Lecture 5

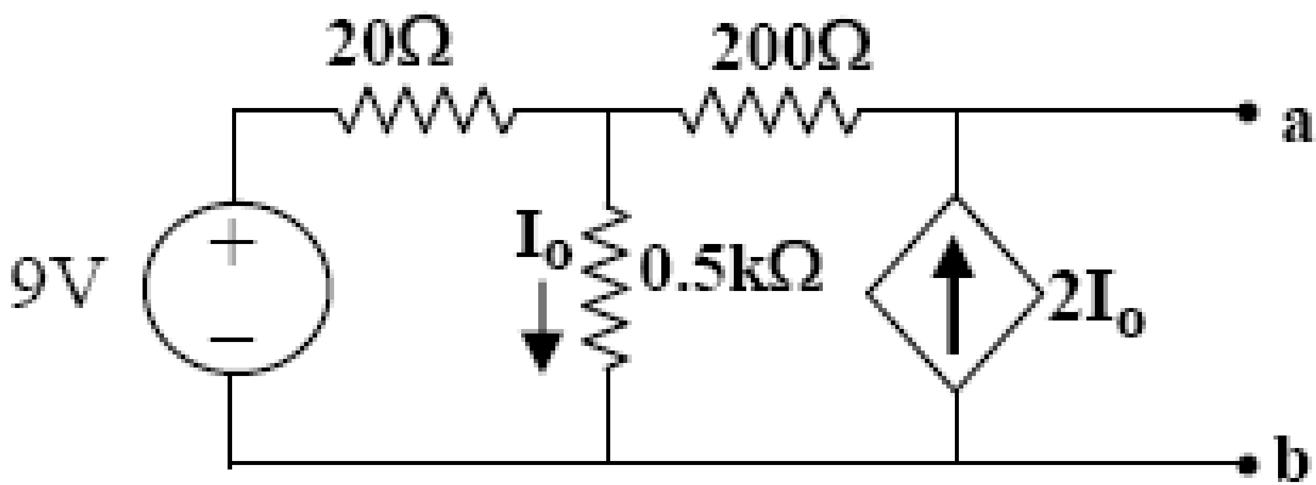
Revision for Quiz

and

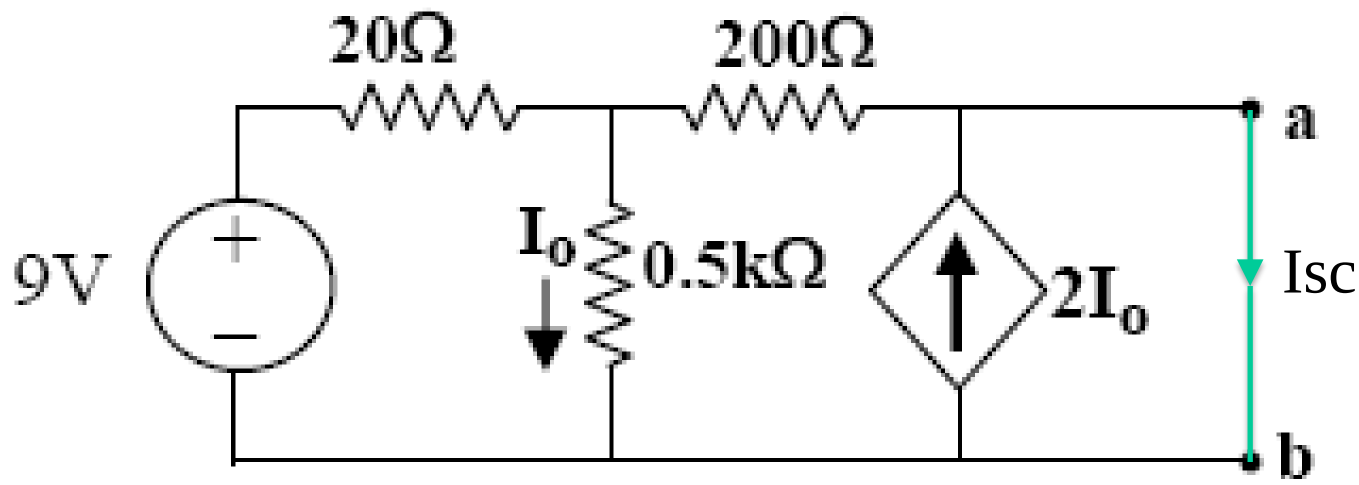
RL Circuit

Revision for Quiz

Q.1 Find the Norton's equivalent current (Isc) of the circuit between the terminals **a-b**.



To find Norton’s current Isc, terminals **a** and **b** are short-circuited

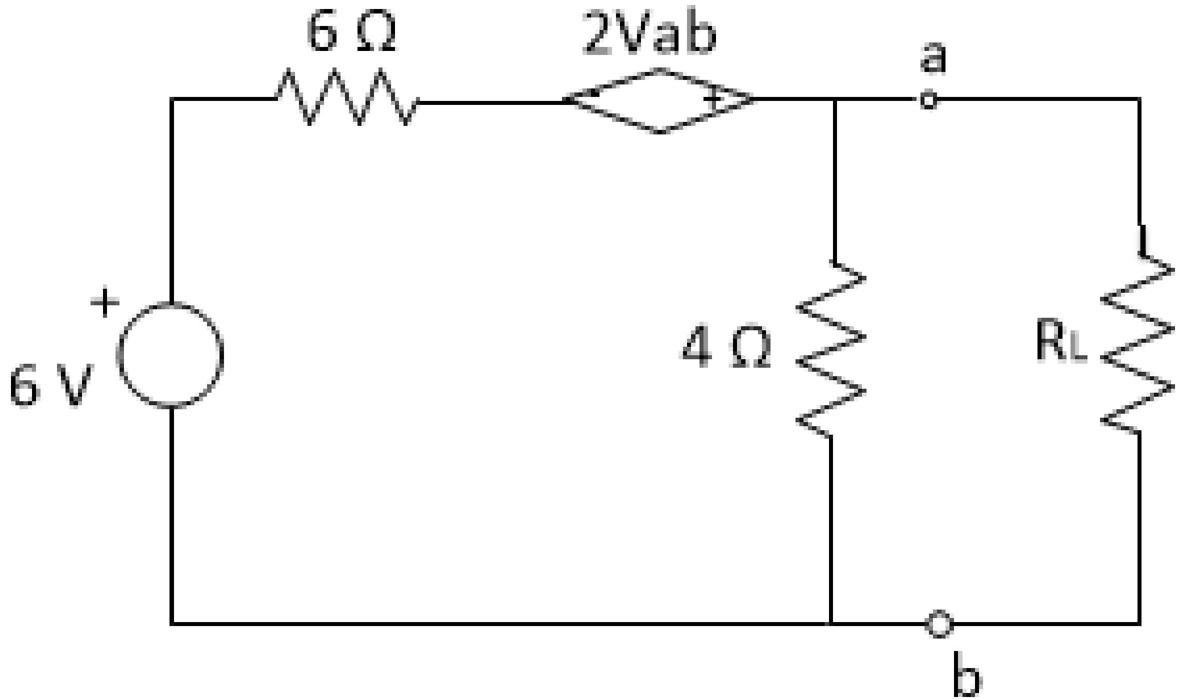


Apply Nodal/Mesh analysis and find Isc.

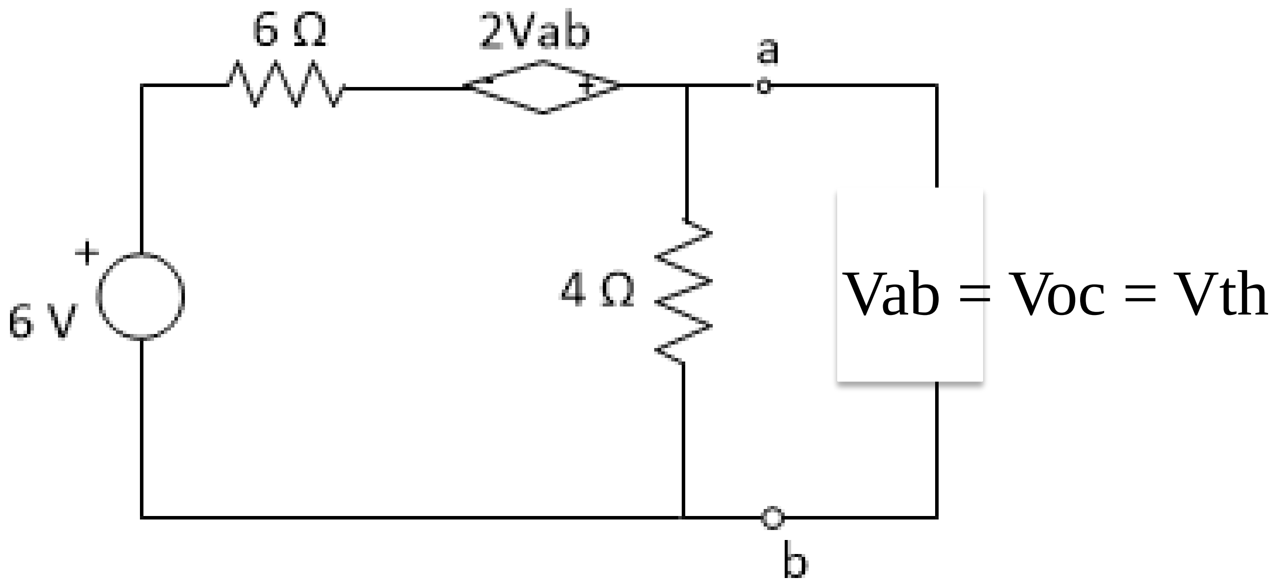
Answer : Isc = 0.07104 A (approx)

Revision for Quiz

Q.2 Find maximum power delivered to the load **R_L**.



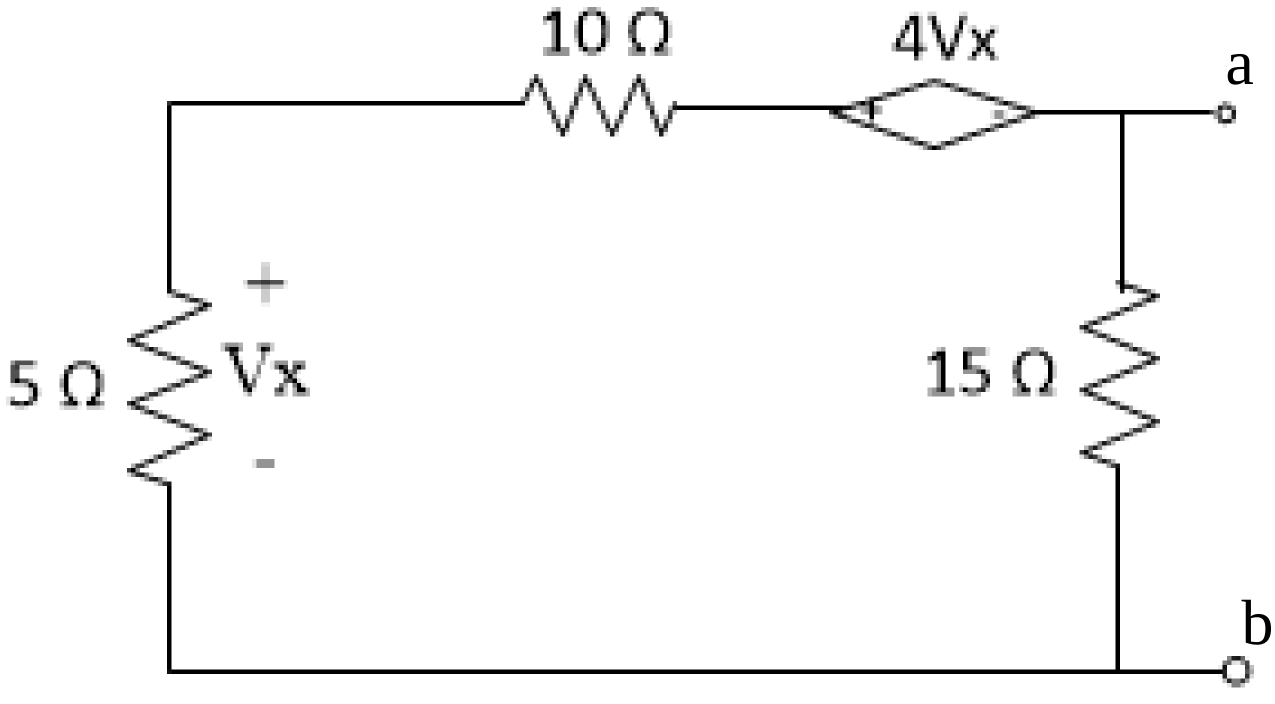
Thevenin’s voltage and equivalence resistance are to be estimated first. Remove RL to find Vab=Vth



Answer : Pmax = 3 W

Revision for Quiz

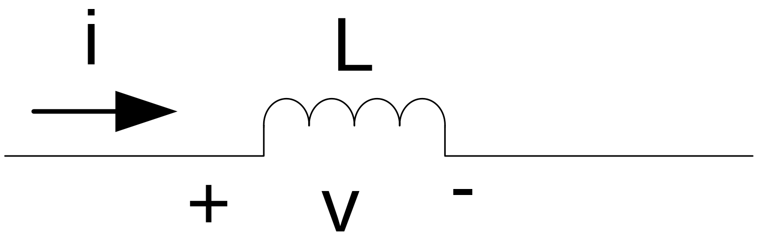
Q.3 Find Thevenin's equivalent of the following circuit.



Answer : Vth = 0 V and Rth = - 7.5 Ω

RL and RC Circuits

The Inductor

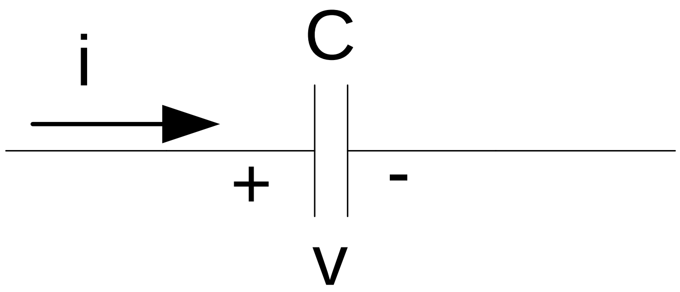


$$v = L\frac{di}{dt}$$

If current i is constant, voltage v is 0

So, inductor behaves as a short circuit to dc input current

The Capacitor

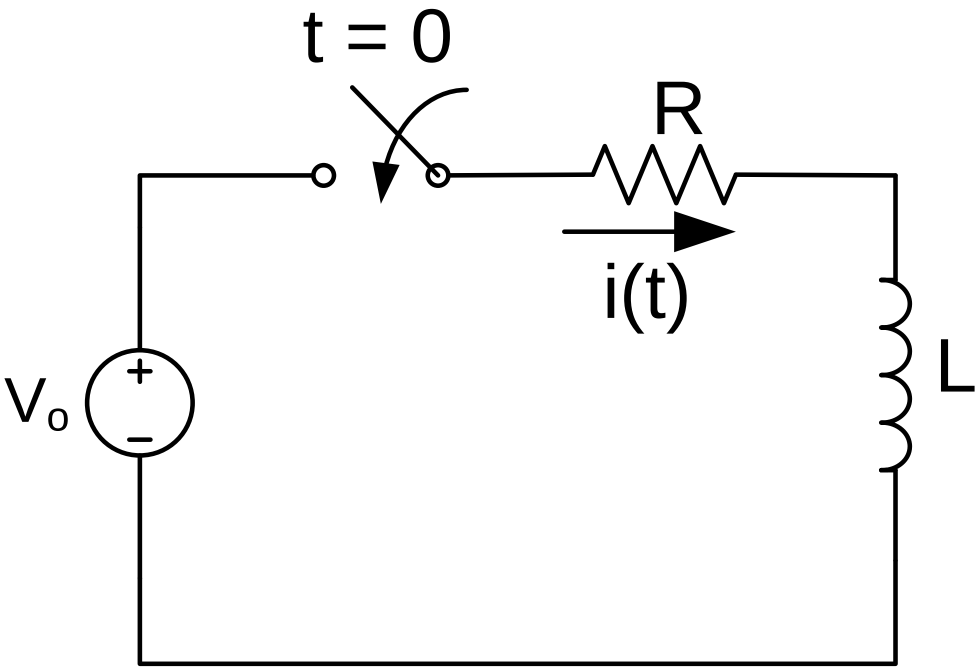


$$i = C\frac{dv}{dt}$$

If voltage v is constant, then i is 0

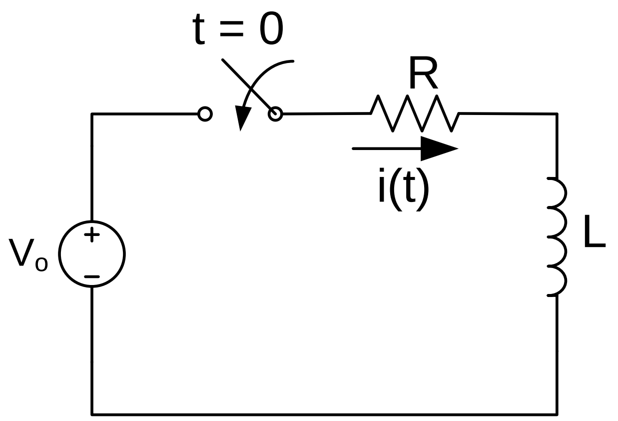
So, capacitor behaves as an open circuit to dc input voltage

Response of RL Series Circuit



Find i(t)

Why RL load?



For t < 0, i(t) = 0

$$Ri + L\frac{di}{dt} = V_0 \quad \text{for } t > 0$$ *and using KVL*

$$\text{or, } L\frac{di}{dt} = V_0 - Ri$$

$$\text{or, } \frac{Ldi}{V_0 - Ri} = dt$$

Integrating, we get

$$\int \frac{L\,di}{V_0 - Ri} = \int dt$$

$$-\frac{L}{R}\ln\left(V_0 - Ri\right) = t + K$$

Integrating, we get

$$\int \frac{L\,di}{V_0 - Ri} = \int dt$$

$$-\frac{L}{R}\ln\left(V_0 - Ri\right) = t + K$$

Setting $i = 0$ at $t = 0$, we have

$$-\frac{L}{R}\ln V_0 = K$$

Hence, we have $$-\frac{L}{R}\ln\left(V_0 - Ri\right) = -\frac{L}{R}\ln V_0 + t$$

$$or,\ -\frac{L}{R}\ln\left(\frac{V_0 - Ri}{V_0}\right) = t$$

$$\text{or, } \ln\left(\frac{V_0 - Ri}{V_0}\right) = -\frac{Rt}{L}$$

$$or,\ \frac{V_0 - Ri}{V_0} = e^{-\frac{Rt}{L}}$$

$$V_0 - Ri = V_0 e^{-\frac{R}{L}t}$$

$$or, \;\; Ri = V_0 - V_0 e^{-\frac{R}{L}t}$$

$$or, \;\; i = \frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t}$$

The expression for the response for all t is

$$\boxed{i(t) = \left(\frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t} \right)}$$

$$\frac{L}{R} = \tau = \text{Time Constant}$$

$$i(t) = \left(\frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{t}{\tau}} \right)$$

Another way to solve

$$Ri + L\frac{di}{dt} = V_0 \quad \text{for } t > 0$$

Solution -

Complimentary Function + Particular Integral

Solution -

Natural Response (i_n) from

$$Ri + L\frac{di}{dt} = 0$$

Forced Response (i_f) from

$$Ri + L\frac{di}{dt} = V_0$$

Complete Solution is

$$i = i_n + i_f$$

For Natural Response

$$Ri + L\frac{di}{dt} = 0$$

$$\text{or, } \frac{di}{dt} + \frac{R}{L}i = 0$$

$$or, \;\; i_n = Ae^{-\frac{R}{L}t}$$

For Forced Response

$$Ri + L\frac{di}{dt} = V_0$$

$$\mathit{But} \quad L\frac{di}{dt} = 0$$

$$\text{So,} \quad i_f = \frac{V_0}{R}$$

$$i(t) = Ae^{-\left(\frac{R}{L}\right)t} + \frac{V_0}{R} \quad t > 0$$

For Finding A   :   Apply the condition

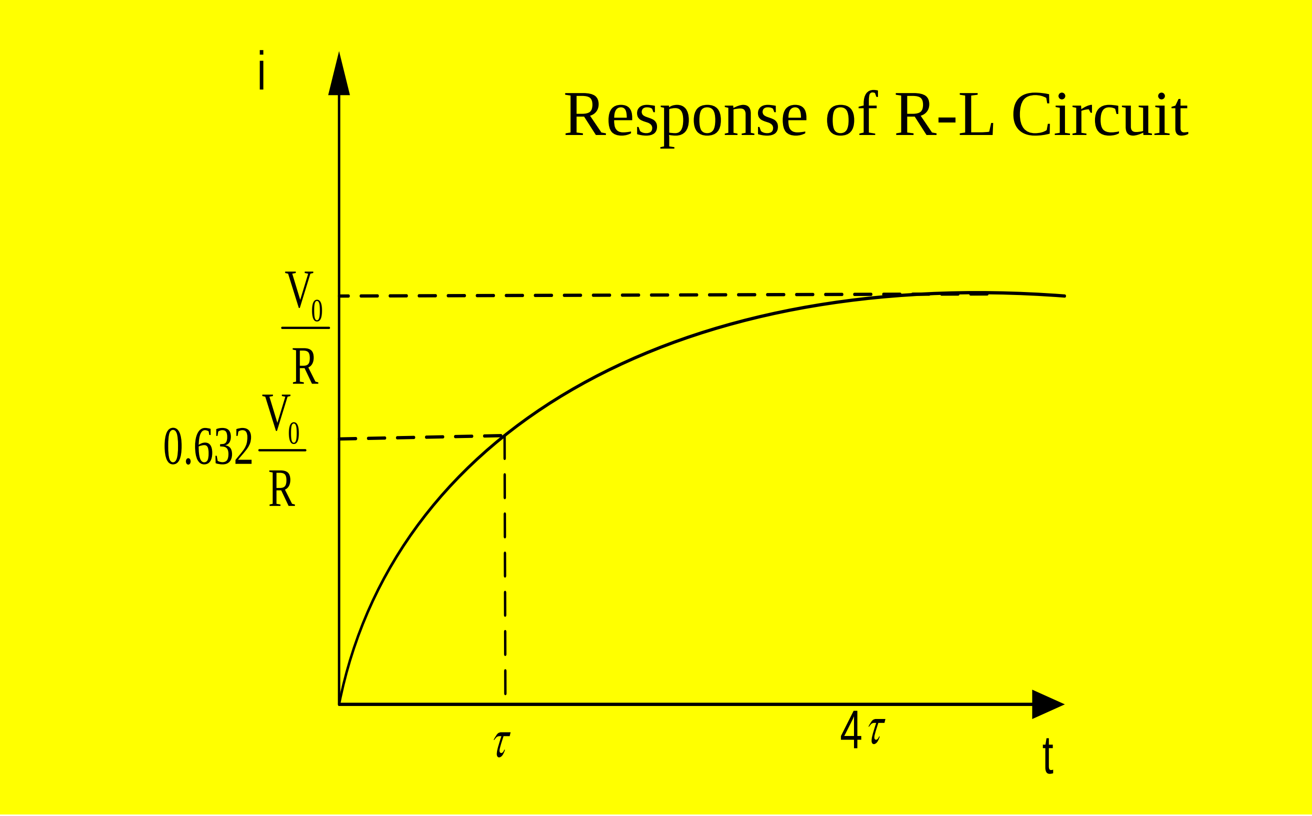
$i(t = 0) = i(t = 0^-) = i(t = 0^+)$

$$0 = A + \frac{V_0}{R}$$

$$A = -\frac{V_0}{R}$$

$$i(t) = \left(\frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t} \right) \quad t > 0$$

$$or, \;\; i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \quad t > 0 \qquad \tau = \frac{L}{R} \quad \text{Time Constant}$$



At t=4τ 98% of final response of i(t) happens