Lectures 12-14 (Engineering Mechanics) - Friction

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Friction - Introduction

- We have usually assumed (uptil now) that the forces of action and reaction between contacting surfaces act normal to the surfaces
- This assumption characterizes the interaction between smooth surfaces
- There are many problems in which we must consider the ability of contacting surfaces to support tangential as well as normal forces
- Tangential forces generated between contacting surfaces are called friction forces
- Whenever a tendency exists for one contacting surface to slide along another surface, the friction forces developed are always in a direction to oppose this tendency
- In some cases, we want friction to be minimized (Bearings, propulsion of aircraft and missiles) and in others we want it to be maximized (brakes, clutches, belt drives and wedges)
- Where there is sliding motion between parts, the friction forces result in a loss of energy which is dissipated in the form of heat. Wear is another effect of friction

Frictional Phenomena

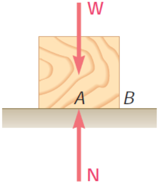
Dry Friction

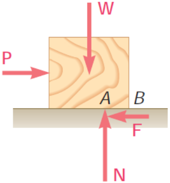
- Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide
- Friction force tangent to surfaces of contact occurs during interval leading upto impending slippage and while slippage takes place
- The direction of this friction force always opposes the motion or impending motion

Fluid Friction

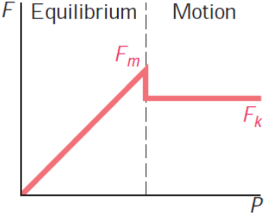
- Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities
- This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers

Laws of Dry Friction - Coefficients of Friction



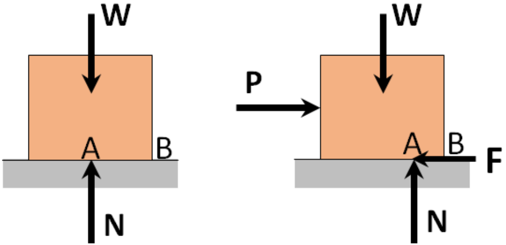


- If P is small, block will not move - balanced by static-friction force F
- F - resultant of number of forces acting over surface of contact: irregularities and molecular attraction



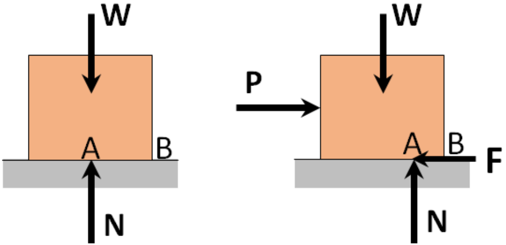
- If $P\uparrow$, $F\uparrow$ until magnitude reaches F_m
- If $P\uparrow$ further, F cannot balance anymore and block starts sliding
- As soon as block has been set in motion, magnitude of F drops from F_m to F_k - **WHY?**
- From then on, block keeps sliding with increasing velocity while F_k(kinetic-friction force), remains approximately constant

Sliding or Tipping ??

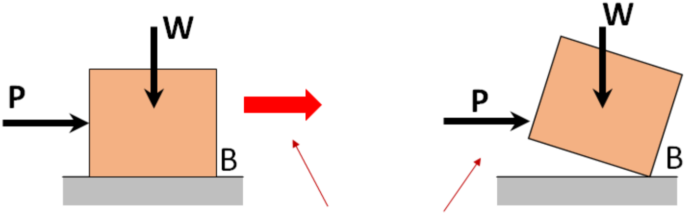


Why does position of N change???

Sliding or Tipping ??



Why does **position of N change**???

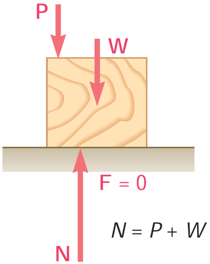


Will it **Slide** or **Tip** ???

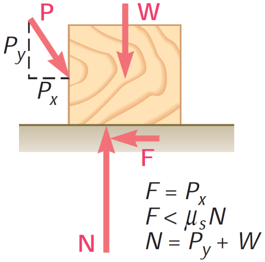
TABLE 8.1 Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

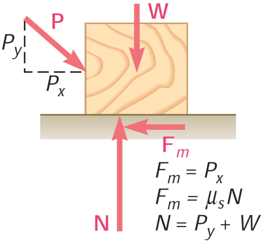
- Range of Static friction - upto point of slippage or impending motion
- F - determined by equilibrium, $F_m = \mu_s N$
- If motion is not impending - $F < \mu_s N$
- Range of kinetic friction - after motion ensues
- F_k - equilibrium does not apply, $F_k = \mu_k N$
- μ_k is 25 percent smaller than μ_s. However, if a body is moving at very low velocity over the surface of another, then $\mu_s \approx \mu_k$
- F_m and F_k are - (i) proportional to normal force (ii) dependent on type and condition of contact surfaces (iii) independent of contact area **iff** normal pressure is such that it cannot deform or crush contacting surface



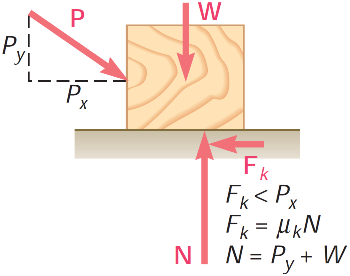
(a) No friction ($P_x = 0$)



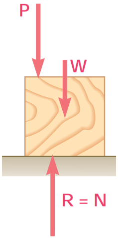
(b) No motion ($P_x < F_m$)



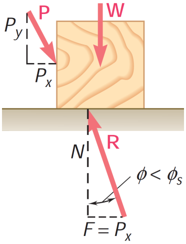
(c) Motion impending $\longrightarrow$ ($P_x = F_m$)



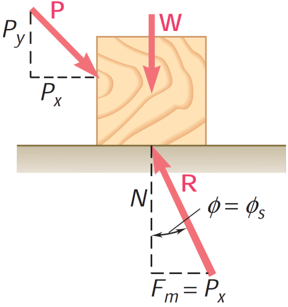
(d) Motion $\longrightarrow$ ($P_x > F_m$)



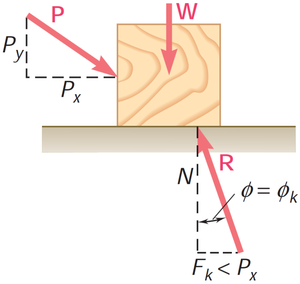
(a) No friction



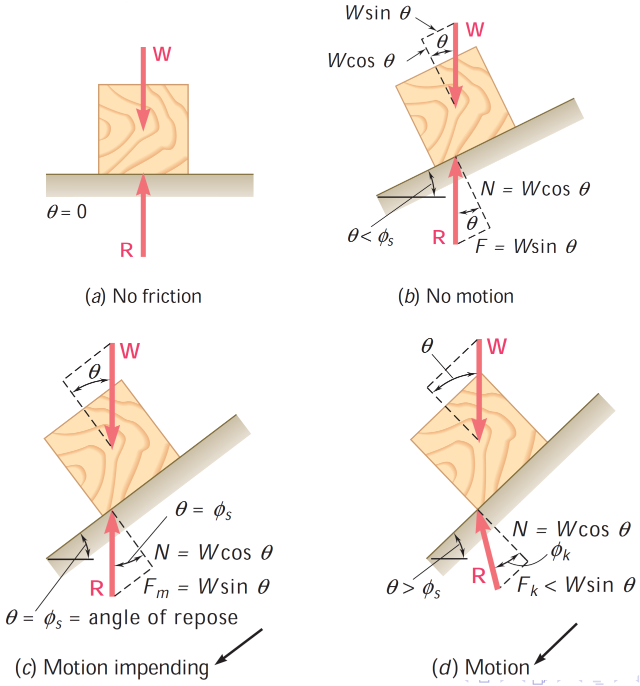
(b) No motion



(c) Motion impending ⟶



(d) Motion ⟶



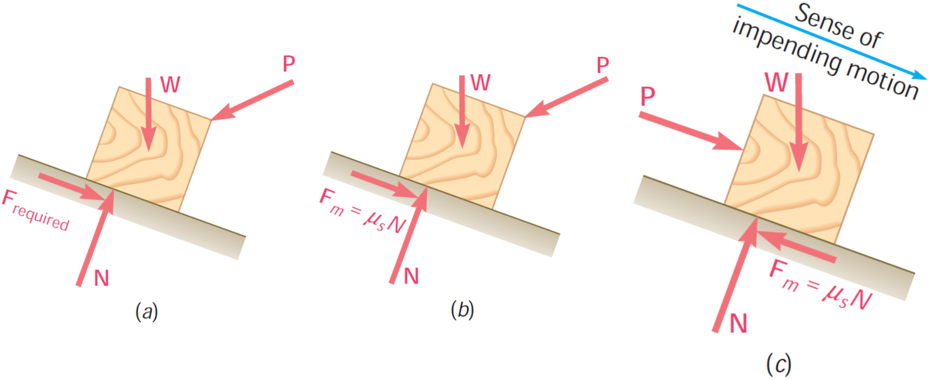
(*a*) No friction

(*b*) No motion

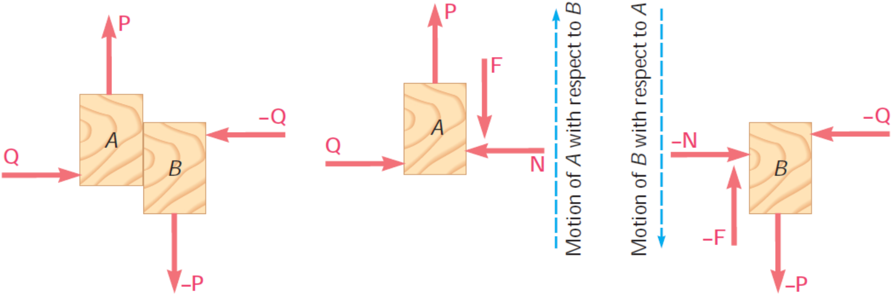
(*c*) Motion impending

(*d*) Motion

Types of Problems

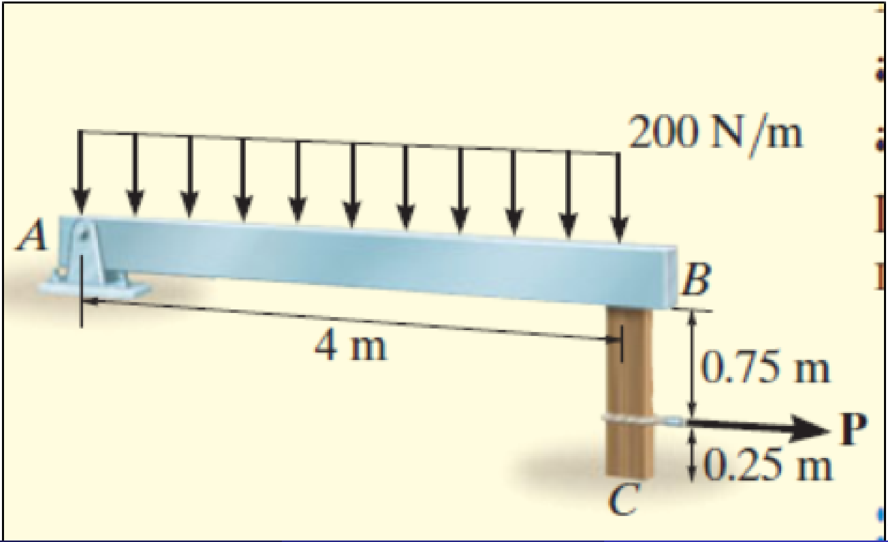


Sense of Friction



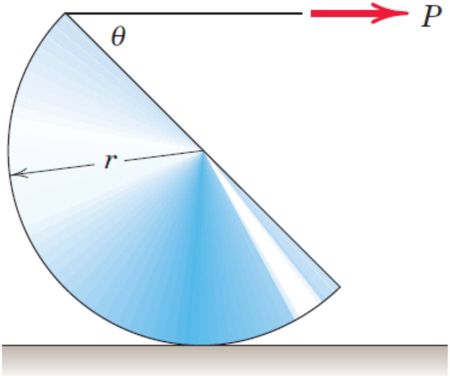
(a) (b) (c)

Problem 1 - A beam AB is subjected to a uniform load of 200 N/m and is supported at B by post BC. If the co-efficients of static friction at B and C are 0.2 and 0.5 respectively, determine force **P** needed to pull the post from under the beam. Neglect weight of members and thickness of beam.

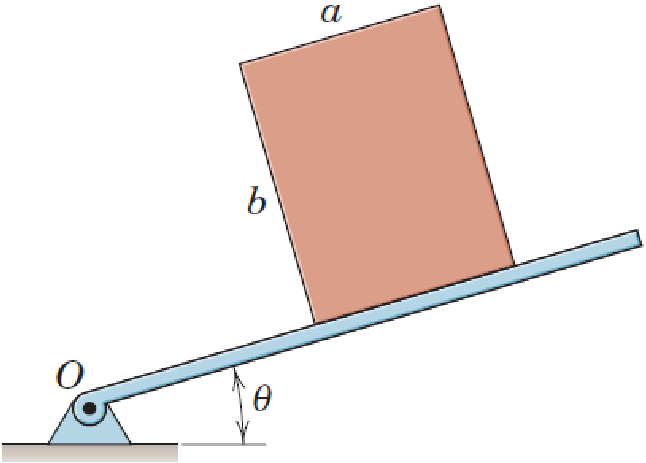


TRY ON YOUR OWN

Problem 2 - The solid semicylinder of mass m and radius r is rolled through an angle by the horizontal force P. If the coefficient of static friction is μ_s, determine the angle θ at which the cylinder begins to slip on the horizontal surface as P is gradually increased. What value of μ_s would permit θ to reach 90^0?



Problem 3 - The homogeneous rectangular block of mass m rests on the inclined plane which is hinged about a horizontal axis through O. If the coefficient of static friction between the block and the plane is μ, specify the conditions which determine whether the block tips before it slips or slips before it tips as the angle θ is gradually increased.

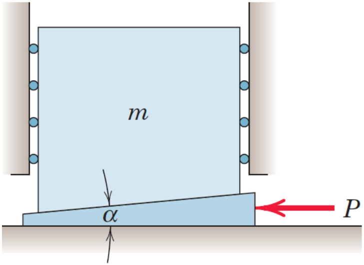


Wedges

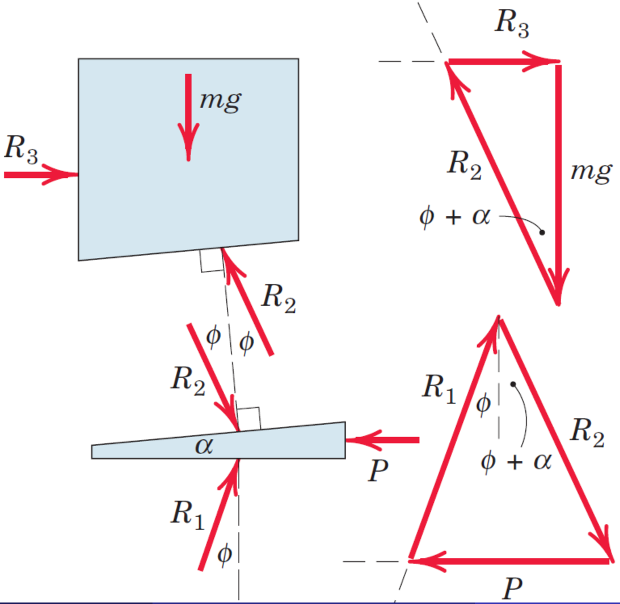
- A wedge is used to produce small adjustments in the position of a body or to apply large forces. Largely depend on friction

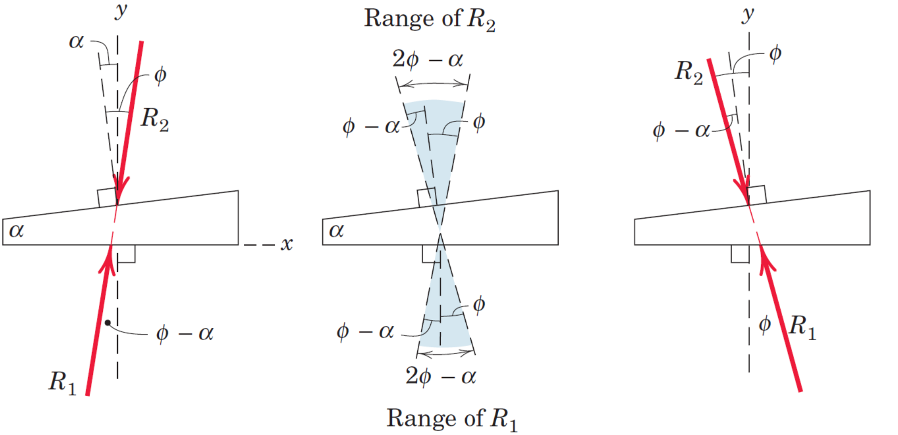
Wedges are used as shown to split tree trunks because the normal forces exerted by the wedges on the wood are much larger than the forces required to insert the wedges

- When sliding of a wedge is impending, the resultant force on each sliding surface of the wedge will be inclined from the normal to the surface by an amount equal to the friction angle
- The component of the resultant along the surface is the friction force, which is always in the direction to oppose the motion of the wedge relative to the mating surfaces



Forces to raise the load



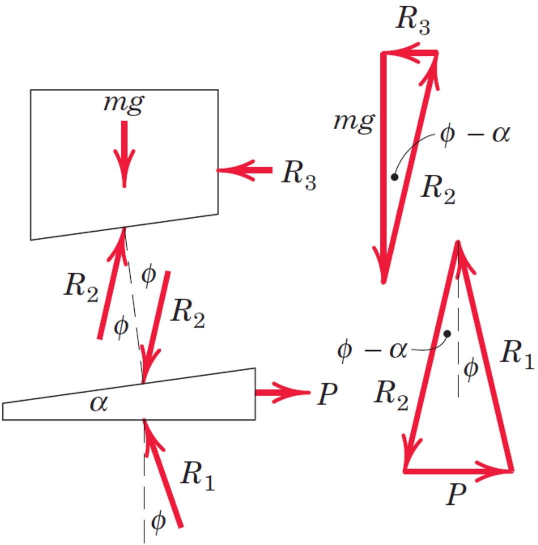


(a) Slipping impending at upper surface

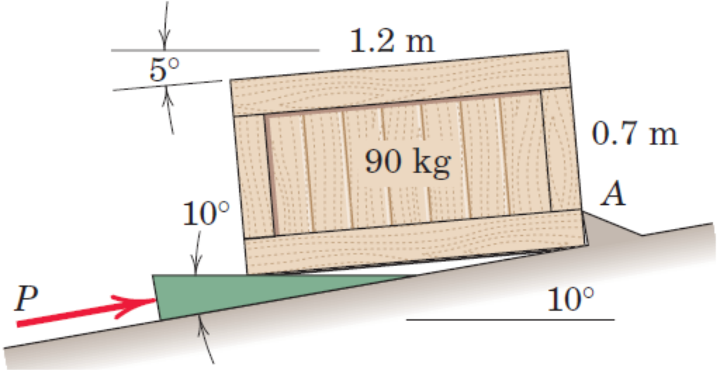
(b) Range of $R_1 = R_2$ for no slip

(c) Slipping impending at lower surface

Forces to lower the load

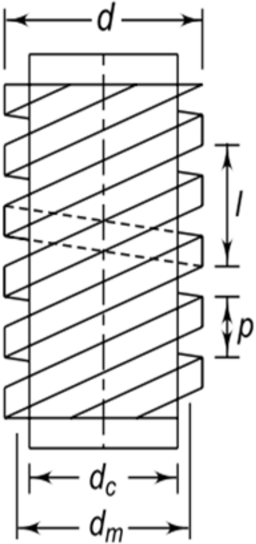


Problem 4 - Determine the force P required to force the 10^0 wedge under the 90-kg uniform crate which rests against the small stop at A. The coefficient of friction for all surfaces is 0.40.



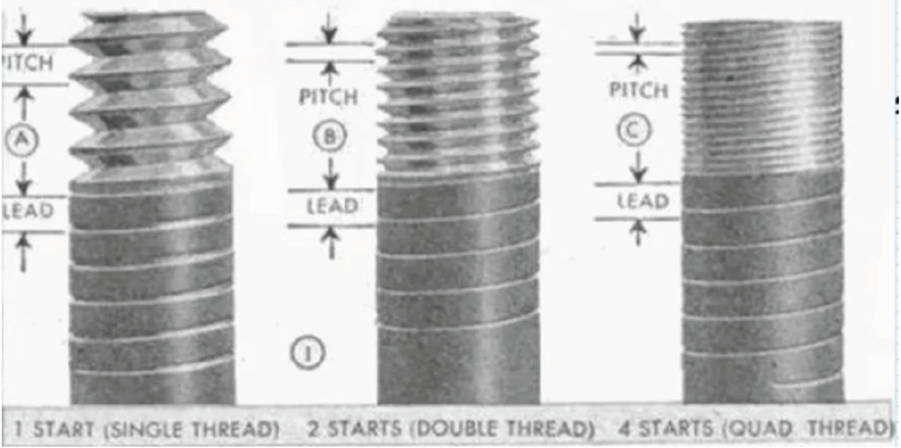
Screwjack

- Basic use- Raise Heavy weights by small distances.
- Screws are used for fastening and for transmitting power or motion
- Square-threaded screws are frequently used in jacks, presses, and other mechanisms.
- Friction developed in the threads largely determines the action of the screw.
- Different Types of Threads which can be used are- Square, V-Shape, Buttress, ACME, Whitworth, etc.
- For Screw Jacks, Square threads are found to me most efficient.

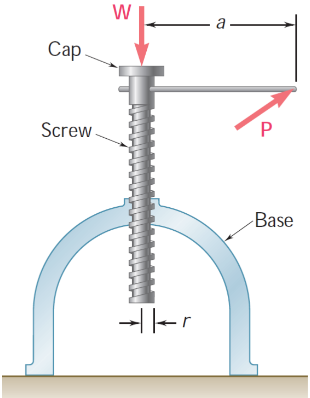


- Pitch - Distance measured parallel to the axis of the screw from a point on one thread to the corresponding point on the adjacent thread.
- Nominal Diameter - largest diameter of the screw. Also called major diameter.
- Core diameter- smallest diameter of the screw thread. Also called minor diameter

Pitch and Lead



Force Analysis



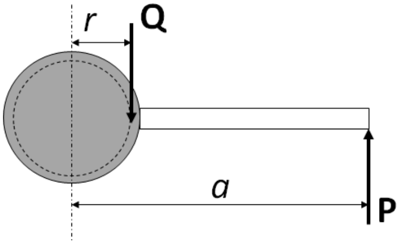
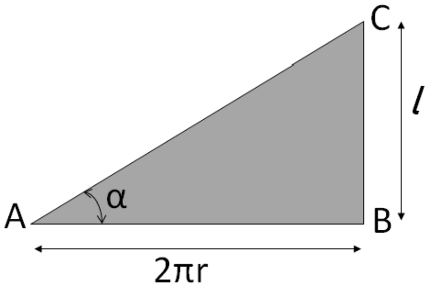


Figure: Q - Equivalent effort applied at mean radius of screwjack

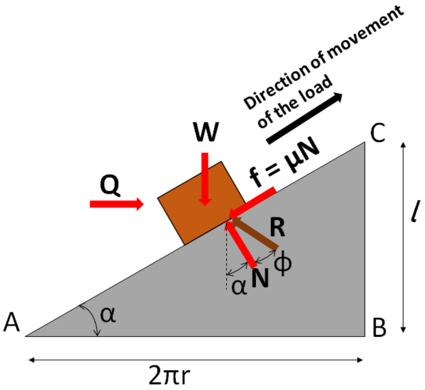
- The screw carries a load W and is supported by the base of the jack.
- Screw-base contact takes place along a portion of their threads. By applying a force P on the handle, the screw can be made to turn and to raise the load W.
- $Q \times r = P \times a, \Rightarrow \quad Q = \dfrac{Pa}{r}$

Equivalence with inclined plane problem



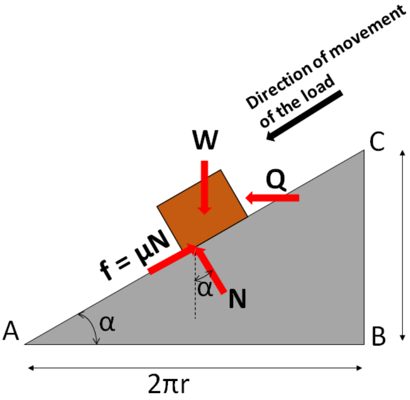
- When handle is rotated by one turn, screw is also rotated by one turn and the screw will have vertical displacement equal to lead l.
- **Lifting of Weight "W" in Screw Jack can be compared with a Weight "W" being lifted on an angular incline.**
- Lead of a screw is the Lift of Weight because of one complete revolution of Screw. Lead – 1) Single thread= pitch, 2) Multiple thread $= n\times$pitch
- α - angle of the screw

Force Analysis - Raising the Load



- Resolving forces along the inclined plane - $f + W\sin\alpha = Q\cos\alpha$
- Resolving forces normal to inclined plane - $N = W\cos\alpha + Q\sin\alpha$
- Replacing $f = \mu N$ and $\mu = \tan\phi$ and solving, we have $Q = W\tan(\alpha+\phi)$
- Replacing $Q = \dfrac{Pa}{r}$, we have $P = \dfrac{r}{a} \times W\tan(\alpha+\phi)$
- Torque required to raise the load - $T = P \times a = rW\tan(\alpha+\phi)$

Force Analysis - Lowering the Load



- Resolving forces along the inclined plane -
  $f = W\sin\alpha + Q\cos\alpha$
- Resolving forces normal to inclined plane -
  $N = W\cos\alpha - Q\sin\alpha$
- Replacing $f = \mu N$ and $\mu = \tan\phi$ and solving, we have
  $Q = W\tan(\phi - \alpha)$
- Replacing $Q = \dfrac{Pa}{r}$, we have
  $P = \dfrac{r}{a} \times \tan(\phi - \alpha)$
- Torque required to lower the load -
  $T = P \times a = rW\tan(\phi - \alpha)$

Lowering the Load - Two cases, $T = rW\tan(\phi - \alpha)$

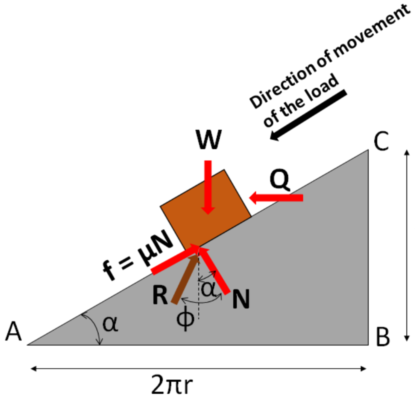


Figure: $\phi \geq \alpha$ - Self locking, a net positive torque is required to lower load, and it will not lower automatically.
$T = rW\tan(\phi - \alpha)$

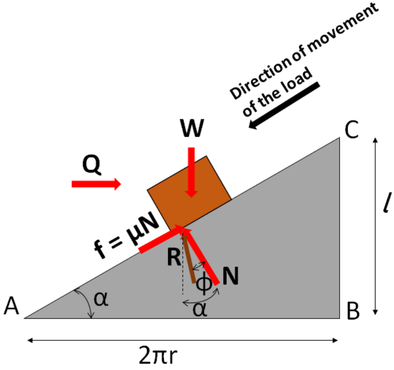


Figure: $\phi < \alpha$ - value of torque required to lower load in negative. $\Rightarrow$ lowers automatically as no effort required. This is undesirable (called as "Overhauling").
$T = rW\tan(\alpha - \phi)$

Efficiency, Velocity Ratio and Mechanical Advantage

Let P_{ideal} be ideal effort (absence of friction) required to lift load **W**.

$$P_{ideal} = \frac{r}{a} W \tan\alpha.$$

Efficiency of a screw jack is given by

$$\eta = \frac{\textit{Ideal Effort}}{\textit{Actual Effort}} = \frac{P_{ideal}}{P} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

Maximum Efficiency - happens when $2\alpha + \phi = 90^0$, $\eta_{max} = \dfrac{1-\sin\phi}{1+\sin\phi}$

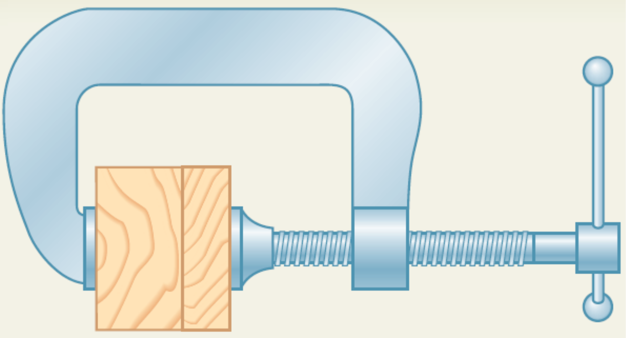
Velocity Ratio (V.R.) - Ratio of distance moved by effort **P** to distance moved by load

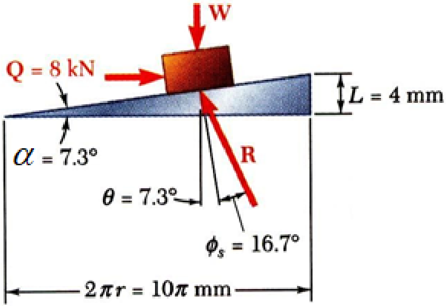
Mechanical Advantage (M.A.) - Ratio of Load lifted to effort applied

Try at home

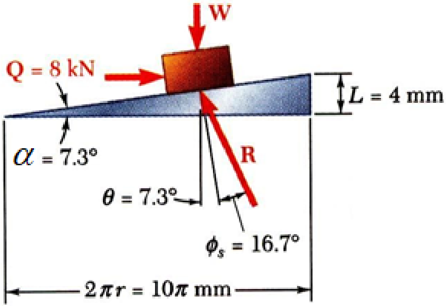
Is efficiency related to M.A. and V.R. ???
What is efficiency for lowering load - ???

Problem 5 - A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread of mean diameter equal to 10 mm with a pitch of 2 mm. The coefficient of friction between threads is $\mu_s = 0.3$. If a maximum couple of 40 N-m is applied in tightening the clamp, determine (a) the force exerted on the pieces of wood, (b) the couple required to loosen the clamp.





- Tightening clamp is analogous to moving up an inclined plane - **WHY ??**
- $\alpha = \tan^{-1}\left(\frac{L}{2\pi r} = \frac{2(2)}{10\pi} = 0.1273\right) = 7.3^0$
- $\phi_s = \tan^{-1}\mu_s = 16.7^0$



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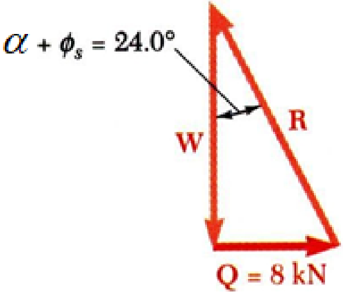
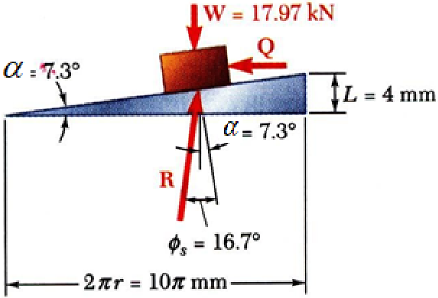
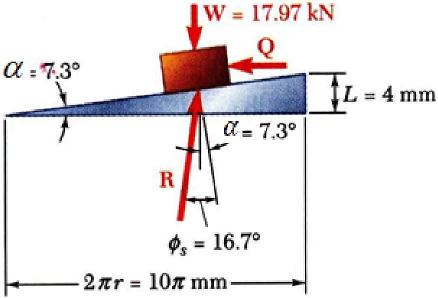


- Using block and plane analogy with impending motion up the plane, calculate clamping force with force triangle
- $Qr = 40$ N-m, $\rightarrow$ Q=8 kN
- $\tan(\alpha + \phi_s) = \frac{Q}{W}, \;\Rightarrow\; W = \frac{8\ kN}{\tan 24^0} = 17.97\ kN$



- Loosening clamp is analogous to moving down an inclined plane - **WHY ??**
- With impending motion down the plane, calculate the force and torque required to loosen the clamp



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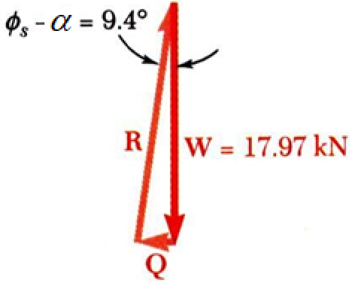
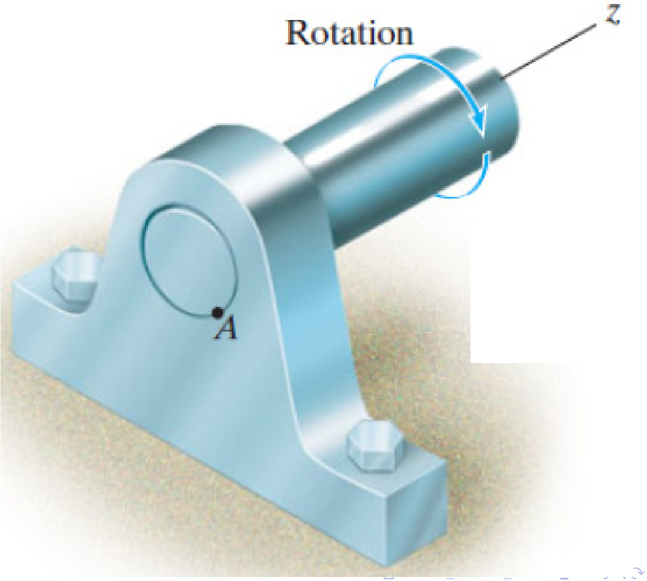


- $\tan(\phi_s - \alpha) = \frac{Q}{W}$, $\Rightarrow$ $Q =$ $(17.97\ kN) \tan\ 9.4^0 = 2.975\ kN$
- Torque $=$ $Qr = (2.975\ kN)(5\ mm)$=14.87 N-m

Journal Bearing (Axle friction)

Figure: Journal bearings are used to provide lateral support to rotating shafts and axles



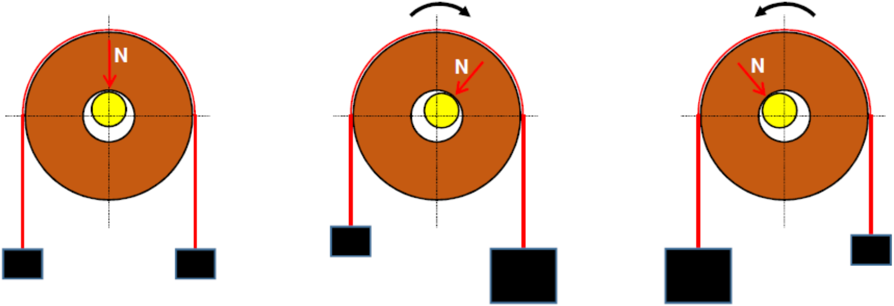


Figure: Rotation causes journal to slide within bearing. Point of application of the normal reaction is where the journal begins to slip

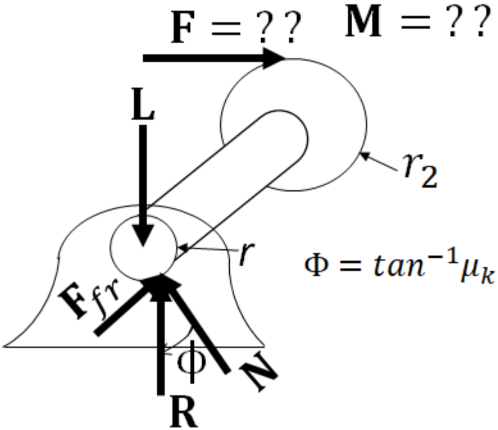
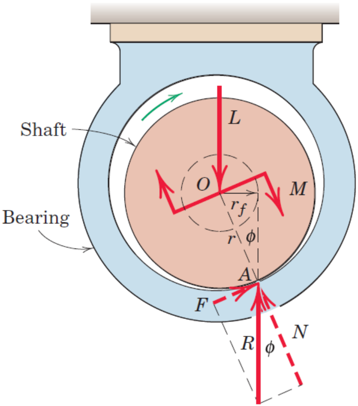


Figure: Assumptions: Bearing only partially or NOT lubricated, so that bearing and axle are in direct contact along a single straight line

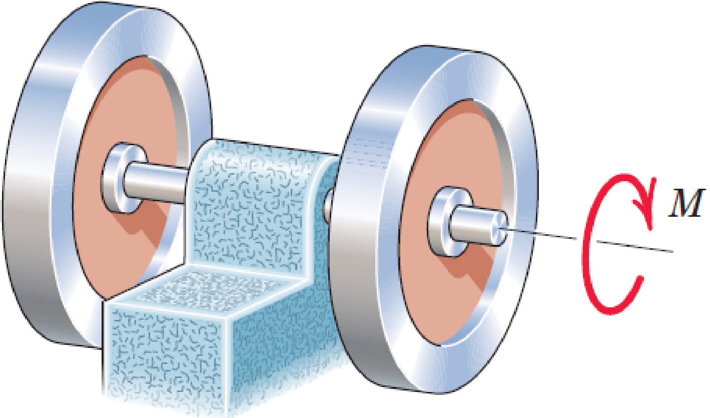
FBD

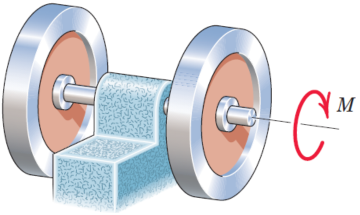


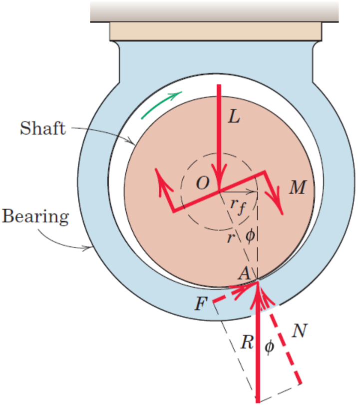
- Torque **M** required to maintain motion and load **L** on the shaft will cause reaction at A.
- For vertical equilibrium **R** must equal **L** but will not be collinear with it
- Thus, **R** will be tangent to a small circle of radius r_f called the friction circle
- Equating sum of moments about A to zero gives - $M = Lr_f = Lr \sin \phi$
- For small coefficient of friction, ϕ is small. Hence, we have $M = \mu Lr$

This moment $M = \mu Lr$ must be the same (but opposite in sense) to moment required to overcome the friction. Hence, $M = \mu Lr = Fr_2$.

Problem 6 - The two flywheels are mounted on a common shaft which is supported by a journal bearing between them. Each flywheel has a mass of 40 kg, and the diameter of the shaft is 40 mm. If a 3-N m couple **M** on the shaft is required to maintain rotation of the flywheels and shaft at a constant low speed, compute (a) the coefficient of friction in the bearing and (b) the radius *r* of the friction circle.







Solution: Draw the FBD of the shaft and the bearing

(a) Moment equilibrium at O

$M = Rr_f = Rrsin\phi$

$M = 3$ Nm, $R = 2x40x9.81 = 784.8$ N, $r = 0.020$ m

$\rightarrow sin\phi = 0.1911 \rightarrow \phi = 11.02^o$

(b) $r_f = rsin\phi = 3.82$ mm