AI1110
Assignment 8

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Outline

Exercise 6.42

Q: x and y are independent random variables with geometric p.m.f.

$$\Pr(x = k) = pq^k$$

k= 0,1,2,...

$$\Pr(y = m) = pq^m$$

m= 0,1,2,... Find the p.m.f. of

(a) x+y

(b) x-y

Solution

X,Y are independent geometric random variables.Thus

$$\Pr(X = k, Y = m) = \Pr(X = k) \times \Pr(Y = m) \tag{1}$$
$$= (p \times q^k) \times (p \times q^m) \tag{2}$$
$$= p^2 \times q^(k + 2) \tag{3}$$

(a) Let

$$Z = X + Y$$

$$\Pr(Z = n) = \Pr(X + Y = n) \tag{4}$$
$$= \sum_{k=0}^{n} \Pr(X = k, Y = n - k) \tag{5}$$
$$= \sum_{k=0}^{n} \Pr(X = k) \times \Pr(Y = n - k) \tag{6}$$

Computation

$$\Pr(Z = n) = \sum_{k}^{n} (p \times q^k) \times (p \times q^(n - k)) \tag{7}$$

$$= (n + 1)p^2 q^n n = 0, 1, 2, 3, ... \tag{8}$$

Let W=X-Y

Case-1:

$$W \geq 0 \implies X \geq Y \tag{9}$$

Thus for $m \geq 0$

$$\Pr(W = m) = \Pr(X - Y = m) \tag{10}$$

$$= \sum_{k=0}^{\infty} \Pr(X = m + k, Y = k) \tag{11}$$

$$= \sum^{\infty} \Pr(X = m + k, Y = k) \tag{12}$$

Computation

$$= \sum_{k=0}^{\infty} \Pr\left(X = m + k\right) Y = k \tag{14}$$

$$= \sum_{k=0}^{\infty} (pq^m + k) \times (pq^k) \tag{15}$$

$$= p^2 q^m \sum_{k=0}^{\infty} q^2 k \tag{16}$$

$$= p^2 q^m \times (1 + q^2 + q^4 + ...) \tag{17}$$

$$= \frac{p^2 q^m}{1 - q^2} \tag{18}$$

Since $p = 1 - q$ and $1 - q^2 = (1 - q) \times (1 + q)$

Computation

$$\Pr\left(W = m\right) = \frac{p^2 q^m}{1+q} m = 0, 1, 2, ... \tag{19}$$

Let $W = X - Y$
Case -2:

$$W < 0 \tag{20}$$
$$\implies X < Y \tag{21}$$

Thus for $m < 0$

$$\Pr\left(W = m\right) = \Pr\left(X - Y = m\right) \tag{22}$$
$$= \sum_{k=0}^{\infty} \Pr\left(X = k, Y = k - m\right) \tag{23}$$

Computation

$$= \sum_{k=0}^{\infty} \Pr\left(X = k, Y = k - m\right) \tag{24}$$

$$= \sum_{k=0}^{\infty} \Pr\left(X = k\right) Y = k - m \tag{25}$$

$$= \sum_{k=0}^{\infty} (pq^{k}) \times (pq^{k-m}) \tag{26}$$

$$= p^2 q^{-m} \sum_{k=0}^{\infty} q^2 k \tag{27}$$

$$= p^2 q^{-m} \times (1 + q^2 + q^4 + ...) \tag{28}$$

$$= \frac{p^2 q^{-m}}{1 - q^2} \tag{29}$$

Conclusion

Since $p = 1 - q$ and $1 - q^2 = (1 - q) \times (1 + q)$

$$\Pr(W = m) = \frac{p^2 q^{-m}}{1 + q} m = -1, -2, ... \tag{30}$$

Hence from equations (19) and (30) ,We can write that

$$\Pr(W = m) = \frac{p q^{|x|}}{1 + q} m = 0, \pm 1, \pm 2, ... \tag{31}$$

Output graph for p=0.7

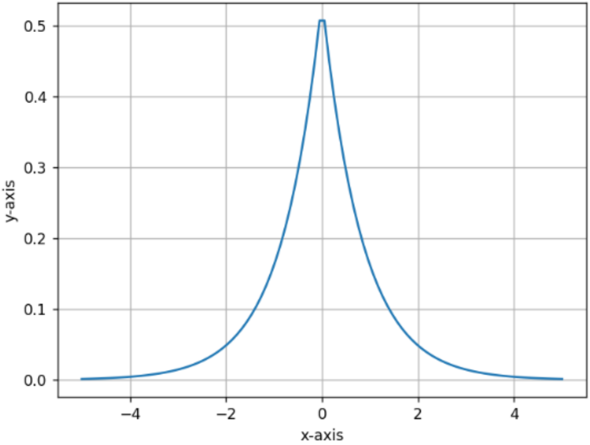

Figure: fig 1