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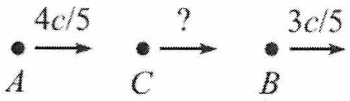
Name	Pradnesh P. Kalkar	Roll No	190123046

2. Two observers A and B travel at speed $4c/5$ and $3c/5$, respectively, with respect to the ground, as shown in the Figure. How fast should the observer C travel with respect to the ground so that she sees A and B are approaching towards her at the same speed u? Compute the value of u. [8]



Let the velocity of C w.r.t ground be V.
Let S' be the frame attached to C.

W.r.t. C the figure will look like

i.e. $V_{A/C} = u$, $V_{B/C} = -u$ —— (1)

For Obs. A:

u_A → vel of A in ground frame $= 4c/5$

vel. of S' = vel of C = V ; $\Rightarrow \beta = V/c$

u_A' → vel of A in S' frame i.e. w.r.t C.

$$\therefore \quad u_A' = \frac{u_A - V}{1 - \frac{V u_A}{c^2}}$$

$$\therefore u_A' = \frac{\frac{4c}{5} - V}{1 - \frac{V \cdot 4c}{5c^2}} = \frac{\frac{4c}{5} - V}{1 - \frac{4V}{5c}} = \left(\frac{4c - 5V}{5c - 4V}\right) c$$

$$u_A' = \left(\frac{4 - 5\beta}{5 - 4\beta}\right) c = u \quad \text{...(from (1))}$$

$$\therefore \left(\frac{4 - 5\beta}{5 - 4\beta}\right) c = u \quad \text{—— (2)}$$

For obs B:

Let the notations be similar to as defined for A.

$\therefore u_B = \frac{3c}{5} \Rightarrow u_B' = \frac{u_B - V}{1 - \frac{V u_B}{c^2}}$

$\therefore u_B' = \frac{\frac{3c}{5} - V}{1 - \frac{V \cdot 3c}{5c^2}} = \frac{\frac{3c}{5} - V}{1 - \frac{3V}{5c}} = \left(\frac{3c - 5V}{5c - 3V}\right) c$

$\therefore u_B' = \left(\frac{3 - 5\beta}{5 - 3\beta}\right) c = -u \quad \cdots$ (from (1))

$\therefore \left(\frac{3 - 5\beta}{5 - 3\beta}\right) c = -u \quad \text{---} (3)$

$\therefore$ Adding (2) and (3), we get

$\frac{4 - 5\beta}{5 - 4\beta} + \frac{3 - 5\beta}{5 - 3\beta} = 0$

$\therefore (5 - 3\beta)(4 - 5\beta) + (5 - 4\beta)(3 - 5\beta) = 0$

$\therefore 35\beta^2 - 74\beta + 35 = 0$

$(7\beta - 5)(5\beta - 7) = 0$

$\beta = 5/7$ or $\beta = 7/5$ (as V has to be less than c)

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$\therefore \beta = 5/7 \Rightarrow \boxed{V = \frac{5c}{7}}$

From (2), $u = \left(\frac{4 - 5\beta}{5 - 4\beta}\right) c = \left(\frac{4 - \frac{25}{7}}{5 - \frac{20}{7}}\right) c = \boxed{\frac{c}{5}}$

Ans: Velocity of obs C w.r.t ground $= \boxed{\frac{5c}{7}}$ in forward direction.

$\boxed{u = \frac{c}{5}}$