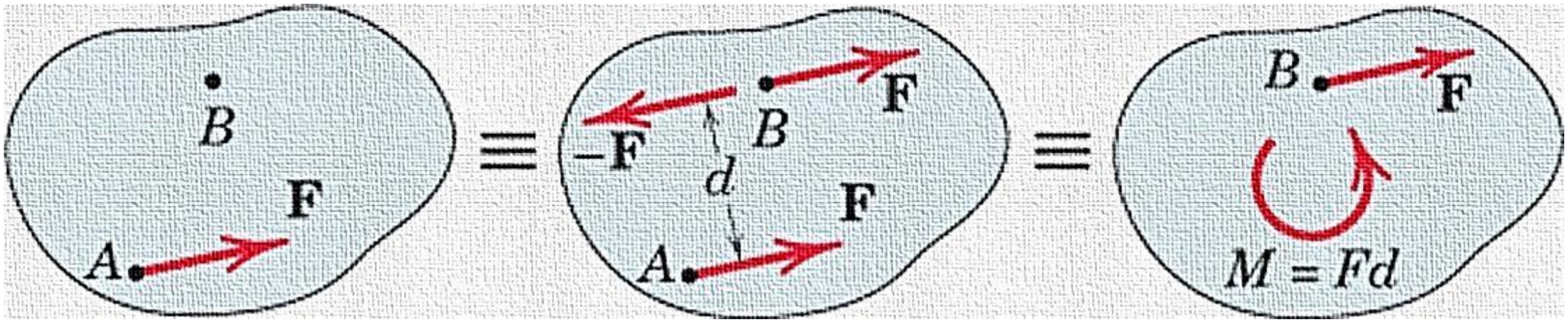
ME101: Engineering Mechanics

2019-20 (II Semester)

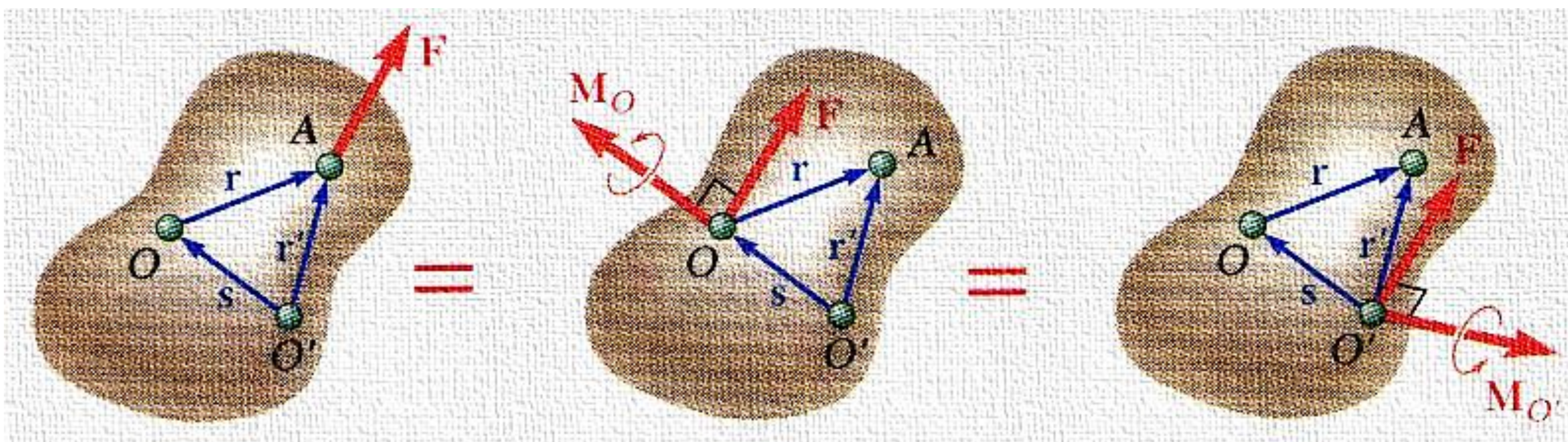
LECTURE: 3

Equivalent Force and Couple

- Two dimensional plane

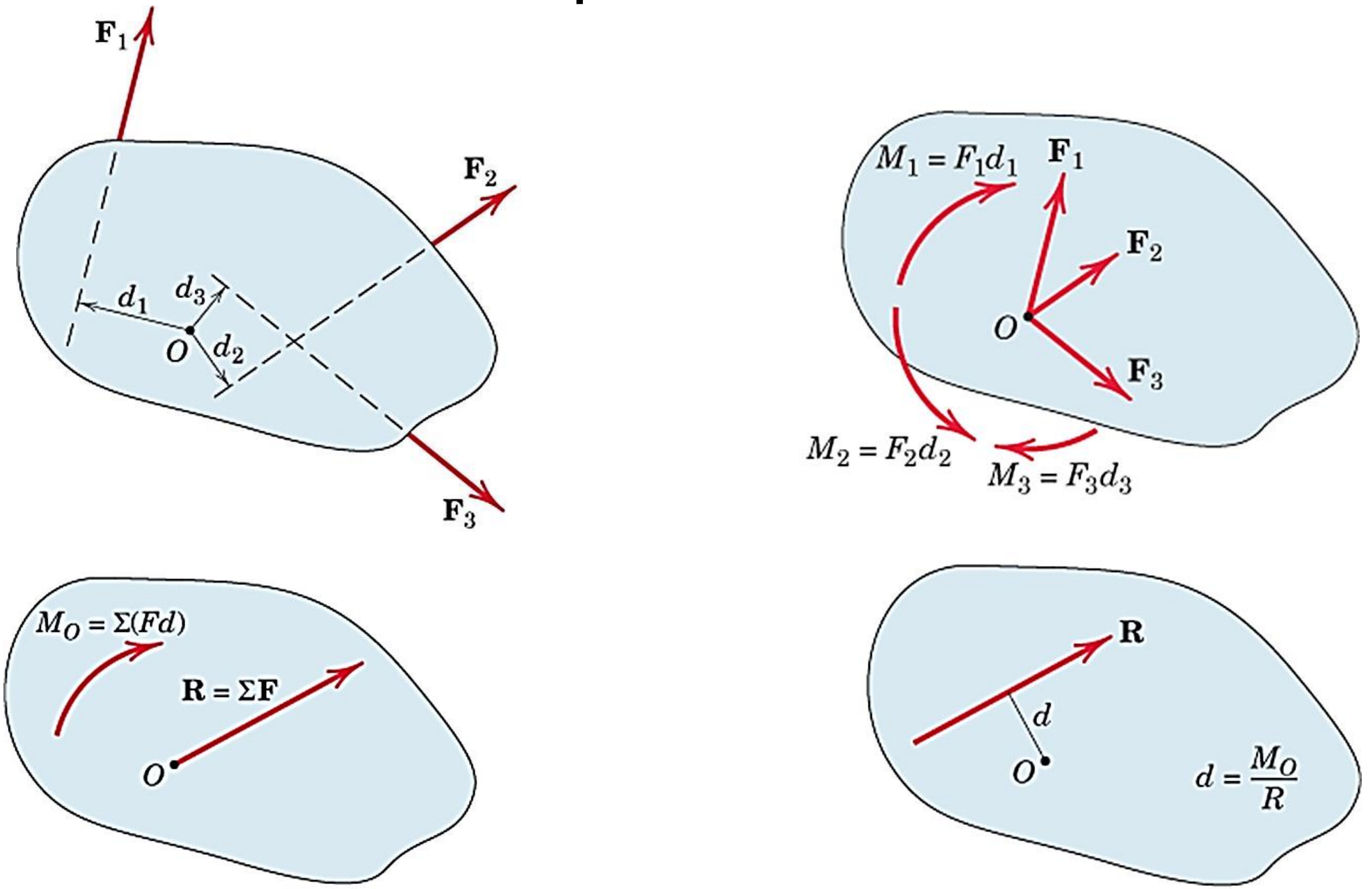


- Three dimensional space



Resultant of Concurrent Forces

- Two dimensional plane



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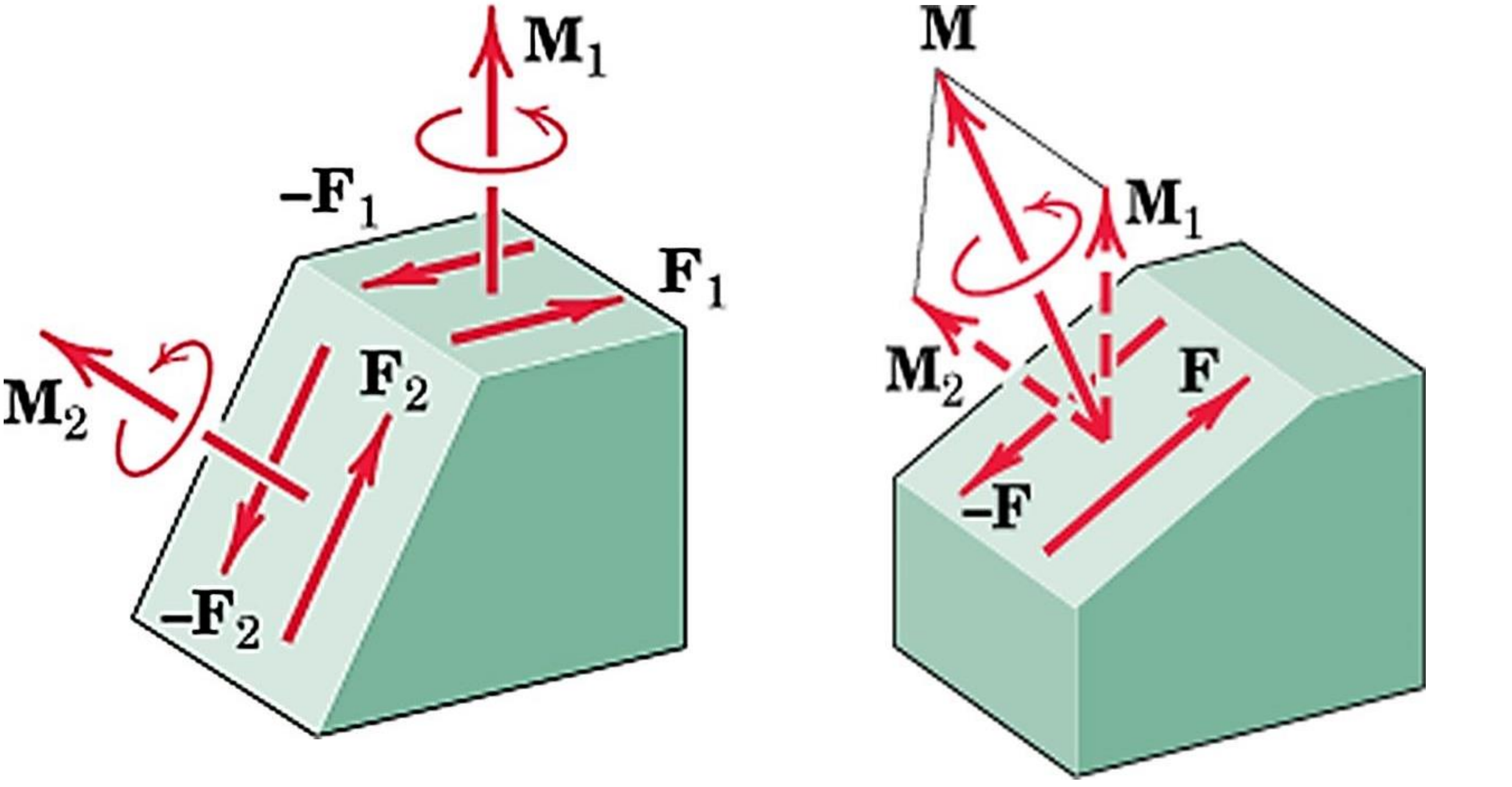
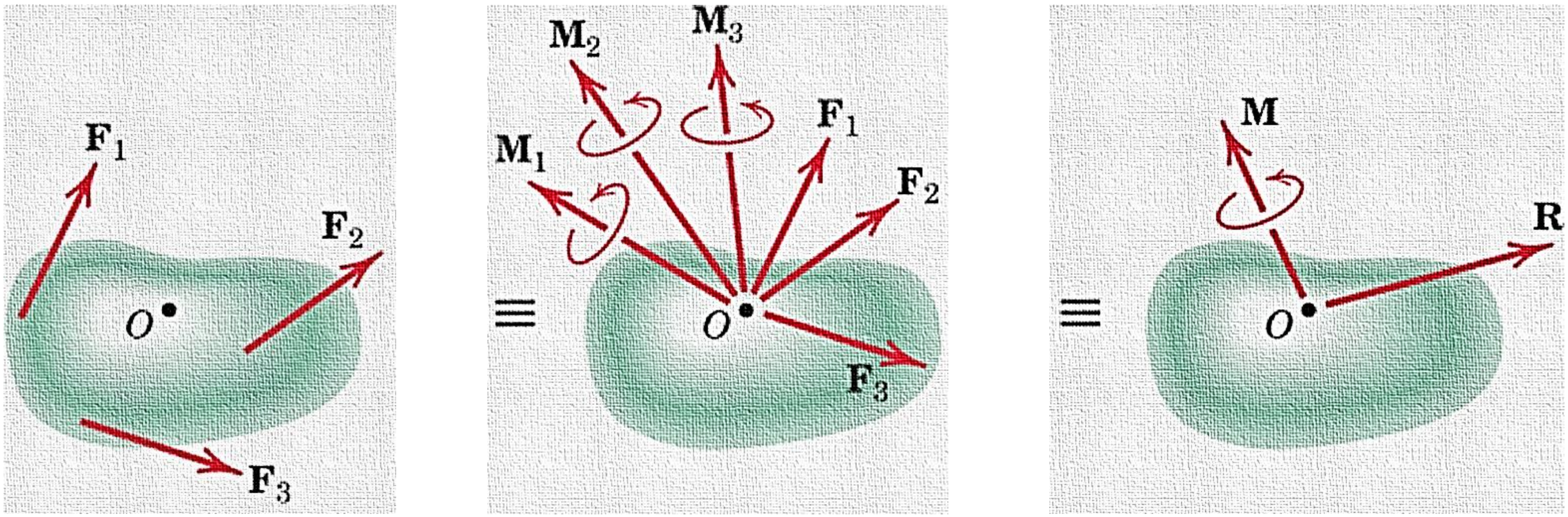
Resultant of Two Couples



$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$

Resultant of Force System:: 3D

- Three dimensional space

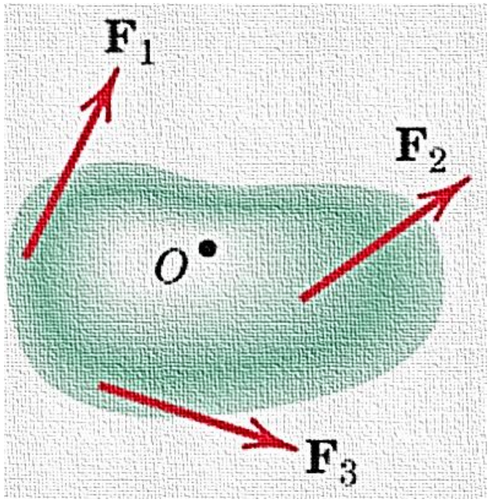


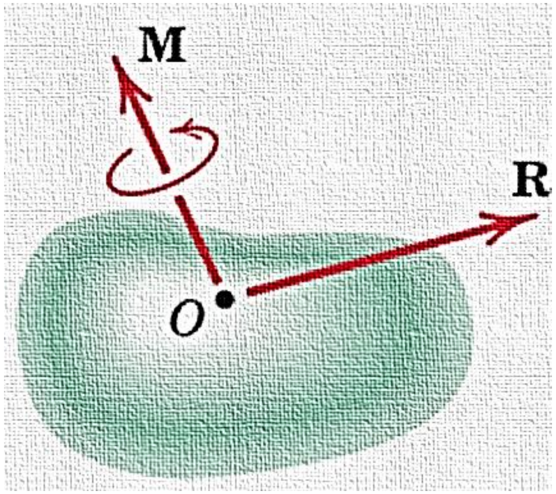
Equivalent force-couple systems for each force

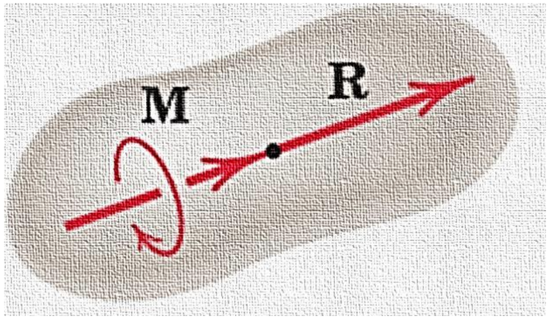
Resultant equivalent force-couple system

Wrench Action

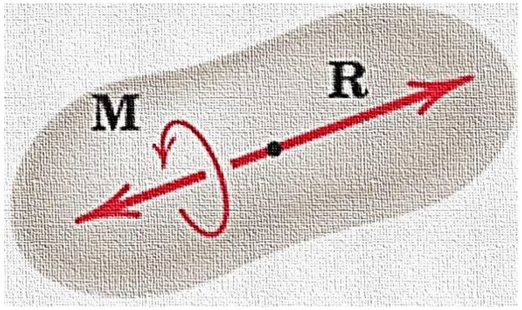
- Coincidence of resultant **R** and **M** axes







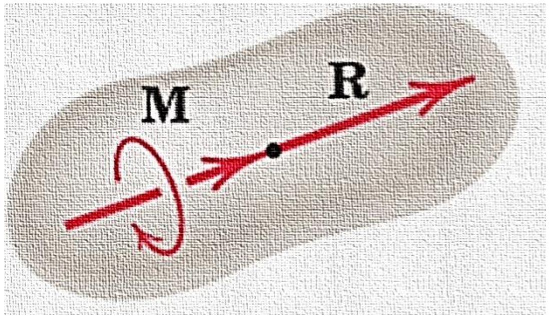
Positive Wrench



Negative Wrench

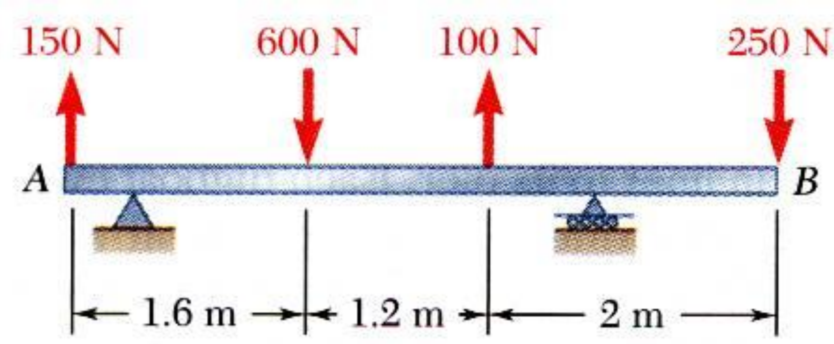
Wrench Action

- Wrench action or *Screw Driver* action



Positive Wrench

Example on Equivalent System



For the beam, reduce the system of forces shown to

(a) an **equivalent force-couple** system at ***A***,

(b) an **equivalent force couple** system at ***B***, and

(c) a **single force** or **resultant**

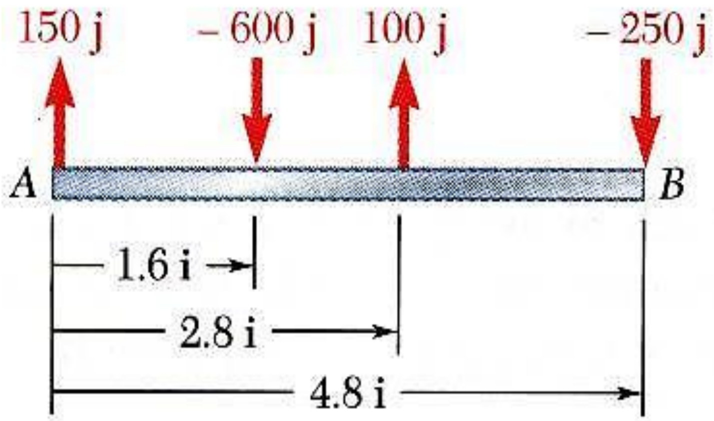
Solution:

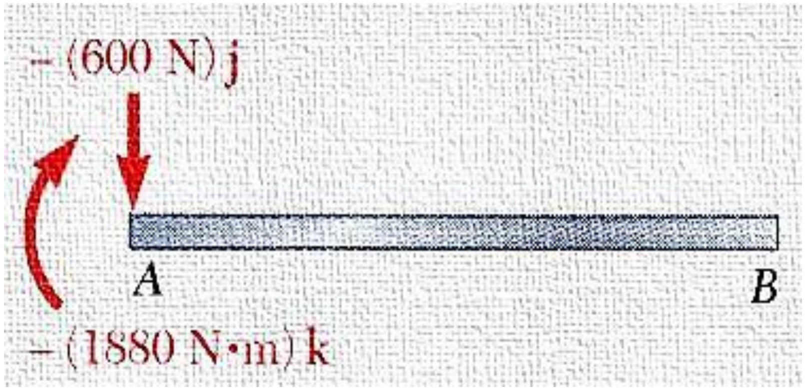
a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about *A*.

b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

c) Determine the point of application for the resultant force such that its moment about *A* is equal to the resultant couple at *A*.

Example on Equivalent System





SOLUTION:

a) Compute the resultant force and the resultant couple at *A*.

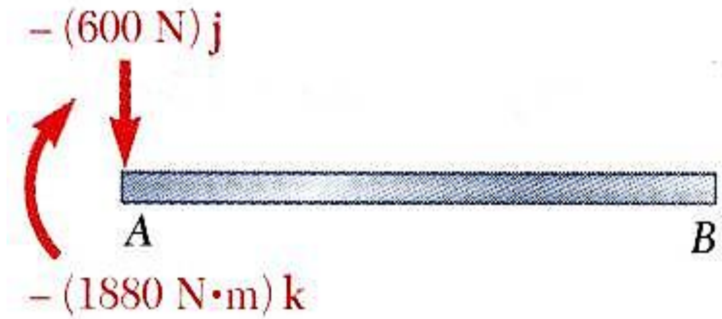
$$\vec{R} = \sum \vec{F}$$
$$= (150\ \mathrm{N})\vec{j} - (600\ \mathrm{N})\vec{j} + (100\ \mathrm{N})\vec{j} - (250\ \mathrm{N})\vec{j}$$

$$\boxed{\vec{R} = -(600\ \mathrm{N})\vec{j}}$$

$$\vec{M}_A^R = \sum (\vec{r} \times \vec{F})$$
$$= (1.6\vec{i}) \times (-600\vec{j}) + (2.8\vec{i}) \times (100\vec{j})$$
$$+ (4.8\vec{i}) \times (-250\vec{j})$$

$$\boxed{\vec{M}_A^R = -(1880\ \mathrm{N \cdot m})\vec{k}}$$

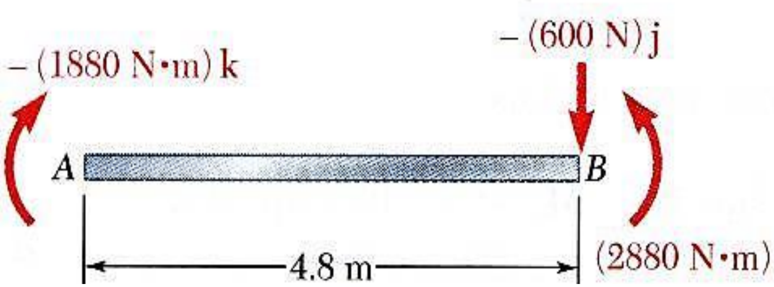
Example on Equivalent System



b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

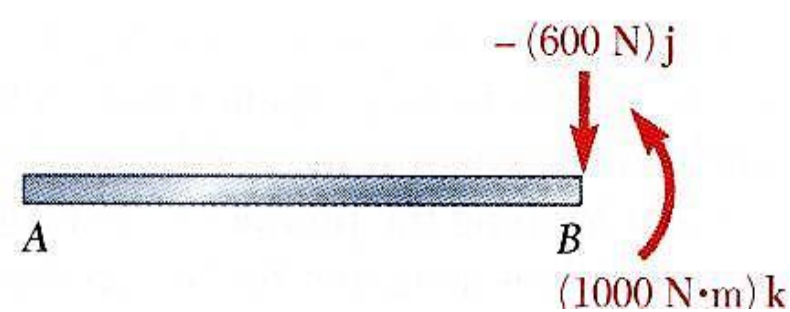
The force is unchanged by the movement of the force-couple system from *A* to *B*.

$$\boxed{\vec{R} = -(600\ \mathrm{N})\vec{j}}$$



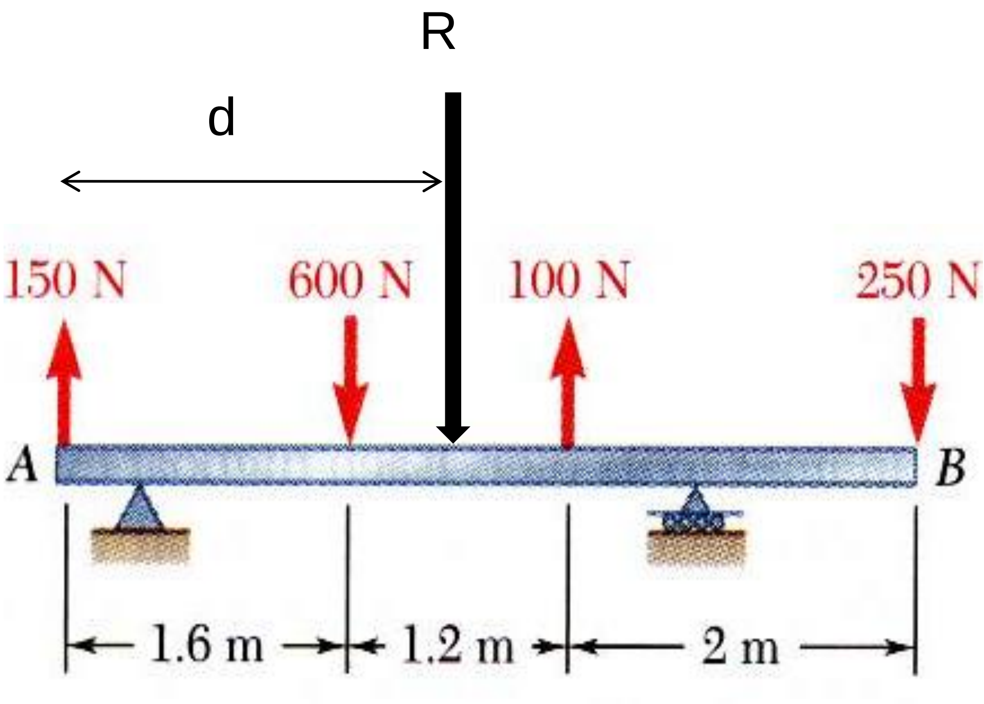
The couple at *B* is equal to the moment about *B* of the force-couple system found at *A*.

$$\begin{aligned}\vec{M}_B^R &= \vec{M}_A^R + \vec{r}_{B/A} \times \vec{R} \\ &= -(1880\ \mathrm{N\cdot m})\vec{k} + (-4.8\ \mathrm{m})\vec{i} \times (-600\ \mathrm{N})\vec{j} \\ &= -(1880\ \mathrm{N\cdot m})\vec{k} + (2880\ \mathrm{N\cdot m})\vec{k}\end{aligned}$$



$$\boxed{\vec{M}_B^R = +(1000\ \mathrm{N\cdot m})\vec{k}}$$

Example on Equivalent System



c)

$F_R = F_1 + F_2 + F_3 + F_4$

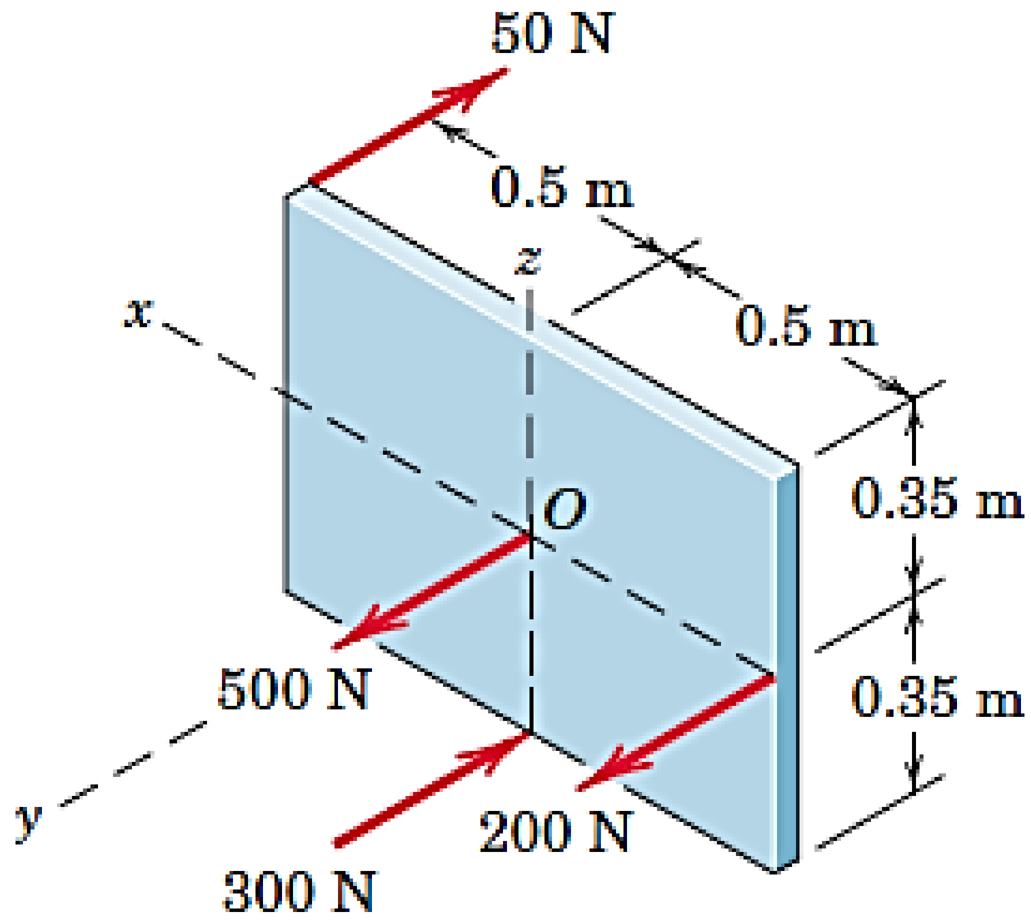
R = 150-600+100-250 = -600 N

$F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4$

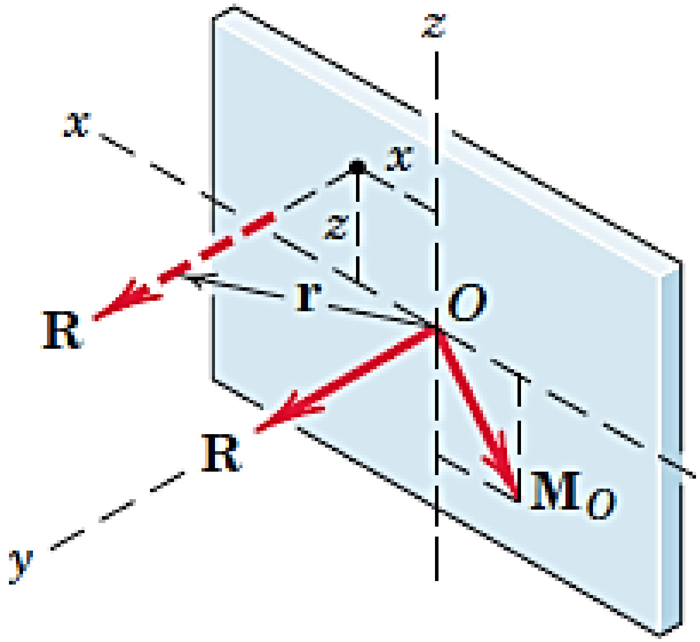
d = 3.13 m

Example on resultant of a force system

- Determine the resultant of the system of parallel forces acting on the plate. Solve with a vector approach.



Example on resultant of a force system



Solution

Transfer of all forces to point O results in the force–couple system

$$\mathbf{R} = \Sigma\mathbf{F} = (200 + 500 - 300 - 50)\mathbf{j} = 350\mathbf{j}\ \text{N}$$

$$\begin{aligned}\mathbf{M}_O &= [50(0.35) - 300(0.35)]\mathbf{i} + [-50(0.50) - 200(0.50)]\mathbf{k} \\ &= -87.5\mathbf{i} - 125\mathbf{k}\ \text{N}\cdot\text{m}\end{aligned}$$

Example on resultant of a force system

Solution

The placement of $\mathbf{R}$ so that it alone represents the above force–couple system is determined by the principle of moments in vector form

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$

$$(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times 350\mathbf{j} = -87.5\mathbf{i} - 125\mathbf{k}$$

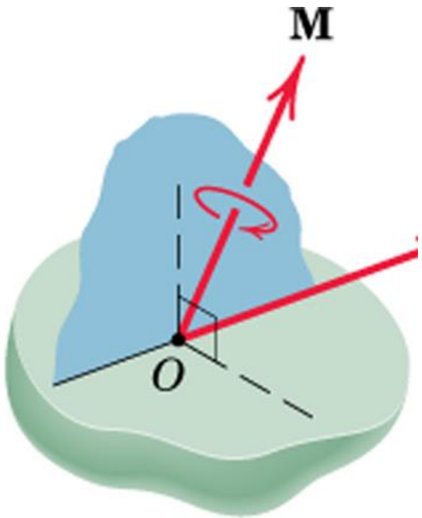
$$350x\mathbf{k} - 350z\mathbf{i} = -87.5\mathbf{i} - 125\mathbf{k}$$

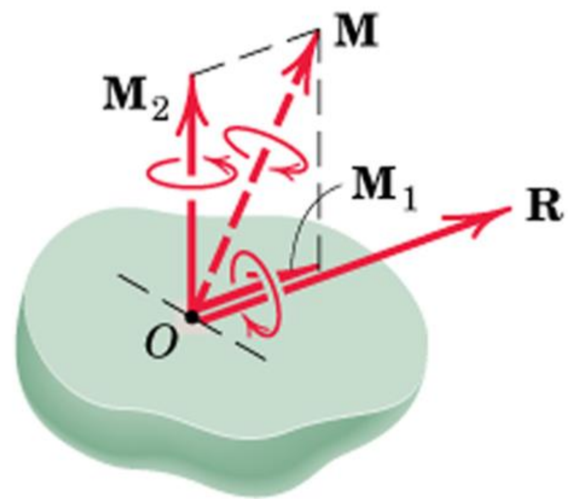
From the one vector equation we may obtain the two scalar equations

$$350x = -125 \qquad \text{and} \qquad -350z = -87.5$$

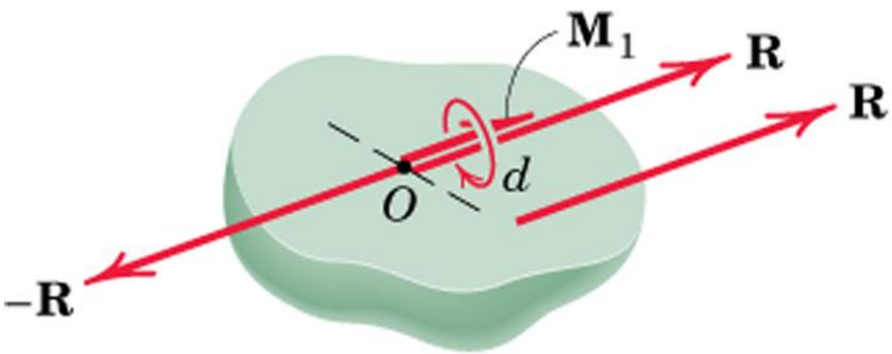
Hence, $x = -0.357$ m and $z = 0.250$ m are the coordinates through which the line of action of $\mathbf{R}$ must pass. The value of y may, of course, be any value, as permitted by the principle of transmissibility. Thus, as expected, the variable y drops out of the above vector analysis.

Replacement of force system by wrench

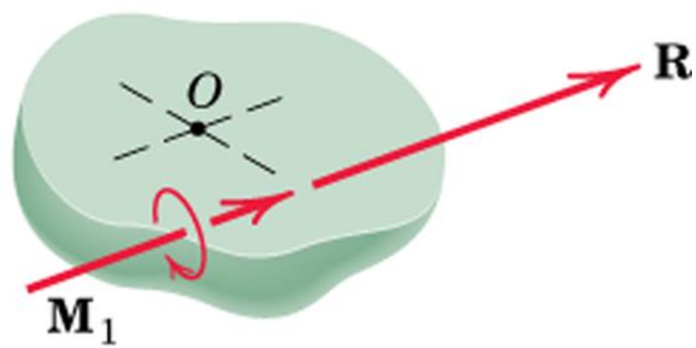




Resolve $\mathbf{M}$ into components



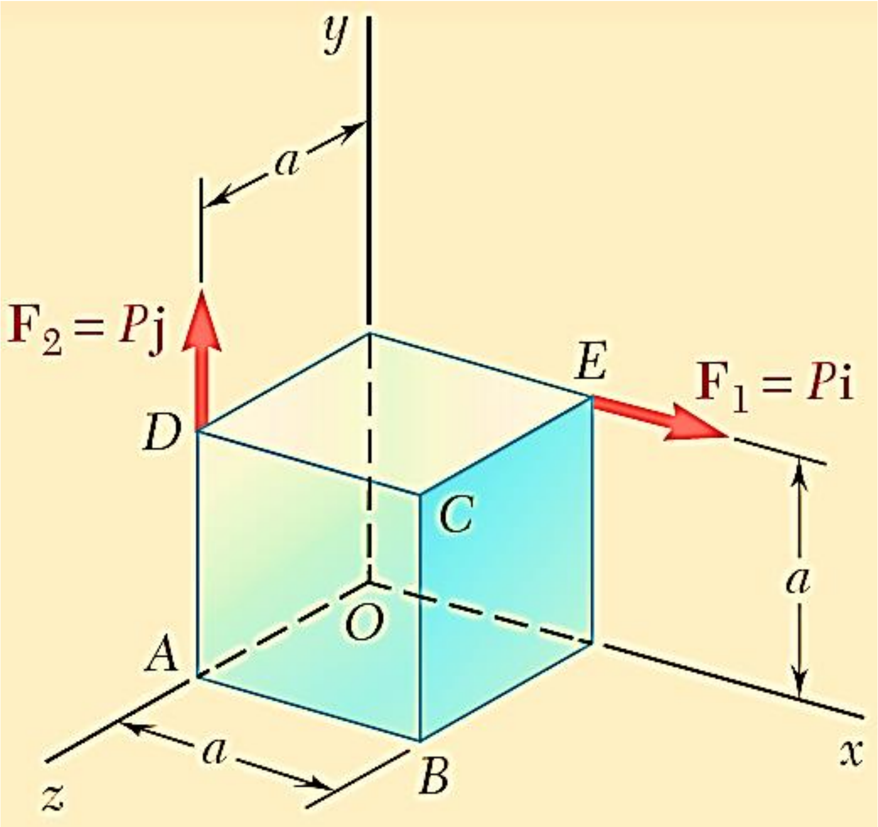
Resolve $\mathbf{M}_2$ into a couple



Resultant Wrench

Example on Wrench

Replace the two forces by an equivalent wrench, and determine (*a*) the magnitude and direction of the resultant force **R** , (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *yz* plane.



Example on Wrench

Step-1) Equivalent Force-Couple System at *O*

Determine the equivalent force-couple system at *O* .

The position vectors of the points of application of the two given forces are $\mathbf{r}_E = a\mathbf{i} + a\mathbf{j}$ and $\mathbf{r}_D = a\mathbf{j} + a\mathbf{k}$

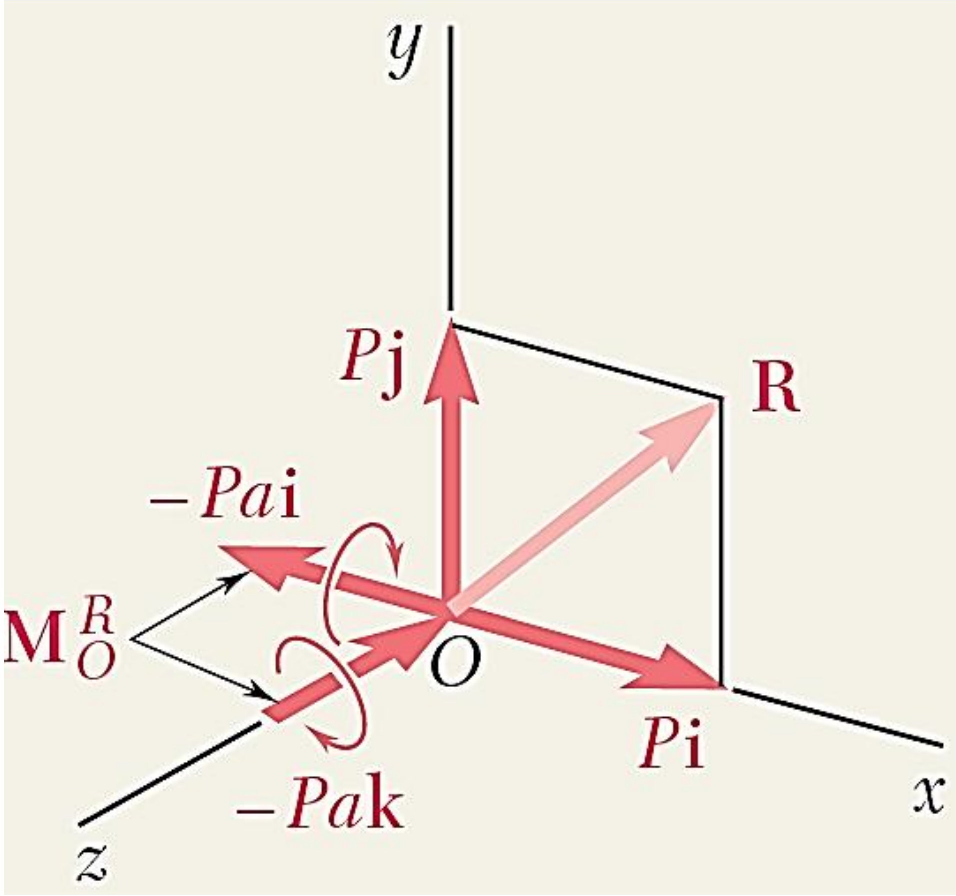
$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = P\mathbf{i} + P\mathbf{j} = P(\mathbf{i} + \mathbf{j})$$

$$\begin{aligned}\mathbf{M}_O^R &= \mathbf{r}_E \times \mathbf{F}_1 + \mathbf{r}_D \times \mathbf{F}_2 \\ &= (a\mathbf{i} + a\mathbf{j}) \times P\mathbf{i} + (a\mathbf{j} + a\mathbf{k}) \times P\mathbf{j} \\ &= -Pa\mathbf{k} - Pa\mathbf{i} = -Pa(\mathbf{i} + \mathbf{k})\end{aligned}$$

Example on Wrench

Resultant force

R has the magnitude $R = P\sqrt{2}$, lies in the xy plane and forms angles of 45° with the x and y axes. Thus



Example on Wrench

Resultant force

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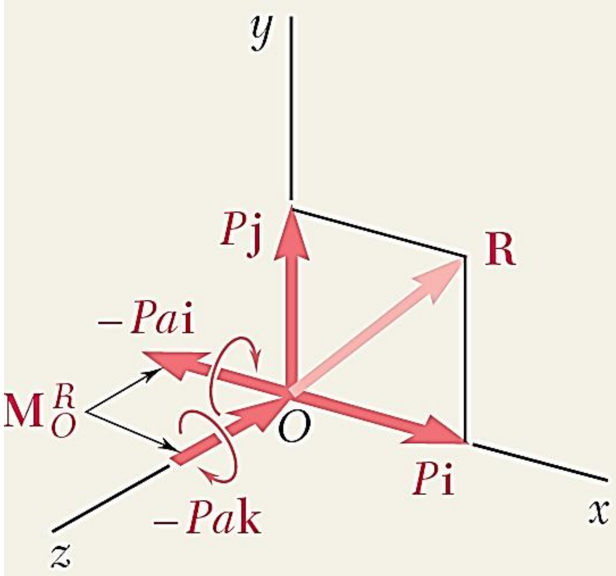


$$R = P\sqrt{2} \qquad \theta_x = \theta_y = 45° \qquad \theta_z = 90°$$

Pitch of wrench

$$p = \frac{\mathbf{R}\cdot\mathbf{M}_O^R}{R^2} = \frac{P(\mathbf{i}+\mathbf{j})\cdot(-Pa)(\mathbf{i}+\mathbf{k})}{(P\sqrt{2})^2}$$

$$= \frac{-P^2a(1+0+0)}{2P^2} \qquad p = -\frac{a}{2}$$

Example on Wrench

Axis of wrench

$$\mathbf{M}_1 + \mathbf{r} \times \mathbf{R} = \mathbf{M}_O^R$$

or, noting that $\mathbf{r} = y\mathbf{j} + z\mathbf{k}$ and substituting for $\mathbf{R}$, $\mathbf{M}_O^R$, and $\mathbf{M}_1$ from Eqs. (1), (2),

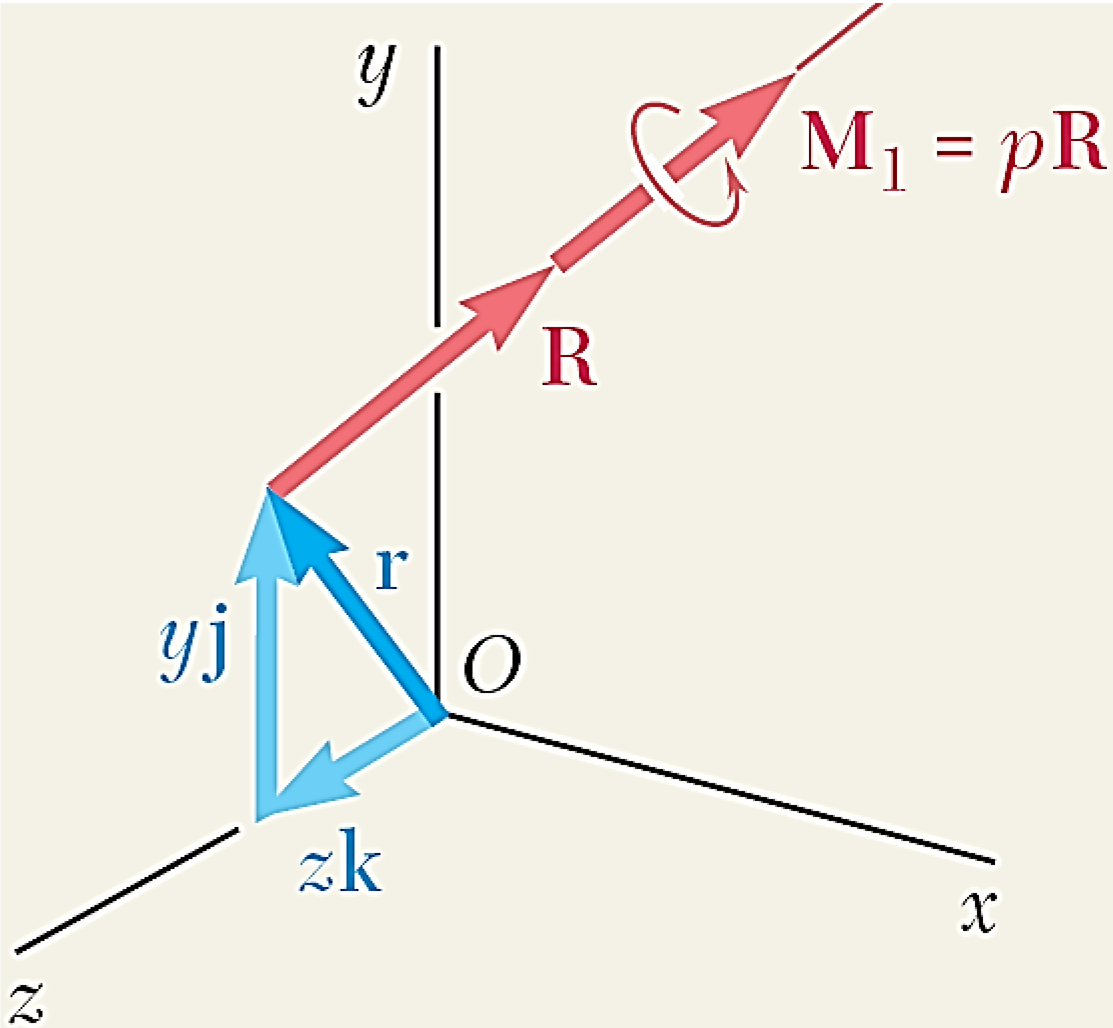
$$-\frac{Pa}{2}(\mathbf{i} + \mathbf{j}) + (y\mathbf{j} + z\mathbf{k}) \times P(\mathbf{i} + \mathbf{j}) = -Pa(\mathbf{i} + \mathbf{k})$$

$$-\frac{Pa}{2}\mathbf{i} - \frac{Pa}{2}\mathbf{j} - Py\mathbf{k} + Pz\mathbf{j} - Pz\mathbf{i} = -Pa\mathbf{i} - Pa\mathbf{k}$$

Equating the coefficients of $\mathbf{k}$, and then the coefficients of $\mathbf{j}$, we find

$$y = a \qquad z = a/2$$

Example on Wrench



$$y = a \qquad z = a/2$$