Balanced Three-Phase System

A balanced three-phase four-wire system has a balanced three phase source and a balanced three-phase load. The loads in all the three phases are same. The fourth wire is the neutral wire. Fig. 1 shows a balanced three-phase system with a Y-connected load.

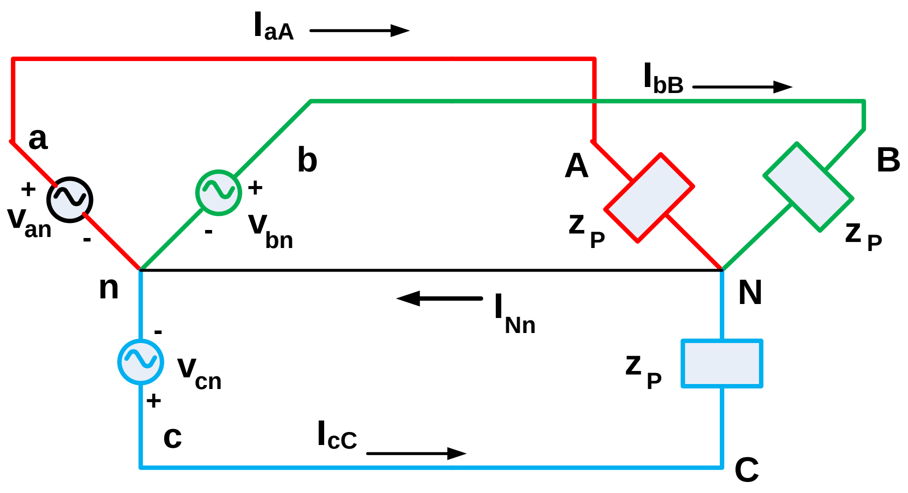

Fig. 1

The three line currents are $I_{aA,}\, I_{bB}$ and I_{cC} . In case of an Y-connected system the line currents are equal / same the phase currents (they are not distinguishable). Using KVL in the three single phases separately

$$I_{aA} = \frac{V_{an}}{z_P}, \quad I_{bB} = \frac{V_{bn}}{z_P} = \frac{V_{an}\angle -120^0}{z_P} = I_{aA}\angle - 120^0,$$

Similarly $I_{cC} = I_{aA}\angle -240^0$

The three line currents have equal magnitude and they differ from each other by a phase angle of 120 degree. The current in the neutral wire is

$$I_{Nn} = I_{aA} + I_{bB} + I_{cC} = 0$$

The zero current in the neutral wire can be visualized as having an infinite impedance or it is an open circuit condition. There will be no effect on the line currents if the neutral wire is taken out from the circuit. This can save the cost of the transmission line.

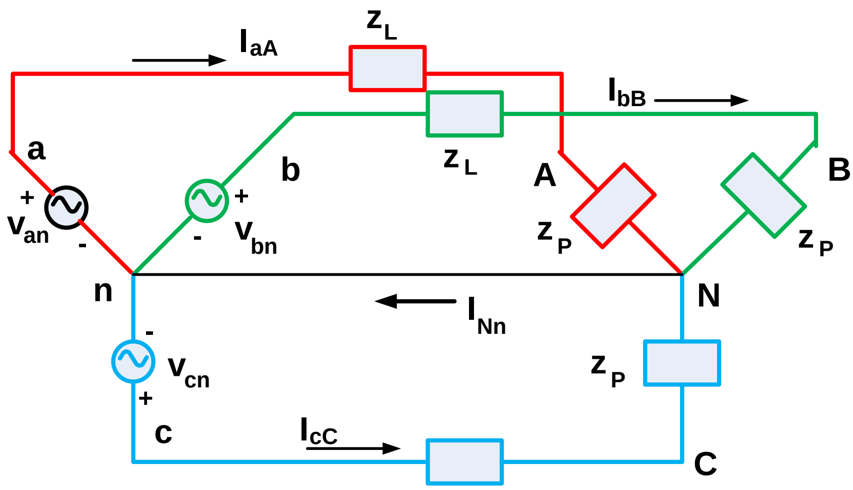

Fig. 2

If we introduce a line impedance (Fig.2), the currents will be

$$I_{aA} = \frac{V_{an}}{Z_L + Z_p}$$

$$I_{bB} = \frac{V_{bn}}{Z_L + z_p} = I_{aA} L - 120^0$$

$$I_{cC} = I_{aA} L - 240^0$$

$$I_{Nn} = I_{aA} + I_{bB} + I_{cC} = 0$$

The three line currents have the same relationship as seen earlier. Now considering an impedance in the neutral wire (Fig. 3), the circuit can be analyzed using Kirchoff's current law at node N. The voltage at node n is zero as this is the common reference point for the three voltage sources.

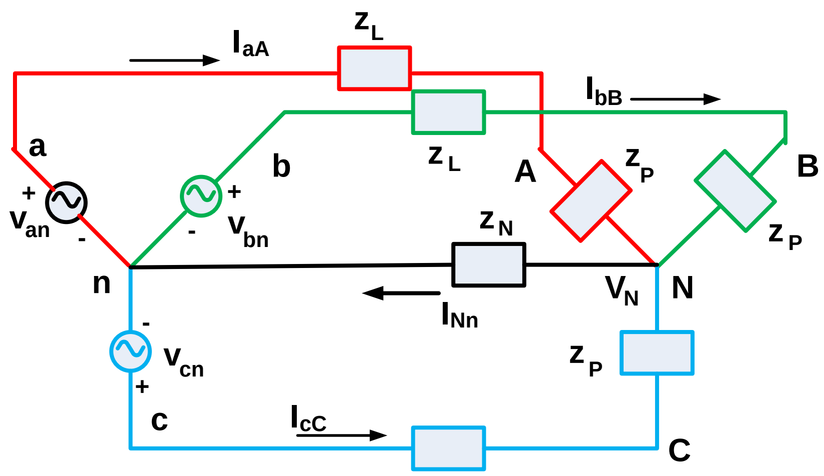

Fig. 3

$$\frac{\boldsymbol{V_N}}{\boldsymbol{Z_N}} = \frac{\boldsymbol{V_{an}} - \boldsymbol{V_N}}{\boldsymbol{Z_L} + \boldsymbol{Z_P}} + \frac{\boldsymbol{V_{bn}} - \boldsymbol{V_N}}{\boldsymbol{Z_L} + \boldsymbol{Z_p}} + \frac{\boldsymbol{V_{cn}} - \boldsymbol{V_N}}{\boldsymbol{Z_L} + \boldsymbol{Z_p}}$$
$$\Rightarrow \boldsymbol{V_{\mathrm{N}}}\left[\frac{\mathbf{1}}{\boldsymbol{Z_N}} + \frac{\mathbf{3}}{\boldsymbol{Z_L} + \boldsymbol{Z_p}}\right] = \frac{\boldsymbol{V_{an}} + \boldsymbol{V_{bn}} + \boldsymbol{V_{cn}}}{\boldsymbol{Z_L} + \boldsymbol{Z_p}}$$
$$\boldsymbol{V_{an}} + \boldsymbol{V_{bn}} + \boldsymbol{V_{cn}} = \mathbf{0} \Rightarrow \boldsymbol{V_{\mathrm{N}}} = \mathbf{0}$$

The voltage of node N is zero. The current in the neutral wire will be zero in this case.

These analysis show that the current in the neutral wire is always zero for a balanced three phase system. This is true for any impedance in the neutral wire even for short circuit (zero impedance) or open circuit (infinite impedance) case. A short circuit or zero impedance neutral wire enables us to analyse the balanced polyphase circuit on a per phase basis. The phase-B (B) equivalent circuit for analysis is shown in Fig. 4.

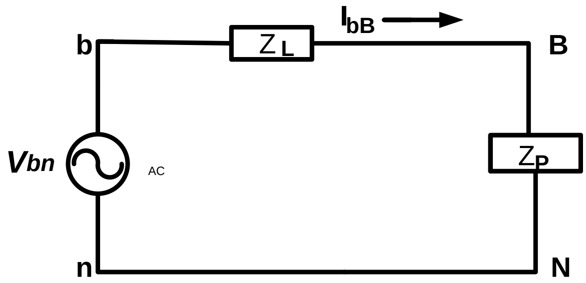

Fig. 4

In a star connected network the line and phase currents are same, $\boldsymbol{I_L} = \boldsymbol{I_P}$. In fact, they are not distinguishable. In a Y-connected system, the line voltage is $\sqrt{3}$ times the phase voltage ($\boldsymbol{V_L} = \sqrt{\mathbf{3}}\,\boldsymbol{V_P}$).

Three-phase System with Delta Load

For a Y-connected load the line voltage is √3 times the phase voltage and line current is same as the phase current. In a delta connected system, the loads are connected back to back as shown in Fig. 5.

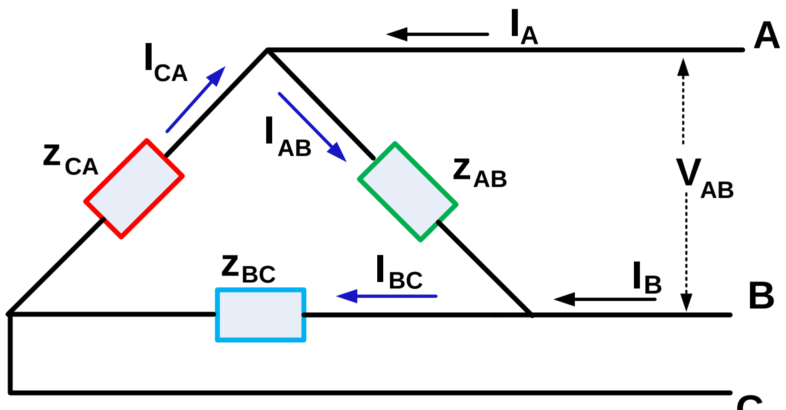

Fig. 5

The line voltages and the phase voltages are same. They are not distinguishable. The relation between the line and the phase voltages is

$$|V_{AB}| = |V_{BC}| = |V_{CA}| = |V_L| = |V_p|$$

The line voltages or the phase voltages in the delta connection are equal in magnitude and they differ from each other by an angle of 120^0. If we consider a balanced three phase delta load, i.e., $Z_{AB} = Z_{BC} = Z_{CA} = Z_P$ and V_{AB} as the reference voltage ($V_{AB} = V_P\angle 0^0$), the phase currents will be

$$I_{AB} = \frac{V_{AB}}{Z_P} = \frac{V_P\angle 0^0}{Z_P} = I_P\angle 0^0,\ I_{BC} = I_P\angle -120^0 \text{ and } I_{CA} = I_P\angle -240^0$$

The line currents can be found as follows:

$$I_A = I_{AB} - I_{CA}$$

$$= I_p\angle 0^o - I_p\angle -240^\circ$$

$$= I_p\sin\omega t - I_p\sin(\omega t - 240^0)$$

$$= I_p[\sin\omega t - (\sin\omega t.cos240^0 - \cos\omega t \cdot sin240^0)$$

$$= I_p\left[\sin\omega t + \frac{1}{2}\sin\omega t - \frac{\sqrt{3}}{2}\cos\omega t\right]$$

$$= \sqrt{3}I_P\left[\frac{\sqrt{3}}{2}\sin\omega t - \frac{1}{2}cos\omega t\right] = \sqrt{3}I_p sin(\omega t - 30^0)$$

Hence, $\boldsymbol{I_A = \sqrt{3}I_P\angle -30^0}$, $\boldsymbol{I_B = \sqrt{3}I_P\angle -150^0}$ and $\boldsymbol{I_C = \sqrt{3}I_P\angle -270^0}$

In delta connection, the phase voltage and the line voltage are same. The magnitude of the line current is √3 times the magnitude of the phase current. One can convert a delta connected load to a star connected load by using star-delta transformation.

Example: A balanced three-phase three-wire system has a line voltage of 500 V. Two balanced Y-connected loads are present. One is a capacitive load with 7-j2 per phase and the other is an inductive load of 4+j2 a per phase. Find (a) the phase voltage, (b) the line current, (c) the total power drawn by the load. (d) the power factor at which the source is operating.

Solution:

(a) As this is a Y-connected system, the phase voltage will be

$$\boldsymbol{v_p = \frac{V_L}{\sqrt{3}} = \frac{500}{\sqrt{3}}V = 288.67V}$$

(b) Two loads are connected in parallel. The per phase load $\mathbf{Z_P}$ can be estimated as

$$\boldsymbol{Z_1 = 7 - j2 \qquad Z_2 = 4 + j2}$$

$$\boldsymbol{Z_P = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{32 + j6}{11}}$$

$$\boldsymbol{I_P = I_L = \frac{V_P}{Z_P} = 97.53L - 10 \cdot 6^0}$$

(c) Total power is

$$\boldsymbol{P = 3V_p I_p \cos\theta = 3 \times 288 \cdot 67 \times 97 \cdot 53 \cos(-10.6^0)}$$
$$\mathbf{= 83\ KW}$$

(d) The source power factor is

$$\boldsymbol{P \cdot F. = cos\theta = cos(-10 \cdot 6^0)}$$
$$\mathbf{= 0 \cdot 983\ lagging}$$

In part (a) and part (b) of the above question, the magnitudes of voltage and current were asked. This question can be modified by asking the phasor voltage, $\boldsymbol{V_{BN}}$ and the phasor current $\boldsymbol{I_B}$. Given line voltage is $\boldsymbol{V_{AB}}$ is equal to 500 v.

Solution: The magnitude of the voltages and currents will be same as found in the previous case. The difference will be the extra information which is the phase angle of the voltage and the current. In this question $\boldsymbol{V_{AB}}$ is the reference phasor as its angle is zero. The phase voltage V_{BN} will lag the line voltage by an angle of 150 degree. Similarly, the line current I_B will lag the line voltage V_{AB} by an angle of 160.6 degree. Hence

$$\boldsymbol{V_{BN} = 288 \cdot 67 L - 150^0 V}$$
$$\boldsymbol{I_B = 97 \cdot 53 L - 160.6^0 A}$$

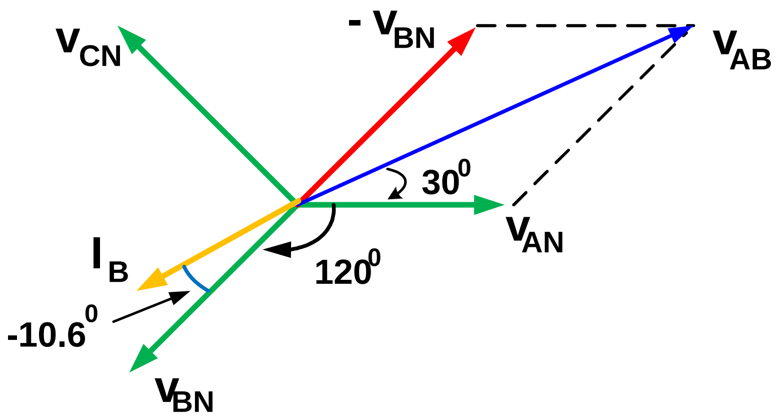

Fig. 6

Example: Each phase of a balanced three-phase Δ-connected load consists of a 0.2 H inductor in series with the parallel combination of a 5 μF capacitor and a 200 Ω resistance. Assume zero line resistance and a phase voltage of 200 V at ω=400 rad/s. Find (a) the phase current; (b) the line current; (c) the total power absorbed by the load.

Solution:

$$\boldsymbol{X_L = j\omega L = j400x0.2 = j80\ \Omega}$$

$$\boldsymbol{X_C = \frac{1}{j\omega C} = -j500\ \Omega}$$

Phase impedance of the circuit = $\boldsymbol{j\ 80 + \frac{-j500\times 200}{-j500+200} = 172.413 + j\ 11\ \Omega}$

a. Magnitude of phase current = $\boldsymbol{\frac{V_{ph}}{I_{ph}} = \frac{200}{172.764} = 1.158\ A}$

b. Magnitude of line current = $\boldsymbol{\sqrt{3} \times I_{ph} = \sqrt{3} \times 1.158 = 2.01\ A}$

c. Total real power consumed = $\boldsymbol{3xI_{ph}^2 xR = 3x1.158^2 x172.413 = 693.6\ W}$