Lectures 6-8 (Engineering Mechanics) - Structures (continuation), Distributed Forces and Center of Gravity

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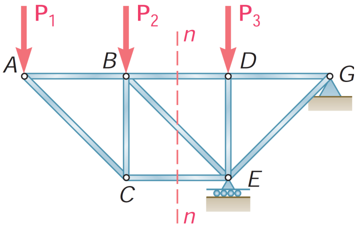
IIT Guwahati

20 - 22 Jan 2020

Truss Analysis - Method of Sections



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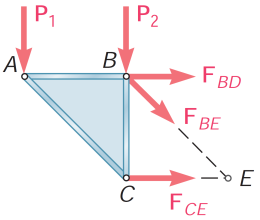
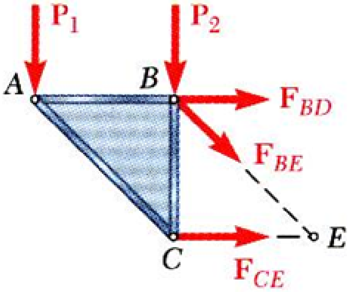
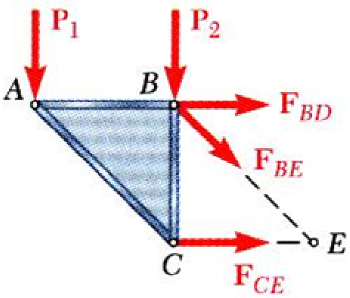


(b)

- When the force in only one member or the forces in a very few members are desired, the method of sections works well
- To determine force in member BD, form a section by "cutting" the truss at $n - n$ and create a free body diagram for the left side
- FBD - right or left side ???
- Notice that exposed internal forces - all assumed to be in tension
- With only three members cut by section, equations for static equilibrium may be applied to determine unknown member forces



Using the left-side FBD, write **one** equilibrium equation that can be solved to find FBD.



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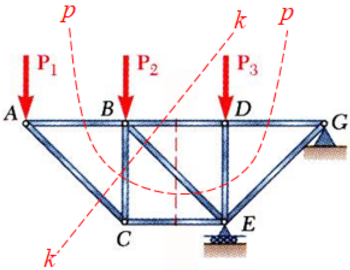
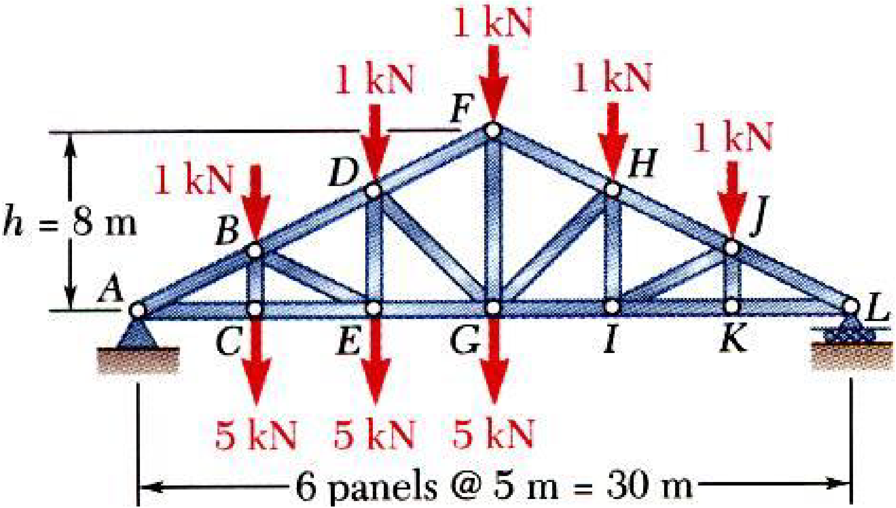
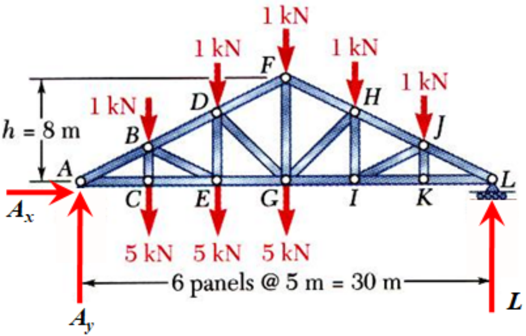


- Assume that the initial section cut was made using line k-k. **Why would this be a poor choice?**
- Notice that any cut may be chosen, so long as the cut creates a separated section.
- So, for example, this cut with line p-p is acceptable.



Problem 1 - Using the method of sections, Determine the force in members FH , GH , and GI



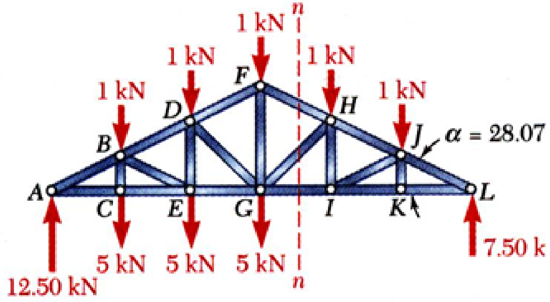
Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at A and L.

$\sum M_A = 0 = -(5\ m)(6\ kN) - (10\ m)(6\ kN) - (15\ m)(6\ kN) - (20\ m)(1\ kN) - (25\ m)(1\ kN) + (25\ m)L,$

$\Rightarrow \ \ L = 7.5\ kN$

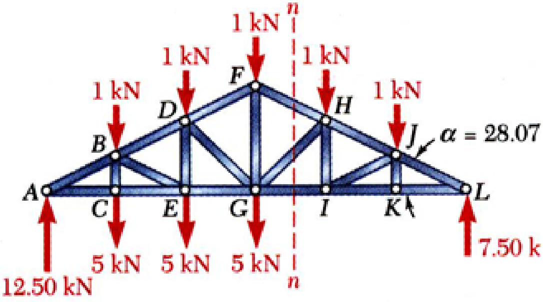
$\sum F_y = 0 = -20\ kN + L + A_y, \Rightarrow \ \ A_y = 12.5\ kN$

$\sum F_x = 0 = A_x$



Make a cut through members FH, GH, and GI and take the right-hand section as a free body. Draw this FBD.

Which one equilibrium equation that could be solved to find F_{GI}?



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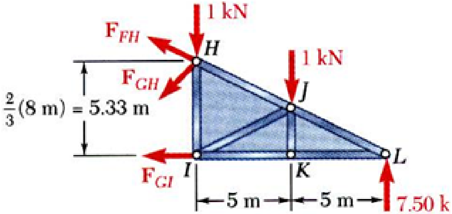
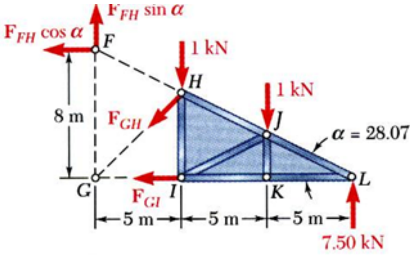


Sum of the moments about point H.

$$\sum M_H = 0 = 7.5\ kN(10\ m) - (1\ kN)(5\ m) - F_{GI}(5.33\ m)$$

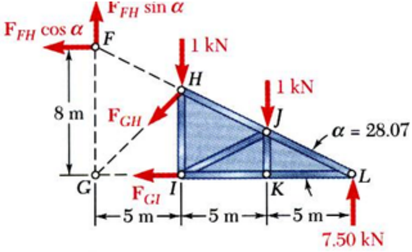
$$\Rightarrow F_{GI} = +13.13\ kN \text{ (T)}$$



F_{FH} is shown as its components. **What one equilibrium equation will determine F_{FH}?**

$$\alpha = tan^{-1}\left(\frac{FG}{GL} = \frac{8\ m}{15\ m}\right) = 28.07^0$$

$\sum M_G = 0 =$
$(7.5\ kN)(15\ m) - (1\ kN)(10\ m) -$
$(1\ kN)(5\ m) + (F_{FH}\cos\alpha)(8\ m),$
$F_{FH} = 13.81\ kN\ (C)$



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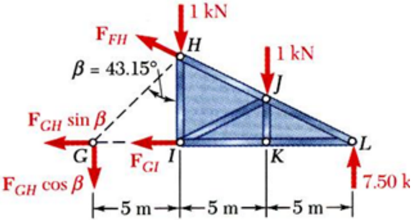
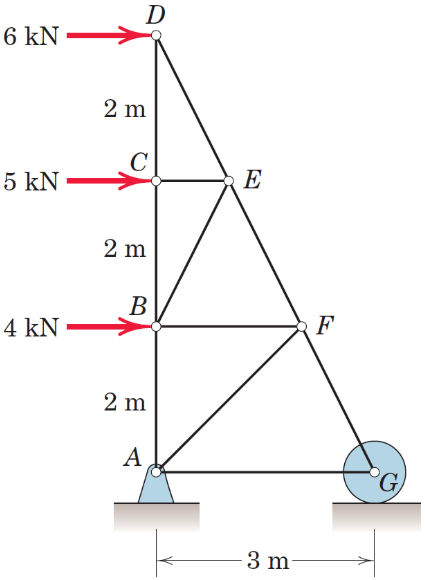


Many options of funding F_{GH}.

$$\beta = tan^{-1}\left(\frac{GI}{HI} = \frac{5\ m}{(2/3)(8\ m)}\right) =$$
43.15^0

$\sum M_L = 0 = (1\ kN)(10\ m) +$
$(1\ kN)(5\ m) + (F_{GH} \cos\beta)(15\ m)$,
$F_{GH} = 1.371\ kN\ (C)$

Solve in Class (I) - Find force in member BE



Solve in Class (II) - Find forces in member BC, BE, BF

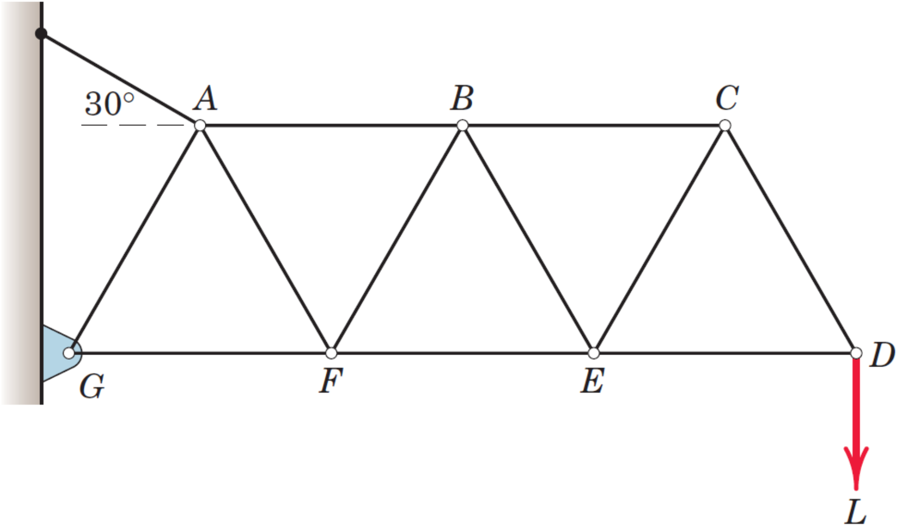
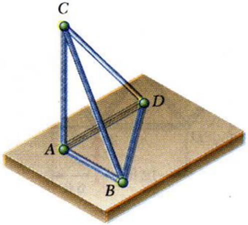
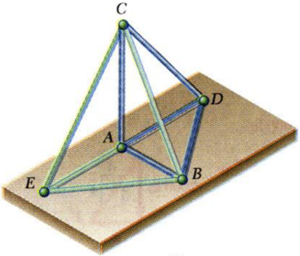


Figure: Triangles are equilateral

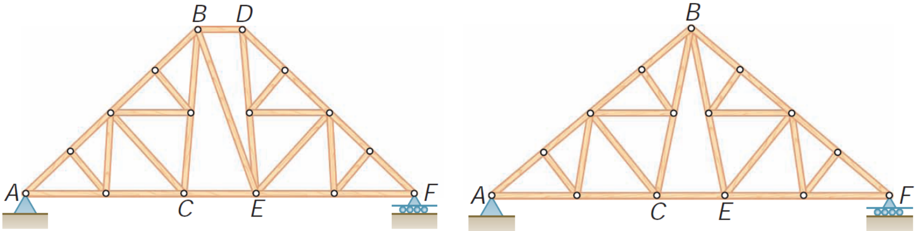
Space Trusses





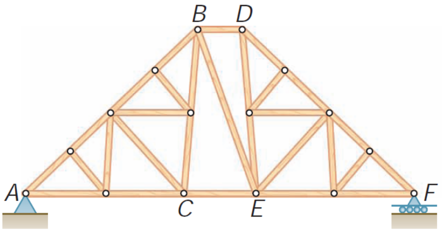
- An elementary space truss consists of 6 members connected at 4 joints to form a tetrahedron.
- A simple space truss is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss, $m = 3n - 6$ where m is the number of members and n is the number of joints.
- Conditions of equilibrium for joints provide $3n$ equations. For simple truss, $3n = m + 6$ and equations can be solved for m member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.

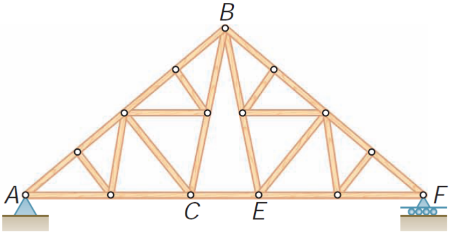
Compound Trusses



- several simple trusses rigidly connected - compound trusses
- Trusses shown are not simple trusses; they cannot be constructed from a triangular truss by adding successive pairs of members.

Compound Trusses





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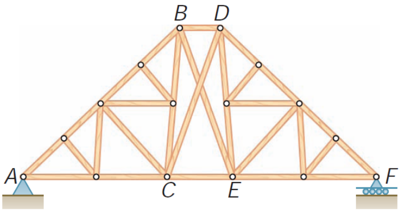
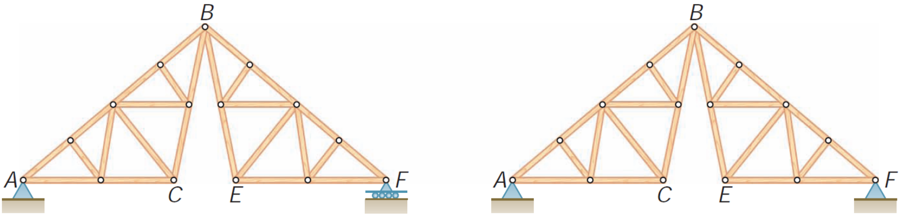


- Compound trusses are statically determinate, rigid, and completely constrained - m = 2n-3

  Statically indeterminate - m>2n-3



(a) (b)

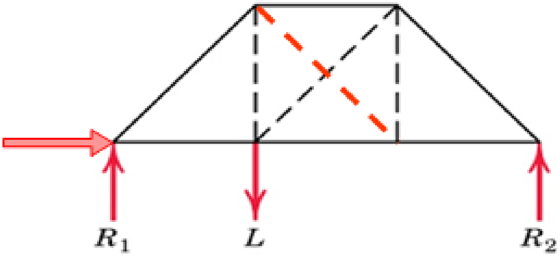
- In (a), m<2n-3. Since truss supported by pin and roller, number of unknowns is m+3. Hence m+3<2n $\Rightarrow$ **Non-rigid**
- In (b), number of unknowns is m+4 and is equal to 2n.
- Necessary but insufficient condition for a compound truss to be statically determinate, rigid, and completely constrained - m+r=2n

When more number of members/supports are present than are needed to prevent collapse/stability

Statically Inderterminate Truss - cannot be analysed using equations of equilibrium alone!

External Redundancy - Extra supports than required. Degree of indeterminacy from available equilibrium equations

Internal Redundancy - Extra Members than required

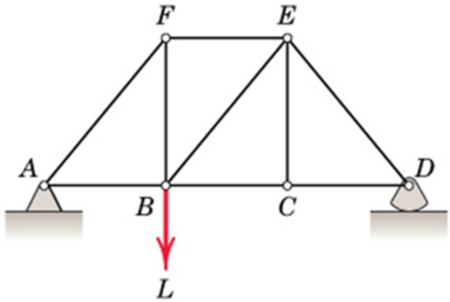


Equilibrium of each joint can be specified by two scalar force equations - $2j$ equations for a truss with "j" number of joints $\Rightarrow$ **KNOWN QUANTITIES**

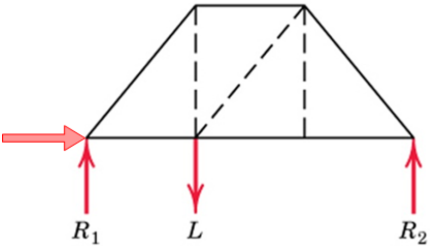
For a truss with "m" number of two force members, and maximum 3 unknown support reactions $\rightarrow$ **Total Unknowns** = m + 3 ("m" member forces and 3 reactions for externally determinate truss)

- $m + 3 = 2j \rightarrow$ Statically Determinate Internally
- $m + 3 > 2j \rightarrow$ Statically Indeterminate Internally
- $m + 3 < 2j \rightarrow$ Unstable Truss

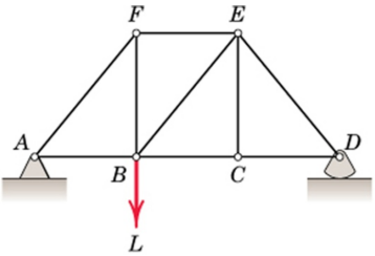
Plane Truss - Determinacy

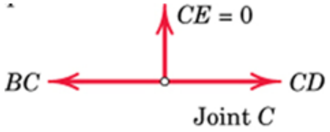


No. of unknown reactions = 3
No. of equilibrium equations = 3
Statically Determinate (External)



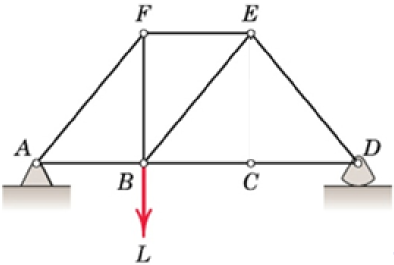
No. of members (m) = 9
No. of joints (j) = 6
No. of unknown reactions = 3
Therefor, **m+R=2j**
Statically Determinate (Internal)

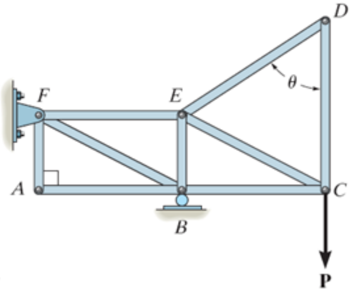


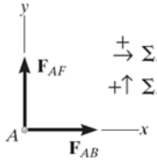


At joint C, summation of forces in y direction gives $F_{CE} = 0$

Simplified Structure $\Rightarrow$

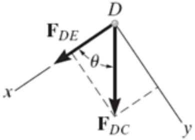






$\xrightarrow{+} \Sigma F_x = 0;\ \boxed{F_{AB} = 0}$

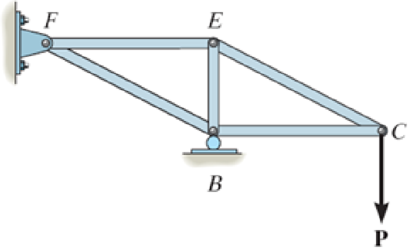
$+\uparrow \Sigma F_y = 0;\ \boxed{F_{AF} = 0}$



$+\searrow \Sigma F_y = 0;\ F_{DC} \sin\theta = 0;\ \boxed{F_{DC} = 0}$ since $\sin\theta \neq 0$

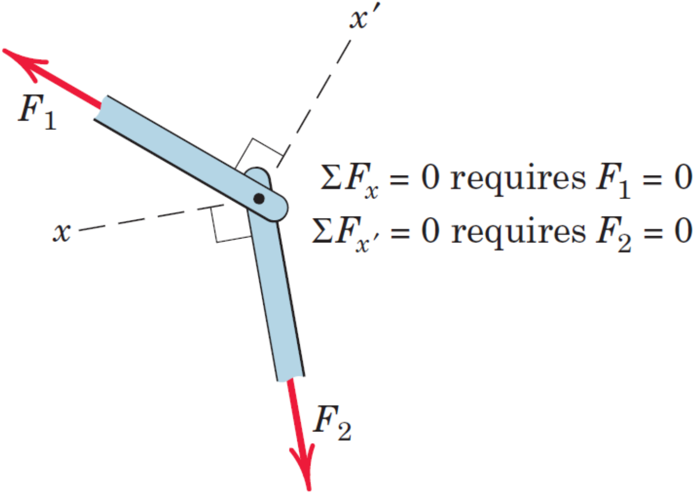
$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\ \boxed{F_{DE} = 0}$

Simplified Structure $\Rightarrow$



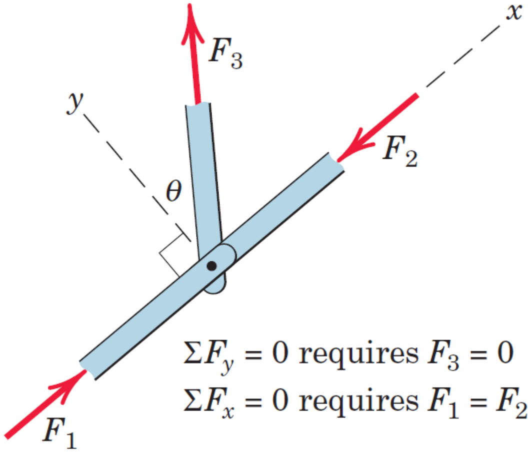
ZERO FORCE MEMBERS - CONDITIONS

If only two noncollinear members form a truss joint and **no external load** or support reaction is applied to the joint, the two members must be zero force members



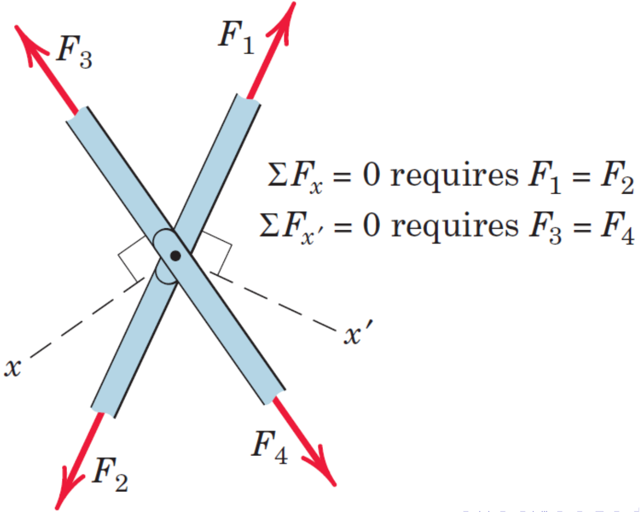
ZERO FORCE MEMBERS - CONDITIONS

If three members form a truss joint for which two of the members are **collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint**

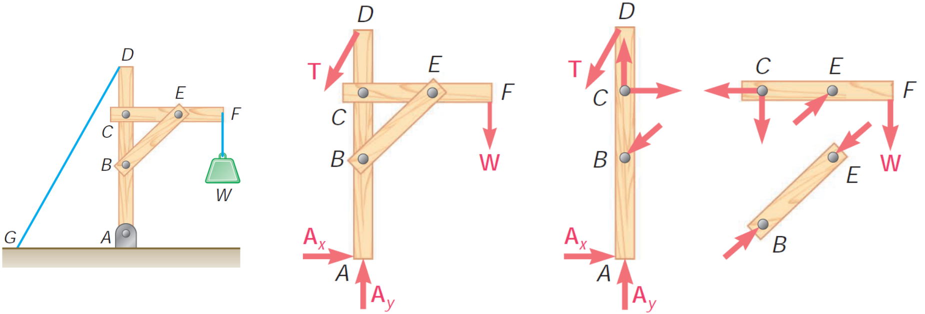


SPECIAL CONDITION

When two pairs of collinear members are joined as shown in figure, the forces in each pair must be equal and opposite.

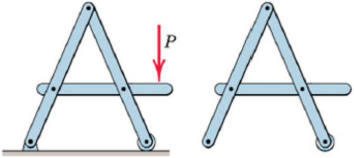


Analysis of a Frame



- Frames and machines are structures with at least one multiforce (>2 forces) member. FBD of complete frame is used to determine external forces acting on frame
- Internal forces are determined by dismembering frame and creating FBDs for each component
- Forces on multiforce members have unknown magnitude and line of action - represented by 2 unknown components

- **Rigid Non-collapsible** - structure constitutes a rigid unit by itself when removed from its supports
- first find all forces external to the structure treated as a single rigid body
- then dismember the structure & consider equilibrium of each part



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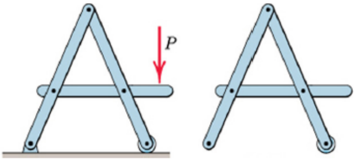
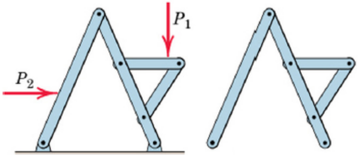
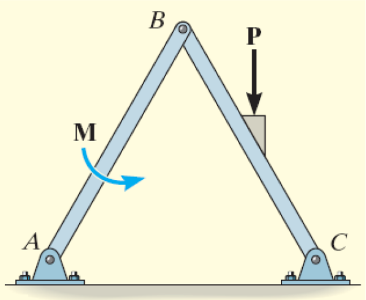
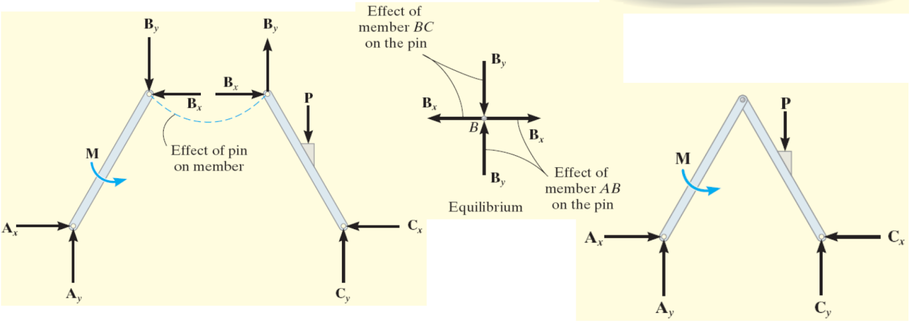


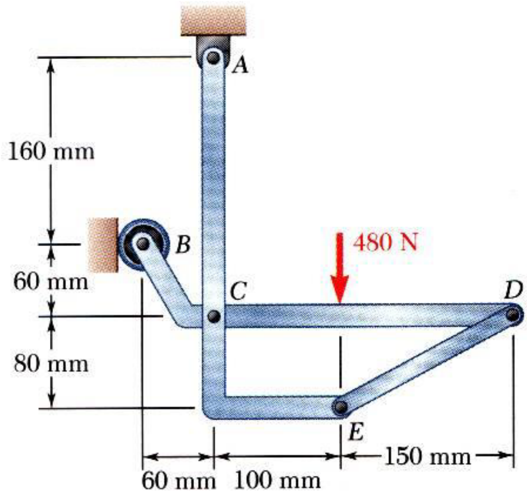
- **Non-Rigid Collapsible** - structure is not a rigid unit by itself but depends on its external supports for rigidity
- calculation of external support reactions cannot be completed until the structure is dismembered and individual parts are analysed



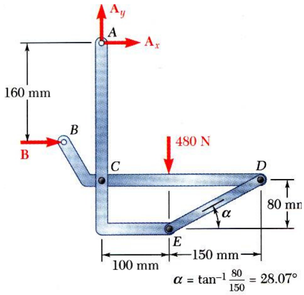
- **FREE BODY DIAGRAMS**
- Draw FBD of (a) Each member, (b) Pin at B, and (c) Whole system







Problem 3 - Members ACE and BCD are connected by a pin at C and by the link DE . For the loading shown, determine the force in link DE and the components of the force exerted at C on member BCD



SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.

$$\sum F_y = 0 = A_y - 480\ \text{N} \qquad \boxed{A_y = 480\ \text{N}\ \uparrow}$$

$$\sum M_A = 0 = -(480\ \text{N})(100\ \text{mm}) + B(160\ \text{mm})$$

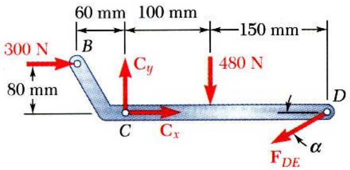
$$\boxed{B = 300\ \text{N}\ \rightarrow}$$

$$\sum F_x = 0 = B + A_x \qquad \boxed{A_x = -300\ \text{N}\ \leftarrow}$$

Note:

$$\alpha = \tan^{-1}\frac{80}{150} = 28.07°$$

- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.



$$\sum M_C = 0 = (F_{DE} \sin\alpha)(250\,\text{mm}) + (300\,\text{N})(60\,\text{mm}) + (480\,\text{N})(100\,\text{mm})$$

$$F_{DE} = -561\,\text{N} \qquad \boxed{F_{DE} = 561\,\text{N}\;\; C}$$

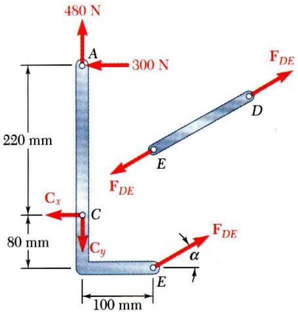
- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

$$\sum F_x = 0 = C_x - F_{DE}\cos\alpha + 300\,\text{N}$$

$$0 = C_x - (-561\,\text{N})\cos\alpha + 300\,\text{N} \qquad \boxed{C_x = -795\,\text{N}}$$

$$\sum F_y = 0 = C_y - F_{DE}\sin\alpha - 480\,\text{N}$$

$$0 = C_y - (-561\,\text{N})\sin\alpha - 480\,\text{N} \qquad \boxed{C_y = 216\,\text{N}}$$



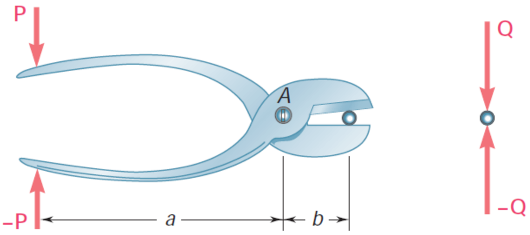
- With member *ACE* as a free-body, check the solution by summing moments about *A*.

$$\begin{aligned}\sum M_A &= (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})\\ &= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0\end{aligned}$$

(checks)

Machines

- Machines are structures designed to transmit and modify forces. Typically they transform input forces (**P**) into output forces (**Q**).
- Given the magnitude of **P**, determine the magnitude of **Q**



(a) (b)

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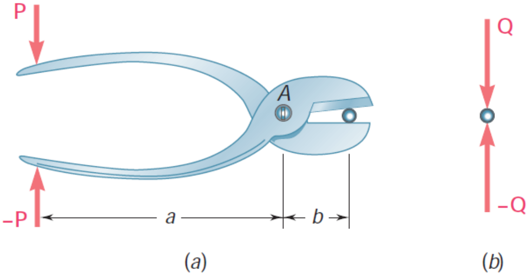
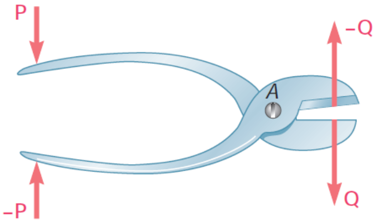
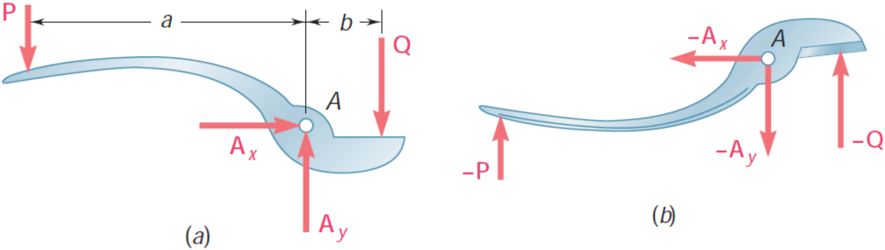


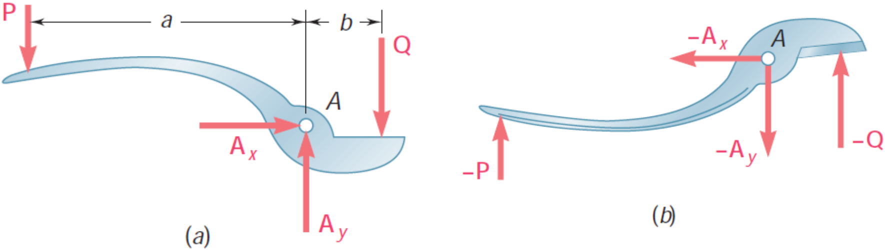
Create a free-body diagram of the complete machine, including the reaction that the wire exerts.





Machines are nonrigid structures. Use one of the components as a free-body.

Sum moments about A, $\sum M_A = 0 = aP - bQ, \Rightarrow Q = \dfrac{a}{b}P$



Machines are nonrigid structures. Use one of the components as a free-body.

Sum moments about A, $\sum M_A = 0 = aP - bQ, \Rightarrow Q = \frac{a}{b}P$

Info on Machines

- Free bodies should be chosen to include input forces and reactions to output forces, and total number of unknown force components should not exceed number of available independent equations.
- It is advisable, before attempting to solve a problem, to determine whether structure considered is determinate

Trusses and frames - Recollection

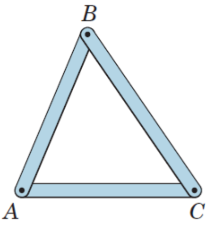


Figure: Truss

- Members - rigid
- Connections between members behave like frictionless pins
- Each member is connected at exactly 2 locations
- External loads are applied ONLY at joints.

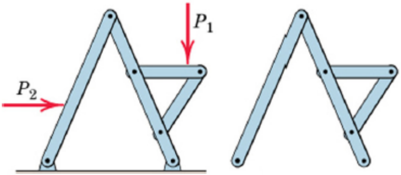
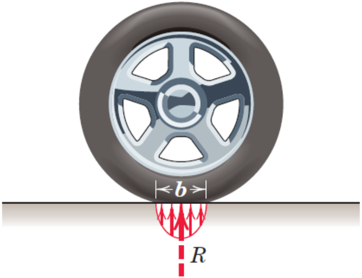


Figure: Frame

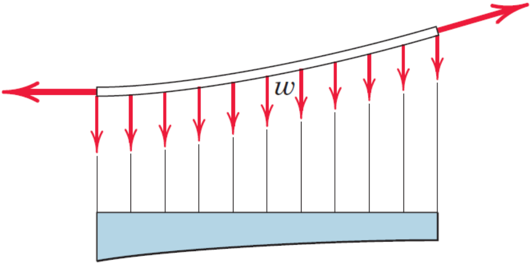
Distributed Forces

- The force exerted by the pavement on an automobile tire is applied to the tire over its entire area of contact, which may be appreciable if the tire is soft
- When analyzing the forces acting on the car as a whole, if the dimension b of the contact area is negligible compared with the other pertinent dimensions, such as the distance between wheels, then we may replace the actual distributed contact forces by their resultant R treated as a concentrated force

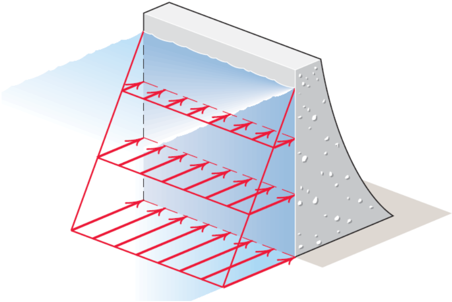


- When forces are applied over a region whose dimensions are not negligible compared with other pertinent dimensions, then we must account for the actual manner in which the force is distributed.
- We do this by summing the effects of the distributed force over the entire region using mathematical integration.
- This requires that we know the intensity of the force at any location.
- There are three categories of such problems

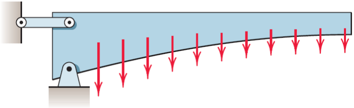
Line Distribution - When a force is distributed along a line, as in the continuous vertical load supported by a suspended cable, the intensity *w* of the loading is expressed as force per unit length of line, newtons per meter (N/m)



Area Distribution - When a force is distributed over an area, as with the hydraulic pressure of water against the inner face of a section of dam, the intensity (Pressure) is expressed as force per unit area



Volume Distribution - A force which is distributed over the volume of a body is called a body force. The most common body force is the force of gravitational attraction, which acts on all elements of mass in a body. The determination of the forces on the supports of the heavy cantilevered structure, would require accounting for the distribution of gravitational force throughout the structure

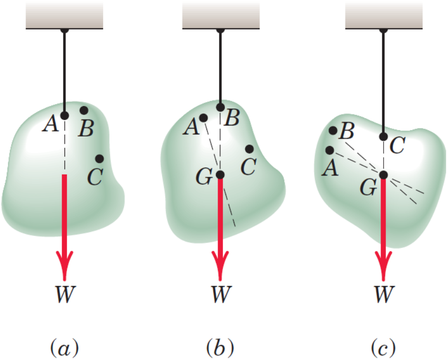


The body force due to the gravitational attraction of the earth (weight) is by far the most commonly encountered distributed force.

How to find the point in a body through which the resultant gravitational force acts - ??

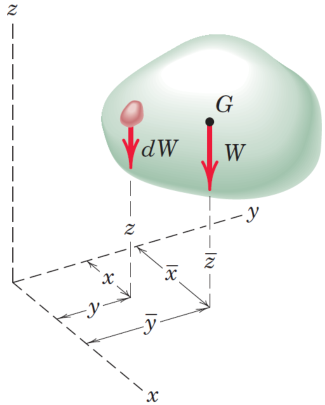
How to find associated geometric properties of lines, areas, and volumes - ??

The body will be in equilibrium under the action of the tension in the cord and the resultant **W** of the gravitational forces acting on all particles of the body **lines of action will be concurrent at a single point G, which is called the center of gravity of the body**



Determining the C.G.

- **Principle of Moments applied**
- The moment of the resultant gravitational force W about any axis equals the sum of the moments about the same axis of the gravitational forces dW acting on all particles treated as infinitesimal elements of the body



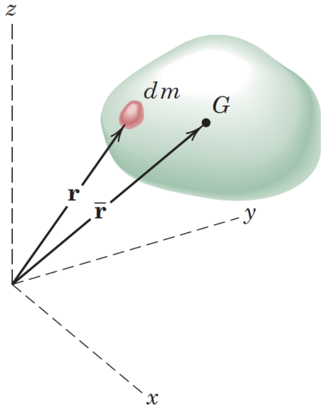
- Resultant of gravitational forces acting on all elements is weight - $W = \int dW$
- Moment Principle - eg. choosing y axis, moment about y axis of elemental weight is xdW and sum for all elements in $\int xdW$. This must equal $W\bar{x}$. Thus $W\bar{x} = \int xdW$

Thus, $\bar{x} = \frac{\int x dW}{W}$. Similarly, $\bar{y} = \frac{\int y dW}{W}$, $\bar{z} = \frac{\int z dW}{W}$.

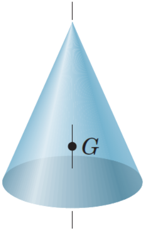
Substitution - $W = mg$ and $dW = g\ dm$,

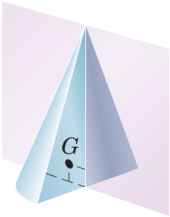
$$\Rightarrow \bar{x} = \frac{\int x dm}{m}, \bar{y} = \frac{\int y dm}{m}, \bar{z} = \frac{\int z dm}{m}$$

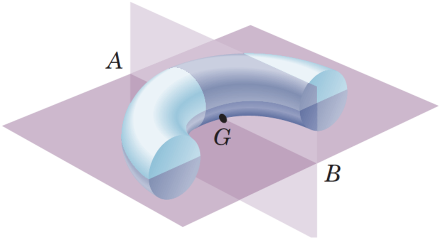
- elemental mass and the mass center G are located by position vectors, $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\bar{\mathbf{r}} = \bar{x}\hat{i} + \bar{y}\hat{j} + \bar{z}\hat{k}$
- Above expressions are components of vector equation $\bar{\mathbf{r}} = \frac{\int \mathbf{r} dm}{m}$
- Since $dm = \rho dV$, we can rewrite above expressions as $\bar{x} = \frac{\int x\rho dV}{\int \rho dV}$



- Above expressions are independent of gravitational effects since g no longer appears. They therefore define a unique point in the body which is a function solely of the distribution of mass.
- This point is called the **center of mass** (C.M.), and clearly it coincides with the center of gravity as long as the gravity field is treated as uniform and parallel
- Choice of reference axes important in calculating C.M.







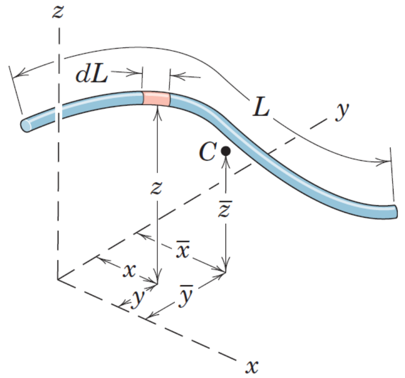
Centroids of Lines, Areas and Volumes

When density ρ of body is uniform throughout - remaining expressions define a purely geometrical property $\Rightarrow$ **CENTROID**

When ρ is uniform throughout, Centroid and Center of mass are identical

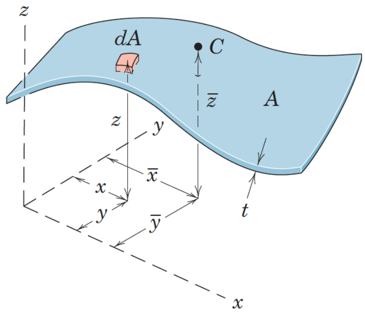
Centroid of lines

- For slender rod or wire of length L, body approximates a line segment - $dm = \rho A dL$.
- If ρ and A are constant over length of the rod, coordinates of C.M. become same as centroid of line segment
- $\bar{x} = \dfrac{\int x dL}{L}$, $\bar{y} = \dfrac{\int y dL}{L}$, $\bar{z} = \dfrac{\int z dL}{L}$



Centroid of Areas

- $dm = \rho t dA$
- If ρ and t are constant over entire area, coordinates of C.M. become same as centroid of area
- $\bar{x} = \dfrac{\int x dA}{A}, \bar{y} = \dfrac{\int y dA}{A}, \bar{z} = \dfrac{\int z dA}{A}$
- $\int x dA$ - **First moment of area**
- Centroids of curved surfaces will (in general) not lie on surface

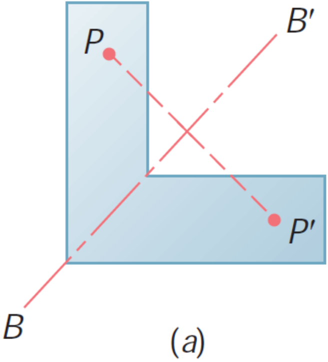


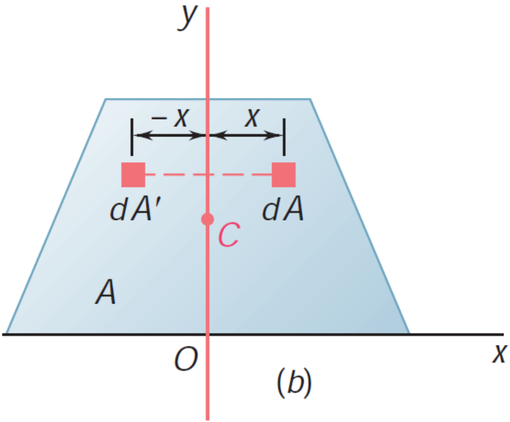
Centroid of Volumes

- $dm = \rho dV$
- If ρ is constant over entire volume, coordinates of C.M. become same as centroid of volume
- $\bar{x} = \dfrac{\int x dV}{V}, \bar{y} = \dfrac{\int y dV}{V}, \bar{z} = \dfrac{\int z dV}{V}$

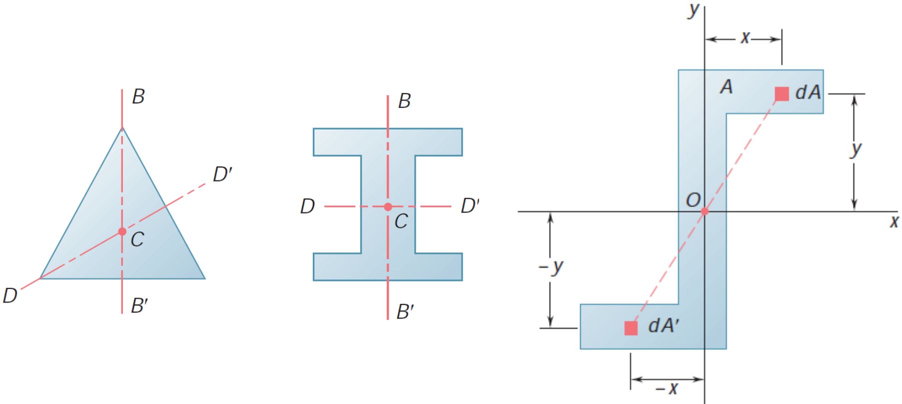
First Moments of Areas

- An area is symmetric with respect to an axis $BB^{'}$ if for every point P there exists a point $P^{'}$ such that $PP^{'}$ is perpendicular to $BB^{'}$ and is divided into two equal parts by $BB^{'}$.

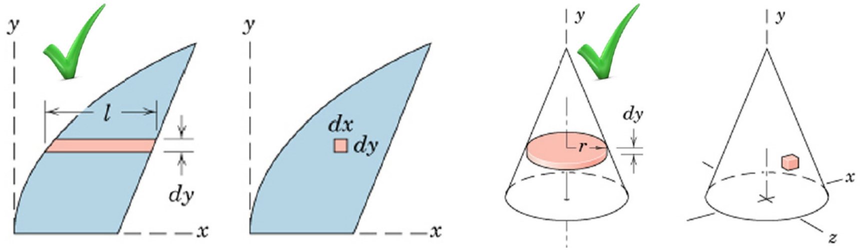




- First moment of an area with respect to line of symmetry is zero.
- If an area possesses a line of symmetry, its centroid lies on that axis.
- If an area possesses two lines of symmetry, its centroid lies at their intersection.
- An area is symmetric with respect to a center O if for every element *dA* at (x,y) there exists an area *dA* of equal area at (-x,-y).
- The centroid of the area coincides with the center of symmetry.

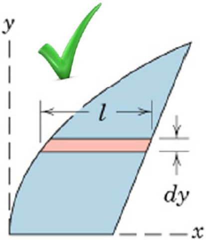


Guidelines for Choice of Elements for Integration

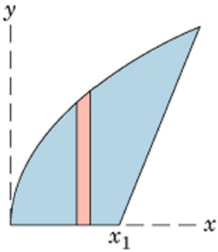


$A = \int dA = \int l\,dy$ $\quad$ $A = \int\int dx\,dy$ $\quad$ $V = \int dV = \int \pi r^2\,dy$ $\quad$ $V = \int\int\int dxdydz$

Order of Element Selected for Integration - A first order differential element should be selected in preference to a higher order element



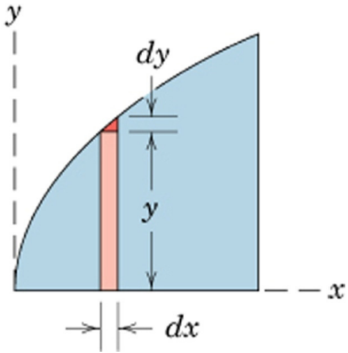
Continuity in the expression for the width of the strip



Discontinuity in the expression for the height of the strip at $x = x_1$

Continuity - Choose an element that can be integrated in one continuous operation to cover the entire figure

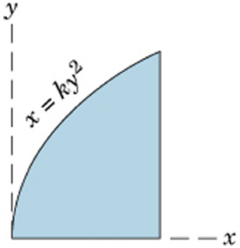
Guidelines for Choice of Elements for Integration



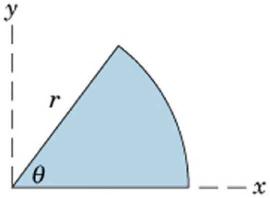
Discarding Higher Order Terms - Higher order terms may always be dropped compared with lower order terms.

Vertical strip of area under the curve is given by first order term $dA = ydx$. The second order triangular area $0.5dxdy$ may be discarded.

Guidelines for Choice of Elements for Integration



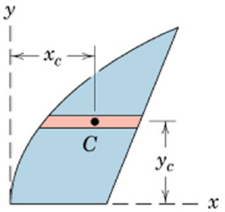
Boundaries of this area (not circular) can be easily described in rectangular coordinates

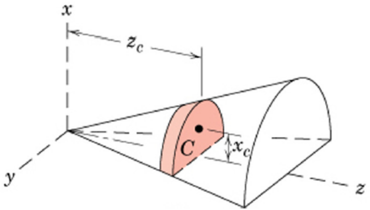


Boundaries of this circular sector are best suited to polar coordinates

Coordinate system should best match the boundaries of the figure

Guidelines for Choice of Elements for Integration





Modified Equations

$$\bar{x} = \frac{\int x_c dA}{A} \quad \bar{y} = \frac{\int y_c dA}{A} \quad \bar{z} = \frac{\int z_c dA}{A}$$

$$\bar{x} = \frac{\int x_c dV}{V} \quad \bar{y} = \frac{\int y_c dV}{V} \quad \bar{z} = \frac{\int z_c dV}{V}$$

While expressing moment of differential elements, take coordinates of the centroid of the differential element as lever arm (not the coordinate describing the boundary of the area)

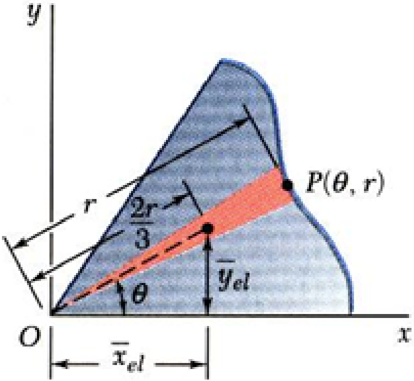
$$\bar{x}A = \int x dA = \int\int x dx dy = \int \bar{x}_{el} dA$$

$$\bar{y}A = \int y dA = \int\int y dx dy = \int \bar{y}_{el} dA$$

Double integration to find the first moment may be avoided by defining dA as a thin rectangle or strip.

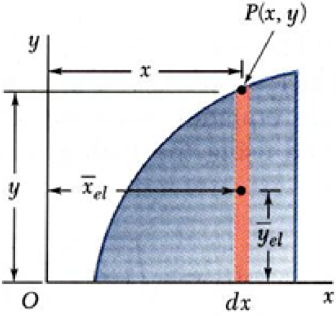
$$\bar{x}A = \int \bar{x}_{el} dA = \int \frac{2r}{3}\cos\theta \left(\frac{1}{2}r^2 d\theta\right)$$

$$\bar{y}A = \int \bar{y}_{el} dA = \int \frac{2r}{3}\sin\theta \left(\frac{1}{2}r^2 d\theta\right)$$



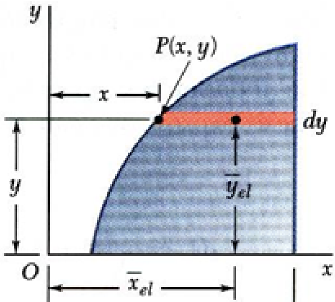
$$\bar{x}A = \int \bar{x}_{el} dA = \int x(ydx)$$

$$\bar{y}A = \int \bar{y}_{el} dA = \int \frac{y}{2}(ydx)$$

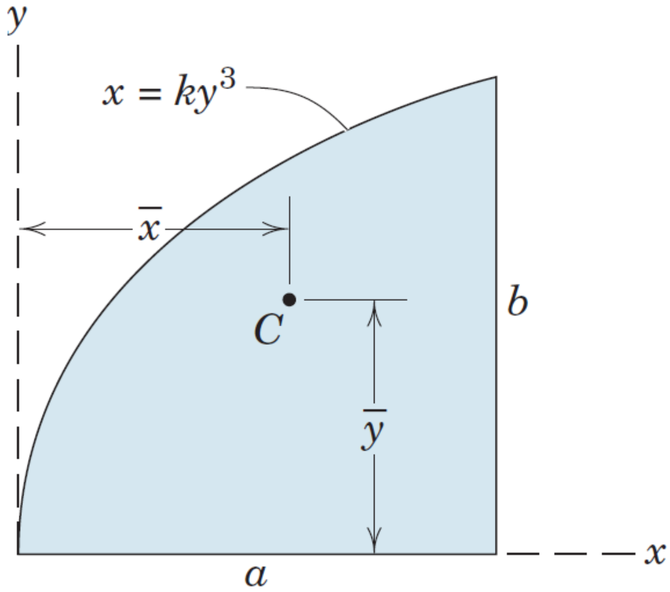


$$\bar{x}A = \int \bar{x}_{el} dA = \int \frac{a+x}{2}[(a-x)dx]$$

$$\bar{y}A = \int \bar{y}_{el} dA = \int y[(a-x)dx]$$



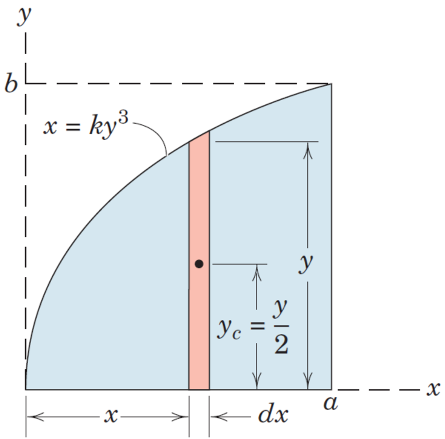
Problem 4 - Locate the centroid of the area under the curve $x = ky^3$ from x = 0 to x = a.



A vertical element of area $dA = ydx$ is chosen. The x-coordinate of the centroid is found from $\bar{x}A = \int x_c dA$

$\Rightarrow \bar{x} \int_0^a ydx = \int_0^a xydx$

Substituting $y = (x/k)^{1/3}$ and $k = a/b^3$ and integrating gives $\frac{3ab}{4}\bar{x} = \frac{3a^2 b}{7} \Rightarrow \bar{x} = \frac{4}{7}a$



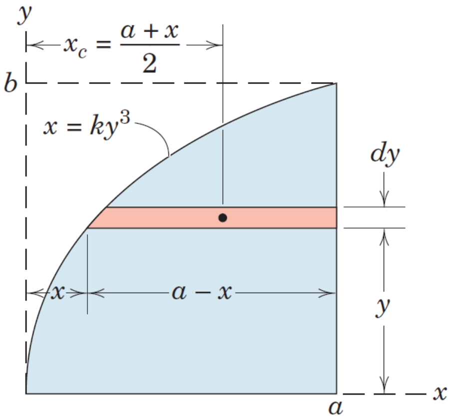
The y-coordinate of the centroid is found from $\bar{y}A = \int y_c dA$

$\Rightarrow \bar{y} \int_0^a ydx = \int_0^a \left(\frac{y}{2}\right) ydx$

Substituting $y = b(x/a)^{1/3}$ and integrating, $\frac{3ab}{4}\bar{y} = \frac{3ab^2}{10} \Rightarrow \bar{y} = \frac{2}{5}b$

A horizontal element of area may be used in place of a vertical element. x-coordinate to centroid of the rectangular element is $x_c = x + (a - x)/2 = (a + x)/2$, which is simply the average of the coordinates a and x of the ends of the strip. Hence, $\bar{x}A = \int x_c dA$

$\Rightarrow \bar{x} \int_0^b (a - x)dy = \int_0^b \left(\frac{a + x}{2}\right)(a - x)dy$

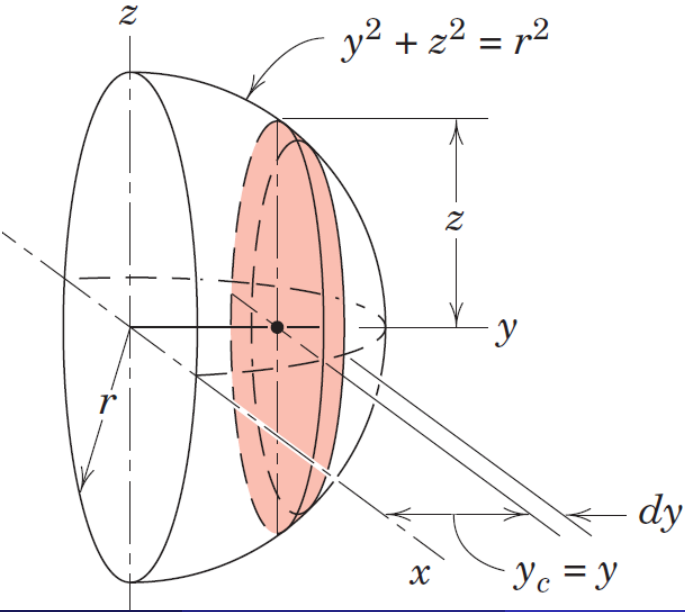


The y-coordinate of the centroid is found from $\bar{y}A = \int y_c dA$

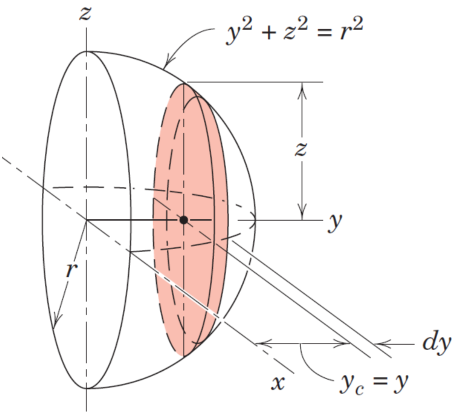
$\Rightarrow \bar{y} \int_0^b (a - x)dy = \int_0^b y(a - x)dy$

Check if answer matches with previously calculated solutions

Problem 5 - Locate the centroid of the volume of a hemisphere of radius r with respect to its base.



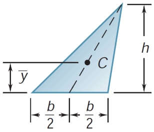
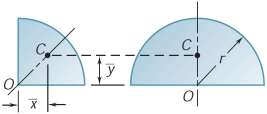
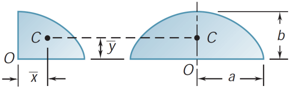
With the axes chosen as shown in the figure, $\bar{x} = \bar{z} = 0$ by symmetry. The most convenient element is a circular slice of thickness dy parallel to the $x - z$ plane. Since the hemisphere intersects the $y - z$ plane in the circle $y^2 + z^2 = r^2$, the radius of the circular slice is $z = +\sqrt{r^2 - y^2}$. The volume of the elemental slice becomes $dV = \pi(r^2 - y^2)dy$



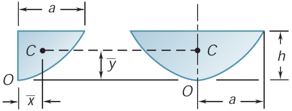
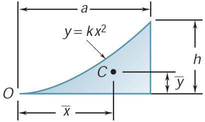
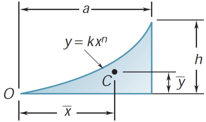
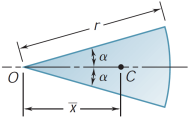
$$\bar{y}V = \int y_c dV, \Rightarrow \bar{y}\int_0^r \pi(r^2 - y^2)dy = \int_0^r y\pi(r^2 - y^2)dy.$$

Integrating gives, $\dfrac{2}{3}\pi r^3\bar{y} = \dfrac{\pi r^4}{4}, \Rightarrow \bar{y} = \dfrac{3}{8}r$

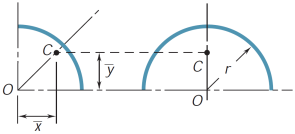
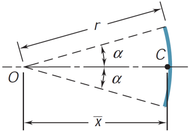
Centroids of common shapes of areas

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

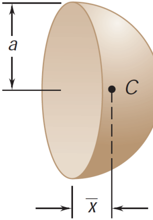
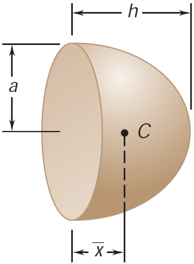
Centroids of common shapes of areas

Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

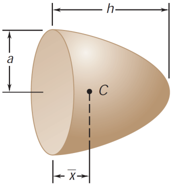
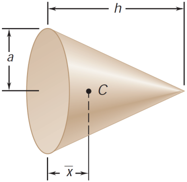
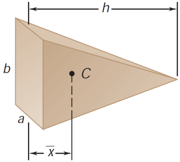
Centroids of common shapes of lines

Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

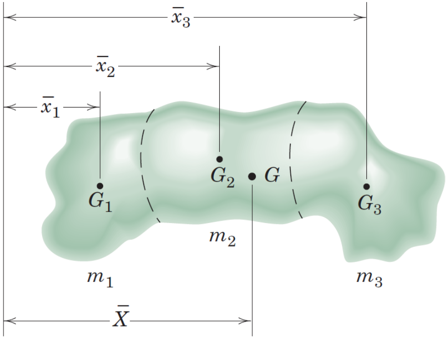
Centroids of common 3D shapes

Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$

Centroids of common 3D shapes

Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$

When a body or figure can be conveniently divided into several parts whose mass centers are easily determined, we use the principle of moments and treat each part as a finite element of the whole. Its parts have masses m_1, m_2, m_3 with the respective mass-center coordinates $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$ in the x-direction.

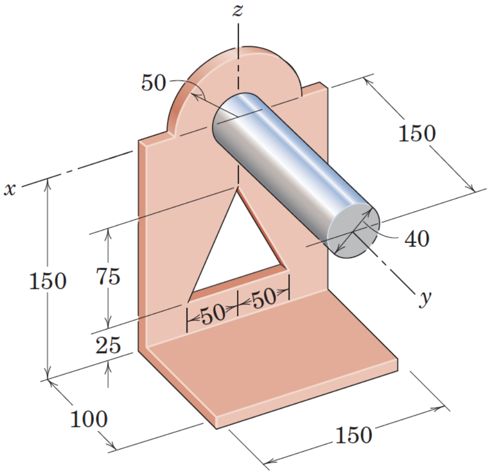


The moment principle gives $(m_1 + m_2 + m_3)\bar{X} = m_1\bar{x}_1 + m_2\bar{x}_2 + m_3\bar{x}_3$

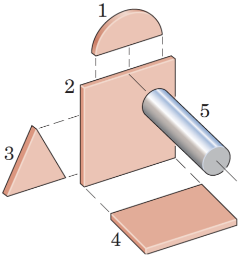
Generalization - $\bar{X} = \dfrac{\sum m\bar{x}}{\sum m}$, $\bar{Y} = \dfrac{\sum m\bar{y}}{\sum m}$, $\bar{Z} = \dfrac{\sum m\bar{z}}{\sum m}$

Analogous relations hold for composite lines, areas, and volumes, where the $m's$ are replaced by $L's$, $A's$, and $V's$, respectively.

Problem 6 - Locate the center of mass of the bracket-and-shaft combination. The vertical face is made from sheet metal which has a mass/unit area of 25 kg/m^2. The material of the horizontal base has a mass/unit area of 40 kg/m^2, and the steel shaft has a density of 7.83 Mg/m^3.



The composite body may be considered to be composed of the five elements. The triangular part will be taken as a negative mass. **For the reference axes indicated it is clear by symmetry that the x-coordinate of the center of mass is zero** $\Rightarrow$ **IS IT TRUE ???**



PART	m (kg)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{z}$ (mm)	$m\bar{x}$ (kg.mm)	$m\bar{y}$ (kg.mm)	$m\bar{z}$ (kg.mm)
1	0.098	0	0	21.2	0	0	2.08
2	0.562	0	0	-75	0	0	-42.19
3	-0.094	0	0	-100	0	0	9.38
4	0.6	0	50	-150	0	30	-90
5	1.476	0	75	0	0	110.7	0
TOTAL	2.642				0	140.7	-120.73

$$\bar{Y} = \frac{\sum m\bar{y}}{\sum m} = 53.3 \text{ mm}, \ \bar{Z} = \frac{\sum m\bar{z}}{\sum m} = -45.7 \text{ mm}$$

Theorems of Pappus-Guldinus

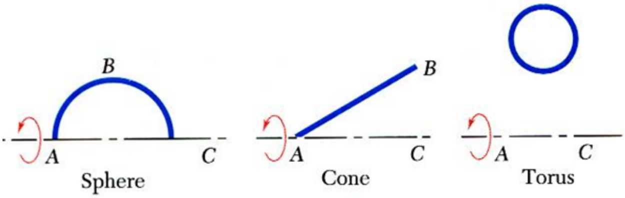


Figure: Surface of revolution is generated by rotating plane curve about fixed axis.

Theorems of Pappus-Guldinus

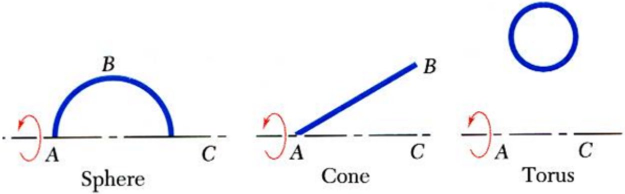
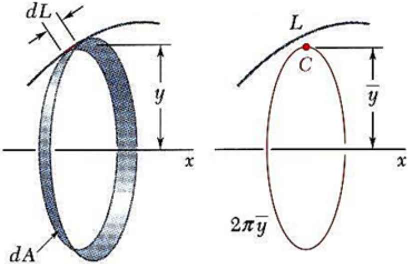


Figure: Surface of revolution is generated by rotating plane curve about fixed axis.

THEOREM I - Area of a surface of revolution is equal to the length of the generating curve times the distance traveled by the centroid through the rotation. $A = 2\pi \bar{y} L$



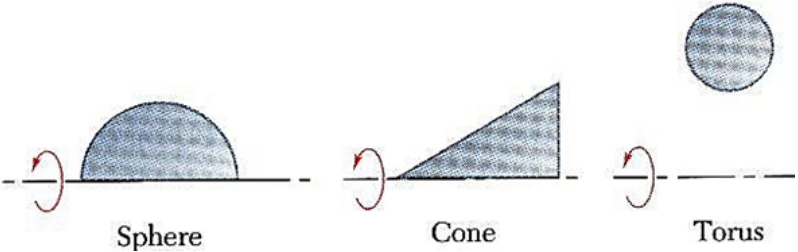


Figure: Body of revolution is generated by rotating plane area about fixed axis.

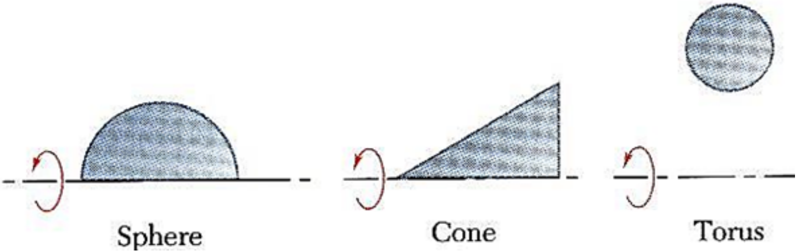
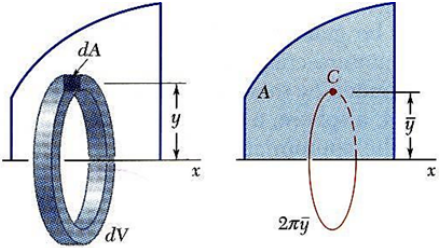
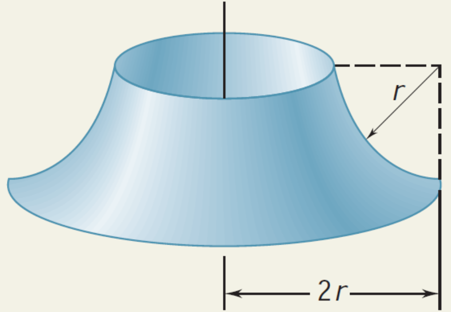


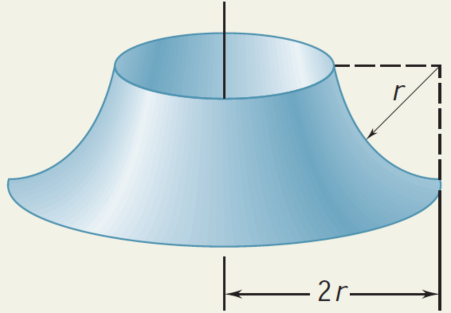
Figure: Body of revolution is generated by rotating plane area about fixed axis.

THEOREM 2 - Volume of a body of revolution is equal to the generating area times the distance traveled by the centroid through the rotation.
$V = 2\pi\bar{y}A$





Problem 7 -Determine the area of the surface of revolution shown, which is obtained by rotating a quarter-circular arc about a vertical axis.



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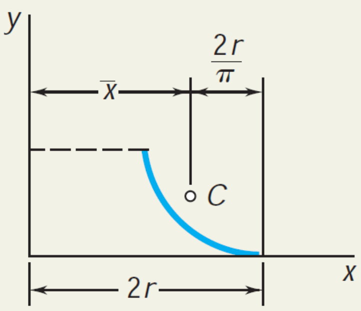
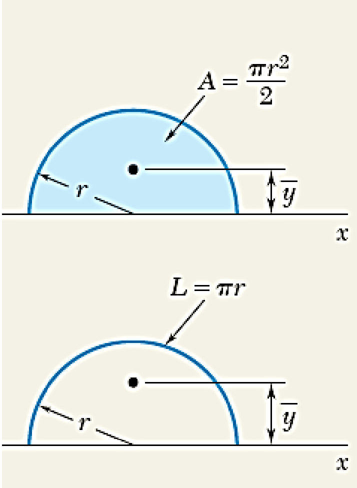


According to Theorem I of Pappus-Guldinus, the area generated is equal to the product of the length of the arc and the distance traveled by its centroid. Hence, $\bar{x} = 2r - \dfrac{2r}{\pi}$ and area $A = 2\pi\bar{x}L =$

$$2\pi\left(2r - \frac{2r}{\pi}\right)\left(\frac{\pi r}{2}\right) = 2\pi r^2(\pi - 1)$$

Problem 8 - Using Theorems of Pappus-Guldinus, determine (a) the centroid of a semicircular area, (b) centroid of a semicircular arc. Recall that the volume and surface area of a sphere are $(4/3)\pi r^3$ and $4\pi r^2$ respectively.

Problem 8 - Using Theorems of Pappus-Guldinus, determine (a) the centroid of a semicircular area, (b) centroid of a semicircular arc. Recall that the volume and surface area of a sphere are $(4/3)\pi r^3$ and $4\pi r^2$ respectively.



Volume of a sphere is the product of the area of a semicircle and distance travelled by centroid in one revolution about x-axis.

Hence, $V = 2\pi\bar{y}A$,

$$\Rightarrow \frac{4}{3}\pi r^3 = 2\pi\bar{y}(\pi r^2/2) \Rightarrow \bar{y} = \frac{4r}{3\pi}$$

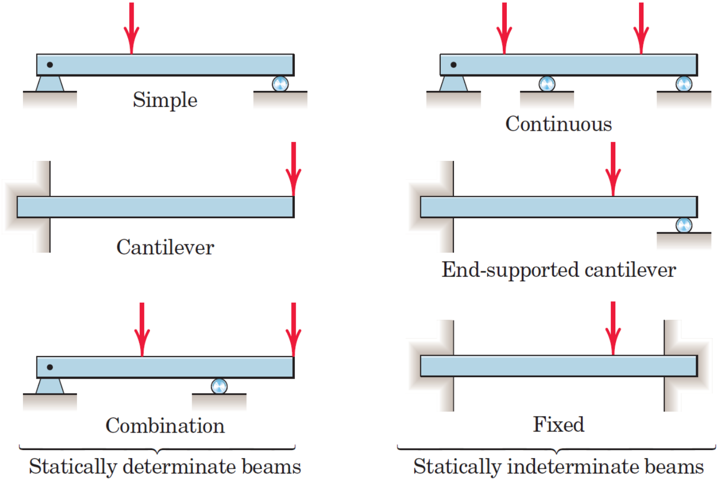
Area of a sphere is the product of the length of a semicircle and distance travelled by centroid in one revolution about x-axis.

Hence, $A = 2\pi\bar{y}L$,

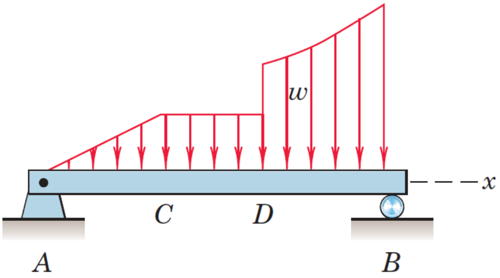
$$\Rightarrow 4\pi r^2 = 2\pi\bar{y}(\pi r) \Rightarrow \bar{y} = \frac{2r}{\pi}$$

BEAMS

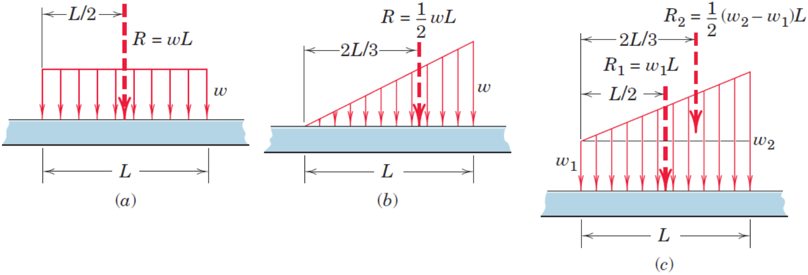
Beams are structural members which offer resistance to bending due to applied loads. Most beams are long prismatic bars, and the loads are usually applied normal to the axes of the bars



Distributed Load on a Beam



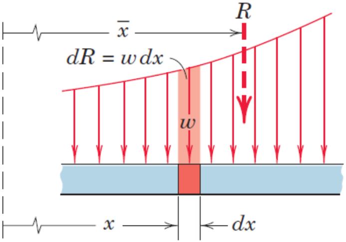
- The intensity w of a distributed load may be expressed as force per unit length of beam.
- The intensity may be constant or variable, continuous or discontinuous
- The intensity of the loading is constant from C to D and variable from A to C and from D to B.
- Intensity is discontinuous at D, where it changes magnitude abruptly.
- Although the intensity itself is not discontinuous at C, the rate of change of intensity dw/dx is discontinuous



- In cases (a) and (b), resultant load R is represented by area formed by the intensity w and the length L over which the force is distributed. The resultant passes through the centroid of this area.
- In part (c), the trapezoidal area is broken into a rectangular and a triangular area, and the corresponding resultants R_1 and R_2 of these subareas are determined separately.
- Note that a single resultant could be determined by using the composite technique for finding centroids. Usually, however, the determination of a single resultant is unnecessary.

General Load Distribution

- Start with a differential increment of force $dR = wdx$.
- The total load R is then the sum of the differential forces, $R = \int wdx$



- Resultant R is located at the centroid of the area under consideration.
- The x-coordinate of this centroid is found by the principle of moments
  $R\bar{x} = \int xwdx, \Rightarrow \ \bar{x} = \dfrac{\int xwdx}{R}$
- For the distribution shown, the vertical coordinate of the centroid need not be found
- Once the distributed loads have been reduced to their equivalent concentrated loads, the external reactions acting on the beam may be found by a straightforward static analysis

Problem 9 - Determine the reactions at A and B for the beam subjected to a combination of distributed and point loads.