Q1 $T(n) = 3T(n/2) + n^2$

$a=3 \quad b=2 \quad f(n)=n^2$

∵ a & b are constant & f(n) is the function

∴ master's theorem is applicable.

$c = \log_b a$

$= \log_2 3 = 1.58$

$n^c = n^{1.58}$ which is $< n^2$

∴ case 3 is applicable

$T(n) = O(n^2)$

Q2 $T(n) = 4T(n/2) + n^2$

$a=4 \quad b=2 \quad f(n)=n^2$

$c = \log_b a = \log_2 4 = 2.$

$n^c = n^2 \quad n^c = f(n)$

case 2. $T(n) = O(n^2 \log n)$

Q3 $T(n) = T(n/2) + 2^n$

$a=1 \quad b=2 \quad f(n) = 2^n$

$c = \log_b a \Rightarrow \log_2 1 = 0 \quad n^c = n^0 = 1$

$f(n) > n^c$

∴ case 3

$T(n) = O(2^n)$

Q4 $T(n) = 2^n T(n/2) + n^n$

$a = 2^n \quad b=2 \quad f(n) = n^n$

∵ a is not constant, its value depends on n

∴ master's theorem not applicable.

Q5 $T(n) = 16T(n/4) + n$

$a = 16$, $b = 4$ $f(n) = n$

$c = \log_b a = \log_4 16 = 2$

$n^c > f(n)$

case 1

$T(n) = \Theta(n^2)$

Q6 $T(n) = 2T(n/2) + n\log n$

$a = 2$, $b = 2$ $f(n) = n\log n$

$c = \log_b a = \log_2 2 = 1$

$n^c = n$

$f(n) > n^c$

case 3 is applied

$\therefore T(n) = \Theta(n\log n)$

Q7 $T(n) = 2T(n/2) + n/\log n$

$a = 2$, $b = 2$ $f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n$

non polynomial diff b/w n^c & $f(n)$,

$\therefore$ masters theorem not applicable

Q8 $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4 \quad f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$\therefore f(n) > n^c$

case 3 is applied

$T(n) = \Theta(n^{0.51})$

Q9. $T(n) = 0.5\, T(n/2) + \frac{1}{n}$

$a < 1 \therefore$ master's theorem applicable

Q10 $T(n) = 16\, T(n/4) + n!$

$a = 16, b = n, f(n) = n!$

$c = \log_4 16 = 2$

$n^c = n^2$

$f(n) > n^c$ case 3.

$T(n) = \Theta(n!)$

Q11 $T(n) = 4\, T(n/2) + \log n$

$a = 4, b = 2 \quad f(n) = \log n$

$c = \log_2 4 = 2.$

$n^c = n^2$

$n^c > f(n)$

$\therefore$ case 1 is applied

$T(n) = \Theta(n^2)$

Q12 $T(n) = \sqrt{n}\, T\left(\frac{n}{2}\right) + \log n$

a is not constant, therefore master's theorem not applicable.

Q13 $T(n) = 3T(n/2) + n$

$a = 3$, $b = 2$, $f(n) = n$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58} > f(n)$

case 1

$T(n) = \Theta(n^{1.58})$

Q14 $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3$, $b = 3$, $f(n) = \sqrt{n}$

$c = \log_b a = 1$

$n^c = n > \sqrt{n}$

case 1 is applied $T(n) = \Theta(n)$

Q15 $T(n) = 4T(n/2) + c \cdot n$

$a = 4$, $b = 2$, $f(n) = c \cdot n$

$n^c = n^2 > f(n)$

$\therefore$ case 1 is applied

$T(n) = \Theta(n^2)$

Q16 $T(n) = 3T(n/4) + n \log n$

$a = 3$, $b = 4$, $f(n) = n \log n$

$c = \log_b a = \log_4 3 = 0.78$

$n^c = n^{0.79} < f(n)$

$\therefore$ case 3 is applied $T(n) = \Theta(n \log n)$

Q17 $T(n) = 3T(n/3) + \frac{n}{2}$

$a = 3, b = 3 \quad f(n) = \frac{n}{2}$

$c = \log_b a = 1$

$n^c = n > \cancel{f}(n)$

case 1: $T(n) = \Theta(n)$

Q18 $T(n) = 6T(\frac{n}{3}) + n^2 \log n$

$c = \log_3 6 \Rightarrow 1.63$

$n^c = n^{1.63} < f(n)$

case 3 is applied $\therefore T(n) = \Theta(n^2 \log n)$

Q19 $T(n) = 4T(n/2) + n\log n$

$c = \log_2 4 = 2$

$n^c = n^2 \quad f(n) = n \log n$

$n^c \cancel{>} f(n)$

$\therefore$ case 1 is applied $T(n) = \Theta(n^2)$

Q20 $T(n) = 64T(n/8) + n^2 \log n$

$a = 64 \quad b = 8 \quad \cancel{f}(n) = n^2 \log n$

$c = \log_b a = 2$

$n^{\cancel{c}} = n^2 < f(n)$

case 3 is applied $T(n) = \Theta(n^2 \lg n)$

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Q21 $T(n) = 7T(n/3) + n^2$

$a = 7$, $b = 3$, $f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77} < f(n)$

case 3 is applied $T(n) = \Theta(n^2)$

Q22 $T(n) = T(\frac{n}{2}) + n(2 - \cos n)$

$\because$ $f(n)$ is not a regular function

$\therefore$ Masters theorem cann't applied

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