Travelling Salesman Problem (TSP)

Introduction

The Travelling Salesman Problem (TSP) is a classical optimization problem in computer science and operations research. In this problem, a salesman must visit a set of cities exactly once and return to the starting city while minimizing the total travel cost. Although the problem is simple to understand, it is computationally difficult to solve optimally for large inputs.

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Problem Definition

Given:

- A set of n cities
- A distance (or cost) between every pair of cities

Objective:

Find a minimum cost tour that visits each city exactly once and returns to the starting city.

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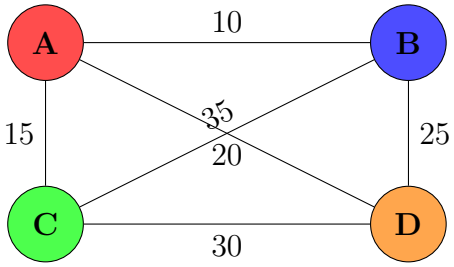
Graph Diagram



TSP Calculation

All possible tours starting from city A are evaluated:

$$A \to B \to C \to D \to A = 95$$
$$A \to B \to D \to C \to A = \mathbf{80}$$
$$A \to C \to B \to D \to A = 95$$
$$A \to C \to D \to B \to A = \mathbf{80}$$
$$A \to D \to B \to C \to A = 95$$
$$A \to D \to C \to B \to A = 95$$

Optimal Solution

Optimal Tour:

$$A \to B \to D \to C \to A$$

Minimum Cost:

$$\boxed{80}$$

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Key Properties of TSP

- Each city is visited exactly once
- The tour must return to the starting city
- The order of visiting cities affects the total cost

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Types of TSP

- **Symmetric TSP:** Distance from city i to j equals distance from j to i
- **Asymmetric TSP:** Distance from city i to j differs from distance from j to i

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Computational Complexity

- Number of possible tours: $(n-1)!$
- Time complexity grows exponentially
- TSP is an **NP-Hard** problem

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Methods to Solve TSP

Exact Methods

- Brute Force Method
- Dynamic Programming (Held–Karp Algorithm)
- Branch and Bound

Approximation and Heuristic Methods

- Nearest Neighbor Algorithm
- Minimum Spanning Tree based approach
- Christofides Algorithm

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Applications of TSP

- Logistics and delivery routing
- Airline scheduling
- Circuit board design
- Network optimization
- Manufacturing planning

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Conclusion

The Travelling Salesman Problem is one of the most important problems in algorithm design. Although finding the optimal solution is computationally expensive, approximation and heuristic methods provide efficient solutions for real-world applications.