Assignment-5
Numerical Differential

Q1 Values for $f(x) = xe^x$ are given below

x	1.8	1.9	2.0	2.1	2.2
$f(x)$	10.889365	12.703199	14.778122	17.148057	19.855030

Use all applicable three point formulas and five point formulas to approximate $f'(2.0)$

Ans Three point endpoint formula

$$f'(x_0) = \frac{1}{2h}[-3f(x_0)+4f(x_0+h) - f(x_0+2h)]$$

Here $h = 0.1$

So

$$f'[2.0] = \frac{1}{2 \times (0.1)}[-3f(2.0)+4f(2.1) - f(2.2)]$$

$$= 5[-3\times(14.778122)+4\times(17.148057) - (19.855030)]$$

$$f'[2.0] = 22.03216$$

→ Three point midpoint formula

$$f'(x_0) = \frac{1}{2h}(-f(x_0-h)+f(x_0+h))$$

$$f'(2.0) = 5[-f(1.9) + f(2.1)]$$

$= 5[-12.703199 + 17.148957]$

$f'(2.0) = 22.22879$

→ Three point endpoint formula

$f'(x_0) = \frac{1}{2h}[f(x_0 - 2h) - 4f(x_0 - h) + 3f(x_0)]$

$f'(2.0) = 22.054675$

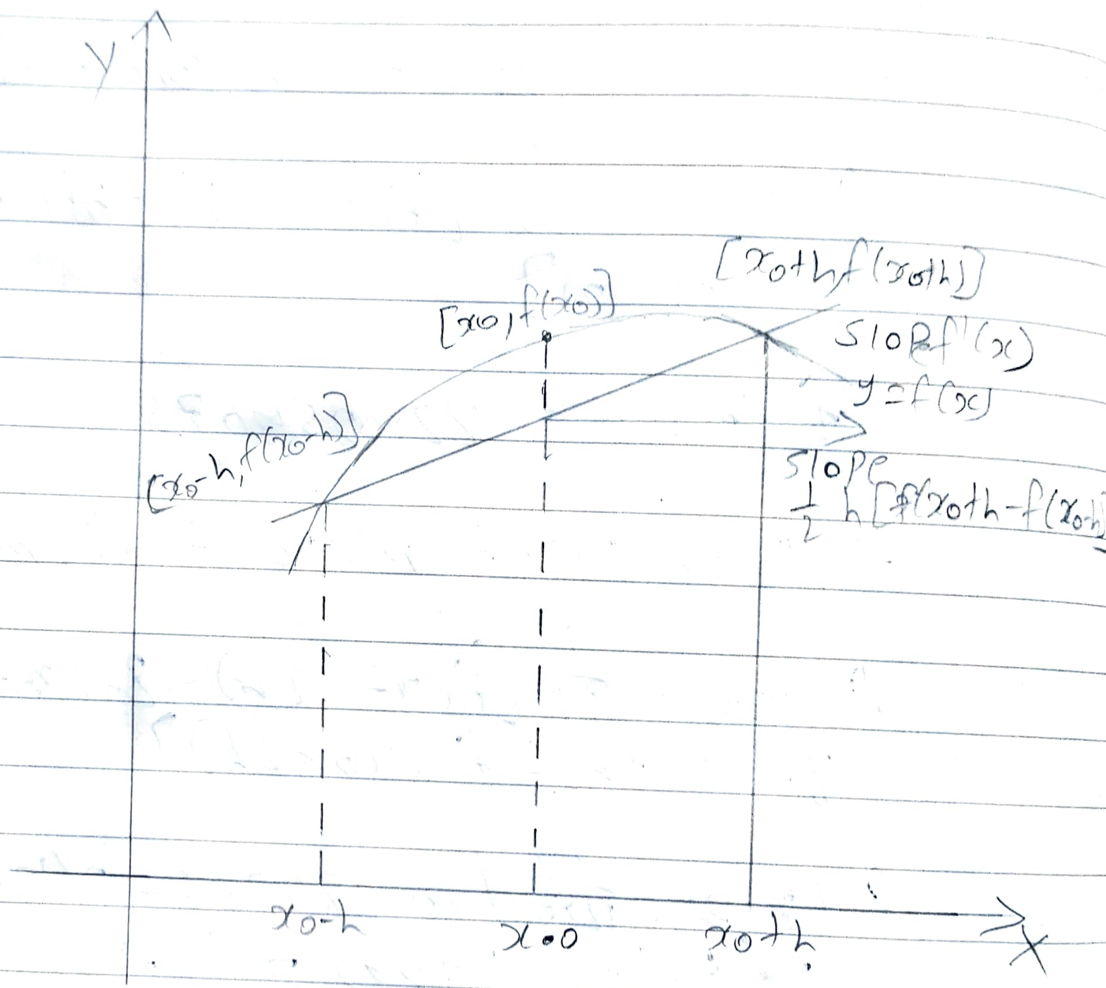
→ Five point midpoint formula.

$f'(x_0) = \frac{1}{12h}[f(x_0 - 2h) - 8f(x_0 - h) + 8f(x_0 + h) - f(x_0 + 2h)]$

$f'(2.0) = \frac{1}{12 \times 0.1}[10.889365 - 8(12.703199) + 8(17.148937) - 19.855030]$

$f'(2.0) = 22.1689992$

Q2 Give Graphical representation of three point midpoint formula.



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Q3 Explain how h too small is not advantageous in numerical differentia

Ans In each of the numerical Differential formulas, h occurs in denominator, As h is made smaller and smaller to increase accuracy, division by small number causes round off errors to increase. Moreover in numerator also, difference of almost equal values occurs, which also contributes to round off errors. As a result, beyond a certain value, h can not be reduced further, as round off errors. As a result beyond a certain value, h can not be reduced further, as round off errors start dominating and accuracy cannot be improved further.

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Q5. In a circuit with impressed voltage $E(t)$ and inductane L, Kirchhoff's first law gives the relationship

$$E(t) = L\frac{di}{dt} + Ri,$$

where R is the resistance in the circuit and i is the current. Suppose we measure the current for several values of t and obtain.

t	1.00	1.01	1.02	1.03	1.04
i	3.10	3.12	3.14	3.18	3.24

where t is measured in seconds, in is in ameres, the inductance L is a constant 0.98 hennaries and the resistance is 0.142 ohms. Approximate the voltage $E(t)$ when $t = 1.00, 1.01, 1.03, 1.04$.

Ans Here

$$E(t) = L\frac{di}{dt} + Ri \quad \text{where } L = 0.98$$

$$h = 0.01, \quad R = 0.142$$

To find $\mathcal{E}(t)$ at $t = 1.00$ we need to find $\frac{di}{dt}$ at 1.00

By three point end point formula,
$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)]$

$\therefore f'(1.00) = \frac{1}{2 \times 0.01}[-3(3.10) + 4(3.12) - 3.14]$

$= 50 \times (0.04) = 2$

So, $\mathcal{E}(1.00) = 0.98 \times 2 + 0.142 \times (3.10)$

$\mathcal{E}(1.00) = 2.4002$ — (1)

→ now at $t = 1.01$
By three point midpoint formula,

$f'(x_0) = \frac{1}{2h}[-f(x_0-h) + f(x_0+h)]$

$\therefore f(1.01) = 50[-3.10 + 3.14]$
$f(1.01) = 2$

So,
$\mathcal{E}(1.01) = 0.98 \times 2 + 0.142 \times (3.12)$
$= 2.40304$ ← (2)

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→ now at $t = 1.02$
By five point midpoint formula,

$$f'(1.0) = \frac{1}{12 \times h}[f(x_0 - 2h) - 8f(x_0 - h) + 8f(x_0 + h) - f(x_0 + 2h)]$$

$$f'(1.02) = \frac{1}{0.12}[3.10 - 8(3.12) + 8(3.18) - (3.24)]$$

$$f'(1.02) = 2.83$$

So

$$E(1.02) = 0.98 \times 2.83 + 0.142 \times (3.14)$$
$$E(1.02) = 3.21928 \text{ --- } (3)$$

→ now at $t = 1.03$
By three point midpoint formula,

$$f'(1.03) = \frac{1}{2 \times 0.01}[-3.14 + 3.24]$$
$$= 5$$

So

$$E(1.03) = 0.98 \times 5 + 0.142 \times 3.18$$
$$= 5.35156 \text{ --- } (4)$$

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→ now at $t = 1.04$
By three point endpoint formula,

$$f'(x_0) = \frac{1}{2h}[f(x_0-2h) - 4f(x_0-h) + 3f(x_0)]$$

$$f'(1.04) = 50[3.14 - 4(3.18) + (3.24)]$$

$$= 7$$

so

$$E(1.04) = 0.98 \times 7 + 0.142 \times 3.24$$
$$E(1.04) = 7.32008 \quad \text{——} \quad (5)$$