Lecture 6

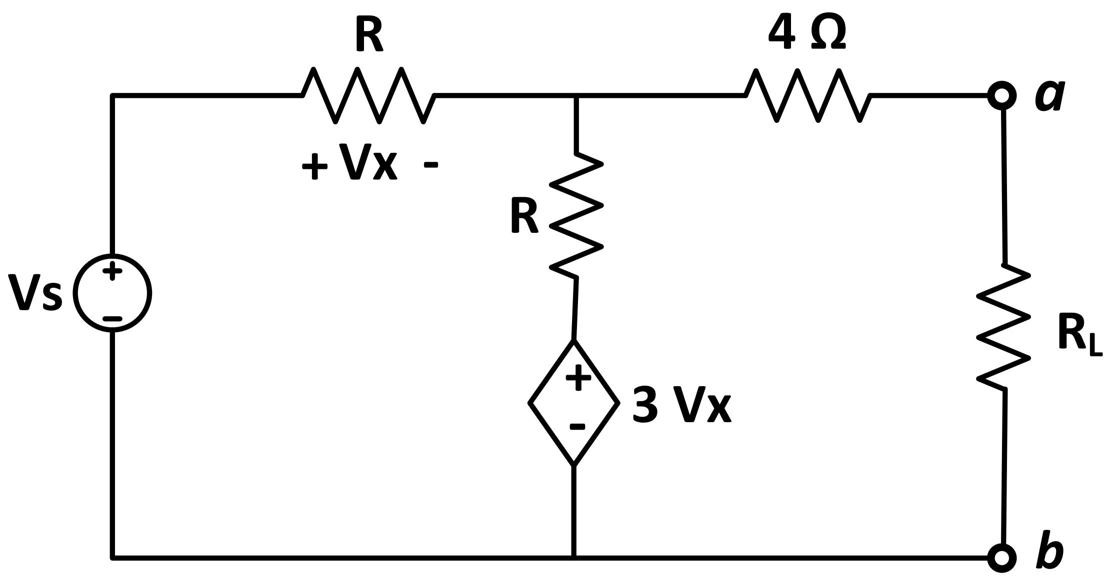
Quiz Question+Solution

and

RL and RC Circuits

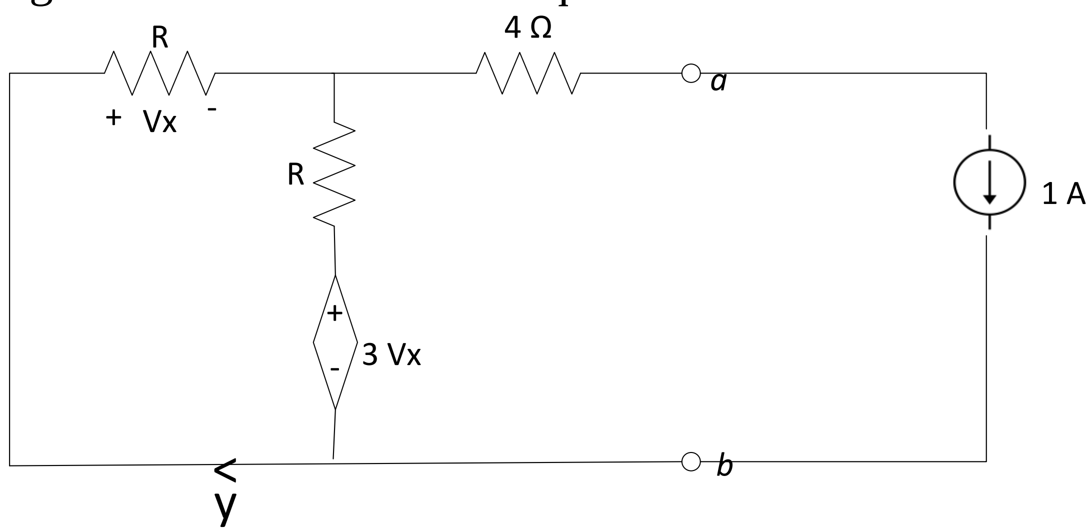
<u>Quiz Question</u>

Using Thevenin's voltage (Vth) and equvalent resistance (Rth) across ***a-b***, find maximum power delivered to the load $\mathbf{R_L}$.



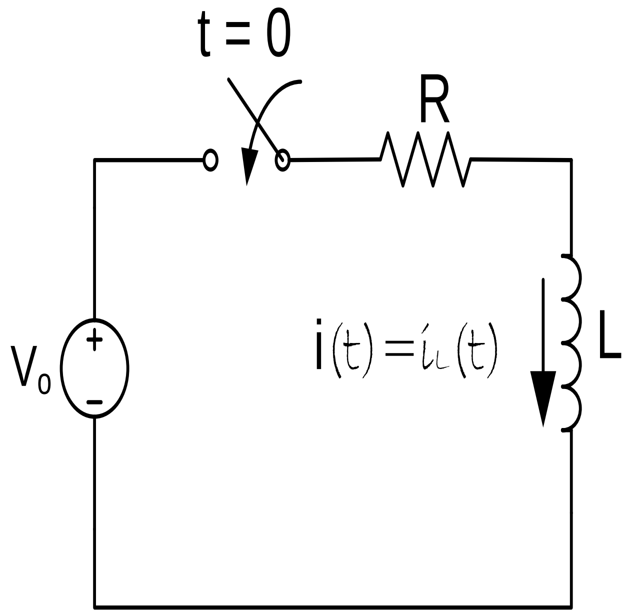
Solution

After removing the load resistor RL, let the mesh current be ‘i’ in clockwise direction.
Applying KVL in the mesh, one gets $Vs – 4Vx – Ri = 0$.
But, $Vx = Ri$, therefore $Vs – 5Ri = 0$ giving $i = Vs / (5R)$ ……… …(1)
Vth = Vab $= 3\,Ri + Ri = 4Ri = (4/5)\,Vs =$ **0.8 Vs** Volt ……………….(2)
The following figure is used to determine equivalent resistance.

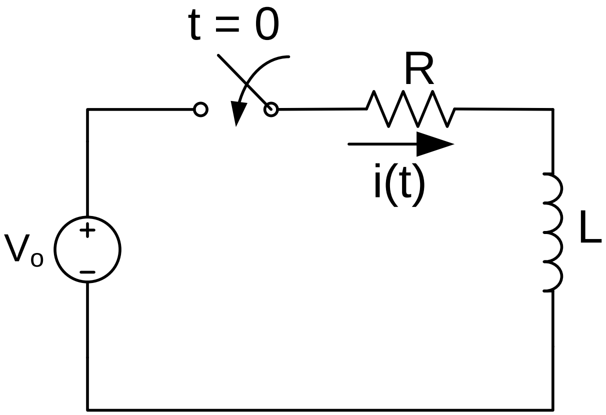


Applying KVL in the LHS mesh yields $Ry – (1-y)R + 3\,Ry = 0$ giving $y = 0.2$ ……. (3)
Voltage across the externally connected 1A source is $Ve = Ry + 4$.
Therefore, from (3) **Req = Ve/1 = (0.2R +4)** Ohm …………………………………...(4)
Finally, maximum power delivered, **Pm = $(\mathbf{Vth})^2/(\mathbf{4Req})$** Watt …………………….(5)

Response of RL Series Circuit



Find i(t)



For t < 0, i(t) = 0

$$Ri + L\frac{di}{dt} = V_0 \quad \text{for } t > 0$$ *and using KVL*

$$\text{or, } L\frac{di}{dt} = V_0 - Ri$$

$$\text{or, } \frac{Ldi}{V_0 - Ri} = dt$$

Integrating, we get

$$\int \frac{L\,di}{V_0 - Ri} = \int dt$$

$$-\frac{L}{R}\ln\left(V_0 - Ri\right) = t + K$$

Integrating, we get

$$\int \frac{L\,di}{V_0 - Ri} = \int dt$$

$$-\frac{L}{R}\ln\left(V_0 - Ri\right) = t + K$$

Setting $i = 0$ at $t = 0$, we have

$$-\frac{L}{R}\ln V_0 = K$$

Hence, we have

$$-\frac{L}{R}\ln\left(V_0 - Ri\right) = -\frac{L}{R}\ln V_0 + t$$

$$or,\ \ -\frac{L}{R}\ln\left(\frac{V_0 - Ri}{V_0}\right) = t$$

$$\text{or, }\ \ln\left(\frac{V_0 - Ri}{V_0}\right) = -\frac{Rt}{L}$$

$$or,\ \ \frac{V_0 - Ri}{V_0} = e^{-\frac{Rt}{L}}$$

$$V_0 - Ri = V_0 e^{-\frac{R}{L}t}$$

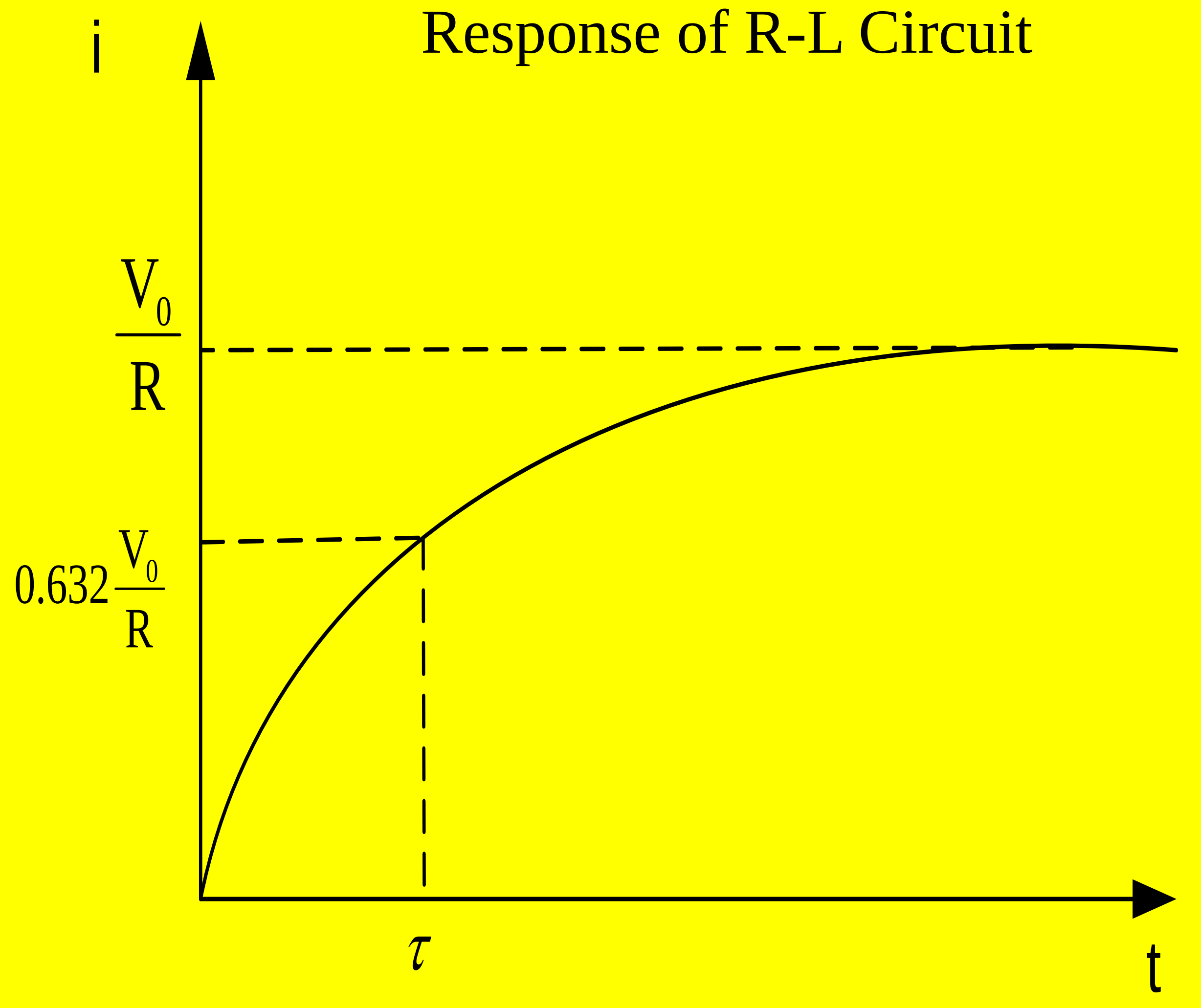
$$or,\ \ Ri = V_0 - V_0 e^{-\frac{R}{L}t}$$

$$or,\ \ i = \frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t}$$

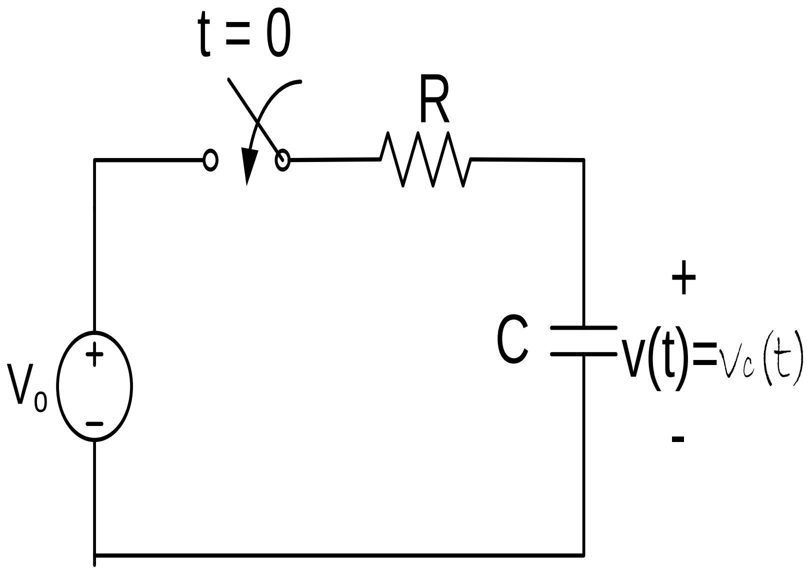
The expression for the response for all t is

$$\boxed{i(t) = \left(\frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t} \right)}$$

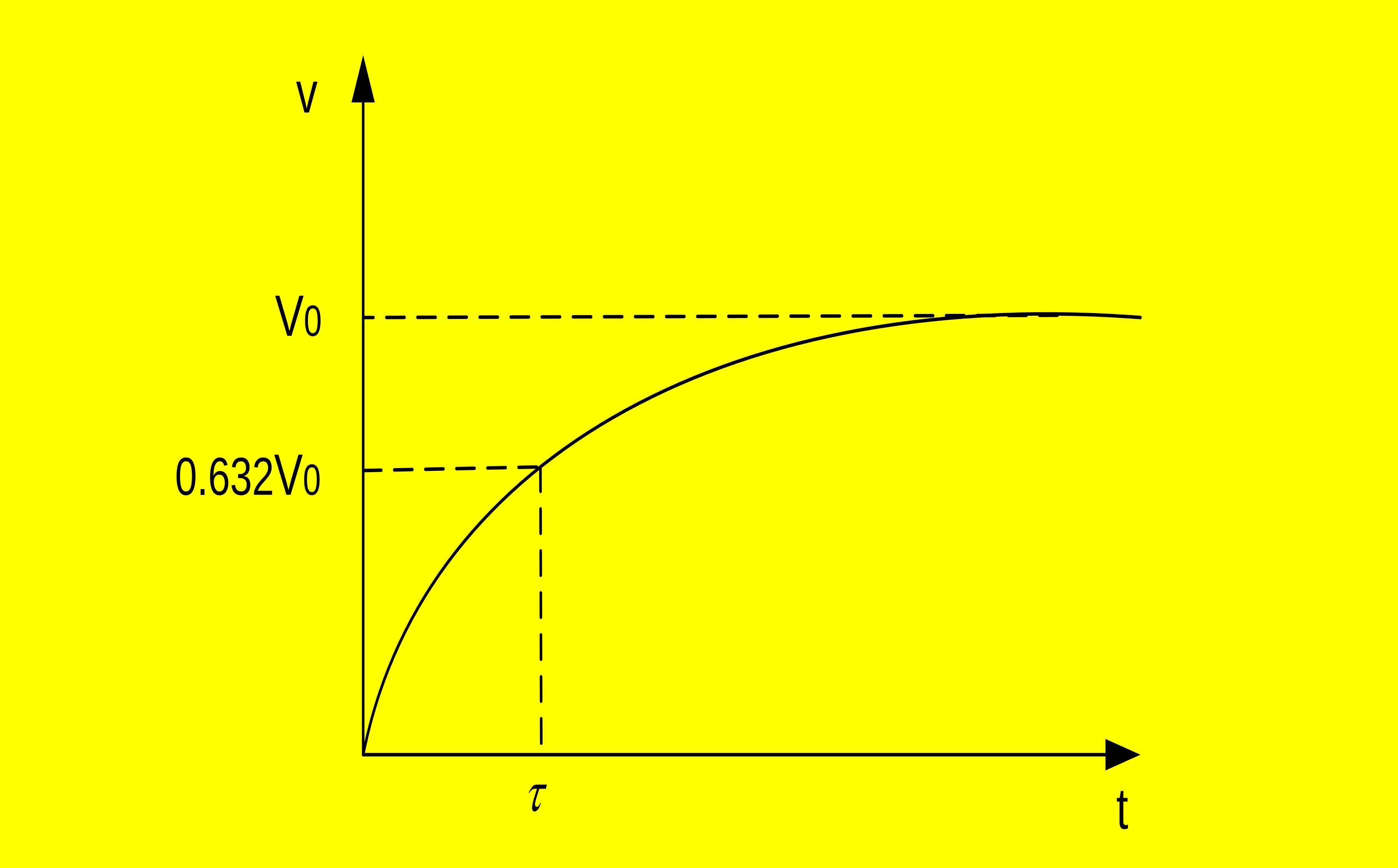
$$\frac{L}{R} = \tau = \text{Time Constant}$$


Response of R-L Circuit
i
$\frac{V_0}{R}$
$0.632\frac{V_0}{R}$
τ
t

Response of RC Series Circuit



Find v(t)


v
V0
0.632V0
τ
t

First order Differential Equation: $dx(t)/dt+ax(t)=b$

Can also be written as: $dx(p)/dp + ax(p) = b$

Multiplying both sides by e^{ap} gives:

$$e^{ap}\, dx(p)/dp + e^{ap}\, ax(p) = e^{ap}\, b$$

$$\Rightarrow \quad d(e^{ap}\, x(p))/dp = e^{ap}\, b$$

$$\Rightarrow \quad d(e^{ap}\, x(p)) = e^{ap}\, b\, dp$$

$$\Rightarrow \int_o^t d(e^{ap}\, x(p)) = \int_o^t e^{ap}\, b\, dp$$

$$\Rightarrow \quad e^{ap}\, x(p) \,\Big|_{o}^{t} = (b/a)\, e^{ap} \,\Big|_{o}^{t}$$

Substitution of limits of integral => $e^{at}\, x(t) - x(0) = (b/a)\,(e^{at} - 1)$

$$\Rightarrow \quad e^{at}\, x(t) = x(0) + (b/a)\,(e^{at} - 1)$$

Multiplying both sides of above equation by e^{-at}

$$\Rightarrow x(t) = e^{-at}\, x(0) + (b/a)\,(1 - e^{-at})$$

$$= (x(0) - b/a)\, e^{-at} + b/a$$

Solution of first order DE $\dot{x}(t) + ax(t) = b$ is

$$x(t) = [x(0) - (b/a)]\, e^{-at} + (b/a)$$

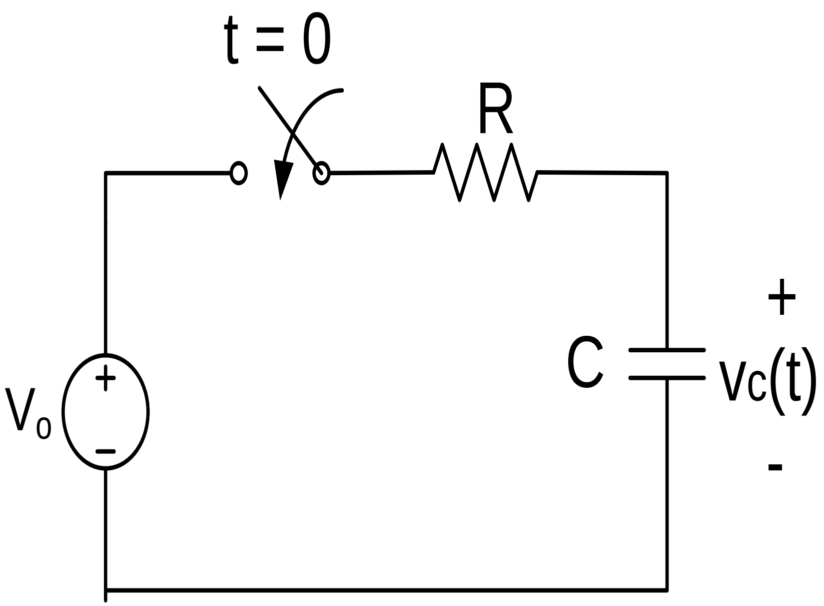
For series RL circuit : $x(t) = i(t)$, $a = R/L$ and $b = V_0 /L$

So, $i_L(t) = [i(0) - (V_0/R)]\, e^{-Rt/L} + (V_0/R)$

For series RC circuit : $x(t) = v(t)$, $a = 1/(RC)$ and $b = V_0 /(RC)$

Then, $v_c(t) = [v_c(0) - V_0]\, e^{-t/(RC)} + V_0$

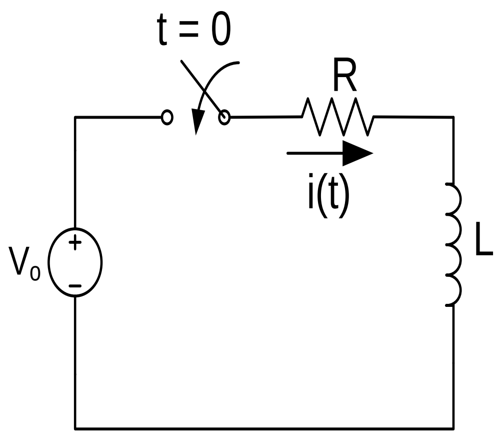
Determine and plot voltage $v_c(t)$ across the capacitor for $0 \le t \le 2$ sec.



V_0=12 V, R = 1 kΩ, C = 470 μF
and $v_c(0)$ = 5 V

$$v_c(t) = [v_c(0) - V_0]\, e^{-t/(RC)} + V_0 = (-7)\, e^{-t/0.47} + 12$$

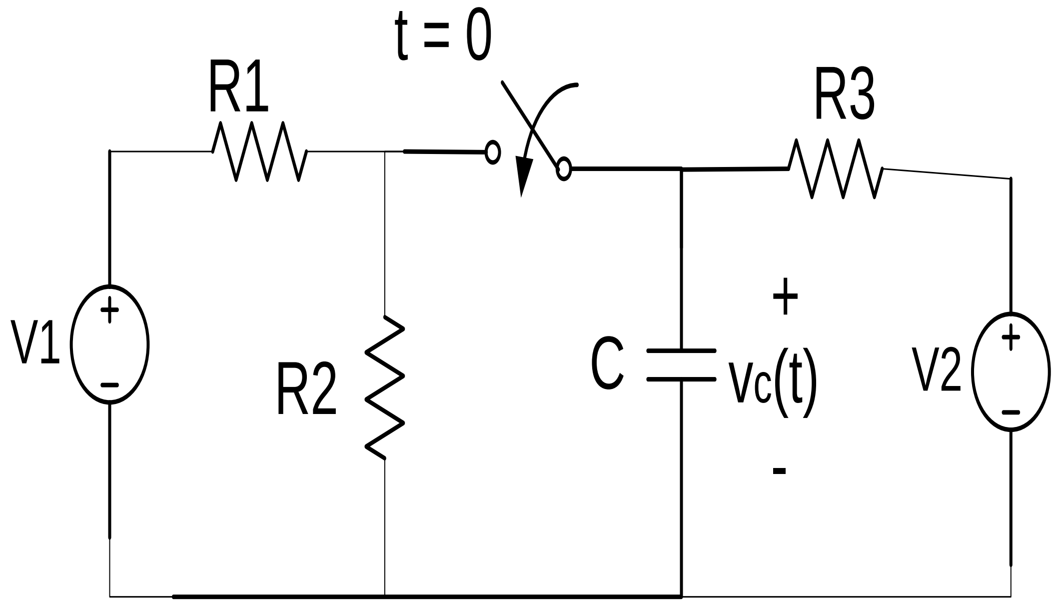
Determine and plot inductor current $i_L(t)$ for $t \leq 0.1$ sec.



Vo = 100 V, R = 8Ω, L = 0.2 H

$$i_L(t) = [0 - 12.5]\, e^{-40t} + 12.5 = 12.5(1 - e^{-40t})$$

Find expression for Vc(t) for t > 0.



Vc(0) = V2, Rth= R1||R2||R3, V_0=Vth=Rth(V1/R1 + V2/R3)

Feedback

- Go slow
- Move faster
- More examples
- Tutorial problems (easy/tough)