

# Asymmetric Encryption

## Introduction

Asymmetric encryption, also known as public key encryption, is a revolutionary method in cryptography that uses a pair of keys: a public key for encryption and a private key for decryption. Unlike symmetric encryption, these keys are mathematically related but distinct, allowing secure communication without the need for shared keys.

### 1. Core Mathematical Principles Behind Asymmetric Encryption

(a) Key Pair Generation: Asymmetric encryption relies on generating a pair of keys: a public key (shared openly for encryption) and a private key (kept secret for decryption).

(b) Prime Numbers and Modular Arithmetic: The security of many asymmetric algorithms, such as RSA, relies on the difficulty of certain mathematical problems.

\* Integer Factorization Problem: Factoring large numbers into primes is computationally infeasible.

\* Discrete Logarithm Problem: Used in algorithms like Diffie-Hellman and ElGamal.

(c) RSA Encryption Formula: Encryption with a public key

$$C = P^e \bmod n$$

P: Plaintext, e: Public exponent, n: Product of two large primes  
Decryption with private key:

$$P = C^d \bmod n$$

d: Private exponent derived from e, p, and q.



① Elliptic Curve Cryptography (ECC): ECC relies on the mathematical properties of elliptic curves over finite fields.

$$y^2 = x^3 + ax + b$$

The difficulty of solving the elliptic curve discrete logarithm problem ensures safety.

## ② Key Management challenges

① Key Pair Distribution: While the public key can be shared openly ensuring its authenticity is critical to prevent man-in-the-middle attacks.

② Private key security: The private key must be securely stored to avoid compromise. Loss of private key results in data being unrecoverable.

③ Scalability: Unlike symmetric encryption, key management is simpler as each user only requires one key pair. However, managing digital certificates can still be complex.

④ Certificate Authorities (CAs): Public Key Infrastructures (PKI) depends on trusted certificate authorities to verify public keys, adding administrative overhead.

## ③ Performance characteristics

a) Speed: Asymmetric encryption is computationally intensive compared to symmetric encryption due to complex mathematical operations.



(b) Resource Usage: High CPU and memory usage makes asymmetric encryption less suited for resource constrained environments like IoT devices.

(c) Hybrid Systems: Many systems combine asymmetric and symmetric encryption for overall better performance.  
Example. TLS/SSL protocols.

(4) Security Strength and Vulnerabilities

(a) Strength

- \*. No key sharing: It eliminates the need for securely sharing a secret key.
- \*. Scalability: Suitable for large-scale systems with many users.
- \*. Digital signatures: Supports authentication and non-repudiation.

(b) Vulnerabilities

- \*. Public key compromise: loss of these of a private key compromises security.
- \*. Man-in-the middle Attacks: If the public key is tampered encryption can be bypassed.
- \*. Quantum computing Threats: Algorithms like RSA and to quantum computers.

## (5) Real - world Applications and Use cases

- (a) Secure Communication: Protocols like TLS/SSL use a symmetric encryption to secure internet communication. Email encryption PGP is pretty good. Privacy relies on asymmetric keys.
- (b) Digital Signature: Asymmetric encryption verifies the authenticity and integrity of documents, contracts, and software (e.g., in blockchain systems).
- (c) Authentication: Used in multi-factor authentication systems and SSH connections to ensure secure access.
- (d) Crypto currency: Cryptographic wallets use asymmetric encryption to manage private and public keys for secure transactions.