Tutorial -3
ME-101, (2019-2020 Semester-II)
Feb 7 2020: Time: 7-55 to 8-50.am

1. A thin, homogeneous wire is bent to form the perimeter of the Fig.1 indicated. Locate the center of gravity of the wire figure thus formed.

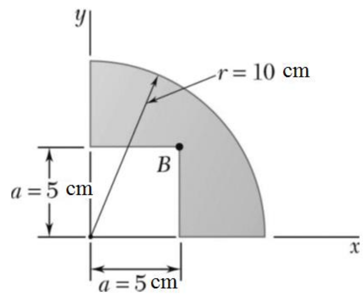


Figure. 1

2. Determine the volume and the surface area of the solid obtained by rotating the area of as shown in Fig. 2 about (*a*) the *x*-axis, (*b*) the *y*-axis.

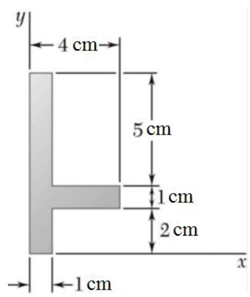


Figure. 2

3. Determine the reaction at the support *A* of the loaded cantilever beam as shown in Fig.3.

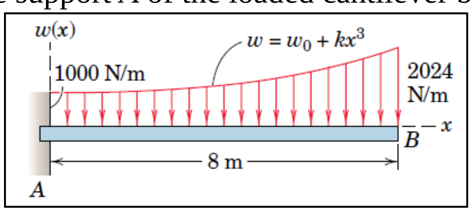


Figure. 3

4. Determine by direct integration the centroid of the area shown in Fig. 4 below.

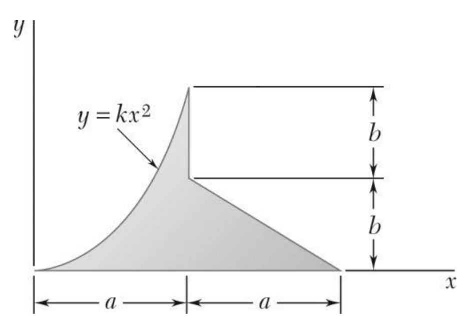


Figure. 4

5. Determine the shear-force and bending-moment diagrams for the cantilever beam shown in Fig.5.

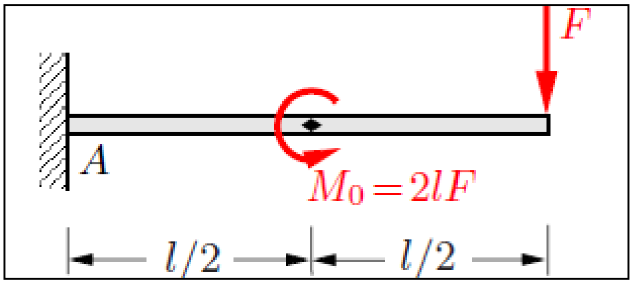


Figure. 5

6. Determine the shear-force and bending-moment diagrams for the beam shown in Fig.6.

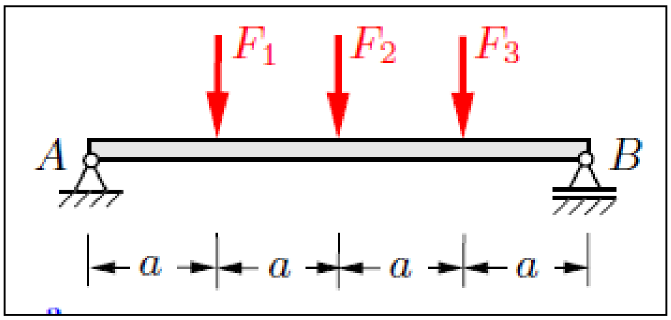
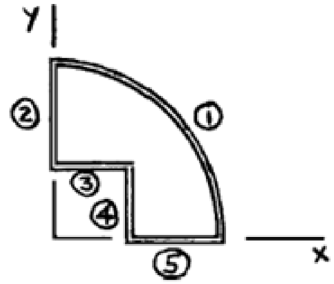


Figure. 6

1. Solution of Problem 1:

By symmetry, $\bar{X} = \bar{Y}$.



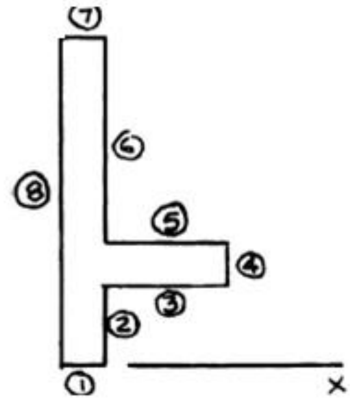
	L, cm	$\bar{x}$, cm	$\bar{y}L$, cm ^{2}
1	$\frac{1}{2}\pi(10) = 15.7080$	$\frac{2(10)}{\pi} = 6.3662$	100
2	5	0	0
3	5	2.5	12.5
4	5	5	25
5	5	7.5	37.5
Σ	35.708		175

Then $\bar{X}\,\Sigma L = \Sigma \bar{x}\, L$ $\quad \bar{X}(35.708) = 175$ $\quad \bar{X} = \bar{Y} = 4.90$ cm

2. Solution of Problem 2:

we have

$$A = 11 \text{ cm}^2$$
$$\Sigma \bar{x}A = 11.5 \text{ cm}^3$$
$$\Sigma \bar{y}A = 39.5 \text{ cm}^3$$



Applying the theorems of Pappus-Guldinus, we have

(a) Rotation about the x-axis:

$$\text{Volume} = 2\pi \bar{y}_{\text{area}} A = 2\pi \Sigma \bar{y}A$$
$$= 2\pi(39.5 \text{ cm}^3)$$

or $\text{Volume} = 248 \text{ cm}^3$

$$\text{Area} = 2\pi \bar{y}_{\text{line}} L = 2\pi \Sigma(\bar{y}_{\text{line}})L$$
$$= 2\pi(\bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4 + \bar{y}_5 L_5 + \bar{y}_6 L_6 + \bar{y}_7 L_7 + \bar{y}_8 L_8)$$
$$= 2\pi[(1)(2) + (2)(3) + (2.5)(1) + (3)(3) + (5.5)(5) + (8)(1) + (4)(8)]$$

or $\text{Area} = 547 \text{ cm}^2$

(b) Rotation about the y-axis:

$$\text{Volume} = 2\pi \bar{x}_{\text{area}} A = 2\pi \Sigma \bar{x}A$$
$$= 2\pi(11.5 \text{ cm}^3)$$

or $\text{Volume} = 72.3 \text{ cm}^3$

$$\text{Area} = 2\pi \bar{x}_{\text{line}} L = 2\pi \Sigma(\bar{x}_{\text{line}})L$$
$$= 2\pi(\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3 + \bar{x}_4 L_4 + \bar{x}_5 L_5 + \bar{x}_6 L_6 + \bar{x}_7 L_7)$$
$$= 2\pi[(0.5)(1) + (1)(2) + (2.5)(3) + (4)(1) + (2.5)(3) + (1)(5) + (0.5)(1)]$$

or $\text{Area} = 169.6 \text{ cm}^2$

3. Solution of Problem 3:

The constants in the load distribution are found to be $w_0 = 1000$ N/m and $k = 2$ N/m^4. The load R is then

$$R = \int w\,dx = \int_0^8 (1000 + 2x^3)\,dx = \left(1000x + \frac{x^4}{2}\right)\Bigg|_0^8 = 10\,050 \text{ N}$$

The x-coordinate of the centroid of the area is found by

$$\bar{x} = \frac{\int xw\,dx}{R} = \frac{1}{10\,050}\int_0^8 x(1000 + 2x^3)\,dx$$

$$= \frac{1}{10\,050}(500x^2 + \tfrac{2}{5}x^5)\Big|_0^8 = 4.49 \text{ m}$$

From the free-body diagram of the beam, we have

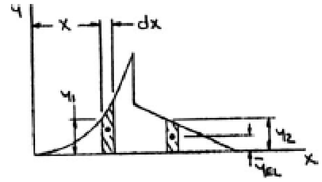
$[\Sigma M_A = 0]$ $\quad M_A - (10\,050)(4.49) = 0$

$$M_A = 45\,100 \text{ N}\cdot\text{m}$$

$[\Sigma F_y = 0]$ $\quad A_y = 10\,050$ N

Note that $A_x = 0$ by inspection.

4. Solution of Problem 4:



For y_1 at $x = a$, $\quad y = 2b, \quad 2b = ka^2, \quad \text{or} \quad k = \frac{2b}{a^2}$

Then $\quad y_1 = \frac{2b}{a^2}x^2$

By observation, $\quad y_2 = -\frac{b}{a}(x+2b) = b\left(2-\frac{x}{a}\right)$

Now $\quad \overline{x}_{EL} = x$

and for $0 \le x \le a$, $\quad \overline{y}_{EL} = \frac{1}{2}y_1 = \frac{b}{a^2}x^2 \quad \text{and} \quad dA = y_1\,dx = \frac{2b}{a^2}x^2 dx$

For $a \le x \le 2a$, $\quad \overline{y}_{EL} = \frac{1}{2}y_2 = \frac{b}{2}\left(2-\frac{x}{a}\right) \quad \text{and} \quad dA = y_2\,dx = b\left(2-\frac{x}{a}\right)dx$

Then

$$A = \int dA = \int_0^a \frac{2b}{a^2}x^2 dx + \int_a^{2a} b\left(2-\frac{x}{a}\right)dx$$

$$= \frac{2b}{a^2}\left[\frac{x^3}{3}\right]_0^a + b\left[-\frac{a}{2}\left(2-\frac{x}{a}\right)^2\right]_0^{2a} = \frac{7}{6}ab$$

and

$$\int \overline{x}_{EL}\,dA = \int_0^a x\left(\frac{2b}{a^2}x^2 dx\right) + \int_a^{2a} x\left[b\left(2-\frac{x}{a}\right)dx\right]$$

$$= \frac{2b}{a^2}\left[\frac{x^4}{4}\right]_0^a + b\left[x^2 - \frac{x^3}{3a}\right]_0^{2a}$$

$$= \frac{1}{2}a^2b + b\left\{\left[(2a)^2 - (a)^2\right] + \frac{1}{3a}\left[(2a^2) - (a)^3\right]\right\}$$

$$= \frac{7}{6}a^2b$$

$$\int \overline{y}_{EL}\,dA = \int_0^a \frac{b}{a^2}x^2\left[\frac{2b}{a^2}x^2 dx\right] + \int_0^{2a} \frac{b}{2}\left(2-\frac{x}{a}\right)\left[b\left(2-\frac{x}{a}\right)dx\right]$$

$$= \frac{2b^2}{a^4}\left[\frac{x^5}{5}\right]_0^a + \frac{b^2}{2}\left[-\frac{a}{3}\left(2-\frac{x}{a}\right)^3\right]_a^{2a}$$

$$= \frac{17}{30}ab^2$$

Hence,

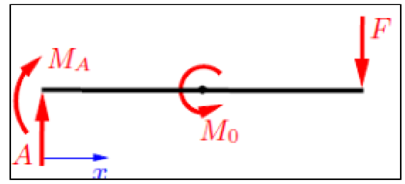
$$\overline{x}A = \int \overline{x}_{EL}\,dA: \quad \overline{x}\left(\frac{7}{6}ab\right) = \frac{7}{6}a^2b \qquad \overline{x} = a$$

$$\overline{y}A = \int \overline{y}_{EL}\,dA: \quad \overline{y}\left(\frac{7}{6}ab\right) = \frac{17}{30}ab^2 \qquad \overline{y} = \frac{17}{35}b$$

5. Solution of Problem 5:

$$\uparrow: \quad A - F = 0 \qquad \rightarrow \quad A = F\,,$$

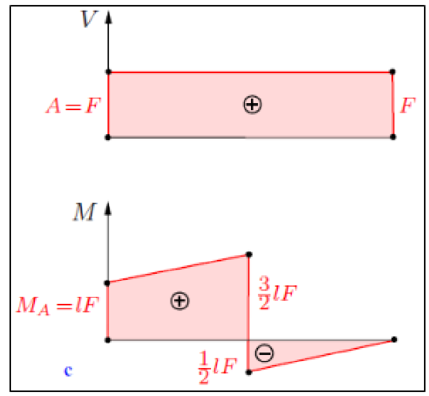
$$\overset{\curvearrowleft}{A}: \quad -M_A + M_0 - lF = 0 \quad \rightarrow \quad M_A = M_0 - lF = lF\,.$$



- ❑ The shear force follows from the equilibrium conditions of the forces in the vertical direction.

$$\underline{\underline{V}} = A = \underline{\underline{F}} \quad \text{for} \quad 0 < x < l\,.$$

- ❑ Because of the couple M_0 at the center of the beam, two regions of x must be considered when the bending moment is calculated. Accordingly, we pass a cut in the region given by 0 < x < l/2 and another one in the span l/2 < x < l. The equilibrium of the moments yields



$$\underline{\underline{M}} = M_A + xA = \underline{\underline{(l+x)F}} \qquad \text{for} \quad 0 < x < \frac{l}{2}\,,$$

$$\underline{\underline{M}} = M_A + xA - M_0 = \underline{\underline{(x-l)F}} \quad \text{for} \quad \frac{l}{2} < x \leq l\,.$$

6. Solution of Problem 6:

$\sum M_A = 0 = B(4a) - F(a) - 2F(2a) - F(3a) \xrightarrow{\text{yields}}$ B = 2F

Similarly, A = 2F

For 0 $\leq x \leq$ a
V = F, M = 2F×(x)

For a $\leq x \leq$ 2a
V = 2F-F = F, M = 2F×(x)-F× (x-a)

For 2a $\leq x \leq$ 3a
V = 2F-F-2F = -F, M = 2F×(x)-F× (x-a)-2F× (x-2a)

For 3a $\leq x \leq$ 4a
V = 2F-F-2F-F = -2F, M = 2F×(x)-F× (x-a)-2F× (x-2a)-F× (x-3a)