Department of Mathematics
Indian Institute of Technology Guwahati

MA 101: Mathematics I

Tutorial Sheet-4

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1. Examine whether the following series are convergent.

   (a) $\sum_{n=1}^{\infty} \frac{n^n}{2^{n^2}}$

   (b) $\sum_{n=1}^{\infty} (\frac{n}{n+1})^{n^2}$

   (c) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$

2. Examine whether the series $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$ is convergent.

3. Examine whether the following series are conditionally convergent.

   (a) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2+1} - n)$

   (b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+(-1)^n}$

   (c) $\sum_{n=1}^{\infty} (-1)^n \frac{a^2+n}{n^2}$, where $a \in \mathbb{R}$

4. Find all $x \in \mathbb{R}$ for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2^n n^2}$ converges.

5. Show that the series $\sum_{n=1}^{\infty} \frac{a^n}{a^n+n}$ is convergent if $0 < a < 1$ and is not convergent if $a > 1$.

6. If $\alpha\ (\neq 0) \in \mathbb{R}$, then show that the series $\sum_{n=1}^{\infty} (-1)^n \sin(\frac{\alpha}{n})$ is conditionally convergent.

7. For $p \in \mathbb{R}$, the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p}$ is convergent iff $p > 0$.

8. (Rearrangement of series). If $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = s$, then prove that $1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \cdots = \frac{3}{2}s$.