PH 101: Physics I

Module 2: Special Theory of Relativity-Basics

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Recap from the last class

- Galilean Relativity:

1) While space coordinates change when seen from different inertial frames, Time is absolute, and is the same in all inertial frames.
2) Galiean relativity fails to accomodate the idea of the contancy of speed of light (in all inertial frame of references).

- Modification required in Galilean relativity:

1)Speed of light is constant in all frame of references (as required by Maxwell EM waves).

2)Consequently Time is not absolute and should be treated as a variable same as its space counterpart.

3)New ideas are required in order to compare the things in different frame of references.

Need of Special Theory of Relativity (STR)

- 1879: Born in Ulm, Germany.
- 1901: Worked at Swiss patent office.
  - Unable to obtain an academic position.
- 1905: Published 4 famous papers.
  - Paper on photoelectric effect (Nobel prize).
  - Paper on Brownian motion.
  - 2 papers on Special Relativity.
  - Only 26 years old at the time!!
- 1915: General Theory of Relativity published.
- 1933: Einstein left Nazi-occupied Germany.
  - Spent remainder of time at Institute of Advanced Study in Princeton, NJ.
  - Attempted to develop unified theory of gravity and electromagnetism (unsuccessful).

Postulates of STR

With the belief that Maxwell's equations must be valid in all inertial frames, Einstein proposes the following postulates:

The principle of relativity: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.

The constancy of the speed of light: Observers in all inertial systems measure the same value for the speed of light in a vacuum ($c=3 \times 10^8$ m/s)

Relativity of Light

Consider Two frames, S and S'.

S' is moving with speed v along x-axis

Consider Two events: Event 1: Light pulse emitted Event 2: This light pulse detected.

In frame S, let the coordinates of these events are: $(0, 0, \ 0, \ 0)$ and $(t, x, y, \ z)$

The same events as recorded in S': $(0, 0, \ 0, \ 0)$ $(t', x', y', \ z')$

The two frames are synchronised at the first event.

The second event correspond to detecting the light pulse => $x^2 + y^2 + z^2 = c^2 t^2$

$$x'^2 + y'^2 + z'^2 = c'^2 t'^2 = c^2 t'^2$$

Lorentz transformation

Homogeneity of time and space: The properties of empty space are same everywhere and for all time.

In general, x' will be a function of x and t, i.e. $x' = f(x,t)$ so that we would have $dx' = f_x dx + f_t dt$

Homogeneity of space and time implies that

$$x' = a_1 x + a_2\, ct$$

$$ct' = b_1 ct + b_2 x$$

$$y' = y$$

$$z' = z$$

The origin of S' is moving with speed v => $x' = 0 \Rightarrow x = vt$

$$\Rightarrow 0 = a_1 vt + a_2\, ct; \quad a_2 = -\frac{v}{c} a_1$$

Observation:

y and z coordinates (of each event) are the same in both the frames, but x coordinate would be different.

From the point of view of S', the frame S is moving with speed v in the -ve x direction.

The relation between x and x' should be invertible, and should look similar. **Should be a linear relation**

Lorentz transformation

$$x' = a_1(x - vt)$$

$$t' = b_1 x + b_2 t$$

$$y' = y$$

$$z' = z$$

$$x^2 + y^2 + z^2 = c^2 t^2$$

$$x'^2 + y'^2 + z'^2 = c'^2 t'^2 = c^2 t'^2$$

$$a_1^2(x - vt)^2 + y^2 + z^2 = c^2(b_1 x + b_2 t)^2$$

Rearranging the terms gives

$$(a_1^2 - c^2 b_1^2)x^2 + y^2 + z^2 - 2(va_1^2 + c^2 b_1 b_2)xt = (c^2 b_2^2 - v^2 a_1^2)t^2$$

Equating this with $x^2 + y^2 + z^2 = c^2 t^2$ gives

$$\boxed{\begin{array}{l} c^2 b_2^2 - v^2 a_1^2 = c^2 \\ a_1^2 - c^2 b_1^2 = 1 \\ va_1^2 + c^2 b_1 b_2 = 0 \end{array}}$$

$$\boxed{\Rightarrow a_1 = b_2 = 1/\sqrt{1 - v^2/c^2} \;=\; \gamma \text{ and } b_1 = -\tfrac{v}{c^2}/\sqrt{1 - v^2/c^2} = -\gamma\beta/c \text{ with } \beta = v/c}$$

Lorentz transformation

Lorentz transformation: relating the coordinates and time in two different coordinates.

$$x' = \gamma(x - \beta x_0); \qquad y' = y; \quad z' = z$$

$$x'_0 = \gamma(x_0 - \beta x)$$

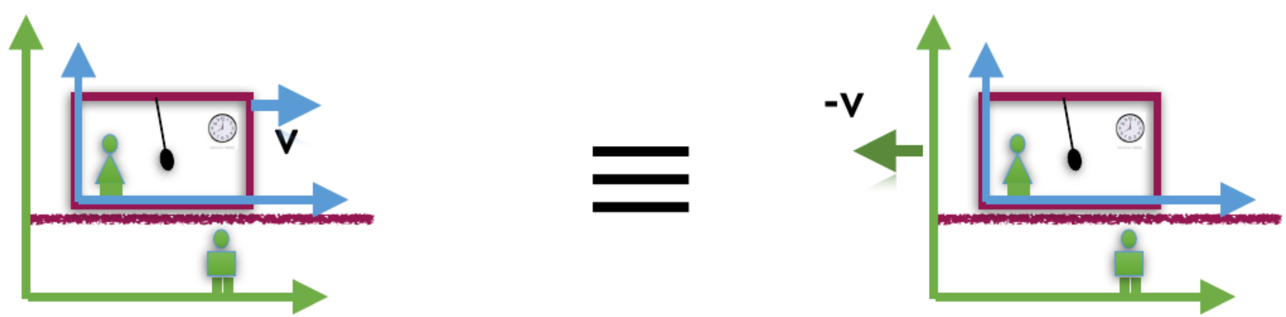
We introduce $x_0 = ct$

Inverting the relation:

$$x = \gamma(x' + \beta x'_0); \qquad y' = y; \quad z' = z$$

$$x_0 = \gamma(x'_0 + \beta x')$$

Exercise



Variation of relativistic factor γ with speed

Recall $\beta = v/c < 1$ for all observers.

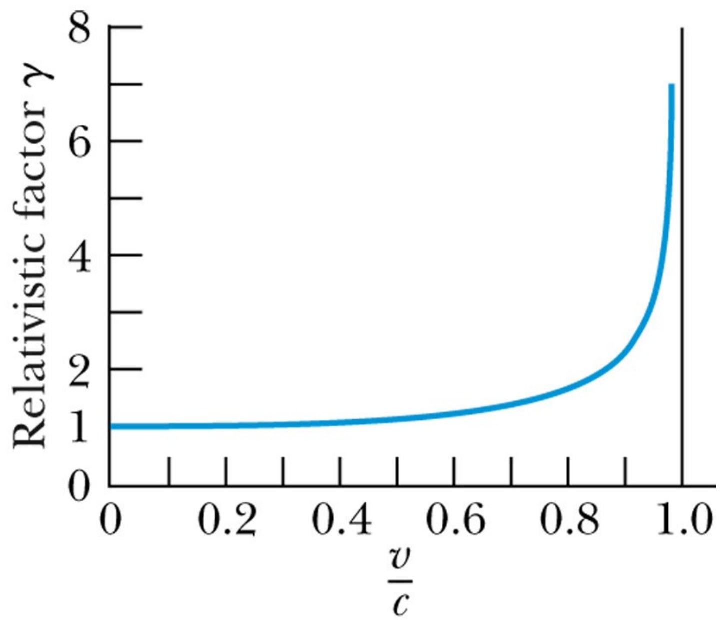
$\gamma \geq 1$: equals 1 only when $v = 0$.

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \beta x/c)$$

Graph of β:
(note $v \neq c$)



1) If $v << c$, i.e., $\beta \approx 0$ and $\gamma \approx 1$, we see these equations reduce to the familiar Galilean transformation.

2) Space and time are now not separated.

Examples (Home work)

Consider frame S and another frame S'.

Their origins coincide at $t = t' = 0$. Their axes are parallel to each other. The origin of S' moves with a constant speed $v = 10\,km/s$ along the x-axis.

An event occurs at $x = 1\,m, \quad y = 2\,m, \quad z = 10\,m$ at $t = 8\,s$ in S.

What are its coordinates in S'?

Examples (Home work)

Consider frame S and another frame S'.

Their origins coincide at $t = t' = 0$. Their axes are parallel to each other. The origin of S' moves with a constant speed $v = 0.2c$ along the x-axis.

An event occurs at $x = 1m, \quad y = 2m, \quad z = 10\ m$ at $t = 8s$ in S.

What are its coordinates in S'?

Examples (Home work)

Consider frame S and another frame S'.

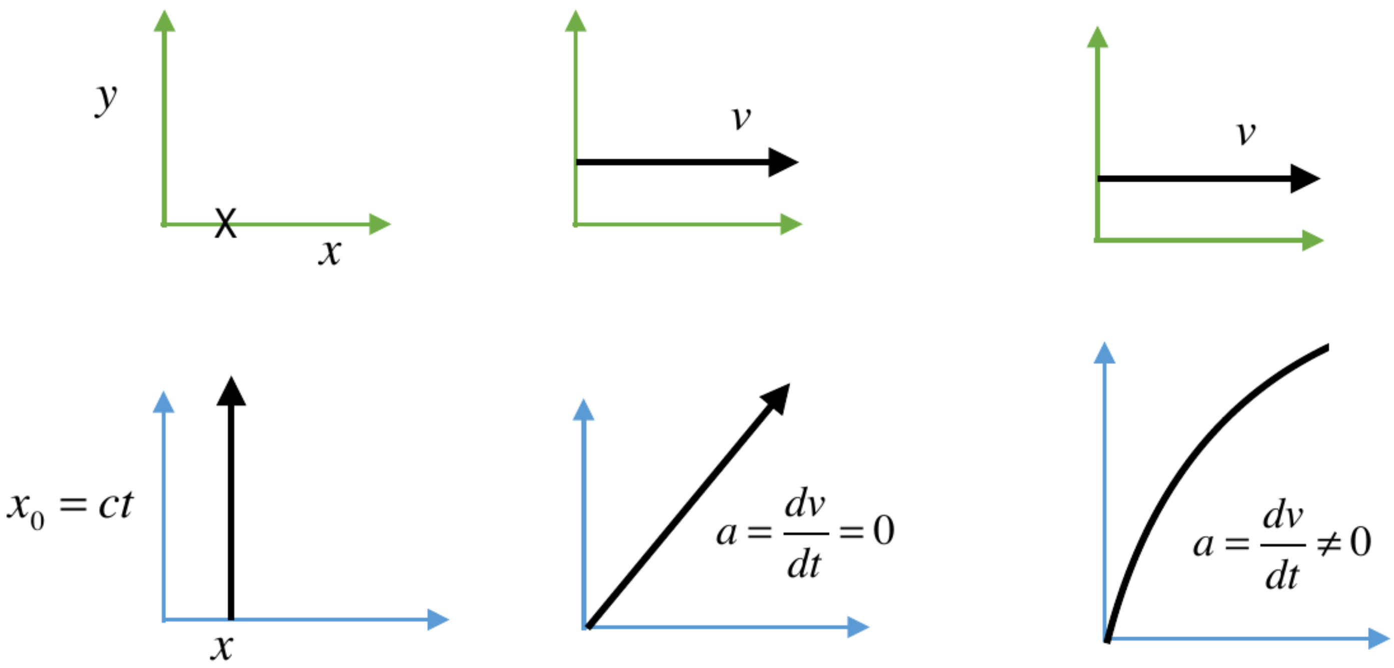
Their origins coincide at $t = t' = 0$. Their axes are parallel to each other. The origin of S' moves with a constant speed $v = 0.7c$ along the x-axis.

An event occurs at $x = 1m,\quad y = 2m,\quad z = 10\ m$ at $t = 8\,s$ in S.

What are its coordinates in S'?

World Line

Meaning of trajectory in the usual sense is lost in STR, as the space and time are interlinked.
The concept of **world line** is introduced instead.



Path of particles
in the usual sense.

World lines in STR

Object at rest

Moving with const
speed along x

Accelerating
object

Characetrization of events

Consider the invariant interval (between two events, E1 and E2): $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$

Three possibilities:

$$ds^2 = 0$$

Events are called light-like events.

For example, E1 is the flashing of light at $(0, 0, 0, 0)$
and E2 is detecting it at (t, x, y, z)

Distance covered by the light pulse in time $t = ct$

This is equal to the spatial distance between the points.

$$ds^2 > 0$$

Events are called time-like events.

For example, E1 is firing a bullet at $(0, 0, 0, 0)$
and E2 is it hitting a target at (t, x, y, z)

Distance covered by the bullet in time $t = vt < ct$

The spatial distance between the points $vt = \sqrt{x^2 + y^2 + z^2}$

$$ds^2 < 0$$

Events are called space-like

This cannot be the case with normal events.
Possible for particles moving faster than c

Such particles with $v > c$ are called Tachyons

Examples

Consider two time-like events E1: (t_1, x_1, y_1, z_1) E2: (t_2, x_2, y_2, z_2)

with $dt = t_2 - t_1, \quad dx = x_2 - x_1, \quad dy = y_2 - y_1, \quad dz = z_2 - z_1$

Since the interval is the same when seen from different inertial frames,

$$ds^2 = c^2\, dt^2 - dx^2 - dy^2 - dz^2$$
$$= c^2\, dt'^2 - dx'^2 - dy'^2 - dz'^2 = ds'^2$$

The events will remain time-like in all inertial frames.

Similarly, light-like and space-like events also would remain so when seen from any inertial frame.

Similarly, light-like and space-like events also would remain so when seen from any inertial frame.

Show explicitly that $ds^2 = ds'^2$

Illustration of World line