Computer Oriented Statistical Methods

Assignment - 2 Variation

Q1

x :	11	0	36	21	31	23	24	-11	-11	-21
y :	10	-2	29	14	22	18	14	-2	-3	10

(a) compute Σx, Σx^2, Σy and Σy^2

→

x	y	x^2	y^2
11	10	121	100
0	-2	0	4
36	29	1296	841
21	14	441	196
31	22	961	484
23	18	529	324
24	14	576	196
-11	-2	121	4
-11	-3	121	9
-21	10	441	100
103	90	4607	2258

so, $\Sigma x = 103$ $\quad \Sigma x^2 = 4607$

$\Sigma y = 90$ $\quad \Sigma y^2 = 2258$

(b) use the results of part (a) to compute the sample mean, variance and standard deviation for x and for y.

→ Mean, variance and standard Deviation for x:

x	$\bar{x}$ (Mean)	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
11	10.3	0.7	0.49
0	10.3	−10.3	106.09
36	10.3	25.7	660.49
21	10.3	10.7	114.49
31	10.3	20.7	428.49
23	10.3	12.7	161.29
24	10.3	13.7	187.69
−11	10.3	−21.3	453.69
−11	10.3	−21.3	453.69
−21	10.3	−31.3	979.69
103		0	3546.10

Mean: $\bar{x} = \frac{103}{10} = 10.3$

Variance: $s^2 = \frac{3546.10}{9} = 394.01$

Standard Deviation: $s = \sqrt{394.01}$

$= 19.85$

Mean, Variance and Standard Deviation for y:

y	$\bar{x}$ (Mean)	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
10	9	1	1
-2	9	-11	121
29	9	20	400
14	9	5	25
22	9	13	169
18	9	9	81
14	9	5	25
-2	9	-11	121
-3	9	-12	144
-10	9	-19	361
90		0	1448

Mean: $\bar{X} = \frac{90}{10} = 9$

Variance: $S^2 = \frac{1448}{9} = 160.89$

Standard Deviation: $S = \sqrt{160.89}$

$= 12.68$

(c) compute a 75% chebyshev interval around the mean for x values and also for y values.
Use the intervals to compare the two funds.

→ we know that 75% of the observation have values within two standard deviations of the mean.

So, $\bar{x} \pm 2s = 10.3 - 2(19.85)$
$= -29.4$
and $10.3 + 2(19.85)$
$= 50$

So, for x it is -29.4 to 50.

For y $\bar{x} \pm 2s = 9 - 2(12.68)$
$= -16.36$
and $9 + 2(12.68)$
$= 34.36$

So, for y it is -16.36 to 34.36.

Vanguard Balanced Index (y) has a smaller spread than Vanguard Total Stock Index (x).

(d) compute the coefficient of variation for each funds. Use the coefficient of variation to compare The two funds. If s represents risks and $\bar{x}$ represents expected return, then s/ $\bar{x}$ can be thought of as a measure of risk per unit of expected return.
In This case, why is smaller CV better? Explain.

→ co-efficient of variation

$$= \frac{\text{Standard Deviation}}{\text{Mean}} \times 100\%$$

For x:

$$CV = \frac{19.85}{10.3} \times 100\%$$

$$= 192.7\%$$

For y:

$$CV = \frac{12.68}{9} \times 100\%$$

$$= 140.9\%$$

If s represents risks and $\bar{x}$ represents expected return, then as the s/ $\bar{x}$ is lesser, lower is the risk.
So, a smaller CV is better because it means a lower risk.

Q2 Let x be a random variable representing time to failure (in hours) at 90% breaking strength.

0.54	1.80	1.52	2.05	1.03	1.18	0.80	1.33	1.29
1.11	3.34	1.54	0.08	0.12	0.60	0.72		
0.92	1.05	1.43	3.03	1.81	2.17	0.63	0.09	
0.56	0.03	0.09	0.18	0.34	1.51	1.45		
1.52	0.19	1.55	0.02	0.07	0.65	0.40		
0.24	1.51	1.45	1.60	1.8	4.69	0.08		
7.89	1.58	1.64	0.03	0.23	0.72			

62.11

(a) Find the range.

→ Range = Maximum value − Minimum value
= 7.89 − 0.02
= 7.87

(b) compute Mean, variance and Standard Deviation.

→ Mean : $\bar{x} = \frac{\Sigma x}{n} = \frac{62.11}{50} = 1.24$

Mean = 1.24

x_i	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$	x_i	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$
0.54	−0.7	0.49	0.09	−1.15	1.32
1.80	0.56	0.31	0.18	−1.06	1.12
1.52	0.28	0.08	0.34	−0.9	0.81
2.05	0.81	0.66	1.51	0.27	0.07
1.03	−0.21	0.04	1.45	0.21	0.04
1.18	−0.06	0.0036	1.52	0.28	0.09
0.80	−0.44	0.19	0.19	−1.05	1.10
1.33	0.09	0.0081	1.55	0.31	0.10
1.29	0.05	0.0025	0.02	−1.22	1.49
1.11	−0.13	0.02	0.07	−1.17	1.37
3.34	2.1	4.41	0.65	−0.59	0.35
1.54	0.3	0.09	0.40	−0.84	0.71
0.08	−1.16	1.35	0.24	−1	1
0.12	−1.12	1.25	1.51	0.27	0.07
0.60	−0.64	0.41	1.45	0.21	0.04
0.72	−0.52	0.27	1.60	0.36	0.13
0.92	−0.32	0.10	1.80	0.56	0.31
1.05	−0.19	0.04	4.69	3.45	11.90
1.43	0.19	0.04	0.08	−1.16	1.35
3.03	1.79	3.20	7.89	6.65	44.22
1.81	0.57	0.32	1.58	0.34	0.12
2.17	0.93	0.86	1.64	0.4	0.16
0.63	−0.61	0.37	0.03	−1.21	1.46
0.56	−0.68	0.46	0.23	−1.01	1.02
0.03	−1.21	1.46	0.72	−0.52	0.27
					89

62.11

Variance : $s^2 = \frac{89}{50} = 1.78$

Standard Deviation: $s = \sqrt{1.78} = 1.33$

(d) Compute the coefficient of variation. What does this number say about time to failure? Why does a small CV indicate more consistent data, whereas a larger CV indicates less consistent data? Explain.

→ Coefficient of variation

$$= \frac{\text{Standard Deviation}}{\text{Mean}} \times 100\ \%$$

$$= \frac{1.33}{1.24} \times 100\ \% = 107\ \%$$

$CV = 107\ \%$

The standard Deviation of the time to failure is just slightly larger than the average time.

A smaller CV indicates more consistent data because the value of s (standard deviation) in the numerator is smaller.

Q3/ Given:-

Pax world Balanced:

Mean = 9.58 %

Standard Daviation S = 14.05 %

Vanguard Balanced Index:

Mean = 9.02 %

Standard Deviation S = 12.50 %

(a) compute the coefficient of variation for each fund.

→ For pax World Balanced:

$$\text{Coefficient of variation } CV = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100\%$$

$$CV = \frac{14.05}{9.58} \times 100\%$$

$$\boxed{CV = 146.7\%}$$

For Vanguard Balanced Index:

$$CV = \frac{12.50}{9.02} \times 100\%$$

$$\boxed{CV = 138.6\%}$$

Here we are using the coefficient of variation to represent risk per unit of return because coefficient of variation indicates how large is the Standard Deviation relative to the Mean.

From this point of view, the risk per unit of return of Vanguard Balanced Index is slightly lesser than that of Pax World Balanced.

So, the fund of Vanguard Balanced Index appears to be better.

Q4 Given average number of physician visits by males per year.

Mean = 2.2
coefficient of variation = 1.5 %
Determine Standard of Deviation.

→ $CV = \frac{S}{x} \times 100\%$

So, $S = 0.015 \times 2.2 = 0.033$

So, the standard deviation of the annual number of visits to physicians made by males is 0.033