Assignment-3 (Robotics)

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Ans-1

$$\dot{X} = J(q)\,\dot{q}$$

the solution $\dot{X}$ depends on the Jacobian $J(q)$. Jacobian $J(q)$ is a function of joint angle q. Joint angles q can configure such a way that reduces the rank of the matrix. Those configurations are termed as singularities or singular configurations.

Reasons for identifying manipulator singularities:

1. Certain directions of motions that are unattainable can be known.
2. At singularities bounded gripper velocities, forces and torques may correspond to unbounded joint, velocities and torques respectively.
3. Singularities most of the time correspond to points of the maximum reach of the manipulator in the manipulator workspace.
4. They also correspond to unreachable points on the manipulator workspace under normal conditions.
5. Near singularities, there may be no solution or infinitely many solutions.

The singular configuration can be decoupled into two singularities.

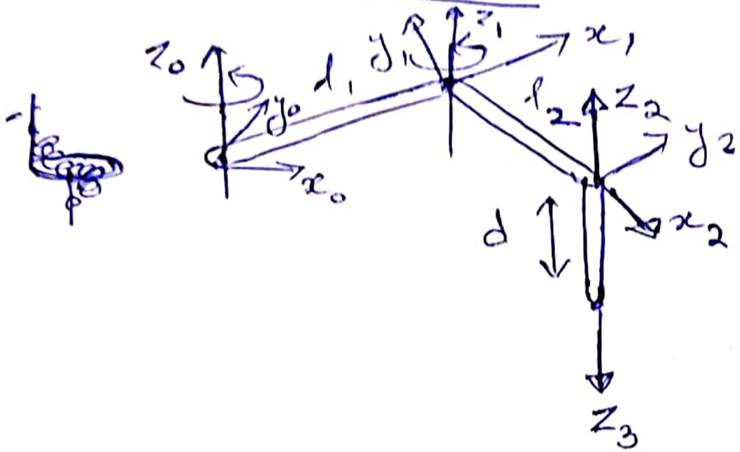
① Arm singularities

② Wrist singularities

Singular configuration can be found by making $\det(J(q)) = 0$.

Singular configuration can be determined by taking a set of arm singularity and wrist singularity and making their determinant 0.

Ans-4 | RRP Scara Robot



DH parameters

link	θ_i	d_i	a_i	α_i
1	q_1	0	l_1	0
2	q_2	0	l_2	0
3	0	$-d$	0	180°

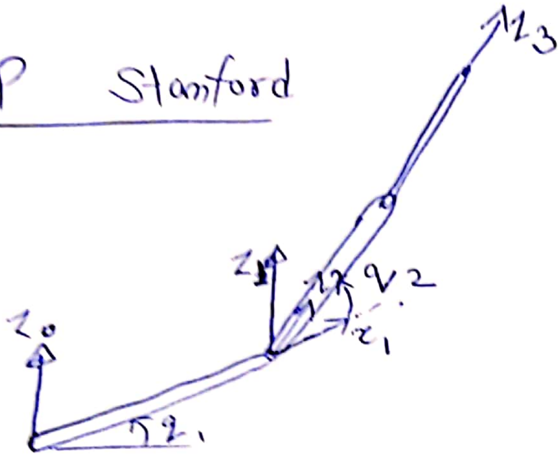
Let $q_1 = 45^\circ$, $q_2 = 90^\circ$, $l_1 = 10$, $l_2 = 10$, $d = 25$

then using Code – End-effector position – $[0, 14.14, -25]$

$$J = \begin{bmatrix} -14.14 & -7.07 & 0 \\ 0 & -7.07 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

these values matches with textbook derivations and my previously derived expressions.

RRP Stanford



D-H parameters

θ_i	d_i	a_i	α_i
q_1	0	l_1	0
$-(\frac{\pi}{2}-q_1)$	0	0	$-\frac{\pi}{2}$
0	l_2+d	0	0

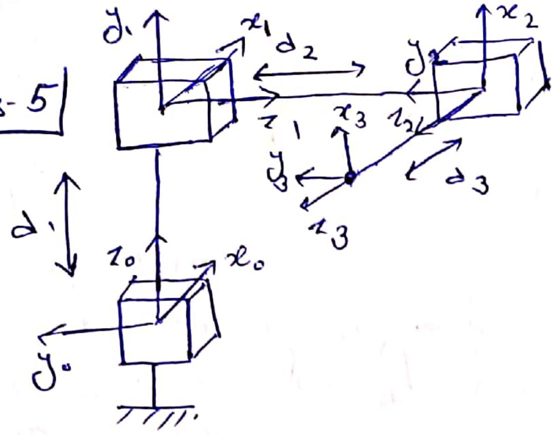
let $l_1 = 5$, $l_2 = 10$, $q_1 = 45$, $q_2 = 45$ & $d = 20$

then using code End-effector position = [3.53, 33.56, 0]

$$J = \begin{bmatrix} -33.53 & -30 & 0 \\ 3.56 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

these values matches with textbook derivations and my previously derived expressions.

Ans-5



DH Parameters

Link	θ_i	d_i	a_i	α_i
1	0	d_1	0	90°
2	90	d_2	0	-90°
3	0	d_3	0	0

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

putting DH parameters in this equation.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

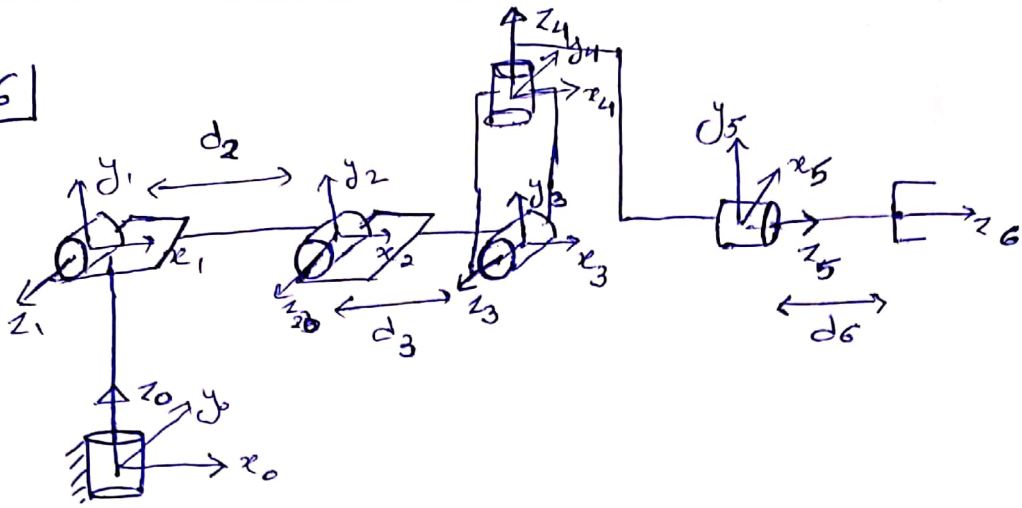
$$T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & -d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let us take $d_1 = 2$, $d_2 = 5$, $d_3 = 3$

$$T_0^3 = \begin{bmatrix} 0 & 0 & -1 & -3 \\ 0 & 1 & 0 & -5 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Code also gives the same result.
Hence verified.

Ans-6



DH Parameters

Link L	θ_i	d_i	a_i	α_i
1	θ_1	0	0	90
2	θ_2	0	d_2	0
3	θ_3	0	d_3	0
4	θ_4	0	0	−90
5	$90° + \theta_5$	0	0	90
6	θ_6	d_6	0	0

$\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ & θ_6 are variable angles. Let all of them be 0.

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Putting all DH parameters —

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = A_1 A_2 A_3 A_4 A_5 A_6 = \begin{bmatrix} 0 & 0 & 1 & d_2+d_3+d_6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

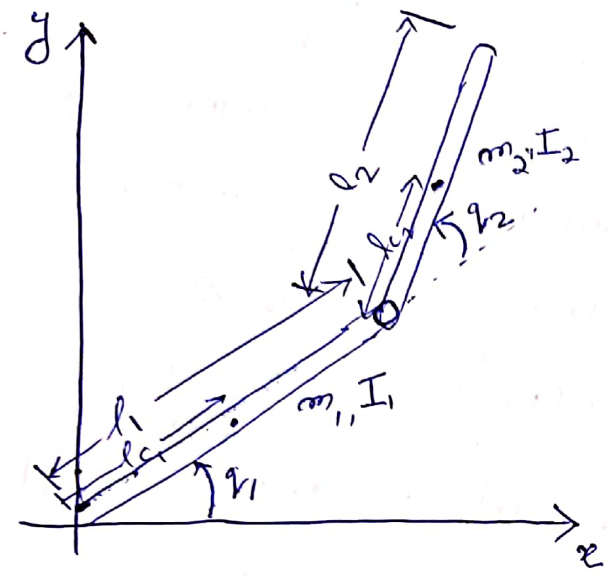
Let us take $d_2 = d_3 = d_6 = 3$

$$T_0^6 = \begin{bmatrix} 0 & 0 & 1 & 9 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From code also same value is obtained. Hence verified.

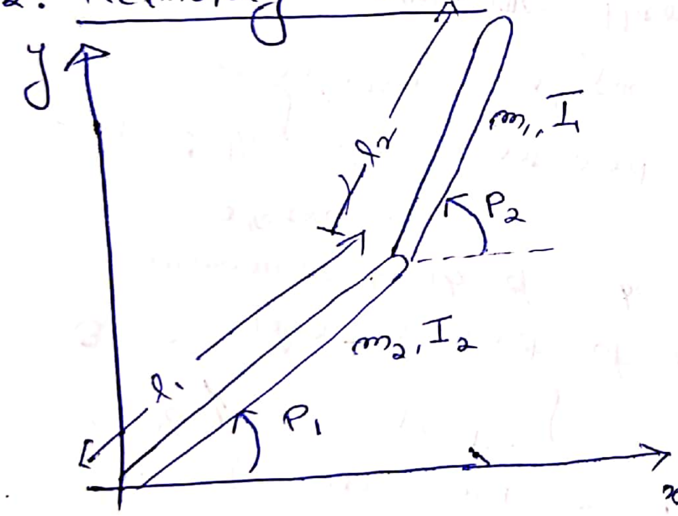
Ans-7 | Three Different Configuration of 2R Manipulator :

1. Direct Drive



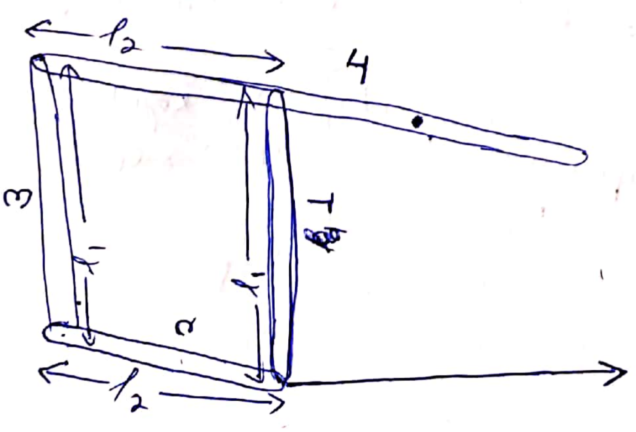
- In this configuration, one motor at the base coordinate frame drives the first link. The second motor is located in the frame of the 1^{st} link. Thus, the second motor gives rotation relative to the first link.
- The jacobian equation derived in the textbook can be directly used.
- Motor on the first link makes the manipulator heavy.

2. Remotely Driven



- Both motors are present in the base frame.
- One motor directly drives the first link. Other motor drives the second link with the help of gearing mechanism or a belt from the base
- The joint angles are absolute and hence jacobian derived in the textbook can not be used.
- Makes the manipulator light.
- Adds addition cost of materials for driving second link from base.

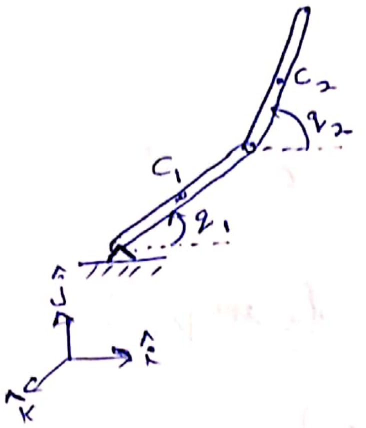
3. Five-Bar linkage



- One motor drives the link 1 and the othe motor drives link 2.
- Parallelogram law enables link 2 and link 4 have same velocity.
- A Closed Kinematic Chain mechanism.
- Jacobian derived in the textbook can not be used. Equation of motion needs to be derived from scratch.
- The system remains 2 DOF only with 5 bars.

Ans-8) Dynamic equation of 2R manipulator :-

Let's assume the links are remotely driven.



c_1 & c_2 are centre of mass of each link.

$$V_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 \\ \frac{l_1}{2}\cos q_1 \\ 0 \end{bmatrix} \dot{q}_1 \quad , \quad V_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1 \cos q_1 & \frac{l_2}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \quad , \quad \omega_2 = \dot{q}_2 \hat{k}$$

$$K = \frac{1}{2}\sum_{i=1}^{m} m_i V_{c_i}^T \cdot V_{c_i} + \frac{1}{2}\sum_{i=1}^{m} \omega_i^T I_i \omega_i$$

$$V_{c_i} = J_{V_{c_i}}(q)\dot{q} \qquad \omega_i = R_i^T J_{\omega_i}(q)\dot{q}$$

$$\therefore \quad K = \frac{1}{2}\dot{q}^T \sum_{i=1}^{m}\left[m_i J_{V_{c_i}}(q)^T J_{V_{c_i}}(q) + J_{\omega_i}(q)^T R_i(q) I_i R_i(q)^T J_{\omega_i}(q)\right]\dot{q}$$

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q}$$

$$D(q) = \begin{bmatrix} m_1\frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2}\cos(q_2-q_1) & m_2\frac{l_2^2}{4} + I_2 \end{bmatrix}$$

$$I_1 = \frac{m_1 l_1^2}{12} \qquad I_2 = \frac{m_2 l_2^2}{12}$$

Computing the christoffel symbols -

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

$$c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_1} - \frac{\partial d_{12}}{\partial q_1}\right]$$

$$= \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \cancel{\frac{1}{2}\frac{\partial d_{22}}{\partial q_1}}^{\,0} = -m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \cancel{\frac{1}{2}\frac{\partial d_{11}}{\partial q_2}}^{\,0} = m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)$$

$$C_{212} = C_{112} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0 \quad , \quad C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Potential energy, $V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$

$$\phi_1 = \frac{\partial V}{\partial q_1} = m_1 g \frac{l_1}{2} \cos q_1 + m_2 g \left(l_1 \cos q_1\right)$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 g \frac{l_2}{2} \cos q_2$$

Therefore, $d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_2^2 + \phi_1 = \tau_1$ — (A)

& $d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2$ — (B)

$$D(q) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = \begin{bmatrix} \frac{m_1 l_1^2}{3} + m_2 l_1^2 & m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) & \frac{m_2 l_2^2}{3} \end{bmatrix}$$

$$\therefore \left(\frac{m_1 l_1^2}{3} + m_2 l_1^2\right)\ddot{q}_1 + m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1)\ddot{q}_2 - m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)\dot{q}_2^2 + m_1 g \frac{l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1 = \tau_1$$

$$m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1)\ddot{q}_1 + \frac{m_2 l_2^2}{3}\ddot{q}_2 + m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)\dot{q}_1^2 + m_2 g \frac{l_2}{2} \cos q_2 = \tau_2$$

the above equations are almost similar to those derived in miniproject.

In miniproject, 1st equation has $m_2 l_1 \frac{l_2}{2} \dot{q}_1 \dot{q}_2 \sin(q_2 - q_1)$ as extra term.

& IInd eqn has $\frac{m_2 l_2^2}{4}\ddot{q}_2 - m_2 \frac{l_1 l_2}{2} \dot{q}_1 \dot{q}_2 \sin(q_2 - q_1)$ as extra term.

Ans-10 | Derivation of equation of motion -

Langrange's equations

Generalized coordinates q_i, $i = 1, \ldots, n$ — joint variables

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \tau_k \qquad k = 1, \ldots, n.}$$

$L = K - V$ $\quad$ K — total K.E. of robot

V — total P.E of robot

for a robot with n rigid links

$$K = \sum_{i=1}^{n} \left[\frac{1}{2} m_i V_{c_i}^T V_{c_i} + \frac{1}{2} \omega_i^T I_i \omega\right]$$

Jacobian magic $\quad K = \frac{1}{2} \sum_{i,j}^{n} d_{ij}(q)\, \dot{q}_i \dot{q}_j$

$$= \frac{1}{2} \dot{q}^T D \dot{q}$$

$$L = K - V$$

$$= \frac{1}{2} \sum_{i,j}^{n} d_{ij}(q)\, \dot{q}_i \dot{q}_j - V(q)$$

$$\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q)\, \dot{q}_j$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_j \frac{d}{dt}\left(d_{kj}(q)\right) \dot{q}_j$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i} \dot{q}_i \dot{q}_j$$

$$\frac{\partial L}{\partial q_k} = \frac{1}{2} \sum_{i,j} \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

Therefore, Euler - Lagrange's Equations

$$\sum_j d_{kj}\ddot{q}_j + \sum_{i,j}\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k \qquad k=1,\dots,n$$

By symmetry

$$\sum_{i,j}\left(\frac{\partial d_{kj}}{\partial q_i}\right)\dot{q}_i\dot{q}_j = \frac{1}{2}\sum_{i,j}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j}\right]\dot{q}_i\dot{q}_j$$

Hence

$$\sum_{i,j}\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j = \sum_{i,j}\frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j$$

the terms

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

c_{ijk} are known as christoffel symbols of first kind.

Therefore Euler lagrange's equations become

$$\boxed{\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k \qquad k=1,\dots,n}$$

where $\phi_k(q) = \dfrac{\partial V(q)}{\partial q_k}$

the above equation can also be written as

$$\boxed{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau}$$

therefore, the above equation can be used to get equation of motion when we are already provided $D(q)$ & $V(q)$.