Assignment 8

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Outline

Question

Papoulis 5.44

The random variable x has zero mean, central moments μ_n, and cumulants λ_n. Show that $\lambda_3 = \mu_3$, $\lambda_4 = \mu_4 - 3{\mu_2}^2$.

Solution

If $\eta = 0$, then $m_n = \mu_n$

$$\lambda_1 = \eta = 0 \tag{1}$$

$$\phi(s) = \sum_{n=0}^{\infty} \frac{\mu_n}{n!} s^n \tag{2}$$

$$\psi(s) = \sum_{n=2}^{\infty} \frac{\lambda_n}{n!} s^n \tag{3}$$

$$1 + \frac{\mu_2}{2!} s^2 + \frac{\mu_3}{3!} s^3 + \cdots = exp\left(\frac{\lambda_2}{2!} s^2 + \frac{\lambda_3}{3!} s^3 + \ldots\right) \tag{4}$$

Solution

Expanding the exponential and equating powers of s, we obtain

$$\mu_2 = \lambda_2 \tag{5}$$
$$\mu_3 = \lambda_3 \tag{6}$$
$$\frac{\mu_4}{4!} = \frac{\lambda_4}{4!} + \frac{1}{2!}\left(\frac{\lambda_2}{2!}\right)^2 \tag{7}$$
$$\implies \lambda_4 = \mu_4 - 3{\lambda_2}^2 \tag{8}$$
$$\implies \lambda_4 = \mu_4 - 3{\mu_2}^2 \tag{9}$$