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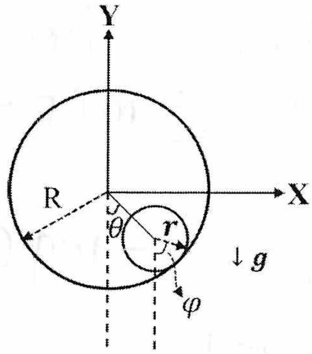
Q5	**[5 marks]** A solid homogeneous cylinder of radius r and mass m rolls without slipping on the inside surface of a stationary large cylinder of radius R as shown in the figure. Acceleration due to gravity (g) acts downwards. Choose appropriate generalized coordinates from the variables marked in the figure. (i) Express the kinetic and potential energies using generalized coordinates and generalized velocities. (ii) Determine the equation of motion of the inside cylinder using Lagrangian formalism. (iii) Assuming θ to be small (*i.e.*, $\sin\theta \approx \theta$), find the time period of oscillation about the stable equilibrium.	

Sol^n $\theta \to$ Angle rotated by position vector of COM of small cylinder from vertical.

$\phi \to$ Angle rotated by small cylinder from vertical

No. of generalised coordinates = 1 ; Let θ be the generalised coordinate.

$I = \frac{mr^2}{2}$, Let (x,y) be the cartesian coordinates of COM of small cylinder.

$\therefore x = (R-r)\sin\theta, \quad y = -(R-r)\cos\theta$

Now, as the cylinder rolls without slipping

$\therefore (R-r)d\theta = -r\,d\phi \Rightarrow \frac{d\theta}{d\phi} = \frac{-r}{R-r} = \frac{\dot\theta}{\dot\phi}$

$\Rightarrow (R-r)\dot\theta = -r\dot\phi$ —— (1)

$T = \frac{1}{2}m\left[\dot x^2 + \dot y^2\right] + \frac{1}{2}I\dot\phi^2$

$\dot x = (R-r)\cos\theta\,\dot\theta, \quad \dot y = (R-r)\sin\theta\,\dot\theta$

$\therefore T = \frac{1}{2}m(R-r)^2\dot\theta^2 + \frac{mr^2}{4}\frac{(R-r)^2\dot\theta^2}{r^2}$

$\therefore T = \frac{1}{2}m(R-r)^2\dot\theta^2 + \frac{m(R-r)^2\dot\theta^2}{4}$ —— (2)

$$U = mgy = -mg(R-r)\cos\theta \quad — (3)$$

$$\therefore L = T - U = \frac{1}{2}m(R-r)^2\dot{\theta}^2 + \frac{m(R-r)^2\dot{\theta}^2}{4} + mg(R-r)\cos\theta \quad — (4)$$

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{3}{2}m(R-r)^2\dot{\theta} \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{3}{2}m(R-r)^2\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -mg(R-r)\sin\theta$$

$\therefore$ We get

$$\frac{3}{2}m(R-r)^2\ddot{\theta} + mg(R-r)\sin\theta = 0 \quad — (5)$$

$$\Rightarrow \ddot{\theta}\left[\frac{3(R-r)}{2}\right] + g\sin\theta = 0$$

↑ eq^n of motion

Now, for oscillation,

$\sin\theta \approx \theta$

$$\therefore \ddot{\theta}\left[\frac{3(R-r)}{2}\right] = -g\theta$$

$$\therefore \ddot{\theta} = \frac{-2g\theta}{3(R-r)}$$

$$\therefore \omega \text{ (Angular frequency)} = \sqrt{\frac{2g}{3(R-r)}}$$

$$\therefore 2\pi f = \sqrt{\frac{2g}{3(R-r)}}$$

$$\therefore \boxed{f = \frac{1}{2\pi}\sqrt{\frac{2g}{3(R-r)}} \text{ Hz}}$$