Assignment 4

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1 Question-

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

2 Answer-

Let us consider random variables X and Y.
X is for boys.
Y is for girls.
For random variable X,Y 0,1 are possible values.
X = 0 $\implies$ youngest is boy
X = 1 $\implies$ eldest is boy
Similarly,
Y = 0 $\implies$ youngest is girl
Y = 1 $\implies$ eldest is girl
So, (as each born child is equally likely to be a boy or girl)

$$P_X(0) = P_X(1) = P_Y(0) = P_Y(1) = \frac{1}{2} \tag{2.1}$$

Let A denote an event,
A: 'Both are girls'
Then,

$$A = (Y = 0) \cap (Y = 1) \tag{2.2}$$
$$\implies P(A) = P_Y(0) \times P_Y(1) \tag{2.3}$$
$$\implies P(A) = \frac{1}{4} \tag{2.4}$$

2.1. Part (i) -
Let B denote following event:
B: 'Youngest is girl'
Then,

$$P(B) = P_Y(0) \tag{2.1.1}$$
$$\implies P(B) = \frac{1}{2} \tag{2.1.2}$$

Now,

$$P(A|B) \times P(B) = P(B|A) \times P(A) \tag{2.1.3}$$
$$P(B|A) = 1 \tag{2.1.4}$$
$$\implies P(A|B) \times \frac{1}{2} = 1 \times \frac{1}{4} \tag{2.1.5}$$
$$\implies P(A|B) = \frac{1}{2} \tag{2.1.6}$$
$$\implies P(A|B) = 0.5 \tag{2.1.7}$$

Therefore, the conditional probability of A given that B occured is 0.5.

2.2. Part (ii) -
Let C denote following event:
C: 'Atleast one is girl'
Then,

$$P(C) = P_Y(0) \times P_Y(1) + P_Y(0) \times P_X(1) + P_X(0) \times P_Y(1) \tag{2.2.1}$$
$$\implies P(C) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \tag{2.2.2}$$
$$\implies P(C) = \frac{3}{4} \tag{2.2.3}$$

Now,

$$P(A|C) \times P(C) = P(C|A) \times P(A) \tag{2.2.4}$$
$$P(C|A) = 1 \tag{2.2.5}$$
$$\implies P(A|C) \times \frac{3}{4} = 1 \times \frac{1}{4} \tag{2.2.6}$$
$$\implies P(A|C) = \frac{1}{3} \tag{2.2.7}$$
$$\implies P(A|C) = 0.33 \tag{2.2.8}$$

Therefore, the conditional probability of A given that C occured is 0.33.