Digital Signal Processing

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Contents

***Abstract*—This manual provides a simple introduction to digital signal processing.**

1 Software Installation

Run the following commands

```
sudo apt−get update
sudo apt−get install libffi−dev libsndfile1 python3
    −scipy python3−numpy python3−matplotlib
sudo pip install cffi pysoundfile
```

2 Digital Filter

2.1 Download the sound file from

```
wget https://raw.githubusercontent.com/
    gadepall/
EE1310/master/filter/codes/Sound_Noise.wav
```

2.2 You will find a spectrogram at https://academo.org/demos/spectrum-analyzer. Upload the sound file that you downloaded in Problem 2.1 in the spectrogram and play. Observe the spectrogram. What do you find?
Solution: There are a lot of yellow lines between 440 Hz to 5.1 KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.

2.3 Write the python code for removal of out of band noise and execute the code.
Solution: We can get code from below link-

```
https://github.com/PradeepMundlik/
    EE3900/tree/master/Assignment1/
    codes/q2/Cancel−noise.py
```

2.4 The output of the python script in Problem 2.3 is the audio file Sound_With_ReducedNoise.wav. Play the file in the spectrogram in Problem 2.2. What do you observe?
Solution: The key strokes as well as background noise is subdued in the audio. Also, the signal is blank for frequencies above 5.1 kHz.

3 Difference Equation

3.1 Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{3.1}$$

Sketch $x(n)$.
Solution:

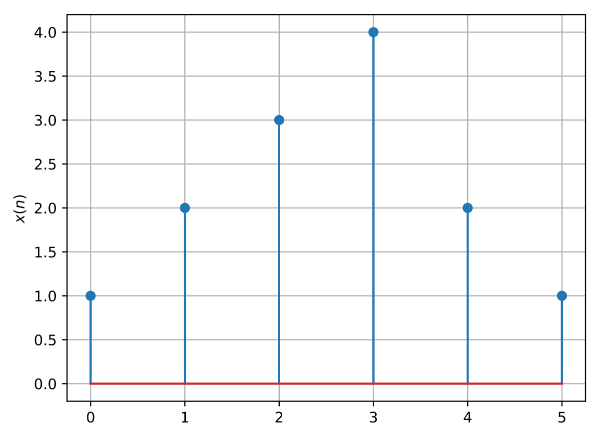

Fig. 3.1

```
https://github.com/PradeepMundlik/
    EE3900/blob/master/Assignment1/
    codes/q3/xn.py
```

3.2 Let

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2),$$
$$y(n) = 0, n < 0 \tag{3.2}$$

Sketch $y(n)$.
Solution: The following code yields Fig. 3.2.

```
https://github.com/PradeepMundlik/
    EE3900/blob/master/Assignment1/
    codes/q3/yn.py
```

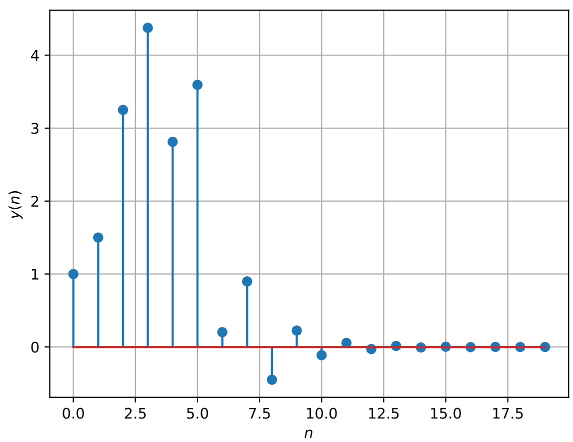

Fig. 3.2

4 Z-transform

4.1 The Z-transform of $x(n)$ is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \tag{4.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{4.3}$$

Solution: From (4.1),

$$\mathcal{Z}\{x(n-1)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{1-n} \tag{4.4}$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1}\sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.5}$$

$$= z^{-1}X(z) \tag{4.6}$$

resulting in (4.2). Similarly, it can be shown that

$$\mathcal{Z}\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-k)z^{k-n} \tag{4.7}$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-k} \tag{4.8}$$

$$= z^{-k}\sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.9}$$

$$= z^{-k}X(z) \tag{4.10}$$

4.2 Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.11}$$

from (3.2) assuming that the Z-transform is a linear operation.
Solution: Applying (4.7) in (3.2),

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{4.12}$$

$$\implies \frac{Y(z)}{X(z)} = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{4.13}$$

4.3 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.14}$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.15}$$

is

$$U(z) = \frac{1}{1-z^{-1}}, \quad |z| > 1 \tag{4.16}$$

Solution:

$$\mathcal{Z}\{\delta(n)\} = \sum_{n=-\infty}^{\infty} \delta(n)z^{-n} \tag{4.17}$$

$$= 1 \tag{4.18}$$

$$\delta(n) \overset{\mathcal{Z}}{\rightleftharpoons} 1 \tag{4.19}$$

and from (4.15),

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{4.20}$$

$$= \frac{1}{1-z^{-1}}, \quad |z| > 1 \tag{4.21}$$

using the fomula for the sum of an infinite geometric progression.

4.4 Show that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.22}$$

Solution:

$$\mathcal{Z}\{a^n u(n)\} = \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n} \tag{4.23}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n} \tag{4.24}$$

$$= \sum_{n=0}^{\infty} \left(az^{-1}\right)^n \tag{4.25}$$

$$= \frac{1}{1 - az^{-1}}, \quad \left|az^{-1}\right| < 1 \tag{4.26}$$

using the fomula for the sum of an infinite geometric progression.

4.5 Let

$$H\left(e^{j\omega}\right) = H\left(z = e^{j\omega}\right). \tag{4.27}$$

Plot $|H\left(e^{j\omega}\right)|$. Comment. $H(e^{j\omega})$ is known as the *Discret Time Fourier Transform* (DTFT) of $x(n)$.

Solution: The following code plots Fig. 4.5.

```
code of dtft
```

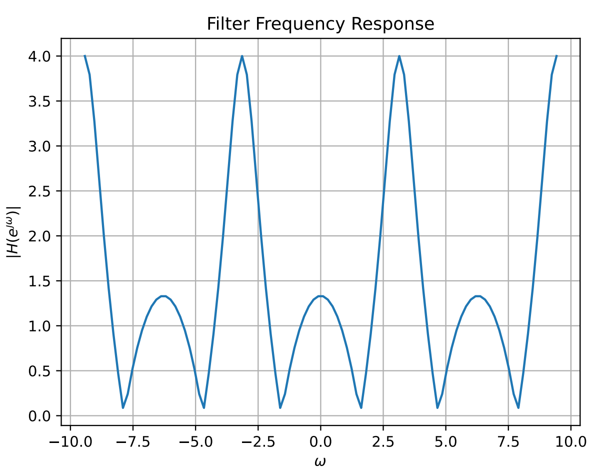

Fig. 4.5: $|H\left(e^{j\omega}\right)|$