Assignment 3
AI1110

I Sai Pradeep
AI21BTECH11013

***Abstract*—This document contains the solution to problem 8 of exercise 13.3 in cbse 12th class**

Question 8,Exercise 13.3: A factory has two machines A and B. Past record shows that machine A produced 60% of th items of output and machine B produced 40% of the items. Further,2% of the items produced by machine A and 1% produced by machine B were defective. All the itmes are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

Solution: Let $X = \{0, 1\}$ be a random variable representing the machine from which the item is selected and let $Y = \{0, 1\}$ be a random variable representing the chances that the selected item is defective. Let P_7 be the probability that the selected item is defective, and let P_8 be the probability that the selected item is defective and is from machine B. See Tables (I) and (II) for the input probabilities.

Event	Description
$X = 0$	Item selected is from Machine A
$X = 1$	Item selected is from Machine B
$Y = 0$	Selected item is defective
$Y = 1$	Selected item is not defective

TABLE I

Probability	Value
P_1= $\Pr(X = 0)$	$\frac{60}{100} = \frac{3}{5}$
P_2= $\Pr(X = 1)$	$\frac{40}{100} = \frac{2}{5}$
P_3= $\Pr(Y = 0\|X = 0)$	$\frac{2}{100} = \frac{1}{50}$
P_4= $\Pr(Y = 0\|X = 1)$	$\frac{1}{100}$
P_5= $\Pr(Y = 0\|X = 0) \times \Pr(X = 0)$	$\frac{1}{50} \times \frac{3}{5} = \frac{3}{250}$
P_6= $\Pr(Y = 0\|X = 1) \times \Pr(X = 1)$	$\frac{1}{100} \times \frac{2}{5} = \frac{1}{250}$
P_7= $\Pr(Y = 0)$	?
P_8= $\Pr(X = 1\|Y = 0)$	?

TABLE II

The desired probability is then obtained from Table (II) as

$$P_7 = \Pr(Y = 0) \tag{1}$$
$$= P_5 + P_6 \tag{2}$$
$$= \frac{3}{250} + \frac{1}{250} \tag{3}$$
$$= \frac{4}{250} \tag{4}$$
$$= \frac{2}{125} \tag{5}$$

From Bayes' theorem,

$$\Pr(X = 1|Y = 0) = \frac{\Pr(X = 1) \times \Pr(Y = 0|X = 1)}{\Pr(Y = 0)} \tag{6}$$

Hence,

$$P_8 = \Pr(X = 1|Y = 0) \tag{7}$$
$$= \frac{\Pr(X = 1) \times \Pr(Y = 0|X = 1)}{\Pr(Y = 0)} \tag{8}$$
$$= \frac{\frac{2}{5} \times \frac{1}{100}}{\frac{2}{125}} \tag{9}$$
$$= \frac{\frac{2}{500}}{\frac{2}{125}} \tag{10}$$
$$S = \frac{1}{4} \tag{11}$$

Therefore, the probability that the defective item is selected from machine B is $\frac{1}{4}$.