Department of Mathematics
Indian Institute of Technology Guwahati

MA 101: Mathematics I

Tutorial Sheet-2

July-December 2019

---

1. Let (x_n) be a convergent sequence in $\mathbb{R}$ with limit $\ell \in \mathbb{R}$ and let $\alpha \in \mathbb{R}$.
   (a) If $x_n \geq \alpha$ for all $n \in \mathbb{N}$, then show that $\ell \geq \alpha$.
   (b) If $\ell > \alpha$, then show that there exists $n_0 \in \mathbb{N}$ such that $x_n > \alpha$ for all $n \geq n_0$.
   (c) If (x_n) and (y_n) are convergent sequences and $x_n \geq y_n$ for all $n \in \mathbb{N}$, then $\lim\limits_{n\to\infty} x_n \geq \lim\limits_{n\to\infty} y_n$.

   (Note that ℓ can be equal to α in (a) even if $x_n > \alpha$ for all n.)

2. Let (x_n) be a convergent sequence of positive real numbers such that $\lim\limits_{n\to\infty} x_n < 1$. Show that $\lim\limits_{n\to\infty} x_n^n = 0$.

3. If $|\alpha| < 1$, then the sequence (α^n) converges to 0.

4. Show that the sequence $((2^n + 3^n)^{\frac{1}{n}})$ converges to 3.

5. Let (a_n) be a sequence of real numbers such that each of the subsequences (a_{2n}), (a_{2n-1}) and (a_{3n}) converges. Show that (a_n) is convergent.

6. If (a_n) is a bounded sequence and (b_n) is another sequence which converges to 0, show that the product $(a_n b_n)$ converges to 0.

7. Let (a_n) be a sequence of real numbers. Define the sequence (s_n) by $s_n = \dfrac{1}{n}\sum\limits_{i=1}^{n} a_i$.
   (a) If (a_n) is bounded, then show that (s_n) is also bounded.
   (b) If (a_n) is monotone, then show that (s_n) is also monotone.
   (c) If (a_n) converges to ℓ, then show that the sequence (s_n) also converges to ℓ.

8. Show that the sequence (x_n) defined by $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ diverges to infinity.

9. Let the sequence (a_n) be defined by
$$a_1 = 1, a_{n+1} = \left(\frac{3 + a_n^2}{2}\right)^{1/2}, \quad n \geq 1.$$
   Show that (a_n) converges to $\sqrt{3}$.

10. Let $a_1 > 0$ and for $n \geq 1$, $a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{2}{a_n}\right)$. Show that the sequence $\{a_n\}$ is convergent and find the limit.

11. For $a \in \mathbb{R}$, let $x_1 = a$ and $x_{n+1} = \dfrac{1}{4}(x_n^2 + 3)$ for all $n \geq 2$. Examine the convergence of the sequence $\{x_n\}$ for different values of a. Also, find $\lim\limits_{n\to\infty} x_n$ whenever it exists.

12. Let $x_1 = 6$ and $x_{n+1} = 5 - \frac{6}{x_n}$ for all $n \in \mathbb{N}$. Examine whether the sequence (x_n) is convergent. Also, find $\lim\limits_{n\to\infty} x_n$ if (x_n) is convergent.