SIMPLE PULLEY

A simple pulley is a wheel of metal or wood, with a groove around its circumference, to receive rope or chain. The pulley rotates freely about its axle, which passes through its centre and is perpendicular to its surface plane. This axle is supported by a metal or a wooden frame, called block as show in Fig. 11.9. Following assumptions are made in the study of pulley system, which are quite reasonable from the practical point of view :

1. The weight of the pulley block is small as compared to the weight to be lifted, and thus may be neglected in calculations.
2. The friction between the pulley surface and the string is negligible, and thus the tension in the two sides of the rope, passing round the pulley, may be taken to be equal.

A little consideration will show, that in a simple pulley, its mechanical advantage as well as velocity ratio is 1 under the assumed conditions mentioned above. The only advantage of a simple pulley is that the effort can be applied Fig. 11.10 (*a*), (*b*) and (*c*). Simple pulleys are generally used in certain mechanical advantage and efficiency.

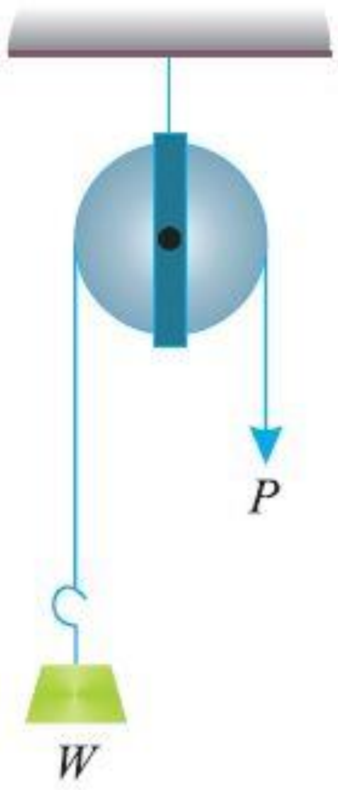


Fig. 11.9. Simple pulley.

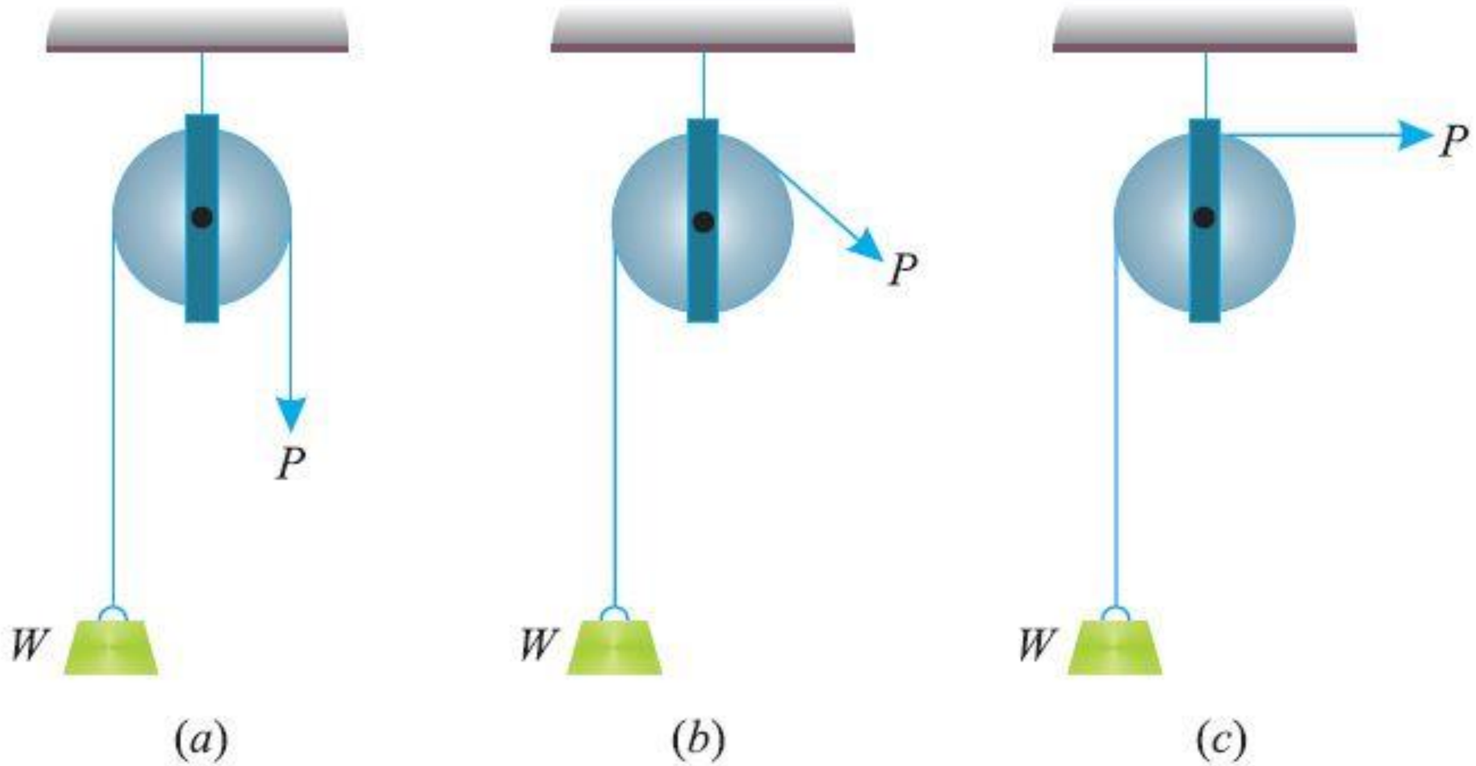


Fig. 11.10. Force applied in different directions.

Though there are many typed of pulleys used by engineers, yet the following system of pulleys are commonly used :

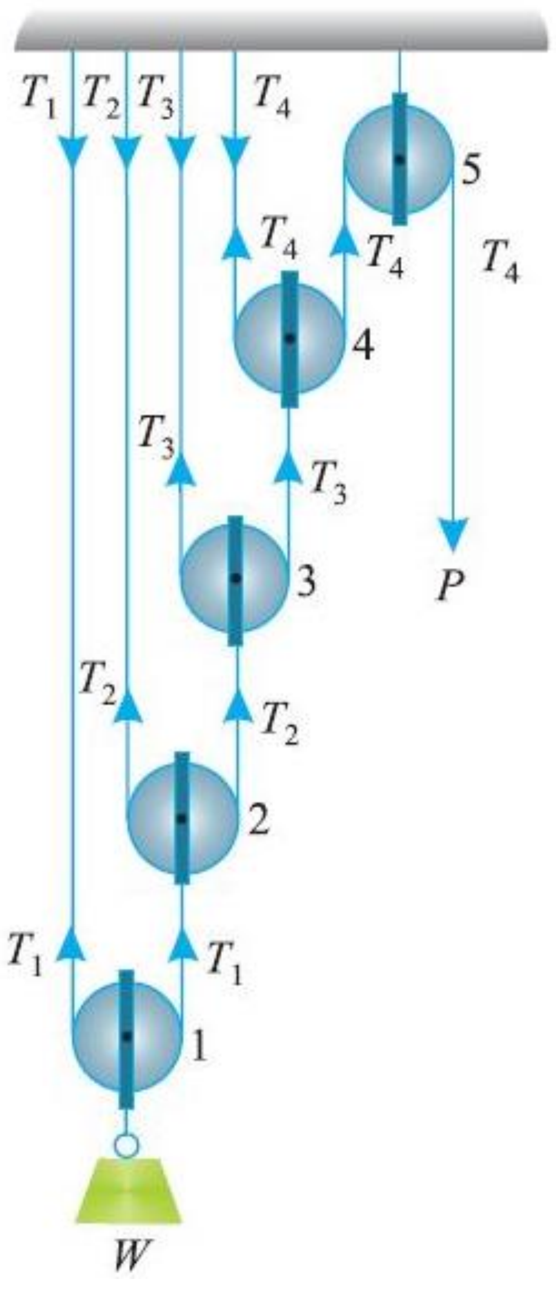
1. First system of pulleys.
2. Second system of pulleys.
3. Third system of pulleys.

FIRST SYSTEM OF PULLEYS

In Fig. 11.11 is shown the first system of pulleys. In this system, the pulleys are so arranged that there are as many strings as there are pulleys. The end of each string is fastened to a rigid ceiling; while the other end passing round the bottom periphery of the pulley, is fastened to the next higher pulley.

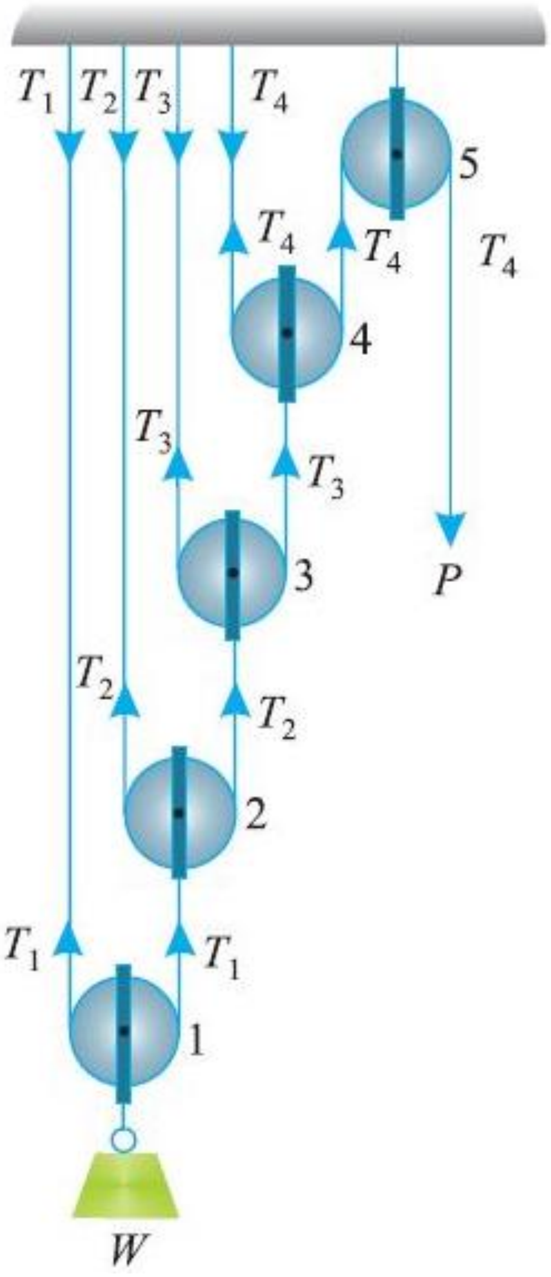
The load is attached to the bottom-most pulley ; whereas the effort is applied to the far end of the string passing round the last pulley. Another pulley (no. 5) is used just to change the direction of the effort.

The velocity ratio of the system may be obtained by considering a unit motion of the load. In this case, let the weight W be raised by x metres. Since the loads is supported on both sides of the string, thus this slackness of x metres will have to be taken up by the pulley 2. If the relative position of the pulley 2, with respect to the pulley 1, is to remain undisturbed, then the pulley 2 should move upwards through a distance of $2x$ metres.



Now this upward movement of pulley 2 through a distance of $2x$ metres will cause a total slackness of $2 \times (2x) = 2^2x$ metres in the string, which has to be taken up by the pulley 3. Thus the pulley 3 should move upwards through a distance of 2^2x metres, thus causing a slackness of $2 \times (2^2x) = 2^3x$ in the string passing round the pulley 4. Thus the pulley 4 should move upwards through a distance of 2^3x metres causing a slackness of $2 \times (2^3x) = 2^4x$ meters, which must be taken up by the free end of the string to which the effort is applied.

Fig.



$$\therefore \qquad \text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

Thus, in general, is there are n pulley in the system, then

$$\text{V.R.} = 2^n$$

Now
$$\text{M.A.} = \frac{W}{P}$$

and efficiency,
$$\eta = \frac{\text{M.A.}}{\text{V.R.}}$$

SECOND SYSTEM OF PULLEYS

In Fig. 11.12 (a) and (b) is shown second system of pulleys containing two blocks, one upper and the other lower, both carrying either equal number of pulleys or the upper block may have one pulley more than the lower one.

In both the cases, the upper block is fixed and the lower one is movable. There is obly one string, which passes round all the pulleys one end of which is fixed to the upper block (when both the blocks have the same no. of pulleys) or to the lower block (when the upper block has one pulley more than the lower one). The other end of the string is free and the effort is applied to this free end as shown in Fig. 11.12 (a) and (b). In both the cases the load is attached to lower block.

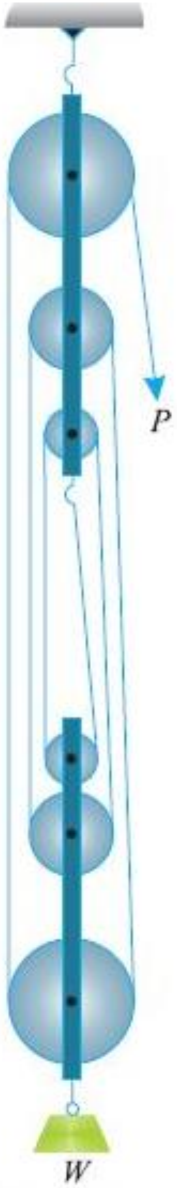
A little consideration will show, that for x displacement of the weight, the effort will move through a distance to nx, where n is the number of pulleys in both the blocks. Thus velocity ratio

$$\text{V.R.} = \frac{n\,x}{x} = n$$

Now $$\text{M.A.} = \frac{W}{P} \quad \text{...as usual}$$

and efficeincy, $$\eta = \frac{\text{M.A.}}{\text{V.R.}} \quad \text{...as usual}$$

(a) Different no. of pulleys



(b) Same no. of pulleys

THIRD SYSTEM OF PULLEYS

In Fig. 11.13, is shown a third system of pulleys. In this system, like the first system of pulleys, the pulleys are arranged in such a way that there are as many strings as there are pulleys. One end of each string is fixed to a block *B* –*B*, to which the load is attached. The other end of each string, passing round the upper periphery of the pulley, is fastened to the next lower pulley as shown in 11.13.

The velocity ratio, of the system, may be obtained by considering a unit motion of the load. In this case, let the weight *W* be raised by *x* metres. Since the llad is supported on all the strings, therefore all the strings will be slackened by *x* metres.

Now consider the pulley 1, which is fixed to the ceiling. The slackness of string s_1 equal to *x* metres will have to be taken up by the pulley 2, which should come down through a distance of $2x$ metres. But as the string s_2 also slacks by *x* metres, therefore the string s_1 will be pulled through a distance of $(2x - x) = x$ metre. Now consider the pulley 2. As the string s_1 has been pulled through a distance *x* metres, therefore the string s_2 will be pulled through a

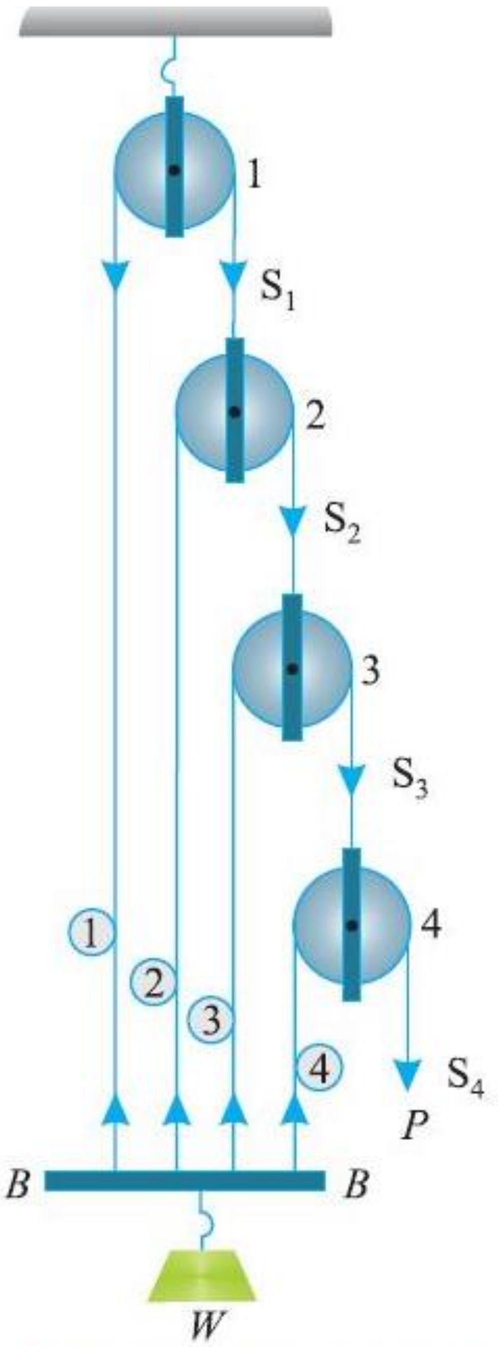


Fig. 11.13. Third system of pulleys.

distance of $2x + x = 3x = (2^2 - 1)x$. Similarly, in order to keep the relative position of the pulley 3 undisturbed, the string s_3 will be pulled through a distance of $(2 \times 3x + x) = 7x = (2^3 - 1)\,x$ and the string s_4 *i.e.*, effort will be pulled through a distance of $(2 \times 7x + x) = 15x = (2^4 - 1)\,x$.

$$\therefore \qquad \text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by load}}$$

$$= \frac{(2^4 - 1)\,x}{x} = (2^4 - 1)$$

Thus, in genera, if there are n pulleys in this system, then

$$\text{V.R.} = 2^n$$

Now $$\text{M.A.} = \frac{W}{P} \qquad \text{... as usual}$$

and efficiency, $$\eta = \frac{\text{M.A.}}{\text{M.R.}} \qquad \text{... as usual}$$

Example *Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 r.p.m. The coefficient of friction between the belt and pulley is 0·25, angle of lap 160° and maximum tension in the belt is 2·5 kN.*

Solution. Given: Diameter of pulley (d) = 600 mm = 0·6 m ; Speed of the pulley (N) = 200 r.p.m.: Coefficient of friction (μ) = 0·25: Angle of lap (θ) = 160° $= 160° \times \dfrac{\pi}{180°} = 2{\cdot}79$ rad and maximum tension (T_1) = 2·5 kN.

Let T_2 = Tension in the belt in slack side

We know that speed of the belt,

$$v = \frac{\pi d N}{60} = \frac{\pi \times 0{\cdot}6 \times 200}{60} = 2\pi \text{ rad/s}$$

and $$2{\cdot}3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta = 0{\cdot}25 \times 2{\cdot}79 = 0{\cdot}6975$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0{\cdot}6975}{2{\cdot}3} = 0{\cdot}3033$$

or $$\frac{2{\cdot}5}{T_2} = 2{\cdot}01 \quad \text{...(Taking antilog of 0·3033)}$$

$\therefore$ $$T_2 = \frac{2{\cdot}5}{2{\cdot}01} = 1{\cdot}24 \text{ kN}$$

We know that power transmitted by the belt,

$$P = (T_1 - T_2)\, v = (2{\cdot}5 - 1{\cdot}24)\, 2\pi = 7{\cdot}92 \text{ kW}$$ **Ans.**

Example. *In a system of pulleys of the first type, there are three pulleys, and a weight of 320 N can be lifted by an effort of 50 N.*

Find the efficiency of the machine and the amount of friction.

Solution. Given : No. of pulleys (n) = 3 ; Weight lifted (W) = 320 N and effort (P) = 50 N.

Efficeincy of the machine

We know that velocity ratio of first system of pulleys.

$$\text{V.R.} = 2^n = 2^3 = 8$$

and

$$\text{M.A.} = \frac{W}{P} = \frac{320}{50} = 6.4$$

$\therefore$ Efficiency, $$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{6.4}{8} = 0.80 = 80\% \quad \textbf{Ans.}$$

Amount of friction

We know that amount of friction in terms of load,

$$F_{(\text{load})} = (P \times \text{V.R.}) - W = (50 \times 8) - 320 = 80 \text{ N} \quad \textbf{Ans.}$$

and amount of friction in terms of effort,

$$F_{(\text{effort})} = P - \frac{W}{\text{V.R.}} = 50 - \frac{320}{8} = 10 \text{ N} \quad \textbf{Ans.}$$

Example *A weight of 1 kN is lifted by an effort of 125 N by second system of pulleys, having 5 pulleys in each block.*

Calculate the amount of effort wasted in friction and the frictional load.

Solution. Given: Weight lifted (W) = 1 kN = 1000 N ; Effort (P) = 125 N and no. of pulleys (n) = 2 × 5 = 10.

Amount of effort wasted in friction

We know that velocity ratio

$$\text{V.R.} = n = 10$$

and amount of effort wasted in friction,

$$F_{(\text{effort})} = P - \frac{W}{\text{V.R.}} = 125 - \frac{1000}{10} = 25 \text{ N} \qquad \textbf{Ans.}$$

Amount of frictional load

We also know that amount of frictional load,

$$F_{(\text{load})} = (P \times \text{V.R.}) - W = (125 \times 10) - 1000 = 240 \text{ N} \qquad \textbf{Ans.}$$

Example. *In a third system of pulleys, there are 4 pulleys. Find the effort required to lift a load of 1800 N, if efficiency of the machine is 75%.*

Calculate the amount of effort wasted in friction.

Solution. Given: No. of pulleys (n) = 4 ; Load lifted (W) = 1800 N and efficiency (η) = 75% = 0.75.

Effort required to lift the load

Let P = Effort required in newton to lift the load.

We know that velocity ratio of third system of pulleys.

$$\text{V.R.} = 2^n - 1 = 2^4 - 1 = 15$$

and $$\text{M.A.} = \frac{W}{P} = \frac{1800}{P}$$

We also know that efficiency

$$0.75 = \frac{\text{M.A.}}{\text{V.R.}} = \frac{\frac{1800}{P}}{15} = \frac{120}{P}$$

or $$P = \frac{120}{0.75} = 160 \text{ N} \qquad \textbf{Ans.}$$

Effort wasted in friction

We know that effort wasted in friction,

$$F_{(\text{effort})} = P - \frac{W}{\text{V.R.}} = 160 - \frac{1800}{15} = 40 \text{ N} \qquad \textbf{Ans.}$$