Assignment 12

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Outline

Question

Papoulis 9.36

Using (9.135), show that

$$R(0) - R(\tau) \geq \frac{1}{4^n}[R(0) - R(2^n\tau)] \tag{1}$$

9.135

$$S(w) = \int_{-\infty}^{\infty} R(\tau)\cos(\omega\tau)\, d\tau \tag{2}$$

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(w)\cos(\omega\tau)\, d\omega \tag{3}$$

Solution

We will use :

$$1 - \cos(\theta) = 2\sin^2(\frac{\theta}{2}) \geq 2\sin^2(\frac{\theta}{2})\cos^2(\frac{\theta}{2}) = \frac{1}{4}(1 - cos(2\theta)) \tag{4}$$

we will conclude with 9.135 that

$$R(0) - R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(w)(1 - \cos(\omega\tau))\,d\omega \tag{5}$$

$$\geq \frac{1}{8\pi}\int_{-\infty}^{\infty} S(w)(1 - \cos(\omega\tau))\,d\omega \tag{6}$$

$$= \frac{1}{4}[R(0) - R(2\tau)] \tag{7}$$

$$R(0) - R(\tau) \geq \frac{1}{4}[R(0) - R(2\tau)] \tag{8}$$

the result follows n = 1. Reapeating the above, we obtain general result

$$R(0) - R(\tau) \geq \frac{1}{4^n}[R(0) - R(2^n\tau)] \tag{9}$$