

AI1110

Assignment 4

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Outline

1 Question

2 Solution

EXAMPLE 2.20

Q:- In a group of n people,

- (a) what is the probability that two or more persons will have the same birthday (month and date)?
- (b) What is the probability that someone in that group will have birthday that matches yours?

Solution

Let $X = \{0, 1\}$ be a random variable representing the chances about people having same birthday. Let $Y = \{0, 1\}$ be a random variable representing the chances that someone has birthday same as mine. See Table (1) and (2)

Events and random variables

Event	Description
$X = 0$	Two or more persons have the same birthday
$X = 1$	No two persons have the same birthday

Table 1

Event	Description
$Y = 0$	Someone has birthday same as mine.
$Y = 1$	No one has birthday same as mine.

Table 2

Computation for part (a)

There are $N = 365$ equally likely ways (number of days in a year) for each person, where their birthdays fall independently. there are N ways for the first person to have a birthday, $N - 1$ ways for the second person without matching the first person, and $N - n + 1$ ways for the last person to have non matching birthday.

So there are $N(N-1)\dots(N-n+1)$ favourable outcomes. For n persons, there are N^n possible outcomes.

$$\Pr(X = 0) = \frac{\prod_{i=0}^{n-1} (N - i)}{N^n} = \prod_{i=0}^{n-1} \left(1 - \frac{i}{N}\right)$$

Hence,

$$\Pr(X = 1) = 1 - \prod_{i=0}^{n-1} \left(1 - \frac{i}{365}\right)$$

Computation for part (b)

There are $N - 1$ unfavorable days among N days for each person to not have same birthday as mine. probability that a person does not have same birthday as mine is $\frac{N-1}{N}$. For a group of n persons,

$$\Pr(Y = 1) = \left(\frac{N-1}{N}\right)^n = \left(1 - \frac{1}{N}\right)^n$$

Hence,

$$\Pr(Y = 0) = 1 - \left(1 - \frac{1}{365}\right)^n$$