

Assignment -3

Solution of ordinary

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Numerical methods

Q1 Define a differential equation.

What is meant by solution of a differential equation? Verify that

(i) $y = C_1 e^x + C_2 e^{-x}$ is a solution of the differential $y'' - 2y' + y = 0$.

(ii) $y = 2 \sqrt{C-x}$ is a solution of the differential equation $y' + \frac{1}{y} = 0$.

Ans Define a differential eq.

→ A differential equation is an equation involving independent variable, dependent variable and its one or more derivatives

ex

$$y'' + y = 0$$

Here y is dependent variable,
 y is function of x

⇒ Solution of differential equation

A solution of a differential equation is a specific function

that satisfies the equation.

(i) Her differential equation,

$$y'' - 2y' + ty = 0 \quad (1)$$

$$\text{let } y(x) = C_1 e^x + (C_2 x e^x) \quad (1)$$

$$y'(x) = C_1 e^x + (C_2 \frac{d}{dx}(x e^x))$$

$$= C_1 e^x + (C_2 (e^x x + e^x))$$

$$\frac{d}{dx}(x)$$

$$= C_1 e^x + (C_2 (x e^x + e^x))$$

$$y'(x) = C_1 e^x + C_2 e^x + (C_2 x e^x) \quad (2)$$

$$y''(x) = C_1 e^x + C_2 e^x + (C_2 (e^x + x e^x))$$

$$y''(x) = C_1 e^x + 2(C_2 e^x) + (C_2 x e^x) \quad (3)$$

Substituting (1) (2) & (3) in (1),

$$0 = C_1 e^x + 2(C_2 e^x) + (C_2 x e^x) - 2(C_1 e^x)$$

$$(C_2 e^x + (C_2 x e^x)) + (C_1 e^x + (C_2 x e^x))$$

$$\begin{aligned}
 &= 2C_1 e^x + 2C_2 e^x + 2C_2 x e^x \\
 &- 2C_1 e^x - 2C_2 e^x - 2C_2 x e^x \\
 &\equiv 0
 \end{aligned}$$

$\therefore y = C_1 e^x + C_2 x e^x$ is a solution

$$\text{of } y'' - 2y' + y = 0$$

(ii) Here differential between equation is $y' + \frac{1}{y} = 0$

$$\text{Let } y = 2\sqrt{c-x} = 2(c-x)^{\frac{1}{2}} \quad \text{--- (1)}$$

$$y' = 2 \times \frac{1}{2} (c-x)^{\frac{1}{2}-1} \times \frac{d}{dx}(c-x)$$

$$= (c-x)^{\frac{1}{2}} \times -1 = -\frac{1}{\sqrt{c-x}} \quad \text{--- (2)}$$

$$\text{②} \Rightarrow \frac{-1}{\sqrt{c-x}} + \frac{1}{2\sqrt{c-x}} \neq 0$$

$\therefore y = 2\sqrt{c-x}$ is not a soln.

$$\text{of } y' + \frac{1}{y} = 0$$

Q2 how is ordinary differential equation different from Partial differential equation? Give one example of each.

Anc.

When dependent variable is a function of only one variable, then all the derivatives involved in the equation are ordinary and the equation is called ordinary differential equation.

- if dependent variable is a function of more than one independent variable, that is, it is a function of several variables, then the equation contains partial derivations with respect to different independent variables and the equation is called Partial Differential Equations.

⇒ Example of ordinary

$$y'' + by = 0$$

$$y' + y = 0$$

⇒ Example of Partial

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (\text{Laplace's Eq})$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y) \quad (\text{Poisson's Eq})$$

Q3 Differentiate between initial value problem and Boundary value problem. classify following differential equation in initial value problem and Boundary value problem.

Anc.

→ If the condition are specified at a single point these condition are called initial condition and the differential eq combined with conditions is called initial value problem.

2 If the condition are specified at more than one point these condition are called Boundary condition and the differential Eq combined with conditions is called Boundary value problem

$$(i) y' = t - y, y(0) = 1$$

→ Initial Value Problem

$$(ii) y'' + 9y = e^t \sin 3t, y(0) = 1, y'(0) = 2$$

→ Boundary Value Point problem

$$(iii) y'' + y = 0, y(0) = 2, y'(0) = 2$$

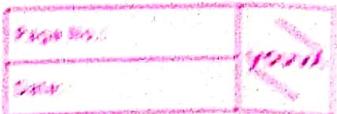
→ Boundary Value Point problem

$$(iv) y'' + y' = t^2 + y^2, y(0) = 1, \\ y'(0) = 1$$

→ Boundary Value Point problem

$$(v) y' = t^2 + y^2, y(0) = 1$$

→ Initial Value problem



Ques. determine Order and degree
of following differential eq.

Ans

$$(i) y'' + 4y = e^x$$

→ order = 2, degree = 1

$$(ii) y'' + 4(y')^3 + y^2 = x^3 + y^2$$

→ order = 2, degree = 1

$$(iii) (y'')^2 + (y')^3 + 3y = 5x$$

→ order = 2, degree = 2

$$(iv) y' + 2y^2 = x^2$$

→ order = 1, degree = 1

Q5. What are the characteristics of single step numerical methods to find solution of first order, first degree IVP?

Ans. These are the characteristics of single step numerical methods.

- ① It is direct
- ② It is non iterative
- ③ It is based on Taylor series method.
- ④ It is used estimating at previous step & function information.
- ⑤ Practically, error cannot be estimated.

Q6 Differentiate between single step and multistep numerical methods to obtain solution of IVP.

Ans

Single Step methods

→ Direct

→ Not iterative

→ Based on Taylor Series method

→ Self starting.

→ Practically, accuracy cannot be estimated.

Multistep method

→ More than one previous step

→ Iterative.

→ Predictor Corrector formulas.

→ Not self starting

→ Practically, accuracy can
be estimated.