Quantum Mechanics (PH101)
Course Instructors: Pankaj Mishra and Tapan Mishra
Tutorial-9
due on Wednesday, 13th of November, 2019 (8:00Hrs IST)

1. Consider a particle of mass $m$ confined in a potential given by

$$V(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq a \\ \infty, & \text{if } x < 0 \quad \text{or} \quad x > a. \end{cases}$$

The energy eigen states of the particle is given by $\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$ with eigen energy $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$, where $n = 1, 2, 3, \ldots$

**(a)**At $t = 0$ an initial wave function is the mixture of two stationary states (first ($\psi_1(x)$) and second ($\psi_2(x)$) energy eigen-states) and given by $\psi(x, t = 0) = \sqrt{\frac{1}{3}}\psi_1(x) + \sqrt{\frac{2}{3}}\psi_2(x)$.

(i) What is the average energy, $< E >$ of the particle at $t = 0$?

(ii)In a measurement of energy, what is the probability to get the energy $E = E_1 = \frac{\pi^2\hbar^2}{2ma^2}$?

(iii) What will be the form of state $\psi(x, t)$ at later time t? Note that $\psi(x, t)$ is the stationary state, so one can write $\psi(x, t) = \psi(x, t = 0)e^{-iEt/\hbar} = \sum_n C_n\psi_n(x, t = 0)e^{-iE_nt}$, where $C_n$ is the probability amplitude of the finding the state in the $n^{th}$ eigen state.

(iv)What is the value of $< E >$ of the particle at time $t$?

**(b)**Consider a particle with energy $E_1$ in the box.

(i) What is expectation value of the position of the particle?
(ii) What is the probability to find it in the region $0 \leq x \leq \frac{a}{2}$?

**(c)** Repeat the above for the particle of energy $E_2$.

2. The probability of finding a particle of mass $m$ confined in an infinitely deep potential well of width $2a$ in the ground state, first excited state and second excited state are 60%, 30% and 10% respectively. Find the normalized wave function of the particle and the energy expectation value.

3. The wave function of a particle of mass $m$ moving in one dimension is $\Psi(x,\ t) = Ae^{-\frac{m\omega}{\hbar}x^2}e^{-i\omega t}$, where $A$ and $\omega$ are constants. Find the potential energy of the particle.

4. Find the probability that a particle trapped in a box $L$ wide can be found between $0.45L$ and $0.55L$ for the ground and first excited states.