PH 102, Electromagnetism,

Post Mid Semester

Lecture 12

Electromagnetic Waves

in vacuum

Reflection and Transmission at
Oblique Incidence

D. J. Griffiths: 9.3

Sovan Chakraborty, Department of Physics, IITG

Electromagnetic Waves

Electromagnetic Waves in Matter:

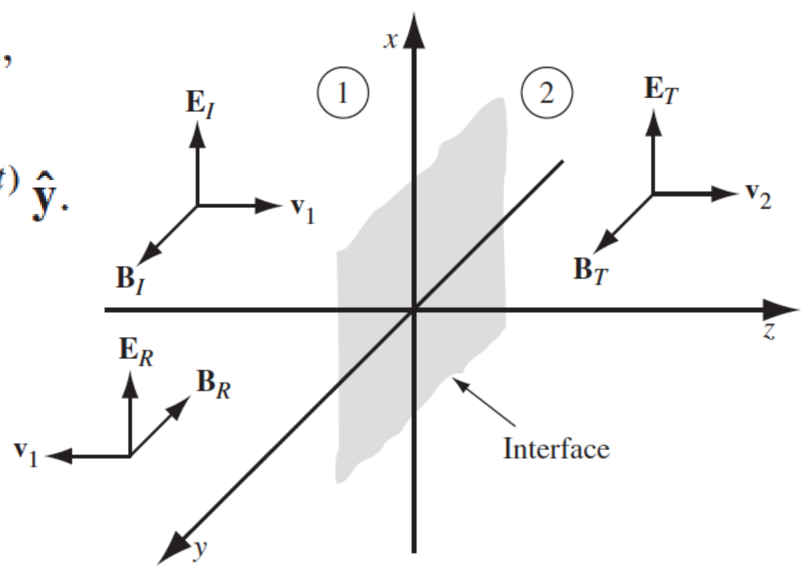
Reflection and Transmission at Normal Incidence:

- Suppose, the xy plane forms the boundary between two linear media.
- A plane wave of frequency **ω**, traveling in the ***z*** direction and polarized in the ***x*** direction, approaches the interface from the left

Incident Wave:

$$\tilde{\mathbf{E}}_I(z,t) = \tilde{E}_{0_I} e^{i(k_1 z - \omega t)}\,\hat{\mathbf{x}},$$

$$\tilde{\mathbf{B}}_I(z,t) = \frac{1}{v_1}\tilde{E}_{0_I} e^{i(k_1 z - \omega t)}\,\hat{\mathbf{y}}.$$



Electromagnetic Waves

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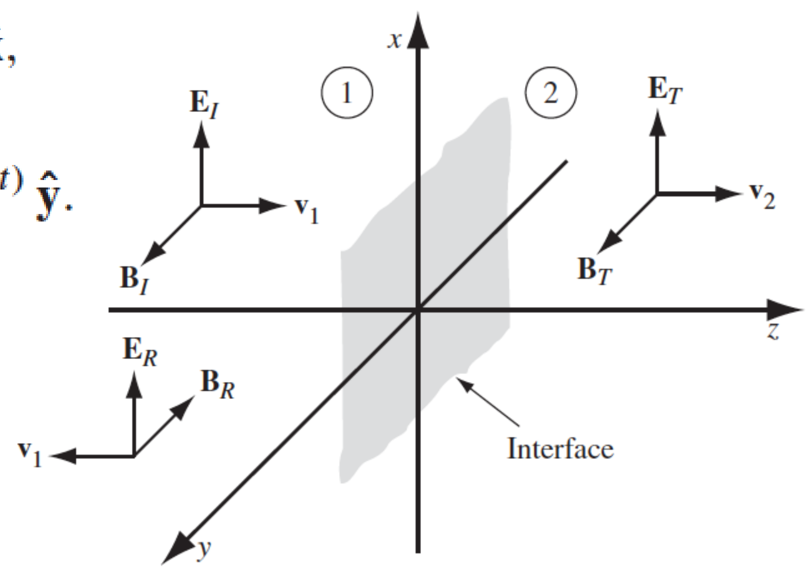

Reflected Wave:

$$\tilde{\mathbf{E}}_R(z,t) = \tilde{E}_{0_R} e^{i(-k_1 z - \omega t)}\,\hat{\mathbf{x}},$$

$$\tilde{\mathbf{B}}_R(z,t) = -\frac{1}{v_1}\tilde{E}_{0_R} e^{i(-k_1 z - \omega t)}\,\mathbf{y},$$

Note the minus sign in $\tilde{\mathbf{B}}_R$

$$\tilde{\mathbf{B}}(\mathbf{r},t) = \frac{1}{c}\tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}(\hat{\mathbf{k}} \times \hat{\mathbf{n}})$$

Electromagnetic Waves

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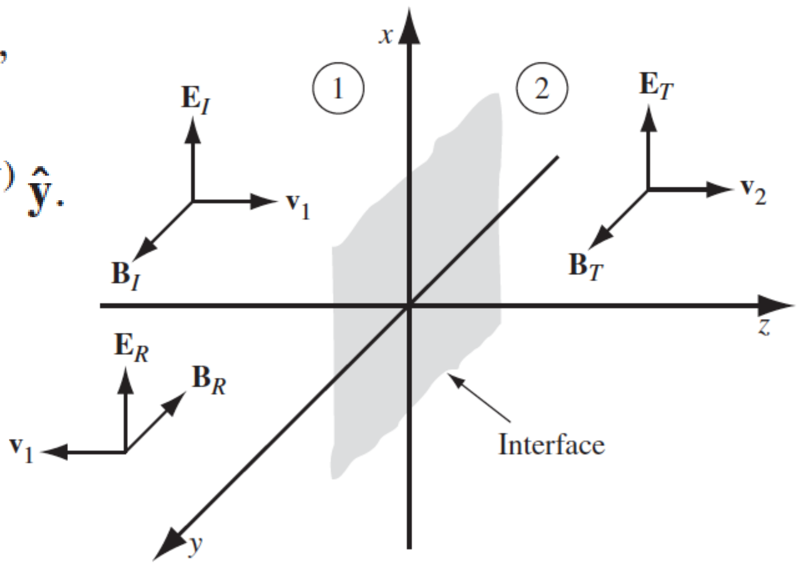

Transmitted Wave:

$$\tilde{\mathbf{E}}_T(z,t) = \tilde{E}_{0_T} e^{i(k_2 z - \omega t)}\,\hat{\mathbf{x}},$$

$$\tilde{\mathbf{B}}_T(z,t) = \frac{1}{v_2}\tilde{E}_{0_T} e^{i(k_2 z - \omega t)}\,\hat{\mathbf{y}},$$

Reflected Wave:

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Note the minus sign in $\tilde{\mathbf{B}}_R$

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Electromagnetic Waves

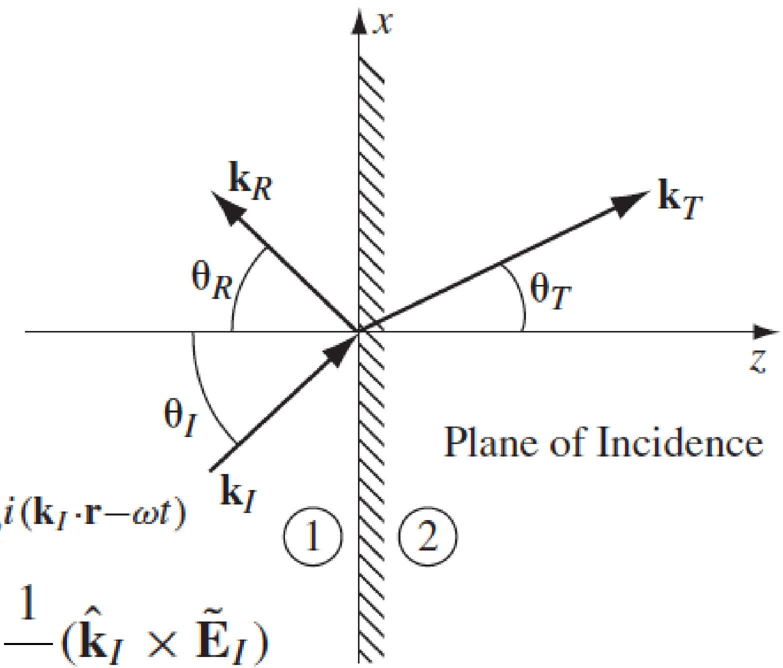
Reflection and Transmission at Oblique Incidence :

Reflection & Transmission at

Normal Incidence: Incoming wave hits the interface head-on.

Oblique incidence: General case, incoming wave meets the boundary @ angle θ_I

Normal Incidence: Oblique incidence, $\theta_I = 0$.



$$\tilde{\mathbf{E}}_I(\mathbf{r}, t) = \tilde{\mathbf{E}}_{0_I} e^{i(\mathbf{k}_I \cdot \mathbf{r} - \omega t)}$$

$$\tilde{\mathbf{B}}_I(\mathbf{r}, t) = \frac{1}{v_1}(\hat{\mathbf{k}}_I \times \tilde{\mathbf{E}}_I)$$

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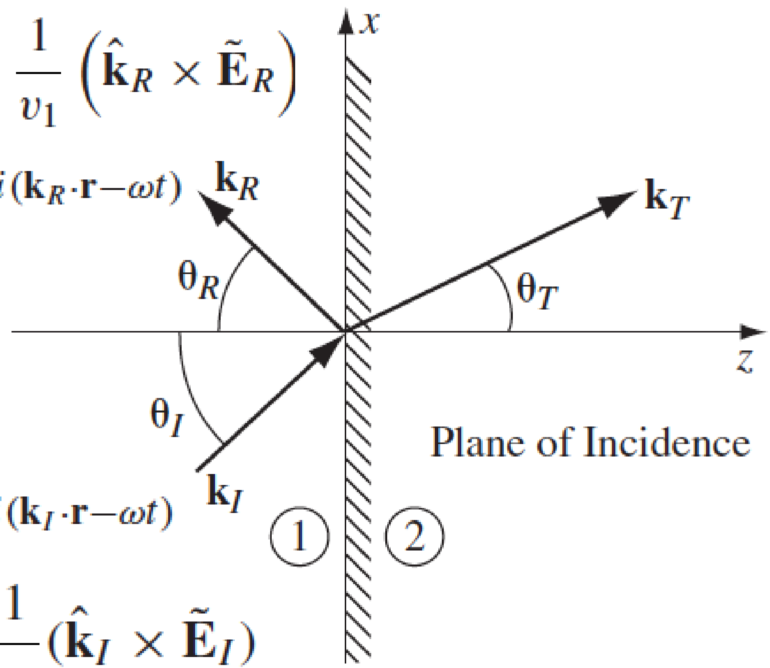

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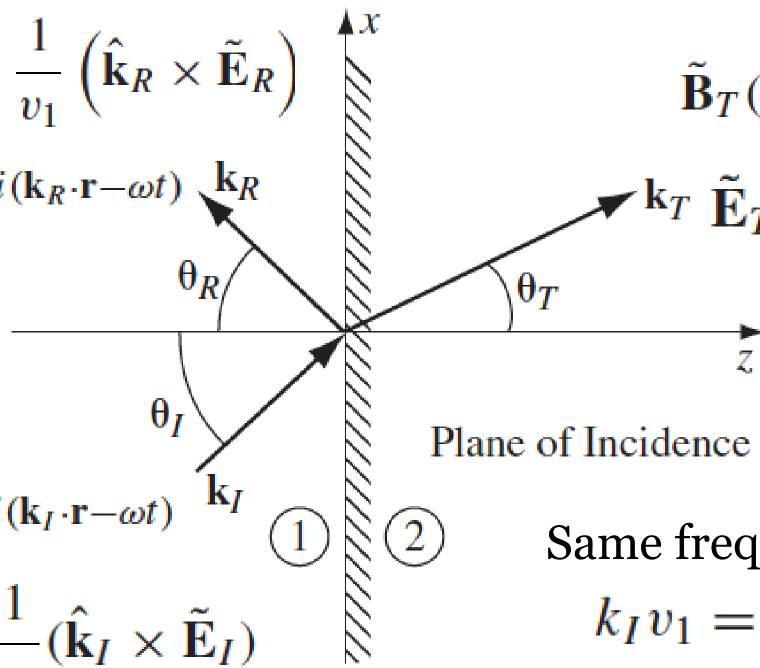


$$\tilde{\mathbf{E}}_I(\mathbf{r},t) = \tilde{\mathbf{E}}_{0_I} e^{i(\mathbf{k}_I\cdot\mathbf{r}-\omega t)}$$

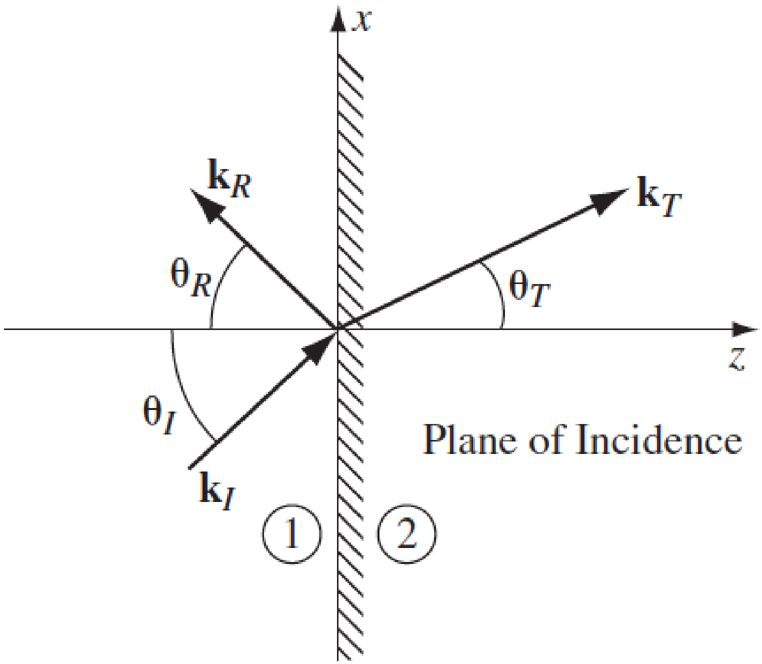
$$\tilde{\mathbf{B}}_I(\mathbf{r},t) = \frac{1}{v_1}(\hat{\mathbf{k}}_I \times \tilde{\mathbf{E}}_I)$$

Same frequency ω : Linear media

$$k_I v_1 = k_R v_1 = k_T v_2 = \omega$$

$$k_I = k_R = \frac{v_2}{v_1} k_T = \frac{n_1}{n_2} k_T$$

Electromagnetic Waves



$\tilde{\mathbf{E}}_I + \tilde{\mathbf{E}}_R$ & $\tilde{\mathbf{B}}_I + \tilde{\mathbf{B}}_R$, joined to $\tilde{\mathbf{E}}_T + \tilde{\mathbf{B}}_T$

by boundary conditions

$$\text{(i)}\quad \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}, \qquad \text{(iii)}\quad \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel},$$

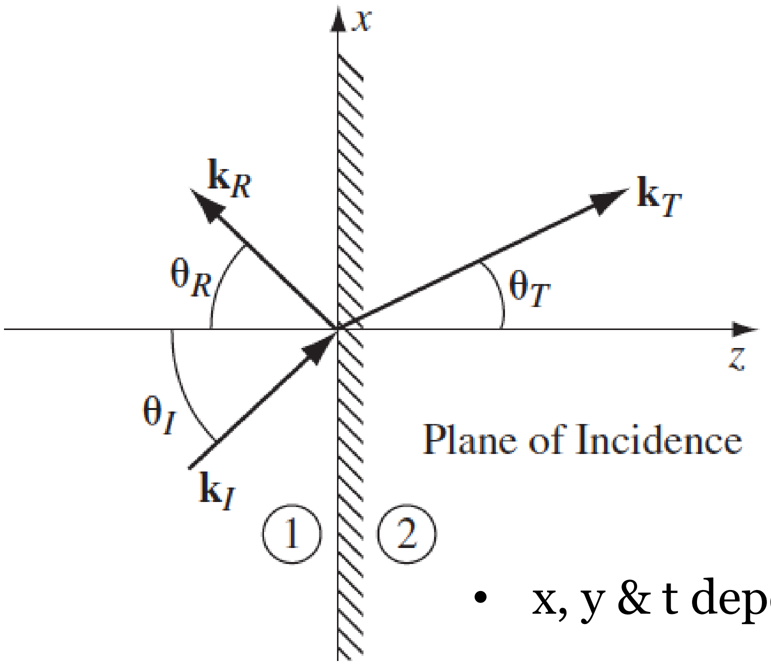
$$\text{(ii)}\quad B_1^{\perp} = B_2^{\perp}, \qquad \text{(iv)}\quad \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel}.$$

These all share the generic structure at z = 0 .

$$(\ )\exp[i\,(\mathbf{k_I}.\mathbf{r}-\omega t) + (\ )\exp[i\,(\mathbf{k_R}.\mathbf{r}-\omega t)]$$

$$= (\ )\exp[i\,(\mathbf{k_T}.\mathbf{r}-\omega t)$$

Electromagnetic Waves



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These all share the generic structure at z = 0 .

$(\ )\exp[i\,(\mathbf{k_I}.\mathbf{r}-\omega t) + (\ )\exp[i\,(\mathbf{k_R}.\mathbf{r}-\omega t)]$

$= (\ )\exp[i\,(\mathbf{k_T}.\mathbf{r}-\omega t)$

- x, y & t dependence is confined to the exponents

- The boundary conditions hold at ***all*** points on the plane, and for ***all*** times,
- Thus the exponential factors must be ***equal***

$$\mathbf{k}_I \cdot \mathbf{r} = \mathbf{k}_R \cdot \mathbf{r} = \mathbf{k}_T \cdot \mathbf{r}, \quad \text{when } z = 0,$$

Electromagnetic Waves

$$\mathbf{k}_I \cdot \mathbf{r} = \mathbf{k}_R \cdot \mathbf{r} = \mathbf{k}_T \cdot \mathbf{r}, \quad \text{when } z = 0,$$

and for all x and all y.

$$x(k_I)_x + y(k_I)_y = x(k_R)_x + y(k_R)_y = x(k_T)_x + y(k_T)_y$$

Holds: only if the components are separately equal,

For x= 0, we get,

$(k_{\mathrm{I}})_{\mathrm{y}} = (k_R)_{\mathrm{y}} = (k_T)_{\mathrm{y}}$,

while y = 0, gives

$(k_{\mathrm{I}})_x = (k_R)_x = (k_T)_x$,

Electromagnetic Waves

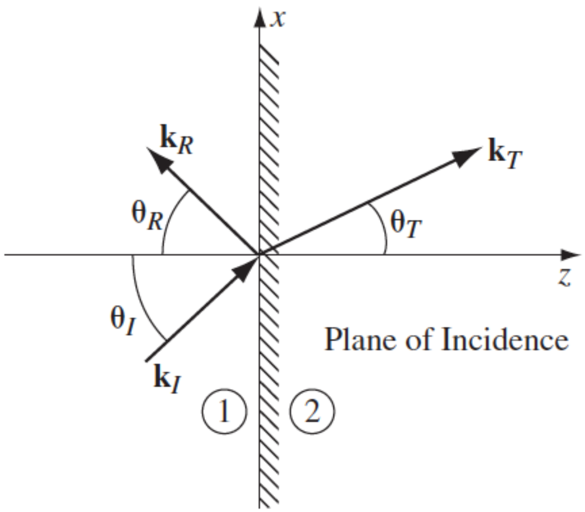
$$\mathbf{k}_I \cdot \mathbf{r} = \mathbf{k}_R \cdot \mathbf{r} = \mathbf{k}_T \cdot \mathbf{r}, \quad \text{when } z = 0,$$

Holds: only if the components are separately equal,

$$(k_I)_y = (k_R)_y = (k_T)_y$$

One can orient the axes such that
$\boldsymbol{k}_\mathbf{I}$ lies in the $x\,z$ plane (i.e. $(k_I)_y = 0$);

$\boldsymbol{k}_\mathbf{R}$ and $\boldsymbol{k}_\mathbf{T}$ also lies in the same plane
($(k_I)_y = (k_R)_y = (k_T)_y = 0$).



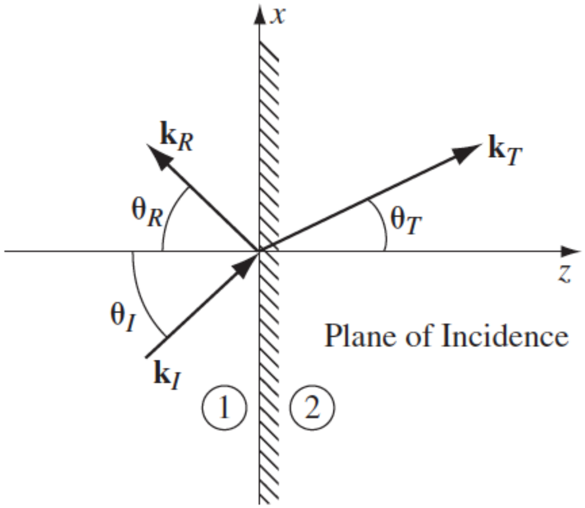
First Law: The incident, reflected, and transmitted wave vectors form a plane (plane of incidence), which also includes the normal to the surface (here, z axis).

Electromagnetic Waves

$$\mathbf{k}_I \cdot \mathbf{r} = \mathbf{k}_R \cdot \mathbf{r} = \mathbf{k}_T \cdot \mathbf{r}, \quad \text{when } z = 0,$$

Holds: only if the components are separately equal,

$$(k_\mathrm{I})_x = (k_R)_x = (k_T)_x$$



Now, $k_I \sin\theta_I = k_R \sin\theta_R = k_T \sin\theta_T$,

θ_I, θ_R & θ_T, are the angle of incidence, reflection & transmission (refraction) respectively.
and measured with respect to the normal..

$$\boxed{k_I \sin\theta_I = k_R \sin\theta_R}$$

$$\boxed{k_I = k_R = \frac{v_2}{v_1} k_T = \frac{n_1}{n_2} k_T}$$

Second Law: The angle of incidence is equal to the angle reflection,

$\theta_\mathrm{I} = \theta_\mathrm{R}$. (**law of reflection**)

Electromagnetic Waves

$$\mathbf{k}_I \cdot \mathbf{r} = \mathbf{k}_R \cdot \mathbf{r} = \mathbf{k}_T \cdot \mathbf{r}, \quad \text{when } z = 0,$$

Holds: only if the components are separately equal,

$$(k_\text{I})_x = (k_R)_x = (k_T)_x$$

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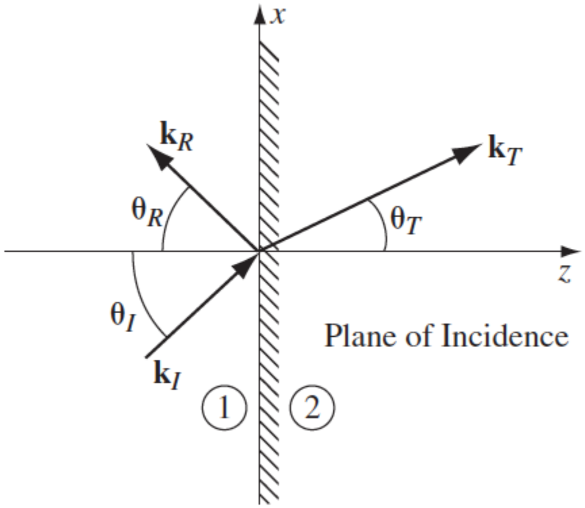

$$\boxed{k_R \sin\theta_R = k_T \sin\theta_T}$$

$$\boxed{k_I = k_R = \frac{v_2}{v_1} k_T = \frac{n_1}{n_2} k_T}$$

Third Law: $\dfrac{\sin\theta_T}{\sin\theta_I} = \dfrac{n_1}{n_2}$. **(law of refraction/Snell's Law)**

Electromagnetic Waves

$$\tilde{\mathbf{B}}_R(\mathbf{r},t) = \frac{1}{v_1}\left(\hat{\mathbf{k}}_R \times \tilde{\mathbf{E}}_R\right)$$

$$\tilde{\mathbf{B}}_T(\mathbf{r},t) = \frac{1}{v_2}\left(\hat{\mathbf{k}}_T \times \tilde{\mathbf{E}}_T\right)$$

$$\tilde{\mathbf{E}}_R(\mathbf{r},t) = \tilde{\mathbf{E}}_{0_R} e^{i(\mathbf{k}_R\cdot\mathbf{r}-\omega t)}$$

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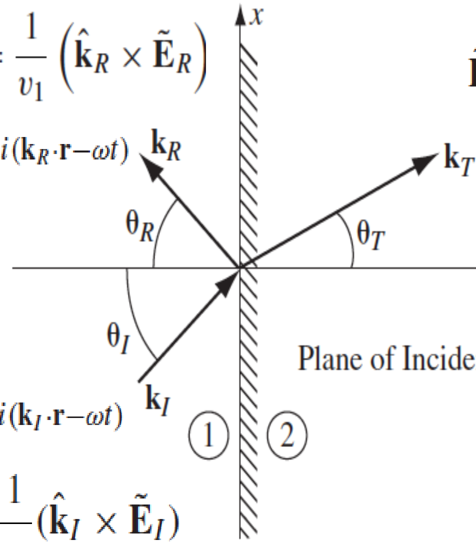

$$\tilde{\mathbf{E}}_I(\mathbf{r},t) = \tilde{\mathbf{E}}_{0_I} e^{i(\mathbf{k}_I\cdot\mathbf{r}-\omega t)}$$

$$\tilde{\mathbf{B}}_I(\mathbf{r},t) = \frac{1}{v_1}(\hat{\mathbf{k}}_I \times \tilde{\mathbf{E}}_I)$$

$$\mathbf{k}_I \cdot \mathbf{r} = \mathbf{k}_R \cdot \mathbf{r} = \mathbf{k}_T \cdot \mathbf{r}, \quad \text{when } z = 0,$$

Boundary conditions :

$$\text{(i)}\ \ \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}, \qquad \text{(iii)}\ \ \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel},$$

$$\text{(ii)}\ \ B_1^{\perp} = B_2^{\perp}, \qquad \text{(iv)}\ \ \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel}.$$

It is only Amplitudes + Exponentials cancelled,

Electromagnetic Waves

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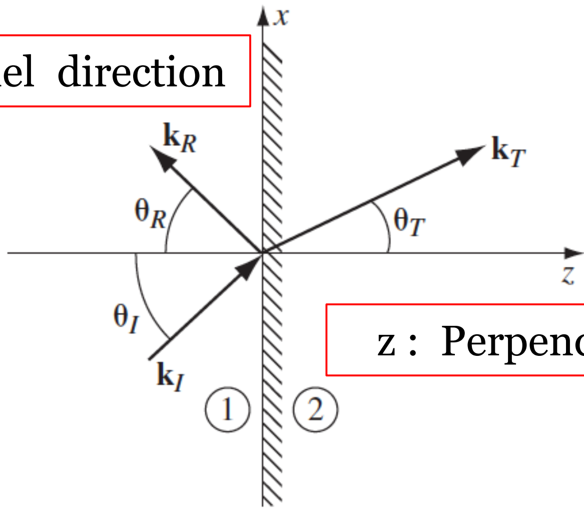
x & y: Parallel direction



z : Perpendicular direction

Boundary conditions :

$$\text{(i)}\quad \epsilon_1\left(\tilde{\mathbf{E}}_{0_I} + \tilde{\mathbf{E}}_{0_R}\right)_z = \epsilon_2\left(\tilde{\mathbf{E}}_{0_T}\right)_z \qquad \text{(iii)}\quad \left(\tilde{\mathbf{E}}_{0_I} + \tilde{\mathbf{E}}_{0_R}\right)_{x,y} = \left(\tilde{\mathbf{E}}_{0_T}\right)_{x,y}$$

$$\text{(ii)}\quad \left(\tilde{\mathbf{B}}_{0_I} + \tilde{\mathbf{B}}_{0_R}\right)_z = \left(\tilde{\mathbf{B}}_{0_T}\right)_z \qquad \text{(iv)}\quad \frac{1}{\mu_1}\left(\tilde{\mathbf{B}}_{0_I} + \tilde{\mathbf{B}}_{0_R}\right)_{x,y} = \frac{1}{\mu_2}\left(\tilde{\mathbf{B}}_{0_T}\right)_{x,y}$$

Here, $\tilde{\mathbf{B}}_0 = (1/v)\hat{\mathbf{k}} \times \tilde{\mathbf{E}}_0$ and (iii)/(iv) are pairs of eqs, for x & y component.

Electromagnetic Waves

Boundary conditions :

(i) $\epsilon_1\left(\tilde{\mathbf{E}}_{0_I}+\tilde{\mathbf{E}}_{0_R}\right)_z=\epsilon_2\left(\tilde{\mathbf{E}}_{0_T}\right)_z$

(ii) $\left(\tilde{\mathbf{B}}_{0_I}+\tilde{\mathbf{B}}_{0_R}\right)_z=\left(\tilde{\mathbf{B}}_{0_T}\right)_z$

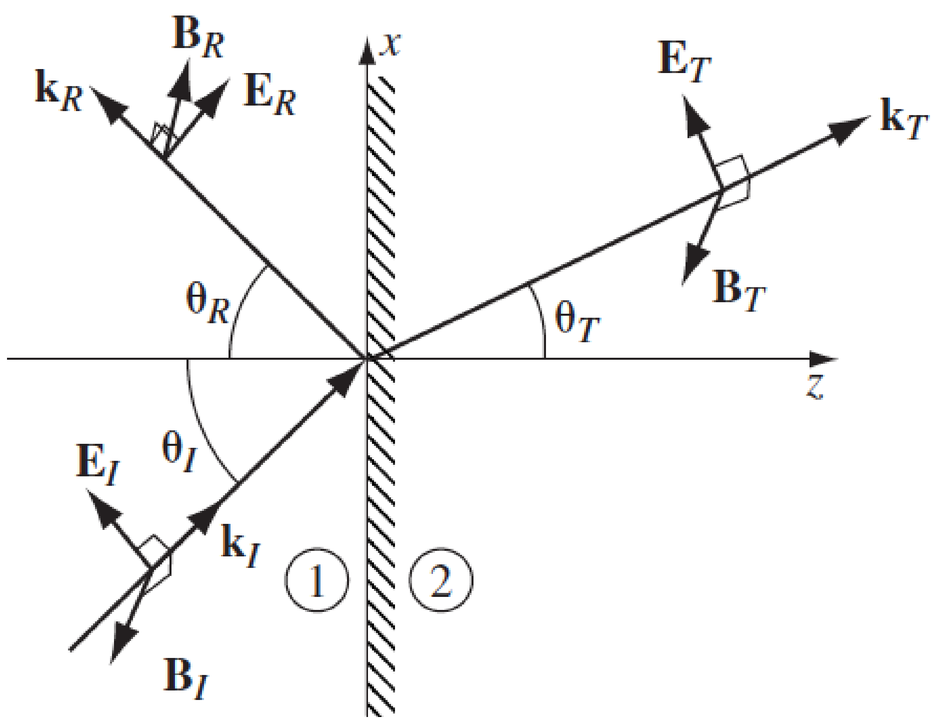
(iii) $\left(\tilde{\mathbf{E}}_{0_I}+\tilde{\mathbf{E}}_{0_R}\right)_{x,y}=\left(\tilde{\mathbf{E}}_{0_T}\right)_{x,y}$

(iv) $\frac{1}{\mu_1}\left(\tilde{\mathbf{B}}_{0_I}+\tilde{\mathbf{B}}_{0_R}\right)_{x,y}=\frac{1}{\mu_2}\left(\tilde{\mathbf{B}}_{0_T}\right)_{x,y}$

Polarization of the incident wave is parallel to the plane of incidence

(***E**$_I$* confined in xz plane)

The reflected and transmitted waves are also polarized in this plane



Electromagnetic Waves

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(i) $\epsilon_1 \left(\tilde{\mathbf{E}}_{0_I} + \tilde{\mathbf{E}}_{0_R} \right)_z = \epsilon_2 \left(\tilde{\mathbf{E}}_{0_T} \right)_z$

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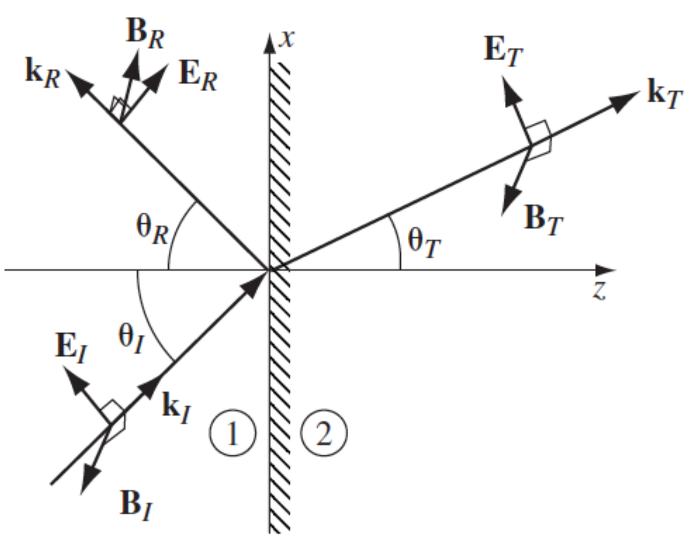
(iii) $\left(\tilde{\mathbf{E}}_{0_I} + \tilde{\mathbf{E}}_{0_R} \right)_{x,y} = \left(\tilde{\mathbf{E}}_{0_T} \right)_{x,y}$

(iv) $\frac{1}{\mu_1} \left(\tilde{\mathbf{B}}_{0_I} + \tilde{\mathbf{B}}_{0_R} \right)_{x,y} = \frac{1}{\mu_2} \left(\tilde{\mathbf{B}}_{0_T} \right)_{x,y}$



(i) $\epsilon_1 \left(-\tilde{E}_{0_I} \sin\theta_I + \tilde{E}_{0_R} \sin\theta_R \right) = \epsilon_2 \left(-\tilde{E}_{0_T} \sin\theta_T \right)$

(ii) No z-component for ***B***

(iii) $\tilde{E}_{0_I} \cos\theta_I + \tilde{E}_{0_R} \cos\theta_R = \tilde{E}_{0_T} \cos\theta_T$ No y-comp

(iv) $\frac{1}{\mu_1 v_1} \left(\tilde{E}_{0_I} - \tilde{E}_{0_R} \right) = \frac{1}{\mu_2 v_2} \tilde{E}_{0_T}$ No x-comp

Electromagnetic Waves

Boundary conditions :

(i) $$\epsilon_1\left(-\tilde{E}_{0_I}\sin\theta_I+\tilde{E}_{0_R}\sin\theta_R\right)=\epsilon_2\left(-\tilde{E}_{0_T}\sin\theta_T\right)$$

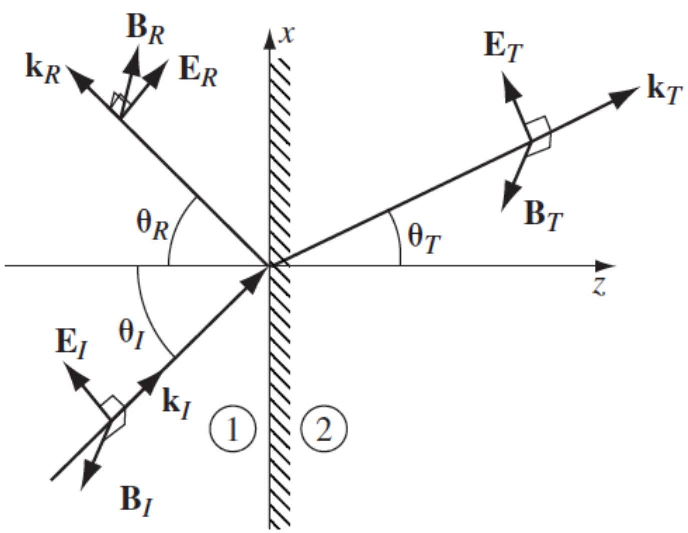
$$\tilde{E}_{0_I}-\tilde{E}_{0_R}=\beta\tilde{E}_{0_T}$$

(ii) No z-component for $\boldsymbol{B}$

$$\beta\equiv\frac{\mu_1 v_1}{\mu_2 v_2}=\frac{\mu_1 n_2}{\mu_2 n_1}$$

(iii) $$\tilde{E}_{0_I}\cos\theta_I+\tilde{E}_{0_R}\cos\theta_R=\tilde{E}_{0_T}\cos\theta_T$$

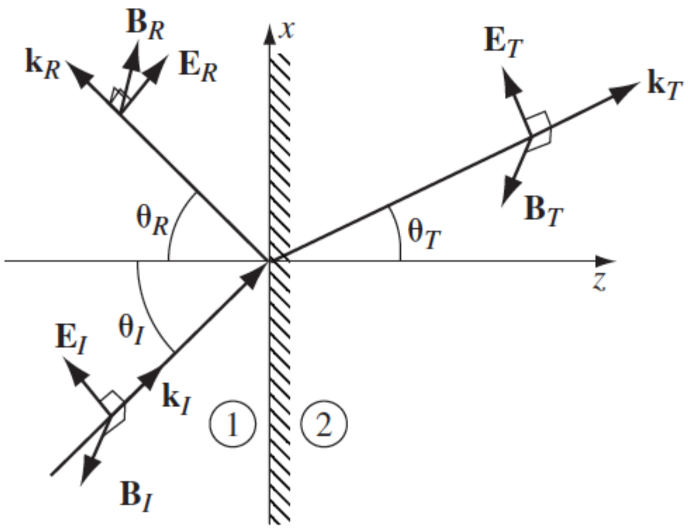
$$\tilde{E}_{0_I}+\tilde{E}_{0_R}=\alpha\tilde{E}_{0_T}\qquad \alpha\equiv\frac{\cos\theta_T}{\cos\theta_I}$$



(iv) $$\frac{1}{\mu_1 v_1}\left(\tilde{E}_{0_I}-\tilde{E}_{0_R}\right)=\frac{1}{\mu_2 v_2}\tilde{E}_{0_T}$$

$$\tilde{E}_{0_I}-\tilde{E}_{0_R}=\beta\tilde{E}_{0_T}$$

Electromagnetic Waves



$$\tilde{E}_{0_I} - \tilde{E}_{0_R} = \beta \tilde{E}_{0_T} \qquad \tilde{E}_{0_I} + \tilde{E}_{0_R} = \alpha \tilde{E}_{0_T}$$

$$\boxed{\tilde{E}_{0_R} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right) \tilde{E}_{0_I}, \quad \tilde{E}_{0_T} = \left(\frac{2}{\alpha + \beta}\right) \tilde{E}_{0_I}.}$$

Fresnel's equations, for the case of polarization in the plane of incidence.

- Transmitted wave is in phase with the incident one.
- Reflected wave is either in phase, $\alpha > \beta$ or 180° out of phase, if $\alpha < \beta$.
- Amplitudes of the transmitted & reflected waves depend on the θ_I, as $\alpha \equiv \dfrac{\cos\theta_T}{\cos\theta_I}$.

$$\alpha = \frac{\sqrt{1 - \sin^2\theta_T}}{\cos\theta_I} = \frac{\sqrt{1 - [(n_1/n_2)\sin\theta_I]^2}}{\cos\theta_I}.$$

Normal incidence ($\theta_I = 0$), $\alpha = 1$.

Grazing incidence ($\theta_I = 90^\circ$), α diverges, the wave is totally reflected.

Electromagnetic Waves

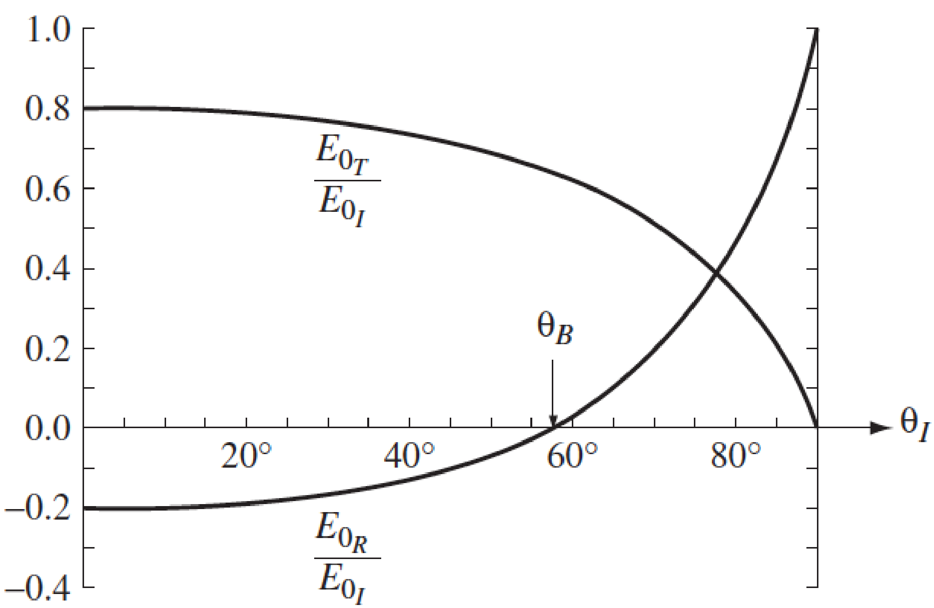
Brewster's Angle:

$$\tilde{E}_{0_R} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right) \tilde{E}_{0_I}$$

Intermediate angle, θ_B (Brewster's angle).

Reflected wave is completely extinguished, $\alpha = \beta$, or $\sin^2\theta_B = \frac{1-\beta^2}{(n_1/n_2)^2 - \beta^2}$.

For $\mu_1 \cong \mu_2$, so $\beta \cong n_2/n_1$, $\sin^2\theta_B \cong \beta^2/(1+\beta^2)$, $\tan\theta_B \cong \frac{n_2}{n_1}$



Transmitted and reflected amplitudes as functions of θ_I , for light incident on glass (n_2=1. 5) from air (n_1=1)

-ve ratio: 180 degree out of phase, Absolute value for amplitude

Electromagnetic Waves

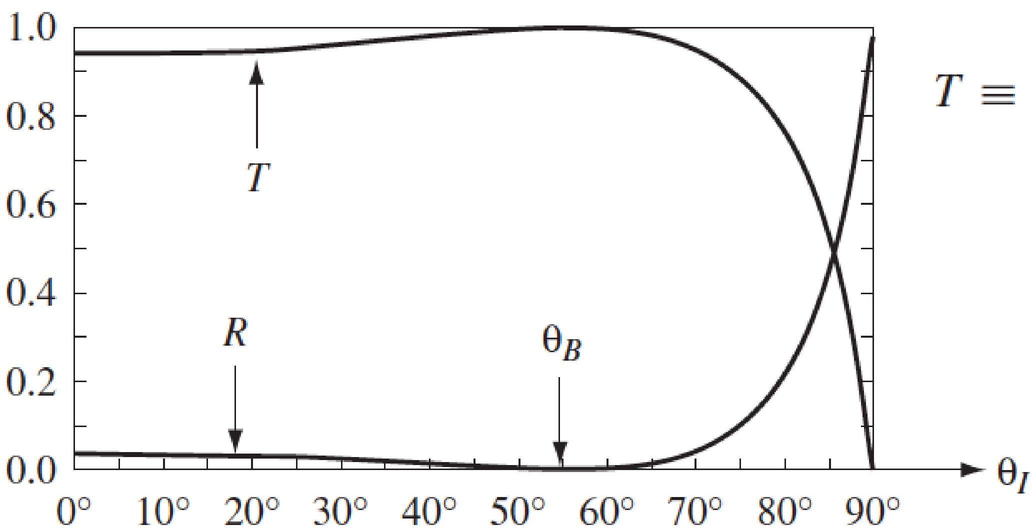
The power per unit area striking the **interface** is **S.ẑ**.

@ Interface, giving the cosθ

Thus the incident intensity is, $I_I = \frac{1}{2}\epsilon_1 v_1 E_{0_I}^2 \cos\theta_I$

The reflected and transmitted intensities are

$$I_R = \frac{1}{2}\epsilon_1 v_1 E_{0_R}^2 \cos\theta_R \quad \text{and} \quad I_T = \frac{1}{2}\epsilon_2 v_2 E_{0_T}^2 \cos\theta_T$$

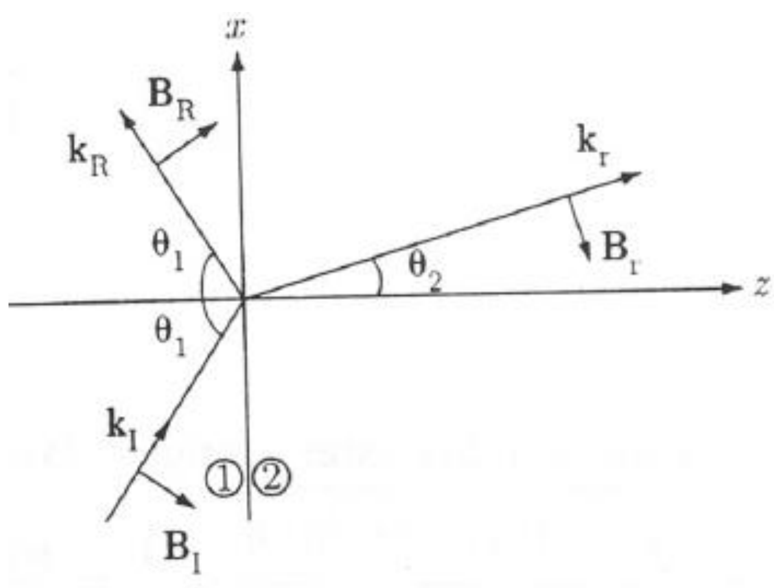


$$T \equiv \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{E_{0_T}}{E_{0_I}}\right)^2 \frac{\cos\theta_T}{\cos\theta_I} = \alpha\beta\left(\frac{2}{\alpha+\beta}\right)^2$$

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0_R}}{E_{0_I}}\right)^2 = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2$$

Electromagnetic Waves

Problem 9.17 Analyze the case of polarization *perpendicular* to the plane of incidence (i.e. electric fields in the y direction, in Fig. 9.15). Impose the boundary conditions (Eq. 9.101), and obtain the Fresnel equations for $\tilde{E}_{0_R}$ and $\tilde{E}_{0_T}$. Sketch $(\tilde{E}_{0_R}/\tilde{E}_{0_I})$ and $(\tilde{E}_{0_T}/\tilde{E}_{0_I})$ as functions of θ_I, for the case $\beta = n_2/n_1 = 1.5$. (Note that for this β the reflected wave is *always* 180° out of phase.) Show that there is no Brewster's angle for *any* n_1 and n_2: $\tilde{E}_{0_R}$ is *never* zero (unless, of course, $n_1 = n_2$ and $\mu_1 = \mu_2$, in which case the two media are optically indistinguishable). Confirm that your Fresnel equations reduce to the proper forms at normal incidence. Compute the reflection and transmission coefficients, and check that they add up to 1.



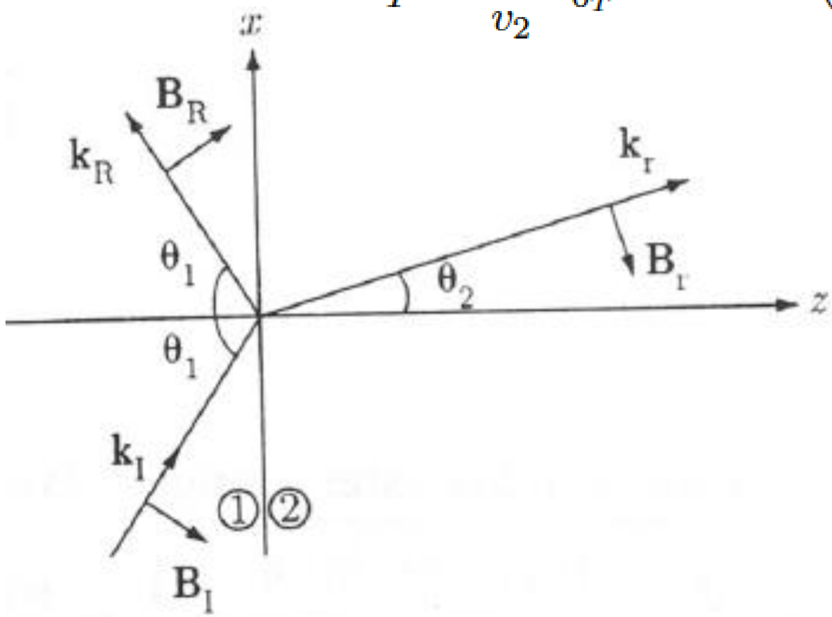
Electromagnetic Waves

$$\text{(i) } \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}, \quad \text{(iii) } \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel},$$

$$\text{(ii) } B_1^{\perp} = B_2^{\perp}, \quad \text{(iv) } \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel}.$$

$$\begin{aligned} \tilde{\mathbf{E}}_R &= \tilde{E}_{0_R} e^{i(\mathbf{k}_R \cdot \mathbf{r} - \omega t)} \hat{\mathbf{y}}, \\ \tilde{\mathbf{B}}_R &= \frac{1}{v_1} \tilde{E}_{0_R} e^{i(\mathbf{k}_R \cdot \mathbf{r} - \omega t)} (\cos\theta_1 \,\hat{\mathbf{x}} + \sin\theta_1 \,\hat{\mathbf{z}}); \end{aligned}$$

$$\begin{aligned} \tilde{\mathbf{E}}_T &= \tilde{E}_{0_T} e^{i(\mathbf{k}_T \cdot \mathbf{r} - \omega t)} \hat{\mathbf{y}}, \\ \tilde{\mathbf{B}}_T &= \frac{1}{v_2} \tilde{E}_{0_T} e^{i(\mathbf{k}_T \cdot \mathbf{r} - \omega t)} (-\cos\theta_2 \,\hat{\mathbf{x}} + \sin\theta_2 \,\hat{\mathbf{z}}) \end{aligned}$$



$$\begin{aligned} \tilde{\mathbf{E}}_I &= \tilde{E}_{0_I} e^{i(\mathbf{k}_I \cdot \mathbf{r} - \omega t)} \hat{\mathbf{y}}, \\ \tilde{\mathbf{B}}_I &= \frac{1}{v_1} \tilde{E}_{0_I} e^{i(\mathbf{k}_I \cdot \mathbf{r} - \omega t)} (-\cos\theta_1 \,\hat{\mathbf{x}} + \sin\theta_1 \,\hat{\mathbf{z}}); \end{aligned}$$

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}. \ [\text{Note: } \mathbf{k}_I \cdot \mathbf{r} - \omega t = \mathbf{k}_R \cdot \mathbf{r} - \omega t = \mathbf{k}_T \cdot \mathbf{r} - \omega t, \text{ at } z = 0]$$

Electromagnetic Waves

Boundary conditions :

(i) Trivial

(ii) & (iii) $\tilde{E}_{0_I} + \tilde{E}_{0_R} = \tilde{E}_{0_T}$

$$\alpha \equiv \frac{\cos\theta_2}{\cos\theta_1}; \ \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2}$$

(iv) $\tilde{E}_{0_I} - \tilde{E}_{0_R} = \alpha\beta\tilde{E}_{0_T}$

$$\tilde{E}_{0_T} = \left(\frac{2}{1+\alpha\beta}\right)\tilde{E}_{0_I} \qquad \tilde{E}_{0_R} = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)\tilde{E}_{0_I}$$

In phase, $\alpha\beta > 0$

in phase for $\alpha\beta < 1$

$$E_{0_T} = \left(\frac{2}{1+\alpha\beta}\right)E_{0_I} \qquad E_{0_R} = \left|\frac{1-\alpha\beta}{1+\alpha\beta}\right|E_{0_I}$$

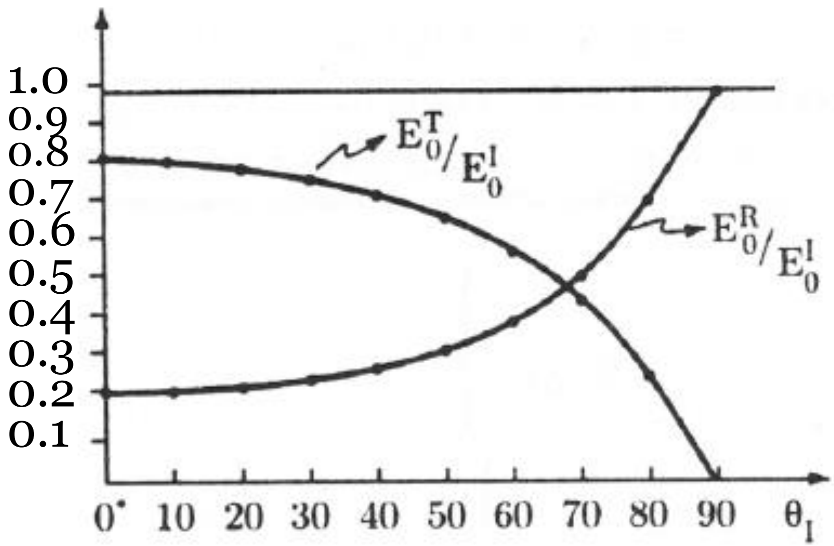
Fresnel equations for polarization **perpendicular** to the plane of incidence.

Electromagnetic Waves

Now,

$$\alpha\beta = \beta\frac{\sqrt{1-\sin^2\theta/\beta^2}}{\cos\theta} = \frac{\sqrt{\beta^2-\sin^2\theta}}{\cos\theta}$$, θ is the angle of incidence!

$$\beta = 1.5,\ \alpha\beta = \frac{\sqrt{2.25-\sin^2\theta}}{\cos\theta}$$



Electromagnetic Waves

Brewster's angle: $E_{0_R} = 0 \text{ would mean that } \alpha\beta = 1$

$$\alpha = \frac{\sqrt{1-(v_2/v_1)^2\sin^2\theta}}{\cos\theta} = \frac{1}{\beta} = \frac{\mu_2 v_2}{\mu_1 v_1}$$

or, $$1 - \left(\frac{v_2}{v_1}\right)^2 \sin^2\theta = \left(\frac{\mu_2 v_2}{\mu_1 v_1}\right)^2 \cos^2\theta$$

or, $$1 = \left(\frac{v_2}{v_1}\right)^2 \left[\sin^2\theta + (\mu_2/\mu_1)^2\cos^2\theta\right]$$

$\mu_1 \approx \mu_2$, would mean $1 \approx (v_2/v_1)^2$

i.e. optically indistinguishable media!

[find angle if μ's are not same]

Electromagnetic Waves

Fresnel's equations:

$$E_{0_T} = \left(\frac{2}{1+\alpha\beta}\right) E_{0_I} \qquad E_{0_R} = \left|\frac{1-\alpha\beta}{1+\alpha\beta}\right| E_{0_I}$$

At normal incidence, $\alpha = 1$,

$$E_{0_T} = \left(\frac{2}{1+\beta}\right) E_{0_I} \qquad E_{0_R} = \left|\frac{1-\beta}{1+\beta}\right| E_{0_I}$$

The reflection and transmission coefficients:

$$R = \left(\frac{E_{0_R}}{E_{0_I}}\right)^2 = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2$$

R + T = 1

$$T = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\alpha\left(\frac{E_{0_T}}{E_{0_I}}\right)^2 = \alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2$$