Tutorial # 3

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PH 101: Physics I (2019)

Due on: 21st August, 2019

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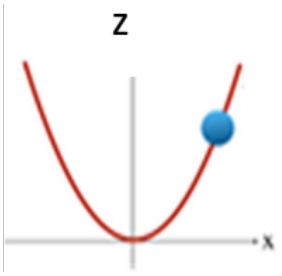
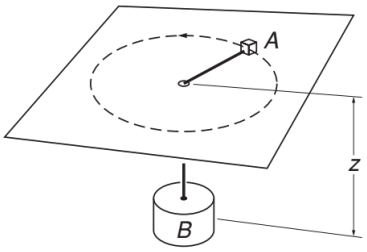
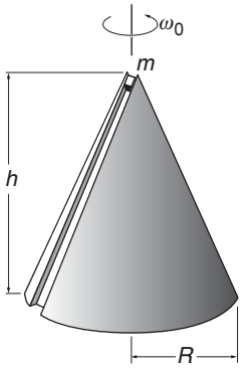
1. A bead of mass m slides without friction on a frictionless parabolic wire, $z = ax^2$ under gravity. The wire is kept vertical as shown in the figure. Find (a) the degrees of freedom, (b) the Lagrangian function, and (c) the equation of motion.



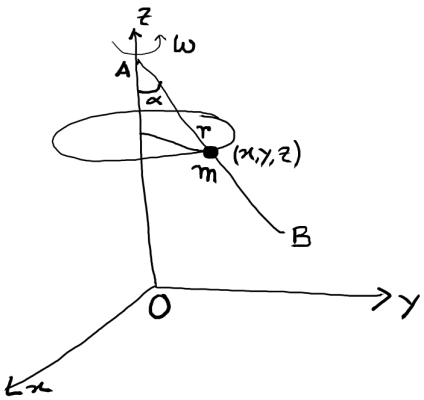
2. A particle of mass M (marked A) is confined to move the horizontal $x - y$ plane. Another mass m (marked B) which is free to move on the vertical $z-$axis is tied to the mass A through a light string of length l. Find (a) the degrees of freedom, (b) the Lagrangian function, and (c) the equation of motion.



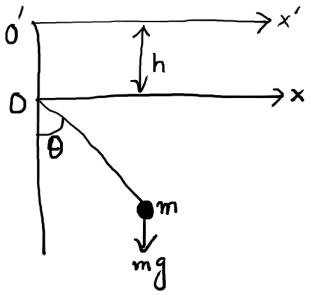
3. A solid cone of height h and radius R is free to rotate about the vertical z-axis. At time $t = 0$ small mass m starts sliding down from the apex of the cone along a straight groove cut on its surface, under gravity. Let the moment of inertia of the cone about the z-axis is I. If at time, $t = 0$, the angular velocity of the cone is ω_0, find the angular velocity of the cone when the mass m leaves it. Also, show that the angular momentum of the system along the z-axis is conserved.



4. Refer to the adjacent figure, where AB is a straight frictionless wire held fixed at point A on a vertical axis OA and the wire AB rotates about OA with constant angular velocity ω. A bead of mass m is constrained to move on the wire. Find (a) the Lagrangian, and (b) Lagrange's equation of motion.



5. Refer to the adjacent figure. The point of suspension of a simple pendulum moves harmonically in the vertical direction (along OO') between OX and $O'X'$. Find (a) the Lagrangian, and (b) the equation of motion,



6. **Home work to students**: Formulate the Lagrangian ($\mathcal{L}$) for the following systems.

   (a) A projectile of mass, m , moving on the vertical xy-plane under gravity. Workout considering Cartesian coordinates as well as plane polar coordinates.

   (b) A bead of mass, m, moving on elliptic wire, given by, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are the semi-major and semi-minor axes, respectively. (Ignore gravity)

   (c) A point mass m constrained to move on the surface of a fixed gravitating solid sphere of mass M and radius R.