A Beam Summation Scheme for Ultra-wideband RCS Calculations in the High–frequency Regime

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***Abstract* — An ultra-wideband (UWB) Gaussian beam (GB) summation scheme for calculating the radar cross section (RCS) of arbitrary shaped scatterers is introduced. To allow high frequency approximations, the scatterers are assumed to be piecewise-smooth comprising surfaces with radii of curvature much larger than the wavelength. The formulation is based on the windowed Fourier transform (WFT) frame theory applied to the incident wave on the expansion plane. The incident plane wave is expanded into a frequency independent iso-diffracting Gaussian beam (ID-GB) lattice, emerging from the phase-space (PS) lattice. The choice of iso-diffracting Gaussian window provides the optimal frame representation for all frequencies, thus generating stable and localized expansion coefficients. Furthermore, these beams are fully characterized by frequency independent parameters hence need to be traced and calculated once for all the frequencies in the band. The field of each GB is then tracked through multiple reflections by multiple surfaces within the scatterer domain until the beam emerges out and propagates to the scattering zone, where the beam contributions are summed up to obtain the scattered field, hence eventually radar cross section.**

***Index Terms* — Wave scattering, Gaussian beam, radar cross section.**

I. Introduction

This work is concerned with the RCS calculations for complex targets in the high-frequency regime. Such targets combine, in general large smooth surfaces, regions with intricate details, as well as multiple scattering phenomena. Exact numerical solutions cannot be used in the high-frequency regime due to their high complexity, hence the calculations are usually done by physical optics (PO) integrations or ray methods, or a hybrid combination of these approaches. Two major concerns are the scaling of the complexity of these algorithms with frequency, and their ultra-wide-band (UWB) features, i.e., the additional cost of calculating the RCS not only at a single frequency but over a wide frequency band.

The main advantage of the ray approach is that its numerical complexity remains constant with frequency. Its implementation, however, requires complicated algorithms to identify and classify all the specular points on the targets for all possible illumination and observation directions. There contributions include single and multiple scattering events together with the respective scattering (diffraction) coefficients and the transition function (e.g., between the lit and shadow zones) that becomes quite complicated in tracing the multiple scattering on the complex target. In this approach, intricate details in the targets should be accommodated in hybrid sense via a PO integration.

An alternative ray-based approach is the shooting and bouncing rays (SBR) algorithm. In this approach, one expressed the illuminating fields as a dense lattice of rays and then traces these rays through the target following the basic rules of geometrical optics (GO). This ray shooting algorithm has many advantages over the conventional "ray search" approach described in the previous paragraph but at an expense of its numerical complexity that scales up linearly with frequency.

II. Beam Summation Formulation

The goal of the present research is to formulate a beam summation (BS) scheme for calculating the RCS of large complex targets. Beam summation (BS) methods have long been utilized for modeling wave propagation in complex environments. Several schemes for expanding time-harmonic source-excited fields in terms of a spectrum of beam waves have been introduced in the past [1]. For the purpose of calculating the RCS where the incident field is a plane wave, the appropriate strategy is the so called ultra-wide band phase-space Gaussian beam summation (UWB-PS-BS) scheme introduced originally in [2]-[4] and subsequently used in various applications, (e.g., [5]-[6] and [7]-[9]). In [10], the UWB-PS-BS has been used for modeling indoor propagation. Gaussian beam summation technique has been also used for the analysis of monostatic scattering from objects comprising flat surfaces in [11]-[14].

The Gaussian beam representation is based on windowed Fourier transform (WFT) frame analysis of the incident plane wave distribution on the aperture, which is sufficient to encompass the target domain. The expansion of the incident field gives rise to a lattice of beams emerging from the phase-space (PS) lattice, which indeed depends on the overcompleteness parameter and the beamwidth.

A unique feature of the UWB-PS-BS is the use of the degree of freedom provided by the frame overcompleteness to construct a *frequency independent* beam lattice. Another unique feature is the use of iso-diffracting Gaussian beams (ID-GB) which are optimal in the sense that they are *matched to the lattice for all frequencies*, thus generating a snug frame for all frequencies in the band. These ID-GB are characterized by a frequency independent collimation distance, hence tracking them in the medium is fully characterized by *frequency independent* parameters (e.g., the Beam Collimation and, Curvature Matrices, the Beam coordinate system, and the target geometry) so that they hence need to be calculated and traced only once for all the frequencies in the band.

The interaction between the ID-GBs and the target boundary can be obtained via "ray-tracking" which provides the specular point, the local outward normal at the specular point and the angle of incidence with respect to the specular point. Therefore, once must compute the direction of the reflected beam via Snell's law. Hence establishing the plane of incidence. Therefore, the characteristics traits of the reflected field are thus computed in the plane of incidence. The curvature matrix obtained for the reflected GB field is identical to the conventional geometrical optics transformation for the phase-front's curvature for the time harmonic ray fields as described in [15]. The reflected fields henceforth are treated as the incident GB for the next target interaction before the GB exits the target domain into the scattering domain. These interactions are purely geometrical in nature. Hence the geometrical characteristics traits (The last interaction of a GB with the scatter within the scatterer domain defined as the "exit point", the beam coordinate system) and the beam characteristics traits (complex amplitude that accounts for all the amplitude and the phase accumulation along its corresponding beam propagation axis, complex curvature matrix at the exit point) of all the beams exiting the target domain into the scattering domain are stored into a file. Each beam contributions are expressed in the angular coordinate system in the geographical coordinate system and summed up along with the beams passing in the vicinity of the observation direction with its corresponding expansion coefficient to obtain the RCS of the target.

The elegance of the algorithm lies in the fact that for UWB-PS-BS method, the beam tracking is only to be done once for all the frequencies in the frequency band. This is due to the features of UWB-PS-BS discussed earlier.

The beam algorithm depends on several parameters such as the beam collimation and the beam lattice sparsity that need to be determined by the wave-modeler to optimize the algorithm. The emphasis in this presentation is on the calibration of the algorithm to achieve a given level of accuracy, compared to the PO. The present focus is on calculating the RCS of an acoustically hard sphere. This target is used to calibrate the UWB-PS-BS's expansion parameters to examine more complicated targets that comprise multiple interaction between scatterers (e.g., two adjacent sphere) and compare the results and the frequency scaling of the complexity with those of the PO.

III. Results

The UWB-PS-BS algorithm depends on the expansion parameters that need to be judiciously chosen by the wave-modeler. In order to calibrate and validate the algorithm we will calculate first the UWB bistatic RCS over a one-octave frequency band, of an acoustically hard sphere of the radius $k_{\max}a = 1000$ located at the origin and illuminated by a plane wave propagating along the positive z axis. The RCS is calculated over a one octave bandwidth. We place the "expansion plane" at a plane $z = -a$ so that it is tangent to the sphere. Our algorithm calibration studies have shown that the beam collimation parameter b should be chosen such that $b<a$ and the overcompleteness at the highest frequency should be no grater than $\nu_{max} = 1/3$. Accordingly we choose $k_{\max}b = 750$ and $\nu_{\max} = 0.20$. This fully determine all the other parameters of the algorithm, and in particular the PS beam lattice [2]. For the sake of demonstration, we depict in Fig. 1 the GBs launched form the expansion plane to interact with the sphere. Only a few beams are shown to demonstrate the phenomenology. Recall that the beam algorithm is a priori localized, as only few beams are launched to simulate the incident plane wave, and only part of them pass near the observation point so that they need to be accounted for in the field calculations.

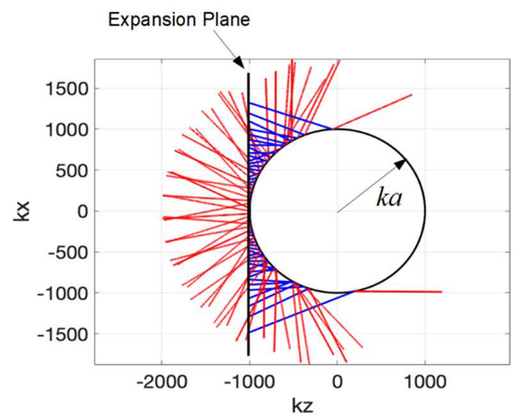

Fig. 1. The incident GBs (blue) launched from the expansion plane and the GBs reflected (red) from a hard sphere located at the origin. (Of all the beams tracked, only few beams are shown).

While computing the bistatic RCS of the sphere, we split the scattered field into its components in the lit zone and the shadow zone of the sphere, specifically reflected fields in the lit zone and the shadow radiation fields in the shadow zone [16], [17]. While reflected fields are computed via the UWB-PS-BS method, the shadow radiation fields are computed via tracking only those incident GB impinging on the sphere while tagging these GBs with a coefficient -1 up to the far zone.

Thus in Figs. 2 and 3, we depicted the reflected field component of the bistatic RCS and the shadow radiation field component of the bistatic RCS as a function of elevation angle θ via UWB-PS-BS (Red solid line) and compared the results with PO integration approximation (Blue dash-dotted line).

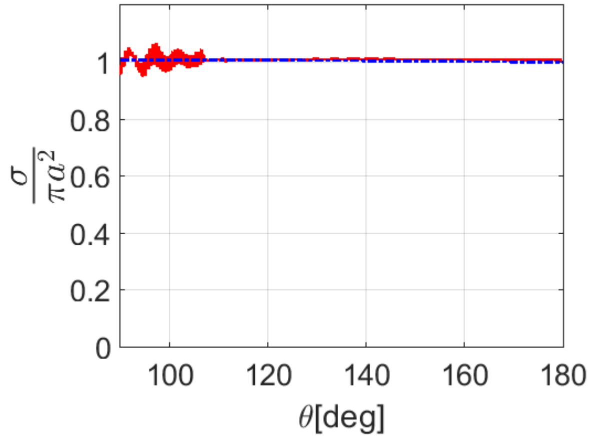

Fig. 2. Reflected field component of the bistatic RCS of an acoustically hard sphere of radius $ka = 1000$ as a function of elevation angle $\theta \in \left[\frac{\pi}{2}, \pi\right]$ computed via PO integration (Blue dash-dotted lines) and UWB-PS-BS (Red solid line) for $kb = 750$.

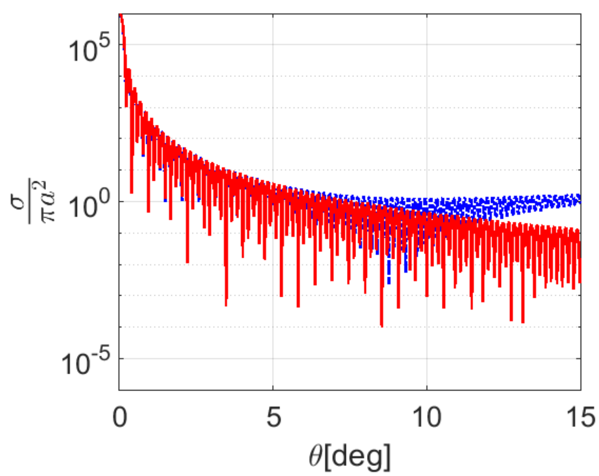

Fig. 3. Shadow radiation field component of the Bistatic RCS of an acoustically hard sphere of radius $ka = 1000$ as a function of elevation angle $\theta \in \left[0, \frac{\pi}{12}\right]$ computed via PO integration (Blue dash-dotted line) and UWB-PS-BS (Red solid line) for $kb = 750$.

IV. Conclusions

A novel discrete UWB-PS-BS representation for the calculation of the far field, specifically the bistatic and monostatic RCS of a smooth complex target was introduced. In the future, our goal is to extend the algorithm for any arbitrary shaped targets described in the CAD geometry (e.g., targets with cavities) while also incorporating the edge effects.

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