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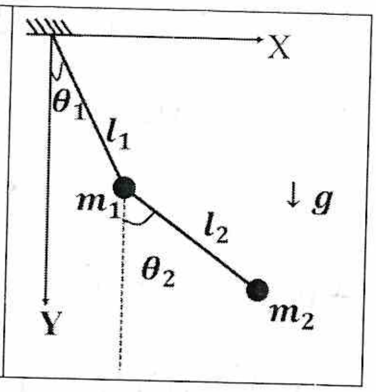
Q1 **[5 marks]** Consider a double pendulum as shown in the figure. Assume that both strings (of lengths l_1 and l_2) are inextensible and massless. Acceleration due to gravity (g) acts downwards.

Choosing the appropriate generalized coordinates from the variables marked in the figure, (i) find the kinetic and potential energies using generalized coordinates and generalized velocities. (ii) Write down the Lagrangian for the system. (iii) Obtain the equation of motion corresponding to any one generalized coordinate.



Sol^n : Number of generalised coordinates $= 2$

Let (θ_1, θ_2) be the generalised coordinates.

Let (x_1, y_1) and (x_2, y_2) be the cartesian coordinates of m_1 and m_2 respectively.

Assume $U = 0$ at origin.

$\therefore\ x_1 = l_1 \sin\theta_1$, $y_1 = l_1\cos\theta_1$, $x_2 = l_1\sin\theta_1 + l_2\sin\theta_2$,

$y_2 = l_1\cos\theta_1 + l_2\cos\theta_2$

$\therefore\ \dot{x}_1 = l_1\cos\theta_1\dot{\theta}_1$, $\dot{y}_1 = -l_1\sin\theta_1\dot{\theta}_1$, $\dot{x}_2 = l_1\cos\theta_1\dot{\theta}_1 + l_2\cos\theta_2\dot{\theta}_2$

$\dot{y}_2 = -l_1\sin\theta_1\dot{\theta}_1 - l_2\sin\theta_2\dot{\theta}_2$ —— (1)

1/2

Now, $T = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2)$

substituting the values from eq^n (1), we get

$$T = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\left[l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)\right] \quad \text{—— (2)}$$

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$U = -m_1 g y_1 - m_2 g y_2$

$$\therefore U = -m_1 g l_1\cos\theta_1 - m_2 g(l_1\cos\theta_1 + l_2\cos\theta_2) \quad \text{—— (3)}$$

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$\therefore L = T - U$

$$= \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\left[l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)\right] + m_1 g l_1\cos\theta_1 + m_2 g(l_1\cos\theta_1 + l_2\cos\theta_2) \quad \text{—— (4)}$$

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Now, we obtain equation of motion for the generalised coordinate θ_2.

$\therefore$ From the Euler-Lagrange equation, we have

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1 - \theta_2)$$ ½

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1 [\sin(\theta_1 - \theta_2)](\dot{\theta}_1 - \dot{\theta}_2)$$

$$\frac{\partial L}{\partial \theta_2} = + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_2 g l_2 \sin\theta_2$$

$\therefore$ Now, substituting in the Lagrange's eqⁿ, we get

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 g l_2 \sin\theta_2 = 0$$

$$\therefore m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + m_2 g l_2 \sin\theta_2 = 0$$

$$\therefore \boxed{l_2 \ddot{\theta}_2 + l_1 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - l_1 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + g \sin\theta_2 = 0}$$

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