Assignment 2 ICSE class 12 2019

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1 Question

If $y = e^{\sin^{-1}(x)}$ and $z = e^{-\cos^{-1}(x)}$, prove that $\frac{dy}{dz} = e^{\frac{\pi}{2}}$.

2 Solution

Given:

$$y = e^{\sin^{-1}(x)} \tag{2.1}$$

$$z = e^{-\cos^{-1}(x)} \tag{2.2}$$

To prove:

$$\frac{dy}{dz} = e^{\frac{\pi}{2}} \tag{2.3}$$

Proof:
Here, $z = e^{-\cos^{-1}(x)} > 0 \ \forall x \in R$
Consider,

$$\frac{y}{z} = \frac{e^{\sin^{-1}(x)}}{e^{-\cos^{-1}(x)}} \tag{2.4}$$

$$\implies \frac{y}{z} = e^{\left(\sin^{-1}(x)+\cos^{-1}(x)\right)} \tag{2.5}$$

We know that,

$$\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2} \tag{2.6}$$

From (2.5) and (2.6),

$$\implies \frac{y}{z} = e^{\frac{\pi}{2}} \tag{2.7}$$

$$\implies y = e^{\frac{\pi}{2}} \times z \tag{2.8}$$

(As $e^{\frac{\pi}{2}} = constant$)

$$\implies dy = e^{\frac{\pi}{2}} \times dz \tag{2.9}$$

$$\implies \frac{dy}{dz} = e^{\frac{\pi}{2}} \tag{2.10}$$

Hence, proved.

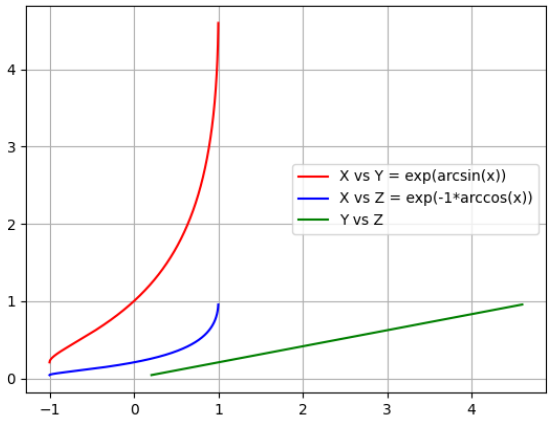

Fig. 0. Graph showing $y = e^{\sin^{-1}(x)}$ and $z = e^{-\cos^{-1}(x)}$ and Y vs Z.