Intro to Robotics Assignment-4
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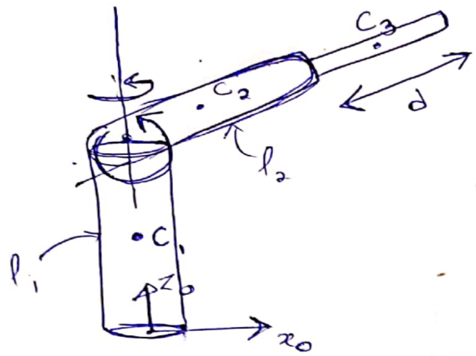
Stanford Manipulator



$$C_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{l_1}{2} \end{bmatrix}$$

$$C_2 = \begin{bmatrix} \frac{l_2}{2}\cos q_2 \cos q_1 \\ \frac{l_2}{2}\cos q_2 \sin q_1 \\ l_1 + \frac{l_2}{2}\sin q_2 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} (\frac{d}{2} + l_2)\cos q_2 \cos q_1 \\ (\frac{d}{2} + l_2)\cos q_2 \sin q_1 \\ l_1 + (l_2 + \frac{d}{2})\sin q_2 \end{bmatrix}$$

$$V_{c_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{d} \end{bmatrix} \Rightarrow J_{v_{c_1}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

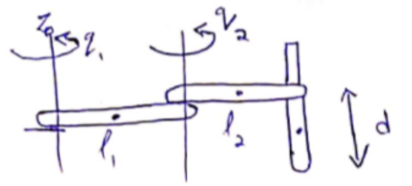
$$J_{v_{c_2}} = \begin{bmatrix} -\frac{l_2}{2}\sin q_1 \cos q_2 & -\frac{l_2}{2}\cos q_1 \sin q_2 & 0 \\ \frac{l_2}{2}\cos q_1 \cos q_2 & -\frac{l_2}{2}\sin q_1 \sin q_2 & 0 \\ 0 & \frac{l_2}{2}\cos q_2 & 0 \end{bmatrix}$$

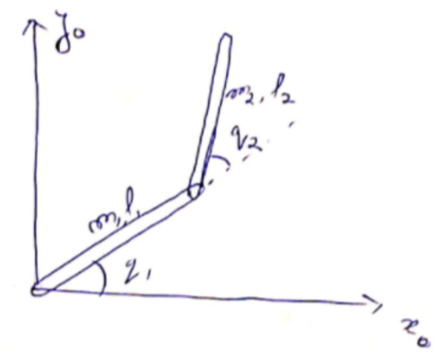
$$J_{v_{c_3}} = \begin{bmatrix} -(\frac{d}{2} + l_2)\sin q_1 \cos q_2 & -(\frac{d}{2} + l_2)\cos q_1 \sin q_2 & \frac{1}{2}\cos q_1 \cos q_2 \\ (\frac{d}{2} + l_2)\cos q_1 \cos q_2 & -(\frac{d}{2} + l_2)\sin q_1 \sin q_2 & \frac{1}{2}\sin q_1 \cos q_2 \\ 0 & (l_2 + \frac{d}{2})\cos q_2 & \frac{1}{2} \times \sin q_2 \end{bmatrix}$$

$$D(q) = m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}} + m_3 J_{v_{c_3}}^T J_{v_{c_3}} + \begin{bmatrix} I_1 + I_2 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

for stanford $\quad I_1 \approx 0 \,, \quad I_2 = m_2 \frac{l_2^2}{3}$

Scara Robot





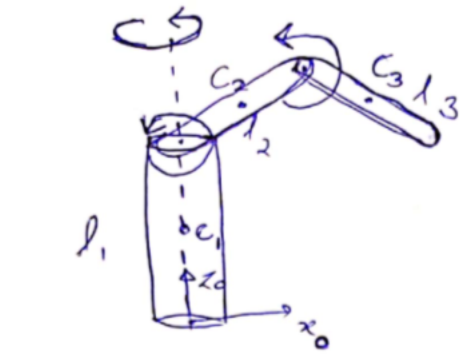
$$J_{v_{c_1}} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 & 0 & 0 \\ \frac{l_1}{2}\cos q_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_{v_{c_2}} = \begin{bmatrix} -l_1 \sin q_1 - \frac{l_2}{2}\sin(q_1+q_2) & -\frac{l_2}{2}\sin(q_1+q_2) & 0 \\ l_1\cos q_1 + \frac{l_2}{2}\cos(q_1+q_2) & \frac{l_2}{2}\cos(q_1+q_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_{v_{c_3}} = \begin{bmatrix} -l_1 \sin q_1 - l_2 \sin(q_1+q_2) & -l_2\sin(q_1+q_2) & 0 \\ l_1\cos q_1 + l_2\cos(q_1+q_2) & l_2\cos(q_1+q_2) & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$$D(q) = m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}} + m_3 J_{v_{c_3}}^T J_{v_{c_3}} + \begin{bmatrix} I_1 + I_2 & I_2 & 0 \\ I_2 & I_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Puma Manipulator



$$C_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{l_1}{2} \end{bmatrix}$$

$$C_2 = \begin{bmatrix} \frac{l_2}{2} \cos q_2 \cos q_1 \\ \frac{l_2}{2} \cos q_2 \sin q_1 \\ l_1 + \frac{l_2}{2} \sin q_2 \end{bmatrix} \qquad C_3 = \begin{bmatrix} \left(\frac{l_3}{2} \cos q_3 + l_2\right) \cos q_2 \cos q_1 \\ \left(\frac{l_3}{2} \cos q_3 + l_2\right) \cos q_2 \sin q_1 \\ l + l_2 \sin q_2 + \frac{l_3}{2} \sin q_3 \end{bmatrix}$$

$$J_{v_{c_1}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_{v_{c_2}} = \begin{bmatrix} -\frac{l_1}{2} \sin q_1 \cos q_2 & -\frac{l_2}{2} \cos q_1 \sin q_2 & 0 \\ \frac{l_2}{2} \cos q_1 \cos q_2 & -\frac{l_2}{2} \sin q_1 \sin q_2 & 0 \\ 0 & \frac{l_2}{2} \cos q_2 & 0 \end{bmatrix}$$

$$J_{v_{c_3}} = \begin{bmatrix} -(\frac{l_3}{2}\cos q_3 + l_2)\sin q_1 \cos q_2 & -(\frac{l_3}{2}\cos q_3 + l_2)\cos q_1 \sin q_2 & -\frac{l_3}{2}\cos q \cos q_2 \sin q_3 \\ (\frac{l_3}{2}\cos q_3 + l_2)\cos q_1 \cos q_2 & -(\frac{l_3}{2}\cos q_3 + l_2)\sin q_1 \sin q_2 & -\frac{l_3}{2}\sin q_1 \cos q_2 \sin q_3 \\ 0 & l_2 \cos q_2 & \frac{l_3}{2}\cos q_3 \end{bmatrix}$$

$$J_{\omega_1} = \dot{q}_1 \hat{k} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_{\omega_2} = \dot{q}_2 \hat{j} + \dot{q}_1 \hat{k} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_{\omega_3} = (\dot{q}_3 + \dot{q}_2)\hat{j} + \dot{q}_1 \hat{k} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$I_1 \approx 0 \quad , \quad I_2 = \frac{m_2 l_2^2}{3} \quad , \quad I_3 = \frac{m_3 l_3^2}{3}$$

$$D(q) = m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}} + m_3 J_{v_{c_3}}^T J_{v_{c_3}} + \begin{bmatrix} I_1+I_2+I_3 & 0 & 0 \\ 0 & I_2+I_3 & I_3 \\ 0 & I_3 & I_3 \end{bmatrix}$$

PI Compensator

$$C(s) = k_P + \frac{k_I}{s}$$

Closed loop system, $\Theta_m(s) = \frac{(K_p s + K_I)\Theta^d(s)}{\Omega_2(s)} - \frac{r s}{\Omega_2(s)} D(s)$

$\Omega_2(s) = J_{eff} s^3 + (B_{eff}) s^2 + K K_p s + K K_I$

Let us take constant desired angles Θ^d and constant disturbance $d = m_{total} g$.

$$\Theta_m(s) = \frac{1}{s}\left(\frac{(K_p s + K_I)\Theta^d}{\Omega_2(s)} - \frac{r s \times d}{\Omega_2(s)} \right)$$

$$= \frac{1}{s}\left(\frac{(K_p \Theta_d - d r)s + K_I \Theta_d}{\Omega_2(s)} \right)$$

applying the routh criterion to $\Omega_2(s)$, it follows that the closed loop system is stable if the gains are positive. and in addition

$$\boxed{K_I < \frac{B_{eff}}{J_{eff}} \times K_p}$$