PH 102, Electromagnetism,

Post Mid Semester

Lecture 5.

Electrodynamics:

Ohm’s law, motional emf
and
Electromotive force

D. J. Griffiths: 7.1

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Ohm's Law

Current flow !! Charges pushed??

What is the this force pushing charges??

They move fast or slow depending on the material!!

Current density $$\mathbf{J} = \sigma \mathbf{f}.$$

f : force per unit charge

σ: What is this proportionality constant?

Ohm's Law

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They move fast or slow depending on the material!!

Current density $$\mathbf{J} = \sigma \mathbf{f}.$$

f : force per unit charge

$\boldsymbol{\sigma}$**:** conductivity

Conductivity ($\boldsymbol{\sigma}$): Empirical constant and depends on material

Resistivity (ρ) = $1/\boldsymbol{\sigma}$**,** is the more familiar description.

Remember in the present context, ρ and $\boldsymbol{\sigma}$ are not the surface or volume charge density!

TABLE 7.1 Resistivities, in ohm-meters (all values are for 1 atm, 20° C). *Data from Handbook of Chemistry and Physics*, 91st ed. (Boca Raton, Fla.: CRC Press, 2010) and other references.

Material	Resistivity	Material	Resistivity
Conductors:		*Semiconductors:*	
Silver	1.59×10^{-8}	Sea water	0.2
Copper	1.68×10^{-8}	Germanium	0.46
Gold	2.21×10^{-8}	Diamond	2.7
Aluminum	2.65×10^{-8}	Silicon	2500
Iron	9.61×10^{-8}	*Insulators:*	
Mercury	9.61×10^{-7}	Water (pure)	8.3×10^{3}
Nichrome	1.08×10^{-6}	Glass	$10^{9} - 10^{14}$
Manganese	1.44×10^{-6}	Rubber	$10^{13} - 10^{15}$
Graphite	1.6×10^{-5}	Teflon	$10^{22} - 10^{24}$

No perfect conductor ($\rho = 0$) or insulator ($\rho = \infty$):

Metals are perfect conductors for practical purposes!

What is this force driving charges!!

In principle, the force can be anything!

In our context, it is an Electromagnetic Force

Hence, force per unit charge is, $(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.

$$\mathbf{J} = \sigma \mathbf{f}. \quad \Longrightarrow \quad \mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

However, the velocity of the charges is sufficiently small that we can ignore the 2nd term, i.e.

$$\boxed{\mathbf{J} = \sigma \mathbf{E}.}$$

Ohm's Law

Note:

$\mathbf{E} = 0$, inside a conductor! How is current density (J) not zero?

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Ohm's Law

Note:

$\mathbf{E} = 0$, inside a conductor! How is current density (J) not zero?

- E=0 for stationary charges here current is flowing!
- For perfect conductors $\mathbf{E} = \mathbf{J}/\boldsymbol{\sigma} \approx 0$, (Metal circuit wires: equipotential)
- Metals: current driven by negligible electric field!

Example 7.1. A cylindrical resistor of cross-sectional area A and length L is made from material with conductivity σ. (See Fig. 7.1; as indicated, the cross section need not be circular, but I *do* assume it is the same all the way down.) If we stipulate that the potential is constant over each end, and the potential difference between the ends is V, what current flows?

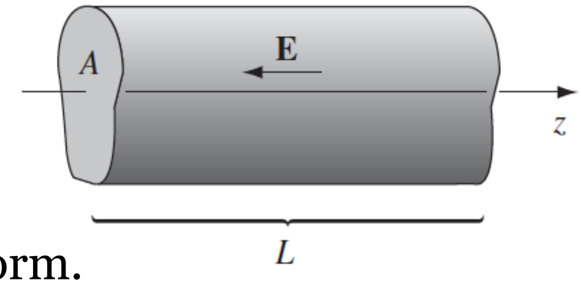

FIGURE 7.1

Solution: The electric field is uniform within the wire,

then the current density $I = JA = \sigma EA = \frac{\sigma A}{L} V$, is also uniform.

Why uniform **E** ?

For steady currents and uniform conductivity, $\nabla \cdot \mathbf{E} = \frac{1}{\sigma} \nabla \cdot \mathbf{J} = 0$

Therefore, the charge density is zero and any unbalanced charge resides on the surface!

Thus the Laplace's eq holds within a homogeneous ohmic material with steady current.

Back to D. J.G 3rd chapter on potentials....

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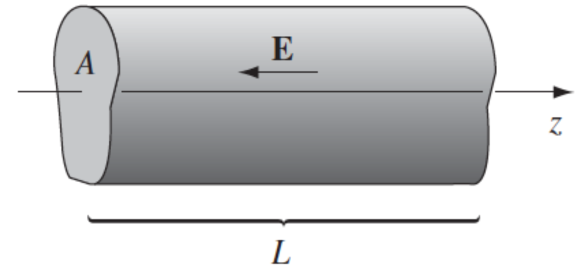


FIGURE 7.1

Solution:

For steady currents and uniform conductivity,

$$\nabla \cdot \mathbf{E} = \frac{1}{\sigma} \nabla \cdot \mathbf{J} = 0$$

$\nabla^2 V = \nabla^2_z V = 0$, with boundary conditions

$$V(z) = \begin{cases} +V_0 & at\ z=0 \\ 0 & at\ z=L \end{cases}$$

and

On the cylindrical surfaces, $\mathbf{J} \cdot \hat{\mathbf{n}} = \mathbf{E} \cdot \hat{\mathbf{n}} = 0$ [No charge leaking out to the nonconducting surrounding]

Hence, $\partial V / \partial n = 0$. Thus V or its normal derivative is specified on all surfaces,

V is unique!

Simple to guess the potential, $V(z) = V_o(1 - z/L)$

Uniqueness theorem!

corresponding, $\mathbf{E} = -\nabla V = \dfrac{V_0}{L}\hat{\mathbf{z}}$, indeed uniform!

Example 7.1. A cylindrical resistor of cross-sectional area A and length L is made from material with conductivity σ. (See Fig. 7.1; as indicated, the cross section need not be circular, but I *do* assume it is the same all the way down.) If we stipulate that the potential is constant over each end, and the potential difference between the ends is V, what current flows?

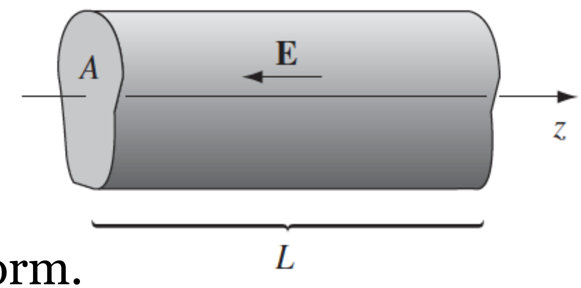


FIGURE 7.1

Solution: The electric field is uniform within the wire,

then the current density $I = JA = \sigma EA = \frac{\sigma A}{L} V$, is also uniform.

Total current flowing is proportional to the potential difference

$$V = IR.$$ Ohm's Law

The constant of proportionality is R *(ohms, Ω)*, is the resistance. R depends on geometry of the arrangement and the conductivity of the medium between the electrodes.

Here, $R = (L/\sigma A)$

Example 7.2. Two long coaxial metal cylinders (radii a and b) are separated by material of conductivity σ (Fig. 7.2). If they are maintained at a potential difference V, what current flows from one to the other, in a length L?

The field between the cylinders, $\mathbf{E} = \dfrac{\lambda}{2\pi\epsilon_0 s}\hat{\mathbf{s}}$,

Here, λ is the charge per unit length on the inner cylinder.

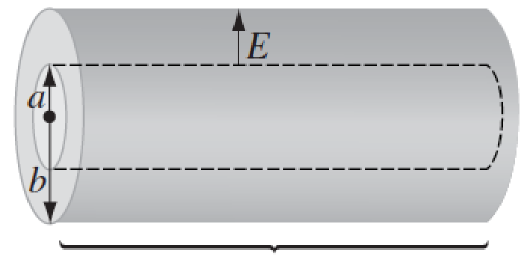


FIGURE 7.2

The current, $I = \int \mathbf{J}\cdot d\mathbf{a} = \sigma \int \mathbf{E}\cdot d\mathbf{a} = \dfrac{\sigma}{\epsilon_0}\lambda L.$

The surface integral is over the surface enclosing the inner cylinder.

The potential difference between the cylinders, $V = -\displaystyle\int_b^a \mathbf{E}\cdot d\mathbf{l} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$,

Substituting λ from the current in terms of V, $I = \dfrac{2\pi\sigma L}{\ln(b/a)}V.$

Therefore, here the resistance is, $R = \ln(b/a)/2\pi\sigma L.$

Microscopic aspects of Electrical conduction

We focus on Metals i.e. good conductors!

- In any metal the *free* electrons exist as a *gas* of charge particle inside the conductor.
- If the metal is in thermal equilibrium with its surrounding local environment then the electron gas is also in thermal equilibrium.
- In Thermal equilibrium each degrees of freedom is associated with $k_B T/2$ of thermal energy, with Boltzmann's constant $K_B = 1.381\times10^{-23}$ Joules/Kelvin degree.

Free electron in metal has 3 d.f, hence $K.E. = 3\, k_B T/2$,

Average thermal speed of the electron gas, $v = \sqrt{\dfrac{3k_B T}{m}}$

$$v(\text{ room temp}) = \sqrt{\frac{3 \times 1.381 \times 10^{-23} \times 300}{9.1 \times 10^{-31}}}$$

$T = 300K$ and $m = 9.1\times10^{-31}$ kg.

$$= 1.2 \times 10^5\ meters/sec$$

Off course, this is not the speed of electrons conducting electrical power.

The free conduction electrons scatter of metal atoms and of each other!

Microscopic aspects of Electrical conduction

Acceleration of free electrons in a wire:

Consider the electric field to be longitudinal, $\overrightarrow{a_e} = -\frac{e\vec{E}}{m}$

However, non zero acceleration would mean current increasing with time!!

Ohm's law suggest constant velocity as a constant field produces constant current!!

The frequent collisions of the electrons mean constant average speed.

These collisions would mean loosing kinetic energy to the atoms resulting in heat.

The electrons drift through conductor with an avg terminal velocity, $\langle \vec{v}_d \rangle = \frac{1}{2} a \langle \tau \rangle$

Here, $\langle \tau \rangle = mean/average\ time\ between\ succesive\ collisions = \frac{\boldsymbol{\lambda}_{mfp}}{\langle v_{thermal} \rangle}$

$\boldsymbol{\lambda}_{mfp} = mean\ free\ path$

$\langle v_{thermal} \rangle = average\ thermal\ speed\ of\ electrons$

Thus, $\langle \vec{v}_d \rangle = \frac{1}{2} a \langle \tau \rangle = \frac{1}{2} a \frac{\boldsymbol{\lambda}_{mfp}}{\langle v_{thermal} \rangle}$

Microscopic aspects of Electrical conduction

If there are 'n' molecules/volume and 'f' free electrons per molecule with charge q and mass m, then the current density is,

$$J_{free} = (n_q)q\langle\overrightarrow{v_d}\rangle \qquad [n_q = nf]$$

$$= (n_q)q\frac{1}{2}a\frac{\boldsymbol{\lambda}_{mfp}}{\langle v_{thermal}\rangle}$$

$$= (n_q)q\frac{1}{2}\frac{q\vec{E}}{m}\frac{\boldsymbol{\lambda}_{mfp}}{\langle v_{thermal}\rangle} = \frac{1}{2}\frac{n_q q^2\boldsymbol{\lambda}_{mfp}}{\langle v_{thermal}\rangle}\frac{\vec{E}}{m}$$

Increases with n_q,
Decreases with T

Example: Cu, electron valency one.

No of free electrons in per gm of Cu $= \frac{N_A}{A_{Cu}} = \frac{6.022\times 10^{23}}{63.54} = 9.477\times 10^{21}$

Also, $\rho_{Cu} = 8.95\ grams/cm^3$

Thus number density of free electrons in Cu,

$$n_e^{Cu} = 9.477\times 10^{21}(electrons/g)\times 8.95\ (g/cm^3)$$
$$= 8.482\times 10^{28}(electrons/m^3)$$

Microscopic aspects of Electrical conduction

Suppose we have a copper wire carrying a steady free current of I_{free} = 1 Ampere, the copper wire e.g. has a cross-sectional area of $A_{\perp}$ = 1 mm^2 = 10^{-6} m^2.

$$\langle \overrightarrow{v_d} \rangle = \frac{\overleftarrow{J_{free}}}{n_e q}$$

$$\langle \overrightarrow{v_d} \rangle = \frac{\overleftarrow{I_{free}}/A_{\perp}}{n_e q}$$

$$\langle \overrightarrow{v_d} \rangle = \frac{\frac{1.0}{10^6}}{8.482 \times 10^{28} \times 1.602 \times 10^{-19}} = 73.6 \frac{\mu m}{sec}$$

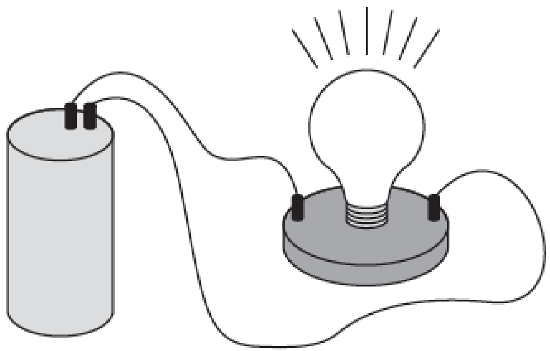
Thus the drift velocity is much smaller!!

Through collisions the work done by electrical force is converted to heat in the resister.

Work done per unit charge is V & charge flowing per unit time is I,

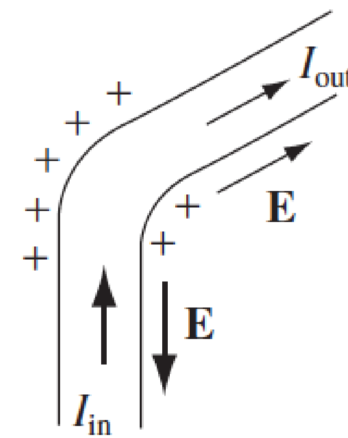
Power delivered: $P = V\,I = I^2R$, *i.e.* the Joule heating law.

Electromotive force



The current is same all the way around the loop!!

- The driving force is inside the battery, then how the pushing is done in the rest of the circuit?
- Also how is the push exactly right to produce the same current in each segment?



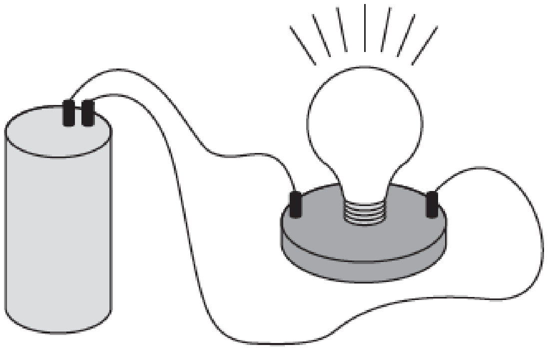
The electrical field of any accumulating charge even out the flow.

$I_{in} > I_{out}$, charge piling up at the knee produces filed away from the Knee/kink.

$\boldsymbol{E}$ opposes I_{in} and helps I_{out}, till there is no more accumulation of charge.

Two forces driving the current in the circuit, the source $\mathbf{f_s}$ (force inside a battery) and the electrostatic force ($\boldsymbol{E}$) smoothing out the flow, $\mathbf{f} = \mathbf{f_s} + \boldsymbol{E}$

Electromotive force



$$\boxed{\mathbf{f} = \mathbf{f_s} + \boldsymbol{E}}$$

Whatever be the mechanism of $\mathbf{f_s}$, the net effect is determined by the line integral around the circuit $\mathcal{E} \equiv \oint \mathbf{f} \cdot d\mathbf{l} = \oint \mathbf{f}_s \cdot d\mathbf{l}.$ [$\oint \mathbf{E} \cdot d\mathbf{l} = 0$ for electrostatic fields]

This integral of force per unit charge is called the Electromotive force or EMF

Within the ideal source (R_{bat}=0) of emf the net force on the charges is zero so $\boldsymbol{E} = -\mathbf{f_s}$.

The potential difference between the terminals

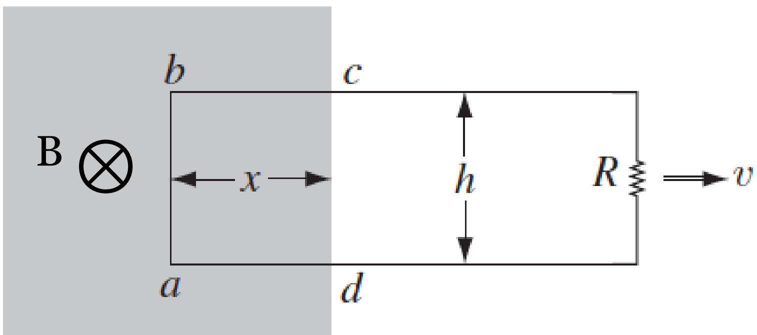
$$V = -\int_a^b \mathbf{E} \cdot d\mathbf{l} = \int_a^b \mathbf{f}_s \cdot d\mathbf{l} = \oint \mathbf{f}_s \cdot d\mathbf{l} = \mathcal{E}$$

Battery maintains Voltage equal to emf

$$\boxed{\text{fs = 0, outside the source}}$$

Motional emf

Generators exploit motinal emf's: Move a wire through a magnetic field.



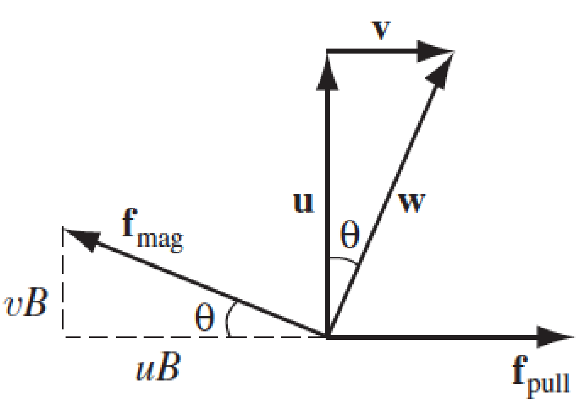
Charges in ab experience a magnetic force qvB
Driving current in loop in the clockwise direction.

The emf is, $\mathcal{E} = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = vBh$

In bc & ad force is $\perp$ to the wire.

This emf is established by magnetic force, but they are not doing any work.

Who is doing the work?



Charges have a vertical velocity **u** & vertical velocity ***v***

Force, quB to the left, $f_{\text{pull}} = uB$

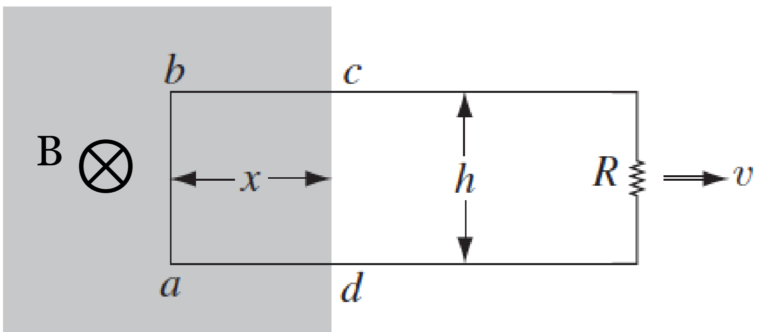
f_{pull} is transmitted by the wire structure to the charge.

The work done per unit charge,

$$\int \mathbf{f}_{\text{pull}} \cdot d\mathbf{l} = (uB)\left(\frac{h}{\cos\theta}\right)\sin\theta = vBh = \mathcal{E}$$

Motional emf

Generators exploit motinal emf's: Move a wire through a magnetic field.



Charges in ab experience a magnetic force qvB
Driving current in loop in the clockwise direction.

The emf is, $\mathcal{E} = \oint \mathbf{f}_{mag} \cdot d\mathbf{l} = vBh$

In bc & ad force is $\perp$ to the wire.

This emf is established by magnetic force, but they are not doing any work.

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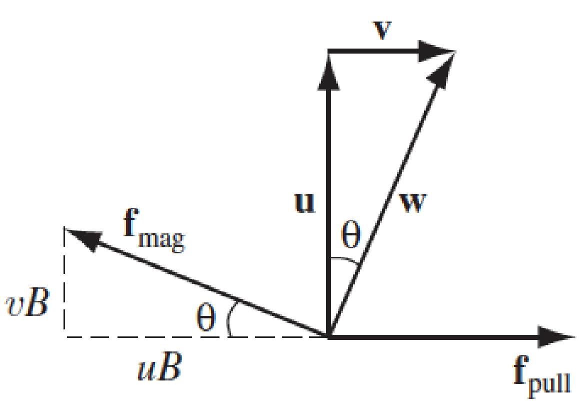


$$\int \mathbf{f}_{pull} \cdot d\mathbf{l} = (uB)\left(\frac{h}{\cos\theta}\right)\sin\theta = vBh = \mathcal{E}$$

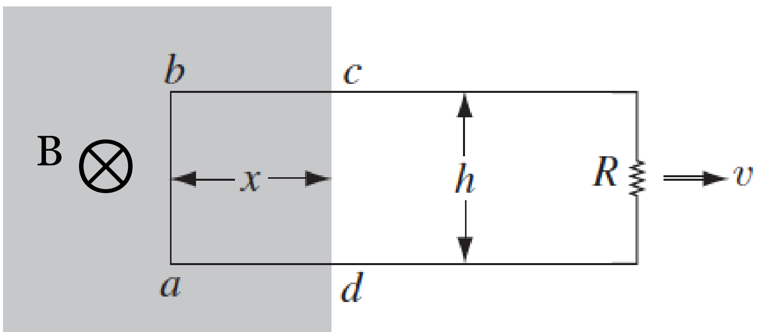
Work done per unit charge is equal to the emf

Integrals are taken on completely different path and force

Here we follow one charge in its motion.

Motional emf

Generators exploit motinal emf's: Move a wire through a magnetic field.



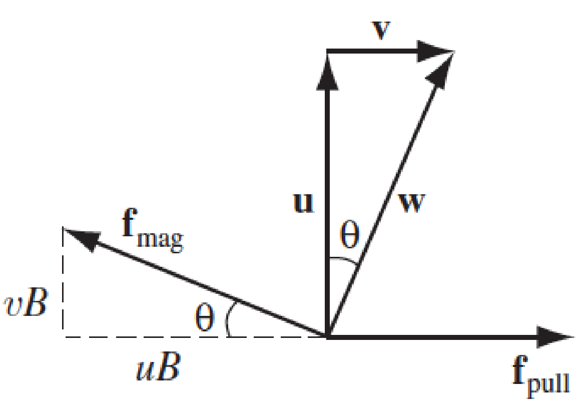
Charges in ab experience a magnetic force qvB
Driving current in loop in the clockwise direction.

The emf is, $\mathcal{E} = \oint \mathbf{f}_{mag} \cdot d\mathbf{l} = vBh$

In bc & ad force is ⊥ to the wire.

This emf is established by magnetic force, but they are not doing any work.

Who is doing the work?



emf generated in a moving loop

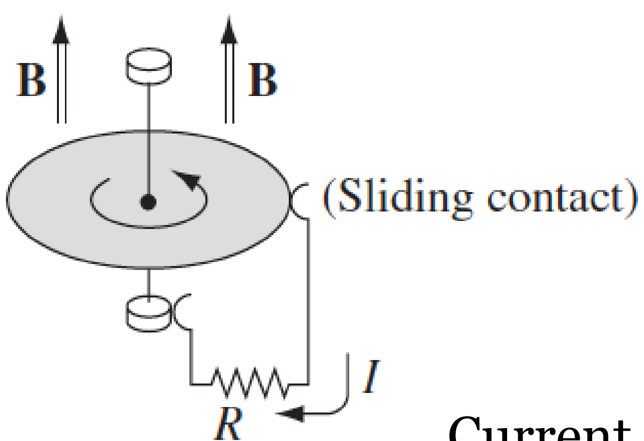
$$\varepsilon = \mathrm{v}\bar{B}h = Bh(-\frac{dx}{dt}) = -\frac{d}{dt}(\underbrace{Bhx}_{\Phi}) = -\frac{d}{dt}\Phi$$

$$\mathcal{E} = -\frac{d\Phi}{dt}.$$

Flux rule for motional emf.

Example 7.4. A metal disk of radius a rotates with angular velocity ω about a vertical axis, through a uniform field $\mathbf{B}$, pointing up. A circuit is made by connecting one end of a resistor to the axle and the other end to a sliding contact, which touches the outer edge of the disk (Fig. 7.15). Find the current in the resistor.

The speed of a point on the disk at a distance s from the axis is $v = \omega s$



force per unit charge is $\mathbf{f}_{mag} = \mathbf{v} \times \mathbf{B} = \omega s B \hat{\mathbf{s}}$.

Emf,

$$\mathcal{E} = \int_0^a f_{mag}\, ds = \omega B \int_0^a s\, ds = \frac{\omega B a^2}{2}$$

Current in the resistor

$$I = \frac{\mathcal{E}}{R} = \frac{\omega B a^2}{2R}.$$

Faraday disk: https://www.youtube.com/watch?v=NJPAX1g1YqI

Eddy Current: https://www.youtube.com/watch?v=Yu1uRvErM80