Assignment 13

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Question

We are given the data x(t) = f(t) + n(t), where $R_n(\tau) = N\delta(\tau)$ and E[(t) = 0]. We wish to estimate the integral

$$g(t) = \int_0^t f(\alpha)\, d\alpha \tag{1}$$

knowing that g(T) = 0. Show that if we use as the estimate of g(t) the process w(t) = z(t) - z(T)t/T, where

$$z(t) = \int_0^t x(\alpha)\, d\alpha \tag{2}$$

then

$$E[w(t)] = g(t) \tag{3}$$

$$\sigma_w^2 = Nt\left(1 - \frac{t}{T}\right) \tag{4}$$

Solution

$$E[z(t)] = g(t) \tag{5}$$
$$E[w(t)] = g(t) - g(T)t/T \tag{6}$$
$$w(t) = \left(1 - \frac{t}{T}\right) \int_0^t x(\alpha)\, d\alpha - \frac{t}{T} \int_t^T x(\alpha)\, d\alpha \tag{7}$$

The above two integrals are uncorrelated because n(t) is white noise.

Hence,

$$\sigma_w^2 = \left(1 - \frac{t}{T}\right)^2 Nt + \frac{t^2}{T^2} N(T - t) \tag{8}$$

$$= Nt\left(1 - \frac{t}{T}\right) \tag{9}$$

NOTE: The above shows that the information that g(T) = 0 can be used to improve the estimate of g(t). Indeed, if use w(t) instead of z(t) for the estimate of g(t) in terms of the data x(t), the variance is reduced from Nt to $Nt\left(1 - \frac{t}{T}\right)$.