March 17, 2020

1. Two long coaxial solenoids each carry current I, but in opposite directions, as shown in figure 1. The inner solenoid (radius a) has  $n_1$  turns per unit length, and the outer one (radius b) has  $n_2$ . Find  $\vec{B}$  in each of the three regions: (i) inside the inner solenoid, (ii) between them, and (iii) outside both.



Figure 1: Figure for problem 1.

- 2. Consider a long current carrying (I) wire of radius R with a volume current density proportional to  $s^2$  and in the  $\hat{z}$  direction. Here, s is the distance from the axis of the wire.
  - (a) Find the proportionality constant.
  - (b) Find the magnetic field and the vector potential inside (s < R) the wire.
  - (c) Find the vector potential outside (s > R) the wire.
- 3. (a) Consider an infinitely long solid cylindrical wire of radius R. Inside the wire, magnetic vector potential is given by  $\vec{A} = A_0 r \sin \phi \hat{z}$ ,  $A_0$  being a constant.
  - (i) Find the magnetic field inside the wire.
  - (ii) Find the volume current density inside the wire.
  - (iii) Find the net current passing through the wire.
  - (iv) Find the magnetic field outside the wire.

(b) Consider an infinitely long cylinder of radius a carrying a uniform volume current  $J_0\hat{z}$ , with z-axis coming out of the plane in figure 2. The wire has an off-axis hole of radius b with centre P at a distance d from the axis of the cylinder, passing through the point O. What is the y component of magnetic field at any point within the hole?

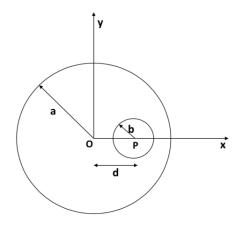


Figure 2: Figure for problem 3 (b)

Consider an infinitely long solid cylinder of radius R to be made up of a dielectric material and to be made permanently polarised so that the polarisation is everywhere radially outward, with a magnitude proportional to the distance from the axis of the finder  $\vec{P} = \frac{1}{2}P_0r\hat{r}$ ,  $P_0$  being a constant. If the cylinder is rotated with a constant angular velocity  $\omega$  about its axis (which coincides with z-axis) without change in P, what is the magnetic field on the axis of the cylinder?

(d) Consider an infinitely long hollow cylinder of radius R carrying uniform surface current  $\vec{K} = K\hat{z}$  with z-axis coinciding with the axis of the cylinder. Find out the magnetic field inside the cylinder  $\vec{B}_{\text{inside}}$  and outside the cylinder  $\vec{B}_{\text{outside}}$ .

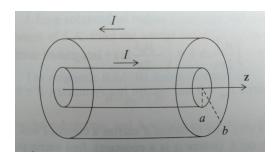


Figure 3: Figure for tutorial problem 4.

4. A long hollow coaxial wire has inner radius a and outer radius b. Uniform current I flows along its inner surface and return through the outer surface as shown in figure 3. Find vector potential at a distance s from its axis.

5. Show that for uniform magnetic field  $\vec{B}$ , the magnetic vector potential can be written as  $\vec{A} = -\frac{1}{2}(\vec{r} \times \vec{B})$ . Is this result unique, or are there other functions with the same properties?