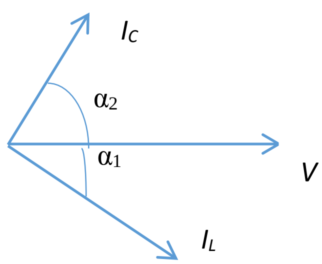
SOLUTIONS

Solution of pre-tutorial:

For the same source, the impedance of LHS figure is $Z_1 = 4+j2\omega$,
whereas that of RHS is either $Z_2 = j8\omega L/(8+j\omega L)$ or $Z_2 = 8/(j8\omega C+1)$.
The circuits are equivalent to each order when the source is subjected to the same impedance i.e. $Z_1 = Z_2$.
In the case of an inductor, $4+j2\omega = j8\omega L/(8+j\omega L)$……………………………(1)
Cross-multiplication of (1) yields $j8\omega L = 32-2\omega^2 L+j(16\omega+4\omega L)$…………..(2)
Equating real and imaginary part of (2) gives $\omega = 2$ rad/s and $L = 4$ H.
In the case of a capacitor, $4+j2\omega = 8/(j8\omega C+1))$…………(3)
Solution of (3) yields a negative value for C.
Therefore, the equivalent circuit will have a sinusoidal source with $\omega = 2$ rad/s and the 8 **Ω** resistor in parallel with the inductor of 4 H.

Solution of problem 2:

Approximate Phasor diagram can be given as follows:



$\alpha_1 = \tan^{-1}(\omega L/R_1)$ and $\alpha_2 = \tan^{-1}(1/(\omega R_2 C))$

For the two currents to be in quadrature, $\alpha_1+\alpha_2 = \pi/2 \Rightarrow \tan^{-1}(\omega L/R_1) + \tan^{-1}(1/(\omega R_2 C)) = \pi/2$ ……(1)

- ⇨ $\tan[\tan^{-1}(\omega L/R_1) + \tan^{-1}(1/(\omega R_2 C))] = \tan(\pi/2)$
- ⇨ $[(\omega L/R_1) + 1/(\omega R_2 C)] / [1-L/(R_1 R_2 C)] = 1/0$ …………………………………………………...(2)

Cross-mutiplication and further simplification of (2) gives the condition: $R_1 R_2 = L/C$.

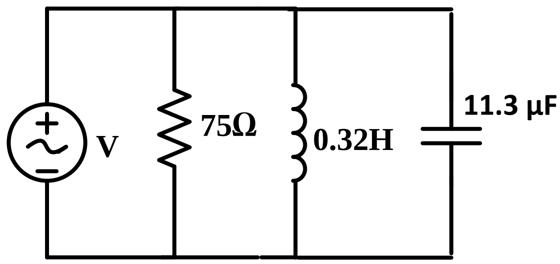
Solution of problem 3:

Given, $\mathbf{V} = 240\sqrt{2}\angle 0^0 = V_m \angle\theta$ and $\mathbf{Z} = (75 \parallel j32\pi) = 48 + j\,36 = 60\angle 36.87^0$ (approx.)
Then, $\mathbf{I} = \mathbf{V}/\mathbf{Z} = 4\sqrt{2}\angle{-36.87^0} = I_m \angle\phi$ and $(\theta-\phi) = 36.87^0$.
The power factor is $\cos(\theta-\phi) = \cos(36.87^0) =$ **0.8 lagging** (as the pf angle is +ve)

(1) Real power = $P = (½)\, V_m I_m \cos(\theta-\phi) = (½)\, 240\sqrt{2} \times 4\sqrt{2} \times \cos(36.87^0) = 768$ W
Also, using RMS values of voltage and current, $P = V_{rms} \times I_{rms} \times \cos(36.87^0) = 240 \times 4 \times 0.8 = 768$ W
Also, $P = V_{rms}^2/R = 240^2/75 = 768$ W

(2) Reactive power = Q = V_{rms}^2 / X_L = V_{rms} x I_{rms} x sin(36.87^0) = 576 VAR
(Note: X_L = 100 Ω, approx)
Also, reactive power = P tan(θ-ϕ) = 576 VAR

In the following circuit, a capacitor of C=11.3 μF is connected in parallel with the source.



Reactive power of the capacitor, Qc = - V_{rms}^2/Xc = **-** 204.04 VAR

Net reactive power experienced by the source = 576 – 204.04 = 371.96 VAR
Net real power experienced by the source = 768 W
Using the power triangle,
tan (θ-ϕ) = 371.96/768 => (θ-ϕ) = 25.842^0 (pf angle is positive)

Therefore, power factor = cos(25.842^0) = 0.9 lagging

Solution of problem 4:

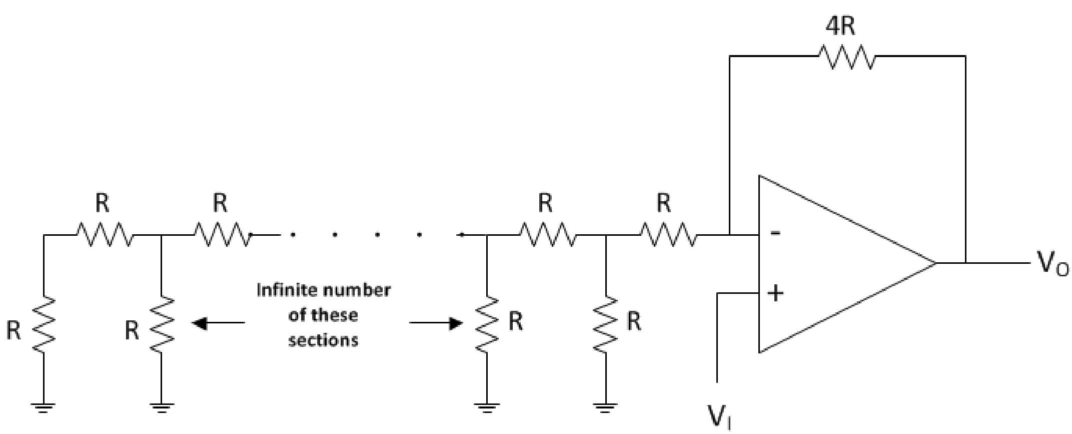
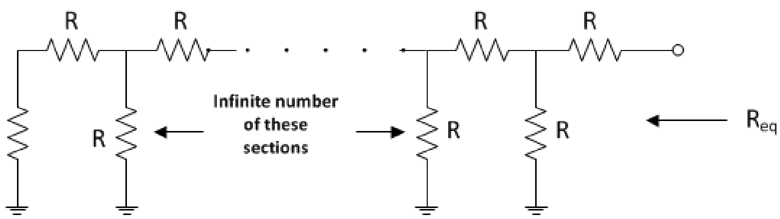
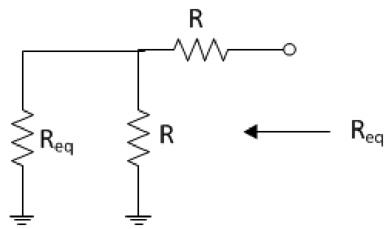


Fig. 3



or, equivalently



$$\left\{ \begin{array}{l} R_{eq} = R + \dfrac{RR_{eq}}{R + R_{eq}} \quad \text{or} \quad R_{eq}^2 + RR_{eq} = R^2 + RR_{eq} + RR_{eq} \\ R_{eq}^2 - RR_{eq} - R^2 = 0 \\ \text{Solving,} \quad R_{eq} = \dfrac{R + \sqrt{R^2 + 4R^2}}{2} = \left(\dfrac{1+\sqrt{5}}{2} \right) R = 1.618R \end{array} \right\}$$

$R_{eq}=(R_{eq}||R)+R$

$$\frac{V_I}{R_{eq}} = \frac{V_O - V_I}{4R}$$

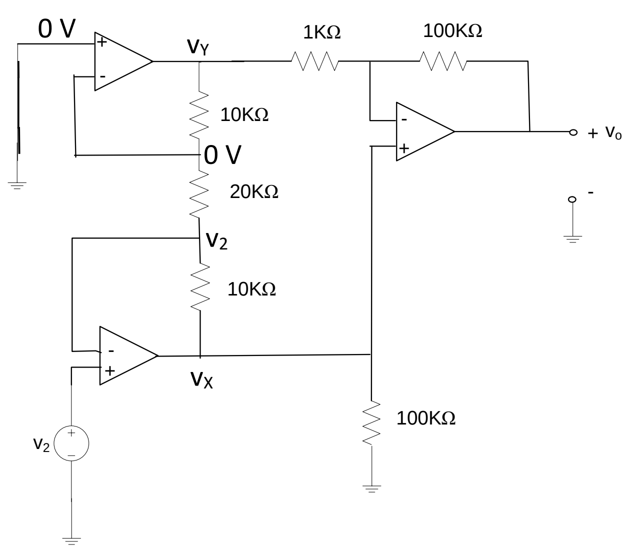
$$\frac{V_O}{V_I} = 1 + \frac{4R}{R_{eq}} = 1 + \frac{4}{1.618} = 3.472$$

A_V=3.472

Solution of problem 5:

We can solve this circuit directly but it would be interesting to consider an unusual way of solving it through superposition.

Setting v_1=0 (i.e. grounding that source), the circuit becomes as shown.

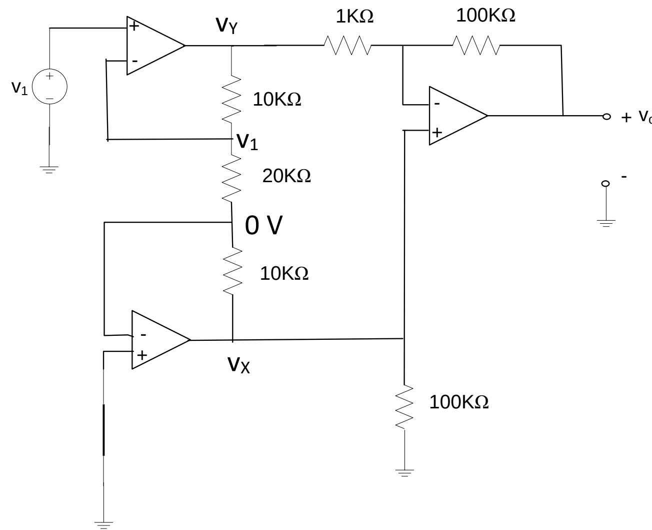


$$\frac{v_x - v_2}{10} = \frac{v_2}{20} \quad \Rightarrow \quad v_x = 1.5v_2$$

$$\frac{v_y}{10} = -\frac{v_2}{20} \quad \Rightarrow \quad v_y = -0.5v_2$$

$$v_o(\,for\ v_1 = 0) = -100v_y + (1+100)v_x = 201.5v_2$$

Now setting $v_2=0$ (i.e. grounding that source), the circuit becomes as shown



$$\frac{v_x}{10} = -\frac{v_1}{20} \quad \Rightarrow \quad v_x = -0.5v_1$$

$$\frac{v_y - v_1}{10} = \frac{v_1}{20} \quad \Rightarrow \quad v_y = 1.5v_1$$

$$v_o = -100v_y + (1+100)v_x = -200.5v_1$$

Combining the two, we get –

$v_o = -200.5v_1 + 201.5v_2$