Q1 fun(int n)

```
{ int j=1, i=0
  while (i<n)
  { i=i+j
    j++ }
}
```

$$\frac{K(K+1)}{2} = n$$

$$K^2 = n$$

$K = \sqrt{n}$ TC: $O(\sqrt{n})$

Q2 $T(0) = 0$

$T(1) = 0$

$T(n) = T(n-1) + T(n-2) + 1$

let $T(n-1) = T(n-2)$

$T(n) = 2T(n-1) + 1$

using back. substn

$T(n) = 2\cdot 2(T(n-2)+1)+1$

$= 4(T(n-2)) + 3$

$T(n-2) = 2T(n-3)+1$

$= 2(2(2(T(n-3)+1)+1)+1$

$= 8T(n-3)+7$

$T(n) = \cancel{2^{k}} \; 2^{k}T(n-k)+2^{k}-1$

$T(0) = 0$

$n-k = 0$

$n = k$

$T(n) = 2^{n}T(n-n)+2^{n}-1$

$= 2^{n}+2^{n}$

TC = $O(2^{n})$

Q3 log(log n)

```
fun (int n)
{ for (int i = n, i >= 2, pow(i, 1/2))
    { some O(1)
    }
}
```

n(log n)

```
for (int i = 1, i <= n, i++)
{ for (int j = 1, j <= n , j = j*2 )
    { for som O(1)
    }
}
```

n^3

```
for ( int i = 1; i < n ; i++ )
  for ( int j = 1 ; j < n ; j++ )
    for ( int k = 1 ; k < n; k++ )
      Som O(1)
```

Q4 $T(n) = T(n/4) + T(n/2) + cn^2$

assume $T(n/2) >= T(n/4)$

$T(n) = 2T(n/2) + cn^2$

$c = \log_b a$

$c = \log_2 2 = 1$

$n^c < f(n)$

$T(= O(n^2)$

Q5

i	j
1	n
2	$n/2$
3	$n/3$
⋮	
n	n/n tim

$T.C = O(n(\log n))$

Q6

$i = 2,\ 2^k, (2^k)^k, ((2^k)^k)^k \ldots\ 2^{k \log_k(\log n)}$

$2^{k \log_k(\log n)} = n$

$T.C = O(\log(\log n))$

~~Q8~~

~~[illegible]~~

Q7

$T(n) = T\left(\frac{99n}{100}\right) + \frac{n}{100}$

$T(1) = 0$

putting $n = \frac{99}{100} n$

$T\left(\frac{99}{100} n\right) = T\left(\left(\frac{99}{100}\right)^2 n\right) + \frac{99n}{(100)^2}$

$\vdots$

$T(n) = T\left(\frac{99}{100}^k n\right) + \frac{(99)^{k-1}}{(100)^k} n$

$\left(\frac{99}{100}\right)^k n = 1$

$n = \left(\frac{100}{99}\right)^k$

$k = \log_{\frac{100}{99}} n$

$$\boxed{T(n) = n \log n}$$

Q8

a) $100 < \log(\log n) < \log n < \sqrt{n} < n\log n = \log(n!)$
$< n^2 < 2^n < 2^{2n} < 4^n < n!$

(b) $1 < \log\log(n) < \sqrt{\log n} < \log(n) < 2n < 4n < 2(2^n)$
$< \log(2n) < 2\log(n) < n < n\log n = \log(n!) < n!$

(c) $96 < \log_2(n) = \log_8(n) < n\log_8(n) = n\log_2(n) < 5n < 8n^2 < 7n^3$
$\cancel{8n}' < 8^{2n}$