Strassen's TC Using Back Substitution

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Given Recurrence Relation
From Strassen's algorithm:

$$T(n) = 7T\left(\frac{n}{2}\right) + cn^2$$

where c is a constant.

Step 1: First Substitution
Substitute $T\left(\frac{n}{2}\right)$:

$$T(n) = 7\left[7T\left(\frac{n}{4}\right) + c\left(\frac{n}{2}\right)^2\right] + cn^2 = 7^2T\left(\frac{n}{4}\right) + \frac{7cn^2}{4} + cn^2$$

Step 2: Second Substitution
Substitute $T\left(\frac{n}{4}\right)$:

$$T(n) = 7^2\left[7T\left(\frac{n}{8}\right) + c\left(\frac{n}{4}\right)^2\right] + \frac{7cn^2}{4} + cn^2 = 7^3T\left(\frac{n}{8}\right) + \frac{7^2cn^2}{16} + \frac{7cn^2}{4} + cn^2$$

Step 3: General Form (After k Substitutions)

$$T(n) = 7^kT\left(\frac{n}{2^k}\right) + cn^2\left(1 + \frac{7}{4} + \frac{7^2}{4^2} + \cdots + \frac{7^{k-1}}{4^{k-1}}\right)$$

Step 4: Stop Condition
Recursion stops when:

$$\frac{n}{2^k} = 1 \Rightarrow k = \log_2 n$$

Step 5: Substitute k

$$T(n) = 7^{\log_2 n}T(1) + cn^2\sum_{i=0}^{\log_2 n-1}\left(\frac{7}{4}\right)^i$$

Step 6: Simplification
First term:

$$7^{\log_2 n} = n^{\log_2 7}$$

Second term (geometric series):

$$\sum_{i=0}^{\log_2 n-1} \left(\frac{7}{4}\right)^i = O\left(\left(\frac{7}{4}\right)^{\log_2 n}\right) = O\left(n^{\log_2 7-2}\right)$$

Multiplying by cn^2:

$$O\left(n^{\log_2 7}\right)$$

Step 7: Final Time Complexity

$$T(n) = O\left(n^{\log_2 7}\right) \approx O\left(n^{2.81}\right)$$