Frequency Response

Frequency response is the response of a system as a function of frequency. In an electrical system or circuit the response corresponds to a voltage or current as a function of frequency.

Fig. 1

Fig. 1 shows a system with ***x(t)*** as the input and ***y(t)*** as the output. The input ***x(t)*** and the output ***y(t)*** can be voltages or currents in the system. Based on the properties of the system, the output or response ***y(t)*** can be different from the input by two aspects. Either the amplitude or the phase or both can change. The system is designed in such a way that the output should be a desired response with a target amplitude on phase. Studying or analysing the frequency response has the goal of finding how the output of a system varies with respect to the frequency of the input sinusoid or the signal.

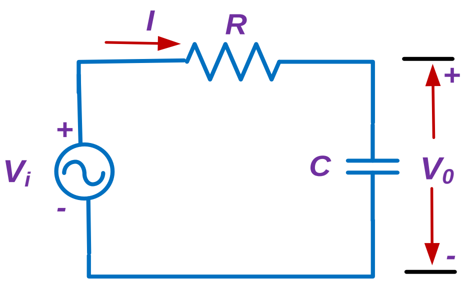

Fig. 2

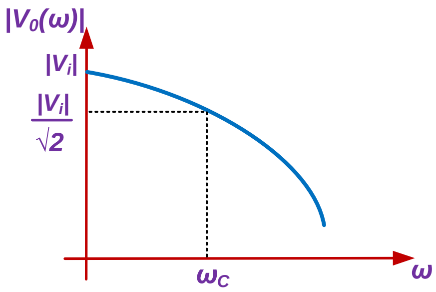

Fig. 3

Fig. 2 shows a RC circuit. ***Vi*** is the input voltage and ***Vo*** is the output voltage. **<u>Vo</u>** can be found out as a function of frequency by using phasor analysis of the circuit.

$$V_i - IR - \frac{1}{j\omega C} I = 0$$

$$V_0 = \frac{\mathrm{I}}{j\omega C} \Rightarrow I = j\omega C V_0$$

$$V_i - j\omega C V_0 R - V_0 = 0$$

$$\Rightarrow V_0 = \frac{V_i}{1 + j\omega CR}$$

$$\Rightarrow \frac{V_0}{V_i} = \frac{1}{1 + j\omega CR}$$

$$\left|\frac{V_0}{V_i}\right| = \frac{1}{\sqrt{1+\omega^2 C^2 R^2}}$$

Fig. 3 shows $|V_0(\omega)|$ as a function of ω. It has a maximum value i.e., $|V_0(\omega)| = |V_i(\omega)|$ when $\omega = 0$. As ω increases, $|V_0(\omega)|$ decreases. This is a low-pass characteristics. This is used as a low-pass filter. It will allow low frequency components to pass through the system. At the same time it will attenuate the high frequency components.

At a frequency ω_c, the magnitude of the response is $\frac{1}{\sqrt{2}}$ times the maximum value. This frequency is called the cutoff frequency or half power frequency. As the power is proportional to the square of the voltage, the power will be halved when the voltage magnitude is attenuated by a factor of $\sqrt{2}$. The magnitude of the voltage at this frequency is dropped by -3 dB ($20log_{10}\left(\frac{1}{\sqrt{2}}\right)$). So this is also called as -3 dB frequency.

$$\frac{1}{\sqrt{1+\omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$
$$\Rightarrow \omega_c = \frac{1}{RC}$$

The time domain analysis of the circuit will provide a different insight.

$$v_i = Ri + \frac{1}{C}\int i dt$$
$$i = C\frac{dv_0}{dt}$$

$$v_i = RC\frac{dv_0}{dt} + v_0$$

If ***RC***, i.e. the time constant of the circuit is chosen as very large then

$$RC >> 1$$
$$v_i = RC\frac{dv_0}{dt}$$
$$\Rightarrow v_0 = \frac{1}{RC}\int v_i dt$$

This low-pass filter circuit can be used as an analog integrator by choosing a high time constant.

Fig. 4 shows another ***RC*** circuit. The output voltage is taken across the resistor ***R***.

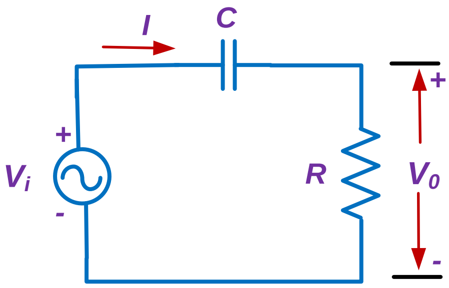

Fig. 4

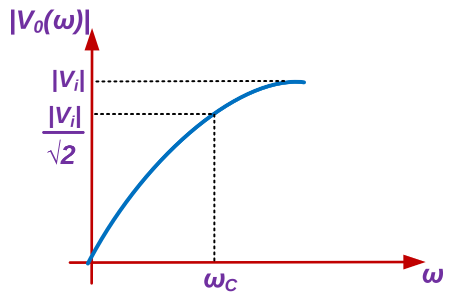

Fig. 5

$$V_0 = IR \Rightarrow I = \frac{V_0}{R}$$

$$V_i = \frac{I}{j\omega C} + V_0$$

$$\Rightarrow V_i = \frac{V_0}{j\omega CR} + V_o$$

$$\Rightarrow \frac{V_0}{V_i} = \frac{1}{1 + \frac{1}{j\omega CR}}$$

$$\left|\frac{V_0}{V_i}\right| = \frac{1}{\sqrt{1 + \frac{1}{\omega^2 C^2 R^2}}}$$

$$At \quad \omega = 0$$

$$\left|\frac{V_0}{V_i}\right| = 0 \Rightarrow |V_0(\omega)| = 0$$

and

$$At \quad \omega = \infty$$

$$\left|\frac{V_0}{V_i}\right| = 1 \Rightarrow |V_0(\omega)| = |V_i(w)|$$

Fig. 5 shows the frequency response of the circuit. The response has a minimum value at $\omega = 0$. The response increases as ω increases and it has a maximum value as ω tends to infinity. The half power frequency or cutoff frequency or -3 dB frequency is given as

$$\omega_c = \frac{1}{RC}$$

Time domain analysis of this circuit will show the following

$$v_o = iR$$
$$i = C\frac{d(v_i - v_0)}{dt}$$
$$\Rightarrow \frac{v_0}{R} = C\frac{dv_i}{dt} - C\frac{dv_0}{dt}$$
$$\Rightarrow v_0 + RC\frac{dv_0}{dt} = RC\frac{dv_i}{dt}$$

If the time constant ***RC*** is very small, the circuit can be used as a differentiator

$$RC \ll 1$$
$$v_0 = RC\frac{dv_i}{dt}$$