Special Theory of Relativity (PH101)
Course Instructors: Pankaj Mishra and Tapan Mishra
Tutorial-7
due on Wednesday, 23rd of October, 2019 (8:00Hrs IST)

1. Two rockets of rest length $L_0$ are approaching the earth from opposite directions at velocities $\pm c/2$. What will be size of each rocket measured from the other rocket frame of reference?

2. A body of rest mass $m_0$ moving at speed $u$ in positive x-direction approaches an identical body of same mass which is at rest. Find the speed $V$ of a frame which is also moving in positive x-direction from which the total momentum of the system would be zero. Further repeat the calculation for non-relativistic range, i.e., $u \ll c$ and see whether your relativistic result reduces to that value or not.

3. A spaceship moves away from Earth with speed $v$ and fires a shuttle craft in the forward direction at a speed $v$ relative to the spaceship. The pilot of the shuttle craft launches a probe in the forward direction at speed $v$ relative to the shuttle craft.

   (i)Determine the speed of the shuttle craft relative to the Earth.
   (ii) Determine the speed of the probe relative to the Earth.

   Analyze your results for the limiting cases $v \ll c$ and $v \to c$.

4. An electron $e^-$ ($m_0 = 0.511MeV/c^2$) with kinetic energy 1 $MeV$ undergoes a head-on collision with a positron $e^+$ at rest (A positron is an antimatter particle that has the same mass as the electron but opposite charge). In the process of collision the two particles annihilate resulting in creation of two photons of equal energy. Consider one photon makes an angle $\theta$ and another makes and $-\theta$ with the electrons direction of motion (A photon $\gamma$ is a massless particle of electromagnetic radiation having energy $E = pc$). The process is denoted as
   $e^- + e^+ \to 2\gamma$

   Determine the energy $E$, momentum $p$ and angle of emission $\theta$ of each photon.

5. A bullet of rest mass $m_0 = 100gm$ fired in the $x$-direction at a speed of $0.5c$ hits and get stuck on to a ball of rest mass $M_0 = 200gm$ sitting at rest with respect to the earth. What is the speed of the combined system after the collision as per special theory of relativity (momentum defined as $\vec{p} = \Gamma_u m_0 \vec{u}$ is conserved)? (Neglect all other effects including that of air drag and gravity.)

6. Consider a shell of total mass $M_0 = 1$ kg at rest with respect to an observer on earth. At time $t = 0$ it explodes into three equal parts, each of which flies away with the same speed $u = 0.2c$, making an angle $120^o$ between each other. The velocities may be considered as $\vec{u}_1 = 0.2c\hat{x}$, $\vec{u}_2 = 0.2c\left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)$ , $\vec{u}_3 = 0.2c\left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$ . For the observer on the earth, what is

the energy and what is momentum of each of the pieces, (let us denote these by $E_1,\ E_2,\ E_3$ and $\vec{p}_1,\ \vec{p}_2,\ \vec{p}_3$ respectively)? Denoting the energy-momentum four-vectors as $p_1,\ p_2,\ p_3$ with $p_1 \equiv \left(\frac{E_1}{c},\ \vec{p}_1\right)$ etc., find $(p_1+p_2+p_3)\cdot(p_1+p_2+p_3)$. (In general, $p \cdot p = \frac{E^2}{c^2} - \vec{p}\cdot\vec{p} = m_0^2 c^2$ . ) Consider another observer moving with velocity $\vec{v} = 0.5c\hat{x}$ with respect to the observer on the Earth. Find $E_1',\ E_2',\ E_3'$ and $\vec{p}\,'_1,\ \vec{p}\,'_2, \vec{p}\,'_3$ . What are the masses of each of these parts as seen from the moving frame? Find the magnitude of the sum of the energy-momentum four-vector in this frame $((p_1' + p_2' + p_3') \cdot (p_1' + p_2' + p_3'))$.