WELCOME TO IIT GUWAHATI

PH 101: PHYSICS 1

Lecture 1

Uday Narayan Maiti
Room No.: 17,*(Center for environment, core III)*
Department of Physics, IIT Guwahati
Email: udaymaiti@iitg.ac.in

Class Divisions

- Division – I**: CE and ME**
- Division –II: **BT, ECE and EEE**
- Division – III: **CSE and M&C**
- Division – IV: **CL, CST and EPH**

Time Line…

- First Day of Instruction: **25th July 2019**
- Quiz-1: **29th August (Tentative)**
- Mid Semester Examinations: **16 September 2019 to 22 September 2019**
- Quiz – II: **6th November (Tentative)**
- End Semester Examinations: **22nd November 2019 to 29th November 2019**

Lecture schedule

Lectures	Div-I / Div-II (AM)	Div-III / Div-IV (PM)
~~Monday~~	~~11:00 - 11:55~~	~~2:00 – 2:55~~
Thursday	9:00 – 9:55	4:00 – 4:55
Friday	10:00 – 10:55	3:00 – 3:55
Wednesday	Tutorial for all during 8:00 – 8:55 AM	

- ❖ Primarily presented with slides. Slides will be available online after the class
- ❖ Do not hesitate to interrupt if you have doubts during the class

http://www.iitg.ac.in/phy/ph101_2019.php

Tutorial Groups

Sl. No.	Group	Discipline	Roll No.	Room No.	Tutor
1	T-1	CSE	190101001 – 190101045	L1	KP
2	T-2	CSE	190101046 – 190101090	L2	PM
3	T-3	CSE	190101091 – 190101101	L3	TM
		ECE	190102001 – 190102034		
4	T-4	ECE	190102035 – 190102079	L4	AKSh
5	T-5	ECE	190102080 – 190102092	1005	UR
		ME	190103001 – 190103032		
6	T-6	ME	190103033 – 190103077	1006	GSS
7	T-7	ME	190103078 – 190103105	1G1	PKP
		CE	190104001 – 190104017		
8	T-8	CE	190104018 – 190104062	1G2	MCK
9	T-9	CE	190104063 – 190104101	1104	BRM
		BT	190106001 – 190106006		

Tutorial Groups

Sl. No.	Group	Discipline	Roll No.	Room No.	Tutor
10	T-10	BT	190106007 – 190106051	1205	BRB
11	T-11	BT	190106051 – 190106075	2203	BB
		CL	190107001 – 190107021		
12	T-12	CL	190107022 – 190107066	2205	SB
13	T-13	CL	190107067 – 190107084	3102	PKG
		EE	190108001 – 190108027		
14	T-14	EE	190108028 – 190108057	3202	AP
		EP	190121001 – 190121015		
15	T-15	EP	190121016 – 190121060	4005	SC
16	T-16	CST	190122001 – 190122045	4006	UNM
17	T-17	CST	190122046 – 190122057	4G1	SBD
		MC	190123001 – 190123033		
18	T-18	MC	190123034 – 190123064	4G2	Sovan
19	T-19	All backlog students		1203	SG

Tutorials

- The main purpose of the tutorial is to provide you with an opportunity to interact with a teacher.
- The teacher will assist you in clearing your doubts and answer your queries regarding the course topics.
- A problem sheet will be given to you for your practice. These problems also indicate the difficulty level of the examinations.
- You are expected to attempt these problems before you come to the tutorial class. Ask your doubts regarding these problems to your teacher during the tutorial class.
- The teacher may or may not solve all the problems in the tutorial class. In case you find a problem very difficult, do ask your teacher to help you.

Syllabus

Topics to be covered

- ❑ Classical Mechanics (midsem)
- ❑ Relativity
- ❑ Quantum Mechanics

- ❑ **Santabrata Das and Uday Narayan Maiti will instruct upto Midsem**
- ❑ **Pankaj Mishra and Tapan Mishra will instruct after Midsem**

Syllabus

Up to midsem

Calculus of variation: Fermat's principle, Principle of least action, Euler-Lagrange equations and its applications.

Lagrangian mechanics: Degrees of freedom, Constraints and constraint forces, Generalized coordinates, Lagrange's equations of motion, Generalized momentum, Ignorable coordinates, symmetry and conservation laws, Lagrange multipliers and constraint forces.

Hamiltonian mechanics: Concept of phase space, Hamiltonian, Hamilton's equations of motion and applications.

Books

Texts:

1. Introduction to Classical Mechanics by R. Takwale and P. Puranik [McGraw Hill Education, 1 st Ed., 2077]
2. Classical mechanics by John Taylor [University Science Books, 2005].

References:

- ❖ A Student's Guide to Lagrangians and Hamiltonians by Patrick Hamill [Cambridge University Press, l-st edition, 20L3].
- ❖ Theoretical Mechanics by M. R. Spiegel [Tata McGraw Hill, 2008].
- ❖ The Feynman Lectures on Physics, Vol. I by R. P. Feynman, R. B. Leighton, and M. Sands [Narosa Publishing House, 1998]

Syllabus

After midsem

Special theory of Relativity: Postulates of STR. Galilean transformation. Lorentz transformation. simultaneity. Length Contraction. Time dilation. Relativistic addition of velocities. Energy momentum relationships.

Quantum Mechanics: Two-slit experiment. De Broglie's hypothesis. Uncertainty Principle, wave function and wave packets, phase and group velocities. Schrtidinger Equation. Probabilities and Normalization. Expectation values. Eigenvalues and eigenfunctions.

Applications in one dimension: Infinite potential well and energy quantization. Finite square well, potential steps and barriers - notion of tunnelling, Harmonic oscillator problem zero point energy, ground state wavefunction and the stationary states.

Books

Texts:

1.Introduction to Classical Mechanics by Takwale R and Puranik P (McGraw Hill Education, 1 st Ed., 2077).

2. Classical mechanics by f John Taylor (University Science Books, 2005).

3. Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles by R. Eisberg and R, Resnick [f ohn-Wiley,2 nd Ed., 2006)

References:

7. A Student's Guide to Lagrangians and Hamiltonians by Patrick Hamill (Cambridge University Press, l-st edition, 20L3).

2. Theoretical Mechanics by M. R. Spiegel [Tata McGraw Hill, 2008).

3. The Feynman Lectures on Physics, Vol. Iby R. P. Feynman, R. B. Leighton, and M. Sands, [Narosa Publishing House, 1998J

Evaluations

- **Quiz-1** of 10% weightage (tentatively on 29th Aug.)
- ***Mid-Semester*** exam of 30% weightage
- **Quiz-2** of 10% weightage
- ***End-Semester*** exam of 50% weightage

Class Manners

- Maintain Silence
- Attendance Rule will be strictly followed
- Mobile Phone Policy

Goal: Beyond Newtonian mechanics

Introduction of new concepts of mechanics beyond Newton’s law: Largangian and Hamiltonian equations of dynamics

Goal: Beyond Newtonian mechanics

Why this is important?

- **An example: Describing the motion of a pendulum**.
- In Newtonian mechanics, we start by drawing a diagram with multiple arrows for all the forces which are acting on the pendulum.

- We then find how the pendulum is moving by using Newton's second law: $\vec{F} = m\vec{a}$

- More generally, we use Newton's three laws as fundamental laws of nature and derive everything else from there. It is centered around forces, since these are ultimately used to figure out the trajectories.

Goal: Beyond Newtonian mechanics

Why this is important?

- In **Lagrangian** mechanics, things work differently.
- We start by calculating the kinetic and potential energies of the pendulum.
- Instead of Newton’s three laws, we assume that there is another fundamental law of nature: **the principle of least action**.
- According to this principle, we calculate the motion by minimizing a certain quantity, called the action, that is related to the two forms of energy mentioned above.
- Lagrangian mechanics therefore is centered around energies. Forces are no longer needed to determine the motion of objects.

Goal: Beyond Newtonian mechanics

Why this is important?

- Both formalisms of course lead to the same trajectory for the pendulum.

- Moreover, it is possible to show more generally that **these two formalism are equivalent**. However, they each are better suited for certain types of problems.

Goal: Beyond Newtonian mechanics

Why this is important?

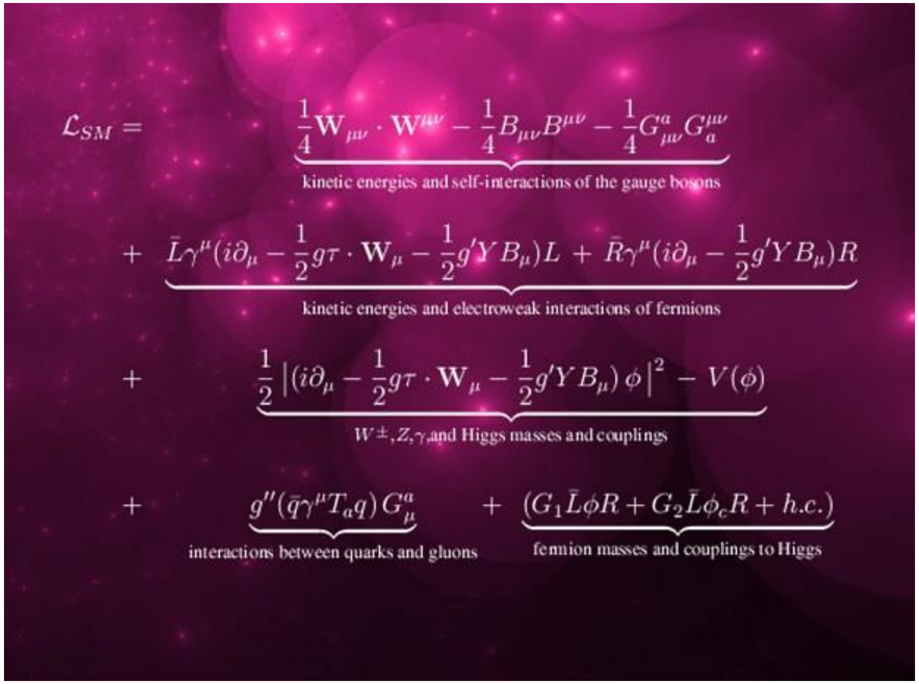
- Overall, Lagrangian mechanics might just seem like a different way to calculate all the boring stuff you already knew anyway.

- However, it is much more! It also is a gateway to discover a lot more of the interesting and amazing properties of nature!

Goal: Beyond Newtonian mechanics

Why this is important?

- For your feeling, the Lagrangian of the standard model that describes the constituents of the Universe is as follows:



Goal: Beyond Newtonian mechanics

Why this is important?

- Besides being more convenient in problem solving, there is a deeper reason: **to introduce Lagrangian mechanics**.
- It turns out that many of the fundamental laws of physics can easily be described by such a principle of least action.
- For example, the concepts are extendable to other modern area of physics like quantum mechanics, field theory etc.

Layout of mechanics course

- Mathematical concepts of partial differentiation and coordinate systems.
- Constrain, degree's of freedom and generalized coordinates.
- Challenges with unknown nature of constrain forces in Newtonian Mechanics.
- D'Alembert's Principle of virtual work to remove the constrain forces from analysis.
- Lagrange's equation: An alternative to Newton's law.
- Variational method and Lagrange's equation from variational principle.
- Hamiltonian equations of motion

Lets start with basic mathematical concepts

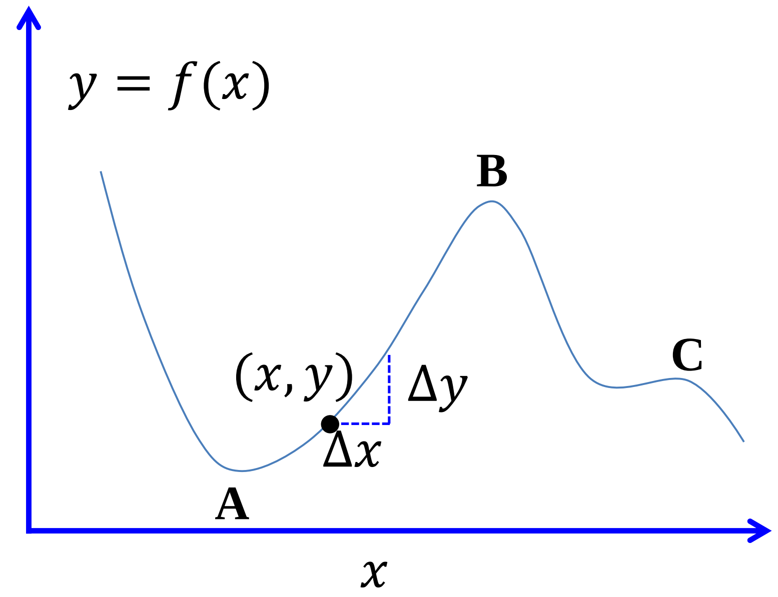
Key to understand classical mechanics

Total Differential: Function of one variable

$\boldsymbol{Given}$: $\boldsymbol{y = f(x)}$ **is a function of one variable** $\boldsymbol{x}$

$$f'(x) = \frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x} = \underset{\Delta x \to 0}{Lt} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

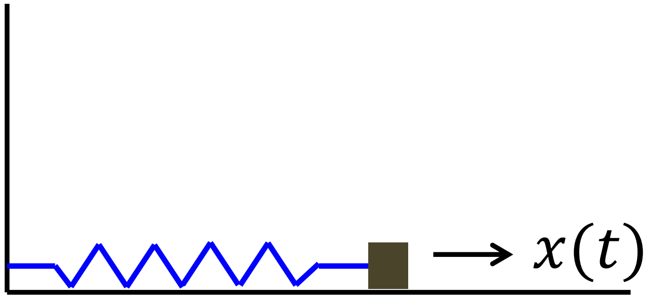
$$\boldsymbol{dy = [f'(x)]\, dx = \left(\frac{dy}{dx}\right) dx}$$



- Infinitesimal change of y around certain point (x) =(rate of change of y around the point) (magnitude of change in x)

- At stationary points (A, B, C), y does not changes [$dy = 0$] even if x is changed infinitesimally, which implies that at those points $f'(x) = 0$ (Stationary points)

Examples of total differential in mechanics



Motion in 1D

- Particle moving in along a line (say x −direction), $x(t)$, the instantaneous position is a function of time.

Velocity $v(x) = \dot{x} = \frac{dx}{dt}$

Acceleration $a(x) = \dot{v} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Examples of total differential in mechanics

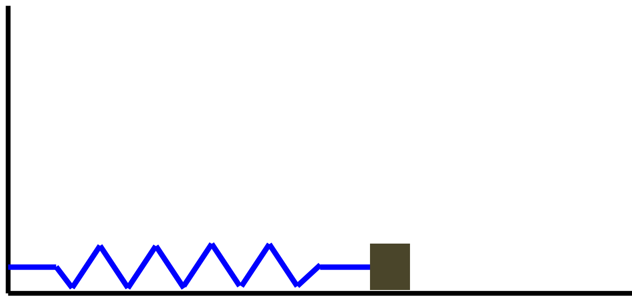
Potential energy is a function of position (spring elongation) $U(x)$,

- Increase in potential energy due to displacement dx;

$$dU = \left(\frac{dU}{dx}\right) dx.$$

- **Work done against the restoring force**:

$-F_x\, dx$. (minus sign for work against the force)

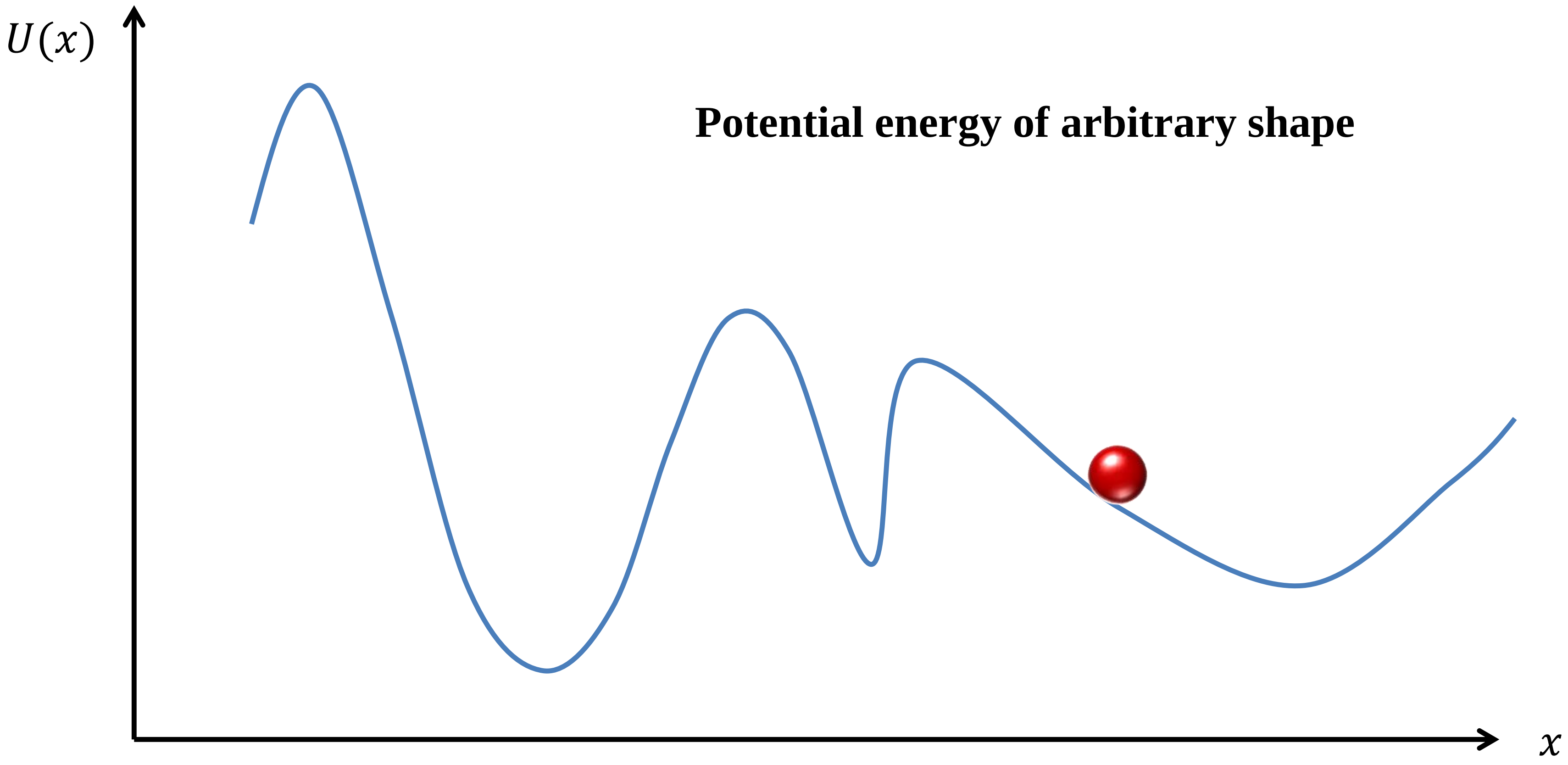


- **Conservation of energy**: Increase of potential energy = work done

$$-F_x dx = dU = \left(\frac{dU}{dx}\right) dx;$$

$$\text{Thus,} \quad F_x = -\frac{dU}{dx}$$

Potential energy in 1D



Restoring force

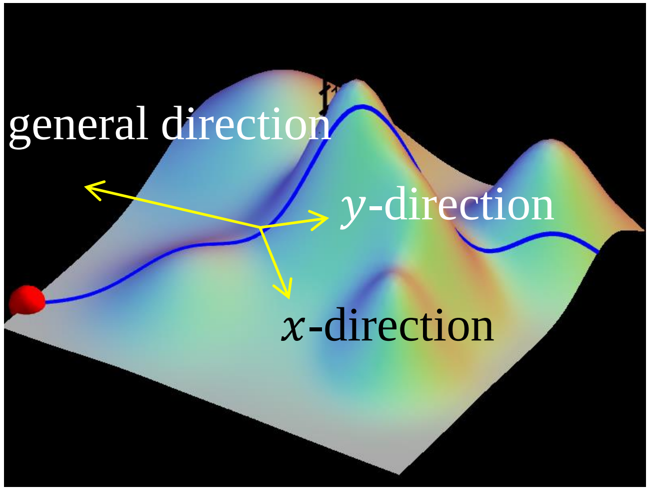
$$F_x = -\left(\frac{dU}{dx}\right)$$

is different at different point

Partial differential: function of more than one variables

$f(x, y)$ depends on two independent variables x and y.

Example: Height (f) of a hill as function of position coordinates (x, y).



- The rate of change of height (slope) in the 'x' *direction*, when ***y* remains constant**, is denoted by

$$\left(\frac{\partial f}{\partial x}\right)_{\boldsymbol{y}} = \underset{\Delta x \to 0}{Lt} \frac{f(x + \Delta x, \boldsymbol{y}) - f(x, \boldsymbol{y})}{\Delta x}$$

- The rate of change in the 'y' *direction*, when ***x* remains constant** is denoted by

$$\left(\frac{\partial f}{\partial y}\right)_{\boldsymbol{x}} = \underset{\Delta y \to 0}{Lt} \frac{f(\boldsymbol{x}, y + \Delta y) - f(\boldsymbol{x}, y)}{\Delta y}$$

Partial differential

- How much height will change if I walk in the 'x' direction [**keeping 'y' fixed**] by 'dx' ?

$$[df]_{dx} = \left(\frac{\partial f}{\partial x}\right) dx$$

$$= (rate\ of\ change\ in\ x\ direction)(amount\ of\ change\ in\ x)$$

- Similarly,

$$[df]_{dy} = \left(\frac{\partial f}{\partial y}\right) dy$$

- How much height will change if I go in the arbitrary direction so that 'x' changes by 'dx' and 'y' also changes by 'dy'

$$df = \left(\frac{\partial f}{\partial x}\right) dx + \left(\frac{\partial f}{\partial y}\right) dy \quad = [df]_{dx} + [df]_{dy}$$

- Generalization for a function which depends on several variables $f(x_1, x_2, x_3 \ldots.\ x_n)$

$$df = \left(\frac{\partial f}{\partial x_1}\right) dx_1 + \left(\frac{\partial f}{\partial x_2}\right) dx_2 + \ldots\ldots. + \left(\frac{\partial f}{\partial x_n}\right) dx_n = \sum_i \left(\frac{\partial f}{\partial x_i}\right) d\, x_i$$

A proof

- For a function $f(x,y)$, if x changes by Δx and y changes by Δy, then total change in f

$$\Delta f = f(x+\Delta x, y+\Delta y) - f(x,y)$$

$$\Delta f = f(x+\Delta x, y+\Delta y) - f(x+\Delta x, y) + f(x+\Delta x, y) - f(x,y)$$

$$\Delta f = \left[\frac{f(x+\Delta x, y+\Delta y) - f(x+\Delta x, y)}{\Delta y}\right]\Delta y + \left[\frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}\right]\Delta x$$

$\Delta x, \Delta y \to 0$

$$df = \left(\frac{\partial f}{\partial y}\right) dy + \left(\frac{\partial f}{\partial x}\right) dx$$

Reciprocity relation

- If $f(x, y)$, then one can also express x as a function of y and f.

To emphasize that x, y, f are all in the same footing, replace f $with$ z.

This does not imply that x, y, z are coordinate positions (might be),

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz \qquad \text{[if you express } x(y,z)\text{]}$$

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz \qquad \text{[if you express y}(x,z)\text{]}$$

Substituting,

$$dx = \left(\frac{\partial x}{\partial y}\right)_z \left[\left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz\right] + \left(\frac{\partial y}{\partial z}\right)_x dz$$

If $z = x^2 y$
You can also

write $x = \left(\frac{z}{y}\right)^{1/2}$

Or

$y = \frac{z}{x^2}$

- If z remain constant, $dz = 0$,

then we obtain *reciprocity relation*

$\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial y}{\partial x}\right)_z^{-1}$, for simplification of writing, we use $\left(\frac{\partial x}{\partial y}\right)$=$\left(\frac{\partial y}{\partial x}\right)^{-1}$

Differentiation of function of functions

- So far we have considered the differentiation of a function $f(x, y)$ w. r. t. x, y.

Now consider the case where x and y are function of another variable say, u.

We wish to find the derivative $\frac{df}{du}$.

Example: $f = xy + \ln y^2$ [f *is* **Explicit function** of **independent** variables x, y

and **implicit function** of u],

where $x = a \cos u$ and $y = a \sin u$

How to calculate $\frac{df}{du}$?

Method 1: Direct substitution

Step 1:Substitution: $f = (a \cos u)(a \sin u) + \ln(a \sin u)^2$

Step 2: Find $\frac{df}{du}$

$$\frac{df}{du} = a^2(cos^2u - sin^2u) + 2\cot u$$

Chain rule of partial differential

Method 2: Chain rule

You know, $df = \left(\frac{\partial f}{\partial x}\right) dx + \left(\frac{\partial f}{\partial y}\right) dy$

Thus $\frac{df}{du} = \left(\frac{\partial f}{\partial x}\right)\frac{dx}{du} + \left(\frac{\partial f}{\partial y}\right)\frac{dy}{du}$,

Given: $f = xy + \ln y^2; And\ x = a\ \cos u$ and $y = a\ \sin u$

$$\left(\frac{\partial f}{\partial x}\right)_y = y\ ;\left(\frac{\partial f}{\partial y}\right)_x = x + \frac{2}{y}\ ;\ \frac{dx}{du} = -a \sin u;$$

$$\frac{dy}{du} = a \cos u$$

Find the differentials $\left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{dx}{du}, \frac{dy}{du}\right]$ individually and then substitute in the above relation.

$$\frac{df}{du} = y(-a \sin u) + \left(x + \frac{2}{y}\right)(a \cos u) = a^2(cos^2 u\ - sin^2 u) + 2 \cot u$$

- Generalization for a function depends **explicitly** on several independent variables $f(x_1, x_2, x_3 \ldots.\ x_n)$, and the variables are function of another set of independent variables,

i. e., $x_i\ (u_1, u_2, \ldots.\ u_n)$

$$df = \left(\frac{\partial f}{\partial x_1}\right) dx_1 + \left(\frac{\partial f}{\partial x_2}\right) dx_2 + \ldots\ldots. + \left(\frac{\partial f}{\partial x_n}\right) dx_n = \sum\left(\frac{\partial f}{\partial x_i}\right) dx_i$$

$$\frac{\partial f}{\partial u_1} = \left(\frac{\partial f}{\partial x_1}\right)\frac{\partial x_1}{\partial u_1} + \left(\frac{\partial f}{\partial x_2}\right)\frac{\partial x_2}{\partial u_1} + \ldots\ldots. + \left(\frac{\partial f}{\partial x_n}\right)\frac{\partial x_n}{\partial u_1} = \sum\left(\frac{\partial f}{\partial x_i}\right)\frac{\partial x_i}{\partial u_1}$$

$$\frac{\partial f}{\partial u_j} = \sum_1^n \left(\frac{\partial f}{\partial x_i}\right)\frac{\partial x_i}{\partial u_j}$$

Use of chain rule

If a set of coordinates x_i is function of another **set of independent coordinates** q_j and time (t).
Prove that : $\frac{\partial \dot{x}_i}{\partial \dot{q}_j} = \frac{\partial x_i}{\partial q_j}$ [Cancellation of dots]; [Where $\dot{x}_i$ and $\dot{q}_j$ are time derivatives of x_i and q_j respectively.

$$\boldsymbol{Given}\ ,\ x_i = x_i\ (q_1, q_2, \ldots, q_n, \mathrm{t})$$

$$\dot{x}_i = \frac{dx_i}{dt} = \frac{\partial x_i}{\partial q_1}\frac{dq_1}{dt} + \frac{\partial x_i}{\partial q_2}\frac{dq_2}{dt} + \ldots\ldots\ldots + \frac{\partial x_i}{\partial q_n}\frac{dq_n}{dt} + \frac{\partial x_i}{\partial t}\frac{dt}{dt}$$

$$\dot{x}_i = \frac{\partial x_i}{\partial q_1}\dot{q}_1 + \frac{\partial x_i}{\partial q_2}\dot{q}_2 + \ldots\ldots\ldots + \frac{\partial x_i}{\partial q_n}\dot{q}_n + \frac{\partial x_i}{\partial t}$$

$$\frac{\partial \dot{x}_i}{\partial \dot{q}_1} = \frac{\partial x_i}{\partial q_1}$$

In general, $\frac{\partial \dot{x}_i}{\partial \dot{q}_j} = \frac{\partial x_i}{\partial q_j}$

Summary of partial differential

- If a function 'f' depends on several variables; I,e $f(x_1, x_2, x_3 \ldots. x_n)$, then

$$df = \left(\frac{\partial f}{\partial x_1}\right)dx_1 + \left(\frac{\partial f}{\partial x_2}\right)dx_2 + \ldots\ldots + \left(\frac{\partial f}{\partial x_n}\right)dx_n = \sum\left(\frac{\partial f}{\partial x_i}\right)dx_i$$

- Reciprocity relation: $\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial y}{\partial x}\right)_z^{-1}$

- If a function depends on several variables $f(x_1, x_2, x_3 \ldots. x_n)$ and the variables are functions of another set of variables, I,e $x_i\ (u_1, u_2, \ldots. u_n)$

$$\frac{\partial f}{\partial u_j} = \sum_{1}^{n}\left(\frac{\partial f}{\partial x_i}\right)\frac{dx_i}{du_j}$$

Questions please