Stamatics Mini Project - I

1. Algorithm for obtaining MLE of β (vector of Regression Coefficients).

Let $i = 1 \cdots n$, let $x_i = (1, x_{i2} \cdots, x_{ip})$ be vector of covariates For i^{th} observation. $\beta \in R^p$. Suppose y_i is realization of Y_i with

$$Y_i \sim \text{Bern}(\phi(x_i^T\beta))$$

Where $\phi(\cdot)$ is the CDF of Standard normal distribution.

Here Likelihood function L is

$$L = \prod_{i=1}^{n} (\phi(x_i^T\beta))^{y_i} (1 - \phi(x_i^T\beta))^{(1-y_i)}$$

Taking log likelihood

$$\log(L) = \sum_{i=1}^{n} y_i \log(\phi(x_i^T\beta)) + (1-y_i)\log(1 - \phi(x_i^T\beta))$$

To find MLE of β we need to find β such that likelihood function L is maximum. As it can be seen that maximum value of L cannot be greater than 1, hence $\log(L)$ can not be greater than 0. Also we know that log-likelihood function is globally concave in β (see references). Hence to find MLE of β we need to find β such that $\frac{\partial \log(L)}{\partial \beta_j} = 0$ where $\beta_j \in \beta$, $1 \le j \le p$. To achieve this We would find β such β_j one by one by a technique very similar to Newton Raphson Method.

Now consider j such that $1 \le j \le p$. And initialize β to any vector. also $\beta_j \in \beta$

now let

$m_1 = 0$ {it can be any value, we took it 0}.

and

$$m_2 = m_1 - \frac{\left(\frac{\partial \log(L)}{\partial \beta_j}\Big|_{\beta_j = m_1}\right)}{\left(\log(L)\Big|_{\beta_j = m_1}\right)}$$

Now iterate this process till $\frac{\left(\frac{\partial \log(L)}{\partial \beta_j}\Big|_{\beta_j = m_1}\right)}{\left(\log(L)\Big|_{\beta_j = m_1}\right)} \to 0$

or $m_2 \to m_1$. Just like we do in newton raphson method. Finally obtained m_2 is optimal value of β_j. Repeat this process of all values of j to get MLE of β.

We were able to apply this technique because our function was concave and less than or equal to zero. This method is known to work in such special cases. (Well I don't know about other cases as found this technique myself).

So now we have our MLE of β.

References :

1. Course Material provided by mentors.
2. Wikipedia page about Probit Model : https://en.wikipedia.org/wiki/Probit_model
3. Brilliant Page about Newton Raphson Method. : https://brilliant.org/wiki/newton-raphson-method/

Result of Code of Part(2) and Part(3)

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MLE estimates of beta coefficients are :  [-0.28899739 -0.51030861  0.24067783  0.00770233  0.04720332  0.29149678]
Score is: 41/50
Hence accuracy is : 82.0%
Log liklihood is:  -382.0021416062404
Probablity of survival of Jack :  0.27972159780777384
Probablity of survival of Rose :  0.9956798837985218
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