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5. A particle in the infinite square well potential of width 'a' has its initial wave function which is a linear combination of the first two stationary eigenstates, namely the ground state ($\psi_1(x)$) and the first excited state ($\psi_2(x)$) given by: [1+4+3]

$$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)].$$

(a) Find the value of A.

(b) Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Express the latter in terms of *sinusoidal* functions of time, eliminating the exponentials with the help of formula: $e^{i\theta} = \cos\theta + i\sin\theta$.

(c) Compute the expectation value of the Hamiltonian operator $<\hat{H}>$ at $t = 0$ and at t in the state $\Psi(x,t)$.

a) $\psi(x,0) = c_1\psi_1(x) + c_2\psi_2(x)$ where $c_1 = c_2 = A$

For $\psi(x,0)$ to be normalized, we have $|c_1|^2 + |c_2|^2 = 1$ (since ψ_1 and ψ_2 are orthonormal and both are normalised)

$\therefore 2A^2 = 1 \Rightarrow A^2 = \frac{1}{2} \Rightarrow \boxed{A = \frac{1}{\sqrt{2}}}$ 1

b) $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} \Rightarrow E_1 = \frac{\pi^2\hbar^2}{2ma^2}, \quad E_2 = \frac{2\pi^2\hbar^2}{ma^2}$

$$\psi(x,t) = \frac{1}{\sqrt{2}}\psi_1(x)e^{\frac{-iE_1t}{\hbar}} + \frac{1}{\sqrt{2}}\psi_2(x)e^{\frac{-iE_2t}{\hbar}}$$ 1

For infinite square well, $\psi_1(x) = \psi_1^*(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)$

$\psi_2(x) = \psi_2^*(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{2\pi x}{a}\right)$

$$\therefore \boxed{\psi(x,t) = \frac{1}{\sqrt{a}}\sin\left(\frac{\pi x}{a}\right)e^{-\frac{i\pi^2\hbar t}{2ma^2}} + \frac{1}{\sqrt{a}}\sin\left(\frac{2\pi x}{a}\right)e^{-\frac{i2\pi^2\hbar t}{ma^2}}}$$ 1

$$\psi^*(x,t) = \frac{1}{\sqrt{2}}\psi_1^*(x)e^{\frac{iE_1t}{\hbar}} + \frac{1}{\sqrt{2}}\psi_2^*(x)e^{\frac{iE_2t}{\hbar}}$$

$$\psi^*(x,t) = \frac{1}{\sqrt{2}}\psi_1(x)e^{iE_1t/\hbar} + \frac{1}{\sqrt{2}}\psi_2(x)e^{iE_2t/\hbar}$$

$$= |c_1|^2 \cdot E_1 + |c_2|^2 E_2$$

$$= A^2 (E_1 + E_2)$$

$$= A^2 \left(\frac{5}{2} \frac{\pi^2 \hbar^2}{ma^2} \right)$$

$$= \frac{1}{2} \cdot \frac{5}{2} \frac{\pi^2 \hbar^2}{ma^2} = \boxed{\frac{5}{4} \frac{\pi^2 \hbar^2}{ma^2}}$$

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The value of $\langle H \rangle$ does not change with time t as energy of the system is always conserved.