PH 102 : Electromagnetism **Department of Physics**
Tutorial 5 **Indian Institute of Technology Guwahati**
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1. In complex notation, we use the Complex wave function which, as discussed in the class, is given by $\tilde{f}(z,t) = \tilde{A}e^{i(kz-\omega t)}$ with $\tilde{A} = Ae^{i\delta}$ being the complex amplitude. Use the method of separation of variables to solve the wave equation and to show that any wave can be expressed as a linear combination of sinusoidal waves:

$$\tilde{f}(z,t) = \int_{-\infty}^{\infty} \tilde{A}(k)e^{i(kz-\omega t)}dk.$$

2. Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is (a) travelling in the negative x direction and polarised in the z direction; (b) travelling in the direction from the origin to the point $(1,1,1)$, with polarisation parallel to the $x-z$ plane. In each case, sketch the wave, and give the explicit Cartesian components of $\vec{k}$ and $\hat{n}$.

3. A paradoxical case of Poynting's theorem occurs when a static electric field is applied perpendicularly to a static magnetic field, as in the case of a pair of electrodes placed within a magnetic circuit (see figure 1).
(a) What are $\vec{E}, \vec{H}$ and $\vec{S}$?
(b) What is the energy density stored in the system?
(c) Verify Poynting's theorem.

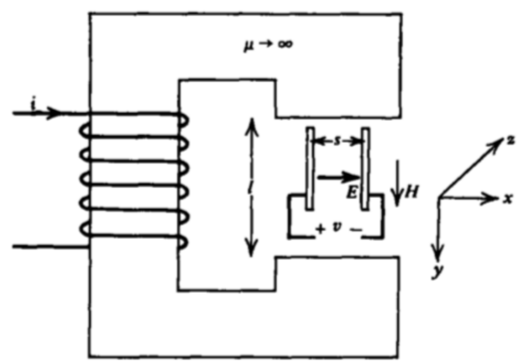

Figure 1: Figure for problem 3.

4. A uniformly distributed volume current of thickness $2d$, $J_o \cos(\omega t)\hat{x}$ is a source of plane waves (see figure 2).
(a) From Maxwell's equations obtain a single differential equation relating E_x to J_x.
(b) Find the electric and magnetic fields within and outside the current distribution.
(c) How much time-averaged power per unit area is delivered by the current?
(d) How does the generated power compare to the electromagnetic time-average power per unit area leaving the volume current at $z = \pm d$?

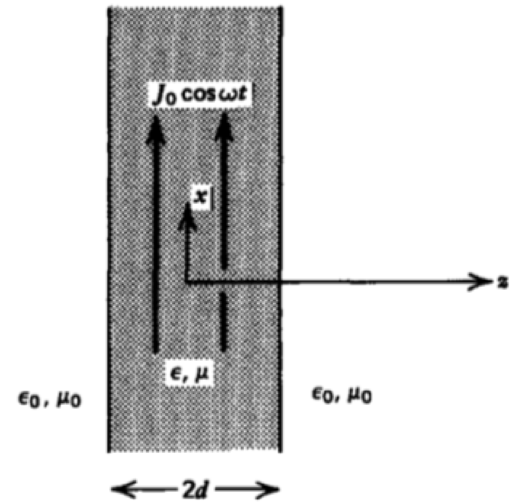

Figure 2: Figure for problem 4.

5. A polarising filter to microwaves is essentially formed by many highly conducting parallel wires whose spacing is much smaller than a wavelength (see figure 3). That polarisation whose electric field field is transverse to the wires passes through. The incident electric field is $\vec{E} = E_x \cos(\omega t - kz)\hat{x} + E_y \sin(\omega t - kz)\hat{y}$.
(a) What is the incident magnetic field and incident power density?
(b) What are the transmitted fields and power density?
(c) Another set of polarising wires are placed parallel but a distance d and oriented at an angle ϕ to the first. What are the transmitted fields?

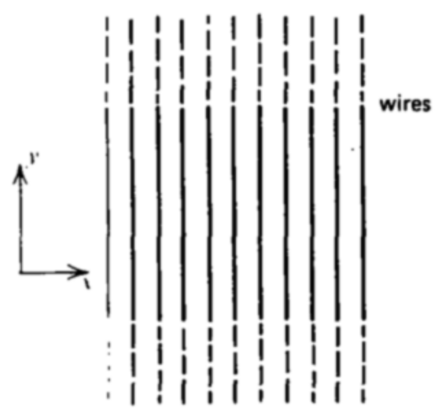

Figure 3: Figure for problem 5.

6. Consider a satellite in a stationary orbit of earth, i.e, to earth based observers the satellite would appear motionless, at a fixed position in the sky. The satellite beams a signal towards earth. The beam covers a region with area A km^2 on earth. Assume the signal to be a monochoromatic plane electromagnetic wave with electric field amplitude E_0. Find the power P delivered at the receiver on earth. What is energy density at the receiver on earth.