4.5

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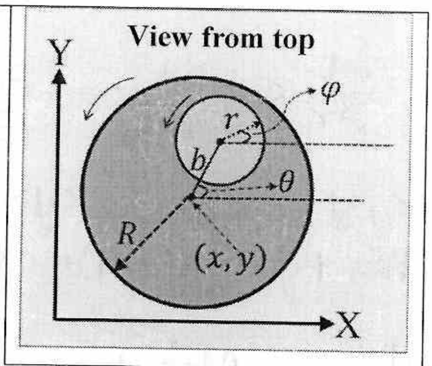
Q3 [**5 marks**] A disc of mass M and radius R slides on a frictionless horizontal surface of infinite extent. A smaller disc of mass m and radius r ($< R$) is pinned through its center to a point on the larger disc. Center-to-center distance between two discs is b as shown in the figure. The smaller disc can rotate freely without friction in the plane of the larger disc.



Choose generalized coordinates from the variables marked in the figure. (*i*) Find the kinetic and potential energies using generalized coordinates and generalized velocities. (*ii*) From the Lagrangian, identify cyclic coordinates and calculate the constant(s) of motion.

Sol^n: No. of generalised coordinates = 4

Let (x, y, θ, ϕ) be generalised coordinates.

$\theta \rightarrow$ angle rotated by bigger disc

$\phi \rightarrow$ angle rotated by smaller disc

Let (x_1, y_1) be the coordinates (cartesian) of small disc's COM.

$\therefore \quad x_1 = x + b\cos\theta \Rightarrow \dot{x}_1 = \dot{x} - b\sin\theta\,\dot{\theta}$

$y_1 = y + b\sin\theta \Rightarrow \dot{y}_1 = \dot{y} + b\cos\theta\,\dot{\theta}$

As, system is moving on horizontal plane $\Rightarrow U = 0$

$I_M = \frac{MR^2}{2}, \quad I_m = \frac{mr^2}{2}$

$\therefore T_M = \frac{1}{2}M(\dot{x}^2+\dot{y}^2) + \frac{1}{2}I_M\dot{\theta}^2$

$\therefore T_M = \frac{1}{2}M(\dot{x}^2+\dot{y}^2) + \frac{MR^2}{4}\dot{\theta}^2 \quad \text{—— (1)}$

$T_m = \frac{1}{2}m(\dot{x}_1^2+\dot{y}_1^2) + \frac{1}{2}I_m\dot{\phi}^2$

$= \frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+b^2\dot{\theta}^2 - 2b\dot{x}\sin\theta\,\dot{\theta} + 2b\dot{y}\cos\theta\,\dot{\theta}\right) + \frac{1}{2}\times\frac{mr^2}{2}\dot{\phi}^2 \quad \text{—— (2)}$

$L = T_M + T_m$

$$\therefore L = \frac{1}{2}M(\dot{x}^2+\dot{y}^2) + \frac{MR^2}{4}\dot{\theta}^2 + \frac{mr^2}{4}\dot{\phi}^2$$
$$+ \frac{1}{2}m(\dot{x}^2+\dot{y}^2+b^2\dot{\theta}^2 - 2\dot{x}b\sin\theta\,\dot{\theta} + 2b\dot{y}\cos\theta\,\dot{\theta}) \quad \text{—— (3)}$$

Now, $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} = \frac{\partial L}{\partial \phi} = 0$, $\frac{\partial L}{\partial \theta} \neq 0$

$\therefore x, y, \phi$ are cyclic coordinates.

$\therefore P_x, P_y$ and P_ϕ are constants of motion. = Row?

$$P_x = \frac{\partial L}{\partial \dot{x}} = \boxed{M\dot{x} + m\dot{x} - mb\sin\theta\,\dot{\theta}}$$

$$P_y = \frac{\partial L}{\partial \dot{y}} = \boxed{M\dot{y} + m\dot{y} + mb\cos\theta\,\dot{\theta}}$$

$$P_\phi = \frac{\partial L}{\partial \dot{\phi}} = \boxed{\frac{mr^2\dot{\phi}}{2}}$$

= Const.