AI1110
Assignment 4

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Outline

EXAMPLE 2.20

Q:- In a group of n people,

(a) what is the probability that two or more persons will have the same birthday (month and date)?

(b) What is the probability that someone in that group will have birthday that matches yours?

Solution

Let $X = \{0, 1\}$ be a random variable representing the chances about people having same birthday. Let $Y = \{0, 1\}$ be a random variable representing the chances that someone has birthday same as mine. See Table (1) and (2)

Events and random variables

Event	Description
$X = 0$	Two or more persons have the same birthday
$X = 1$	No two persons have the same birthday

Table 1

Event	Description
$Y = 0$	Someone has birthday same as mine.
$Y = 1$	No one has birthday same as mine.

Table 2

Computation for part (a)

There are N = 365 equally likely ways (number of days in a year) for each person, where their birthdays fall independently. there are N ways for the first person to have a birthday, $N-1$ ways for the second person without matching the first person, and $N-n+1$ ways for the last person to have non matching birthday.

So there are $N(N-1)...(N-n+1)$ favourable outcomes. For n persons, there are N^n possible outcomes.

$$\Pr(X=0) = \frac{\prod_{i=0}^{n-1}(N-i)}{N^n} = \prod_{i=0}^{n-1}(1-\frac{i}{N})$$

Hence,

$$\Pr(X=1) = 1 - \prod_{i=0}^{n-1}(1-\frac{i}{365})$$

Computation for part (b)

There are N - 1 unfavorable days among N days for each person to not have same birthday as mine. probability that a person does not have same birthday as mine is $\frac{N-1}{N}$. For a group of n persons,

$$\Pr(Y = 1) = (\frac{N-1}{N})^n = (1 - \frac{1}{N})^n$$

. Hence,

$$\Pr(Y = 0) = 1 - (1 - \frac{1}{365})^n$$