PH 102: Physics II

Lecture 11 (Post midsem, Spring 2020)

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LECTURE PLAN (TENTATIVE) OF PH 102 (POST MID-SEM)

SN	Date	Topic	Griffith's section	Division
Lec 1	05-03-2020	Lorentz Force, Biot-Savart law, Divergence & Curl of Magnetostatic Fields	5.1, 5.2, 5.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 2	11-03-2020	Application of Ampere's Law, Magnetic Vector Potential	5.3, 5.4	I, II (4-4:55 pm) III, IV (11-11:55 am)
Tut 1	12-03-2020	Lec 1		
Tut 2	17-03-2020	Lec 2		
Lec 3	18-03-2020	Magnetic dipole, Force & torque on a magnetic dipole, Magnetic materials, magnetization	5.4, 6.1	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 4	19-03-2020	Field of a magnetized object, Boundary conditions	6.2, 6.3, 6.4	I, II (4-4:55 pm) III, IV (11-11:55 am)
Tut 3	24-03-2020	Lec 3, 4		
Lec 5	25-03-2020	Ohm's law, motional emf, electromotive force	7.1	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 6	26-03-2020	Faraday's law, Lenz's law, Self & Mutual inductance, Energy stored in magnetic field	7.2	I, II (4-4:55 pm) III, IV (11-11:55 am)
Tut 4	31-03-2020	Lec 5, 6		
Lec 7	01-04-2020	Maxwell's equations	7.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 8	02-04-2020	Discussions, Problem solving	7.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
	07-04-2020	Quiz II		
Lec 9	08-04-2020	Continuity equation, Poynting theorem	8.1	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 10	16-04-2020	Wave solution of Maxwell's equations, polarisation	9.1, 9.2	I, II (4-4:55 pm) III, IV (11-11:55 am)
Tut 5	21-04-2020	Lec 9, 10		
Lec 11	22-04-2020	Electromagnetic waves in matter, reflection & transmission: normal incidence	9.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 12	23-04-2020	Reflection & transmission: oblique incidence	9.3, 9.4	I, II (4-4:55 pm) III, IV (11-11:55

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Tut 6	28-4-2020	Lec 11, 12		
Lec 13	29-04-2020	Relativity and electromagnetism: Galilean & special relativity	12.1, 12.2, 12.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 14	30-04-2020	Discussions, problem solving	12.1, 12.2, 12.3	I, II (4-4:55 pm) III, IV (11-11:55 am)

EM Waves: Summary

- The wave is transverse; both **E** and **B** fields are perpendicular to the direction of propagation which points in the direction of $\vec{E} \times \vec{B}$
- The **E** and **B** fields are perpendicular to each other, hence their dot product vanishes $\vec{E} \cdot \vec{B} = 0$
- The speed of propagation in vacuum is equal to the speed of light c.
- The ratio of the magnitudes (amplitudes) of the fields is 1/c where c is the speed of propagation in a vacuum.
- Electromagnetic waves obey superposition principle, as the wave equations are linear.

Lecture 10

EM Waves: Summary

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}, \quad \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\vec{\tilde{E}}(\vec{r}, t) = \vec{\tilde{E}}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}, \quad \vec{\tilde{B}}(\vec{r}, t) = \vec{\tilde{B}}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{k} \cdot \vec{\tilde{E}}_0 = 0, \quad \vec{k} \cdot \vec{\tilde{B}}_0 = 0$$

$$\vec{\tilde{B}}(\vec{r}, t) = \frac{1}{c} \hat{k} \times \vec{\tilde{E}}(\vec{r}, t)$$

$$\vec{E}(z, t) = E_0 \cos{(kz - \omega t + \delta)} \hat{x}$$

$$\vec{B}(z, t) = \frac{1}{c} E_0 \cos{(kz - \omega t + \delta)} \hat{y}$$

Lecture 10

Energy in Electromagnetic Waves

The energy per unit volume stored in electromagnetic fields is given by:

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)$$

Lecture 9

For monochromatic plane waves

$$B^2 = \frac{1}{c^2}E^2 = \mu_0\epsilon_0 E^2$$

Lecture 10

Therefore, the electric and magnetic contributions to the energy density are equal:

$$u = \epsilon_0 E^2 = \epsilon_0 E_0^2 \cos^2\left(kz - \omega t + \delta\right)$$

Energy in Electromagnetic Waves

Energy per unit area, per unit time transported by the fields is given by the Poynting vector: $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

Using $\vec{E}(z,t) = E_0 \cos(kz - \omega t + \delta)\hat{x}, \vec{B}(z,t) = \frac{1}{c} E_0 \cos(kz - \omega t + \delta)\hat{y}$

$$\vec{S} = c\epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)\hat{z} = cu\hat{z}$$

Thus, the Poynting vector (energy flux density) is energy density times the velocity of the waves in the direction of propagation, as expected.

In time Δt, a length $c\Delta t$ passes through area A, carrying energy with it equal to $uAc\Delta t$. Thus, the energy per unit time, per unit area is uc.

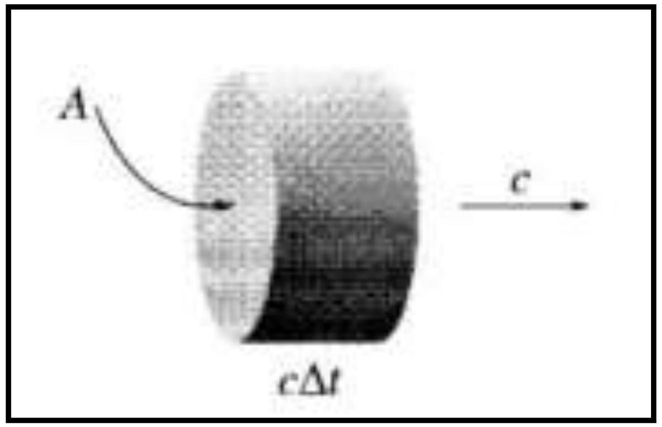

Figure 9.12, Introduction to Electrodynamics, D J Griffiths

Energy in Electromagnetic Waves

For plane electromagnetic wave moving in an arbitrary direction:

$$\vec{E}(\vec{r},t) = E_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \delta)\hat{n}, \;\; \vec{B}(\vec{r},t) = \frac{1}{c} E_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \delta)(\hat{k}\times\hat{n})$$

The Poynting vector is:

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = c\epsilon_0 E_0^2 \cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)[\hat{n}\times(\hat{k}\times\hat{n})]$$

Using $\quad \hat{n}\times(\hat{k}\times\hat{n}) = \hat{k}(\hat{n}\cdot\hat{n}) - \hat{n}(\hat{n}\cdot\hat{k}) = \hat{k}$

we get $\vec{S} = cu\hat{k}$ which is same as before.

vanishes for
transverse wave

Energy in Electromagnetic Waves

Typically for EM waves, the wavelength is so short and the time period is so brief that any macroscopic measurement will encompass many cycles.

Therefore, one should take the average value of the cosine-squared term.

Over a complete cycle, the average value of sine and cosine squared is 1/2.

$$\frac{1}{T}\int_0^T \cos^2\left(kz - \frac{2\pi t}{T} + \delta\right)dt = \frac{1}{2}$$

Therefore,

$$\langle u \rangle = \frac{1}{2}\epsilon_0 E_0^2, \ \langle \vec{S} \rangle = \frac{1}{2}c\epsilon_0 E_0^2 \hat{z}.$$

Intensity: Average power per unit area transported by an EM wave

$$I = \langle S \rangle = \frac{1}{2}c\epsilon_0 E_0^2$$

Exercise: At the upper surface of the Earth's atmosphere, the time-averaged magnitude of the Poynting vector $\langle S \rangle = 1.35 \times 10^3 \ \mathrm{Wm}^{-2}$, is referred to as the solar constant. (a) Assuming that the Sun's EM radiation is a plane sinusoidal wave, what are the magnitudes of the electric and magnetic fields? (b) What is the total time-averaged power radiated by the Sun? The mean Sun-Earth distance is $R = 1.50 \times 10^{11} \ \mathrm{m}$.

Solution: (a) Using $\langle S \rangle = \frac{1}{2} c\epsilon_0 E_0^2$, we get $E_0 = \sqrt{\frac{2\langle S \rangle}{c\epsilon_0}} = 1.01 \times 10^3 \ \mathrm{V/m}$

The corresponding magnetic field: $B_0 = \frac{E_0}{c} = 3.4 \times 10^{-6} \ \mathrm{T}$

which is 10 times smaller than Earth's magnetic field at Earth's surface (0.25-0.65 Gauss).

(b) Total time-averaged power radiated by the Sun at a distance R is

$$\langle P \rangle = \langle S \rangle (4\pi R^2) = 3.8 \times 10^{26} \ \mathrm{W}$$

Intensity at a distance r: $I = \langle S \rangle = \frac{\langle P \rangle}{4\pi r^2}$, typical of a **spherical wave** that originates from a point-like source.

Exercise: Show that the intensity of electromagnetic standing wave given by

$$E_y(x,t) = 2E_0 \cos(kx)\cos(\omega t), \; B_z(x,t) = 2B_0 \sin(kx)\sin(\omega t)$$

is zero. Or, prove that standing waves do not carry any energy, a property that is expected given the fact that standing waves do not propagate.

Electromagnetic Waves in Matter

Inside matter, in the absence of free charge and free current, Maxwell's equations can be written as

$$(i)\ \vec{\nabla}\cdot\vec{D}=0,\ (iii)\ \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t},$$

$$(ii)\ \vec{\nabla}\cdot\vec{B}=0,\ (iv)\ \vec{\nabla}\times\vec{H}=\frac{\partial\vec{D}}{\partial t}.$$

For a linear medium

$$\vec{D}=\epsilon\vec{E}, \epsilon=\epsilon_0(1+\chi_e),$$

$$\vec{H}=\frac{1}{\mu}\vec{B}, \mu=\mu_0(1+\chi_m).$$

Let the medium be homogeneous as well, so that ϵ, μ do not vary from point to point.

Electromagnetic Waves in Matter

For such linear and homogeneous medium, Maxwell's equations become

$$(i)\ \vec{\nabla}\cdot\vec{E}=0,\ (iii)\ \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t},$$

$$(ii)\ \vec{\nabla}\cdot\vec{B}=0,\ (iv)\ \vec{\nabla}\times\vec{B}=\mu\epsilon\frac{\partial\vec{E}}{\partial t}.$$

which differ from the ones in vacuum only in the replacement of $\mu_0\epsilon_0$ by $\mu\epsilon$.

The corresponding EM wave equations in matter are:

$$\nabla^2\vec{E}=\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2},\ \nabla^2\vec{B}=\mu\epsilon\frac{\partial^2\vec{B}}{\partial t^2}$$

Electromagnetic Waves in Matter

As before, the speed of propagation of EM waves in matter can be interpreted to be

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{n}$$

where n is the **index of refraction** of the material given by

$$n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}$$

For most materials $\mu \approx \mu_0$ and hence $n \approx \sqrt{\epsilon_r}$

Since the dielectric constant ϵ_r is almost always greater than 1, light travels more slowly through matter.

Electromagnetic Waves in Matter

All other results derived for EM waves in free space can be carried over to EM waves in matter just by making the simple transcription: $\epsilon_0 \to \epsilon, \mu_0 \to \mu, c \to v$

Energy density: $u = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right)$

Poynting vector: $\vec{S} = \frac{1}{\mu}(\vec{E} \times \vec{B})$

Frequency-wave number: $\omega = kv$

Amplitude of B is 1/v times the amplitude of E.

Intensity: Average power per unit area transported

$$I = \frac{1}{2} v \epsilon E_0^2$$

Reflection & Transmission: Normal Incidence

What happens when an electromagnetic wave passes from one transparent medium to another, say air to water?

The answer to this can simply be found by using the electrodynamic boundary conditions (source-free)

$$(i)\ \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp},\ (iii)\ E_1^{\parallel} = E_2^{\parallel},$$
$$(ii)\ B_1^{\perp} = B_2^{\perp},\ (iv)\ \frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel}.$$

These conditions relate E, B just to the left and just to the right of the interface between two linear media.

Reflection & Transmission: Normal Incidence

A plane EM wave of frequency ω, travelling in the z direction and polarised (**E**) in the x direction, approaches the interface (separating two linear media) from left.

The normally incident fields:

$$\vec{\tilde{E}}_I(z,t) = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{x},$$
$$\vec{\tilde{B}}_I(z,t) = \frac{1}{v_1} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{y}.$$

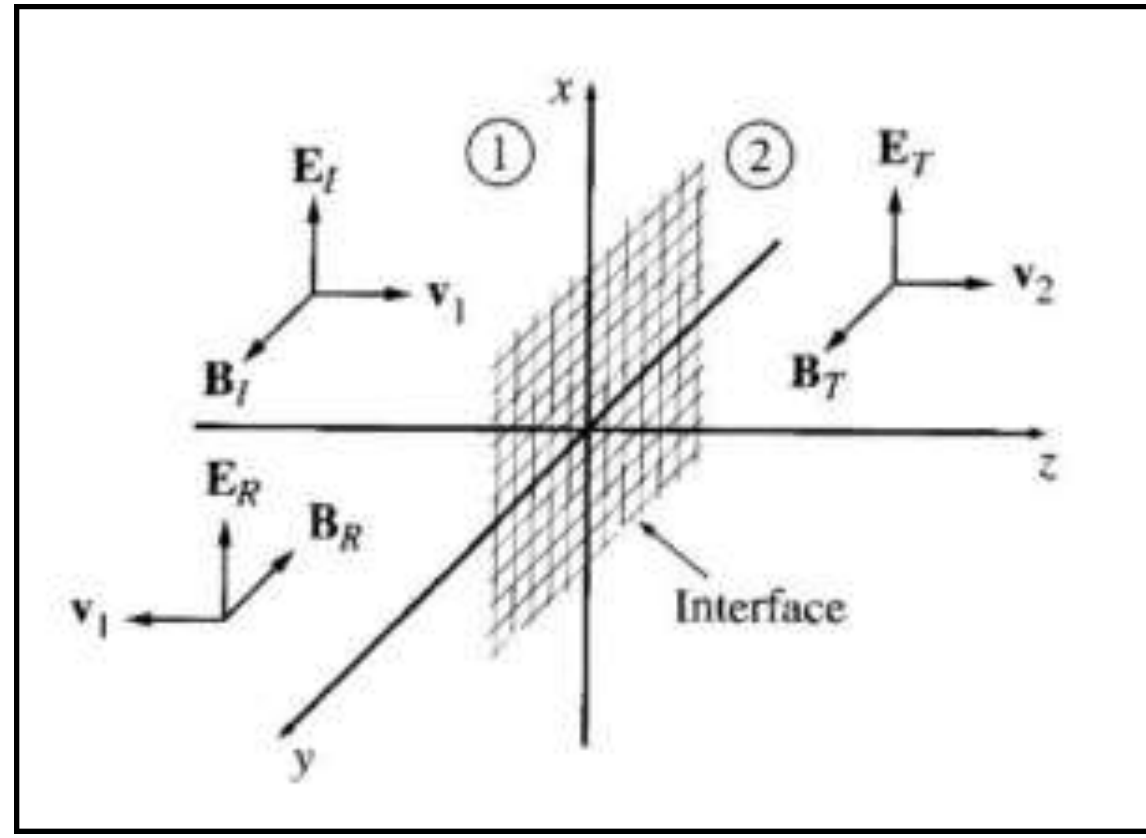

Figure 9.13, Introduction to Electrodynamics, D J Griffiths

This gives rise to a reflected wave travelling in the negative z direction in medium 1. The corresponding fields can be written as

$$\vec{\tilde{E}}_R(z,t) = \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{x},$$
$$\vec{\tilde{B}}_R(z,t) = -\frac{1}{v_1} \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{y}.$$

Same as in incident wave?

Direction of $\vec{B} (\propto \vec{k} \times \vec{E})$ gets reversed as the wave vector direction changes.

One can flip **E** and keep **B** direction same in reflected part. Final result will be same! Verify this!

Reflection & Transmission: Normal Incidence

There also exists a transmitted wave which continues on the right in medium 2:

$$\vec{\tilde{E}}_T(z,t) = \tilde{E}_{0T} e^{i(k_2 z - \omega t)} \hat{x},$$

$$\vec{\tilde{B}}_T(z,t) = \frac{1}{v_2} \tilde{E}_{0T} e^{i(k_2 z - \omega t)} \hat{y}.$$

At the interface separating medium 1 from medium 2 that is, z=0, the combined fields on the left $\vec{\tilde{E}}_I + \vec{\tilde{E}}_R, \vec{\tilde{B}}_I + \vec{\tilde{B}}_R$ must be related to the fields on the right $\vec{\tilde{E}}_T, \vec{\tilde{B}}_T$ by the electromagnetic boundary conditions.

For normal incidence, the first two boundary conditions

$$(i)\ \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp},\ (ii)\ B_1^{\perp} = B_2^{\perp}.$$

are trivial as none of the field components are perpendicular to the interface.

Reflection & Transmission: Normal Incidence

The third boundary condition $(iii)\ E_1^{\parallel} = E_2^{\parallel}$ gives rise to:

$$\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}. \tag{1}$$

The fourth boundary condition $(iv)\ \frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel}$ gives:

$$\frac{1}{\mu_1}\left(\frac{1}{v_1}\tilde{E}_{0I} - \frac{1}{v_1}\tilde{E}_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2}\tilde{E}_{0T}\right)$$

$$\Longrightarrow\ \tilde{E}_{0I} - \tilde{E}_{0R} = \beta\tilde{E}_{0T},\ \beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}. \tag{2}$$

Here $n_1 = c/v_1, n_2 = c/v_2$ are the respective indices of refraction of the two linear media.

Reflection & Transmission: Normal Incidence

Equations (1), (2) can be easily solved to determine the outgoing amplitudes in terms of the incident one:

$$\tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I},\ \tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I}.$$

If $\mu_1 \approx \mu_2 \approx \mu_0$, we can write $\beta = v_1/v_2 = n_2/n_1$and hence

$$\tilde{E}_{0R} = \left(\frac{v_2-v_1}{v_2+v_1}\right)\tilde{E}_{0I},\ \tilde{E}_{0T} = \left(\frac{2v_2}{v_2+v_1}\right)\tilde{E}_{0I},$$

$$\tilde{E}_{0R} = \left(\frac{n_1-n_2}{n_1+n_2}\right)\tilde{E}_{0I},\ \tilde{E}_{0T} = \left(\frac{2n_1}{n_1+n_2}\right)\tilde{E}_{0I}.$$

The expressions for reflected and transmitted amplitudes imply:

If the wave is incident from a rarer medium onto a denser medium ($n_2 > n_1, v_2 < v_1$), the reflected electric field is **out of phase** with the incident one. The negative sign can be interpreted as a phase change of $\pi : e^{i\pi} = -1$.

The reflected electric field is **in phase** with the incident one if the wave is incident from a denser medium onto a rarer medium ($n_2 < n_1, v_2 > v_1$).The transmitted wave is **in phase** with the incident one in both the cases.

The real amplitudes are related as:

$$E_{0R} = \left|\frac{v_2 - v_1}{v_2 + v_1}\right| E_{0I},\ E_{0T} = \left(\frac{2v_2}{v_2 + v_1}\right) E_{0I},$$

$$E_{0R} = \left|\frac{n_1 - n_2}{n_1 + n_2}\right| E_{0I},\ E_{0T} = \left(\frac{2n_1}{n_1 + n_2}\right) E_{0I}.$$

Since the intensity of an electromagnetic wave is given by $I = \epsilon v E_0^2/2$, we can calculate how much of the incident energy is reflected/transmitted.

For $\mu_1 \approx \mu_2 \approx \mu_0$, the ratio of the reflected intensity to the incident intensity is

$$R = \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2.$$

The ratio of the transmitted intensity to the incident intensity is

$$T = \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 = \frac{\mu_2 \epsilon_2 v_2}{\mu_1 \epsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2$$

$$\implies T = \frac{v_1}{v_2}\left(\frac{E_{0T}}{E_{0I}}\right)^2 = \frac{n_2}{n_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

Reflection & Transmission: Normal Incidence

Therefore,

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Reflection Coefficient

$$T = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

Transmission Coefficient

It is straightforward to show that: R+T =1, in accordance with the conservation of energy.

For example, when light passes from air (n=1) to glass (n=1.5), only 4% of it gets reflected (R=0.04, T=0.96).

If $\mu_1 = \mu_2 = \mu_0$ is not assumed, then

$$R = \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{1-\beta}{1+\beta}\right)^2$$

$$T = \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 = \frac{\mu_1}{\mu_2}\frac{\mu_2\epsilon_2 v_2}{\mu_1\epsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 = \beta\left(\frac{2}{1+\beta}\right)^2$$

where $\beta = \dfrac{\mu_1 v_1}{\mu_2 v_2} = \dfrac{\mu_1 n_2}{\mu_2 n_1}$

For the above expressions also, R+T=1, as expected.

In the previous derivations, it has been assumed that the frequency of the wave remains same for incident, reflected and transmitted parts. How to justify that?

Let the incident, reflected and transmitted waves be

$$\vec{\tilde{E}}_I = \tilde{E}_{0I} e^{i(\vec{k}_I \cdot \vec{r} - \omega_I t)} \hat{n}_I, \vec{\tilde{E}}_R = \tilde{E}_{0R} e^{i(\vec{k}_R \cdot \vec{r} - \omega_R t)} \hat{n}_R, \vec{\tilde{E}}_T = \tilde{E}_{0T} e^{i(\vec{k}_T \cdot \vec{r} - \omega_T t)} \hat{n}_T$$

Using the equality of parallel components across the interface (z=0):

$$(\vec{\tilde{E}}_{0I})_{\parallel} e^{i(\vec{k}_I \cdot \vec{r} - \omega_I t)} + (\vec{\tilde{E}}_{0R})_{\parallel} e^{i(\vec{k}_R \cdot \vec{r} - \omega_R t)} = (\vec{\tilde{E}}_{0T})_{\parallel} e^{i(\vec{k}_T \cdot \vec{r} - \omega_T t)}$$

Now this boundary condition should be valid for all time t. This is possible, only when the parts containing time cancel out on both sides. This requires:

$$\omega_I = \omega_R = \omega_T$$