PH 101: Physics I

Module 3: Introduction to Quantum Mechanics

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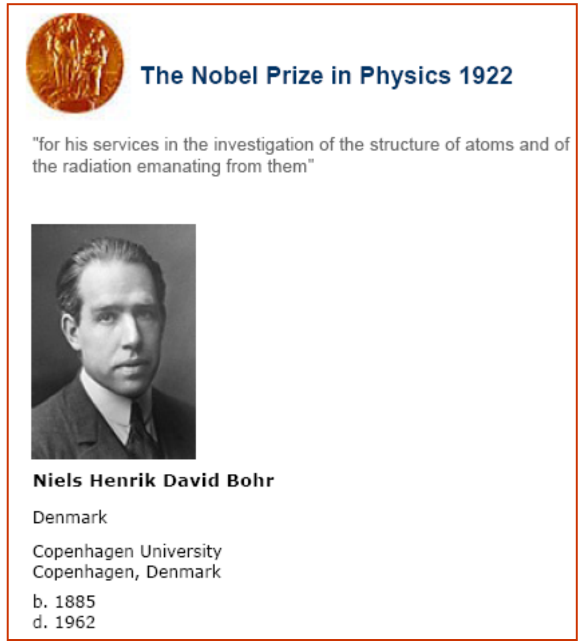


Bohr, Niels Henrik David (1885 -- 1962), Danish physicist.

Bohr's model of atomic structure, 1913

The electron's _orbital angular momentum_ is _quantized_

$$\mathbf{L} = n \cdot \hbar = n \cdot \frac{h}{2\pi}$$

$$E_n = \frac{-13.6\ \mathrm{eV}}{n^2} \qquad = \frac{-m_e q_e^4}{8h^2\epsilon_0^2}\frac{1}{n^2}$$

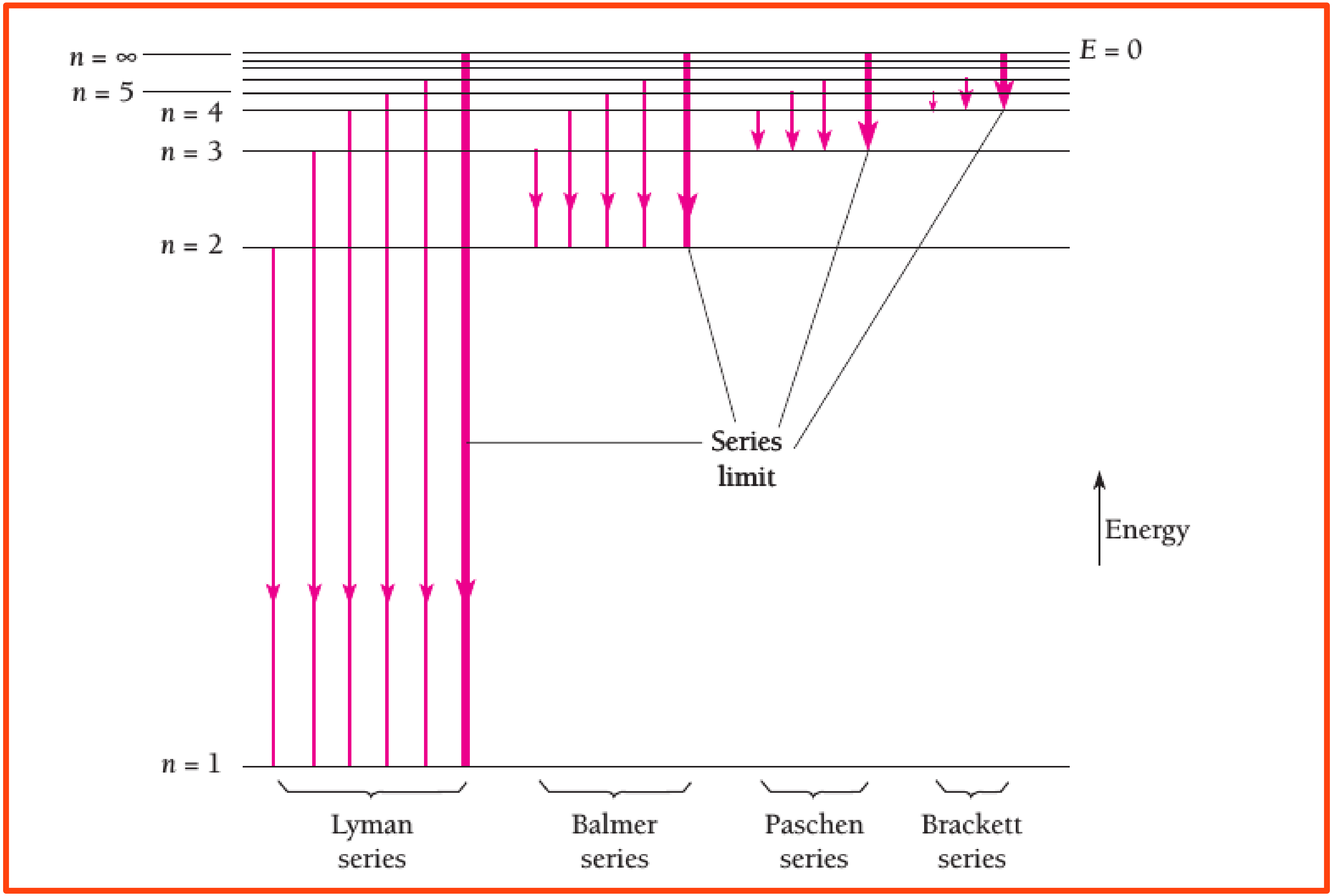
$$E = E_i - E_f = \frac{m_e e^4}{8h^2\epsilon_0^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = \frac{m_e e^4}{8ch^3\epsilon_0^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The theory predicts that _electrons travel in discrete orbits around the atom's nucleus_, with the chemical properties of the element being largely determined by the number of electrons in each of the outer orbits

The idea that an electron could _drop_ from a _higher-energy orbit to a lower one_, _emitting a photon_ (light quantum) of discrete energy (this became the basis for quantum theory).

Atomic transitions



In the limit of very large ‘n’ energy spectrum becomes continuous known *Bohr’s correspondence principle*. We will discuss more about it upcoming lectures.

Atomic transitions: Examples

An electron collides with a hydrogen atom in its ground state and excites it to a state of $n = 3$. How much energy was given to the hydrogen atom in this inelastic (KE not conserved) collision?

Solution: The energy change of a Hydrogen atom that goes from an initial state of quantum number n_i to a final state of quantum number n_f is

$\Delta E = E_f - E_i = \dfrac{-13.6\ eV}{n_f^2} - \dfrac{-13.6\ eV}{n_i^2}$. Here $n_i = 1$ and $n_f = 3$. So

$$\Delta E = -13.6\left(\frac{1}{3^2} - \frac{1}{1^2}\right)eV = 12.1eV$$

Hydrogen atoms in states of high quantum number have been created in the laboratory and observed in space. They are called Rydberg atoms. (a) Find the quantum number of the Bohr orbit in a hydrogen atom whose radius is $0.0100mm$. (b) What is the energy of a hydrogen atom in this state?

Solution: $r_n = 1.00 \times 10^{-5}m$

$n = \sqrt{\dfrac{r_n}{a_0}} = 435$. Therfore, $E_n = -13.6\ eV/(435)^2 = -7.19 \times 10^{-5}\ \ eV$

Evolution of Quantum Mechanics

Stefan (1879)
Wien (1893)
Rayleigh (1900)
&
Jeans (1905)
(Thermal radiation)

Plank(1900)
$$E_n = nh\nu$$

Einstein(1905)
Photons

Quantum Mechanis:
Born(1926), Heisenberg(1925),
Schroedinger(1926), Dirac(1927)

Balmer (1884)
Rydberg (1890)
Zeeman (1896)
(Spectral lines)

Thomson (1897)
(Electron)

Rutherford (1911)
(Nucleus)

Bohr(1913)
Model of atoms

De Broglie(1924)
Wave-particle duality

Bohr's model of the Hydrogen atom

Postulates:

Electron moves in a circular orbit about the nucleus under the influence of the electrostatic attraction.

The orbits are stationary in nature, i.e., the total energy of the electron moving in the orbit remains constant, so it does not emit electromagnetic energy.

Radiation can only take place when a transition is made between the allowed energy levels.

The angular of an electron moving in the stationary orbit is not continuous but can have the discrete values (Angular momentum is quantized).

Bohr's model of the Hydrogen atom

Circular Motion: $$\frac{Ze^2}{(4\pi\epsilon_0)r^2} = \frac{mv^2}{r}$$

Quantization of angular momentum (Bohr's hypothesis):

$$L = mvr = n\hbar, \text{ where,} \qquad n = 1, 2, 3, ...$$

Resulting the quantization of velocity and radius of the orbit:

$$v_n = \frac{Ze^2}{(4\pi\epsilon)\hbar n} \qquad\qquad r_n = \frac{(4\pi\epsilon)\hbar^2 n^2}{Zme^2}$$

Total Energy: E= T(K.E.)+V(Potential energy)

Using the quantization of velocity and radius we have

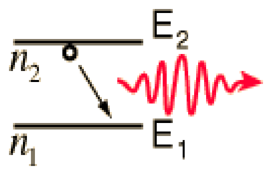
$$V = -\frac{Ze^2}{(4\pi\epsilon_0)r} = -\frac{m}{\hbar^2}\left(\frac{Ze^2}{4\pi\epsilon_0}\right)^2\frac{1}{n^2} = -2T$$ and $$T = \frac{1}{2}mv^2 = \frac{m}{2\hbar^2}\left(\frac{Ze^2}{4\pi\epsilon_0}\right)^2\frac{1}{n^2}$$

Finally we get the quantized energy as

$$E_n = -\frac{m}{2\hbar^2}\left(\frac{Ze^2}{4\pi\epsilon_0}\right)^2\frac{1}{n^2} = -13.6\ Z^2/n^2\quad eV \qquad\qquad n = 1, 2, 3, ...$$

Summary: Bohr's theory

Electron Transitions



A downward transition involves emission of a photon of energy:

$$E_{photon} = h\upsilon = E_2 - E_1$$

Given the expression for the energies of the hydrogen electron states:

$$h\upsilon = \frac{2\pi^2 m e^4}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = -13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]\text{eV}$$

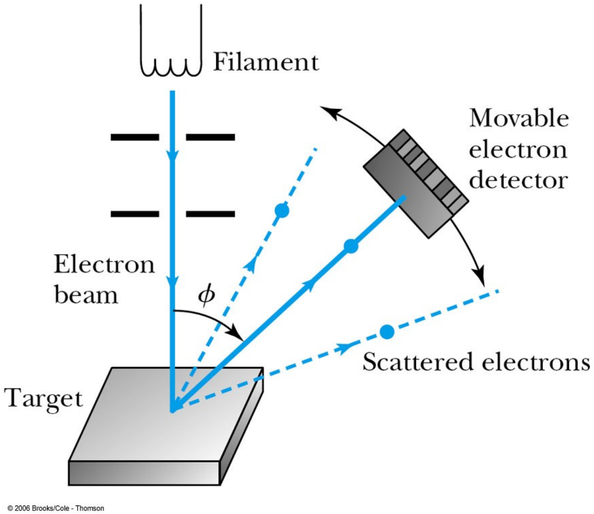
Failures of the Bohr Model

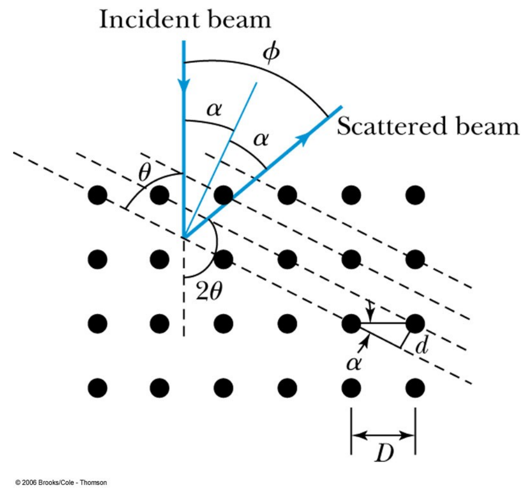
It fails to provide any understanding of why certain spectral lines are brighter than others. There is no mechanism for the calculation of transition probabilities.

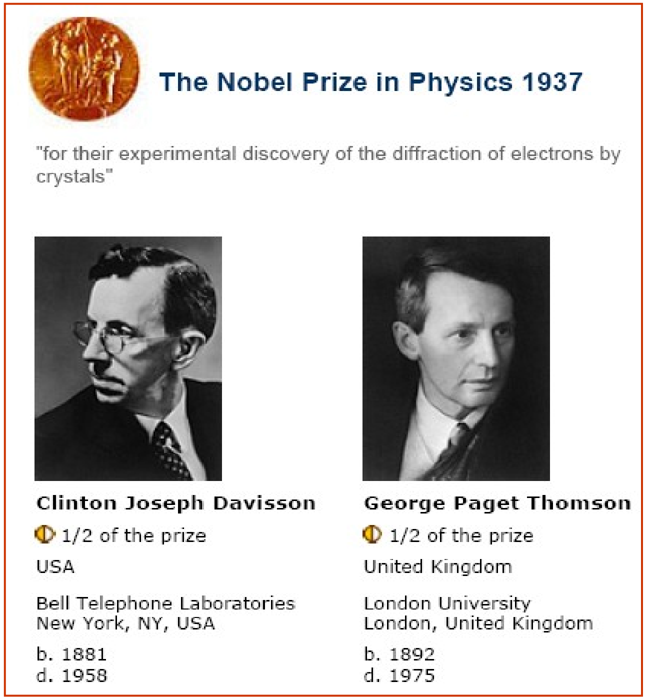
The Bohr model treats the electron as if it were a miniature planet, with definite radius and momentum. This is in direct violation of the uncertainty principle which dictates that position and momentum cannot be simultaneously determined.

The Bohr model gives us a basic conceptual model of electrons orbits and energies. The precise details of spectra and charge distribution must be left to quantum mechanical calculations, as with the Schrödinger equation.

Electron Scattering –> Diffraction.







In 1927, Davisson and Germer experimentally observed that electrons were diffracted much like x rays in nickel crystals, just trying to continue with prior research, no knowledge of De Broglie's hypothesis at that time.

Around the same time George P. Thomson, son of J. J. Thomson, knew about De Broglie's hypothesis and set out to prove (or disprove) it, build the first high energy electron diffraction camera reported seeing the effects of electron diffraction in transmission experiments.

First confirmation of wave nature of particles

This led to the immediate Nobel prize to de Broglie in 1929 and 10 years later to Davisson and Thomson (not to Germer)

Paritcle nature of Wave (Photon)

Compton Effect:

From photoelectric effect we learned that light itself is quantized as proposed by Einstein. This idea was similar to the quantization of oscillator energies in the case of blackbody radiation as proposed by Plank.

The photoelectric effect would only give us the maximal kinetic energy of the photoelectrons, a more direct proof that light wave consists of small small packets of photons with discrete energy was needed !!!

A direct collision experiment of sufficiently energetic light particles with some small particle that possesses mass would establish this in a more concrete way.

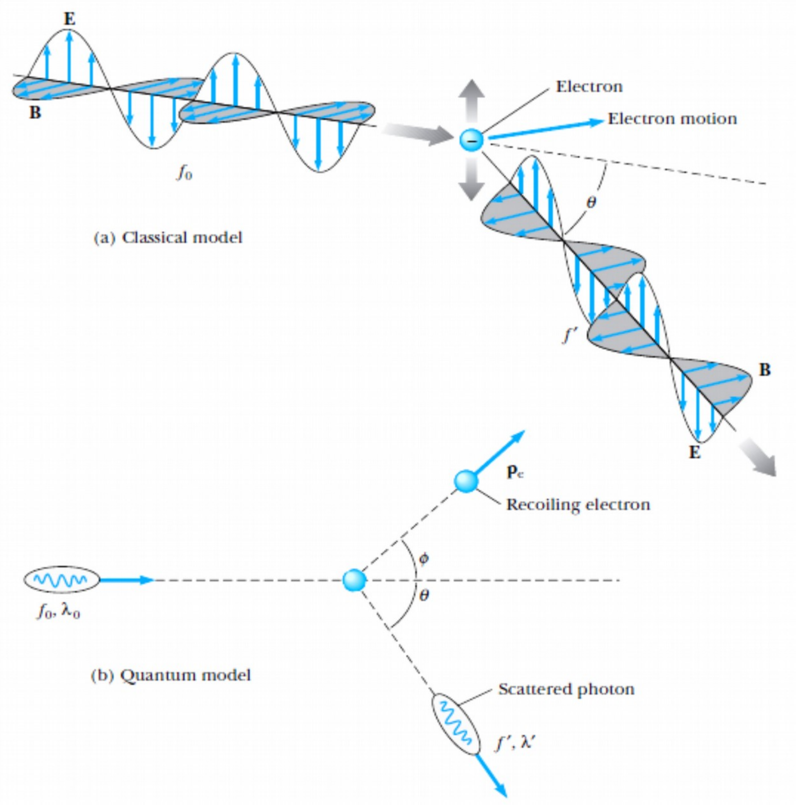


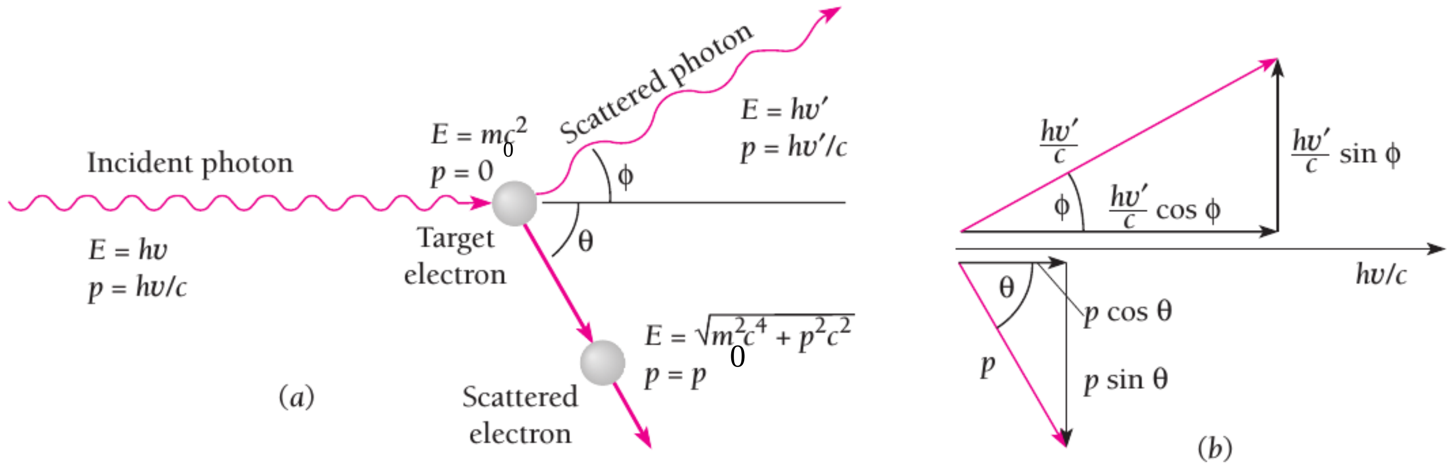
Figure 3.22 X-ray scattering from an electron: (a) the classical model, (b) the quantum model.

Classical physics says: The E vector would disturb the electron and after loosing some energy and a new wave of longer wavelength will be created which will propagate outward in a spherical manner.

If light has also particle properties: A photon should collide with the electron in a relativistic way and the total energy and momentum needs to be conserved as the light particle moves with the speed of light.

Paritcle nature of Wave (Photon)

Compton Effect:



$$\text{Loss in photon energy} = \text{gain in electron energy}$$
$$h\nu - h\nu' = \text{KE}$$

Photon momentum $$p = \frac{E}{c} = \frac{h\nu}{c}$$

Along the original photon direction

$$\text{Initial momentum} = \text{final momentum}$$

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos \phi + p \cos \theta$$

Along the perpendicular direction

$$\text{Initial momentum} = \text{final momentum}$$

$$0 = \frac{h\nu'}{c} \sin \phi - p \sin \theta$$

$$E = \text{KE} + m_0c^2$$
$$E = \sqrt{m_0^2c^4 + p^2c^2}$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos \phi)}$$

Excercise

Nobel prize 1927 for Arthur Holly Compton

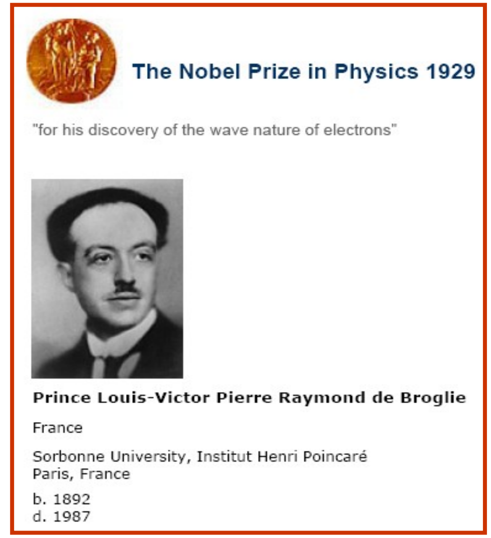
The steps forward...

1. J. J. Thomson's experiment	Thomson's measurements with cathode rays showed that the same particle (the electron), with e/m about 2000 times that of ionized hydrogen, exists in all elements.	
2. Quantization of electric charge	$e = 1.60217653 \times 10^{-19}$ C	
3. Blackbody radiation		
Stefan-Boltzmann law	$R = \sigma T^4$	**3-4**
Wein's displacement law	$\lambda_m T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$	**3-5**
Planck's radiation law	$u(\lambda) = \dfrac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$	**3-18**
Planck's constant	$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$	**3-19**
4. Photoelectric effect	$eV_0 = hf - \phi$	**3-21**
5. Compton effect	$\lambda_2 - \lambda_1 = \dfrac{h}{mc}(1 - \cos\theta)$	**3-25**
6. Photon-matter interaction	The (1) photoelectric effect, (2) Compton effect, and (3) pair production are the three ways of interaction.	

Several other experimental evidences can be added to the list at this point.

The main lesson we learnt that the electromagnetic radiation behaves like particles and particles behave like wave.

Wave-Particle Duality



Louis de Broglie [1892 – 1987] was a French physicist who made ground breaking contributions to quantum theory. In his 1924 PhD thesis he postulated the wave nature of electrons and suggested that all matter has wave properties. This concept is known as the de Broglie hypothesis, an example of wave-particle duality, and forms a central part of the theory of quantum mechanics.

De Broglie won the Nobel Prize for Physics in 1929, after the wave-like behaviour of matter was first experimentally demonstrated in 1927.

"The fundamental idea of [my 1924 thesis] was the following: The fact that, following Einstein's introduction of photons in light waves, one knew that light contains particles which are concentrations of energy incorporated into the wave, suggests that all particles, like the electron, must be transported by a wave into which it is incorporated... My essential idea was to extend to all particles the coexistence of waves and particles discovered by Einstein in 1905 in the case of light and photons." "With every particle of matter with mass m and velocity v a real wave must be 'associated'", related to the momentum by the equation:

$$\lambda = \frac{h}{p} \quad where \quad p = \frac{m_0\,\mathrm{v}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}$$

Wave Particle Duality

Momentum of a photon

Relativity: $E = mc^2 = \sqrt{p^2c^2 + m_0^2c^4}$

$$p = \frac{E}{c}$$

$$\lambda = \frac{h}{p}$$

For photons

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\frac{h}{\lambda} = \frac{E}{c}$$

Photolectric effect: $E = h\nu = \frac{hc}{\lambda}$

Is it valid for particles?

Wavelength-Energy relation

de Broglie *matter wave hypothesis* (1924):

All matter has a wave-like nature (*wave-particle duality*) and that the wavelength and momentum of a particle are related by the simple relation.

de Broglie's idea and Bohr's postulate

The de Broglie idea given immediate explanation of Bohr's postulate for electron moving in a circular orbit.

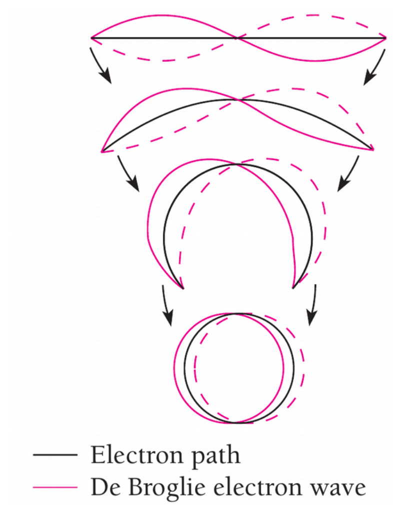
If the state is stationary and the electron possess wave nature and if the electron does not overlap with itself after a complete cycle then a whole number of waves should be accomodated in the circle.

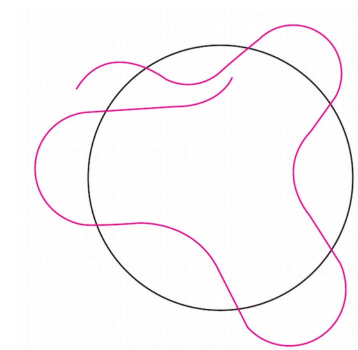
$$n\lambda = 2\pi r$$

$$\text{Now since } \lambda = \frac{h}{p},$$

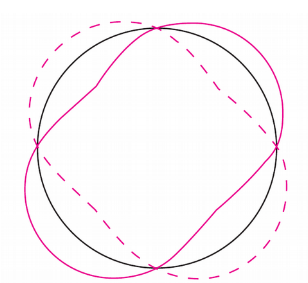
$$L = rp$$

$$\therefore \quad L = \frac{n\lambda}{2\pi}\frac{h}{\lambda} = \frac{nh}{2\pi} = n\hbar$$

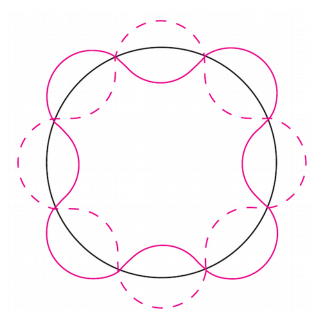




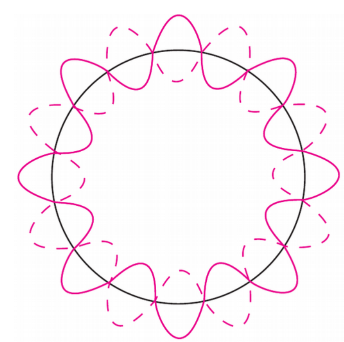
Fractional wavelength will lead to destructive interference and hence the system will not exist.



Circumference = 2 wavelengths



Circumference = 4 wavelengths



Circumference = 8 wavelengths

Modes of vibration of a wire look where a whole number of waves gets fitted into a circle.

Getting a feel for the wave nature of matter

Q. Find the wavelength of the matter wave associated with a tennis ball moving at 110 km/h

A. First, $v \ll c$ so that, $p \approx m_0 v$ Take the mass of the tennis ball to be $m_0 \approx 58\ g$

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ J.s}{58 \times 10^{-3}\ kg \times 110 \times 10^{3} m/(3600 s)}$$

$$= 3.73881 \times 10^{-34} m$$

Compare this with the size of the hydrogen atom

$$= 53 \times 10^{-12}\ m$$

The de Broglie wavelength of a tennis ball can be taken to be zero. This means a tennis ball has negligible wave-like properties.

Q. Find the wavelength of the matter wave associated with an electron moving at 110 km/h

A. First, $v \ll c$ so that, $p \approx m_0 v$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{(m_0 c^2)(\frac{v}{c})}$$

$$hc = 1.24\ eV.\mu m$$

$$m_0 c^2 = 0.511\ MeV$$

$$\frac{v}{c} = 10^{-7}$$

$$\lambda = 24.3\ \mu m$$

The typical size of a bacterium is 6 µm.

1 eV = 1.6 X 10^{-19} J

Getting a feel for the particle nature of waves

Q. Waves on a rope have a wavelength of 10 cm. Find the momentum corresponding to a particle description of these waves. Given that the mass of the rope is 500g what is the speed of this "particle" ?

A.

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ J.s}{0.1\ m}$$

$$= 6.626\ \times 10^{-33}\ kg.m/s$$

The speed is $v\ =\ 1.32\ \times 10^{-32}\ m/s$ which is imperceptible with current technology.

Q. In an electron diffraction experiment, the wavelength of the (matter wave corresponding to) electrons are measured to be 10 μm. Find the speed of the electrons.

A.

$$pc = \frac{hc}{\lambda} = \frac{1.24\ eV\mu m}{10\ \mu m} = 0.124\ eV$$

But,

$$pc = m_0\ v\ c = m_0 c^2\ \left(\frac{v}{c}\right) = 0.511\ MeV\left(\frac{v}{c}\right)$$

Hence,

$$\frac{v}{c} = \ 2.43\ \times 10^{-7}\ \ or\ \ v = 73\ m/s$$

Forthcoming Topics:

Double slit experiments, Understanding the particles nature as group of waves, phase velocity, group velocity, Uncertainty principle, Quantum wave mechanics (Schrodinger Equation) and many more fascinating ideas....