AI1110
Assignment 6

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Outline

EXAMPLE 5.28

Q:- We shall show that the characteristic function of an $N(\eta, \sigma)$ random variable x equals $\Phi_x(\omega) = exp(j\eta\omega - \frac{1}{2}\sigma^2\omega^2)$

Solution

The random variable $x = \dfrac{x-\eta}{\sigma}$ is N(0,1) and its moment function equals

$$\Phi_z(s) = \int_{-\infty}^{\infty} \frac{e^{z\times s} \times e^{\dfrac{-z^2}{2}}}{\sqrt{2\times\pi}} dz \tag{1}$$

$$s \times z - \frac{z^2}{2} = -\frac{(z-s)^2}{2} + \frac{s^2}{2} \tag{2}$$

$$\Phi_z(s) = e^{\dfrac{z^2}{2}} \times \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{-(z-s)^2}2}{\sqrt{2\times\pi}} dz \tag{3}$$

$$\text{Since,} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{x^2}2}{\sqrt{2\times\pi}} dx = 1 \tag{4}$$

$$\Phi_z(s) = e^{\dfrac{z^2}{2}} \tag{5}$$

Computation

As we know that, if y=ax+b then $\Phi_y(w) = e^{jbw}\Phi_x(a\omega)$
Since, $x = \sigma \times z + \eta$

$$\Phi_x(w) = e^{jw\eta}\Phi_z(\sigma\omega) \tag{6}$$

$$\Phi_z(j\omega) = \Phi_z(w) \tag{7}$$

From equation (5)

$$\Phi_z(\sigma\omega) = e^(-\frac{\sigma^2\omega^2}{2}) \tag{8}$$

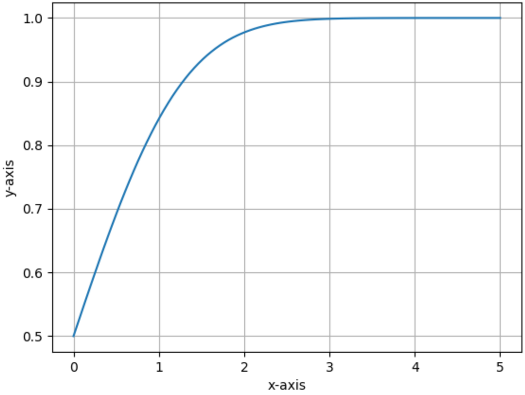
From equations (6) and (8)

$$\Phi_x(w) = e^{jw\eta} \times e^(-\frac{\sigma^2\omega^2}{2}) \tag{9}$$

$$\Phi_x(w) = e^(jw\eta - \frac{\sigma^2\omega^2}{2}) \tag{10}$$

Conclusion

Hence proved that the characteristic function of an $N(\eta, \sigma)$ random variable x equals $\Phi_x(\omega) = exp(j\eta\omega - \frac{1}{2}\sigma^2\omega^2)$



Figure