Digital Signal Processing

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1 Question

The input to a casual linear time-invariant system is

$$x[n] = u[-n-1] + \left(\frac{1}{2}\right)^n u[n] \tag{1.1}$$

The z-transform of the output of this system is

$$Y(z) = \frac{-\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+z^{-1}\right)} \tag{1.2}$$

(c) Determine $y[n]$.

2 Solution

$$-\frac{1}{2}z^{-1} = \frac{1}{3}\left(\left(1-\frac{1}{2}z^{-1}\right)-\left(1+z^{-1}\right)\right) \tag{2.1}$$

$$Y(z) = \frac{\frac{1}{3}}{1+z^{-1}} + \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} \tag{2.2}$$

$$Y(z) = \frac{1}{3}\sum_{n=0}^{\infty}(-z^{-1})^n - \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{1}{2}z^{-1}\right)^n \tag{2.3}$$

$$\implies y[n] = \frac{1}{3}(-1)^n u[n] - \frac{1}{3}\left(\frac{1}{2}\right)^n u[n] \tag{2.4}$$

where,

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & else \end{cases} \tag{2.5}$$