PH 101: Physics I

Module 2: Special Theory of Relativity-Basics and Quantum Mechanics

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End Sem exam: 40%

Quiz: 30th October, 2019 (Wednesday)

PH101: Physics I

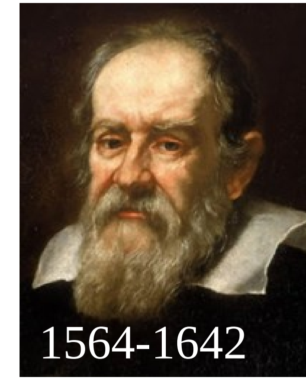
Syllabus:

- Introduction (Galilean Relativity/Necessity of STR)
- Lorentz Transformation, Length contraction, Time Dilation
- Relativistic addition of velocities
- Energy momentum relation and Kinematics

Text Books:

Introduction to Special Relativity by R. Resnick (John Wiley, Singapore, 2000).

Galilean Relativity



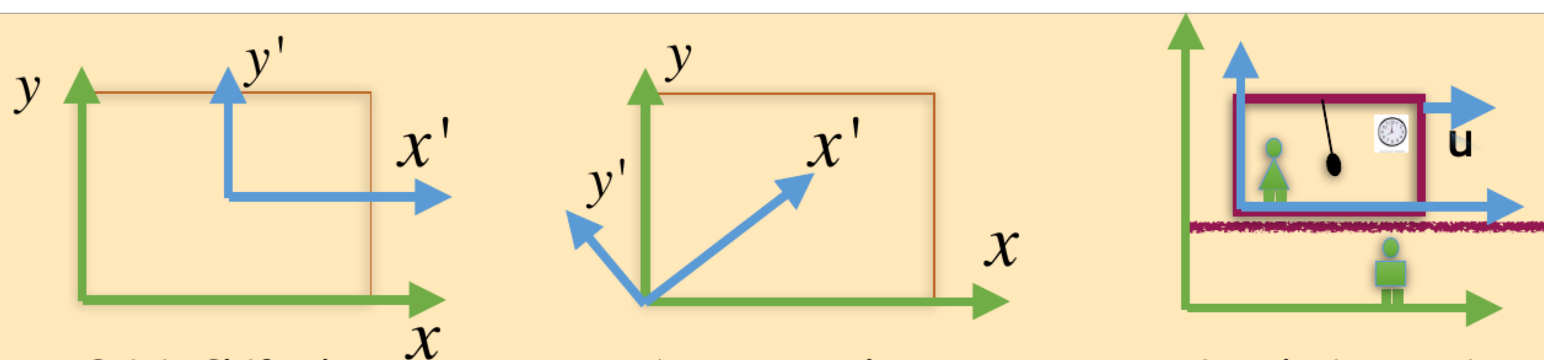
Realtivity broadly refers to the idea that the values of the Physical quantities such as positions, time, velocity, accelerations depend on the reference frame in which it is measured.

Newton's laws of motion are measured with respect to (relative to) inertial frame of reference.

Inertial Frame: A frame in which a particle not acted upon by a force remains at rest or uniform motion.

Event: A happening at a given point in space at a certain instant.
e.g., lighting of a birthday candle, flashing a search light, etc.

Choice of Frames

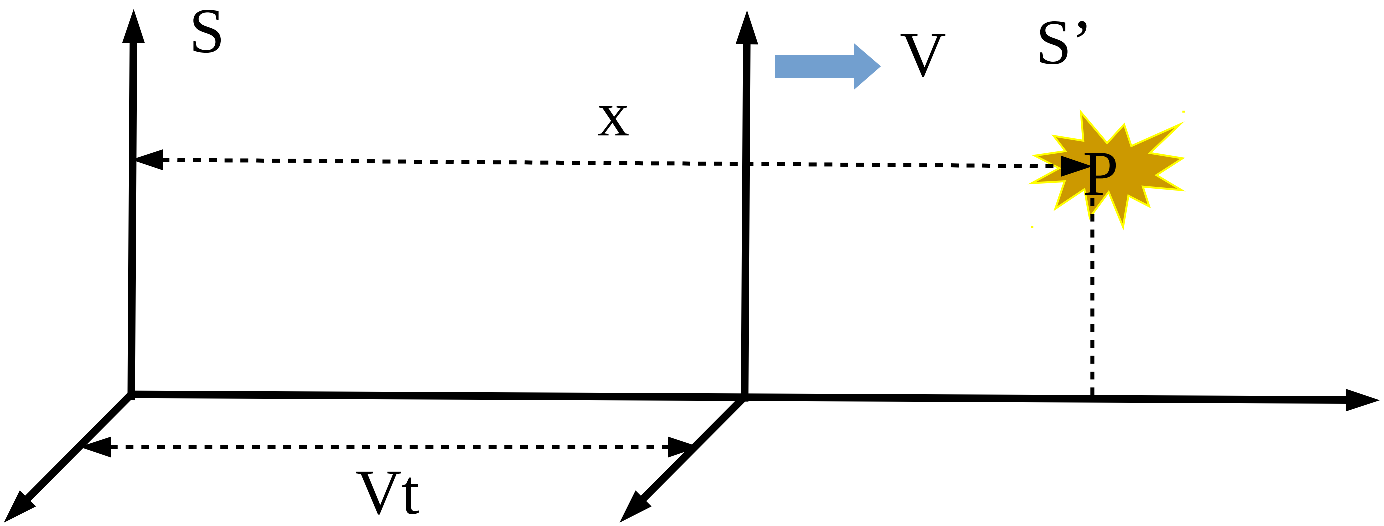


Two frames: Origin Shifted | Axes rotated | In relative motion

Galilean Relativity

In frame S: P=(x, y, z, t)

S': P = (x', y', z', t')



Galilean Transformation

x'=x-Vt
y'=y
z'=z
t'=t

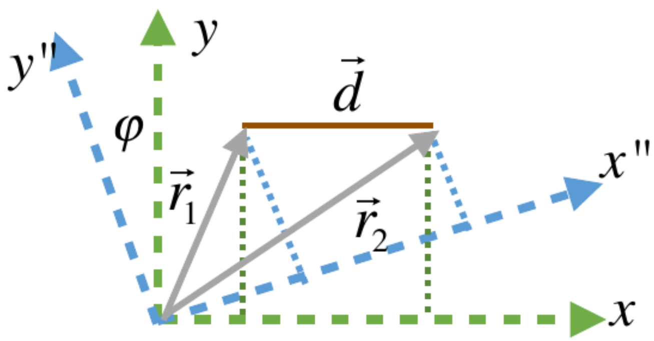
V -> -V

Inverse Galilean Transformation

x=x'+Vt'
y=y'
z=z'
t=t'

Galilean Relativity:
relating the coordinates in the two frames

Space coordiantes: Adding a constant vector to the position vector, or making a rotation about any arbitrary axis leaves the distance between two points same.



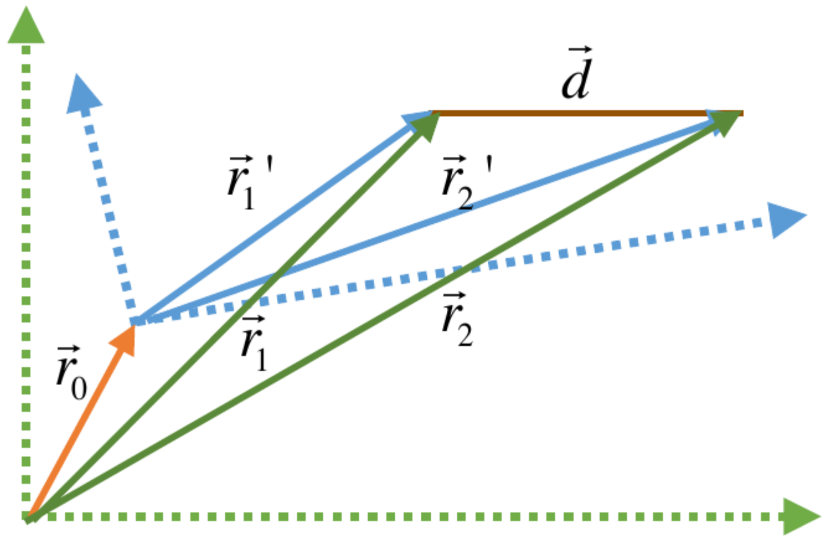
$$\vec{r}\,'' = A\vec{r} \qquad \begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{r}_1\,'' = A\vec{r}_1 \qquad \vec{r}_2\,'' = A\vec{r}_2$$

$$\vec{d} = \vec{r}_2 - \vec{r}_1$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 = (x''_2 - x''_1)^2 + (y''_2 - y''_1)^2$$

Galilean Relativity:
relating the coordinates in the two frames



Adding a constant vector to the position vector.

$$\vec{r}\,' = \vec{r}_0 + \vec{r}\,'' = \vec{r}_0 + A\vec{r}$$

$$\vec{d} = \vec{r}_2\,' - \vec{r}_1\,' = \vec{r}_2\,'' - \vec{r}_2\,''$$

This leads to

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2$$

In addition, one frame could be moving in relative to the other (with constant velocity).

$$\vec{r}\,' = \vec{r}_0 + A(\vec{r} - \vec{v}t)$$

This summarises the most general relation between coordinates in two space time frame in Galilean relativity.

Galilean Relativity:
Relating the coordinates in the two frames

An event seen from two frames, S and S' recorded as (t, x, y, z) and (t', x', y', z')

Consider two such events: (t_1, x_1, y_1, z_1); (t_2, x_2, y_2, z_2) **in S**

and (t'_1, x'_1, y'_1, z'_1); (t'_2, x'_2, y'_2, z'_2) **in S'**

Galilean Relativity: If S and S' are two inertial frames, then distance between two points (where two events take place) do not dependent on who measures them.

Note that, to measure the distance between two points we need to find their positions at the same time.

That means: $t_2 - t_1 = 0, \quad \Rightarrow t'_2 - t'_1 = 0$ Two events simultaneous in S is simultaneous in S' as well.

In general, $t_2 - t_1 = t'_2 - t'_1$ Clocks tick at the same rate.

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2$$

Galilean Relativity:
relating the coordinates in the two frames

Velocity, as seen in S and S'

Consider a particle moving with velocity

$$\vec{u} = \frac{d\vec{r}}{dt} \qquad \text{in frame S.}$$

Its velocity as measured in frame S' is

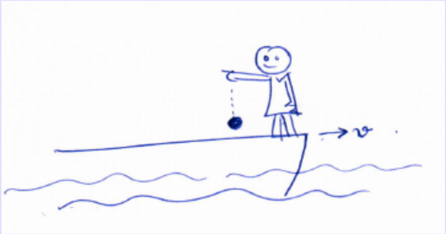
$$\vec{u}' = \frac{d\vec{r}\,'}{dt'} = \frac{d\vec{r}\,'}{dt} = \frac{d\left[\vec{r}_0 + A(\vec{r} - \vec{v}t)\right]}{dt}$$

$$= A\left(\frac{d\vec{r}}{dt} - \vec{v}\right) = A(\vec{u} - \vec{v})$$

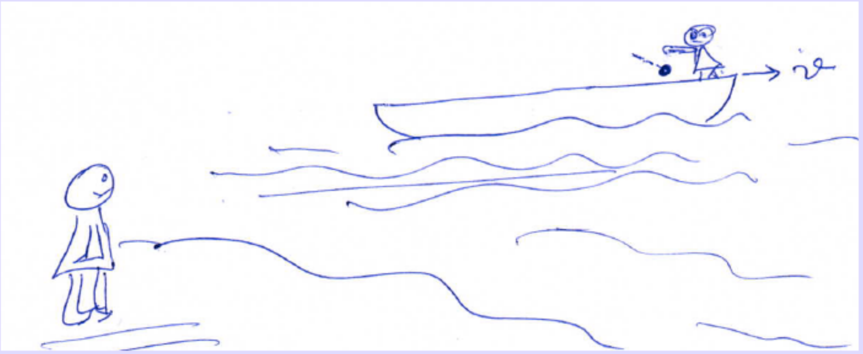
Notice that the matrix **A** is the effect of rotation. In cases where there is no rotation of the axes, but the two frames are in relative motion with respect to each other, A is a unit matrix.

Example

Consider a man in a boat moving with constant speed dropping a ball.



The man will see the ball falling vertically down.



His friend on the shore will see a different trajectory

Galileo's conclusions:

velocities have to be vectorially added

Galilean Relativity: Newtonian mechanics

Newtonian Mechanics is invariant under Galilean Transformation

Consider a particle with mass m.
In frame S, it is seen as moving with velocity $\vec{u}$

Its momentum: $\vec{p} = m\vec{u}$

Force on it is related to change of momentum as $\vec{F} = \frac{d\vec{p}}{dt}$

Seen from frame S' moving with relative velocity $\vec{v}$ $\Rightarrow$ $\vec{p}' = m\left(\vec{u} + \vec{v}\right)$

$$\vec{F}' = \frac{d\vec{p}'}{dt'} = \frac{d\vec{p}}{dt} = \vec{F}$$

The particle experience the same force in all inertial frames.

Galilean Relativity: in case of light

Speed of light and reference frames

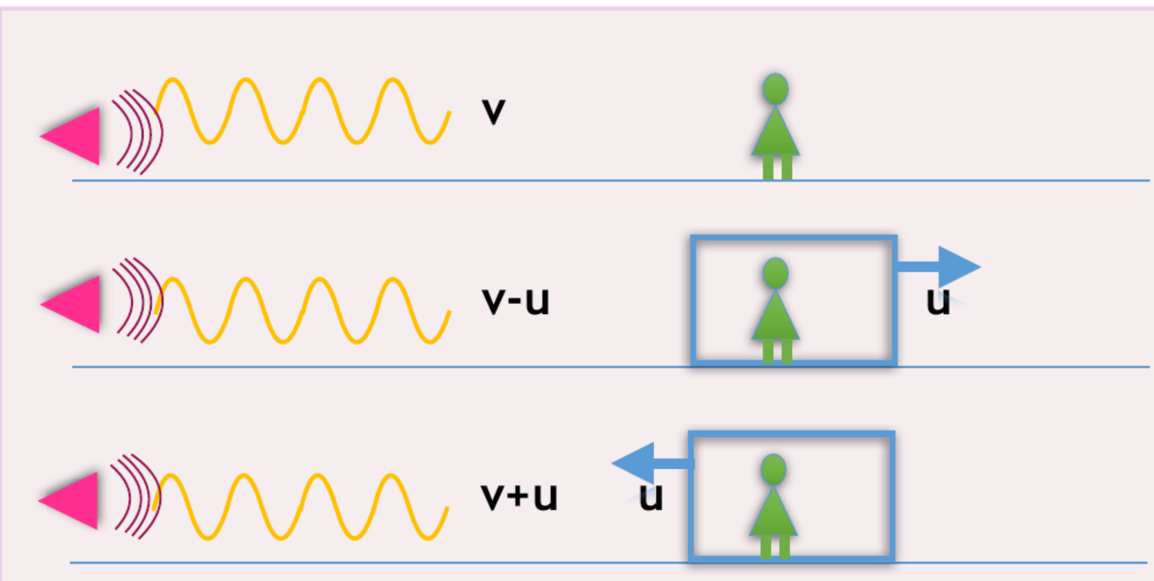
- Speed of sound in air: 340 m/s

  Air is the medium in which it propagates

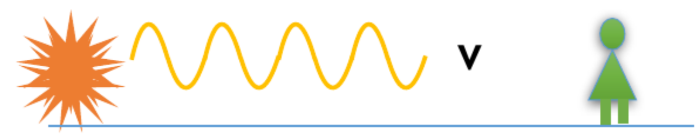
In which reference frame ?

The reference frame in which air is at rest

- For an observer moving w.r.to air



What about light?

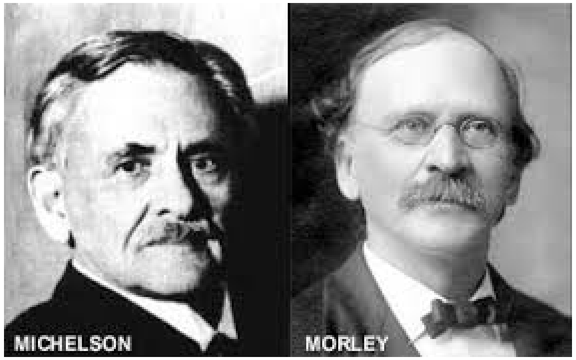


It is thought (Newtonian/Galilean Relativity) that it behaves similar to sound

In such a scenario the light velocity "c" will depend on frame of reference. This is in contradiction with experimental observations!!

Is there a preferable frame of reference for light ? 'Ether' medium !!

Galilean Relativity: in case of light



Does Ether really exist ??

Ether was proposed as an absolute reference system in which the speed of light was this constant and from which other measurements could be made.

The Michelson-Morley experiment was an attempt to show the existence of ether. 1907 Nobel prize was awarded to Michelson.

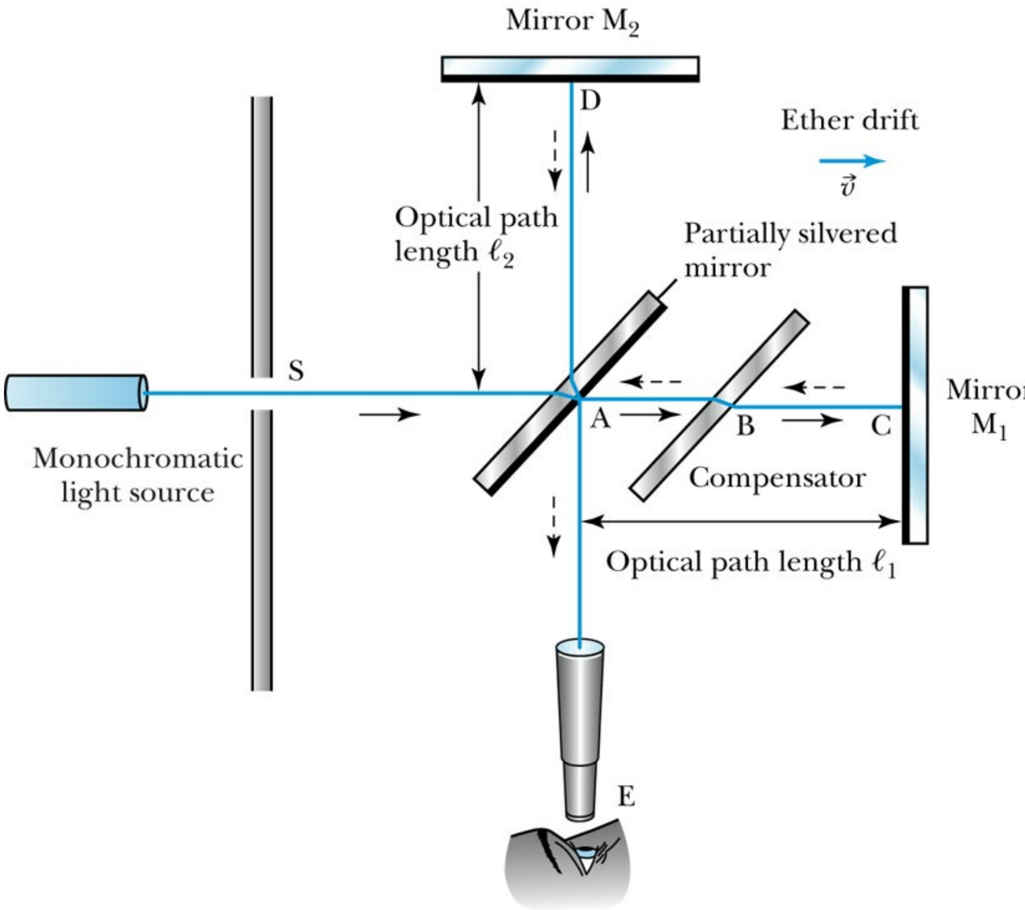
Expected: Shift in the interference pattern as the apparatus is rotated by 90o.

Observation: Null result.

Conclusion: No relative speed for light.

Path breaking finding.

Several other experiments confirmed this in around 50 years.



Galilean Relativity: Electromagnetic Theory

Is Galilean Realtivity compatible with Electromagnetic theory?

Consider two long line charges of linear charge density λ and $-\lambda$ placed parallel to each other separated by distance d.

In frame S, these two line charges are at rest. Force on it $F_C = -\dfrac{\lambda^2}{2\pi\varepsilon_0 d}$

Seen from frame S' moving with relative velocity $\vec{v}$

These two line charges constitute current, say I in addition to having net charge on each other.

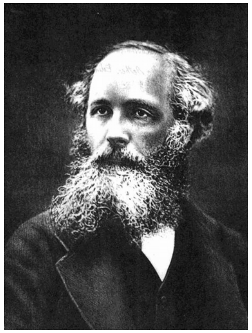
Total force = Coulomb force + Force due to two parallel currents

(one current seeing the magnetic field produced by the other)

$$F' = F_C + F_M$$

Galilean invariance is not applicable to electromagnetic force

Galilean Relativity: Electromagnetic Theory



1831-1879

Maxwell Equations

$$\boxed{\begin{aligned} &\operatorname{div}\mathbf{E} = 0 \\ &\operatorname{curl}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ &\operatorname{div}\mathbf{B} = 0 \\ &\operatorname{curl}\mathbf{B} = \mu_0\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{aligned}}$$

$$\mathbf{E} = E(x,t)\mathbf{j} \quad \text{and} \quad \mathbf{B} = B(x,t)\mathbf{k}$$

From Lorenz Force invariance, we have

$$q\{E - vB\} = q\{E' - v'B'\}$$

$$v' = v - V$$

$$E' - v'B' = (E' + VB') - vB'$$

$$E = E' + VB'$$

$$B = B'.$$

$$x' = x - Vt$$

$$y' = y$$

$$z' = z$$

$$t' = t.$$

$$\boxed{\begin{array}{ll} \dfrac{\partial x'}{\partial x} = 1 & \dfrac{\partial x'}{\partial t} = -V \\ \dfrac{\partial t'}{\partial t} = 1 & \dfrac{\partial t'}{\partial x} = 0. \end{array}}$$

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial B}{\partial t'}\frac{\partial t'}{\partial x}$$

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial t'}\frac{\partial t'}{\partial t} + \frac{\partial E}{\partial x'}\frac{\partial x'}{\partial t}$$

$$\operatorname{curl}\mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \longrightarrow \frac{\partial B}{\partial x} = -\frac{1}{c^2}\frac{\partial E}{\partial t}$$

Following the Galilean transformation we have

$$\frac{\partial B'}{\partial x'} = -\frac{1}{c^2}\frac{\partial E'}{\partial t'} + \frac{1}{c^2}\left\{V\frac{\partial E'}{\partial x'} - V\frac{\partial B'}{\partial t'} + V^2\frac{\partial B'}{\partial x'}\right\}$$

Incompatibility of Maxwell equation with Galilean Relativity !!

Need of Special Theory of Relativity

- 1879: Born in Ulm, Germany.
- 1901: Worked at Swiss patent office.
  - Unable to obtain an academic position.
- 1905: Published 4 famous papers.
  - Paper on photoelectric effect (Nobel prize).
  - Paper on Brownian motion.
  - 2 papers on Special Relativity.
  - Only 26 years old at the time!!
- 1915: General Theory of Relativity published.
- 1933: Einstein left Nazi-occupied Germany.
  - Spent remainder of time at Institute of Advanced Study in Princeton, NJ.
  - Attempted to develop unified theory of gravity and electromagnetism (unsuccessful).

Three possible situations can arise from the discussions so far.

1. If the Galilean transformation is correct then something is wrong with the Maxwell's equations.

2. The Galilean transformation is applicable only for Newtonian mechanics.

3. The Galilean transformation, and the Newtonian principle of relativity based on this transformation were wrong and that there existed a new relativity principle valid for both mechanics and electromagnetism that was not based on the Galilean transformation.

We will continue in the next class.