Assignment 9

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Outline

Question

Papoulis 7.26

Using the Cauchy criterion, show that a sequence x_n tends to a limit in the MS sense iff the limit of $E(x_n, x_m)$ as $n, m \to \infty$ exists.

Solution

If $E\{x_n, x_m\} \to a$ as $n, m \to \infty$, then, given $\epsilon > 0$, we can find a number n_0 such that, if $n, m > 0$

$$E\{x_n, x_m\} = a + \theta(n, m) \ldots (|\theta| < \epsilon) \tag{1}$$

Hence,

$$\begin{aligned} E\{(x_n - x_m)^2\} =& E\{x_n^2\} + E\{x_m^2\} - 2E\{x_n x_m\} && (2) \\ =& a + \theta_1 + a + \theta_2 - 2(a + \theta_3) && (3) \\ =& \theta_1 + \theta_2 - 2\theta_3 && (4) \end{aligned}$$

and since it $|\theta_1 + \theta_2 - 2\theta_3| < 4\epsilon$ for any ϵ, it follows that $E\{(x_n - x_m)^2\} \to 0$, hence (Cauchy) x_n tends to a limit.

Conversly,
If $x_n \to x$ in the MS sense, then $E\{(x_n - x)^2\} \to 0$.
Furthermore,

$$E\{x_n^2\} \to E\{x^2\} \tag{5}$$

$$E\{xx_n\} \to E\{x^2\} \tag{6}$$

$$E^2\{x_n^2 - x^2\} = E^2\{(x_n - x)(x_n + x)\} \tag{7}$$

$$\leq E\{(x_n - x)^2\}E\{(x_n + x)^2\} \to 0 \tag{8}$$

$$E^2\{x(x_n - x)\} \leq E\{x^2\}E(x_n - x)^2 \to 0 \tag{9}$$

Similarly,

$$E^{\{}(x_n - x)(x_m - x)\} \to 0 \tag{10}$$

Hence,

$$E\{x_n x_m\} + E\{x^2\} - Exx_n - Exx_m \to 0 \tag{11}$$

Combining, we connclude that $E\{x_n x_m\} \to E\{x^2\}$.