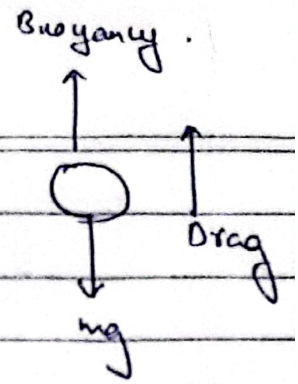
ANALYTICAL ANALYSIS



$$\frac{m\,du}{dt} = W - F_B - F_D$$

$$= mg - V\rho g - 6\pi R\eta u.$$

$\cancel{\text{let } mg = V\rho g = k}$

$$\frac{du}{dt} = g - \frac{V\rho g}{m} - \frac{6\pi R\eta u}{m}$$

$$\text{let } g - \frac{V\rho g}{m} = k$$

$$\text{and, } \frac{6\pi R\eta}{m} = a$$

$$\frac{du}{dt} = k - au$$

$$du = (k - au)\,dt$$

$$\frac{du}{k - au} = dt$$

$$\Rightarrow \int_{V_{t=0}}^{V(t)} \frac{du}{k - au} = \int_{t=0}^{t=t} dt$$

$$k - au = p$$

$$dp = -a\,du$$

$$du = -\frac{1}{a}\,dp$$

$$\int -\frac{1}{a}\frac{dP}{P} = t$$

$$\left[\ln p\right]_{u_t=0}^{u(t)} = -at$$

$$\left[\ln(K - au(t)) - \ln(K)\right]$$

$\Downarrow$ at $t = t$ $\qquad$ $\Downarrow$ at $t = 0$

$$\ln\left(\frac{K - au(t)}{K}\right) = -at$$

$$\frac{K - au(t)}{K} = e^{-at}$$

$$u(t) = \frac{K}{a}\left(1 - e^{-at}\right)$$

Substituting Value of K and a.

$$u(t) = \left[\frac{g - \frac{Vpg}{m}}{\frac{6\pi R\eta}{m}}\right]\left[1 - e^{-\frac{6\pi R\eta}{m}t}\right]$$

Putting Values of ~~K~~ $r = 10^{-5}$ m

$\rho_l = 1000 \, kg/m^3$

$\rho_s = 8050 \, kg/m^3$

$\eta = 10^{3}$ Pa.s

$g = 9.8$

$$K = 9.8 - \frac{1000}{8050} \times 9.8$$

$$= 9.8\left(1 - \frac{100}{865}\right)$$

$$= 9.8\,(1 - .124)$$

$$= \underline{\underline{8.58}}$$

$$a = \frac{{}^{3}\cancel{6}\cancel{\pi} K \eta}{\rho_{solid} \times \cancel{4}/\cancel{3} \cancel{\pi} R^2}$$

$$= \frac{9 \times 10^{-3}}{8050 \times 10^{-10}}$$

$$= 0.00111 \times 10^{7}$$

$$= \underline{0.11 \times 10^{5}}$$

$$= \boxed{11.18 \times 10^{3}}$$

Scanned with CamScanner

Analytical answer

$$u(t) = \frac{8.58}{11.18 \times 10^{-3}}\left(1-e^{-(11.18\times10^{-3}t)}\right)$$

$$u(t) = 767.44\left(1-e^{-11.18\times10^{-3}t}\right)$$

$$u(t=0) = 0$$

Numerical Analysis

↳ Forward Euler Meathod.

Based on plotting curve at fixed intervals using the derivative at that point

We have

$$\frac{du}{dt} = g - \frac{Vpg}{m} - \frac{6\pi R\eta u}{m}$$

We can plot the curve by taking t on x axis and,

h is step

$$t_{i+1} = t_i + h$$

$$u_{i+1} = u_i + h \times \frac{du}{dt}$$

$$u_{i+1} = u_i + g - \frac{Vpg}{m} - \frac{6\pi R\eta u_i}{m}$$

Scanned with CamScanner