PH 102, Electromagnetism,

Post Mid Semester

Lecture 11

Electromagnetic Waves

in vacuum & matter

D. J. Griffiths: 9.2, 9.3

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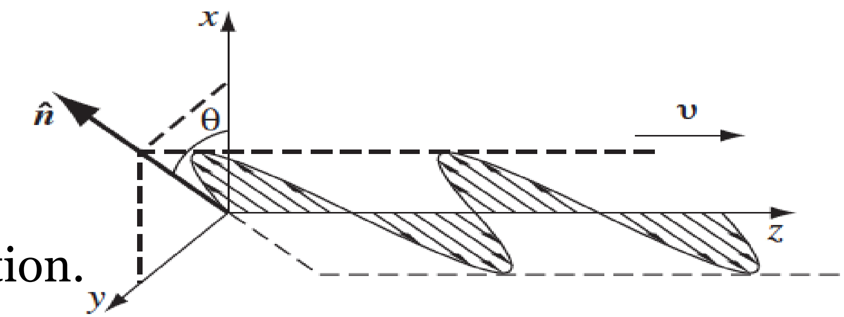
Electromagnetic Waves

Polarization :

Transverse wave: two independent states of polarization (plane of vibration)

along any other direction in the xy plane

$$\tilde{\mathbf{f}}(z,t) = \tilde{A} e^{i(kz-\omega t)}\,\hat{\mathbf{n}}.$$



Polarization vector $\hat{\mathbf{n}}$ defines the plane of vibration.

Transverse waves : $\hat{\mathbf{n}}$ is perpendicular to the direction of propagation

$$\boxed{\hat{\mathbf{n}} \cdot \hat{\mathbf{z}} = 0.}$$

In terms of the polarization angle θ, $\quad \hat{\mathbf{n}} = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}.$

Superposition of two waves—one horizontally polarized and the other one vertically:

$$\tilde{\mathbf{f}}(z,t) = (\tilde{A}\cos\theta)e^{i(kz-\omega t)}\,\hat{\mathbf{x}} + (\tilde{A}\sin\theta)e^{i(kz-\omega t)}\,\hat{\mathbf{y}}.$$

Electromagnetic Waves

The Wave Equation for E and B in vacuum:

For no charge or current, Maxwell's equations: coupled, first-order, pde for ***E*** & ***B*** .

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = 0, \quad \text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0, \quad \text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

and

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \quad \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}.$$

Interdependent,
one is the other!

Price for decoupling: 2nd order equations.

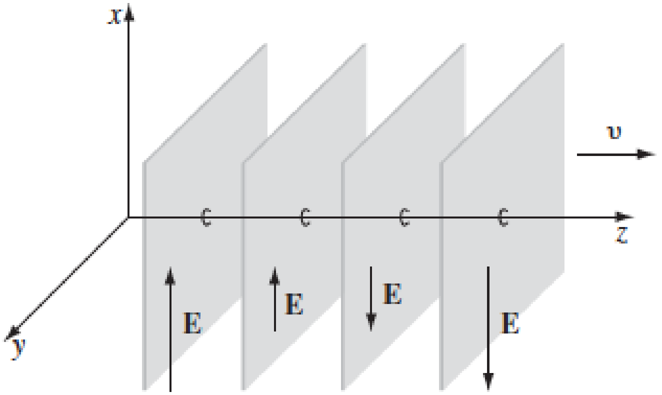
Electromagnetic Waves

Monochromatic Plane Waves :

- Sinusoidal waves of frequency ω: different frequencies in the visible range correspond to different colors, such waves are called ***monochromatic***.

- Waves traveling in the *z* direction, no *x* or *y* dependence: Plane waves

- The fields are ***uniform*** over ***every*** plane perpendicular to the propagation direction.



$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)}, \quad \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(kz-\omega t)},$$

where $\tilde{\mathbf{E}}_0$ and $\tilde{\mathbf{B}}_0$ are the (complex) amplitudes

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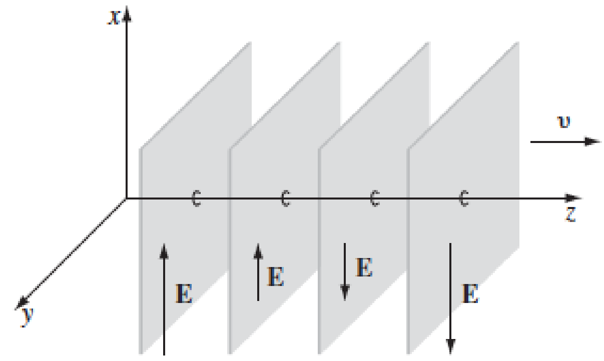

Electromagnetic Waves

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$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)}, \quad \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(kz-\omega t)},$$

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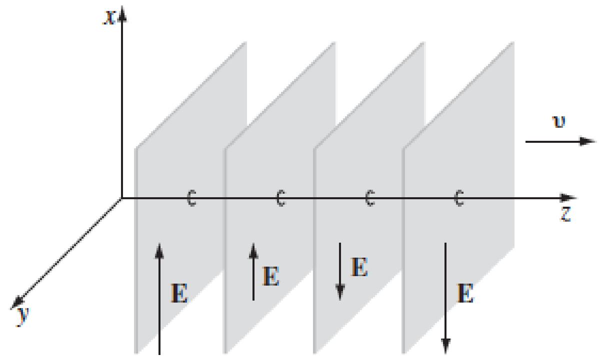

Extra constraints on $\tilde{\mathbf{E}}_0$ and $\tilde{\mathbf{B}}_0$ from Maxwell's equations

$$\nabla \cdot \mathbf{E} = 0 \text{ and } \nabla \cdot \mathbf{B} = 0, \longrightarrow (\tilde{E}_0)_z = (\tilde{B}_0)_z = 0.$$

$\boxed{\boldsymbol{E}, \boldsymbol{B} \perp \mathbf{z}}$

EM plane waves: No longitudinal component.

EM waves are purely transverse!

Electromagnetic Waves

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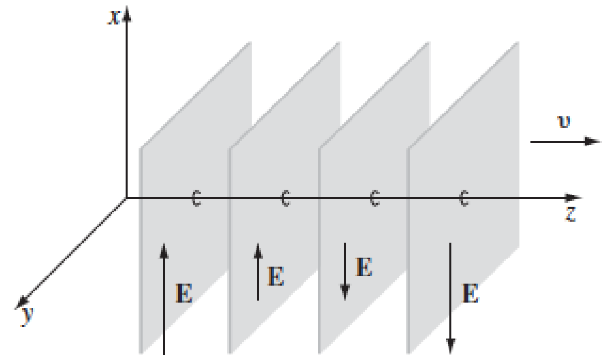
$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)}, \quad \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(kz-\omega t)},$$



where $\tilde{\mathbf{E}}_0$ and $\tilde{\mathbf{B}}_0$ are the (complex) amplitudes

Extra constraints on $\tilde{\mathbf{E}}_0$ and $\tilde{\mathbf{B}}_0$ from Maxwell's equations

Faraday's law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \Rightarrow \quad -k(\tilde{E}_0)_y = \omega(\tilde{B}_0)_x, \quad k(\tilde{E}_0)_x = \omega(\tilde{B}_0)_y,$$

$$\tilde{\mathbf{B}}_0 = \frac{k}{\omega}(\hat{\mathbf{z}} \times \tilde{\mathbf{E}}_0).$$

E & ***B*** in phase & mutually perpendicular.

The real amplitudes, $\boxed{B_0 = \frac{k}{\omega} E_0 = \frac{1}{c} E_0.}$

Electromagnetic Waves

Monochromatic Plane Waves :

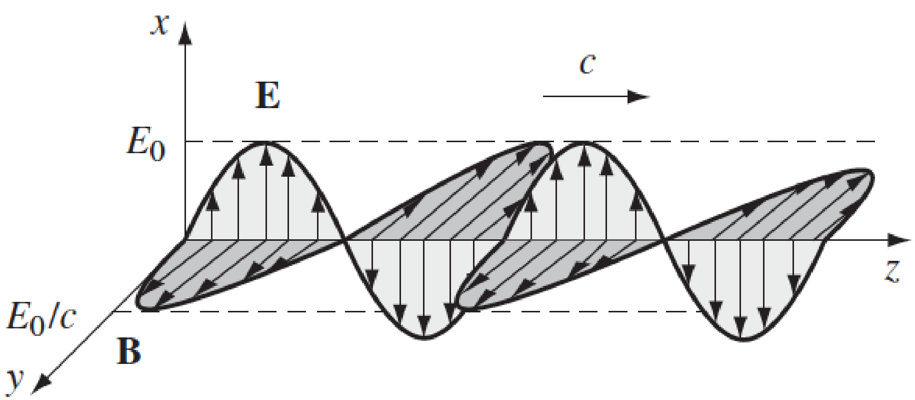
$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)}, \quad \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(kz-\omega t)},$$

Now, if ***E*** points in the *x* direction then ***B*** points in the *y* direction $[\tilde{\mathbf{B}}_0 = \frac{k}{\omega}(\hat{\mathbf{z}} \times \tilde{\mathbf{E}}_0).]$

$$\tilde{\mathbf{E}}(z,t) = \tilde{E}_0 e^{i(kz-\omega t)}\,\hat{\mathbf{x}}, \quad \tilde{\mathbf{B}}(z,t) = \frac{1}{c}\tilde{E}_0 e^{i(kz-\omega t)}\,\hat{\mathbf{y}},$$

The real part of the waves functions

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t + \delta)\,\hat{\mathbf{x}}, \qquad \mathbf{B}(z,t) = \frac{1}{c}E_0 \cos(kz - \omega t + \delta)\,\hat{\mathbf{y}}.$$



Monochromatic plane wave!

Note: Direction of *E* specify polarization direction

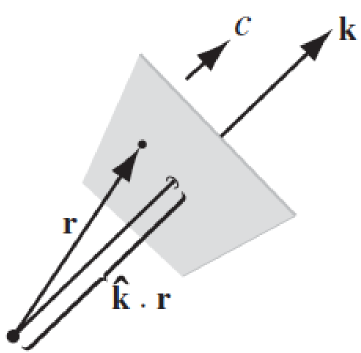
Electromagnetic Waves

Monochromatic Plane Waves :

Generalize to monochromatic plane waves traveling in an arbitrary direction.

(nothing special about the z direction)

Introducing, the wave vector ***k*** (pointing in the direction of propagation)



The scalar product ***k.r*** is the appropriate generalization of *kz* so,

$$\tilde{\mathbf{E}}(\mathbf{r}, t) = \tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\,\hat{\mathbf{n}},$$

$$\tilde{\mathbf{B}}(\mathbf{r}, t) = \frac{1}{c}\tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}(\hat{\mathbf{k}}\times\hat{\mathbf{n}}) = \frac{1}{c}\hat{\mathbf{k}}\times\tilde{\mathbf{E}},$$

$\hat{\mathbf{n}}$ polarization vector

E being transverse, $\hat{\mathbf{n}}\cdot\hat{\mathbf{k}} = 0.$

The real ***E*** & ***B*** fields in a monochromatic plane wave with $(\mathbf{k}, \hat{\mathbf{n}})$

$$\mathbf{E}(\mathbf{r}, t) = E_0\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)\,\hat{\mathbf{n}},$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{c}E_0\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)(\hat{\mathbf{k}}\times\hat{\mathbf{n}}).$$

Electromagnetic Waves

Monochromatic Plane Waves :

Problem 9.9 Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is (a) traveling in the negative x direction and polarized in the z direction; (b) traveling in the direction from the origin to the point (1, 1, 1), with polarization parallel to the xz plane. In each case, sketch the wave, and give the explicit Cartesian components of $\mathbf{k}$ and $\hat{\mathbf{n}}$.

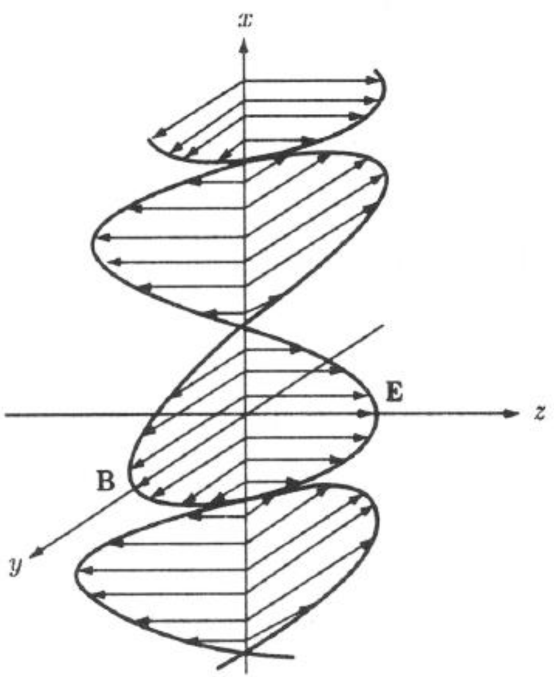
$$\mathbf{E}(\mathbf{r}, t) = E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)\,\hat{\mathbf{n}},$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{c} E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)(\hat{\mathbf{k}}\times\hat{\mathbf{n}}).$$

Answer (a) $\quad \mathbf{k} = -\frac{\omega}{c}\,\hat{\mathbf{x}};\ \ \hat{\mathbf{n}} = \hat{\mathbf{z}}.$

$$\mathbf{k}\cdot\mathbf{r} = \left(-\frac{\omega}{c}\,\hat{\mathbf{x}}\right)\cdot(x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}) = -\frac{\omega}{c}x;$$

$$\mathbf{k}\times\hat{\mathbf{n}} = -\hat{\mathbf{x}}\times\hat{\mathbf{z}} = \hat{\mathbf{y}}.$$



Therefore,

$$\mathbf{E}(x, t) = E_0 \cos\left(\frac{\omega}{c}x + \omega t\right)\hat{\mathbf{z}};$$

$$\mathbf{B}(x, t) = \frac{E_0}{c}\cos\left(\frac{\omega}{c}x + \omega t\right)\hat{\mathbf{y}}.$$

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$$\mathbf{E}(\mathbf{r}, t) = E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)\,\hat{\mathbf{n}},$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{c} E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)(\hat{\mathbf{k}} \times \hat{\mathbf{n}}).$$

Answer (a) $$\mathbf{k} = \frac{\omega}{c}\left(\frac{\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}}{\sqrt{3}}\right);\ \hat{\mathbf{n}} = \frac{\hat{\mathbf{x}} - \hat{\mathbf{z}}}{\sqrt{2}}.$$

Since $\hat{\mathbf{n}}$ is parallel to the $x\,z$ plane, it must have the form $\alpha\,\hat{\mathbf{x}} + \beta\,\hat{\mathbf{z}}$;

$\hat{\mathbf{n}} \cdot \mathbf{k} = 0, \beta = -\alpha$ & since it is a unit vector, $\alpha = 1/\sqrt{2}$.

$$\mathbf{k}\cdot\mathbf{r} = \frac{\omega}{\sqrt{3}c}(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}) \cdot (x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}) = \frac{\omega}{\sqrt{3}c}(x + y + z);$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{n}} = \frac{1}{\sqrt{6}}(-\hat{\mathbf{x}} + 2\,\hat{\mathbf{y}} - \hat{\mathbf{z}}).$$

$$\mathbf{E}(x, y, z, t) = E_0 \cos\left[\frac{\omega}{\sqrt{3}c}(x + y + z) - \omega t\right]\left(\frac{\hat{\mathbf{x}} - \hat{\mathbf{z}}}{\sqrt{2}}\right);$$

Sketch the wave!

$$\mathbf{B}(x, y, z, t) = \frac{E_0}{c} \cos\left[\frac{\omega}{\sqrt{3}c}(x + y + z) - \omega t\right]\left(\frac{-\hat{\mathbf{x}} + 2\hat{\mathbf{y}} - \hat{\mathbf{z}}}{\sqrt{6}}\right).$$

Electromagnetic Waves

Energy and Momentum in Electromagnetic Waves

Energy per unit volume in electromagnetic fields is, $u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$

For monochromatic plane wave, $B_0 = \frac{k}{\omega} E_0 = \frac{1}{c} E_0.$ thus, $B^2 = \frac{1}{c^2} E^2 = \mu_0 \epsilon_0 E^2,$

i.e., the electric and magnetic contributions are equal!!

$$\mathbf{E}(z, t) = E_0 \cos(kz - \omega t + \delta)\,\hat{\mathbf{x}},$$
$$\mathbf{B}(z, t) = \frac{1}{c} E_0 \cos(kz - \omega t + \delta)\,\hat{\mathbf{y}}.$$

The wave carries the energy along with it.

$$u = \epsilon_0 E^2 = \epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)$$

Electromagnetic Waves

Energy flux density (energy per unit area, per unit time) transported by the fields is given by the Poynting vector,

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$$

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t + \delta)\,\hat{\mathbf{x}},$$
$$\mathbf{B}(z,t) = \frac{1}{c} E_0 \cos(kz - \omega t + \delta)\,\hat{\mathbf{y}}.$$

For monochromatic plane waves propagating in the z direction,

$$\mathbf{S} = c\epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)\,\hat{\mathbf{z}} = cu\,\hat{\mathbf{z}}.$$

energy density (u)
times
wave velocity ($c\hat{\mathbf{z}}$)

EM fields also carry momentum,
the momentum density stored in the fields is, $\boxed{\mathbf{g} = \frac{1}{c^2}\mathbf{S}}$

For monochromatic plane waves, $$\mathbf{g} = \frac{1}{c}\epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)\,\hat{\mathbf{z}} = \frac{1}{c} u\,\hat{\mathbf{z}}.$$

Electromagnetic Waves

- For light, the wavelength is very short ($\sim 10^{-7}$ m) and so is the period ($\sim 10^{-15}$ s).
- Any macroscopic measurement will encompass many cycles.
- Time average: more interesting compare to fluctuating cosine-squared:

$$\mathbf{S} = c\epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)\,\hat{\mathbf{z}} = cu\,\hat{\mathbf{z}}. \quad \& \quad \mathbf{g} = \frac{1}{c^2}\mathbf{S}$$

$$\boxed{\frac{1}{T}\int_0^T \cos^2(kz - 2\pi t/T + \delta)\,dt = 1/2.}$$

Time average over a complete cycle

$$\boxed{\langle u \rangle = \frac{1}{2}\epsilon_0 E_0^2,} \quad \boxed{\langle \mathbf{S} \rangle = \frac{1}{2}c\epsilon_0 E_0^2\,\hat{\mathbf{z}},} \quad \boxed{\langle \mathbf{g} \rangle = \frac{1}{2c}\epsilon_0 E_0^2\,\hat{\mathbf{z}}.}$$

Average power per unit area transported by an EM wave: Intensity

$$\boxed{I \equiv \langle S \rangle = \frac{1}{2}c\epsilon_0 E_0^2.}$$

Electromagnetic Waves

Light (normal incidence) on perfect absorber: Delivers momentum to the surface.

In a time Δt, the momentum transfer is $\Delta p = \langle g \rangle A c \Delta t$,

Radiation pressure (average force per unit area)

$$P = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{1}{2}\epsilon_0 E_0^2 = \frac{I}{c}$$

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t + \delta)\,\hat{\mathbf{x}},$$

$$\mathbf{B}(z,t) = \frac{1}{c} E_0 \cos(kz - \omega t + \delta)\,\hat{\mathbf{y}}.$$

Qualitatively :

- ***E*** drives charges in the ***x*** direction
- ***B*** exerts on them a force $q(v \times B)$ in the ***z*** direction.
- The net force on all the charges in the surface produces the pressure!

Electromagnetic Waves

Radiation pressure (average force per unit area)

$$P = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{1}{2}\epsilon_0 E_0^2 = \frac{I}{c}$$

Problem 9.10 The intensity of sunlight hitting the earth is about 1300 $\mathrm{W/m^2}$. If sunlight strikes a perfect absorber, what pressure does it exert? How about a perfect reflector? What fraction of atmospheric pressure does this amount to?

Answer: $$P = \frac{I}{c} = \frac{1.3\times10^3}{3.0\times10^8} = \boxed{4.3\times10^{-6}\,\mathrm{N/m^2}.}$$

For a perfect reflector the pressure is twice as great: $8.6\times10^{-6}\,\mathrm{N/m^2}$.

(momentum switches direction)

Atmospheric pressure is $1.03\times10^5\,\mathrm{N/m^2}$, so the pressure of light on a reflector is

$$(8.6\times10^{-6})/(1.03\times10^5) = 8.3\times10^{-11}\ \text{atmospheres.}$$

Electromagnetic Waves

Electromagnetic Waves in Matter:

Inside matter, no free charge and no free current,

$$\text{(i)} \quad \nabla \cdot \mathbf{D} = 0, \quad \text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0, \quad \text{(iv)} \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}.$$

If the medium is linear and homogeneous,

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B},$$

and

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = 0, \quad \text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0, \quad \text{(iv)} \quad \nabla \times \mathbf{B} = \mu\epsilon \frac{\partial \mathbf{E}}{\partial t},$$

Evidently, EM waves propagate through a linear homogeneous medium at a speed,

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{n},$$

where,
index of refraction $n \equiv \sqrt{\dfrac{\epsilon\mu}{\epsilon_0\mu_0}}$

For most materials, Permeability

$$\mu \equiv \mu_0(1 + \chi_m) \approx \mu_o$$

$$n \cong \sqrt{\epsilon_r} > 1$$

Light travels slowly through matter! Optics!

Electromagnetic Waves

All previous results carry over with
$\boldsymbol{\varepsilon_o} \rightarrow \varepsilon$,
$\mu_o \rightarrow \mu$
&
$c \rightarrow v$

Electromagnetic Waves in Matter:

The energy density, $$u = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right)$$

Poynting vector is $$\mathbf{S} = \frac{1}{\mu}(\mathbf{E} \times \mathbf{B})$$

Monochromatic plane waves: $\omega = kv$, Amplitude of B ~ $1/v$ ×amplitude of E

The intensity is $$I = \frac{1}{2}\epsilon v E_0^2$$

Wave passes from one transparent medium to another,

$$\text{(i)}\ \ \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}, \quad \text{(iii)}\ \ \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel},$$

$$\text{(ii)}\ \ B_1^{\perp} = B_2^{\perp}, \quad \text{(iv)}\ \ \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel}$$

Electromagnetic Waves

Electromagnetic Waves in Matter:

Reflection and Transmission at Normal Incidence:

- Suppose, the xy plane forms the boundary between two linear media.
- A plane wave of frequency $\boldsymbol{\omega}$, traveling in the $\boldsymbol{z}$ direction and polarized in the $\boldsymbol{x}$ direction, approaches the interface from the left

Incident Wave:

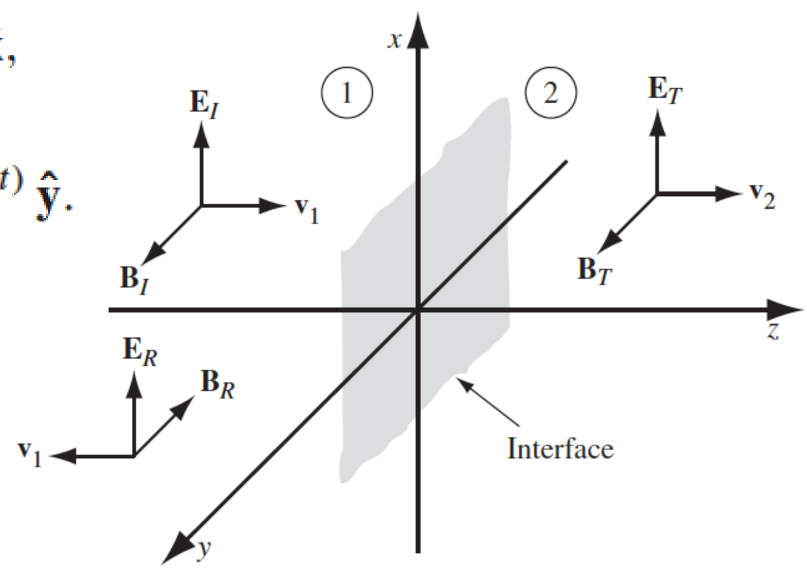
$$\tilde{\mathbf{E}}_I(z,t) = \tilde{E}_{0_I} e^{i(k_1 z - \omega t)}\,\hat{\mathbf{x}},$$

$$\tilde{\mathbf{B}}_I(z,t) = \frac{1}{v_1}\tilde{E}_{0_I} e^{i(k_1 z - \omega t)}\,\hat{\mathbf{y}}.$$

Transmitted Wave:

$$\tilde{\mathbf{E}}_T(z,t) = \tilde{E}_{0_T} e^{i(k_2 z - \omega t)}\,\hat{\mathbf{x}},$$

$$\tilde{\mathbf{B}}_T(z,t) = \frac{1}{v_2}\tilde{E}_{0_T} e^{i(k_2 z - \omega t)}\,\hat{\mathbf{y}},$$



Reflected Wave:

$$\tilde{\mathbf{E}}_R(z,t) = \tilde{E}_{0_R} e^{i(-k_1 z - \omega t)}\,\hat{\mathbf{x}},$$

$$\tilde{\mathbf{B}}_R(z,t) = -\frac{1}{v_1}\tilde{E}_{0_R} e^{i(-k_1 z - \omega t)}\,\mathbf{y},$$

Note the minus sign in $\tilde{\mathbf{B}}_R$

$$\tilde{\mathbf{B}}(\mathbf{r},t) = \frac{1}{c}\tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}(\hat{\mathbf{k}}\times\hat{\mathbf{n}})$$

Electromagnetic Waves

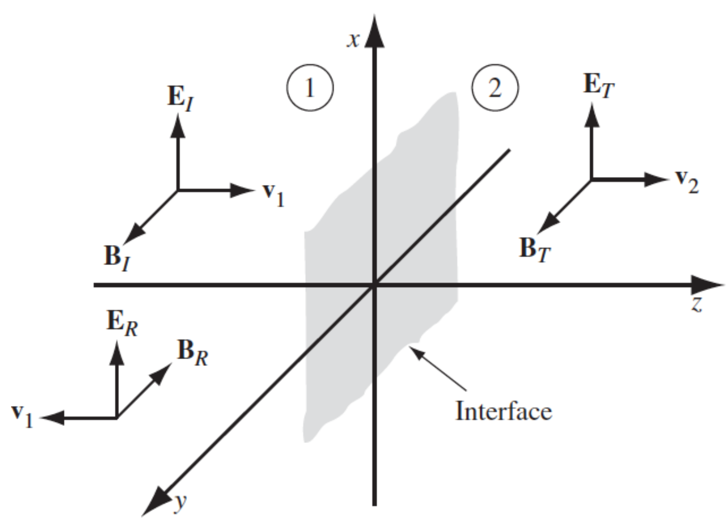
Reflection and Transmission at Normal Incidence:

At z = 0, the E & B fields on left (from both I and R) should join the fields on right (from T) in the boundary conditions.

$$\text{(i)}\ \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}, \qquad \text{(iii)}\ \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel},$$

$$\text{(ii)}\ B_1^{\perp} = B_2^{\perp}, \qquad \text{(iv)}\ \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel}.$$

There are no components perpendicular to the surface, (i) & (ii) trivial.



Electromagnetic Waves

Reflection and Transmission at Normal Incidence:

On the interface,

(i) $\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}$,

(ii) $B_1^{\perp} = B_2^{\perp}$,

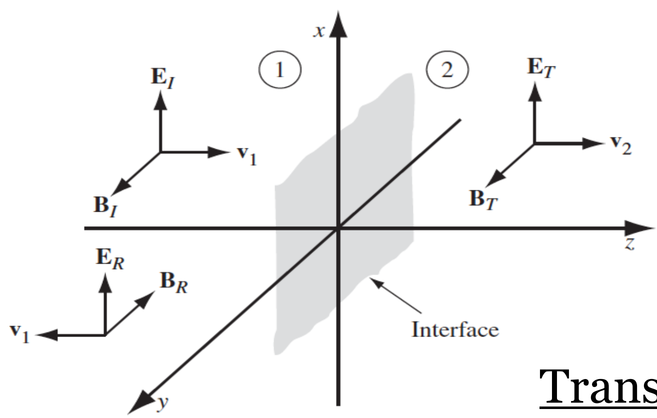
Trivial

(iii) $\mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel}$,

(iv) $\frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel}$.

(iii) $\tilde{E}_{0_I} + \tilde{E}_{0_R} = \tilde{E}_{0_T}$,

(iv) $\frac{1}{\mu_1}\left(\frac{1}{v_1}\tilde{E}_{0_I} - \frac{1}{v_1}\tilde{E}_{0_R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2}\tilde{E}_{0_T}\right)$



<u>Incident Wave:</u>

$$\tilde{\mathbf{E}}_I(z,t) = \tilde{E}_{0_I} e^{i(k_1 z - \omega t)}\,\hat{\mathbf{x}},$$

$$\tilde{\mathbf{B}}_I(z,t) = \frac{1}{v_1}\tilde{E}_{0_I} e^{i(k_1 z - \omega t)}\,\hat{\mathbf{y}}.$$

<u>Reflected Wave:</u>

$$\tilde{\mathbf{E}}_R(z,t) = \tilde{E}_{0_R} e^{i(-k_1 z - \omega t)}\,\hat{\mathbf{x}},$$

$$\tilde{\mathbf{B}}_R(z,t) = -\frac{1}{v_1}\tilde{E}_{0_R} e^{i(-k_1 z - \omega t)}\,\hat{\mathbf{y}},$$

Note the minus sign in $\tilde{\mathbf{B}}_R$

<u>Transmitted Wave:</u>

$$\tilde{\mathbf{E}}_T(z,t) = \tilde{E}_{0_T} e^{i(k_2 z - \omega t)}\,\hat{\mathbf{x}},$$

$$\tilde{\mathbf{B}}_T(z,t) = \frac{1}{v_2}\tilde{E}_{0_T} e^{i(k_2 z - \omega t)}\,\hat{\mathbf{y}},$$

Electromagnetic Waves

Reflection and Transmission at Normal Incidence:

(iv) $$\frac{1}{\mu_1}\left(\frac{1}{v_1}\tilde{E}_{0_I} - \frac{1}{v_1}\tilde{E}_{0_R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2}\tilde{E}_{0_T}\right)$$

(iii) $$\tilde{E}_{0_I} + \tilde{E}_{0_R} = \tilde{E}_{0_T},$$

$$\tilde{E}_{0_I} - \tilde{E}_{0_R} = \beta \tilde{E}_{0_T},$$

where, $$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}.$$

Solving, (iii) & (iv),

$$\boxed{\tilde{E}_{0_R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0_I}, \quad \tilde{E}_{0_T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0_I}.}$$

Permeability, $\mu \equiv \mu_0(1+\chi_m) \approx \mu_o$ $\qquad \beta = v_1/v_2,$

$$\boxed{\tilde{E}_{0_R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)\tilde{E}_{0_I}, \quad \tilde{E}_{0_T} = \left(\frac{2v_2}{v_2 + v_1}\right)\tilde{E}_{0_I},}$$

In phase for $v_2 > v_1$ In phase

Electromagnetic Waves

Reflection and Transmission at Normal Incidence:

(iii) $\tilde{E}_{0_I} + \tilde{E}_{0_R} = \tilde{E}_{0_T}$, (iv) $\tilde{E}_{0_I} - \tilde{E}_{0_R} = \beta \tilde{E}_{0_T}$,

$$\boxed{\tilde{E}_{0_R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) \tilde{E}_{0_I}, \quad \tilde{E}_{0_T} = \left(\frac{2v_2}{v_2 + v_1}\right) \tilde{E}_{0_I},}$$

In terms of the real amplitudes,

$$\begin{aligned} E_{0_R} &= \left|\frac{v_2 - v_1}{v_2 + v_1}\right| E_{0_I}, \quad & E_{0_T} &= \left(\frac{2v_2}{v_2 + v_1}\right) E_{0_I}, \\ &= \left|\frac{n_1 - n_2}{n_1 + n_2}\right| E_{0_I}, & &= \left(\frac{2n_1}{n_1 + n_2}\right) E_{0_I}. \end{aligned}$$

(in terms of refraction indices)

Let us see, what fraction of the incident energy is reflected / transmitted?

Electromagnetic Waves

Reflection and Transmission at Normal Incidence:

$$E_{0_R} = \left|\frac{v_2 - v_1}{v_2 + v_1}\right| E_{0_I}, \quad E_{0_T} = \left(\frac{2v_2}{v_2 + v_1}\right) E_{0_I},$$

$$= \left|\frac{n_1 - n_2}{n_1 + n_2}\right| E_{0_I}, \quad = \left(\frac{2n_1}{n_1 + n_2}\right) E_{0_I}.$$

(in terms of refraction indices)

Let us see, what fraction of the incident energy is reflected / transmitted?

Intensity (average power per unit area), $I = \frac{1}{2}\epsilon v E_0^2.$

Electromagnetic Waves

Reflection and Transmission at Normal Incidence:

$$E_{0_R} = \left|\frac{v_2 - v_1}{v_2 + v_1}\right| E_{0_I}, \quad E_{0_T} = \left(\frac{2v_2}{v_2 + v_1}\right) E_{0_I},$$

$$= \left|\frac{n_1 - n_2}{n_1 + n_2}\right| E_{0_I}, \qquad = \left(\frac{2n_1}{n_1 + n_2}\right) E_{0_I}.$$

(in terms of refraction indices)

Let us see, what fraction of the incident energy is reflected / transmitted?

Intensity (average power per unit area), $I = \frac{1}{2}\epsilon v E_0^2.$

Reflection coefficient (R)

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0_R}}{E_{0_I}}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2,$$

Transmission coefficient (T)

$$T \equiv \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{E_{0_T}}{E_{0_I}}\right)^2 = \frac{4n_1 n_2}{(n_1 + n_2)^2}.$$

Example. light passes from air (n_1= 1) into glass (n_2= 1. 5),
R = 0. 04 and T = 0. 96.

No Surprise! R + T = **1**