ME101: Engineering Mechanics (3 1 0 8)

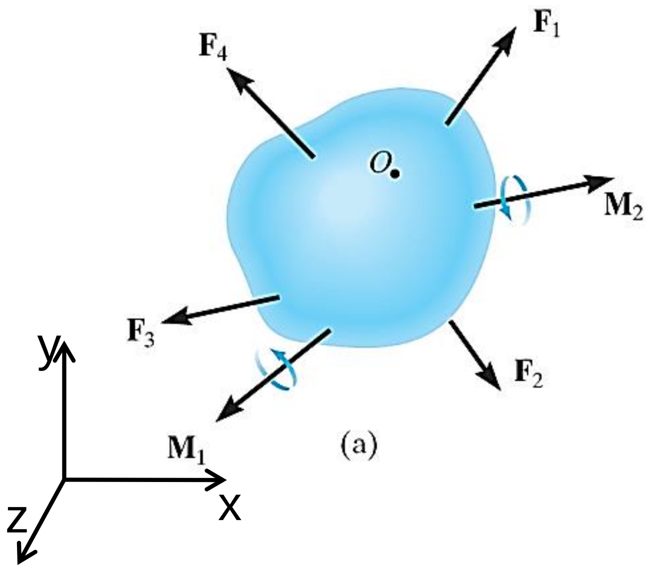
2019-20 (II Semester)

LECTURE: 4

Rigid Body Equilibrium

A rigid body will remain in equilibrium provided

- Sum of all the external forces acting on the body is equal to zero, and
- Sum of the moments of the external forces about a point is equal to zero

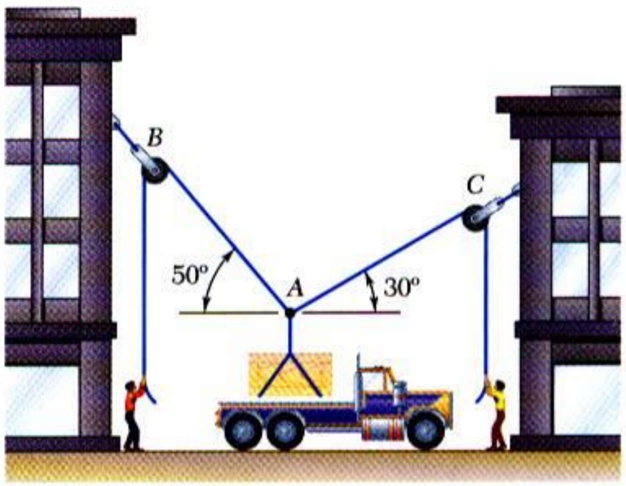


$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

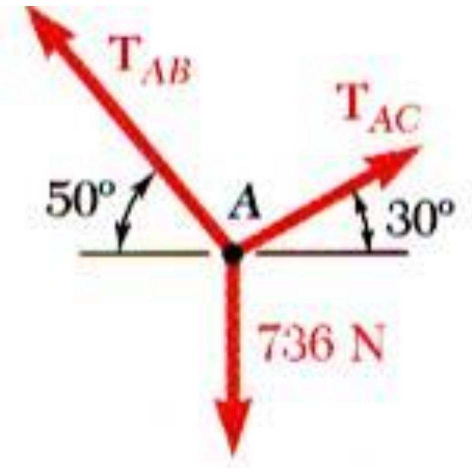
$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0$$

Rigid Body Equilibrium

Free-Body Diagrams



Space Diagram: A sketch showing the physical conditions of the problem.



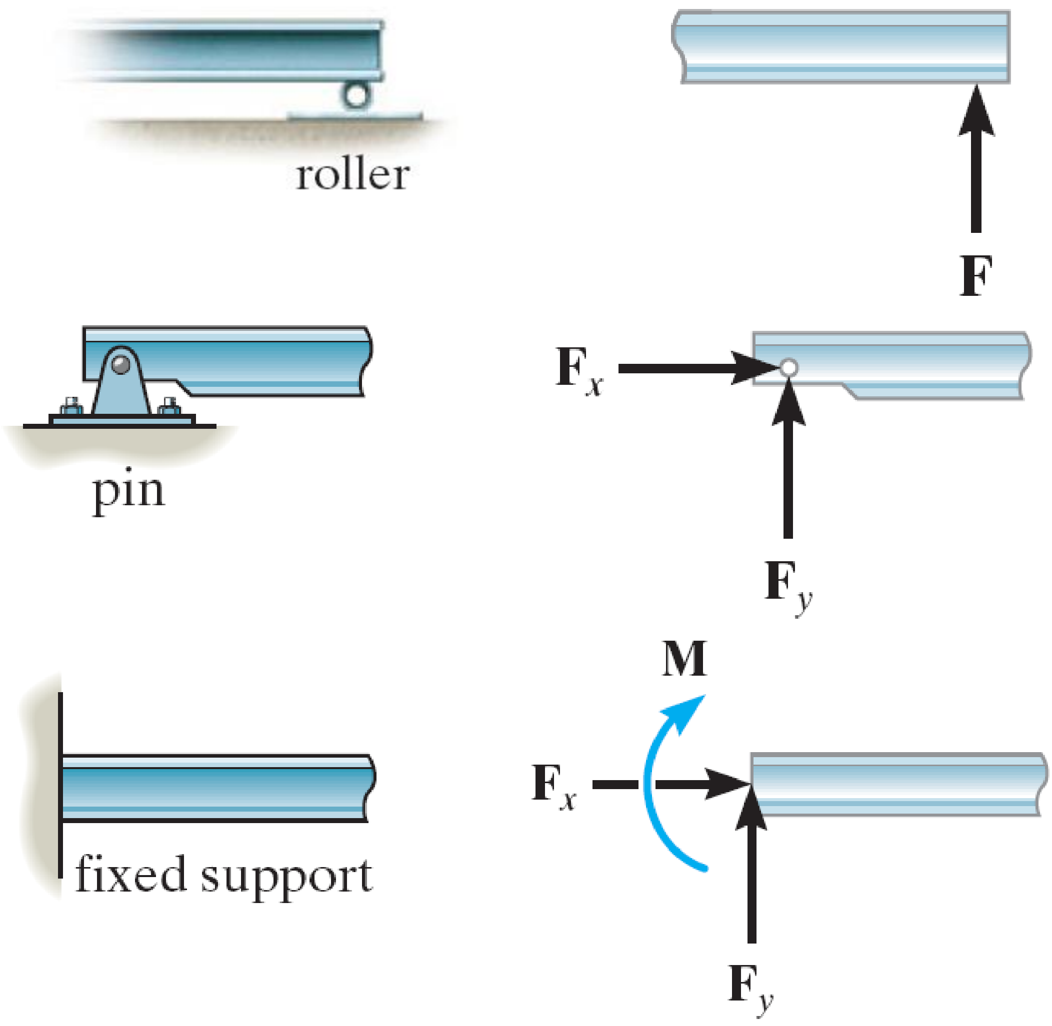
Free-Body Diagram: A sketch showing only the forces on the selected particle.

Rigid Body Equilibrium

Support Reactions

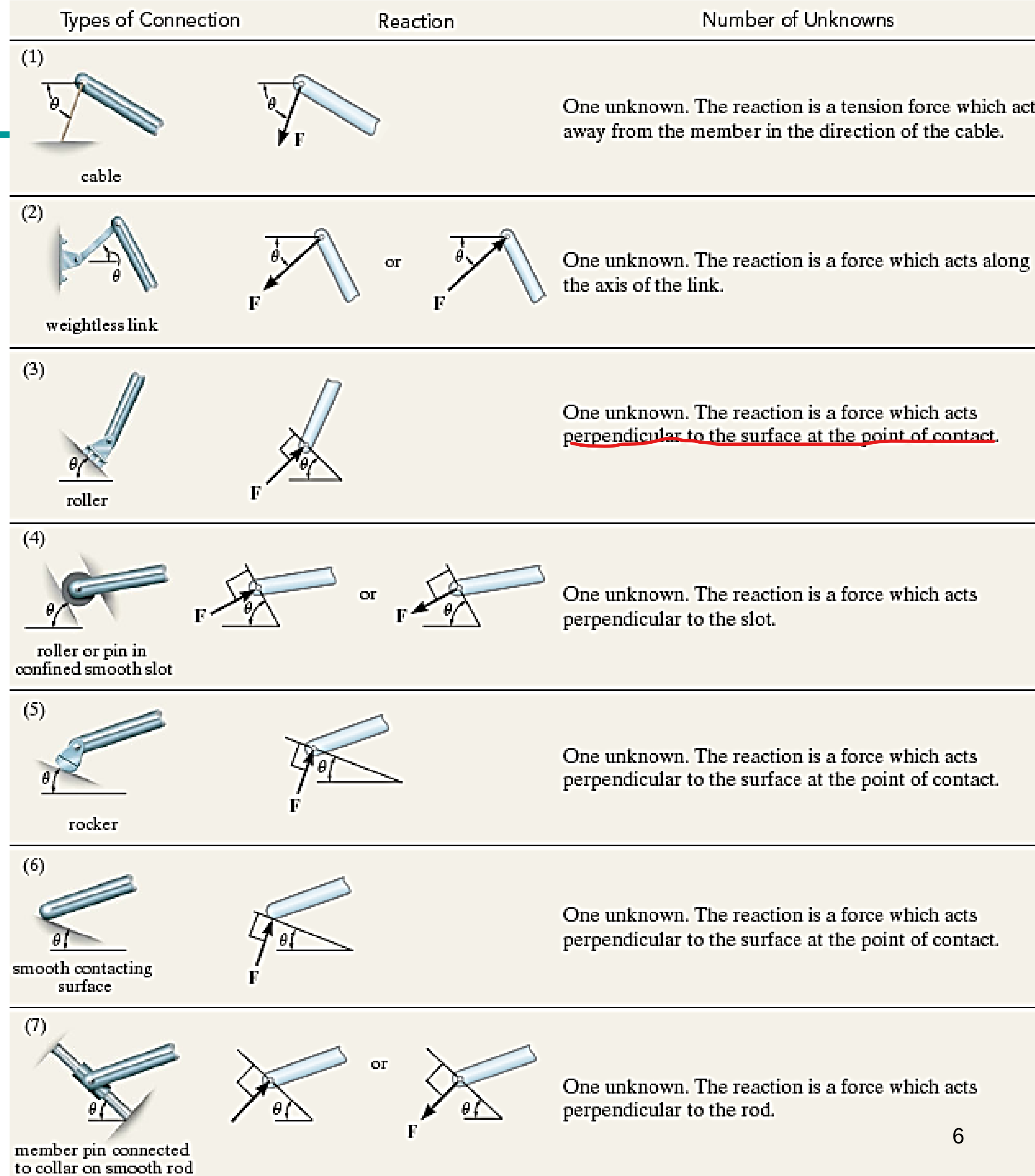
Prevention of Translation or Rotation of a body

Restraints



Rigid Body Equilibrium

Various Supports
2-D Force Systems

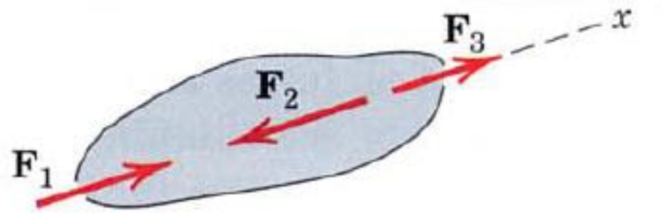
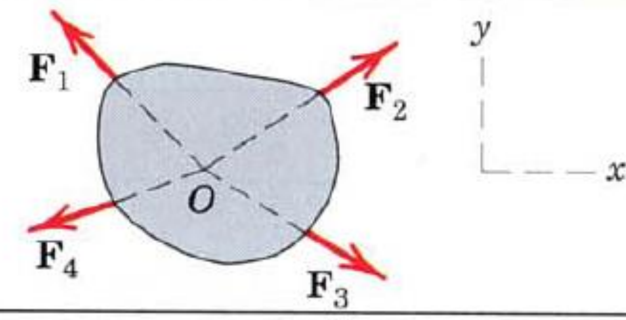
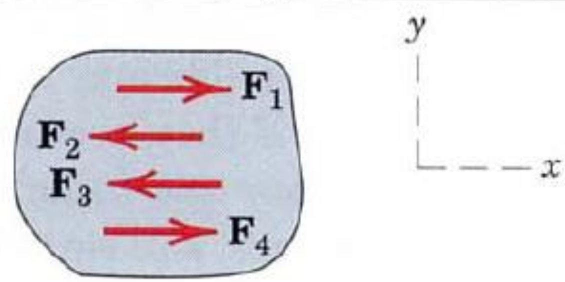
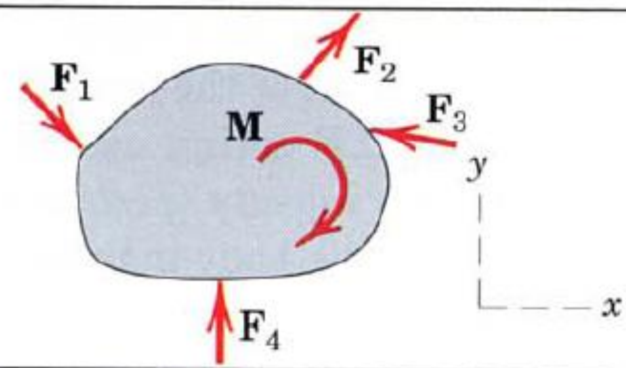


Types of Connection	Reaction	Number of Unknowns
(1) cable	F	One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2) weightless link	F or F	One unknown. The reaction is a force which acts along the axis of the link.
(3) roller	F	One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4) roller or pin in confined smooth slot	F or F	One unknown. The reaction is a force which acts perpendicular to the slot.
(5) rocker	F	One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6) smooth contacting surface	F	One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7) member pin connected to collar on smooth rod	F or F	One unknown. The reaction is a force which acts perpendicular to the rod.

Rigid Body Equilibrium

Categories in 2-D

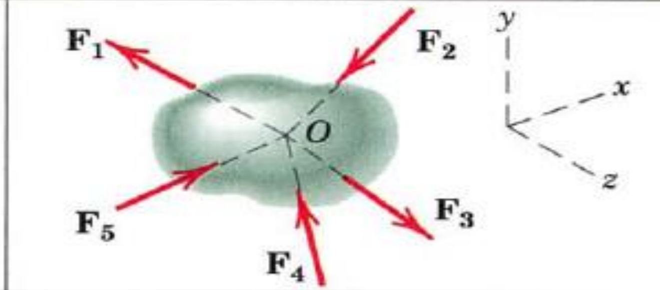
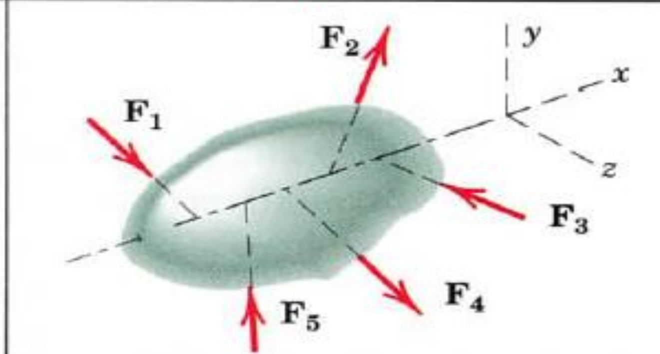
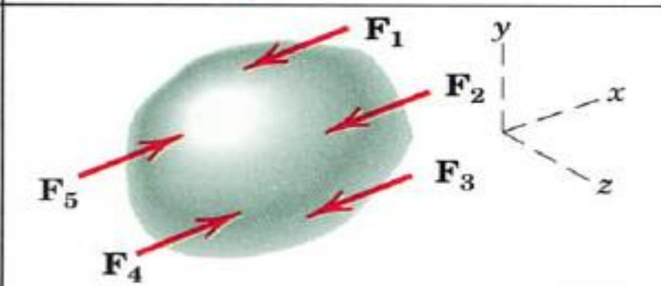
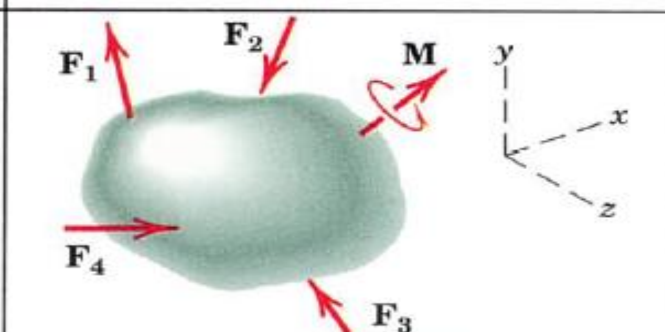
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS

Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

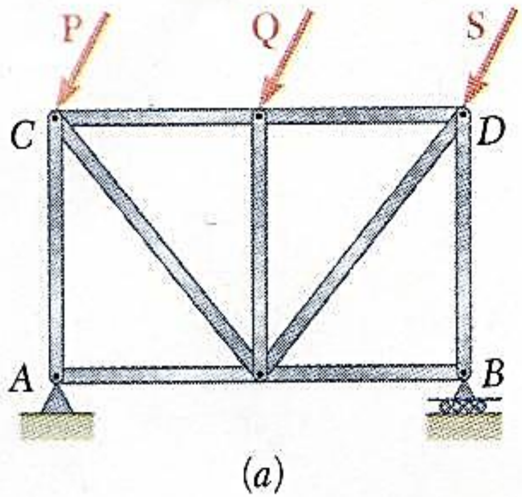
Rigid Body Equilibrium

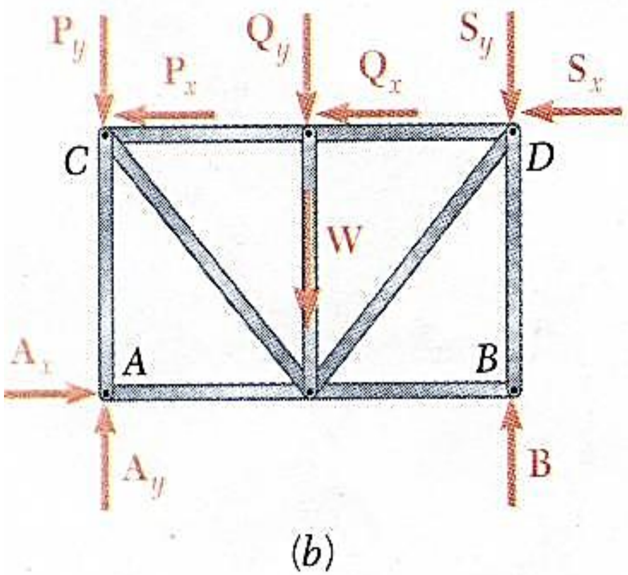
Categories

in 3-D

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\quad$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\quad$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\quad$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\quad$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\quad$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\quad$ $\Sigma M_z = 0$

Equilibrium of a Rigid Body in Two Dimensions





- For all forces and moments acting on a two-dimensional structure,

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

where A is any point in the plane of the structure.

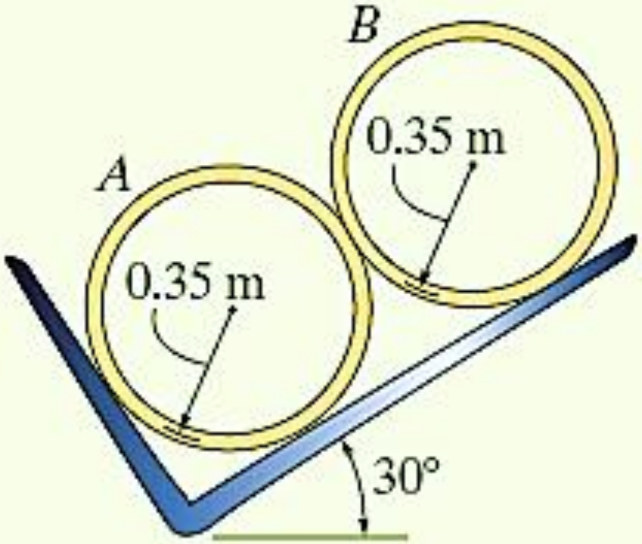
- The 3 equations can be solved for no more than 3 unknowns.

- The 3 equations can not be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

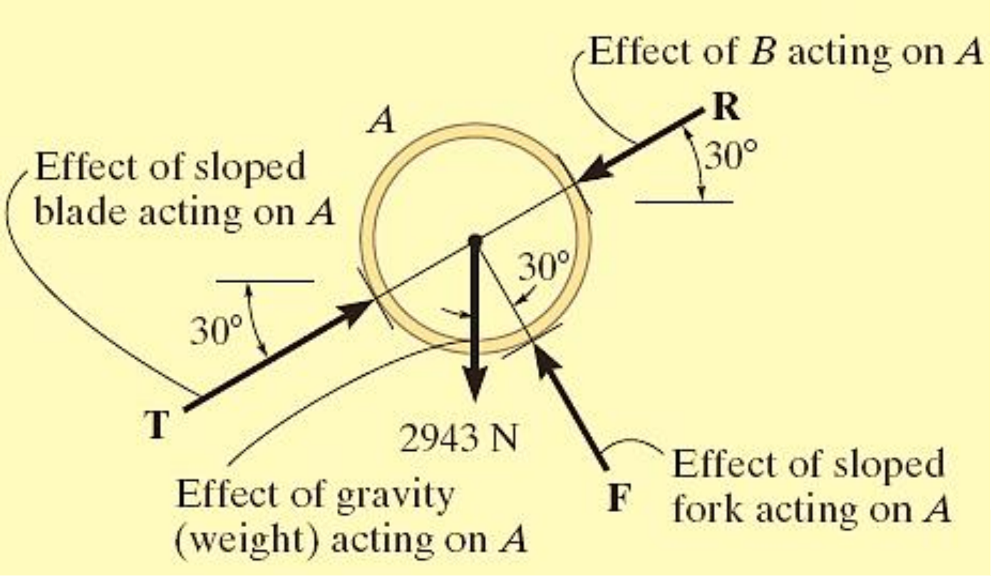
Free body diagram

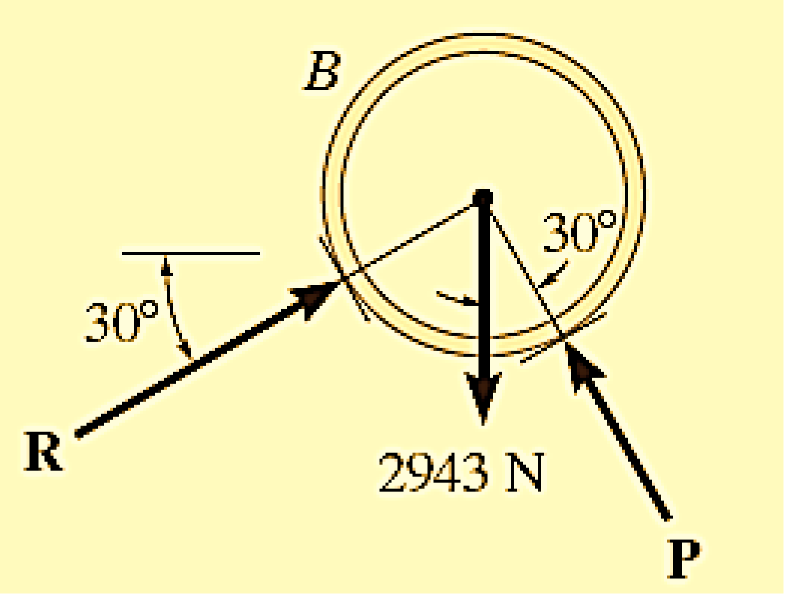

(a)



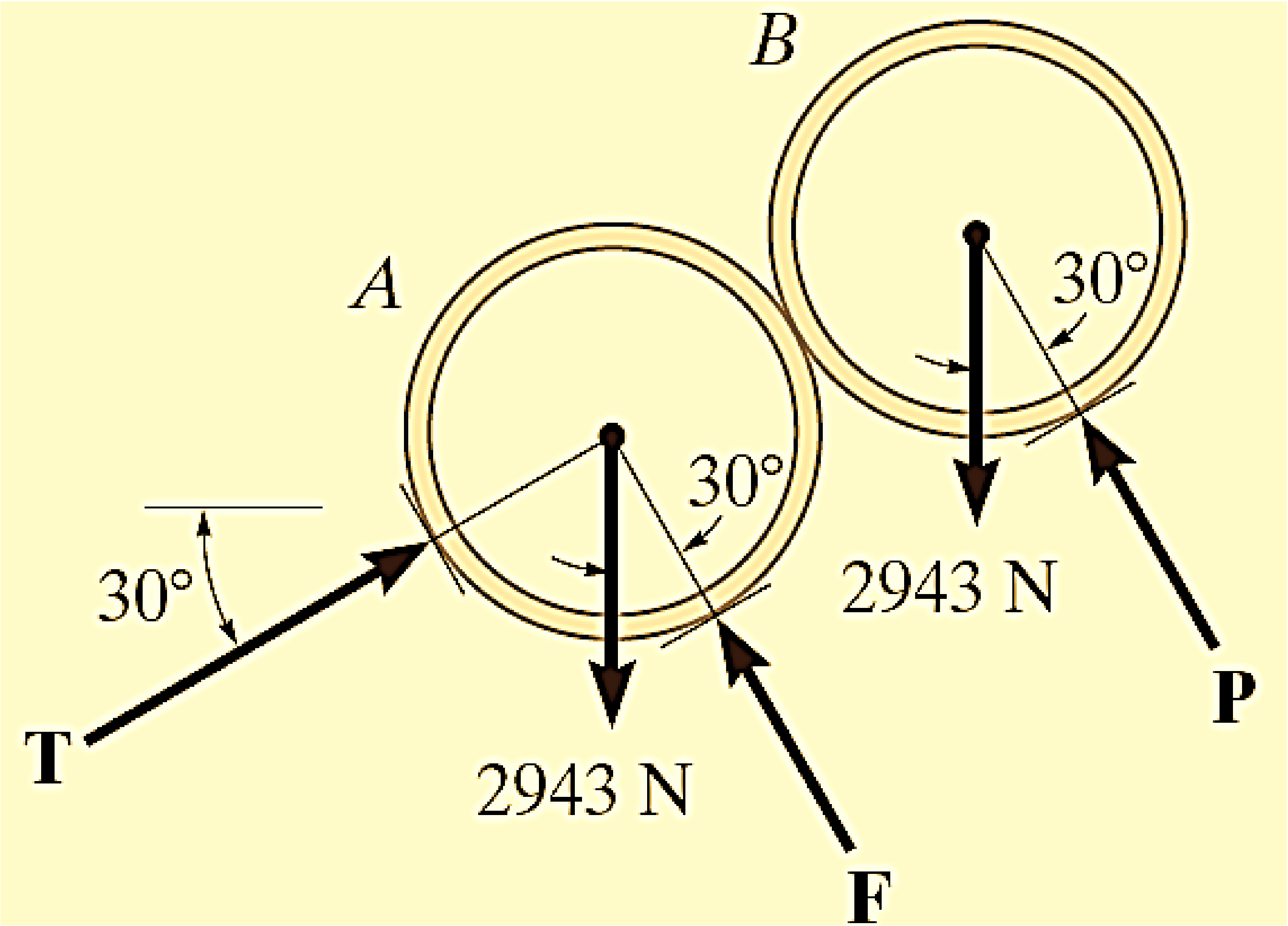
(b)

Free body diagram example

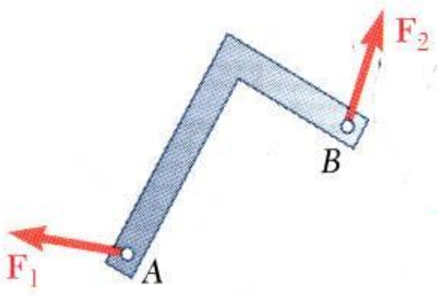




Free body diagram example

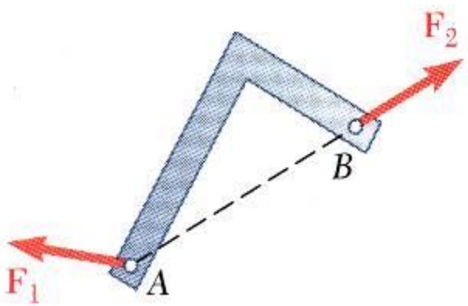


Equilibrium of a Two-Force Body

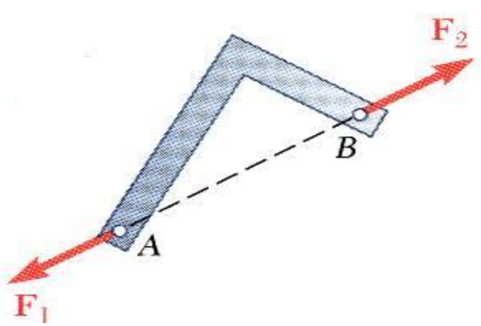


- Consider a plate subjected to two forces $\mathbf{F_1}$ and $\mathbf{F_2}$.

- For static equilibrium, the sum of moments about *A* must be zero. The moment of $\mathbf{F_2}$ must be zero. It follows that the line of action of $\mathbf{F_2}$ must pass through *A*.

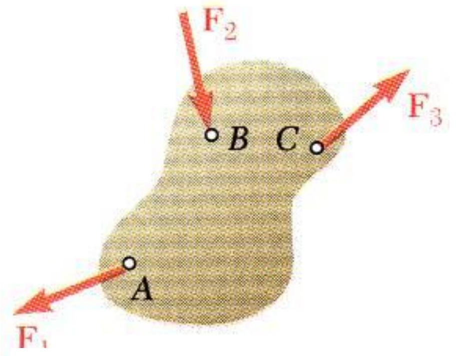


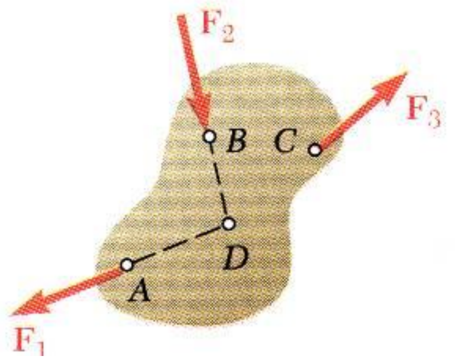
- Similarly, the line of action of $\mathbf{F_1}$ must pass through *B* for the sum of moments about B to be zero.

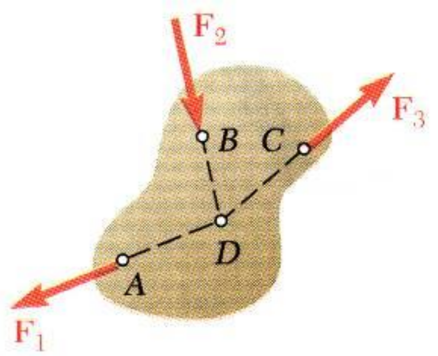


- Requiring that the sum of forces in any direction be zero leads to the conclusion that $\mathbf{F_1}$ and $\mathbf{F_2}$ must have equal magnitude but opposite sense.

Equilibrium of a Three-Force Body

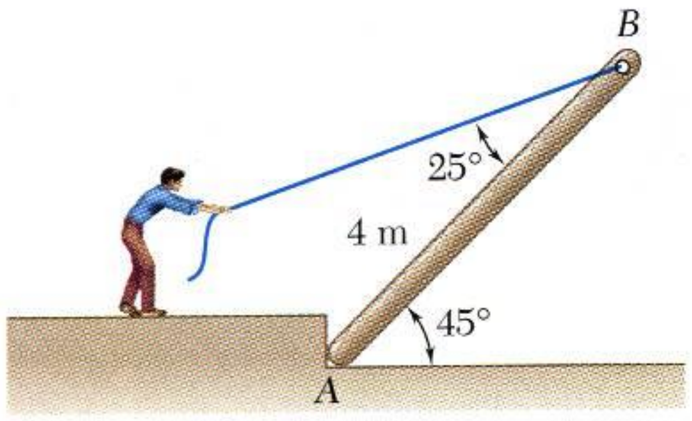






- Consider a rigid body subjected to forces acting at only 3 points.
- Assuming that their lines of action intersect, the moment of **F**$_1$ and **F**$_2$ about the point of intersection represented by *D* is zero.
- Since the rigid body is in equilibrium, the sum of the moments of **F**$_1$, **F**$_2$, and **F**$_3$ about any axis must be zero. It follows that the moment of **F**$_3$ about *D* must be zero as well and that the line of action of **F**$_3$ must pass through *D*.
- The lines of action of the three forces must be concurrent or parallel.

Sample Problem



A man raises a 10-kg joist, of length 4 m, by pulling on a rope.

Find the tension *T* in the rope and the reaction at *A*.

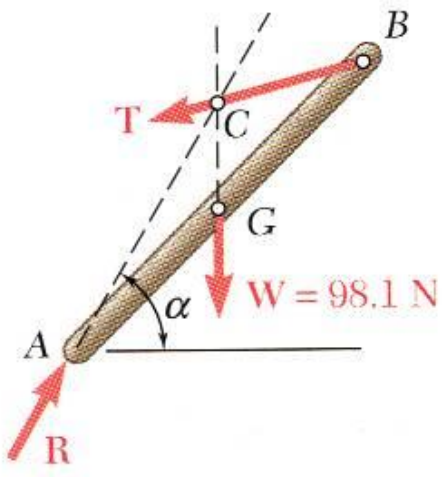
SOLUTION:

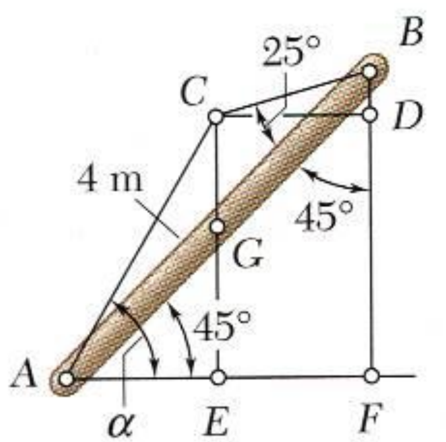
- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at *A*.

- The three forces must be concurrent for static equilibrium. Therefore, the reaction **R** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction **R**.

- Utilize a force triangle to determine the magnitude of the reaction **R**.

Sample Problem





- Create a free-body diagram of the joist.
- Determine the direction of the reaction **R**.

$$BF = AB\cos 45^{\circ} = (4\,\text{m})\cos 45^{\circ} = 2.828\,\text{m}$$

$$CD = AE = \tfrac{1}{2} AF = 1.414\,\text{m}$$

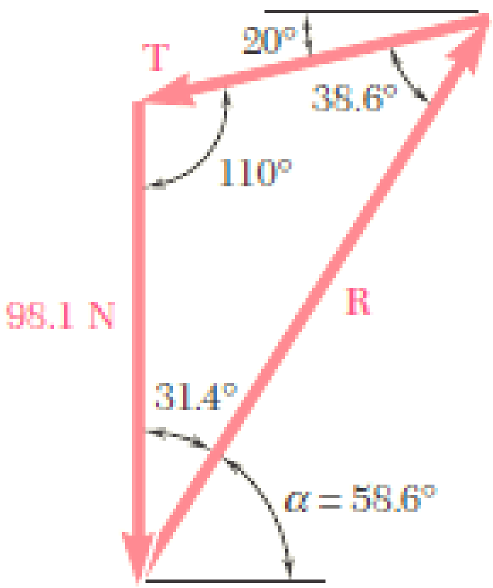
$$BD = CD\cot(45^{\circ} + 25^{\circ}) = (1.414\,\text{m})\tan 20^{\circ} = 0.515\,\text{m}$$

$$CE = BF - BD = (2.828 - 0.515)\,\text{m} = 2.313\,\text{m}$$

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313}{1.414} = 1.636$$

$$\boxed{\alpha = 58.6^{\circ}}$$

Sample Problem



- Determine the magnitude of the reaction **R**.

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\text{N}}{\sin 38.6^\circ}$$

$$\boxed{\begin{aligned} T &= 81.9\,\text{N} \\ R &= 147.8\,\text{N} \end{aligned}}$$

Equilibrium of a Rigid Body in Three Dimensions

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

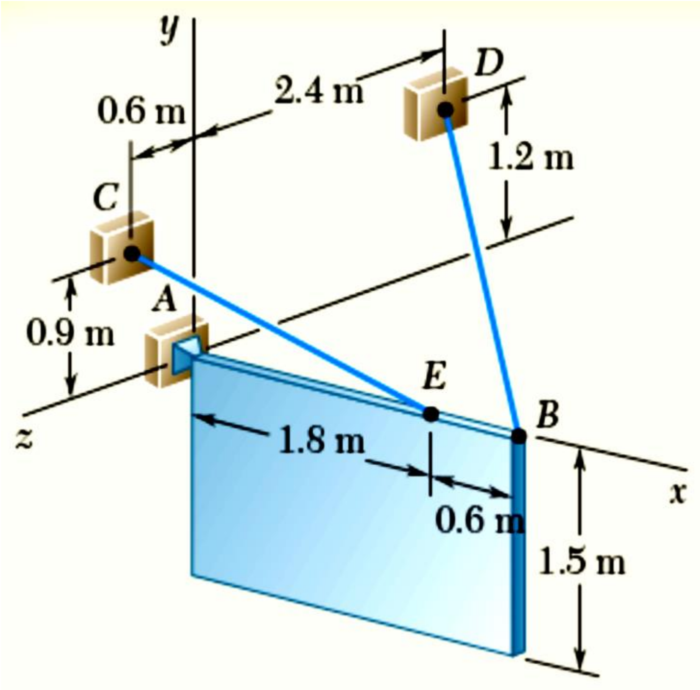
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections or unknown applied forces.

- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \mathbf{F} = 0 \qquad \sum \mathbf{M}_O = \sum (\mathbf{r} \times \mathbf{F}) = 0$$

Sample Problem



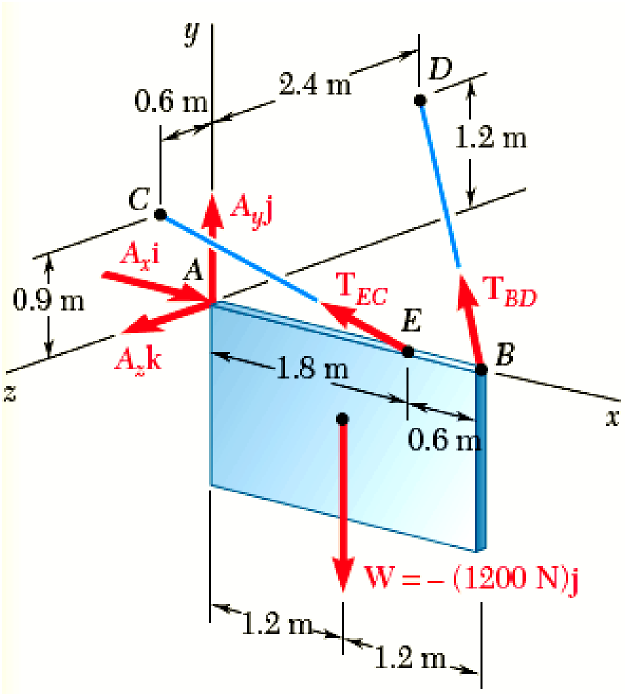
A sign of uniform density weighs 1200 N and is supported by a ball-and-socket joint at *A* and by two cables.

Determine the tension in each cable and the reaction at *A*.

SOLUTION:

- Create a free-body diagram for the sign.
- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

Sample Problem



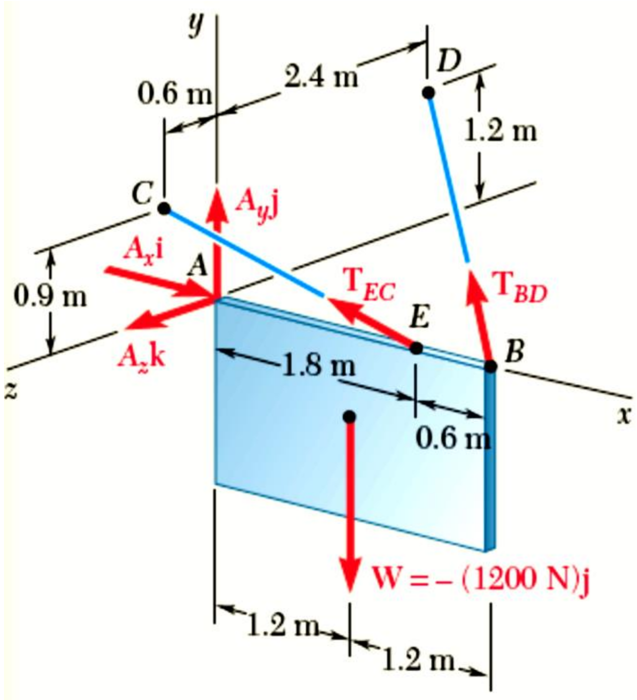
- Create a free-body diagram for the sign.

  Since there are only 5 unknowns, the sign is partially constrained. It is free to rotate about the *x* axis. It is, however, in equilibrium for the given loading.

$$\begin{aligned}\mathbf{T}_{BD} &= T_{BD}\frac{\overrightarrow{BD}}{BD}\\ &= T_{BD}\left(-\tfrac{2}{3}\mathbf{i}+\tfrac{1}{3}\mathbf{j}-\tfrac{2}{3}\mathbf{k}\right)\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{EC} &= T_{EC}\frac{\overrightarrow{EC}}{EC}\\ &= T_{EC}\left(-\tfrac{6}{7}\mathbf{i}+\tfrac{3}{7}\mathbf{j}+\tfrac{2}{7}\mathbf{k}\right)\end{aligned}$$

Sample Problem



$$\sum \mathbf{F} = \mathbf{A} + \mathbf{T}_{BD} + \mathbf{T}_{EC} - (1200\text{ N})\mathbf{j} = 0$$

$$\mathbf{i}: \quad A_x - \tfrac{2}{3}T_{BD} - \tfrac{6}{7}T_{EC} = 0$$

$$\mathbf{j}: \quad A_y + \tfrac{1}{3}T_{BD} + \tfrac{3}{7}T_{EC} - 1200\text{ N} = 0$$

$$\mathbf{k}: \quad A_z - \tfrac{2}{3}T_{BD} + \tfrac{2}{7}T_{EC} = 0$$

$$\sum \mathbf{M}_A = \mathbf{r}_B \times \mathbf{T}_{BD} + \mathbf{r}_E \times \mathbf{T}_{EC} + (1.2\text{ m})\mathbf{i} \times (-1200\text{ N})\mathbf{j} = 0$$

$$\mathbf{j}: \quad 1.6T_{BD} - 0.514T_{EC} = 0$$

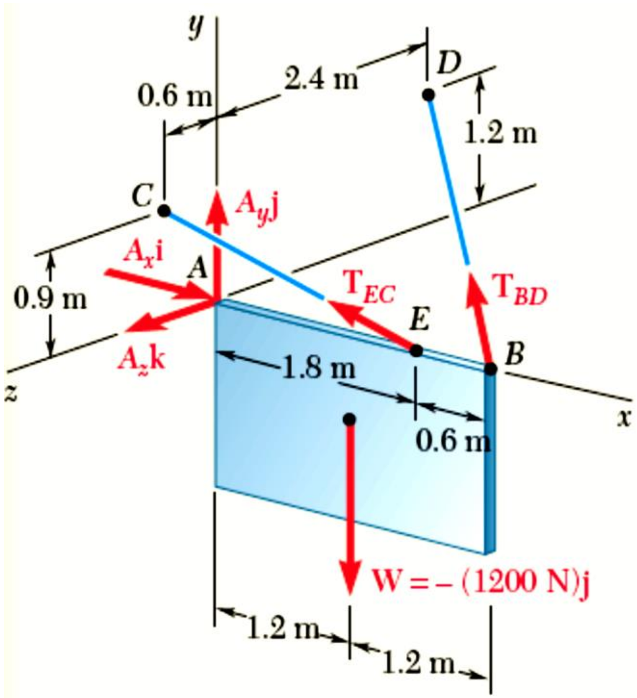
$$\mathbf{k}: \quad 0.8T_{BD} + 0.771T_{EC} - 1440\text{ N . m} = 0$$

- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

Solve the 5 equations for the 5 unknowns,

$$\boxed{\begin{gathered} T_{BD} = 450\text{ N} \qquad T_{EC} = 1400.8\text{ N} \\ \mathbf{A} = (1500.7\text{ N})\mathbf{i} + (449.7\text{ N})\mathbf{j} - (100.2\text{ N})\mathbf{k} \end{gathered}}$$

What if…?



Can the sign be in static equilibrium if cable *BD* is removed?

Discuss, and be sure to provide the reason(s) for your answer.

The sign cannot be in static equilibrium because T_{EC} causes a moment about the *y*-axis (due to the existence of $T_{EC,Z}$) which must be countered by an equal and opposite moment. This can only be provided by a cable tension that has a *z*-component in the negative-*z* direction, such as what T_{BD} has.