Assignment 4

Pradeep Mundlik (AI21BTECH11022)

1 Question-

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

2 Answer-

Let b stands for boy and g stands for girl. The sample space of the experiment is
S = $\{bb, bg, gb, gg\}$
('bg' denotes youngest is boy and older is girl. Similar for others.)
Let A denote an event,
A: 'both are girls'
Then A = $\{gg\}$

1. Part (i) -
   Let B denote following event:
   B: 'youngest is girl'
   Then B = $\{gb, gg\}$
   Now,
   $$A \cap B = \{gg\} \tag{1.1}$$
   $$\implies n(A \cap B) = 1 \tag{1.2}$$
   Also,
   $$n(B) = 2 \tag{1.3}$$
   Therefore, the conditional probability of A given that B occured is
   $$P(A|B) = \frac{n(A \cap B)}{n(B)} \tag{1.4}$$
   $$\implies P(A|B) = \frac{1}{2} \tag{1.5}$$
   $$\implies P(A|B) = 0.5 \tag{1.6}$$
2. Part (ii) -
   Let C denote following event:
   C: 'atleast one is girl'
   Then C = $\{bg, gb, gg\}$
   Now,
   $$A \cap C = \{gg\} \tag{2.1}$$
   $$\implies n(A \cap C) = 1 \tag{2.2}$$
   Also,
   $$n(C) = 3 \tag{2.3}$$
   Therefore, the conditional probability of A given that C occured is
   $$P(A|C) = \frac{n(A \cap C)}{n(C)} \tag{2.4}$$
   $$\implies P(A|C) = \frac{1}{3} \tag{2.5}$$
   $$\implies P(A|C) = 0.33 \tag{2.6}$$