

Boolean function simplification using k-map

- gray code is used to number the rows and columns of a k-map. Consider a 3-variable k-map.

| | | AB | | | |
|---|---|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| C | 0 | 0 | 2 | 6 | 4 |
| | 1 | 1 | 3 | 7 | 5 |

Box numbers are entered in the boxes

- A box which can not be combined with any other box is circled as a single box combination
- 2-box combinations for box-0
 - 0-2 (physically touching horizontally)
 - 0-1 (, , , vertically as well as mirror image)
 - 0-4 (Mirror image)

For box -6

- 6-4 (physically touching horizontally)
 - 6-2 (, , , as well as mirror image)
 - 6-7 (, , , vertically)
- 4 box combination for box-0
 - 0-2-1-3 [1-3 is mirror image of 0-2]
 - 0-2-6-4 [6-4 , , , ,]
 - 0-1-4-5 [4-5 , , , 0-1]

For box -6

- i. 6-4-7-5
- ii. 6-2-3-7
- iii. 6-4-2-0
- Only one 8-box combination 0-1-2-3-4-5-6-7

4- variable k-map

| CD \ AB | | | | |
|---------|----|----|----|----|
| | 00 | 01 | 11 | 10 |
| 00 | 0 | 4 | 12 | 8 |
| 01 | 1 | 5 | 13 | 9 |
| 11 | 3 | 7 | 15 | 11 |
| 10 | 2 | 6 | 14 | 10 |

- A box which can not be combined with any other box is considered as single –box combination.
- 2-box combinations

For box 0

- i. 0-4
- ii. 0-1
- iii. 0-8
- iv. 0-2

For box-7

- 7-15
- 7-5
- 7-6
- 7-3

4-box combinations

For box-0

- (1) 0-4-2-6
- (2) 0-1-8-9
- (3) 0-8-2-10
- (4) 0-4-1-5
- (5) 0-4-12-8
- (6) 0-1-3-2

for box-7

- (1) 7-15-6-14
- (2) 7-15-11-3
- (3) 7-15-5-13
- (4) 7-6-5-4
- (5) 7-3-15-11
- (6) 7-3-1-5

- 8-box combinations

For box-0

- (1) 0-4-12-8-1-5-13-9
- (2) 0-4-12-8-2-6-14-10
- (3) 0-4-1-5-3-7-2-6
- (4) 0-1-3-2-8-9-11-10

For box-7

- (1) 7-3-15-11-2-6-14-10
- (2) 7-5-6-4-0-1-2-3
- (3) 7-3-15-11-1-5-13-9
- (4) 7-4-5-6-12-13-14-15

- only one 16-box combination

Ex:- Simplify the following Boolean function using k-map

$$F(A,B,C) = AB + \bar{A}C + BC$$

$$= \sum m(1, 3, 6, 7)$$

Sol:-

| | | AB | | | |
|---|---|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| C | 0 | | | 1 | |
| | 1 | 1 | 1 | 1 | |

Grouping rules:

- i. All single box combinations
- ii. 2-box single way combination
- iii. 4-box ,, ,, ,,
- iv. 8-box ,, ,, ,,
- v. 16-box ,, ,, ,,
- vi. If any minterms are left, combine those minterms with max possible number of boxes.

∴ The simplified Boolean expression $f(A, B, C) = AB + \bar{A}C$

Ex. : Simplify the following Boolean functions using k-map.

$$f(A,B,C,D)=AB\overline{C}+BCD+B$$

Sol. The given Boolean function can be expressed as sum of minterms as follows:

$$\therefore f(A,B,C,D) = \sum m(4,5,6,7,12,13,14,15)$$

| CD \ AB | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00 | | 1 | 1 | |
| 01 | | 1 | 1 | |
| 11 | | 1 | 1 | |
| 10 | | 1 | 1 | |

\therefore The simplified Boolean expression $f(A,B,C,D) = B$