Special Theory of Relativity (PH101)
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Practice Problems
due on Monday, 30th of October, 2019 (11:00Hrs IST)

1. Electrons in projection television sets are accelerated through a potential difference of 50 kV.

   Calculate the speed of the electrons using the relativistic form of kinetic energy assuming the electrons start from rest. **Answer:** $v = 0.413c$

   Calculate the speed of the electrons using the classical form of kinetic energy. **Answer: v = 0.442c**

2. As seen from Earth, two spaceships A and B are approaching along perpendicular directions. If A is observed by a stationary Earth observer to have velocity $u_y = -0.90c$ and B to have velocity $u_x = 0.90c$, determine the speed of ship A as measured by the pilot of ship B. **Answer:** $u'_x = -0.9c$**,** $u'_y = -0.39c$**,** $u' = 0.98c$

3. A body quadruples its momentum when its speed doubles. What was the initial speed in units of c, i.e., what was u/c? **Answer:** $u/c = \frac{1}{\sqrt{5}}$

4. A body of rest mass m_0 moving at speed v collides with and sticks to an identical body at rest. What is the mass and momentum of the final clump? **Answer:** $M = m_0\sqrt{2(1 + \Gamma_v)}$

5. Two β particles move in opposite direction with velocity $0.6c$ in the laboratory frame. Calculate the velocity of one β particle in the moving frame attached to the other β particle by applying relativistic transformation. Repeat the calculations taking speed of particles as 0.06c . **Answer:** $u'_2 = -0.88c$

6. A 2m long stick, when it is at rest, moves past an observer on the ground with a speed of 0.5c.

   (a) What is the length measured by the observer ? **Answer:** $L = 1.732m$

   (b) If the same stick moves with the velocity of 0.05c what would be its length measured by the observer ? **Answer:** $L = 1.997m$

7. A proton of mass $m_p = 1.67 \times 10^{-27} kg$ moves with a speed of $u = 0.6c$. Compute its relativistic and non-relativistic momentum. **Answer:** $P_{relativistic} = 3.75 \times 10^{-19} kg - m/sec$ **and**

$P_{nonrelativistic} = 3.0 \times 10^{-19} kg - m/sec$

8. The change in frequency of wave that happens due to relative motion between the source and observer is known as the "Doppler effect." In Galilean relativity the modified frequency is given by $\nu = \nu_0 \left(\frac{v \pm v_o}{v \mp v_s} \right)$, where, ν_0 is the original emitted frequency, ν is the observed(detected) frequency, v, v_s, and v_o is the velocity of wave (e.g., sound wave),observer, and source relative to the medium respectively. Here $+(-)$ stands for the situation when observer is approaching (receding) towards (from) source. This relation gets modified in STR as $\nu = \nu_0 \sqrt{\frac{c \pm v}{c \mp v}}$ where, c is the velocity of light and v is the velocity of the light source. Note that here implicitly it is assumed that the observer is in the frame S and light source is in the moving frame S'. Using this information solve the following problem.

   A driver is caught violating the traffic rule by going through a red light signal. The driver claims to the judge that the color she actually saw was green ($\nu = 5.60 \times 10^{14} Hz$) and not red ($\nu = 4.80 \times 10^{14} Hz$) because of the Doppler effect. The judge accepts this explanation and instead fines her for speeding at the rate of 100 (INR) for each km/h that she exceeded the speed limit of 80 km/h. Compute the total fine amount?

   Solution:
   Using the relation $\nu = \nu_0 \sqrt{\frac{c + v}{c - v}}$, as the observer (driver) is approaching towards the source (signal), we have
   $v = c \left(\frac{\nu^2 - \nu_0^2}{\nu^2 + \nu_0^2} \right) = 3 \times 10^8 m/s \left[\frac{(5.60)^2 - (4.80)^2}{(5.60)^2 + (4.80)^2} \right]$
   $= 4.59 \times 10^7 m/sec = 1.65 \times 10^8 km/h$, (since, $1m/sec = 3.6 km/h$).

   The fine will be $(1.65 \times 10^8 - 80) \times 100 (INR) = 16499992000 (INR)$.

9. The light wave equation is given by $\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$, where, E is the electric field and c is the velocity of light. Show that under Galilean transformation the above equation will have the form as $\frac{\partial^2 E'}{\partial x'^2} - \frac{1}{c^2} \frac{\partial^2 E'}{\partial t'^2} - \frac{2v_x}{c^2} \frac{\partial^2 E'}{\partial x' \partial t'} - \frac{v_x}{c^2} \frac{\partial}{\partial x'} \left[v_x \frac{\partial E'}{\partial x'} \right] = 0$, where v_x is the speed of S' frame w.r.t. the S frame. That shows that light wave equation is not compatible with the Galilean relativity. Hint: Use the transformation formula discussed during the first lecture of STR.

   Solution:

   Similar problem has been done in the class where incompatibility of the Ampere's law was shown with the Galilean relativity (GR). Use the relation $E/B = c$ which will give the relation $E' = E$ if it is assumed that for $E = E(x,t)\hat{j}$ and $B = B(x,t)\hat{k}$ the Lorentz force remains invariant which will give the relation $E = E' + v_x B$ (See the lecture note for details).
   hint:

$x' = x - v_x t \quad t' = t \rightarrow \dfrac{\partial x'}{\partial x} = 1 \quad \dfrac{\partial x'}{\partial t} = -v_x \quad \dfrac{\partial t'}{\partial t} = 1 \quad \dfrac{\partial t'}{\partial x} = 0$

using the chain rule, $\dfrac{\partial E}{\partial x} = \dfrac{\partial E}{\partial x'}\dfrac{\partial x'}{\partial x} + \dfrac{\partial E}{\partial t'}\dfrac{\partial t'}{\partial x} = \dfrac{\partial E}{\partial x'}$.

In the similar way it can be shown that $\dfrac{\partial^2 E}{\partial x^2} = \dfrac{\partial^2 E}{\partial x'^2}$.

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial E}{\partial t'}\frac{\partial t'}{\partial t} = \frac{\partial E}{\partial t'} - v_x\frac{\partial E}{\partial x'}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t}\right) &= \frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t'} - v_x\frac{\partial E}{\partial x'}\right) = \frac{\partial}{\partial t'}\left(\frac{\partial E}{\partial t'} - v_x\frac{\partial E}{\partial x'}\right)\frac{\partial t'}{\partial t} + \frac{\partial}{\partial x'}\left(\frac{\partial E}{\partial t'} - v_x\frac{\partial E}{\partial x'}\right)\frac{\partial x'}{\partial t} \\ &= \frac{\partial^2 E}{\partial t'^2} - 2v_x\frac{\partial^2 E}{\partial x'\partial t'} - v_x^2\frac{\partial^2 E}{\partial x'^2}.\end{aligned}$$