PH 102, Electromagnetism,

Post Mid Semester

Lecture 9

Electrodynamics

Maxwell’s Equations:
Conservation Laws,
Poynting’s Theorem.

D. J. Griffiths: 7.3, 8.1

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Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0},$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} - \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}.$$

Maxwell's Equations in Matter

$$\nabla \cdot \mathbf{D} = \rho_f, \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}.$$

$$\boxed{\mathbf{J}_d \equiv \frac{\partial \mathbf{D}}{\partial t}}$$

For linear media:

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \qquad \mathbf{M} = \chi_m \mathbf{H},$$

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \text{and} \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B}$$

where,

$$\epsilon \equiv \epsilon_0(1 + \chi_e)$$

$$\mu \equiv \mu_0(1 + \chi_m)$$

Maxwell's Equations

Boundary Conditions: General Conditions

$$D_1^{\perp} - D_2^{\perp} = \sigma_f \qquad \mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} = \mathbf{0}$$

$$B_1^{\perp} - B_2^{\perp} = 0 \qquad \mathbf{H}_1^{\parallel} - \mathbf{H}_2^{\parallel} = \mathbf{K}_f \times \hat{\mathbf{n}}$$

For linear media: Boundary conditions in terms of **E** and **B** alone,

$$\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f, \qquad \mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} = \mathbf{0},$$

$$B_1^{\perp} - B_2^{\perp} = 0, \qquad \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} - \frac{1}{\mu_2}\mathbf{B}_2^{\parallel} = \mathbf{K}_f \times \hat{\mathbf{n}}.$$

Used in Theory of reflection and refraction!

Continuity equation

Conservation of charge (Global and local)

Total charge in the universe is constant: Global

If the total charge in a volume changes then same amount of charge must have passed through (in or out) the surface.

Let the charge in a volume in V is, $Q(t) = \int_{\mathcal{V}} \rho(\mathbf{r}, t)\, d\tau,$

The current flowing through the its S is $\oint_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{a},$

Local conservation of charge says:

$$\frac{dQ}{dt} = -\oint_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{a}. \xrightarrow{\text{Div Theorem}} \int_{\mathcal{V}} \frac{\partial \rho}{\partial t}\, d\tau = -\int_{\mathcal{V}} \nabla \cdot \mathbf{J}\, d\tau,$$

True for any volume, i.e,

$$\boxed{\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J}.}$$

Not independent, but consequence of the laws of ED

Poynting's Theorem

Continuity Equation for Energy density:

Work to assemble a static charge distribution (against Coulomb repulsion)

$$W_e = \frac{\epsilon_0}{2} \int E^2 \, d\tau, \qquad \text{[E resulting electric field]}$$

Work required to get currents going (against back emf)

$$W_m = \frac{1}{2\mu_0} \int B^2 \, d\tau, \qquad \text{[B resulting magnetic field]}$$

Total energy stored in electromagnetic fields, per unit volume,

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right).$$

- How to Derive energy per unit volume in a general way!
- What is the Energy Conservation Law in ED

Poynting's Theorem

Consider, some charge and current configuration creating the fields E and B at time t.

Work (dW) done by the electromagnetic forces in the interval dt ?

The work done on a charge q is,

$$dW = \mathbf{F} \cdot d\mathbf{l} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v}\, dt = q\mathbf{E} \cdot \mathbf{v}\, dt.$$

Rate of work done on all the charges in a volume V is,

$$\frac{dW}{dt} = \int_{\mathcal{V}} (\mathbf{E} \cdot \mathbf{J})\, d\tau.$$

$q \rightarrow \rho\, d\tau$
and
$\rho v \rightarrow J$

Poynting's Theorem

Rate of work done on all the charges in a volume V is,

$$\frac{dW}{dt} = \int_{\mathcal{V}} (\mathbf{E} \cdot \mathbf{J})\, d\tau.$$

E . J is the power per unit volume and in terms of the fields alone,

$$\boxed{\mathbf{E} \cdot \mathbf{J} = \frac{1}{\mu_0}\mathbf{E} \cdot (\nabla \times \mathbf{B}) - \epsilon_0 \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t}.}$$

$$\boxed{\nabla \times \mathbf{B} - \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}}$$

$$\boxed{\nabla \cdot (\mathbf{E} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{B}).}$$

$$\boxed{\mathbf{E} \cdot (\nabla \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{E}) - \nabla \cdot (\mathbf{E} \times \mathbf{B})}$$

$$\boxed{\mathbf{E} \cdot (\nabla \times \mathbf{B}) = -\mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t} - \nabla \cdot (\mathbf{E} \times \mathbf{B}).}$$

$$\boxed{\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t}$$

$$\mathbf{E} \cdot \mathbf{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\nabla \cdot (\mathbf{E} \times \mathbf{B}).$$

$$\boxed{\mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(E^2),}$$

$$\boxed{\mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(B^2),}$$

Poynting's Theorem

Rate of work done on all the charges in a volume V is,

$$\frac{dW}{dt} = \int_{\mathcal{V}} (\mathbf{E} \cdot \mathbf{J})\, d\tau.$$

E . J is the power per unit volume and in terms of the fields alone,

$$\mathbf{E} \cdot \mathbf{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) - \frac{1}{\mu_0} \nabla \cdot (\mathbf{E} \times \mathbf{B}).$$

$$\frac{dW}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) d\tau - \frac{1}{\mu_0}\oint_{\mathcal{S}} (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{a},$$

Div Theorem!

Poynting's Theorem

Work-Energy theorem of electrodynamics:

$$\frac{dW}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) d\tau - \frac{1}{\mu_0}\oint_{\mathcal{S}} (\mathbf{E}\times\mathbf{B})\cdot d\mathbf{a},$$

Total energy in the fields

Rate of energy transported out of V, *across* S

The 1st term (integral): total energy stored in the fields U_{em}

The 2nd term: rate of energy being transported out of *V*, across *S*, by the EM fields.

Poynting's theorem: *Work done on the charges by the EM force is equal to the decrease in energy stored in the fields, less the energy that flowed out through the surface .*

Poynting vector : Energy per unit time, per unit area, transported by the fields

$$\mathbf{S} \equiv \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}).$$

S* . *da ~ energy per unit time crossing the infinitesimal surface ***da***: Energy Flux

i.e ***S*** is the energy flux density.

Poynting's Theorem

$$\frac{dW}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau - \frac{1}{\mu_0}\oint_{\mathcal{S}} (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{a},$$

$$\frac{dW}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} u\, d\tau - \oint_{\mathcal{S}} \mathbf{S} \cdot d\mathbf{a}.$$

$$\mathbf{S} \equiv \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}).$$
$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right).$$

If no work is done on the charges, $dW/dt = 0$

$$\int \frac{\partial u}{\partial t}\, d\tau = -\oint \mathbf{S} \cdot d\mathbf{a} = -\int (\nabla \cdot \mathbf{S})\, d\tau,$$

Thus we get the "Continuity equation" for energy:

$$\boxed{\frac{\partial u}{\partial t} = -\nabla \cdot \mathbf{S}.}$$

u plays the role of ρ,
S the role of ***J***.

local conservation of electromagnetic energy.

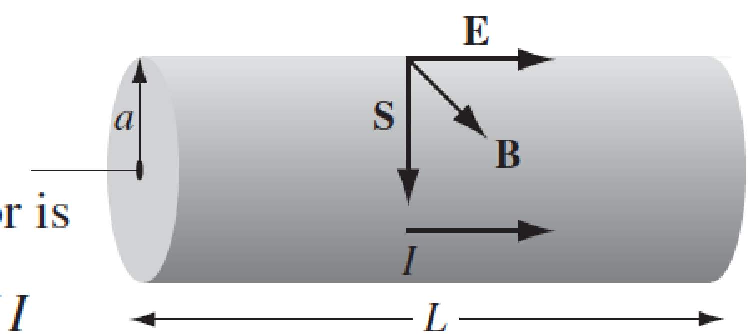
Poynting's Theorem

Example 8.1. When current flows down a wire, work is done, which shows up as Joule heating of the wire $P = VI$ Though there are certainly *easier* ways to do it, the energy per unit time delivered to the wire can be calculated using the Poynting vector. Assuming it's uniform, the electric field parallel to the wire is

$$E = \frac{V}{L},$$

where V is the potential difference between the ends and L is the length of the wire (Fig. 8.1). The magnetic field is "circumferential"; at the surface (radius a) it has the value

$$B = \frac{\mu_0 I}{2\pi a}.$$



Accordingly, the magnitude of the Poynting vector is

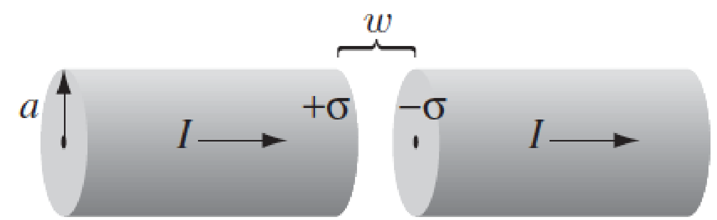
$$S = \frac{1}{\mu_0}\frac{V}{L}\frac{\mu_0 I}{2\pi a} = \frac{VI}{2\pi a L},$$

and it points radially inward. The energy per unit time passing in through the surface of the wire is therefore

$$\int \mathbf{S} \cdot d\mathbf{a} = S(2\pi a L) = VI$$

Poynting's Theorem

Problem 8.2 A fat wire, radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in Fig. 7.45. Find the magnetic field in the gap, at a distance $s < a$ from the axis.



(a) Find the electric and magnetic fields in the gap, as functions of the distance s from the axis and the time t. (Assume the charge is zero at $t = 0$.)

(b) Find the energy density u_{em} and the Poynting vector $\mathbf{S}$ in the gap. Note especially the *direction* of $\mathbf{S}$. Check that Eq. 8.12 is satisfied.

Solution: (a)

$$\mathbf{E}(t) = \frac{\sigma}{\epsilon_0}\,\hat{\mathbf{z}} = \frac{It}{\pi\epsilon_0 a^2}\,\hat{\mathbf{z}}.$$

$$\sigma = \frac{Q}{\pi a^2};\ Q(t) = It$$

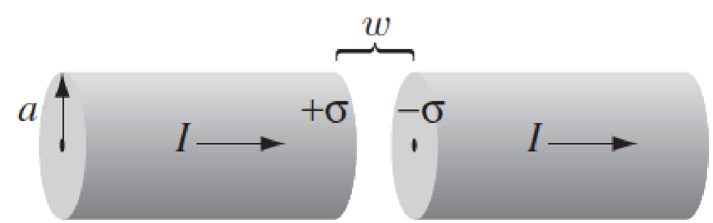
$$B\,2\pi s = \mu_0\epsilon_0 \frac{\partial E}{\partial t}\pi s^2 = \mu_0\epsilon_0 \frac{I\pi s^2}{\pi\epsilon_0 a^2}$$

$$\mathbf{B}(s,t) = \frac{\mu_0 I s}{2\pi a^2}\,\hat{\boldsymbol{\phi}}.$$

Poynting's Theorem

(a) Find the electric and magnetic fields in the gap, as functions of the distance s from the axis and the time t. (Assume the charge is zero at $t = 0$.)

(b) Find the energy density $u_{\rm em}$ and the Poynting vector $\mathbf{S}$ in the gap. Note especially the *direction* of $\mathbf{S}$. Check that Eq. 8.12 is satisfied.



Solution: **(a)**

$$\mathbf{E}(t) = \frac{\sigma}{\epsilon_0}\,\hat{\mathbf{z}} = \frac{It}{\pi\epsilon_0 a^2}\,\hat{\mathbf{z}}.$$

$$\mathbf{B}(s,t) = \frac{\mu_0 I s}{2\pi a^2}\,\hat{\phi}.$$

(b)

$$u_{\rm em} = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) = \frac{1}{2}\left[\epsilon_0\left(\frac{It}{\pi\epsilon_0 a^2}\right)^2 + \frac{1}{\mu_0}\left(\frac{\mu_0 I s}{2\pi a^2}\right)^2\right]$$

$$= \frac{\mu_0 I^2}{2\pi^2 a^4}\left[(ct)^2 + (s/2)^2\right].$$

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{1}{\mu_0}\left(\frac{It}{\pi\epsilon_0 a^2}\right)\left(\frac{\mu_0 I s}{2\pi a^2}\right)(-\hat{\mathbf{s}}) = -\frac{I^2 t}{2\pi^2\epsilon_0 a^4}\,s\,\hat{\mathbf{s}}.$$

Check $\boxed{\dfrac{\partial u}{\partial t} = -\nabla\cdot\mathbf{S}.}$

Maxwell's Equations

Poltential Formulation: Scalar and Vector Potential.

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho, \qquad \text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0, \qquad \text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Given the charge and current, what are the ***E***(r,t) and ***B***(r,t)?
Coulombs's and Bio-Savart law can not give the general answer!!

Fields in terms of potentials.

Electrostaics: Curl E = 0 ➔ E = -grad V [can not be generalized]

However, div B = 0, hence B = Curl A

Faraday's law, $\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A}),$ or $\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}\right) = 0.$

therefore, $\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V.$ or $\boxed{\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}.}$

old form for constant A!

Maxwell's Equations

Potential Formulation: Scalar and Vector Potential.

What about the other 2 laws?

(i) $$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = -\frac{1}{\epsilon_0}\rho;$$

(iv) $$\nabla \times (\nabla \times \mathbf{A}) = \mu_0 \mathbf{J} - \mu_0 \epsilon_0 \nabla \left(\frac{\partial V}{\partial t}\right) - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2},$$

using, $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

$$\left(\nabla^2 \mathbf{A} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2}\right) - \nabla \left(\nabla \cdot \mathbf{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}\right) = -\mu_0 \mathbf{J}.$$

The 4 sets of equations are now contained in 2 sets of eq. only!!

Maxwell's Equations

$$\mathbf{B} = \nabla \times \mathbf{A},$$
$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}.$$

Poltential Formulation: Scalar and Vector Potential.

Gauge Transformations:

The potential equations do not uniquely define the potentials;
You are free to impose extra conditions on V and A , if nothing happens to E and B .

$$\mathbf{A}' = \mathbf{A} + \boldsymbol{\alpha} \quad \text{and} \quad V' = V + \beta.$$

A & A′ gives same ***B*,**

$$\nabla \times \boldsymbol{\alpha} = 0. \qquad \text{hence} \qquad \boldsymbol{\alpha} = \nabla \lambda.$$

V & V′ gives same ***E***

$$\nabla \beta + \frac{\partial \boldsymbol{\alpha}}{\partial t} = 0,$$

$$\nabla \left(\beta + \frac{\partial \lambda}{\partial t} \right) = 0. \qquad \text{so,} \qquad \beta = -\frac{\partial \lambda}{\partial t} + k(t).$$

The following 'gauge transformations' will not affect the Physical quantities **E** and **B**

k(t) can be absorbed into λ,

$$\boxed{\begin{aligned} \mathbf{A}' &= \mathbf{A} + \nabla \lambda, \\ V' &= V - \frac{\partial \lambda}{\partial t}. \end{aligned}}$$

Maxwell's Equations

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = -\frac{1}{\epsilon_0}\rho; \qquad \left(\nabla^2 \mathbf{A} - \mu_0\epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2}\right) - \nabla\left(\nabla \cdot \mathbf{A} + \mu_0\epsilon_0 \frac{\partial V}{\partial t}\right) = -\mu_0 \mathbf{J}.$$

Coulomb Gauge

$$\nabla \cdot \mathbf{A} = 0.$$

$$\nabla^2 V = -\frac{1}{\epsilon_0}\rho.$$

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}', t)}{\mathscr{r}}\, d\tau'.$$

instantaneous!!

$$\nabla^2 \mathbf{A} - \mu_0\epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J} + \mu_0\epsilon_0 \nabla\left(\frac{\partial V}{\partial t}\right).$$

Lorenz Gauge

$$\nabla \cdot \mathbf{A} = -\mu_0\epsilon_0 \frac{\partial V}{\partial t}.$$

$$\nabla^2 \mathbf{A} - \mu_0\epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}.$$

and

$$\nabla^2 V - \mu_0\epsilon_0 \frac{\partial^2 V}{\partial t^2} = -\frac{1}{\epsilon_0}\rho.$$

V and A on an equal footing!!

Maxwell's Equations

Two important consequences of Maxwell's equations:

- Possibility of electromagnetic waves
- Breakdown of Newtonian mechanics, leading to Theory of relativity.

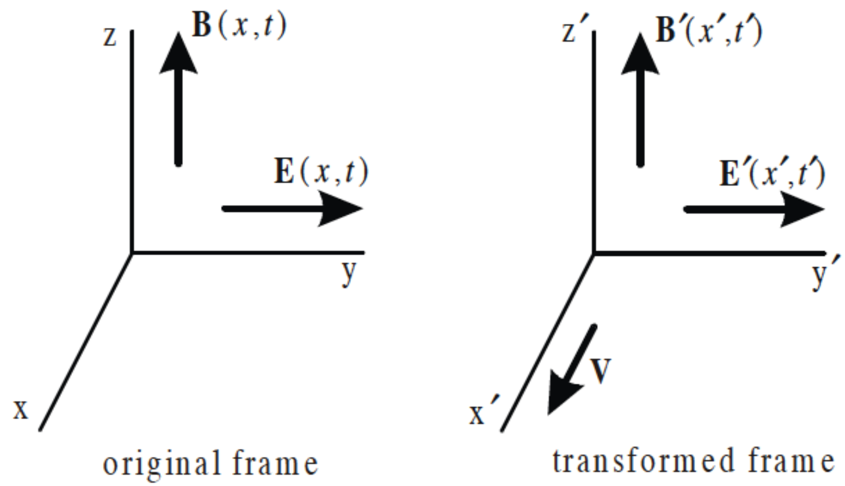
Compatibility of Maxwell's equations and Newton's laws:

Consider, ***E*** and ***B*** in two different reference frames, one moving with a constant velocity *V* with respect to the other.

E & ***B*** satisfy the Maxwell's equations in original frame.

Consider, ***E*** in the y & ***B*** in the z direction and ***E*** & ***B*** depend on x and t, only.

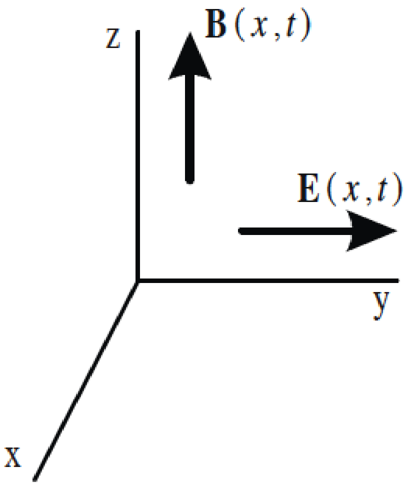
Moving frame quantities signified with primes.



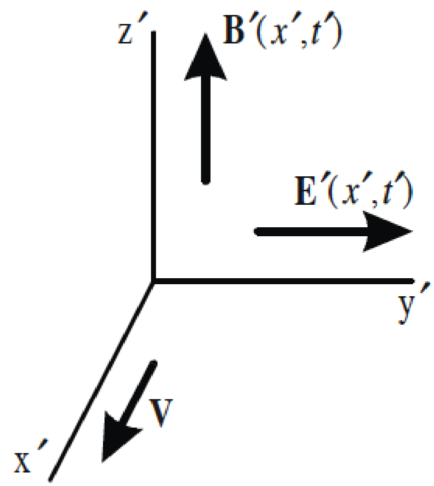
Original frame fields E(x,t) and B(x,t) obey Maxwell's equations

Lets us see, whether the transformed fields E′(x′,t′) and B′(x′, t′) satisfy Maxwell's equations or not!

Maxwell's Equations



original frame



transformed frame

$$x' = x - Vt$$
$$y' = y$$
$$z' = z$$
$$t' = t\ .$$

Newtonian principle: Force same in frames moving with constant velocity with respect to each other.

Invariance of the Lorentz force: Force on a moving charge is independent of ref frame.

Charge q moving with velocity v in the x direction

$$q\{E - vB\} = q\{E' - v'B'\}$$

[-from directions of v and B]

However, the velocity transforms as $v' = v - V$,

$$\text{hence, } E' - v'B' = (E' + VB') - vB' = E - vB.$$

Equating the coefficients of v,

[Sum of a v-independent term and one proportional to v.]

$$E = (E' + VB') \text{ and } B = B'.$$

This is the rule by which the fields transform.

Transformation of Maxwell's Equations

Let us investigate the transformation of the curl B equation (in the absence of any currents).

$$\operatorname{curl}\mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}\,.$$

we assumed, $\mathbf{E} = E(x,t)\,\mathbf{j}$ and $\mathbf{B} = B(x,t)\,\mathbf{k}$

Hence, $\operatorname{curl}\mathbf{B} = -\dfrac{\partial B}{\partial x}\mathbf{j}$ and $\dfrac{\partial \mathbf{E}}{\partial t} = \dfrac{\partial E}{\partial t}\mathbf{j}$

The Maxwell's equation for these fields, $\dfrac{\partial B}{\partial x} = -\dfrac{1}{c^2}\dfrac{\partial E}{\partial t}\,.$

From the co-ordinate transformations.

$$\frac{\partial x'}{\partial x} = 1 \qquad \frac{\partial x'}{\partial t} = -V$$

$$\frac{\partial t'}{\partial t} = 1 \qquad \frac{\partial t'}{\partial x} = 0\,.$$

Transformation of Maxwell's Equations

$$\frac{\partial B}{\partial x}=\frac{\partial B}{\partial x'}\frac{\partial x'}{\partial x}+\frac{\partial B}{\partial t'}\frac{\partial t'}{\partial x}$$

$$\frac{\partial E}{\partial t}=\frac{\partial E}{\partial t'}\frac{\partial t'}{\partial t}+\frac{\partial E}{\partial x'}\frac{\partial x'}{\partial t}\,.$$

$$\frac{\partial x'}{\partial x}=1 \qquad \frac{\partial x'}{\partial t}=-V$$

$$\frac{\partial t'}{\partial t}=1 \qquad \frac{\partial t'}{\partial x}=0\,.$$

$$\frac{\partial B}{\partial x}=\frac{\partial B}{\partial x'}$$

$$\frac{\partial E}{\partial t}=\frac{\partial E}{\partial t'}-V\frac{\partial E}{\partial x'}\,.$$

E = (E′ +VB′) & B = B′, using these relations in $\frac{\partial B}{\partial x}=\frac{\partial B}{\partial x'}$ $\frac{\partial E}{\partial t}=\frac{\partial E}{\partial t'}-V\frac{\partial E}{\partial x'}\,.$ would give us the equation in transformed frame,

$$\frac{\partial B'}{\partial x'}=-\frac{1}{c^2}\frac{\partial E'}{\partial t'}+\frac{1}{c^2}\left\{V\frac{\partial E'}{\partial x'}-V\frac{\partial B'}{\partial t'}+V^2\frac{\partial B'}{\partial x'}\right\}$$

[Extra term ????]

Maxwell's equation are not invariant under Galilean transformation

Transformation of Maxwell's Equations

The Maxwell's equation for these fields, $\frac{\partial B}{\partial x} = -\frac{1}{c^2}\frac{\partial E}{\partial t}$.

$$\frac{\partial B'}{\partial x'} = -\frac{1}{c^2}\frac{\partial E'}{\partial t'} + \frac{1}{c^2}\left\{V\frac{\partial E'}{\partial x'} - V\frac{\partial B'}{\partial t'} + V^2\frac{\partial B'}{\partial x'}\right\}$$

[Extra term ????]

Maxwell's equation are not invariant under Galilean transformation

What about Lorentz transformation?

$$x' = \gamma(x - Vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{V}{c^2}x\right),$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} .$$

What about E and B?

Transformation of Maxwell's Equations

The Maxwell's equation for these fields, $$\frac{\partial B}{\partial x} = -\frac{1}{c^2}\frac{\partial E}{\partial t}.$$

$$\frac{\partial B'}{\partial x'} = -\frac{1}{c^2}\frac{\partial E'}{\partial t'} + \frac{1}{c^2}\left\{V\frac{\partial E'}{\partial x'} - V\frac{\partial B'}{\partial t'} + V^2\frac{\partial B'}{\partial x'}\right\}$$

[Extra term ????]

Maxwell's equation are not invariant under Galilean transformation

What about Lorentz transformation?

$$x' = \gamma(x - Vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{V}{c^2}x\right),$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}}.$$

$$E = \gamma\{E' + VB'\}$$

$$B = \gamma\left\{B' + \frac{V}{c^2}E'\right\};$$

Check for yourself, (Homework)