PH 102, Electromagnetism,

Post Mid Semester

Lecture 4.

Magnetostatics:

Field of a magnetized object,
Boundary Conditions

D. J. Griffiths: 6.2 to 6.4

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Magnetisation

- **Magnetisation**: Net alignment of magnetic dipoles inside a **medium**, in the presence of an **applied magnetic field**.

  - Magnetisation: parallel (paramagnet, Al) or opposite (diamagnets, Cu) to the applied magnetic field. ($\boldsymbol{P} = \varepsilon_0 \chi_e \boldsymbol{E}$, $\chi_e > 0$ ➔ Same as $\boldsymbol{E}$)

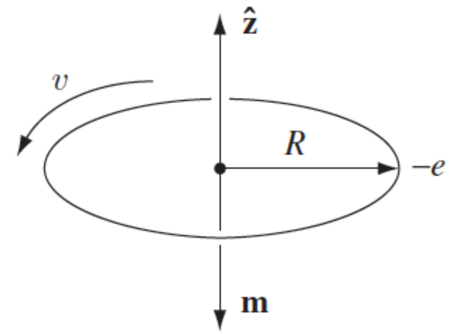
- Ferromagnets (Fe, Ni)**:** Magnetisation retained even after removal of magnetic field.

$\mathbf{N} = \mathbf{m} \times \mathbf{B}$, Torque is in a direction that the line of dipole is parallel to the field and responsible for paramagnetism (odd no of electrons).

Atomic Electrons revolve around the nucleus : Effective magnetic dipole.

Even electrons neutralises the torque on the combination.

Magnetic fields and Atomic Orbits



Current due to orbital motion: $I = \frac{-e}{T} = -\frac{ev}{2\pi R}.$

Is it a steady current? Time period is too small , $T = 2\pi R/v$.

-ve sign for electron.

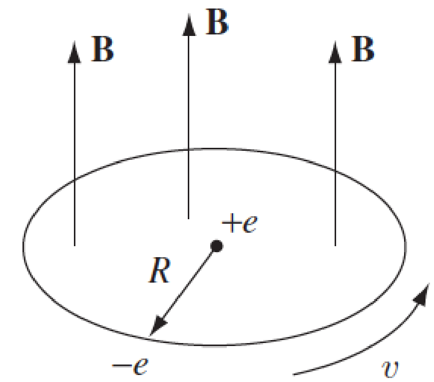
The orbital dipole moment is, $\mathbf{m} = I\pi R^2 = -\frac{1}{2}evR\,\hat{\mathbf{z}}.$

In magnetic field, acting torque = $\mathbf{m} \times \mathbf{B}$, tries to tilt the dipole.

Difficult to tilt the entire orbit!

However,
orientation of the magnetic field can speed up or slow down the electron motion.

Orientation of the magnetic field can speed up or slow down the electron motion.



For magnetic fields perpendicular to the orbital plane,

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} + e\bar{v}B = m_e\frac{\bar{v}^2}{R}.$$

In absence of magnetic fields,

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} = m_e\frac{v^2}{R},$$

The new speed is greater than the zero magnetic field (B = 0) case,

$$e\bar{v}B = \frac{m_e}{R}(\bar{v}^2 - v^2) = \frac{m_e}{R}(\bar{v}+v)(\bar{v}-v),$$

For small change $\Delta v = \bar{v} - v$,

$$\Delta v = \frac{eRB}{2m_e}.$$

[$\bar{v} + v = 2\bar{v} - \Delta v,$
$(\Delta v)^2 \approx 0$]

B turned on and electrons speed up!

Change in orbital speed (i.e., change in current) results in change in dipole moment,

$$\Delta \mathbf{m} = -\frac{1}{2}e(\Delta v)R\,\hat{\mathbf{z}} = -\frac{e^2R^2}{4m_e}\mathbf{B}.$$

Note: Change in dipole moment is opposite to the magnetic field direction.

However, random orientations of electron orbits cancels the orbital dipole moments.

Presence of magnetic field leads to net dipole moment (antiparallel B): Diamagnetism.

Diamagnetism:

- Universal but weaker than Paramagnetism.
- Observed in atoms with even number of electrons (no Paramagnetism)

Paramagnetism vs Diamagnetism

• Easier to tilt the spin than the entire orbit: orbital contribution
to paramagnetism is small!

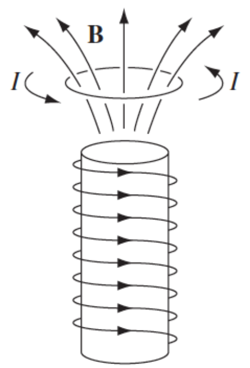
• Paramagnetism: Unpaired electrons, spin dipoles experience a torque trying to align them parallel to the field.

• Diamagnetism: Orbital speed of the electrons is changed, such that the change in orbital dipole moment opposes the direction of the applied field.

Magnetisation (M): magnetic dipole moment per unit volume

(analogous to Polarisation)

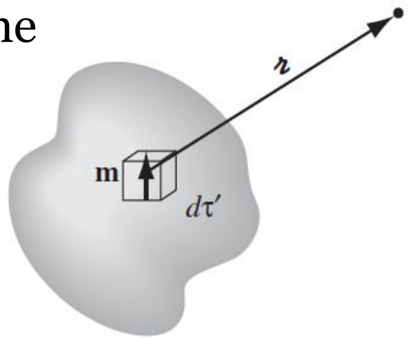
- Paramagnetic: magnetisation upward hence the force is downward.
- Diamagnetic: The magnetisation (force) is downward (upward).
- Paramagnet is attracted into the field but diamagnet is repelled.
- The actual force is much smaller (4-5 order of magnitudes) than an iron sample of similar size



The Field of Magnetized Object

Magnetized material:

Magnetization (**M**) = Magnetic dipole moment per unit volume



Vector potential of a dipole **m** is,

$$\boxed{\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}.}$$

Total vector potential of the object,

Each volume element $d\tau'$ carries dipole moment $\mathbf{M}\,d\tau'$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{M}(\mathbf{r}')\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\,d\tau'.$$

$$= \frac{\mu_0}{4\pi}\int\left[\mathbf{M}(\mathbf{r}')\times\left(\nabla'\frac{1}{\mathscr{r}}\right)\right]d\tau'$$

$$\boxed{\nabla'\frac{1}{\mathscr{r}} = \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}.}$$

The Field of Magnetized Object

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \left[\mathbf{M}(\mathbf{r}') \times \left(\nabla' \frac{1}{\mathscr{r}}\right)\right] d\tau'$$

$$= \frac{\mu_0}{4\pi}\left\{\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]\, d\tau' - \int \nabla' \times \left[\frac{\mathbf{M}(\mathbf{r}')}{\mathscr{r}}\right] d\tau'\right\}$$

volume to surface

$$= \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]\, d\tau' + \frac{\mu_0}{4\pi}\oint \frac{1}{\mathscr{r}}[\mathbf{M}(\mathbf{r}') \times d\mathbf{a}'].$$

$$= \frac{\mu_0}{4\pi}\int_{\mathcal{V}} \frac{\mathbf{J}_b(\mathbf{r}')}{\mathscr{r}}\, d\tau' + \frac{\mu_0}{4\pi}\oint_{\mathcal{S}} \frac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}}\, da'$$

$$\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$\int (\vec{\nabla} \times \vec{v})\, d\tau = -\oint_S \vec{v} \times d\vec{a}$$

To prove, use the div theorem below, with constant vector **c**

$$\int \vec{\nabla} \cdot (\vec{v} \times \vec{c})\, d\tau = \oint (\vec{v} \times \vec{c}) \cdot d\vec{a}$$

$$\mathbf{J}_b = \nabla \times \mathbf{M}, \qquad \mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$$

$\hat{\mathbf{n}}$: normal unit vector

Potential of a volume current

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}}\, d\tau'.$$

Potential of a surface current

$$\mathbf{A} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{K}}{\mathscr{r}}\, da'.$$

The Field of Magnetized Object

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_{\mathcal{V}} \frac{\mathbf{J}_b(\mathbf{r}')}{\mathscr{r}}\, d\tau' + \frac{\mu_0}{4\pi}\oint_{\mathcal{S}} \frac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}}\, da'$$

$$\mathbf{J}_b = \nabla \times \mathbf{M}, \qquad \mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$$

Potential of a volume current

Potential of a surface current

Potential of a magnetized object = Potential from a volume current $\boldsymbol{J_b}$ + Potential from a surface current $\boldsymbol{K_b}$ on boundary

Check the similarity with the electrical case.

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{\sigma_b(\vec{r}\,')}{\mathscr{r}}\, da' + \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho_b(\vec{r}\,')}{\mathscr{r}}\, d\tau'$$

Bound surface charge density

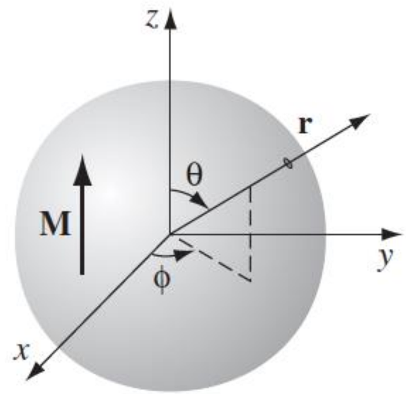
$$\sigma_b = \vec{P}(\vec{r}).\hat{n}$$

Bound volume charge density

$$\rho_b = -\vec{\nabla}.P(\vec{r})$$

The Field of Magnetized Object

Example 6.1. Find the magnetic field of a uniformly magnetized sphere.



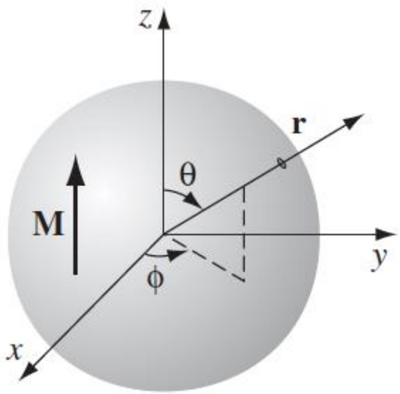
Choose the z-axis along the direction of **M**,

$$\mathbf{J}_b = \nabla \times \mathbf{M} = \mathbf{0}, \quad \mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} = M \sin\theta \, \hat{\boldsymbol{\phi}}$$

Thus, $\mathbf{A}(\mathbf{r}) = \dfrac{\mu_0}{4\pi} \oint_{\mathcal{S}} \dfrac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}} \, da'$

The Field of Magnetized Object

Example 6.1. Find the magnetic field of a uniformly magnetized sphere.



$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \oint_S \frac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}} \, da'$$

with

$$\mathbf{K}_b = M \sin\theta \, \hat{\boldsymbol{\phi}}.$$

For a rotating spherical shell of uniform surface charge σ, the surface current density

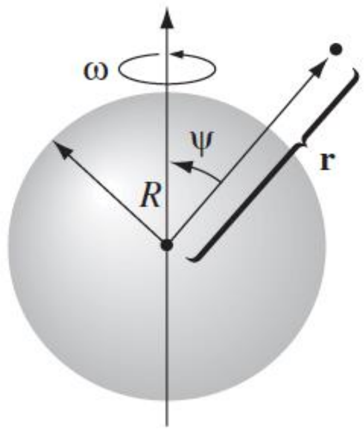
$$\mathbf{K} = \sigma \mathbf{v} = \sigma \omega R \sin\theta \, \hat{\boldsymbol{\phi}}$$

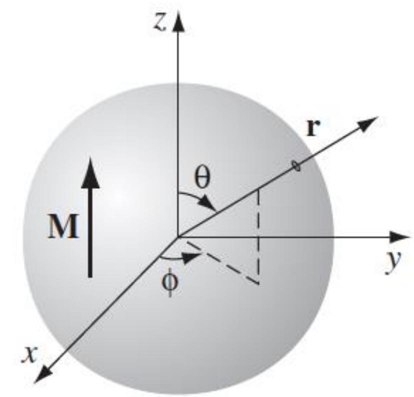
Thus field of a uniformly magnetized sphere is identical to the field of a spinning spherical shell provided, $\sigma R \boldsymbol{\omega} \rightarrow \mathbf{M}$.

Ex. 5.11, L16, Tutorial 8: Q3

The Field of Magnetized Object

Ex. 5.11, L16, Tutorial 8: Q3





$$\mathbf{K} = \sigma \mathbf{v} = \sigma \omega R \sin\theta \,\hat{\boldsymbol{\phi}}$$

$$\mathbf{K}_b = M \sin\theta \,\hat{\boldsymbol{\phi}}.$$

$$\vec{A}(\vec{r}) = \begin{cases} \frac{\mu_0 R\sigma}{3}(\vec{\omega}\times\vec{r}) = \frac{\mu_0 R\omega\sigma}{3} r\sin\theta\hat{\phi} & \text{for } r \le R \\ \frac{\mu_0 R^4\sigma}{3r^3}(\vec{\omega}\times\vec{r}) = \frac{\mu_0 R^4\omega\sigma}{3}\frac{\sin\theta}{r^2}\hat{\phi} & \text{for } r \ge R. \end{cases}$$

$$\boxed{\sigma R\boldsymbol{\omega} \rightarrow \mathbf{M}.}$$

Inside,

$$\vec{B}_{\text{inside}} = \frac{2}{3}\mu_0\vec{M}$$

$$\vec{B}_{\text{inside}} = \frac{2\mu_0 R\omega\sigma}{3}(\cos\theta\hat{r} - \sin\theta\hat{\theta}) = \frac{2}{3}\mu_0\sigma R\vec{\omega}$$

$$\vec{B}_{\text{outside}} = \frac{\mu_0 R\omega\sigma}{3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta}) = \vec{B}_{\text{dipole}}$$

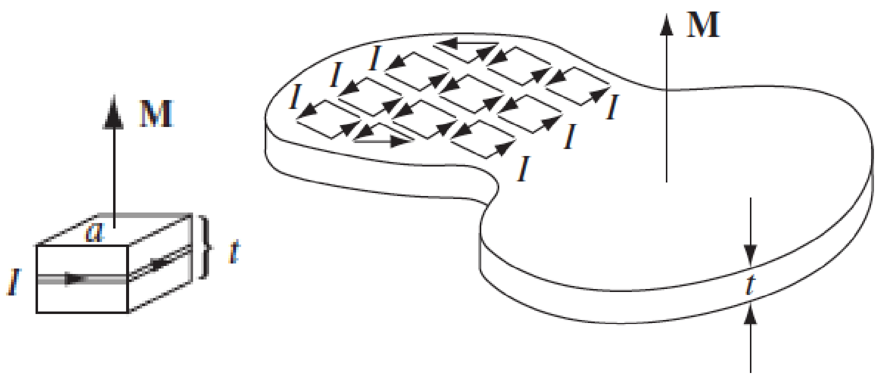
Outside,
field due to a pure dipole

$$\vec{m} = \frac{4}{3}\pi R^3\vec{M}$$

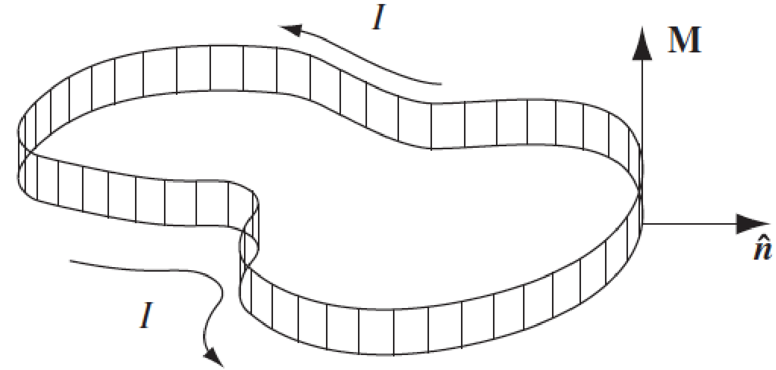
$$\boxed{\mathbf{B}_{\text{dip}}(\mathbf{r}) = \nabla\times\mathbf{A} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}).}$$

Bound Currents: Physical Interpretation

Slab of uniformly magnetised material



Magnetic dipoles: Current loops
Internal currents cancel



No cancellation of currents
at edges, leading to a net current

For each tiny current loop of area a, thickness t, the dipole moment ($\boldsymbol{m}$) in terms of magnetisation ($\boldsymbol{M}$),

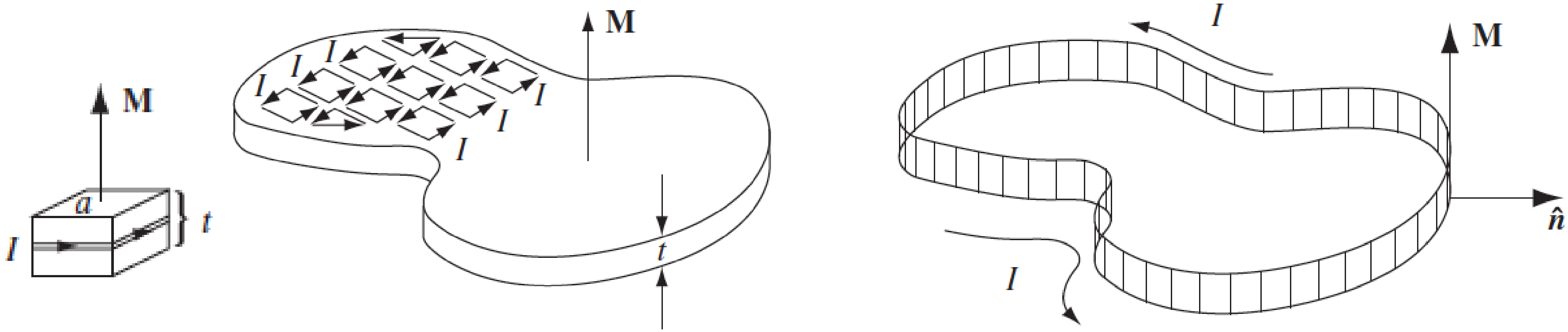
$$m = M\,\mathrm{a}\,t.$$

For current $\boldsymbol{I}$ in each loop, $\quad m = I\,\mathrm{a} \quad \Rightarrow \quad I = Mt$

Hence, the surface current, $K_b = I \,/\, t = M.$

Bound Currents: Physical Interpretation

Slab of uniformly magnetised material



In terms of the outward normal vector, the direction of surface current can be expressed as $\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$

$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$, makes sure that there is no current on top or bottom surface of the slab where magnetisation is parallel to the unit normal vector.

Bound Currents: Physical Interpretation

Surface current:

Bound current as charges taking part in it is attached to a particular atom.

No single charge makes the whole trip over the surface,

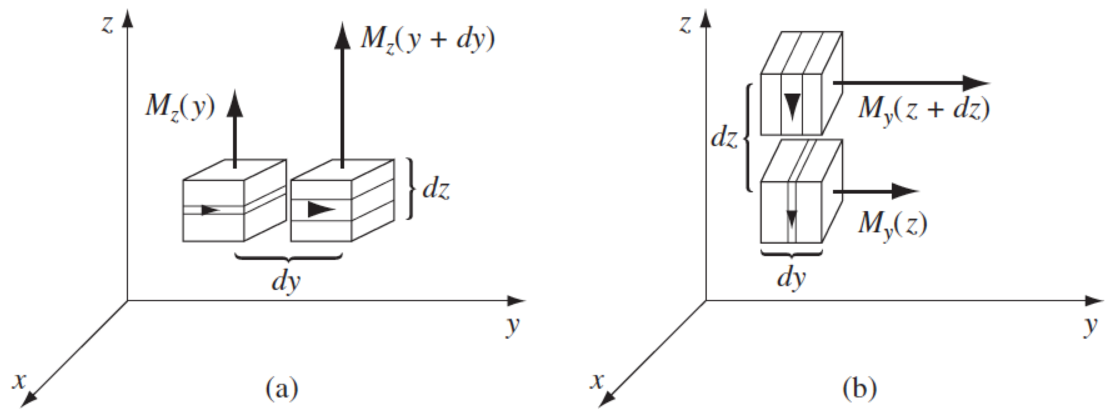
each charge moves only a tiny loop within a single atom.

Bound current is fundamentally different from the usual current.

However, the effects are same (produces magnetic field as usual current)

The field produced this way is the macroscopic field, averaged over a large enough region to contain many atoms or atomic dipoles.

Bound Current: Nonuniform Magnetisation



For non-uniform M, the internal currents no longer cancel.

On the surface where they join, there exists a net current in x-direction (Fig. a)

$$I_x^{a} = \left[M_z(y+dy) - M_z(y)\right]dz = \frac{\partial M_z}{\partial y}dydz$$

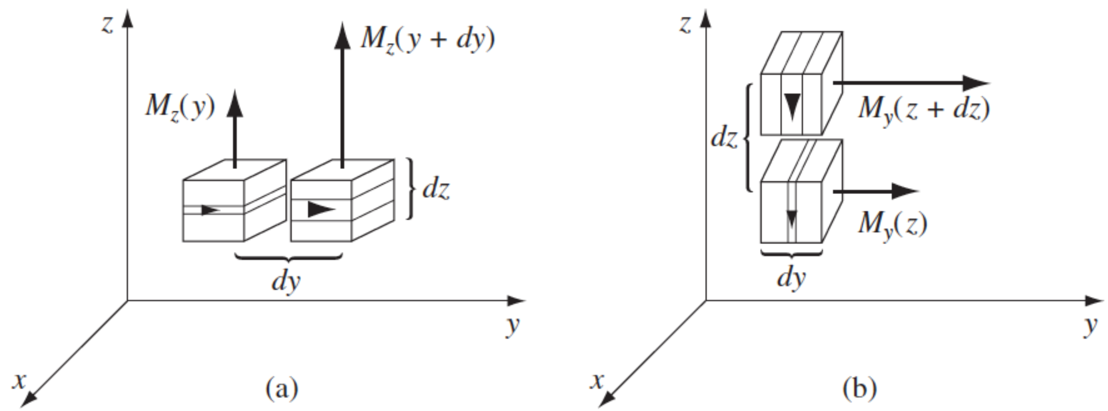
Similarly, non-uniform M in y direction will contribute (Fig. b)

$$-I_x^{b} = \left[M_y(z+dz) - M_y(z)\right]dy = \frac{\partial M_y}{\partial z}dydz$$

Net Current, I_x = dy dz $(J_b)_x = \left(\frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}\right)$ dy dz

$$(J_b)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}$$

Bound Current: Nonuniform Magnetisation



The volume current density corresponding to the currents derived before is,

$$(J_b)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}$$

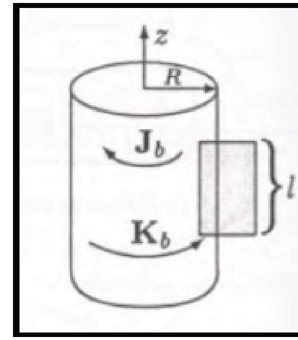
In general, $\mathbf{J}_b = \boldsymbol{\nabla} \times \mathbf{M},$

Like any steady current, this bound current also satisfies the conservation law,

$$\vec{\nabla} \cdot \vec{J}_b = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{M}) = 0$$

Magnetization, establishes bound currents, $\mathbf{J}_b$ within material & $\mathbf{K}_b$ on surface.

Problem 6.12 (Introduction to Electrodynamics, D J Griffiths): An infinitely long cylinder, of radius R, carries a "frozen-in" magnetisation, parallel to the axis $\vec{M} = ks\hat{z}$, where k is a constant, s is the distance from the axis; there is no free current anywhere. Locate the bound currents and calculate the magnetic field.



Solution: The bound currents for the given magnetisation are:

$$\vec{J}_b = \vec{\nabla} \times \vec{M} = -k\hat{\phi}, \quad \vec{K}_b = \vec{M} \times \hat{n} = kR\hat{\phi}$$

Since, current is circumferential, this can be compared with a superposition of solenoids, for which the field outside is zero.

Taking line integral of B along the amperian loop in figure:

$$\begin{aligned}\oint \vec{B} \cdot d\vec{l} = Bl &= \mu_0 \left[\int J_b da + K_b l \right] \\ &= \mu_0 \left[-kl(R-s) + kRl \right] \\ &= \mu_0 kls \implies \vec{B}_{\text{inside}} = \mu_0 ks\hat{z}\end{aligned}$$

Ampere's law in Magnetized Materials

The total current in a medium is the summation of free and bound currents

$$\vec{J} = \vec{J}_b + \vec{J}_f$$

Ampere's law can therefore, be written as

$$\frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = \vec{J} = \vec{J}_f + \vec{J}_b = \vec{J}_f + (\vec{\nabla} \times \vec{M})$$

$$\Longrightarrow \vec{\nabla} \times \left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right) = \vec{\nabla} \times \vec{H} = \vec{J}_f$$

where $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$ plays a role in magnetostatics analogous to $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ in electrostatics.

Ampere's law in integral form for magnetised materials can therefore be written as

$$\oint \vec{H} \cdot d\vec{l} = I_{f_{\rm enc}}$$

$$\oint \vec{D}.d\vec{a} = Q_{\rm f_{enc}}$$

Ampere's law in Magnetized Materials

The free current density is the source of H:

$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

The bound current density is the source of M:

$$\vec{\nabla} \times \vec{M} = \vec{J}_b$$

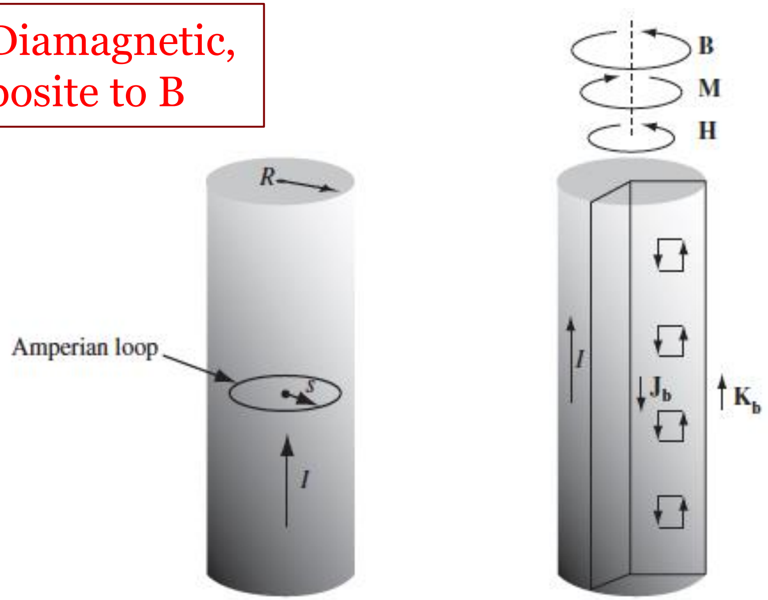
The total current is the source of B:

$$\vec{\nabla} \times \vec{B} = \mu_0(\vec{J}_f + \vec{J}_b)$$

The Auxilary field H

Example 6.2, Introduction to Electrodynamics, D. J. Griffiths: A long copper rod of radius R carries a uniformly distributed (free) current I. Find H inside and outside the rod.

Copper: Diamagnetic, M opposite to B



Using Ampere's law for magnetised object,

$$H(2\pi s) = I_{f_{\text{enc}}} = I\frac{\pi s^2}{\pi R^2}$$

$$\vec{H} = \frac{I}{2\pi R^2} s\hat{\phi} \quad (s \leq R)$$

$$\vec{H} = \frac{I}{2\pi s}\hat{\phi} \quad (s \geq R)$$

Since M is not known, B can not be calculated. It can however be calculated outside the rod where M=0:

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

$$\text{Check } \vec{\nabla}\cdot\vec{H}$$

As noted earlier, divergence of H should be zero in those cases where free current itself determines it.

- Although the Ampere's law for B and H looks similar, they are very different.
- For example, $\vec{\nabla} \cdot \vec{B} = 0$ but $\vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M}$
- Since the divergence of $\vec{H}$ need not be zero always, a vanishing free current does not always imply a vanishing $\vec{H}$. ($\vec{\nabla} \times \vec{H} = 0$ doesn't always mean $\vec{\nabla} \cdot \vec{H} = 0$)
- If the problem has some symmetry, one can use Ampere's law directly to find $\vec{H}$. In such cases divergence of $\vec{M}$ is zero as the free current itself determines $\vec{H}$. (Check divergence of $\vec{H}$ in Ex 6.2)

Boundary Conditions

Using $\vec{\nabla} \cdot (\vec{H} + \vec{M}) = 0 \implies \oint (\vec{H} + \vec{M}) \cdot d\vec{a} = 0$

we have, $H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})$

Using Ampere's law $\oint \vec{H} \cdot d\vec{l} = I_{f_{\text{enc}}}$ we have

$$\vec{H}^{\parallel}_{\text{above}} - \vec{H}^{\parallel}_{\text{below}} = \vec{K}_f \times \hat{n}$$

Which, in the absence of materials, become

$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0 \qquad \vec{B}^{\parallel}_{\text{above}} - \vec{B}^{\parallel}_{\text{below}} = \mu_0(\vec{K}_f \times \hat{n})$

Linear and Nonlinear Media

The proportionality is conventionally denoted by $\vec{M} = \chi_m \vec{H}$

The constant of proportionality χ_m is called the magnetic susceptibility. This dimensionless quantity is positive for paramagnets and negative for diamagnets.

Material	Susceptibility	Material	Susceptibility
Diamagnetic:		*Paramagnetic:*	
Bismuth	-1.7×10^{-4}	Oxygen (O_2)	1.7×10^{-6}
Gold	-3.4×10^{-5}	Sodium	8.5×10^{-6}
Silver	-2.4×10^{-5}	Aluminum	2.2×10^{-5}
Copper	-9.7×10^{-6}	Tungsten	7.0×10^{-5}
Water	-9.0×10^{-6}	Platinum	2.7×10^{-4}
Carbon Dioxide	-1.1×10^{-8}	Liquid Oxygen (−200° C)	3.9×10^{-3}
Hydrogen (H_2)	-2.1×10^{-9}	Gadolinium	4.8×10^{-1}

Linear Media

Materials that obey the proportionality between magnetisation and auxiliary field are called **linear media**. $\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H}$

Thus B is proportional to H that is $\vec{B} = \mu\vec{H}$, where $\mu = \mu_0(1 + \chi_m)$ is called the permeability of the material.

In vacuum, the susceptibility vanishes and hence the permeability is μ_0 (The permeability of free space).

The volume bound current density in a homogeneous linear material is proportional to the free current density

$$\vec{J}_b = \vec{\nabla} \times \vec{M} = \vec{\nabla} \times (\chi_m \vec{H}) = \chi_m \vec{J}_f$$

(Unless free current actually flows through the material, all bound current will be at surface)

Example 6.3, Introduction to Electrodynamics, D. J. Griffiths: An infinite solenoid (n turns per unit length, current I) is filled with linear material of susceptibility χ_m. Find the magnetic field inside the solenoid.

Due to the symmetry of the problem, one can use Ampere's law to find the auxiliary field as $\vec{H} = nI\hat{z}$

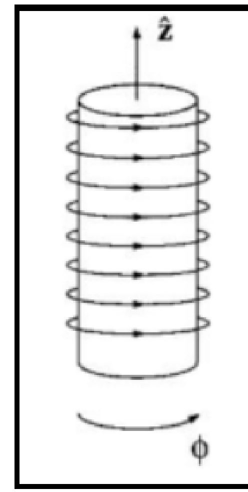


Figure 6.22, Introduction to Electrodynamics, D. J. Griffiths

Therefore $$\vec{B} = \mu_0(1+\chi_m)nI\hat{z}$$

For paramagnetic $(\chi_m > 0)$, field is enhanced.

For diamagnetic $(\chi_m < 0)$, field is somewhat reduced.

This reflects in surface current

$$\vec{K}_b = \vec{M} \times \hat{n} = \chi_m(\vec{H} \times \hat{n}) = \chi_m nI\hat{\phi}$$

which is in same direction as I for paramagnetic and opposite for diamagnetic.

Exercise: A line current I of infinite extent is within a cylinder of radius a that has permeability mu. The cylinder is surrounded by free space. What are the B, H, M fields everywhere? What is the bound current?

Solution: Using ampere's law and taking an amperian circular loop around the current, we can find the auxiliary field as

$$\oint \vec{H} \cdot d\vec{l} = H_\phi(2\pi r) = I \implies H_\phi = \frac{I}{2\pi r}$$

which is same both inside and outside the cylinder. The magnetic field is however different in each region due to the difference in permeability.

$$B_\phi = \begin{cases} \mu H_\phi = \frac{\mu I}{2\pi r}, & 0 < r < a \\ \mu_0 H_\phi = \frac{\mu_0 I}{2\pi r}, & r > a \end{cases}$$

The magnetisation is

$$M_\phi = \frac{B_\phi}{\mu_0} - H_\phi = \begin{cases} \left(\frac{\mu}{\mu_0} - 1\right) H_\phi = \left(\frac{\mu}{\mu_0} - 1\right) \frac{I}{2\pi r}, & 0 < r < a \\ 0, & r > a \end{cases}$$

There is no bound current in the bulk:

$$\vec{J}_b = \vec{\nabla} \times \vec{M} = -\frac{\partial M_\phi}{\partial z}\hat{r} + \frac{1}{r}\frac{\partial}{\partial r}(rM_\phi)\hat{z} = 0, \quad 0 < r < a$$

There exists, a bound line current though at r=0:

$$I_b = \int \vec{J}_b \cdot d\vec{a} = \oint \vec{M} \cdot d\vec{l} = M_\phi(2\pi r) = \left(\frac{\mu}{\mu_0} - 1\right) I$$

There also exists a bound surface current at r=a:

$$K_b = -M_\phi(r = a) = -\frac{I_b}{2\pi a} = -\left(\frac{\mu}{\mu_0} - 1\right)\frac{I}{2\pi a}$$

The negative sign is coming due to $\vec{K}_b = \vec{M} \times \hat{n}$ where $\vec{M}$ is along $\hat{\phi}$ and $\hat{n}$ is along $\hat{r}$.

Absence of bulk current is due to absence of bulk free current. The adjacent dipoles cancel **(?)** the currents except at r=0 and r=a.