1. A rod of proper length l_0 sits at rest in S frame, lying in the x-y plane at an angle of $\theta = tan^{-1}(3/4)$ with the x-axis. A frame S' moves with velocity $\vec{v} = v\hat{x}$ with respect to S. In S', the rod is angled at $45°$ with respect to the x' axis.   [3+5]

   (a) What is v?

   (b) What is the length l' of the rod as measured in S' frame?

a) In S frame:

$$\theta = tan^{-1}\left(\frac{3}{4}\right)$$

In s' frame

Velocity of s' frame w.r.t. $S = v\hat{x}$

∴ Only x-component of the length of the rod undergoes length contraction, and the length of y component remains unchanged

∴ $l_0 \sin\theta = l' \sin 45°$ ——— ①

Now, true length of the x-component of rod will be seen in S.

∴ $l' \cos 45° = \dfrac{l_0 \cos\theta}{\gamma}$ where $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

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$$\therefore \frac{l_0\cos\theta}{\gamma} = l'\cos 45° \longrightarrow ②$$

performing ① ÷ ② gives

$$\gamma\tan\theta = \tan 45° = 1$$

$$\therefore \gamma\cdot\frac{3}{4} = 1 \Rightarrow \gamma = \frac{4}{3}$$

$$\therefore \frac{1}{1-\frac{v^2}{c^2}} = \frac{16}{9}$$

$$\therefore 1-\frac{v^2}{c^2} = \frac{9}{16} \Rightarrow \frac{v^2}{c^2} = \frac{7}{16} \Rightarrow v = \frac{\sqrt{7}}{4}c$$

$$\therefore \boxed{v = \frac{\sqrt{7}\,c}{4} = 0.661\,c} \longrightarrow ③$$

b) From Eq ①, we have

$$l_0\sin\theta = l'\sin 45° = \frac{l'}{\sqrt{2}}$$

as $\tan\theta = \frac{3}{4}$, $\sin\theta = \frac{3}{5}$

$$\therefore l_0\cdot\frac{3}{5} = \frac{l'}{\sqrt{2}}$$

$$\therefore \boxed{l' = \frac{3\sqrt{2}}{5}l_0 = 0.848\,l_0}$$

Ans: a) $v = \frac{\sqrt{7}c}{4} = \boxed{0.661c} = 1.983\times 10^8\, m/s$

b) $l' = \frac{3\sqrt{2}}{5}l_0 = \boxed{0.848\,l_0}$

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