ME101: Engineering Mechanics (3 1 0 8)

2019-20 (II Semester)

LECTURE: 10

Friction

- Types of Friction
- Applications of Friction in Machines
- Wedges
- Screws
- Journal bearing, thrust bearings
- Flexible belts
- Rolling Resistance

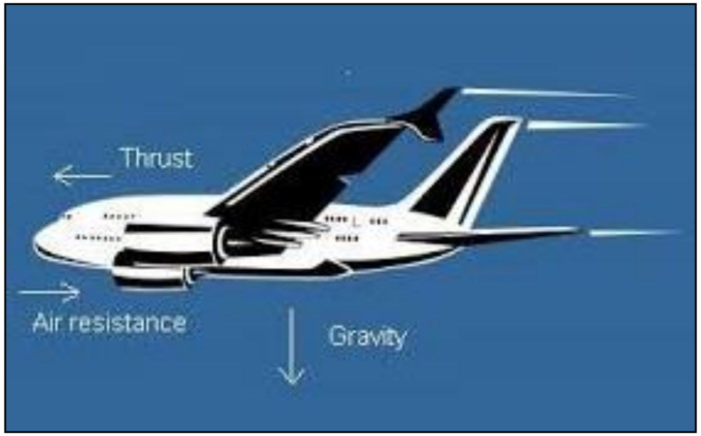
What covered earlier

- In this course so far, it has been assumed that all bodies considered have smooth surfaces. Till date, only forces perpendicular to the contact plane can be transferred between two bodies in contact.

- This is a proper description of the mechanical behavior if the tangential forces occurring in reality due to the roughness of the surfaces can be neglected.

- We will address problems for which this simplification is not valid.

Role of Friction in real life



Friction in Driving a Car

Points about Friction

- However, the friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied.

- There are two types of friction: **dry** or **Coulomb friction** and **fluid friction**. Fluid friction applies to lubricated mechanisms. The present discussion is limited to dry friction between non-lubricated surfaces.

Types of Friction

- **Friction** is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other. There are several types of friction:

- **Dry friction** resists relative lateral motion of two solid surfaces in contact. Dry friction is subdivided into *static friction* ("stiction") between non-moving surfaces, and *kinetic friction* between moving surfaces.

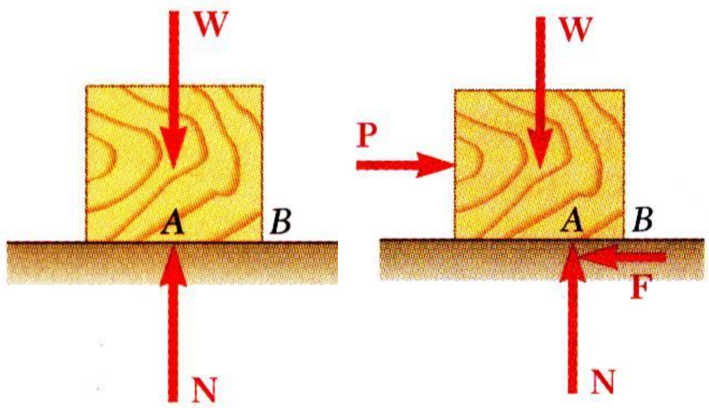
- **Fluid friction** describes the friction between layers of a viscous fluid that are moving relative to each other.

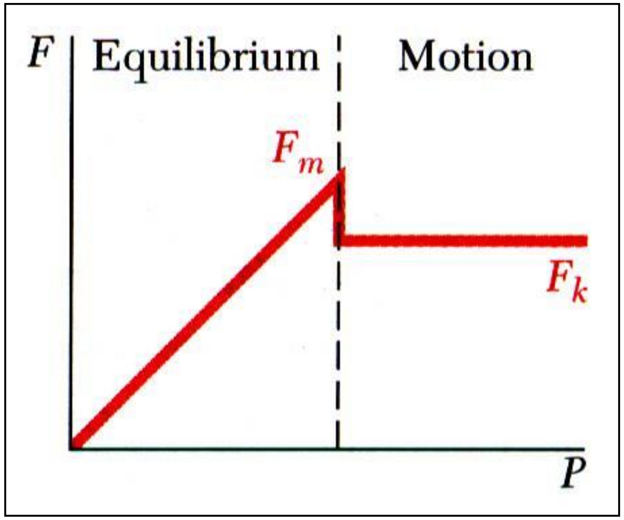
- **Lubricated friction** is a case of fluid friction where a lubricant fluid separates two solid surfaces.

- **Skin friction** is a component of drag, the force resisting the motion of a fluid across the surface of a body.

- **Internal friction** is the force resisting motion between the elements making up a solid material while it undergoes deformation.

Laws of Dry Friction or Coefficients of Friction





- Block of weight W placed on horizontal surface. Forces acting on block are its weight and reaction of surface N.
- Small horizontal force P applied to block. For block to remain stationary, in equilibrium, a horizontal component F of the surface reaction is required. F is a **static-friction force.**
- As P increases, the static-friction force F increases as well until it reaches a maximum value F_m.

$$F_m = \mu_s N$$

- Further increase in P causes the block to begin to move as F drops to a smaller *kinetic-friction force* F_k.

$$F_k = \mu_k N$$

Laws of Dry Friction or Coefficients of Friction *(contd.)*

Table 8.1. Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

- Maximum/limiting static-friction force:

$$F_m = \mu_s N$$

- Kinetic-friction force:

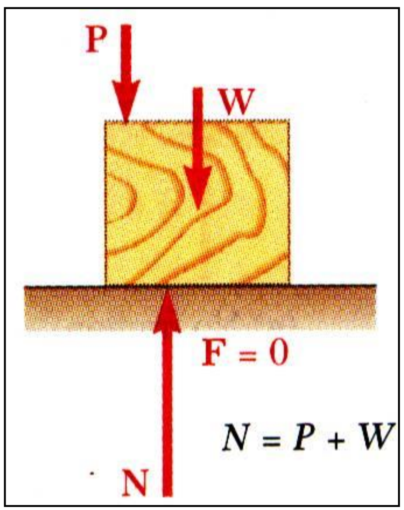
$$F_k = \mu_k N$$

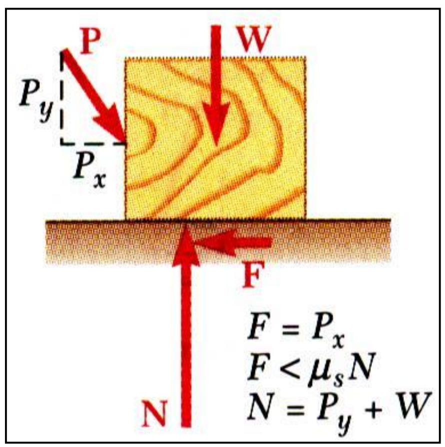
$$\mu_k \cong 0.75 \mu_s$$

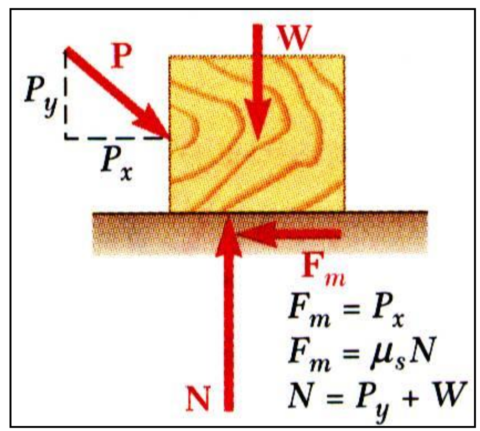
- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

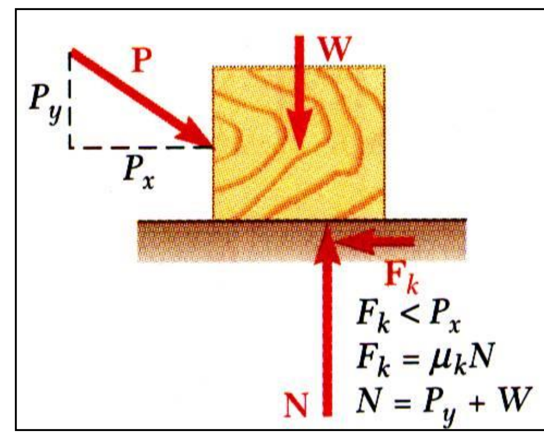
Laws of Dry Friction or Coefficients of Friction *(contd.)*

- Four situations can occur when a rigid body is in contact with a horizontal surface:









- No friction, ($P_x = 0$)

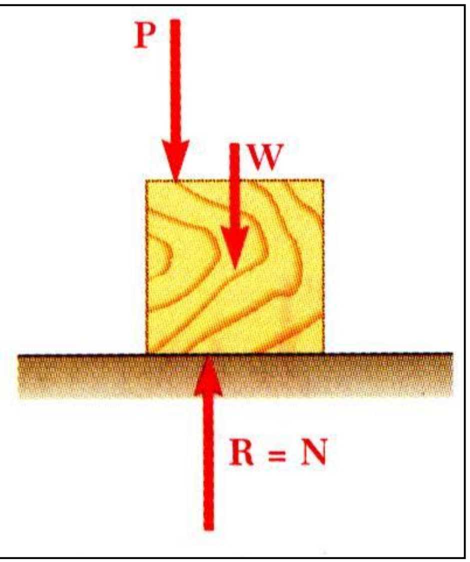
- No motion, ($P_x < F_m$)

- Motion impending, ($P_x = F_m$)

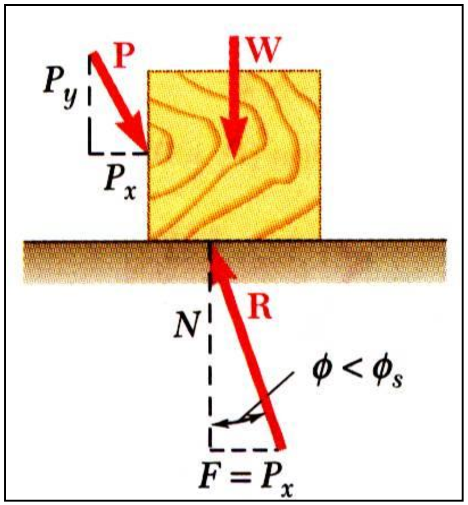
- Motion, ($P_x > F_m$)

Angles of Friction

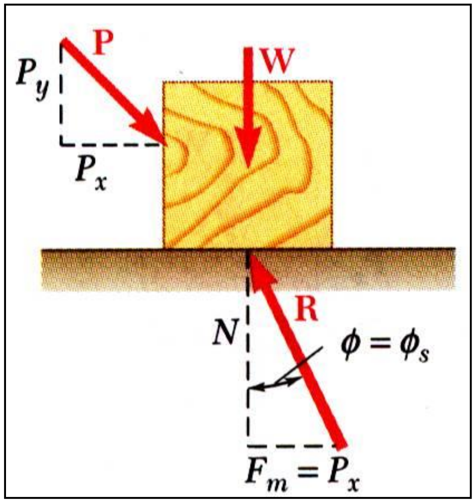
- It is sometimes convenient to replace normal force N and friction force F by their resultant R:



- No friction



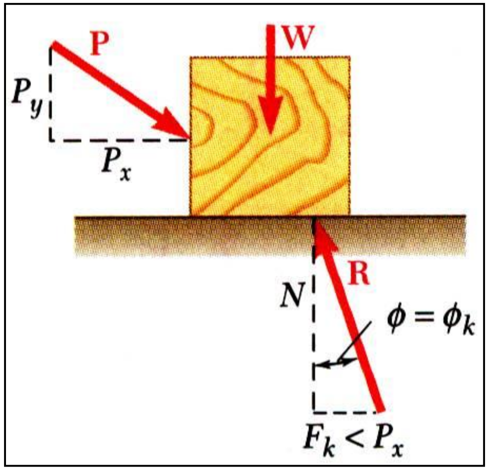
- No motion



- Motion impending

$$\tan\varphi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\varphi_s = \mu_s$$



- Motion

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$

Limitation of Coulomb Friction

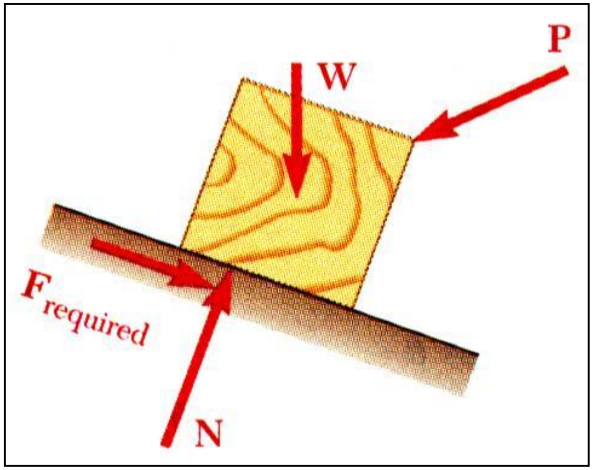
- The limiting frictional force is proportional to the applied normal force, independently of the contact area.

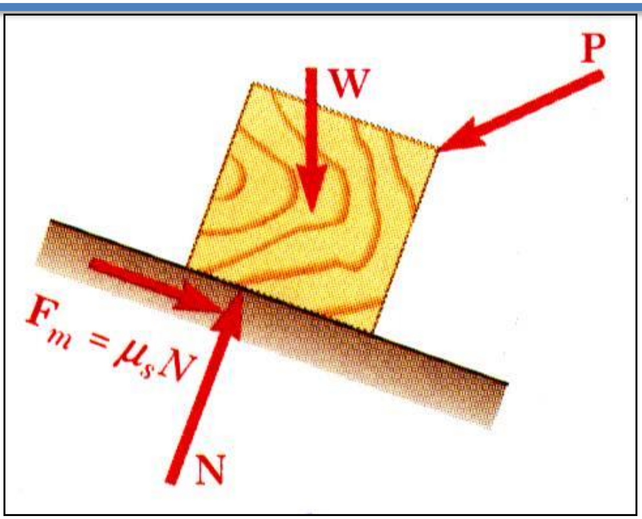
- When the surfaces are conjoined, Coulomb friction becomes a very poor approximation (for example, adhesive tape resists sliding even when there is no normal force, or a negative normal force).

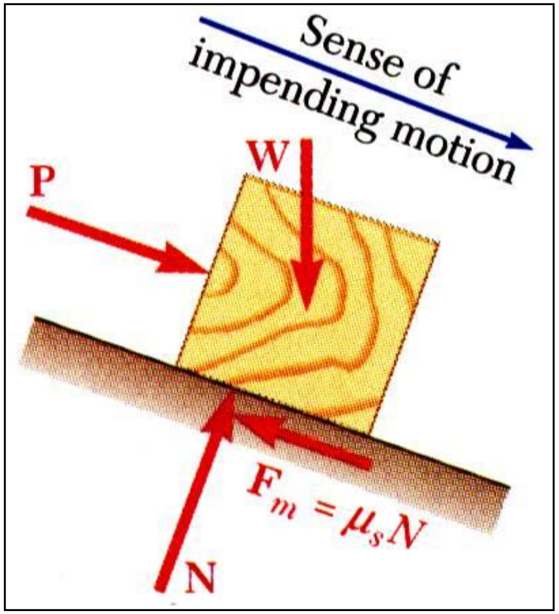
- In this case, the frictional force may depend strongly on the area of contact. Some drag racing tires are adhesive for this reason.

- However, despite the complexity of the fundamental physics behind friction, the relationships are accurate enough to be useful in many applications.

Problems Involving Dry Friction





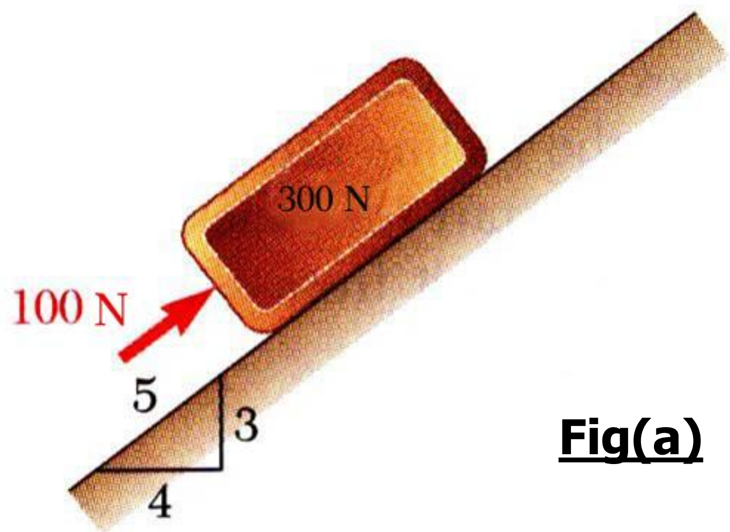


- All applied forces known
- Coefficient of static friction is known
- Determine whether body will remain at rest or slide

- All applied forces known
- Motion is impending
- Determine value of coefficient of static friction.

- Coefficient of static friction is known
- Motion is impending
- Determine magnitude or direction of one of the applied forces

Problem 1



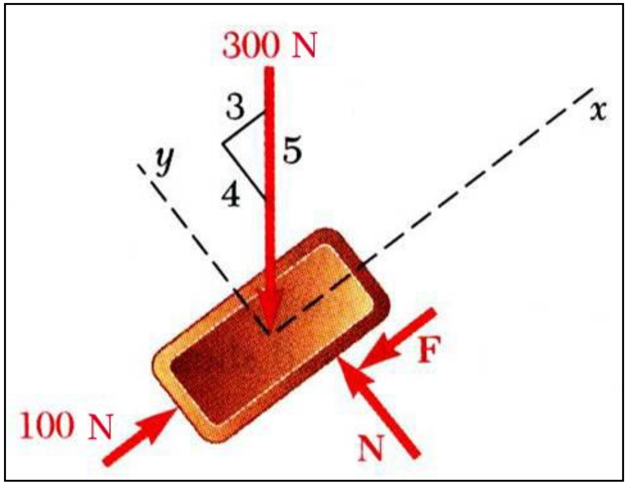
Fig(a)

A 100 N force acts as shown on a 300 N block placed on an inclined plane. The coefficients of friction between the block and plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium and find the value of the friction force as in **Fig(a)**?

SOLUTION:

- ✓ Determine values of friction force and normal reaction force from plane required to maintain equilibrium.
- ✓ Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.
- ✓ If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

Problem 1 *(contd.)*



SOLUTION:

- ✓ Determine values of friction force and normal reaction force from plane required to maintain equilibrium.

$$\sum F_x = 0: \qquad 100\text{ N} - \tfrac{3}{5}(300\text{ N}) - F = 0$$

$$F = -80\text{ N}$$

$$\sum F_y = 0: \qquad N - \tfrac{4}{5}(300\text{ N}) = 0$$

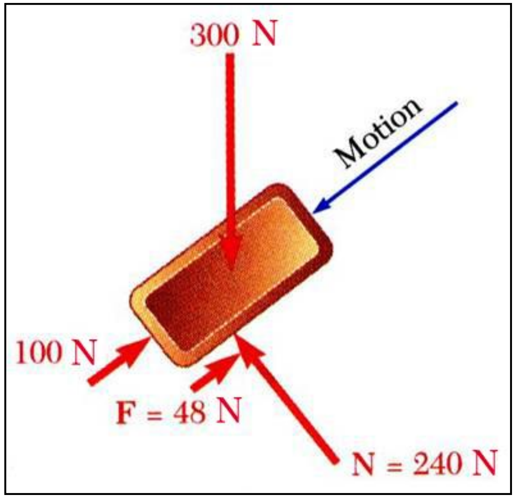
$$N = 240\text{ N}$$

- ✓ Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.

$$F_m = \mu_s N \qquad F_m = 0.25(240\text{ N}) = 48\text{ N}$$

The block will slide down the plane.

Problem 1 *(contd.)*

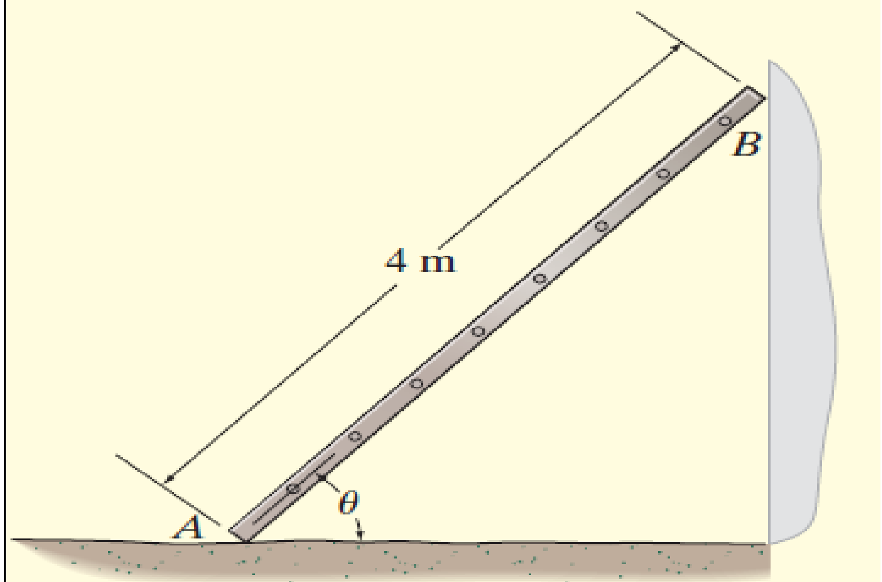


- ✓ If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

$$F_{actual} = F_k = \mu_k N$$
$$= 0.20(240\ \text{N})$$

$$\boxed{F_{actual} = 48\ \text{N}}$$

(Problem-2) The uniform 10-kg ladder in **Fig(b)** rests against the smooth wall at *B* , and the end *A* rests on the rough horizontal plane for which the coefficient of static friction is = 0.3 . Determine the angle of inclination u of the ladder and the normal reaction at *B* if the ladder is on the verge of slipping?



Fig(b)

Problem 2 *(contd.)*

Equations of Equilibrium and Friction. Since the ladder is on the verge of slipping, then $F_A = \mu_s N_A = 0.3N_A$. By inspection, N_A can be obtained directly.

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 10(9.81)\text{ N} = 0 \qquad N_A = 98.1\text{ N}$$

Using this result, $F_A = 0.3(98.1\text{ N}) = 29.43\text{ N}$. Now N_B can be found.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 29.43\text{ N} - N_B = 0$$

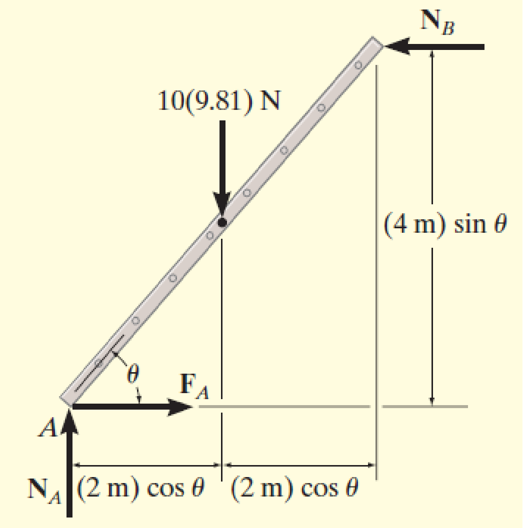
$$N_B = 29.43\text{ N} = 29.4\text{ N} \qquad \textit{Ans.}$$

Finally, the angle θ can be determined by summing moments about point A.

$$\circlearrowleft +\Sigma M_A = 0; \qquad (29.43\text{ N})(4\text{ m})\sin\theta - [10(9.81)\text{ N}](2\text{ m})\cos\theta = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = 1.6667$$

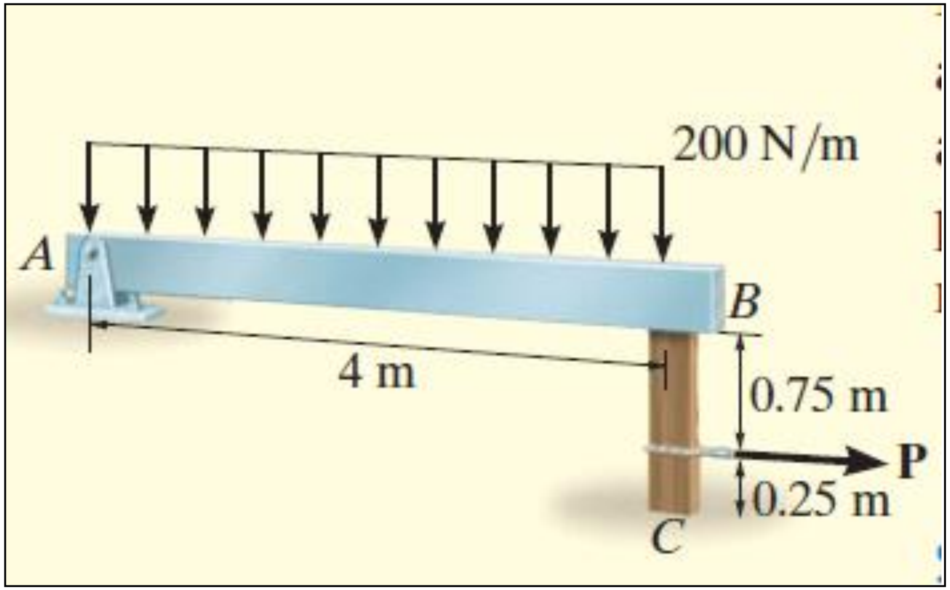
$$\theta = 59.04^\circ = 59.0^\circ \qquad \textit{Ans.}$$



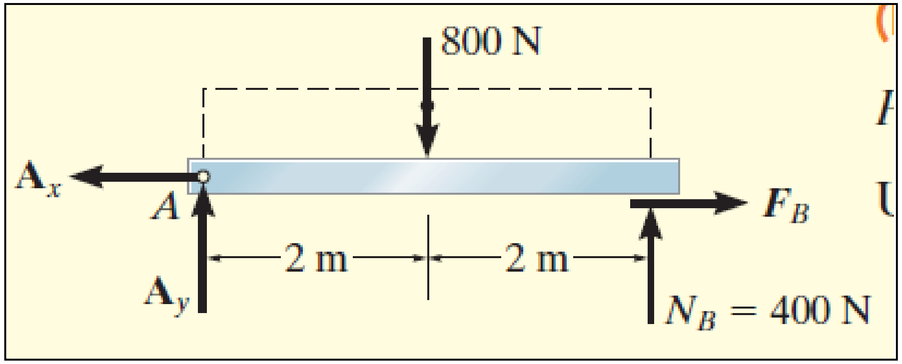
(b)

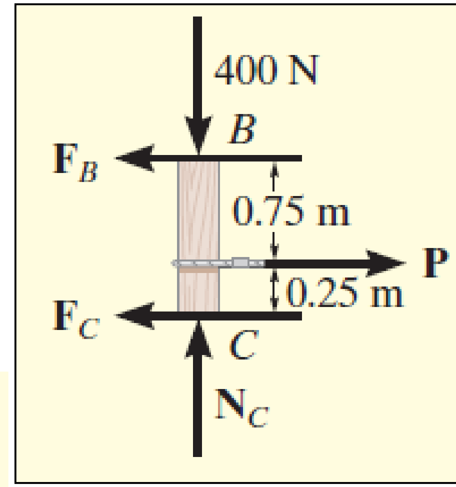
Problem 3

Beam AB is subjected to a uniform load of 200 N/m and is supported at B by post BC, Fig. 8–10a. If the coefficients of static friction at B and C are $\mu_B = 0.2$ and $\mu_C = 0.5$, determine the force $\mathbf{P}$ needed to pull the post out from under the beam. Neglect the weight of the members and the thickness of the beam.



Problem 3 *(contd.)*





Equations of Equilibrium and Friction.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_B - F_C = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_C - 400 \text{ N} = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -P(0.25 \text{ m}) + F_B(1 \text{ m}) = 0 \qquad (3)$$

(Post Slips at *B* and Rotates about C.) This requires $F_C \leq \mu_C N_C$ and

$$F_B = \mu_B N_B; \qquad F_B = 0.2(400 \text{ N}) = 80 \text{ N}$$

Using this result and solving Eqs. 1 through 3, we obtain

N

$$P = 320 \text{ N}$$

$$F_C = 240 \text{ N}$$

$$N_C = 400 \text{ N}$$

Since $F_C = 240 \text{ N} > \mu_C N_C = 0.5(400 \text{ N}) = 200 \text{ N}$, slipping at C occurs. Thus the other case of movement must be investigated.

Problem 3 *(contd.)*

(Post Slips at C and Rotates about B.) Here $F_B \leq \mu_B N_B$ and

$$F_C = \mu_C N_C; \qquad F_C = 0.5 N_C \qquad (4)$$

Solving Eqs. 1 through 4 yields

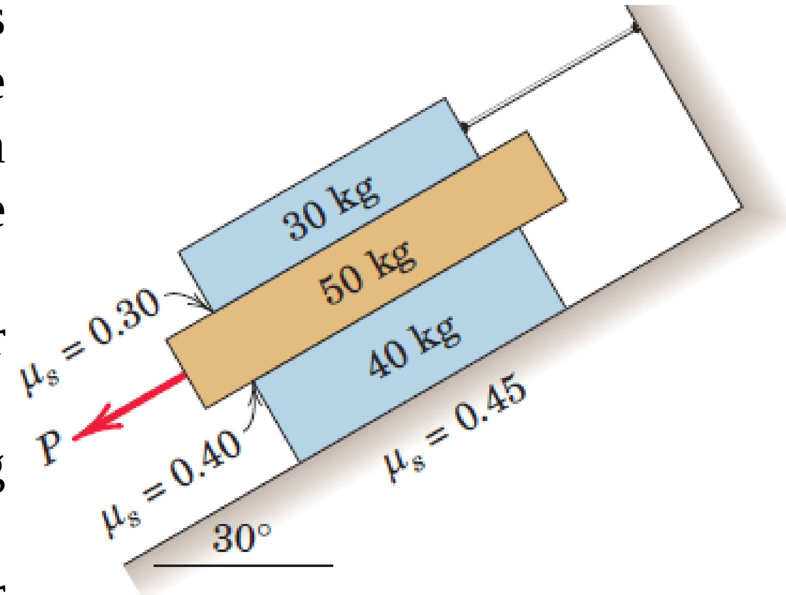
$$P = 267 \text{ N} \qquad \textit{Ans.}$$
$$N_C = 400 \text{ N}$$
$$F_C = 200 \text{ N}$$
$$F_B = 66.7 \text{ N}$$

Obviously, this case occurs first since it requires a *smaller* value for P.

(Problem-4) The three flat blocks are positioned on the 30 incline as shown in **Fig(c)** , and a force *P* parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown**.** Determine the maximum value which *P* may have before any slipping takes place?

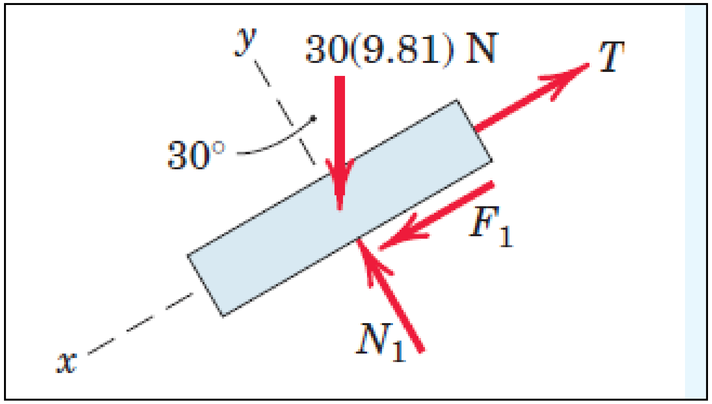
Solution:

- ✓ The free-body diagram of each block is drawn. The friction forces are assigned in the directions to oppose the relative motion which would occur if no friction were present.
- ✓ There are two possible conditions for impending motion.
- ✓ Either the 50-kg block slips and the 40-kg block remains in place, or
- ✓ The 50- and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.



Fig(c)

Problem 4 *(contd.)*

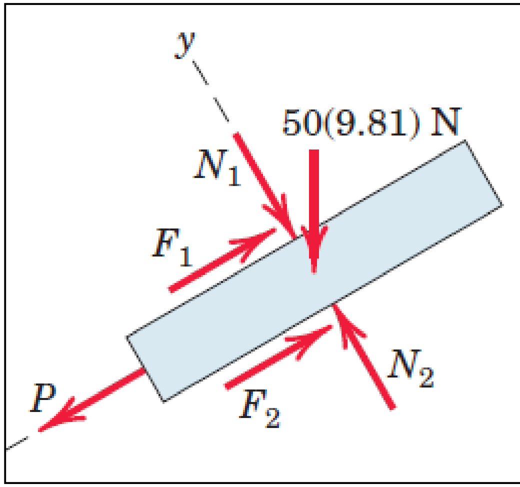


- ✓ The normal forces, which are in the *y*-direction, may be determined without reference to the friction forces, which are all in the *x*-direction.

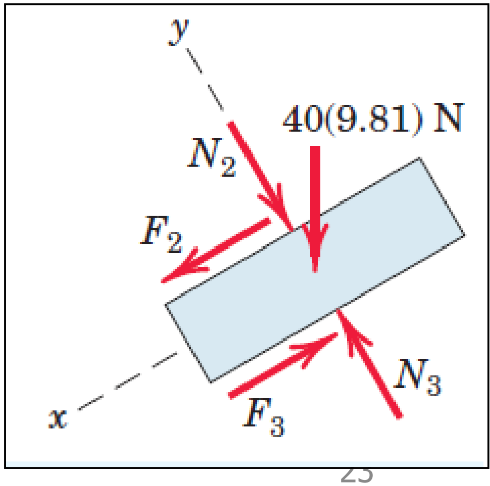
$[\Sigma F_y = 0]$ (30-kg) $N_1 - 30(9.81)\cos 30° = 0$ $N_1 = 255\text{ N}$

(50-kg) $N_2 - 50(9.81)\cos 30° - 255 = 0$ $N_2 = 680\text{ N}$

(40-kg) $N_3 - 40(9.81)\cos 30° - 680 = 0$ $N_3 = 1019\text{ N}$



- ✓ We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg block remains in place. Thus, for impending slippage at both surfaces of the 50-kg block, we have



$[F_{max} = \mu_s N]$ $F_1 = 0.30(255) = 76.5\text{ N}$ $F_2 = 0.40(680) = 272\text{ N}$

The assumed equilibrium of forces at impending motion for the 50-kg block gives

$[\Sigma F_x = 0]$ $P - 76.5 - 272 + 50(9.81)\sin 30° = 0$ $P = 103.1\text{ N}$

Problem 4 *(contd.)*

- ✓ We now check on the validity of our initial assumption. For the 40-kg block with $F2$ =272 N the friction force $F3$ would be given

$$[\Sigma F_x = 0] \qquad 272 + 40(9.81)\sin 30° - F_3 = 0 \qquad F_3 = 468\text{ N}$$

But the maximum possible value of

$$F_3 \text{ is } F_3 = \mu_s N_3 = 0.45(1019) = 459\text{ N}.$$

- ✓ Thus, 468 N cannot be supported and our initial assumption was wrong. We conclude, therefore, that slipping occurs first between the 40-kg block and the incline. With the corrected value $F3$ 459 N, equilibrium of the 40-kg block for its impending motion,

$$[\Sigma F_x = 0] \qquad F_2 + 40(9.81)\sin 30° - 459 = 0 \qquad F_2 = 263\text{ N}$$

Equilibrium of the 50-kg block gives, finally,

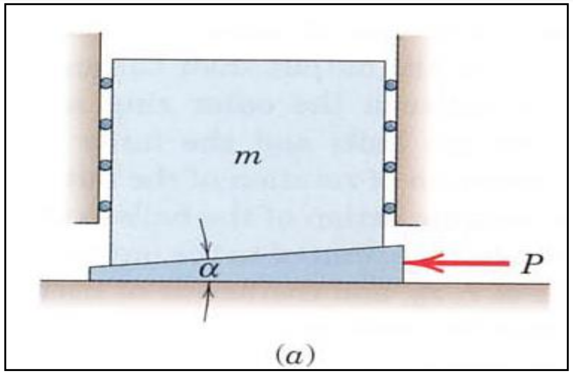
$$[\Sigma F_x = 0] \qquad P + 50(9.81)\sin 30° - 263 - 76.5 = 0$$

$$P = 93.8\text{ N} \qquad \textit{Ans.}$$

Thus, with $P = 93.8$ N, motion impends for the 50-kg and 40-kg blocks as a unit.

Wedge Problem

- ❑ Simple machines used to raise heavy loads.
- ❑ Force required to lift block is significantly less than block weight.
- ❑ Friction prevents wedge from sliding out.
- ❑ Want to find minimum force *P* to raise block.

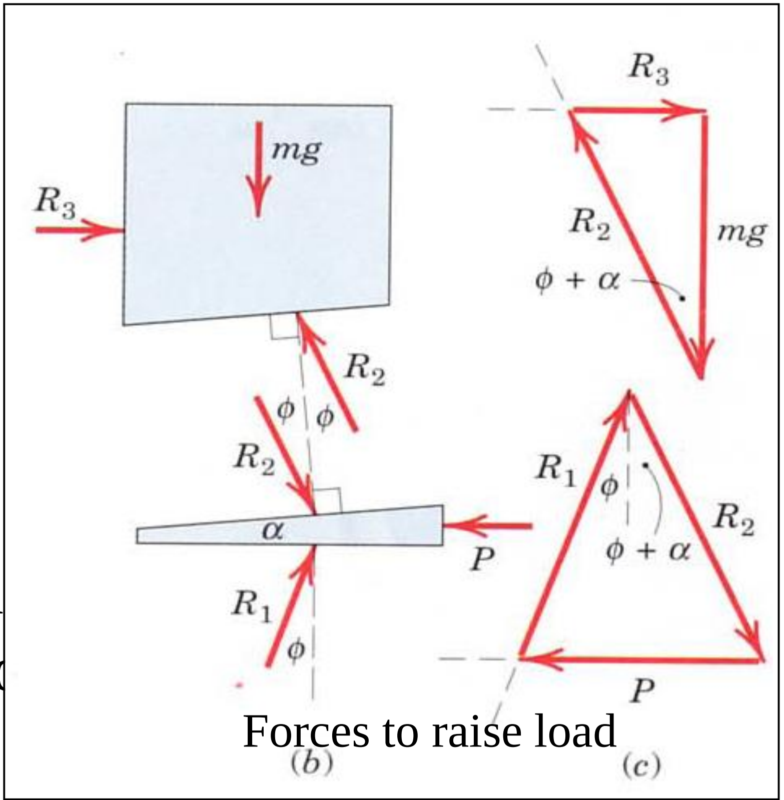


Coefficient of Friction for each pair of surfaces μ = tan (Static/Kinetic

❖ **FBDs**:

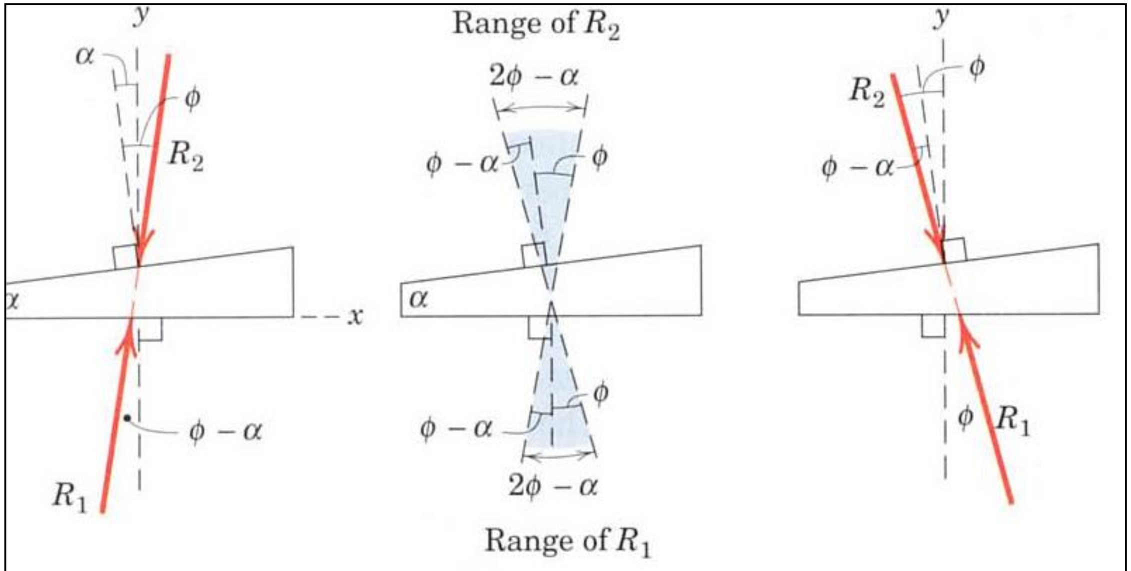
➢ Reactions are inclined at an angle from their respective normal's and are in the direction opposite to the motion. Force vectors acting on each body can also be shown.

➢ *R2* is first found from upper diagram sin *mg* is known. Then *P* can be found out fro the lower diagram since *R2* is known.



Analysis of Wedge Problem

- P is removed and wedge remains in place – Self Locking.
- Equilibrium of wedge requires that the equal reactions R1 and R2 be collinear.
- In the figure, wedge angle α is taken to be less than Φ
  - Impending slippage at the upper surface
  - Impending slippage at the lower surface.
- Slippage must occur at both surfaces simultaneously. In order for the wedge to slide out of its space, else, the wedge is Self-Locking.
- Range of angular positions of R1 and R2 for which the wedge will remain in place is shown in figure (b).

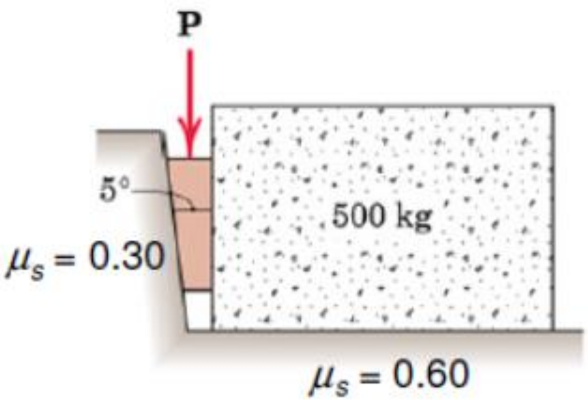


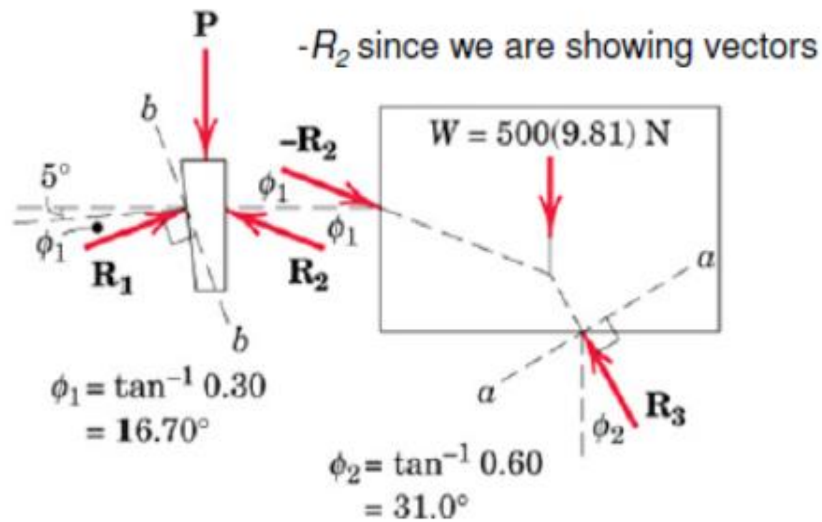
(a) Slipping impending at upper surface
(b) Range of $R_1 = R_2$ for no slip
(c) Slipping impending at lower surface

Problem 5

Coefficient of Static Friction for both pairs of wedge = 0.3
Coefficient of Static Friction between block and horizontal surface = 0.6
Find the least P required to move the block

Solution: Draw FBDs





Problem 5 *(contd.)*

Solution: $W = 500 \times 9.81 = 4905$ N

Three ways to solve

Method 1:

Equilibrium of FBD of the Block

$\sum F_X = 0$

$R_2 \cos \phi_1 = R_3 \sin \phi_2 \rightarrow R_2 = 0.538 R_3$

$\sum F_Y = 0$

$4905 + R_2 \sin \phi_1 = R_3 \cos \phi_2 \rightarrow R_3 = 6970$ N

$\rightarrow R_2 = 3750$ N

Equilibrium of FBD of the Wedge

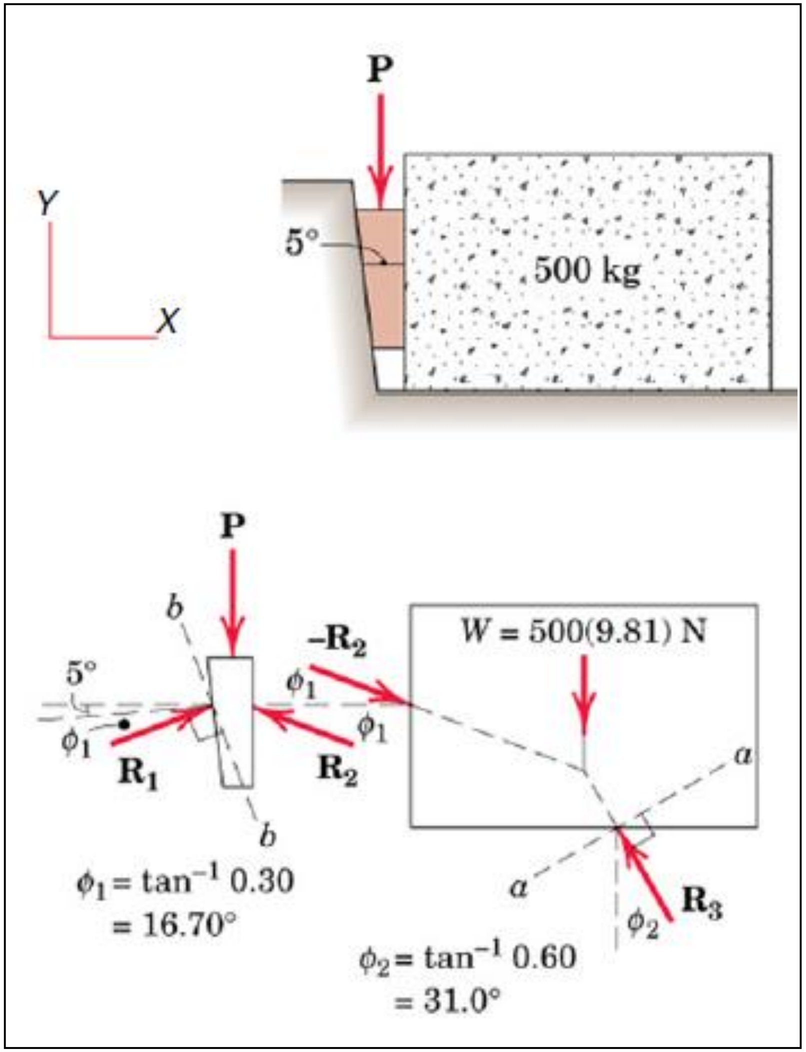
$\sum F_X = 0$

$R_2 \cos \phi_1 = R_1 \cos(\phi_1+5) \rightarrow R_1 = 3871$ N

$\sum F_Y = 0$

$R_1 \sin(\phi_1+5) + R_2 \sin \phi_1 = P$

$\rightarrow P = 2500$ N



Problem 5 *(contd.)*

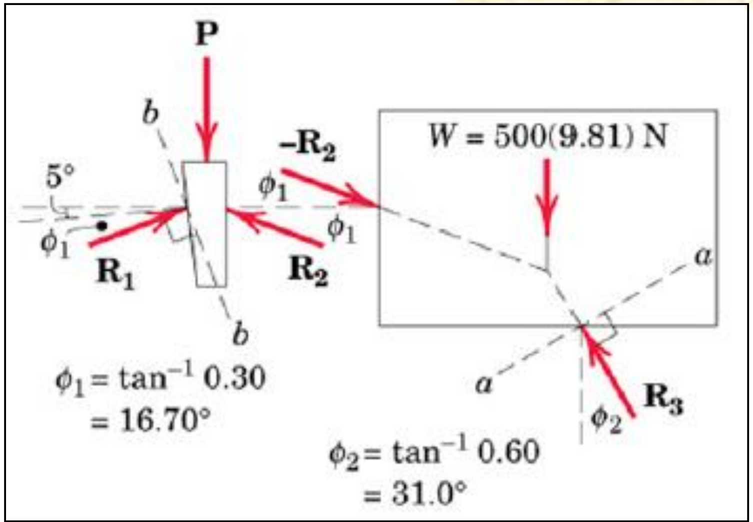
Method 2:
Using Equilibrium equations along reference axes *a-a* and *b-b*
→ No need to solve simultaneous equations
Angle between R_2 and *a-a* axis = 16.70+31.0 = 47.7°

Equilibrium of Block:

$$[\Sigma F_a = 0] \qquad 500(9.81)\sin 31.0° - R_2 \cos 47.7° = 0$$
$$R_2 = 3750 \text{ N}$$

Equilibrium of Wedge:
Angle between R_2 and *b-b* axis = 90-(2Φ_1+5) = 51.6°
Angle between *P* and *b-b* axis = Φ_1+5 = 21.7°

$$[\Sigma F_b = 0] \qquad 3750 \cos 51.6° - P \cos 21.7° = 0$$
$$P = 2500 \text{ N}$$

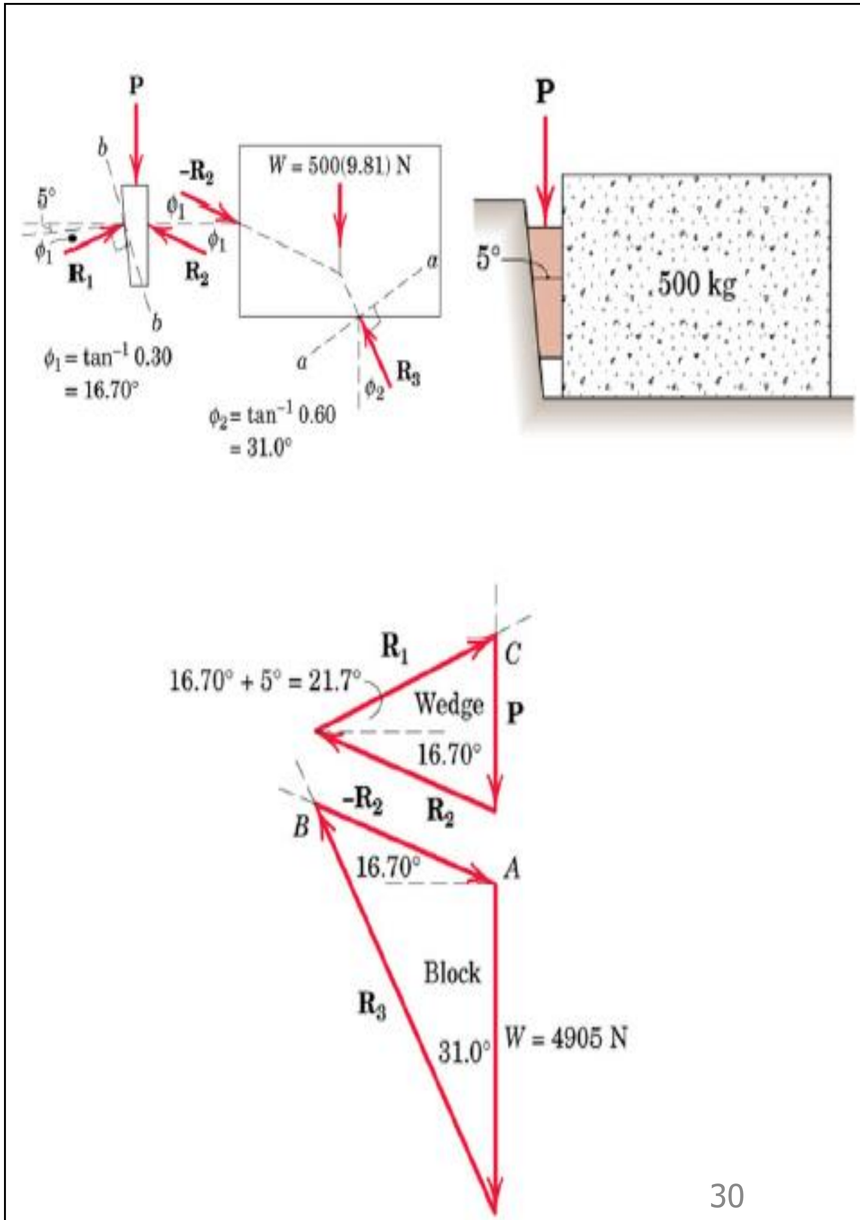


Problem 5 *(contd.)*

Method 3:
Graphical solution using vector polygons

Starting with equilibrium of the block:
W is known, and directions of R_2 and R_3 are known
→ Magnitudes of R_2 and R_3 can be determined graphically

Similarly, construct vector polygon for the wedge from known magnitude of R_2, and known directions of R_2, R_1, *and* P.
→ Find out the magnitude of P graphically



Assignment

Determine the force P required to force the 10° wedge under the 90-kg uniform crate which rests against the small stop at A. The coefficient of friction for all surfaces is 0.40.