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Language Models

CS-585

Natural Language Processing

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Based on slides from Derrick Higgins, Kai Shu

Probabilistic Language Models

Why assign a probability to a sentence or word?

- Machine Translation
  - P("***high*** *winds tonight*") > P("***large*** *winds tonight*")
- Spell checking and text completion
  - *The office is about fifteen* ***minuets*** *from my house*
  - P("*fifteen* ***minutes*** *from*") > P("*fifteen* ***minuets*** *from*")

Probabilistic Language Models

Why assign a probability to a sentence or word? (continued)

- Speech Recognition
  - P("*I saw a van*") >> P("*eyes awe of an*")
- Other Natural Language Generation (NLG) tasks
  - Summarization, question-answering, etc.

Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words:

  $P(W) = P(w_1,w_2,w_3,w_4,w_5 \ldots w_n)$

- Related task: probability of an upcoming word:

  $P(w_5 | w_1,w_2,w_3,w_4)$

- A model that computes either of these:

  $P(W)$ or $P(w_n | w_1,w_2 \ldots w_{n-1})$

  is called a **language model**.

Compute the probability of a sequence

- How to compute this **joint probability** of a sequence of words P(**W**):

  – P(*its, water, is, so, transparent, that*)

- Intuition: Apply the **Chain Rule** of Probability

Reminder: The Chain Rule

- Recall the definition of conditional probabilities

$$\boldsymbol{P(B|A) = \frac{P(A,B)}{P(A)}}$$

Rewriting:

$$\boldsymbol{P(A,B) = P(A)P(B|A)}$$

- More variables:

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$$

- The Chain Rule in General

$$P(x_1, x_2, x_3, \dots, x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2) \dots P(xn|x_1, \dots, x_{n-1})$$

Joint probability of words in sentence

$$P(w_1 w_2 \ldots w_n) = \prod_{i} P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

P("its water is so transparent") =

P(its) × P(water | its)

× P(is | its water)

× P(so | its water is)

× P(transparent | its water is so)

How to estimate these probabilities

- Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

- No! Too many possible sentences!
- We'll never see enough data for estimating these

Markov Assumption

Andrei Markov

- Simplifying assumption:

$$P(\text{the} \mid \text{its water is so transparent that}) \text{ » } P(\text{the} \mid \text{that})$$

- Or maybe

$$P(\text{the} \mid \text{its water is so transparent that}) \text{ » } P(\text{the} \mid \text{transparent that})$$

Markov Assumption

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

- In other words, we approximate each component in the product

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

Simplest case: Unigram model

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

Zero-th order Markov assumption

Some automatically gener

```
fifth, an, of, future
a, the, inflation, most, dollars, quarter, in, is,
mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the
```

Bigram model

- Condition on the previous word:

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \text{ » } P(w_i \mid w_{i-1})$$

First order Markov assumption

```
texaco, rose, one, in, this, issue, is, pursuing, growth, in,
a, boiler, house, said, mr
control, proposal, without                                  d,
fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november
```

N-gram models

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
  - because language has **long-distance dependencies**:

  "The computer(s) which I had just put into the machine room on the fifth floor is (are) crashing."

- But we can often get away with N-gram models

ESTIMATING N-GRAM PROBABILITIES

Estimating bigram probabilities

- The Maximum Likelihood Estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67$ $\quad P(\texttt{Sam} \mid \texttt{<s>}) = \frac{1}{3} = .33$ $\quad P(\texttt{am} \mid \texttt{I}) = \frac{2}{3} = .67$

$P(\texttt{</s>} \mid \texttt{Sam}) = \frac{1}{2} = 0.5$ $\quad P(\texttt{Sam} \mid \texttt{am}) = \frac{1}{2} = .5$ $\quad P(\texttt{do} \mid \texttt{I}) = \frac{1}{3} = .33$

More examples: Berkeley Restaurant Project

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

https://www1.icsi.berkeley.edu/Speech/berp.html

Raw bigram counts

“I want”: 827

“want I”: 2

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

- Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Result:

P(want | I)

P(I | want)

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

P(<s> I want english food </s>) =

P(I | <s>)

× P(want | I)

× P(English | want)

× P(food | english)

× P(</s> | food)

= .000031

What kinds of knowledge?

- P(english | want) = .0011
- P(chinese | want) = .0065
- P(to | want) = .66
- P(eat | to) = .28
- P(food | to) = 0
- P(want | spend) = 0
- P (i | <s>) = .25

Practical Issues

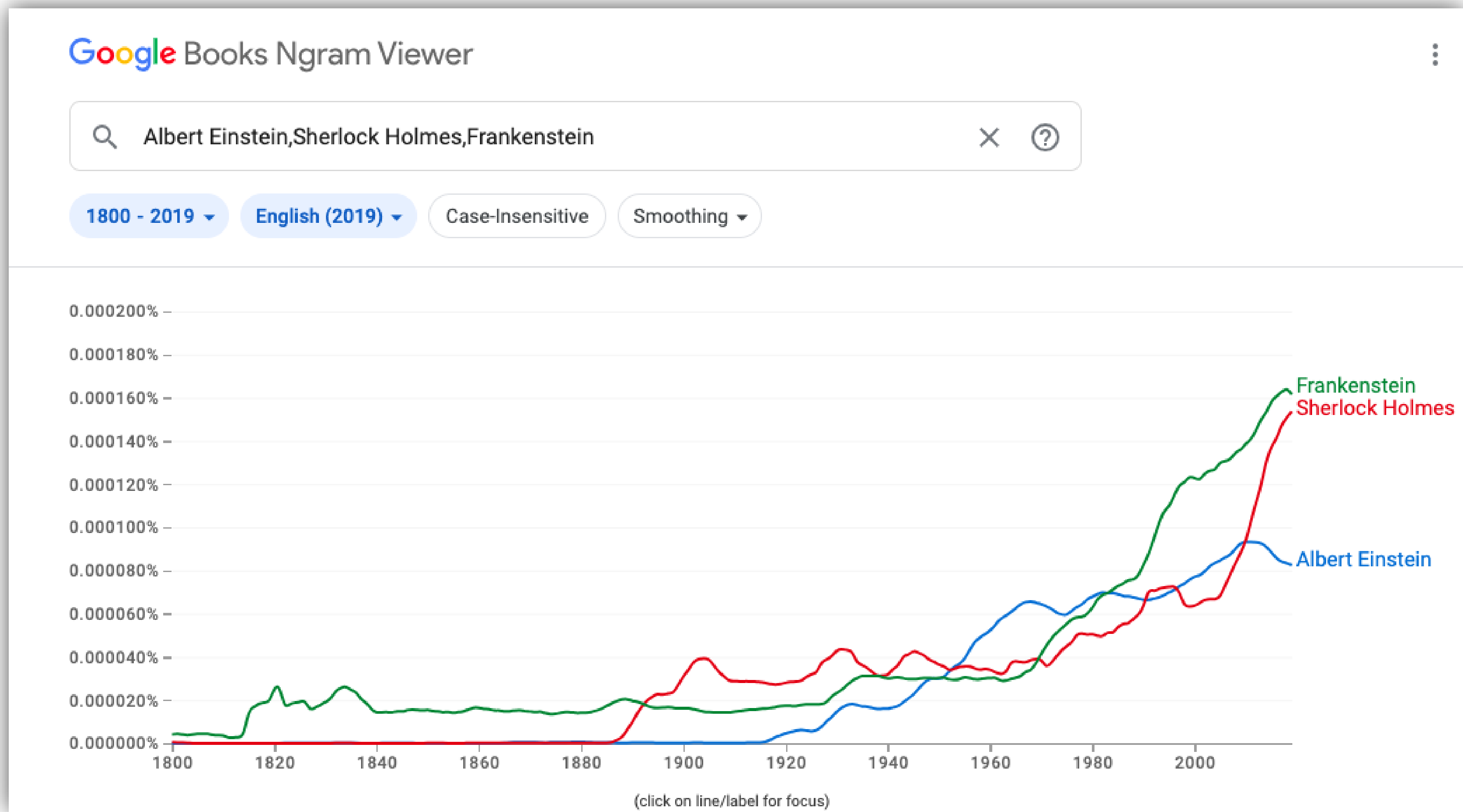
- We do everything in log space
  - Avoid underflow
  - (also adding is faster than multiplying)

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Language Modeling Toolkits

- SRILM (SRI Language Model Toolkit)
  - http://www.speech.sri.com/projects/srilm/
- KenLM (Kenneth Heafield)
  - https://kheafield.com/code/kenlm/
- NLTK Language Modeling Module
  - https://www.nltk.org/api/nltk.lm.html

Google Books Ngrams Viewer



https://books.google.com/ngrams

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EVALUATION AND PERPLEXITY

Evaluation: How good is a model?

- Does our language model prefer good sentences to bad ones?
  - Assign higher probability to "real" or "frequently observed" sentences
- Train model parameters on a **training set**.
- Test model performance on data not seen.
  - A **test set** is an unseen dataset that is different from our training set, totally unused.
  - An **evaluation metric** tells us how well our model does on the test set.

Training on the test set

- We can't allow test sentences into the training set
- We will assign it an artificially high probability when we put it in the test set
- "Training on the test set" →Bad science!

Extrinsic evaluation

- Best evaluation for comparing models A and B
  - Put each model in a task
    - spelling corrector, speech recognizer, MT system
  - Run the task, get an accuracy for A and for B
    - How many misspelled words corrected properly
    - How many words translated correctly
  - Compare accuracy for A and B

Difficulty of extrinsic evaluation

- Extrinsic evaluation
  - Time-consuming; can take days or weeks
- So
  - Sometimes use **intrinsic** evaluation: **perplexity**
  - Bad approximation
    - unless test data looks **just** like the training data
    - So **generally only useful in pilot experiments**

  - But helpful to think about.

Intuition of Perplexity

- The Shannon Game:
  - How well can we predict the next word?

I always order pizza with cheese and ____

The 33rd President of the US was ____

I saw a ____

mushrooms 0.1

pepperoni 0.1

anchovies 0.01

….

fried rice 0.0001

….

and 1e-100

  - Unigrams are terrible at this game. (Why?)
- A better model of a text
  - is one which assigns a **higher probability** to the word that **actually occurs**

Perplexity

IDEA: The best language model is one that:

- Best predicts an unseen test set…
- Gives the highest P(sentence)

Perplexity – transformation of log likelihood from information theory (ENLP-6.42)

$$\text{Perplex}(\boldsymbol{w}) = 2^{-\frac{\ell(\boldsymbol{w})}{M}}$$

M: Number of tokens in held-out dataset

Perplexity

Perplexity – from information theory

$$\text{Perplex}(\boldsymbol{w}) = 2^{-\frac{\ell(\boldsymbol{w})}{M}}$$

- Perfect language model (assigns probability of 1 to held out data) $\rightarrow$ perplexity of 1
- Held out data is assigned probability of zero $\rightarrow$ perplexity is infinite

Lower perplexity = better model

- Training 38 million words, test 1.5 million words, WSJ

N-gram Order	Unigram	Bigram	Trigram
Per-word perplexity	962	170	109

GENERALIZATION

The Shannon Visualization Method

- Choose a random bigram (<s>, w) according to its probability
- Now choose a random bigram (w, x) according to its probability
- And so on until we choose </s>
- Then string the words together

```
<s> I
    I want
      want to
           to eat
              eat Chinese
                  Chinese food
                          food  </s>
I want to eat Chinese food
```

Approximating Shakespeare

1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have –Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. –What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done. –This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in; –It cannot be but so.

https://web.stanford.edu/~jurafsky/slp3/slides/LM_4.pdf

Shakespeare as corpus

- N=884,647 tokens, V=29,066
- Shakespeare produced 300,000 bigram types out of V^2= 844 million possible bigrams.
  - So, 99.96% of the possible bigrams were never seen (have zero entries in the table)
- Quadrigrams worse: What's coming out looks like Shakespeare because it *is* Shakespeare

The Wall Street Journal is not Shakespeare (no offense)

1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
2 gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
3 gram	They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Can you guess the "author" of this random 3-gram sentence?

"You are uniformly charming!" cried he, with a smile of associating and now and then I bowed and they perceived a chaise and four to wish for.

SMOOTHING: ADD-ONE (LAPLACE) SMOOTHING

The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
  - In real life, it often doesn't
  - We need to train robust models that generalize!
- One issue for generalization: Zeros counts
  - OOV words/tokens: occur in test set but not training set ...what will happen?

Zeros

Training set:

… denied the allegations
… denied the reports
… denied the claims
… denied the request

Test set

… denied the offer
… denied the loan

P("offer" | denied the) = 0

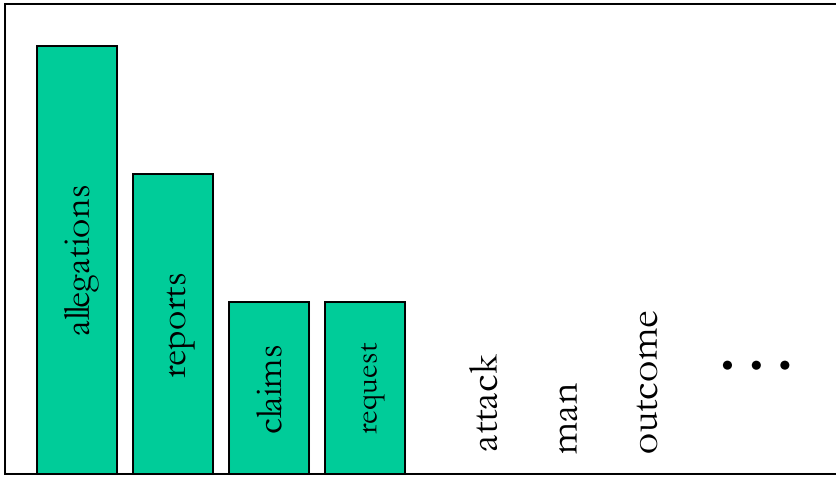
Zero probability bigrams

- Bigrams with zero probability
  - mean that we will assign **0** probability to the test set!
- And hence we cannot compute perplexity (can't divide by 0)!

The intuition of smoothing (from Dan Klein)

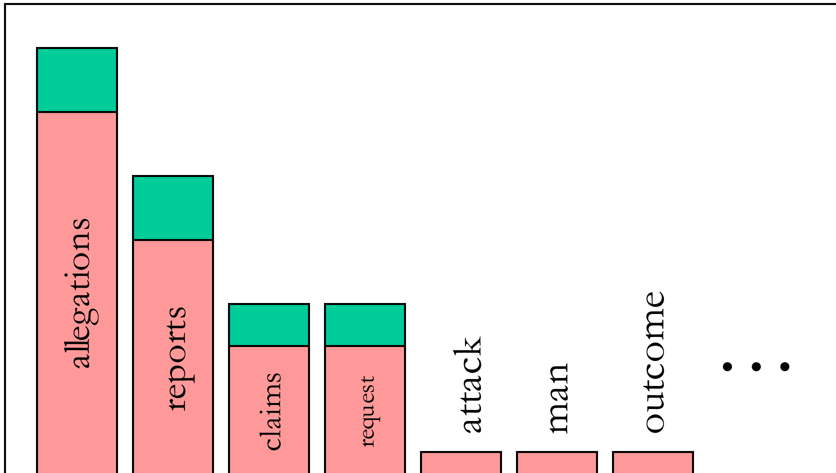
- When we have sparse statistics:

P(w | denied the)
3 allegations
2 reports
1 claims
1 request
7 total



- Steal probability mass to generalize better

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other
7 total



Add-one estimation

- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts!

- MLE estimate:

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add\text{-}1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

Maximum Likelihood Estimates

- The maximum likelihood estimate
  - of some parameter of a model M from a training set T
  - maximizes the likelihood of the training set T given the model M
- Suppose the word "bagel" occurs 400 times in a corpus of a million words
- What is the probability that a random word from some other text will be "bagel"?
- MLE estimate is 400/1,000,000 = .0004
- This may be a bad estimate for some other corpus
  - But it is the **estimate** that makes it **most likely** that "bagel" will occur 400 times in a million word corpus.

Berkeley Restaurant Corpus: Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

https://web.stanford.edu/~jurafsky/icslp-red.pdf

Laplace-smoothed bigrams

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)+1}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Reconstituted counts

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n)+1] \times C(w_{n-1})}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Compare with raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Add-1 estimation is a blunt instrument

- So add-1 isn't generally used for N-grams:
  - Other methods output better estimates
- But add-1 is used to smooth other NLP models
  - For text classification
  - In domains where the number of zeros isn't so huge.

INTERPOLATION AND BACKOFF

Backoff and Interpolation

- Sometimes it helps to use **less** context
  - Condition on less context for contexts you haven't learned much about
- **Backoff:**
  - use trigram if you have good evidence,
  - otherwise bigram, otherwise unigram
- **Interpolation:**
  - mix unigram, bigram, trigram

- Interpolation works better

Linear Interpolation

- Simple interpolation

$$\begin{aligned}\hat{P}(w_n|w_{n-2}w_{n-1}) &= \lambda_1 P(w_n|w_{n-2}w_{n-1}) \\ &+\lambda_2 P(w_n|w_{n-1}) \\ &+\lambda_3 P(w_n)\end{aligned}$$

$$\sum_i \lambda_i = 1$$

- Lambdas conditional on context:

$$\begin{aligned}\hat{P}(w_n|w_{n-2}w_{n-1}) &= \lambda_1(w_{n-2}^{n-1})P(w_n|w_{n-2}w_{n-1}) \\ &+\lambda_2(w_{n-2}^{n-1})P(w_n|w_{n-1}) \\ &+\lambda_3(w_{n-2}^{n-1})P(w_n)\end{aligned}$$

How to set the lambdas?

- Use a **held-out** corpus

Training Data | Held-Out Data | Test Data

- Choose λs to maximize the probability of held-out data:
  - Fix the N-gram probabilities (on the training data)
  - Then search for λs that give largest probability to held-out set:

$$\log P(w_1 \ldots w_n \mid M(\lambda_1 \ldots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \ldots \lambda_k)}(w_i \mid w_{i-1})$$

Unknown Words Token <UNK>

- If we know all the words in advance, so the vocabulary V is fixed
  - Closed vocabulary task
- Often we don't know this
  - **Out Of Vocabulary** = OOV words
  - Open vocabulary task
- Instead: create an unknown word token **<UNK>** that is included in training dataset:
  - Training of <UNK> probabilities
    - Create a fixed lexicon L of size V
    - At text normalization phase, **any training word not in L changed to <UNK>**
    - Now we train its probabilities like a normal word
  - At decoding time
    - If text input: Use UNK probabilities for any word not in training

Kneser-Ney smoothing

- Improved performance over add-one-in → Widely used for n-gram language modeling
- Unigram probabilities depend on how likely to see a word in unfamiliar context
- Computes **continuation probability** proportional to the number of observed contexts in which a word appears
- Example: "*visited Duluth*" *vs* "*visited Francisco*"
  - *"**Francisco**" is more common unigram than "**Duluth**"*
  - *But, "Duluth" appears in many contexts, "Francisco" typically only in one context ("San Francisco")*
  - → **p("*visited Duluth*") > p("*visited Francisco*")**

N-gram Smoothing Summary

- Add-1 smoothing:
  - OK for text categorization, not for language modeling
- Kneser-Ney smoothing is more commonly used
- For very large N-grams like the Web:
  - "Stupid backoff" method – based on counts without discounting
  - https://aclanthology.org/D07-1090.pdf