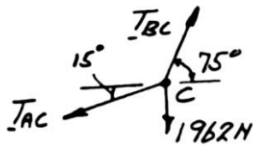
ENGINEERING MECHANICS (ME101) – ANSWERS TO TUTORIAL 1
DATE OF TUTORIAL – 10 Jan 2020.

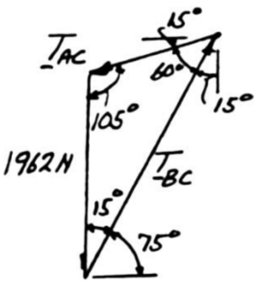
A1.

SOLUTION

Free-Body Diagram



Force Triangle



$$W = mg$$
$$= (200 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 1962 \text{ N}$$

Law of sines: $$\frac{T_{AC}}{\sin 15°} = \frac{T_{BC}}{\sin 105°} = \frac{1962 \text{ N}}{\sin 60°}$$

(a) $$T_{AC} = \frac{(1962 \text{ N}) \sin 15°}{\sin 60°}$$ $T_{AC} = 586$ N ◀

(b) $$T_{BC} = \frac{(1962 \text{ N}) \sin 105°}{\sin 60°}$$ $T_{BC} = 2190$ N ◀

A2.

SOLUTION

$$\mathbf{F} = F\frac{\overrightarrow{BD}}{BD} \quad \text{where} \quad F = 900 \text{ N}$$

$$\overrightarrow{BD} = -(1 \text{ m})\mathbf{i} - (2 \text{ m})\mathbf{j} + (2 \text{ m})\mathbf{k}$$
$$BD = \sqrt{(-1 \text{ m})^2 + (-2 \text{ m})^2 + (2 \text{ m})^2}$$
$$= 3 \text{ m}$$
$$\mathbf{F} = (900 \text{ N})\frac{-\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}}{3}$$
$$= -(300 \text{ N})\mathbf{i} - (600 \text{ N})\mathbf{j} + (600 \text{ N})\mathbf{k}$$
$$\mathbf{r}_{B/O} = (2.5 \text{ m})\mathbf{i} + (2 \text{ m})\mathbf{j}$$

$$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & 2 & 0 \\ -300 & -600 & 600 \end{vmatrix}$$
$$= 1200\mathbf{i} - 1500\mathbf{j} + (-1500 + 600)\mathbf{k}$$

$$\mathbf{M}_O = (1200 \text{ N}\cdot\text{m})\mathbf{i} - (1500 \text{ N}\cdot\text{m})\mathbf{j} - (900 \text{ N}\cdot\text{m})\mathbf{k}$$ ◀

A3.

SOLUTION

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH})$$

Where

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{i}$$

and

$$d_{BH} = \sqrt{(0.375)^2 + (0.75)^2 + (-0.75)^2}$$
$$= 1.125 \text{ m}$$

Then

$$\mathbf{T}_{BH} = \frac{450 \text{ N}}{1.125}(0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k})$$
$$= (150 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} - (300 \text{ N})\mathbf{k}$$

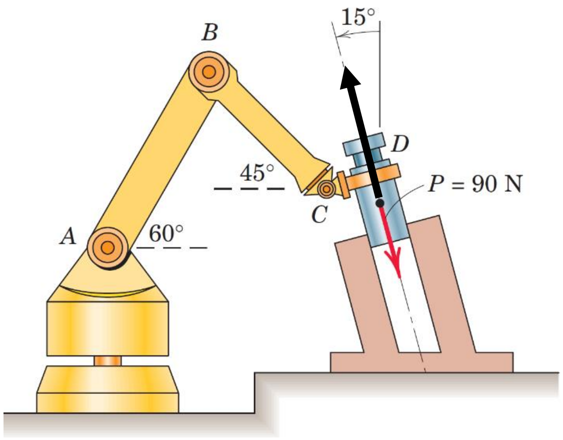
Finally,

$$M_{AD} = \frac{1}{5}\begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix}$$

$$= \frac{1}{5}[(-3)(0.5)(300)]$$

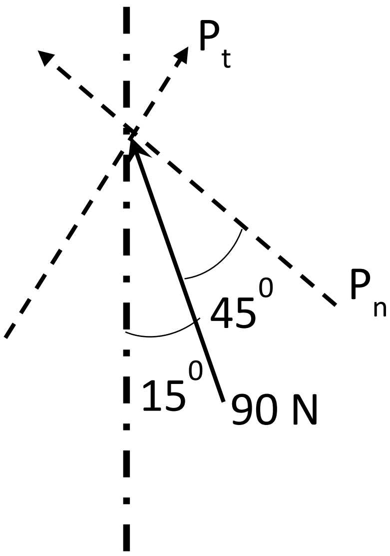
or $M_{AD} = -90.0 \text{ N} \cdot \text{m}$ ◀

A4.



Since, the question asked for the force exerted **ON** the robot, such a force is shown by the black arrow – directed in the opposite direction of the 90 N force.

(a) Considering arm AB, the axes parallel and perpendicular to AB are given by P_t and P_n respectively (Shown below)

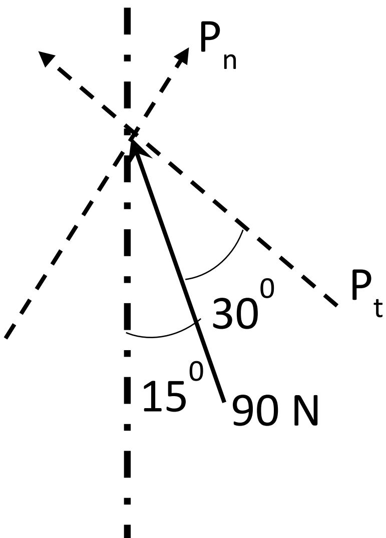


The angles shown above are based on the angle made by arm AB with the horizontal.

The component of force exerted perpendicular to arm AB, i.e. along direction of P_n is given by – 90 cos 45^0 = 63.64 N

The component of force exerted parallel to arm AB, i.e. along direction of P_t is given by – 90 sin 45^0 = 63.64 N

(b) Considering arm AB, the axes parallel and perpendicular to BC are given by P_t and P_n respectively (Shown below)



The angles shown above are based on the angle made by arm BC with the horizontal.

The component of force exerted perpendicular to arm BC, i.e. along direction of P_n is given by – 90 sin 30^0 = 45 N

The component of force exerted parallel to arm BC, i.e. along direction of P_t is given by – 90 cos 30^0 = 77.9 N

A5.

SOLUTION

(*a*) We have

$$\Sigma M_{Bz}: \quad M_{2z} = 0$$

$$\mathbf{k} \cdot (\mathbf{r}_{H/B} \times \mathbf{F}_1) + M_{1z} = 0 \qquad (1)$$

where

$$\mathbf{r}_{H/B} = (0.31 \text{ m})\mathbf{i} - (0.0233)\mathbf{j}$$

$$\mathbf{F}_1 = \boldsymbol{\lambda}_{EH} F_1$$

$$= \frac{(0.06 \text{ m})\mathbf{i} + (0.06 \text{ m})\mathbf{j} - (0.07 \text{ m})\mathbf{k}}{0.11 \text{ m}}(77 \text{ N})$$

$$= (42 \text{ N})\mathbf{i} + (42 \text{ N})\mathbf{j} - (49 \text{ N})\mathbf{k}$$

$$M_{1z} = \mathbf{k} \cdot \mathbf{M}_1$$

$$\mathbf{M}_1 = \boldsymbol{\lambda}_{EJ} M_1$$

$$= \frac{-d\mathbf{i} + (0.03 \text{ m})\mathbf{j} - (0.07 \text{ m})\mathbf{k}}{\sqrt{d^2 + 0.0058} \text{ m}}(31 \text{ N} \cdot \text{m})$$

Then from Equation (1),

$$\begin{vmatrix} 0 & 0 & 1 \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(-0.07 \text{ m})(31 \text{ N} \cdot \text{m})}{\sqrt{d^2 + 0.0058}} = 0$$

Solving for *d*, Equation (1) reduces to

$$(13.0200 + 0.9786) - \frac{2.17 \text{ N} \cdot \text{m}}{\sqrt{d^2 + 0.0058}} = 0$$

from which $d = 0.1350 \text{ m}$ or $d = 135.0 \text{ mm}$ ◀

(*b*)

$$\mathbf{F}_2 = \mathbf{F}_1 = (42\mathbf{i} + 42\mathbf{j} - 49\mathbf{k}) \text{ N} \quad \text{or} \quad \mathbf{F}_2 = (42.0 \text{ N})\mathbf{i} + (42.0 \text{ N})\mathbf{j} - (49.0 \text{ N})\mathbf{k}$$ ◀

$$\mathbf{M}_2 = \mathbf{r}_{H/B} \times \mathbf{F}_1 + \mathbf{M}_1$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(0.1350)\mathbf{i} + 0.03\mathbf{j} - 0.07\mathbf{k}}{0.155000}(31 \text{ N} \cdot \text{m})$$

$$= (1.14170\mathbf{i} + 15.1900\mathbf{j} + 13.9986\mathbf{k}) \text{ N} \cdot \text{m}$$

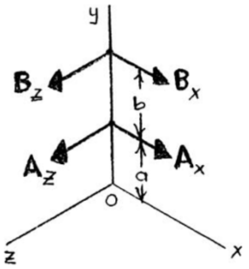
$$+ (-27.000\mathbf{i} + 6.0000\mathbf{j} - 14.0000\mathbf{k}) \text{ N} \cdot \text{m}$$

$$\mathbf{M}_2 = -(25.858 \text{ N} \cdot \text{m})\mathbf{i} + (21.190 \text{ N} \cdot \text{m})\mathbf{j}$$

or $\mathbf{M}_2 = -(25.9 \text{ N} \cdot \text{m})\mathbf{i} + (21.2 \text{ N} \cdot \text{m})\mathbf{j}$ ◀

A6.

SOLUTION



Express the forces at A and B as

$$\mathbf{A} = A_x\mathbf{i} + A_z\mathbf{k}$$
$$\mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Then, for equivalence to the given force system,

$$\Sigma F_x: \quad A_x + B_x = 0 \tag{1}$$
$$\Sigma F_z: \quad A_z + B_z = R \tag{2}$$
$$\Sigma M_x: \quad A_z(a) + B_z(a+b) = 0 \tag{3}$$
$$\Sigma M_z: \quad -A_x(a) - B_x(a+b) = M \tag{4}$$

From Equation (1), $\quad B_x = -A_x$

Substitute into Equation (4):

$$-A_x(a) + A_x(a+b) = M$$
$$A_x = \frac{M}{b} \quad \text{and} \quad B_x = -\frac{M}{b}$$

From Equation (2), $\quad B_z = R - A_z$

and Equation (3), $\quad A_z a + (R - A_z)(a+b) = 0$

$$A_z = R\left(1 + \frac{a}{b}\right)$$

and $\quad B_z = R - R\left(1 + \frac{a}{b}\right)$

$$B_z = -\frac{a}{b}R$$

Then

$$\mathbf{A} = \left(\frac{M}{b}\right)\mathbf{i} + R\left(1 + \frac{a}{b}\right)\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -\left(\frac{M}{b}\right)\mathbf{i} - \left(\frac{a}{b}R\right)\mathbf{k} \blacktriangleleft$$