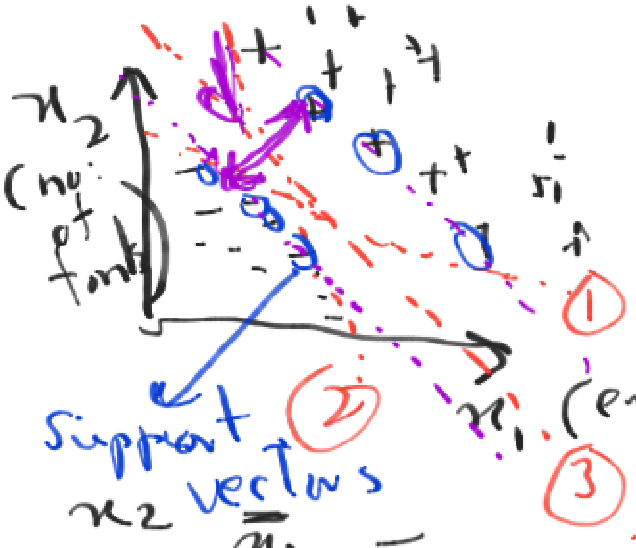
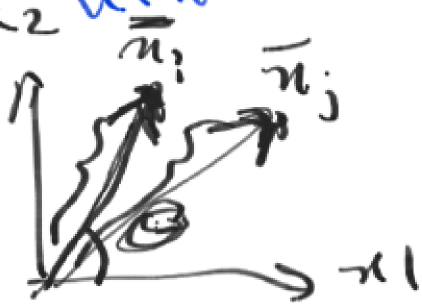
Svm

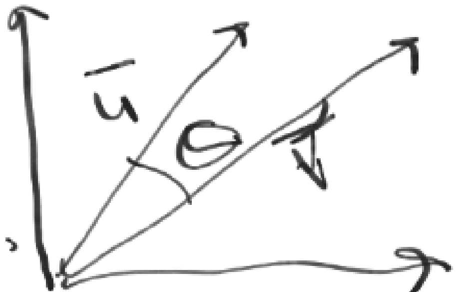


hyperplane



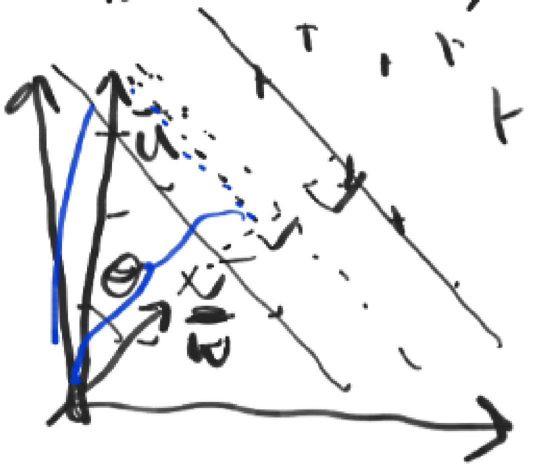
Scalar - ~~mg~~ magnitude

vector - " + direction



$\vec{u} \cdot \vec{v} = |\vec{u}| \times |\vec{v}| \times \cos\theta$

dot product



$|\vec{u}|\cos\theta \; |w| \geq c$ then (+)

$|\vec{u}||\vec{w}|\cos\theta \geq c$

$\bar{u} \cdot \bar{w} \geq c$ then (+)

$\boxed{\bar{u} \cdot \bar{w} + b \geq 0 \text{ then } (+)}$

$\bar{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}$

where $c = -b$

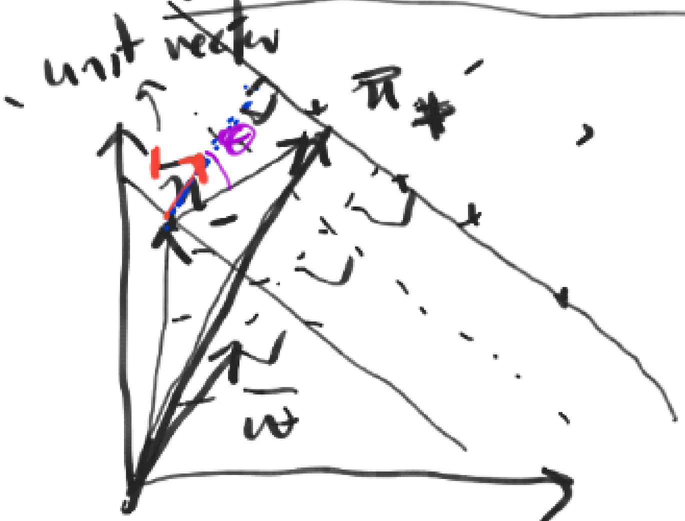
$$\bar{w} \cdot \bar{x}_+ + b \geq 1 \quad \text{then } (+) \qquad \begin{pmatrix} 1 \\ 12 \\ 3 \\ \vdots \end{pmatrix}$$

$$\bar{w} \cdot \bar{x}_- + b \leq -1 \quad \text{then } (-)$$

$$\boxed{y_i (\bar{w} \cdot \bar{x}_i + b) \geq 1} \rightarrow \text{hypothesis}$$

when $y_i = +1$ for $(+)$ & $y_i = -1$ for $(-)$

$$\boxed{y_i (\bar{w} \cdot \bar{x}_i + b) - 1 = 0} \text{ for support vectors.} \rightarrow \text{Constraint}$$



$$(\bar{x}_+ - \bar{x}_-)$$

$$d = |\bar{x}_+ - \bar{x}_-| \cos\theta$$

$$d = |\bar{x}_+ - \bar{x}_-| \times 1 \times \cos\theta$$

$$= (\bar{x}_+ - \bar{x}_-) \cdot \{\text{Any vector of size 1 } \perp \text{ to the boundaries}\}$$

$$= (\bar{x}_+ - \bar{x}_-) \cdot \frac{\bar{w}}{|\bar{w}|}$$

$$= \frac{(\bar{x}_+ \cdot \bar{w} - \bar{x}_- \cdot \bar{w})}{|\bar{w}|} = \frac{(1-b) - (-1-b)}{|\bar{w}|}$$

$+1(\vec{w}\cdot\vec{x}_+ + b) - 1 = 0 \Rightarrow \vec{w}\cdot\vec{x}_+ = 1 - b$

$-1(\vec{w}\cdot\vec{x}_- + b) - 1 = 0 \Rightarrow \vec{w}\cdot\vec{x}_- = (-1-b)$

$d = \frac{2}{|\vec{w}|}$ $\text{Max}_m \frac{2}{|\vec{w}|}$ $\text{Max}_m \frac{1}{|\vec{w}|}$

$\text{Min } |\vec{w}|$

opt. fn $\leftarrow$ $\boxed{\text{Min } \frac{1}{2}|\vec{w}|^2}$

$y_i(\vec{w}\cdot\vec{x}_i + b) - 1 = 0 \rightarrow$ Constraint

Quadratic optimization fn

Max

$L = \frac{1}{2}|\vec{w}|^2 - \sum_i \alpha_i \left[y_i(\vec{w}\cdot\vec{x}_i + b) - 1 \right]$

$\frac{\partial L}{\partial \vec{w}} = 0 = \vec{w} - \sum_i \alpha_i y_i \vec{x}_i \Rightarrow \boxed{\vec{w} = \sum_i \alpha_i y_i \vec{x}_i}$

$\frac{\partial L}{\partial b} = 0 = -\sum_i \alpha_i y_i = 0 \Rightarrow \boxed{\sum_i \alpha_i y_i = 0}$

$\vec{w}\cdot\vec{w} = |\vec{w}| \times |\vec{w}| \times \underbrace{\cos\theta}_{=1}$

$L = \frac{1}{2}\left(\sum_i \alpha_i y_i \vec{x}_i\right)\cdot\left(\sum_j \alpha_j y_j \vec{x}_j\right)$

$$= \sum_i \alpha_i y_i \bar{x}_i \cdot \Big(\sum_j \alpha_j y_j x_j\Big) - \underbrace{\sum_i \alpha_i y_i}_{=0} b + \sum_i \alpha_i$$

Mean

$(1+\bar{x}_i\cdot x_j)^2$ $\quad$ $\phi(\bar{x}_i)\cdot\phi(\bar{x}_j)$

$$L = \sum_i \alpha_i - \frac{1}{2}\sum_i\sum_j \alpha_i \alpha_j y_i y_j \boxed{\bar{x}_i \cdot \bar{x}_j}$$

$(1+\bar{x}_i\cdot\bar{u})^2$

$$\sum_i \alpha_i y_i \boxed{\bar{x}_i \cdot \bar{u}} + b \;\begin{cases} \geq 1 \to (+) \\ \leq -1 \to (-) \end{cases} \qquad \begin{pmatrix} 1 \\ 12 \\ 5 \\ \vdots \end{pmatrix}$$

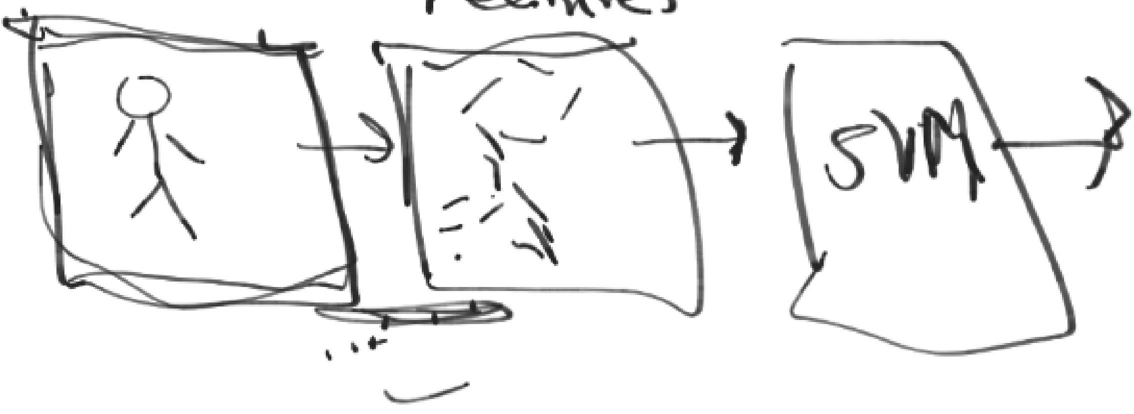
$\phi(\bar{x}_i)\cdot\phi(\bar{u})$

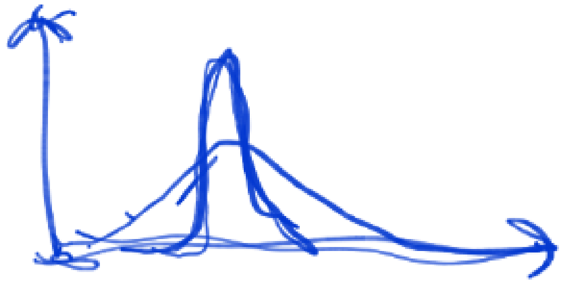
$y_i(\bar{w}\cdot\bar{x}_i + b) \geq 1 \qquad y = 1$ for $+$, -1 for $-$

$y_i(\bar{w}\cdot\bar{x}_i + b) \geq 1 - \varepsilon$

$\bar{w}\cdot\bar{x}_+ + b \geq 1-\varepsilon, \qquad \bar{w}\cdot\bar{x}_- + b \leq -1+\varepsilon$

Features





$\uparrow \text{gamma} = \frac{1}{\sigma \downarrow} \rightarrow$ overfitting

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