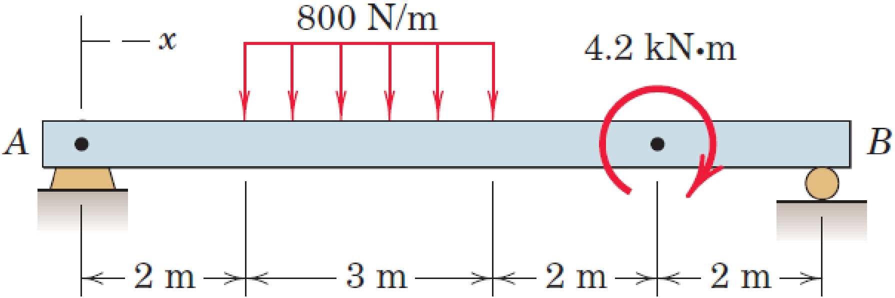
- Where shear force changes sign in SFD, at that point bending moment is maximum or minimum in BMD.
- If bending moment changes sign at a section, then curvature will also change sign at that section. Such a point is called Point of Contraflexure.
- The slope of shear force curve at any section is equal to loading rate at that section.
- Slope of bending moment curve at any section is magnitude of shear force at that section.

Problem 13 - Plot the shear and moment diagrams for the beam loaded with both the distributed and point loads. What are the values of the shear and moment at $x = 6$ m? Determine the maximum bending moment M_{max}



TRY ON YOUR OWN

Problem 14 - What happens if concentrated load is replaced by a 4.2 kN-m couple?

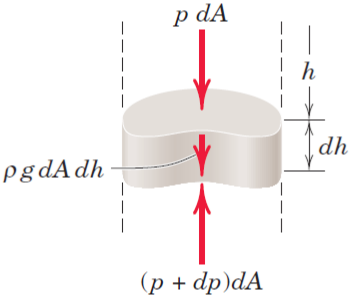


A fluid is any continuous substance which, when at rest, is unable to support shear force

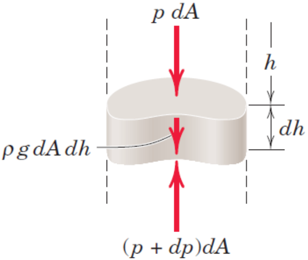
Statics of Fluids $\Rightarrow$ **Hydrostatics**

The pressure at any given point in a fluid is the same in all directions (Pascal's law)

- **In all fluids at rest the pressure is a function of the vertical dimension**
- consider the forces acting on a differential element of a vertical column of fluid of cross-sectional area dA
- The pressure on the upper face is p, and that on the lower face $p + dp$



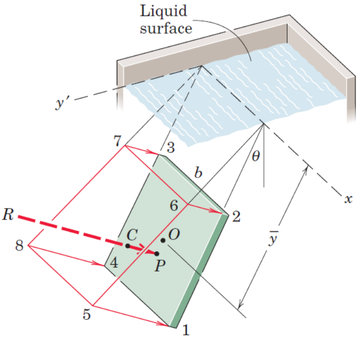
- The weight of the element equals ρg multiplied by its volume. The normal forces on the lateral surface, which are horizontal and do not affect the balance of forces in the vertical direction, are not shown



- Equilibrium of the fluid element in the h-direction - $p\ dA + \rho g\ dA\ dh - (p + dp)dA = 0$
- $dP = \rho g\ dh$
- Holds for both liquids and gases
- Fluids which are essentially incompressible are called liquids, and for most practical purposes we may consider their density ρ constant for every part of the liquid - $p = p_0 + \rho gh$
- p_0 is the pressure on the surface of the liquid where h $= 0$. If p_0 is due to atmospheric pressure and the measuring instrument records only the increment above atmospheric pressure, the measurement gives what is called guage pressure. It is computed from $p = \rho gh$.

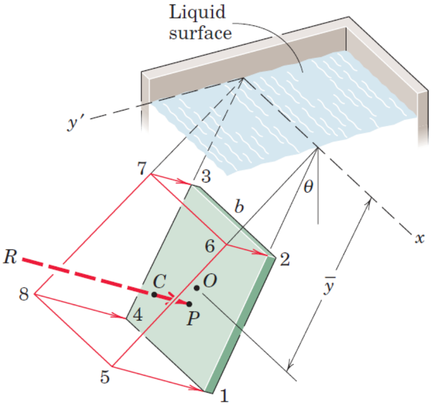
Hydrostatic Pressure on Submerged Rectangular Surfaces

- A body submerged in a liquid, such as a gate valve in a dam or the wall of a tank, is subjected to fluid pressure acting normal to its surface and distributed over its area.

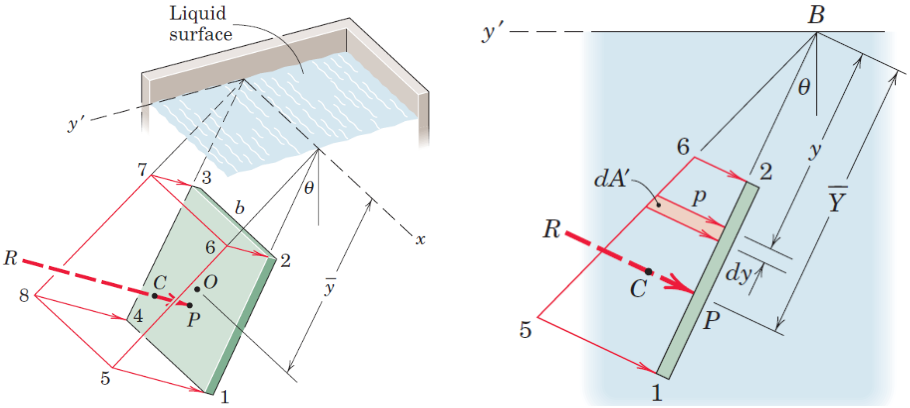


- In problems where fluid forces are appreciable, we must determine the resultant force due to the distribution of pressure on the surface and the position at which this resultant acts.
- For systems open to the atmosphere, the atmospheric pressure p_0 acts over all surfaces and thus yields a zero resultant. In such cases, then, we need to consider only the guage pressure $p = \rho g h$, which is the increment above atmospheric pressure

- plate $1-2-3-4$ with its top edge horizontal and with the plane of the plate making an arbitrary angle θ with the vertical plane. The horizontal surface of the liquid is represented by the $x-y'$ plane

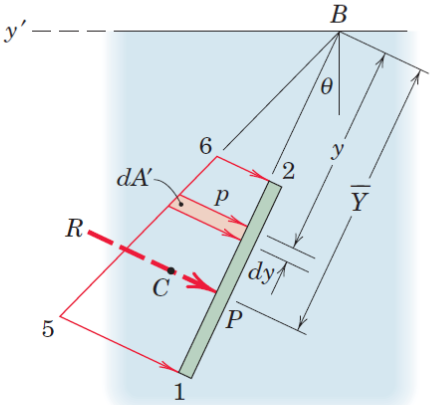


- The fluid pressure (guage) acting normal to the plate at point 2 is represented by the arrow 6-2 and equals ρg times the vertical distance from the liquid surface to point 2 and this pressure is the same at all points along edge 2-3
- At point 1 on the lower edge, the fluid pressure equals ρg times the depth of point 1, and this pressure is the same at all points along edge 1-4



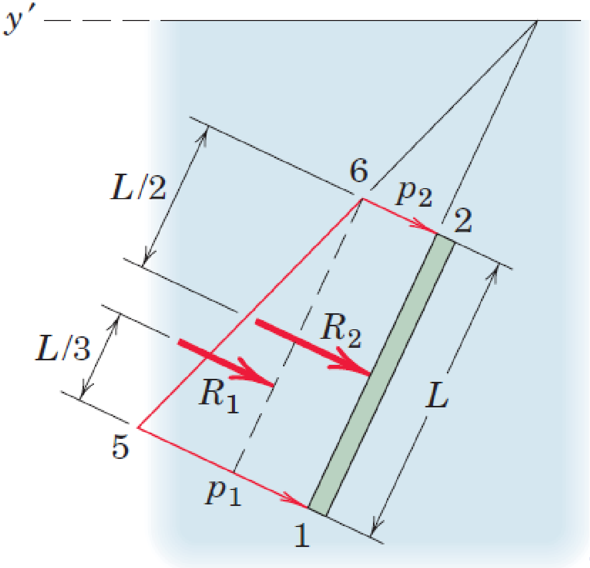
- The variation of pressure p over the area of the plate is governed by the linear depth relationship and therefore it is represented by the arrow p - varies linearly from the value 6-2 to the value 5-1
- The resultant force produced by this pressure distribution is represented by R, which acts at some point P called the center of pressure

If b is the horizontal width of the plate measured normal to the plane of the figure, an element of plate area over which the pressure $p = \rho g h$ acts is $dA = bdy$, and an increment of the resultant force is $dR = pdA = bpdy = bdA'$. Hence, $R = bA'$.

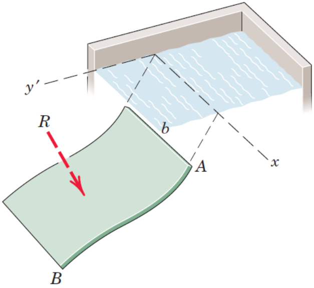


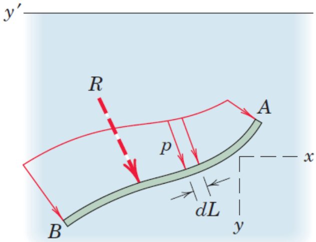
- The trapezoidal area representing the pressure distribution is easily expressed by using its average altitude - $R = p_{av}A = \rho g \bar{h} A$, where $\bar{h} = \bar{y}\cos\theta$.
- line of action of the resultant force R can be obtained from the principle of moments - Using x axis as moment axis, $R\bar{Y} = \int y(pbdy)$.
- We get $\bar{Y} = \dfrac{\int y dA'}{\int dA'}$

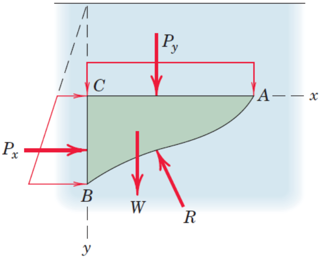
For a trapezoidal distribution of pressure, we may simplify the calculation by dividing the trapezoid into a rectangle and a triangle. $R_2 = p_2 A$ and $R_1 = (1/2)(p_1 - p_2)A$



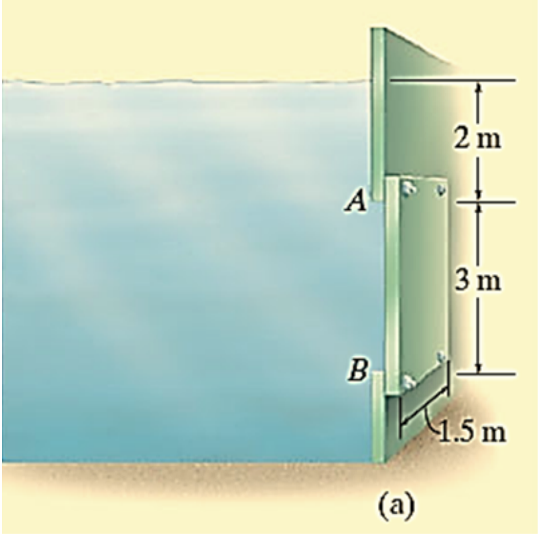
elements of the curved surface are parallel to the horizontal surface $x - y'$ of the liquid

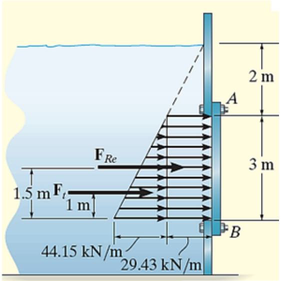






Problem 15 - Determine the magnitude and location of the resultant jhydrostatic force acting on a submerged rectangular plate AB shown. Width of plate is 1.5 m and $\rho_w = 1000\ kg/m^3$





The water pressures at depths A and B are

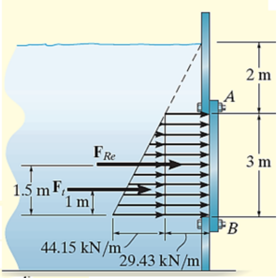
$$p_A = \rho_w g z_A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m}) = 19.62\ \text{kPa}$$

$$p_B = \rho_w g z_B = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m}) = 49.05\ \text{kPa}$$

Since the plate has a constant width, the pressure loading can be viewed in two dimensions as shown in Fig. The intensities of the load at A and B are

$$w_A = bp_A = (1.5\ \text{m})(19.62\ \text{kPa}) = 29.43\ \text{kN/m}$$

$$w_B = bp_B = (1.5\ \text{m})(49.05\ \text{kPa}) = 73.58\ \text{kN/m}$$



Consider two components of $\mathbf{F}_R$, defined by the triangle and rectangle shown in Fig. Each force acts through its associated centroid and has a magnitude of

$$F_{Re} = (29.43\ \text{kN/m})(3\ \text{m}) = 88.3\ \text{kN}$$

$$F_t = \tfrac{1}{2}(44.15\ \text{kN/m})(3\ \text{m}) = 66.2\ \text{kN}$$

Hence,

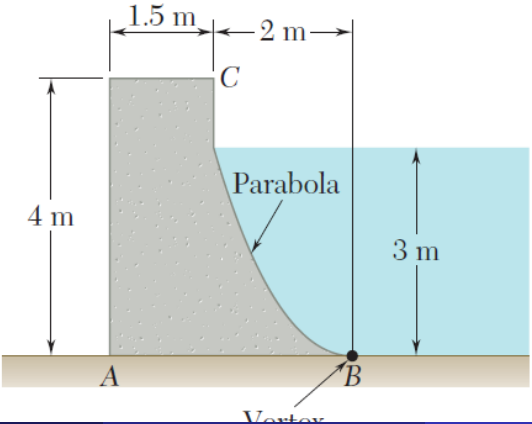
$$F_R = F_{Re} + F_t = 88.3 + 66.2 = 154.5\ \text{kN} \qquad \textit{Ans.}$$

The location of $\mathbf{F}_R$ is determined by summing moments about B,

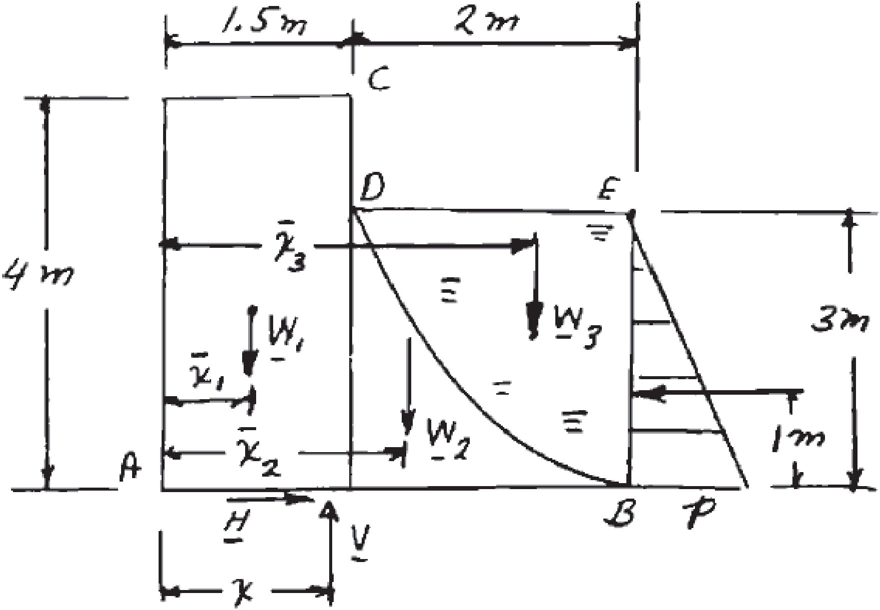
$$\circlearrowleft +(M_R)_B = \Sigma M_B;\ (154.5)h = 88.3(1.5) + 66.2(1)$$

$$h = 1.29\ \text{m} \qquad \textit{Ans.}$$

Problem 16 - The cross section of a concrete dam is as shown. For a 1-m-wide dam section determine (a) the resultant of the reaction forces exerted by the ground on the base AB of the dam, (b) the point of application of the resultant of part a, (c) the resultant of the pressure forces exerted by the water on the face BC of the dam.



Consider FBD of dam and section BDE of water. (Thickness = 1 m)



$$p = (3\text{ m})(10\text{ kg/m}^3)(9.81\text{ m/s}^2)$$

$$W_1 = (1.5\text{ m})(4\text{ m})(1\text{ m})(2.4\times10^3\text{ kg/m}^3)(9.81\text{ m/s}^2) = 144.26\text{ kN}$$

$$W_2 = \frac{1}{3}(2\text{ m})(3\text{ m})(1\text{ m})(2.4\times10^3\text{ kg/m}^3)(9.81\text{ m/s}^2) = 47.09\text{ kN}$$

$$W_3 = \frac{2}{3}(2\text{ m})(3\text{ m})(1\text{ m})(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2) = 39.24\text{ kN}$$

$$P = \frac{1}{2}Ap = \frac{1}{2}(3\text{ m})(1\text{ m})(3\text{ m})(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2) = 44.145\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad H - 44.145\text{ kN} = 0$$

$$H = 44.145\text{ kN} \qquad \mathbf{H} = 44.1\text{ kN} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad V - 141.26 - 47.09 - 39.24 = 0$$

$$V = 227.6\text{ kN} \qquad \mathbf{V} = 228\text{ kN} \uparrow \blacktriangleleft$$

$$\bar{x}_1 = \frac{1}{2}(1.5 \text{ m}) = 0.75 \text{ m}$$

$$\bar{x}_2 = 1.5 \text{ m} + \frac{1}{4}(2 \text{ m}) = 2 \text{ m}$$

$$\bar{x}_3 = 1.5 \text{ m} + \frac{5}{8}(2 \text{ m}) = 2.75 \text{ m}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad xV - \Sigma \bar{x} W + P(1 \text{ m}) = 0$$

$$x(227.6 \text{ kN}) - (141.26 \text{ kN})(0.75 \text{ m}) - (47.09 \text{ kN})(2 \text{ m}) - (39.24 \text{ kN})(2.75 \text{ m}) + (44.145 \text{ kN})(1 \text{ m}) = 0$$

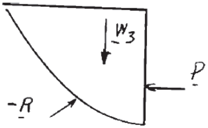
$$x(227.6 \text{ kN}) - 105.9 - 94.2 - 107.9 + 44.145 = 0$$

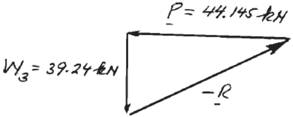
$$x(227.6) - 263.9 = 0$$

$$x = 1.159 \text{ m} \ \text{(to right of } A) \blacktriangleleft$$

Resultant of face *BC*:

Consider free body of section *BDE* of water.





$-\mathbf{R} = 59.1\,\mathrm{kN}$ ∠ $41.6°$

$\mathbf{R} = 59.1\,\mathrm{kN}$ ∠ $41.6°$ ◀