Special Theory of Relativity (PH101)

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Tutorial-6

due on Wednesday, 16th of October, 2019 (8:00Hrs IST)

1. A helicopter is flying with a constant speed of 180 km/hr along the horizontal x-direction with respect to an observer on the ground at a height of 500 m. Consider the y-direction vertically upwards. At certain time, the pilot releases a packet which drops down with zero initial vertical speed. Set the coordinates of the observer and the pilot so that at the time of release, the packet is at the origin in both the reference frames. Set this time to be t=0. Consider acceleration due to gravity $g = 9.8m/sec^2$. Find the position and velocity of the packet in the two frames, 5 seconds after it is dropped, according to Galilean Relativity.

2. Compare the speeds of the following with that of light (express as fraction of c). In each of the cases find the prefactor γ that appear in the Lorentz transformation.
(i) Hima Das running at a speed of 100m in 10 sec.
(ii) Maglev train running at a speed of 500 km/hr.
(iii)Concord aircraft flying with ground speed 1800 km/hr.
(iv) Space-shuttle moving with a speed of 27000km/hr.
(v)Earth orbiting around Sun with a speed of 30km per second.
(vi)Proton making one round of 27km circumference of the Large Hadron Collider (LHC) tunnel in 100 micro second.

3. Two events occur at $\left(t = \frac{X}{2c}, X, 0, 0\right)$ and $\left(t = \frac{X}{c}, 3X, 0, 0\right)$ in a frame S. What is the speed of frame S' (moving along x-axis with constant speed), so that the above two events occur at the same time in this frame? What is the value of this time, and what are the values of x-coordinate in S'?

4. The earth and Sun are 8.3 light-minutes (the distance traveled by light in one minute) apart. Ignore their relative motion for this problem and assume they live in a single inertial frame, the Earth-Sun frame. Events A and B occur at t = 0 on the earth and at t=2 minutes on the Sun respectively. Find the time difference between the events according to an observer moving at $u = 0.8c$ from Earth to Sun. Repeat if observer is moving in the opposite direction at $u = 0.8c$.

5. An observer in frame S who lives on the x-axis sees a flash of red light at $x = 1210m$. After $4.96\mu s$, he sees flash of blue light at $x = 480m$. Use subscripts R and B to label the coordinates of the events related to the red and blue light respectively.
(i) Now suppose there is an observer in S' which is moving with a velocity 'v' with respect to the S frame watches these events. Compute the velocity v for the situation when the observer in S' records both the events occurring at the same place?
(ii) Which event occurs first according to S' and what is the measured time interval between these flashes?

6. The average lifetime of a π meson in its own frame of reference is 26.0 ns. (This is its proper lifetime.) What will be the lifetime of π meson as measured by an observer at rest on earth if it moves with respect to the Earth with a speed $0.95c$. Also compute the average distance it will travel before decaying as measured by an observer at rest on Earth.

7. The change in frequency of wave that happens due to relative motion between the source and observer is known as the "Doppler effect." In Galilean relativity the modified frequency is given by $\nu = \nu_0 \left(\frac{v \pm v_o}{v \mp v_s} \right)$, where, ν_0 is the original emitted frequency, ν is the observed(detected) frequency, v, v_s, and v_o is the velocity of wave (e.g., sound wave),observer, and source relative to the medium respectively. Here $+(-)$ stands for the situation when observer is approaching (receding) towards (from) source. This relation gets modified in STR as $\nu = \nu_0 \sqrt{\frac{c \pm v}{c \mp v}}$ where, c is the velocity of light and v is the velocity of the light source. Note that here implicitly it is assumed that the observer is in the frame S and light source is in the moving frame S'. Using this information solve the following problem.

   A driver is caught violating the traffic rule by going through a red light signal. The driver claims to the judge that the color she actually saw was green ($\nu = 5.60 \times 10^{14} Hz$) and not red ($\nu = 4.80 \times 10^{14} Hz$) because of the Doppler effect. The judge accepts this explanation and instead fines her for speeding at the rate of 100 (INR) for each km/h that she exceeded the speed limit of 80 km/h. Compute the total fine amount?

8. The light wave equation is given by $\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$, where, E is the electric field and c is the velocity of light. Show that under Galilean transformation the above equation will have the form as $\frac{\partial^2 E'}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 E'}{\partial t'^2} - \frac{2v_x}{c^2}\frac{\partial^2 E'}{\partial x' \partial t'} - \frac{v_x}{c^2}\frac{\partial}{\partial x'}\left[v_x \frac{\partial E'}{\partial x'} \right] = 0$, where v_x is the speed of S' frame w.r.t. the S frame. That shows that light wave equation is not compatible with the Galilean relativity. Hint: Use the transformation formula discussed during the first lecture of STR.