PH 102: Physics II

Lecture 8 (Post midsem, Spring 2020)

IIT Guwahati
Debasish Borah

LECTURE PLAN (TENTATIVE) OF PH 102 (POST MID-SEM)

SN	Date	Topic	Griffith's section	Division
Lec 1	05-03-2020	Lorentz Force, Biot-Savart law, Divergence & Curl of Magnetostatic Fields	5.1, 5.2, 5.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 2	11-03-2020	Application of Ampere's Law, Magnetic Vector Potential	5.3, 5.4	I, II (4-4:55 pm) III, IV (11-11:55 am)
Tut 1	12-03-2020	Lec 1		
Tut 2	17-03-2020	Lec 2		
Lec 3	18-03-2020	Magnetic dipole, Force & torque on a magnetic dipole, Magnetic materials, magnetization	5.4, 6.1	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 4	19-03-2020	Field of a magnetized object, Boundary conditions	6.2, 6.3, 6.4	I, II (4-4:55 pm) III, IV (11-11:55 am)
Tut 3	24-03-2020	Lec 3, 4		
Lec 5	25-03-2020	Ohm's law, motional emf, electromotive force	7.1	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 6	26-03-2020	Faraday's law, Lenz's law, Self & Mutual inductance, Energy stored in magnetic field	7.2	I, II (4-4:55 pm) III, IV (11-11:55 am)
Tut 4	31-03-2020	Lec 5, 6		
Lec 7	01-04-2020	Maxwell's equations	7.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 8	02-04-2020	Discussions, Problem solving	7.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
	07-04-2020	Quiz II		
Lec 9	08-04-2020	Continuity equation, Poynting theorem	8.1	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 10	16-04-2020	Wave solution of Maxwell's equations, polarisation	9.1, 9.2	I, II (4-4:55 pm) III, IV (11-11:55 am)
Tut 5	21-04-2020	Lec 9, 10		
Lec 11	22-04-2020	Electromagnetic waves in matter, reflection & transmission: normal incidence	9.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 12	23-04-2020	Reflection & transmission: oblique incidence	9.3, 9.4	I, II (4-4:55 pm) III, IV (11-11:55

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				am)
Tut 6	28-4-2020	Lec 11, 12		
Lec 13	29-04-2020	Relativity and electromagnetism: Galilean & special relativity	12.1, 12.2, 12.3	I, II (4-4:55 pm) III, IV (11-11:55 am)
Lec 14	30-04-2020	Discussions, problem solving	12.1, 12.2, 12.3	I, II (4-4:55 pm) III, IV (11-11:55 am)

Maxwell's Equations

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$	Gauss's (Coulomb's) Law
$\vec{\nabla} \cdot \vec{B} = 0$	No name
$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Faraday's Law
$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$	Ampere's Law with Maxwell's Correction

Sample Problems

(1) An alternating current $I = I_0 \cos(\omega t)$ flows down a long straight wire, and returns along a coaxial conducting tube of radius a. Assuming that the field goes to zero as $s \to \infty$, find E(s, t).

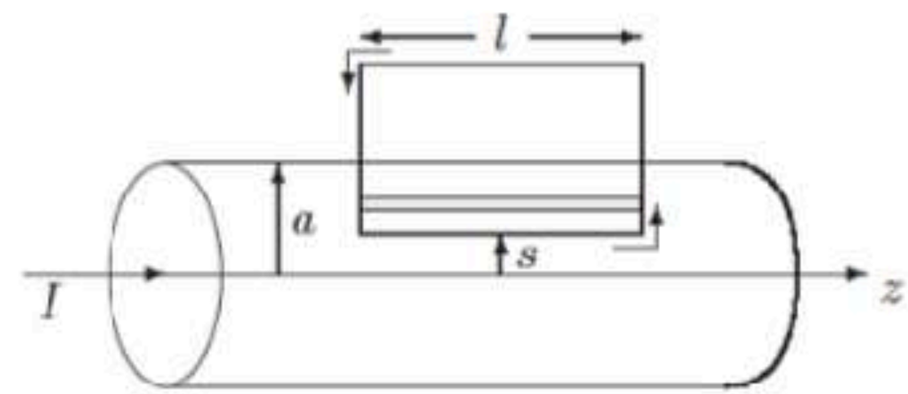
(a) Find the displacement current density J_d.
(b) Integrate it to get the total displacement current, $I_d = \int \mathbf{J}_d \cdot d\mathbf{a}.$

Compare I_d and I. If the outer cylinder were, say, 2 mm in diameter, how high would the frequency have to be, for I_d to be 1% of I?

Solution: Using the "amperian loop" in the figure,

$$\oint \mathbf{E}\cdot d\mathbf{l} = El = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{a} = -\frac{d}{dt}\int_s^a \frac{\mu_0 I}{2\pi s'} l\, ds'$$

$$\therefore E = -\frac{\mu_0}{2\pi}\frac{dI}{dt}\ln\left(\frac{a}{s}\right). \quad \text{and,} \quad \frac{dI}{dt} = -I_0\omega\sin\omega t,$$



Outside, B = 0, so here E = 0 (like B outside a solenoid).

Hence, $$\mathbf{E} = \frac{\mu_0 I_0 \omega}{2\pi}\sin(\omega t)\ln\left(\frac{a}{s}\right)\hat{\mathbf{z}}.$$

(a) $$J_d = \epsilon_0\frac{\partial \mathbf{E}}{\partial t} = \epsilon_0\frac{\mu_0 I_0 \omega^2}{2\pi}\cos(\omega t)\ln(a/s)\,\hat{\mathbf{z}} = \frac{\mu_0\epsilon_0}{2\pi}\omega^2 I\ln(a/s)\,\hat{\mathbf{z}}.$$

What about **E** along the vertical edges of the loop?

(b) $I_d = \int \mathbf{J}_d \cdot d\mathbf{a} = \frac{\mu_0\epsilon_0\omega^2 I}{2\pi}\int_0^a \ln(a/s)(2\pi s\,ds) = \mu_0\epsilon_0\omega^2 I\int_0^a (s\ln a - s\ln s)ds$

$= \mu_0\epsilon_0\omega^2 I\left[(\ln a)\frac{s^2}{2} - \frac{s^2}{2}\ln s + \frac{s^2}{4}\right]\Big|_0^a = \mu_0\epsilon_0\omega^2 I\left[\cancel{\frac{a^2}{2}\ln a} - \cancel{\frac{a^2}{2}\ln a} + \frac{a^2}{4}\right] = \frac{\mu_0\epsilon_0\omega^2 I a^2}{4}.$

(c) $\frac{I_d}{I} = \frac{\mu_0\epsilon_0\omega^2 a^2}{4}$ where, $\mu_0\epsilon_0 = 1/c^2$, $I_d/I = (\omega a/2c)^2$.

Given, $a = 10^{-3}\,\mathrm{m}$, and $\frac{I_d}{I} = \frac{1}{100}$, implies $\frac{\omega a}{2c} = \frac{1}{10}$,

$$\omega = \frac{2c}{10a} = \frac{3\times 10^8\ \mathrm{m/s}}{5\times 10^{-3}\ \mathrm{m}},\ \text{or}\ \omega = 0.6\times 10^{11}/\mathrm{s}$$

$\nu = \frac{\omega}{2\pi} \approx 10^{10}\,\mathrm{Hz}$, Microwave, way above the radio frequencies

Thus the frequency required so that the displacement current is comparable (~ 1%) is very high @ mircowave. Hence, in the early days of Faraday no experimental signal could appear giving clue for the displacement current!

(2) Consider the following fields, $\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\theta(vt - r)\hat{\mathbf{r}}$; $\mathbf{B}(\mathbf{r},t) = \mathbf{0}$

Show that these fields satisfy all of Maxwell's equations, and determine ρ and $\mathbf{J}$. Describe the physical situation that gives rise to these fields.

Solution:

Physically, this is the field of a point charge +q at the origin, out to an expanding spherical shell of radius vt. Outside the shell the field is zero. Evidently the shell carries the opposite charge, -q

Using $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$ and $\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2}\right) = 4\pi\delta^3(\mathbf{r})$.

$$\nabla \cdot \mathbf{E} = \theta(vt - r)\nabla \cdot \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}\right) + \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \cdot \nabla[\theta(vt - r)] = \frac{q}{\epsilon_0}\delta^3(\mathbf{r})\theta(vt - r) + \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}})\frac{\partial}{\partial r}\theta(vt - r).$$

However,

$$\delta^3(\mathbf{r})\theta(vt - r) = \delta^3(\mathbf{r})\theta(t), \text{ and } \frac{\partial}{\partial r}\theta(vt - r) = -\delta(vt - r)$$

Chapter 1
D J Griffiths

so,

$$\rho = \epsilon_0 \nabla \cdot \mathbf{E} = q\delta^3(\mathbf{r})\theta(t) - \frac{q}{4\pi r^2}\delta(vt - r).$$

For t < 0 the field and the charge density are zero everywhere.

$$\nabla \cdot \mathbf{B} = 0, \text{ and } \nabla \times \mathbf{E} = \mathbf{0}$$

(since E has only an r component, and it is independent of θ and ϕ)

$$\nabla \times \mathbf{B} = \mathbf{0} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \partial \mathbf{E} / \partial t.$$

$$\mathbf{J} = -\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = -\epsilon_0 \left\{ \frac{q}{4\pi\epsilon_0 r^2} \frac{\partial}{\partial t} [\theta(vt - r)] \right\} \hat{\mathbf{r}} = -\frac{q}{4\pi r^2} v \delta(vt - r) \hat{\mathbf{r}}.$$

The stationary charge at the origin doesn't contribute to $\mathbf{J}$, of course; for the expanding shell we have $\mathbf{J} = \rho \mathbf{v}$.

Practice similar problems where Maxwell's equations are used to find source terms from the fields and vice versa!

(3) A long straight solenoid having N turns per unit length carries a current $I = I_0 \sin \omega t$. Calculate the displacement current density as a function of s within the solenoid.

Solution: Displacement current is $\vec{J} = \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

Due to the change in current and hence the magnetic field inside the solenoid, there will be induced electric field which can be found out by using Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

Consider a circular loop of radius s coaxial with the solenoid axis

$$\implies E(2\pi s) = -\pi s^2 \frac{dB}{dt} = -\pi s^2 \mu_0 N \frac{dI}{dt}$$

Inside a solenoid, magnetic field is axial, given by $B = \mu_0 N I$

$$\implies E_\phi = -\frac{s}{2} \mu_0 N I_0 \omega \cos \omega t$$

4. (a)

Show that Maxwell's equations with magnetic charge are invariant under the duality transformation,

$$\begin{aligned}
\mathbf{E}' &= \mathbf{E}\cos\alpha + c\mathbf{B}\sin\alpha,\\
c\mathbf{B}' &= c\mathbf{B}\cos\alpha - \mathbf{E}\sin\alpha,\\
cq_e' &= cq_e\cos\alpha + q_m\sin\alpha,\\
q_m' &= q_m\cos\alpha - cq_e\sin\alpha,
\end{aligned}$$

where $c = 1/\sqrt{\epsilon_0\mu_0}$ and α is an arbitrary rotation angle in "**E**/**B**-space." Charge and current densities transform in the same way as q_e and q_m.

4. (b)

Show that the force law, $\mathbf{F} = q_e(\mathbf{E} + \mathbf{v}\times\mathbf{B}) + q_m\left(\mathbf{B} - \frac{1}{c^2}\mathbf{v}\times\mathbf{E}\right)$, is also invariant under the duality transformation.

Solution:

4. (a)

$$\nabla\cdot\mathbf{E}' = (\nabla\cdot\mathbf{E})\cos\alpha + c(\nabla\cdot\mathbf{B})\sin\alpha = \frac{1}{\epsilon_0}\rho_e\cos\alpha + c\mu_0\rho_m\sin\alpha$$

$$= \frac{1}{\epsilon_0}(\rho_e\cos\alpha + c\mu_0\epsilon_0\rho_m\sin\alpha) = \frac{1}{\epsilon_0}(\rho_e\cos\alpha + \frac{1}{c}\rho_m\sin\alpha) = \frac{1}{\epsilon_0}\rho_e'.$$

$$\nabla\cdot\mathbf{B}' = (\nabla\cdot\mathbf{B})\cos\alpha - \frac{1}{c}(\nabla\cdot\mathbf{E})\sin\alpha = \mu_0\rho_m\cos\alpha - \frac{1}{c\epsilon_0}\rho_e\sin\alpha$$

$$= \mu_0(\rho_m\cos\alpha - \frac{1}{c\mu_0\epsilon_0}\rho_e\sin\alpha) = \mu_0(\rho_m\cos\alpha - c\rho_e\sin\alpha) = \mu_0\rho_m'.$$

$$\nabla \times \mathbf{E}' = (\nabla \times \mathbf{E})\cos\alpha + c(\nabla \times \mathbf{B})\sin\alpha = \left(-\mu_0 \mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t}\right)\cos\alpha + c\left(\mu_0 \mathbf{J}_e + \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)\sin\alpha$$

$$= -\mu_0(\mathbf{J}_m \cos\alpha - c\mathbf{J}_e \sin\alpha) - \frac{\partial}{\partial t}\left(\mathbf{B}\cos\alpha - \frac{1}{c}\mathbf{E}\sin\alpha\right) = -\mu_0 \mathbf{J}'_m - \frac{\partial \mathbf{B}'}{\partial t}.$$

$$\nabla \times \mathbf{B}' = (\nabla \times \mathbf{B})\cos\alpha - \frac{1}{c}(\nabla \times \mathbf{E})\sin\alpha = \left(\mu_0 \mathbf{J}_e + \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)\cos\alpha - \frac{1}{c}\left(-\mu_0 \mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t}\right)\sin\alpha$$

$$= \mu_0(\mathbf{J}_e \cos\alpha + \frac{1}{c}\mathbf{J}_m \sin\alpha) + \mu_0\epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}\cos\alpha + c\mathbf{B}\sin\alpha) = \mu_0 \mathbf{J}'_e + \mu_0\epsilon_0 \frac{\partial \mathbf{E}'}{\partial t}.$$

4. (b)
$$\mathbf{F}' = q'_e(\mathbf{E}' + \mathbf{v} \times \mathbf{B}') + q'_m(\mathbf{B}' - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}')$$

$$= \left(q_e \cos\alpha + \frac{1}{c}q_m \sin\alpha\right)\left[(\mathbf{E}\cos\alpha + c\mathbf{B}\sin\alpha) + \mathbf{v} \times \left(\mathbf{B}\cos\alpha - \frac{1}{c}\mathbf{E}\sin\alpha\right)\right]$$

$$+ (q_m \cos\alpha - c q_e \sin\alpha)\left[\left(\mathbf{B}\cos\alpha - \frac{1}{c}\mathbf{E}\sin\alpha\right) - \frac{1}{c^2}\mathbf{v} \times (\mathbf{E}\cos\alpha + c\mathbf{B}\sin\alpha)\right]$$

$$= q_e\Big[(\mathbf{E}\cos^2\alpha + c\mathbf{B}\sin\alpha\cos\alpha - c\mathbf{B}\sin\alpha\cos\alpha + \mathbf{E}\sin^2\alpha)$$

$$+\mathbf{v} \times \left(\mathbf{B}\cos^2\alpha - \frac{1}{c}\mathbf{E}\sin\alpha\cos\alpha + \frac{1}{c}\mathbf{E}\sin\alpha\cos\alpha + \mathbf{B}\sin^2\alpha\right)\Big]$$

$$+q_m\Big[\left(\frac{1}{c}\mathbf{E}\sin\alpha\cos\alpha + \mathbf{B}\sin^2\alpha + \mathbf{B}\cos^2\alpha - \frac{1}{c}\mathbf{E}\sin\alpha\cos\alpha\right)$$

$$+\mathbf{v} \times \left(\frac{1}{c}\mathbf{B}\sin\alpha\cos\alpha - \frac{1}{c^2}\mathbf{E}\sin^2\alpha - \frac{1}{c^2}\mathbf{E}\cos^2\alpha - \frac{1}{c}\mathbf{B}\sin\alpha\cos\alpha\right)\Big]$$

$$= q_e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + q_m\left(\mathbf{B} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}\right) = \mathbf{F}.$$

(5) Consider a parallel plate capacitor of circular cross section of radius a. If the current to the plates is time dependent, obtain the induced magnetic field in between the plates of the capacitor.

Solution: Using Ampere's law (corrected one) in terms of conduction as well as displacement current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I + I_D)$$

In between the plates, the conduction current is obviously zero. However, the displacement current is given by

$$I_D = \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a}$$

Considering a circular loop of radius s<a (inside the capacitor), we can find the magnetic field as

$$B(2\pi s) = \mu_0\epsilon_0\pi s^2\frac{\partial E}{\partial t}$$

$$\implies \vec{B} = \frac{\mu_0\epsilon_0}{2}s\frac{\partial E}{\partial t}\hat{\phi}$$

Constant E in between the plates is assumed ignoring the edge effects!

For points outside the capacitor plates s > a, the magnetic field is

$$B(2\pi s) = \mu_0\epsilon_0\pi a^2\frac{\partial E}{\partial t}$$

$$\implies \vec{B} = \frac{\mu_0\epsilon_0}{2s}a^2\frac{\partial E}{\partial t}\hat{\phi}$$

E outside the plates is taken to be zero!

Ampere’s Law for Finite Current Carrying Wire

- It was argued not to use Ampere's law (in magnetostatics) to find magnetic field of a finite current carrying wire. Biot-Savart law was used instead to find the field.

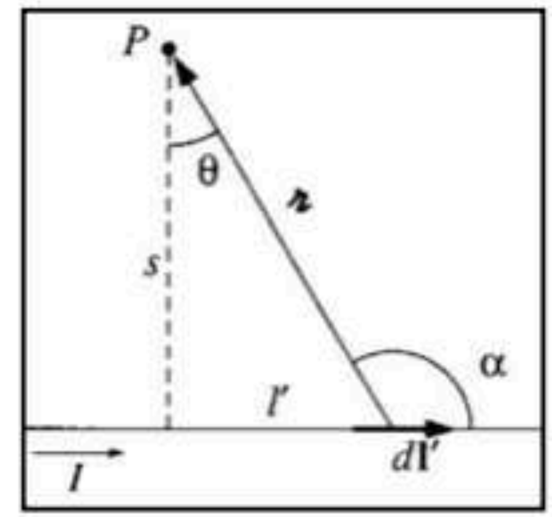
- But why did Ampere’s law give the wrong result?

- Will we get a correct result if we use Ampere’s law with Maxwell’s correction?

Finite Current Carrying Wire

Recall the discussion from Lecture 15

Example 5.5 (Introduction to Electrodynamics, D. J. Griffiths):
Find the magnetic field at a distance s from a long straight wire carrying a steady current I.



Using Biot-Savart law: $B(\vec{s}) = \frac{\mu_0}{4\pi} I \int \frac{|d\vec{l'} \times \hat{\mathfrak{r}}|}{\mathfrak{r}^2}, \vec{\mathfrak{r}} = \vec{s} - \vec{l'}$

Using $|d\vec{l'} \times \hat{\mathfrak{r}}| = dl' \sin\alpha = dl' \cos\theta$

$$l' = s\tan\theta \implies dl' = \frac{s}{\cos^2\theta} d\theta \qquad s = \mathfrak{r}\cos\theta \implies \frac{1}{\mathfrak{r}^2} = \frac{\cos^2\theta}{s^2}$$

we can write,

$$B(\vec{s}) = \frac{\mu_0}{4\pi} I \int_{\theta_1}^{\theta_2} \left(\frac{s}{\cos^2\theta}\cos\theta d\theta\right)\left(\frac{\cos^2\theta}{s^2}\right)$$

$$\implies B(\vec{s}) = \frac{\mu_0 I}{4\pi s} \int_{\theta_1}^{\theta_2} \cos\theta d\theta$$

$$\implies B(\vec{s}) = \frac{\mu_0 I}{4\pi s} (\sin\theta_2 - \sin\theta_1)$$

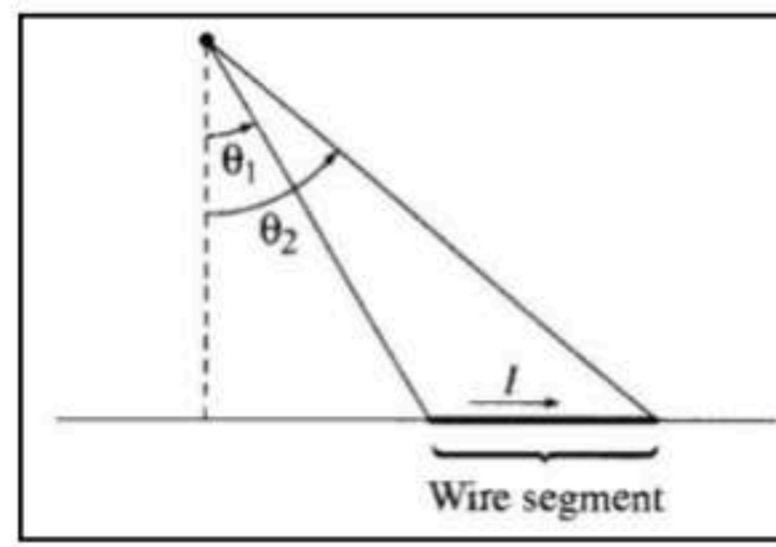


Fig. 5.18, 5.19 (Introduction to Electrodynamics, D. J. Griffiths)

Finite Current Carrying Wire

- If we find the field at a point in a plane half way between the end points of the wire, we will have

$$\theta_1 = -\theta_2 = -\theta \implies B(s) = \frac{\mu_0 I}{2\pi s} \sin\theta$$

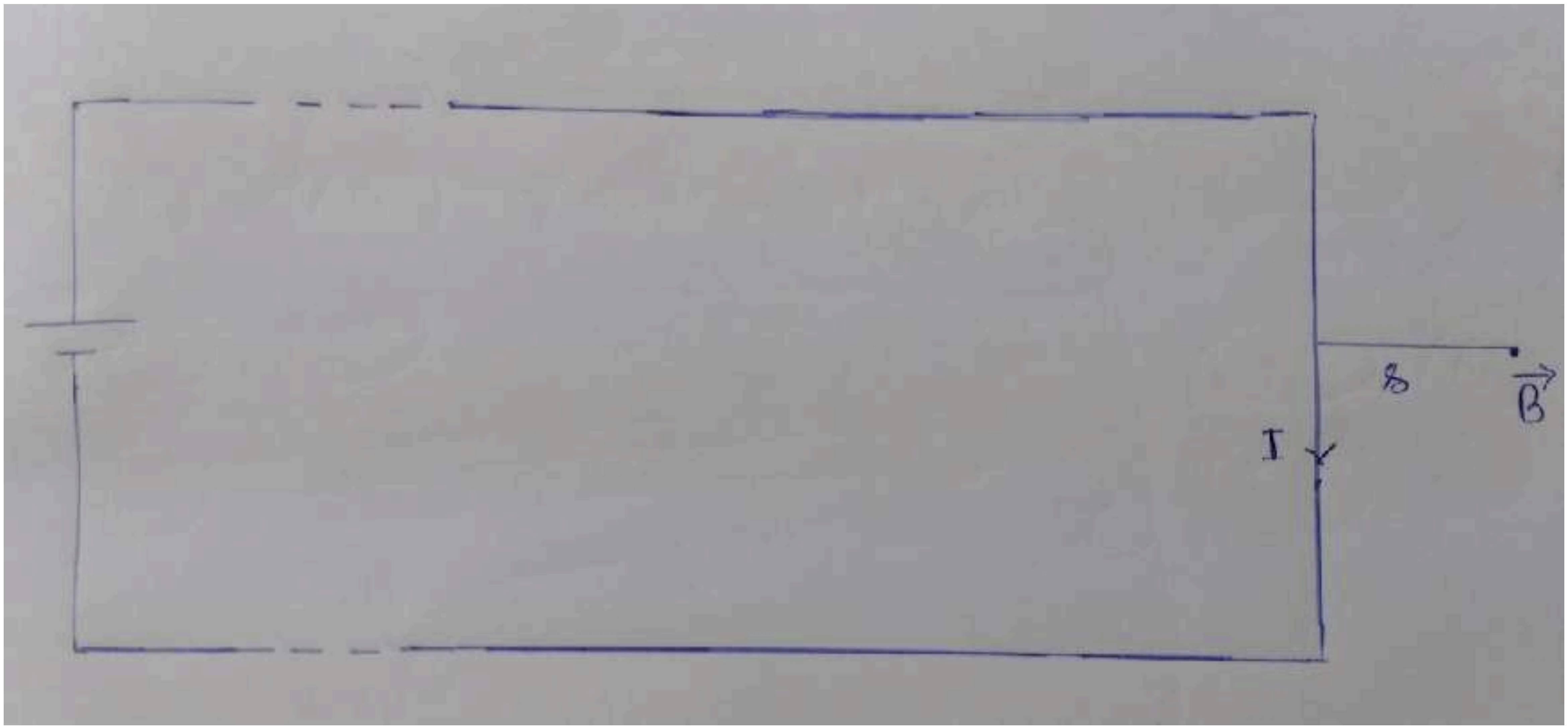
- Use of Ampere's law will give same answer as for an infinite wire $B(s) = \dfrac{\mu_0 I}{2\pi s}$

which is independent of the position of the plane where the point in question is located.

- Thus, Ampere's law and Biot-Savart law give contradictory results: both can not be true!

Finite Current Carrying Wire

- Finite current is an idealised concept. If it is steady current, then it has to be part of a loop:



- But even if we consider the other end of the loop to be infinitely away, yet we lose the azimuthal symmetry, to use Ampere's law.

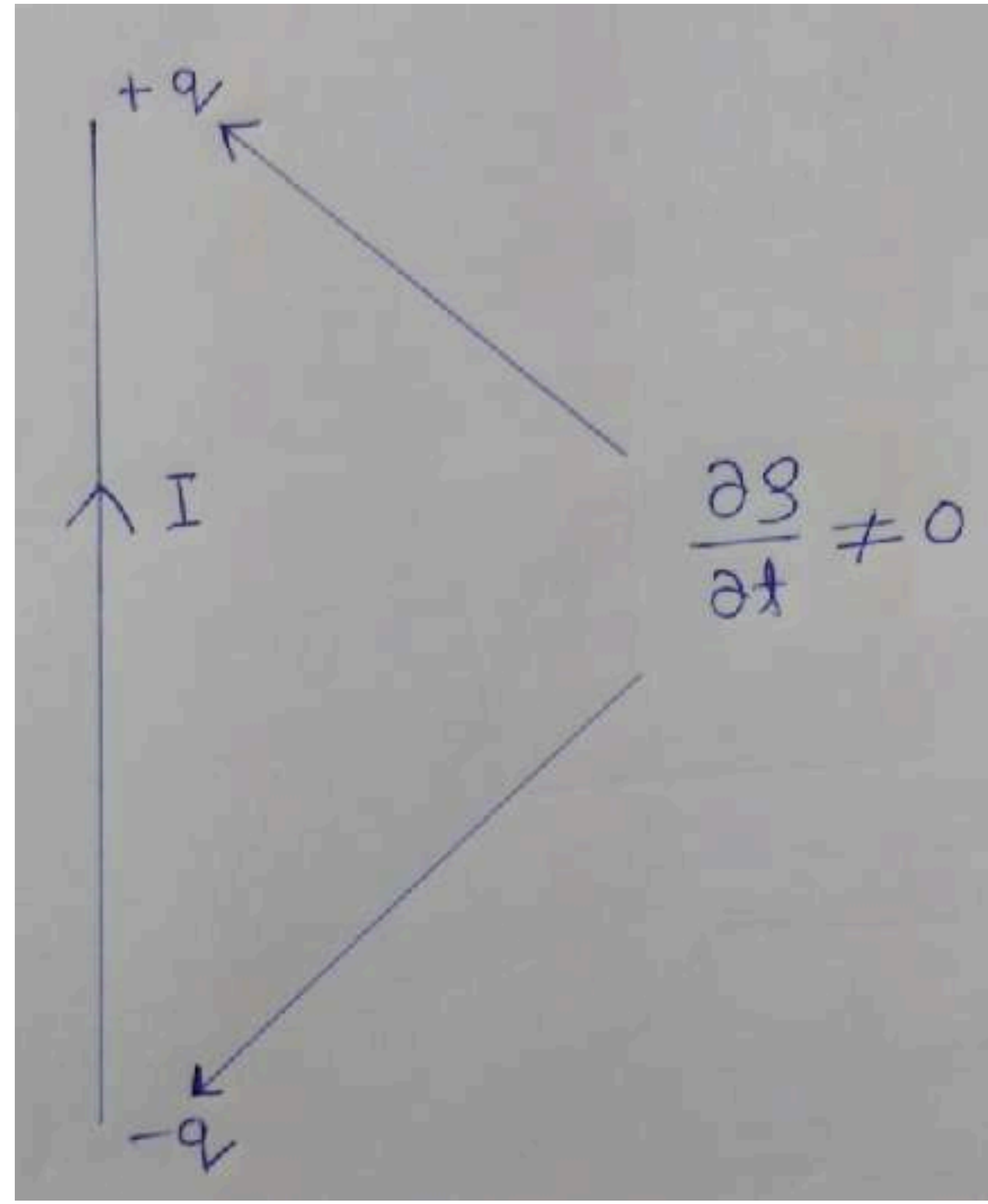
Finite Current Carrying Wire

1. Let us now consider a symmetric situation: the finite wire is not part of a loop and hence has an azimuthal symmetry.

2. But, in accordance with continuity equation,

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

there should be change of charge density at two ends of the wire due to flow of current.



Finite Current Carrying Wire

- Since the current is non-steady, we need to use the corrected version of Ampere's law which is consistent with the continuity equation.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

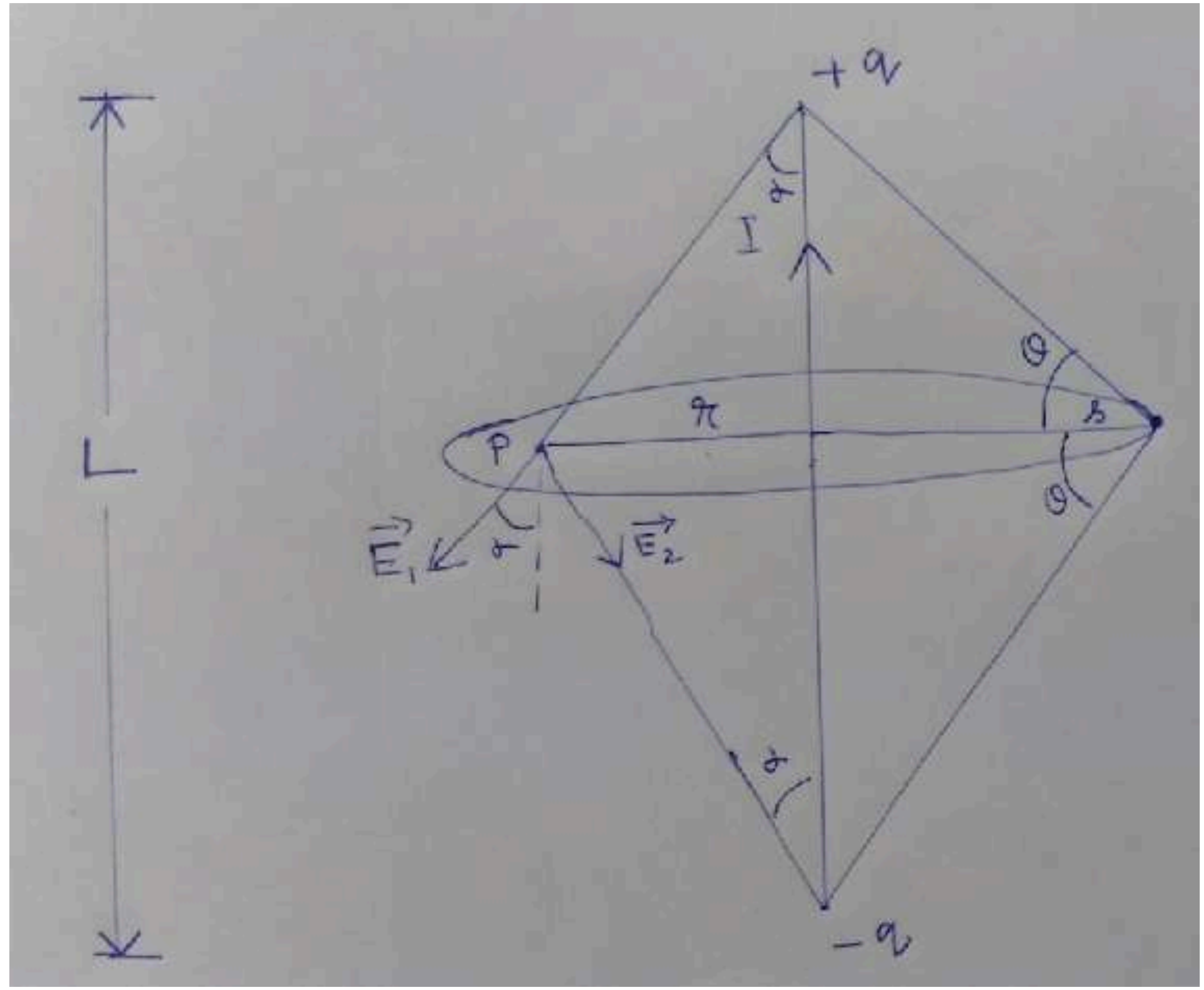
$$\int (\vec{\nabla} \times \vec{B}) \cdot d\vec{a} = \mu_0 \int \vec{J} \cdot d\vec{a} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}, \ \Phi_E = \int \vec{E} \cdot d\vec{a}$$

Electric flux through the surface enclosed by the Amperian loop

Finite Current Carrying Wire

- One has to find the electric flux passing through the surface enclosed by the Amperian loop (See figure on right).
- Let us consider the flux passing through an elemental area da at point P on the surface.



$$d\Phi_E = -\frac{2q}{4\pi\epsilon_0}\frac{\cos\alpha}{(L/2)^2+r^2}da$$

$$da = rdrd\phi \qquad r = \frac{L}{2}\tan\alpha$$

Finite Current Carrying Wire

$$\Phi_E = \int d\Phi_E = -\frac{2q}{4\pi\epsilon_0}\frac{4}{L^2}\int \frac{\cos\alpha}{\sec^2\alpha} r dr d\phi$$

$$\Phi_E = -\frac{2q}{\pi\epsilon_0 L^2}\int \frac{\cos\alpha}{\sec^2\alpha}\frac{L}{2}\tan\alpha\frac{L}{2}\sec^2\alpha d\alpha d\phi$$

$$\Phi_E = -\frac{2q}{4\pi\epsilon_0}\int_0^{\alpha_1}\sin\alpha d\alpha\int_0^{2\pi} d\phi$$

$$\Phi_E = -\frac{q}{\epsilon_0}(1-\cos\alpha_1) = -\frac{q}{\epsilon_0}(1-\sin\theta)$$

Finite Current Carrying Wire

- Using the expression for electric flux, we can find magnetic field from Ampere's law (with Maxwell's correction) as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\frac{q}{\epsilon_0} (1 - \sin\theta) \right)$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I - \mu_0 I + \mu_0 I \sin\theta$$

$$B(2\pi s) = \mu_0 I \sin\theta$$

$$B(s) = \frac{\mu_0 I}{2\pi s} \sin\theta$$

- Which is same as the answer we got using Biot-Savart Law.