Half adder:-

Adds two bits and produces sum and carry

Let the inputs are A and B

outputs are S and C

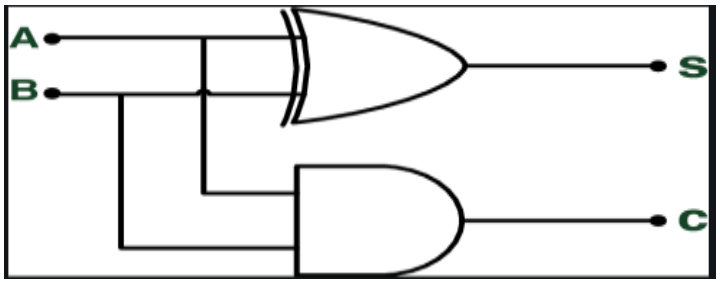
Half Adder Truth table

INPUTS		OUTPUTS	
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$\therefore S = \bar{A}B + A\bar{B} = A \oplus B$$

$$C = AB$$

Circuit



Full adder

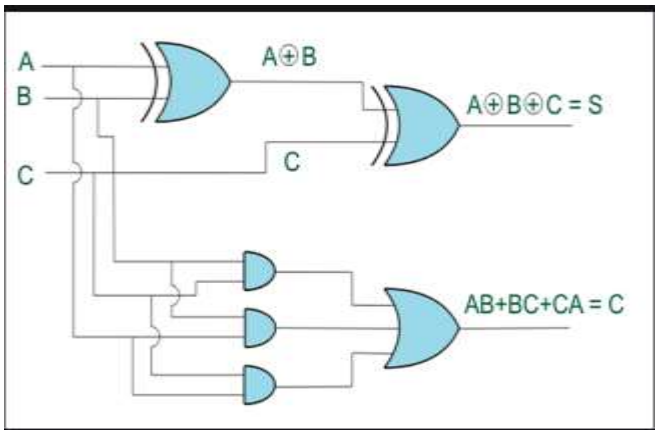
If performs the addition of 3 bits and produces two outputs sum and carry.

Let the inputs are A,B,C and output S and C.

Truth table

A	B	C	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Circuit diagram



$$\therefore S = \sum m(1,2,4,7)$$

C \ AB	00	01	11	10
0	0	(1)	0	(1)
1	(1)	0	(1)	0

$$S = \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC + A\bar{B}\bar{C}$$

$$= \bar{A}(\bar{B}C + B\bar{C}) + A(BC + \bar{B}\bar{C})$$

$$= A \oplus B \oplus C$$

$$C = \sum m(3,5,6,7)$$

C \ AB	00	01	11	10
0	0	0	(1)	0
1	0	(1)	(1)	(1)

$$C = AB + BC + AC$$

Ripple carry adder

Let $A = A_3 A_2 A_1 A_0$

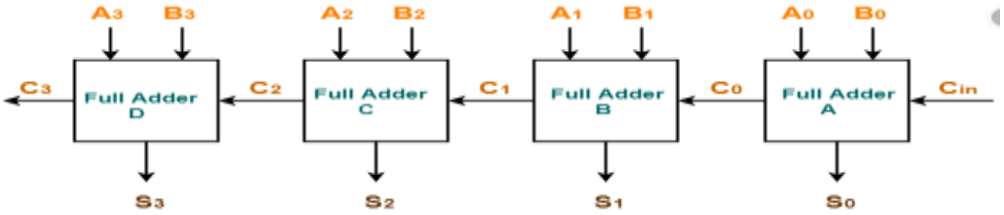
$B = B_3 B_2 B_1 B_0$

To perform A+B, 4 fill adders are required

A+B ⟹

$$\begin{array}{r} A_3 A_2 A_1 A_0 \\ +\, B_3 B_2 B_1 B_0 \\ \hline C_2 C_1 C_0 C_{in} \\ \hline S_3 S_2 S_1 S_0 \end{array}$$

The block diagram of 4-bit ripple carry adder is as follows:



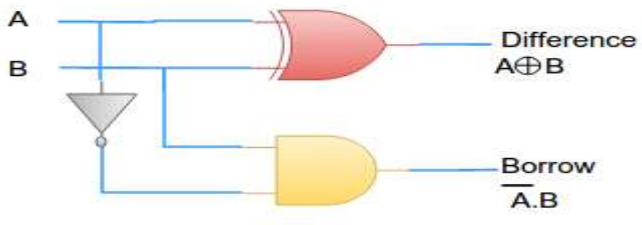
Half Subtractiors

If performs subtraction of two bits and produces two outputs difference and borrow.

Let A,B are the inputs and *Difference*,*Borrow* are the outputs

Input		Output	
A	B	Difference	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$Difference = \bar{A}B + A\bar{B} = A \oplus B$ $\quad Borrow = \bar{A}B$



Full subtractor

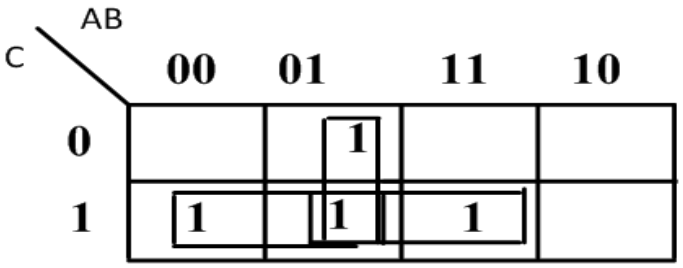
- If performs the subtraction of 3-bit and produces difference and borrow.
- Let the inputs are A,B,C and outputs are D and B.

A	B	C	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$\therefore D = \sum m(1,2,4,7)$$

$$= A \oplus B \oplus C$$

$$B = \sum m(1,2,3,7)$$



$$\therefore B = \overline{A}B + BC + \overline{A}C$$