Pradip Karmakar

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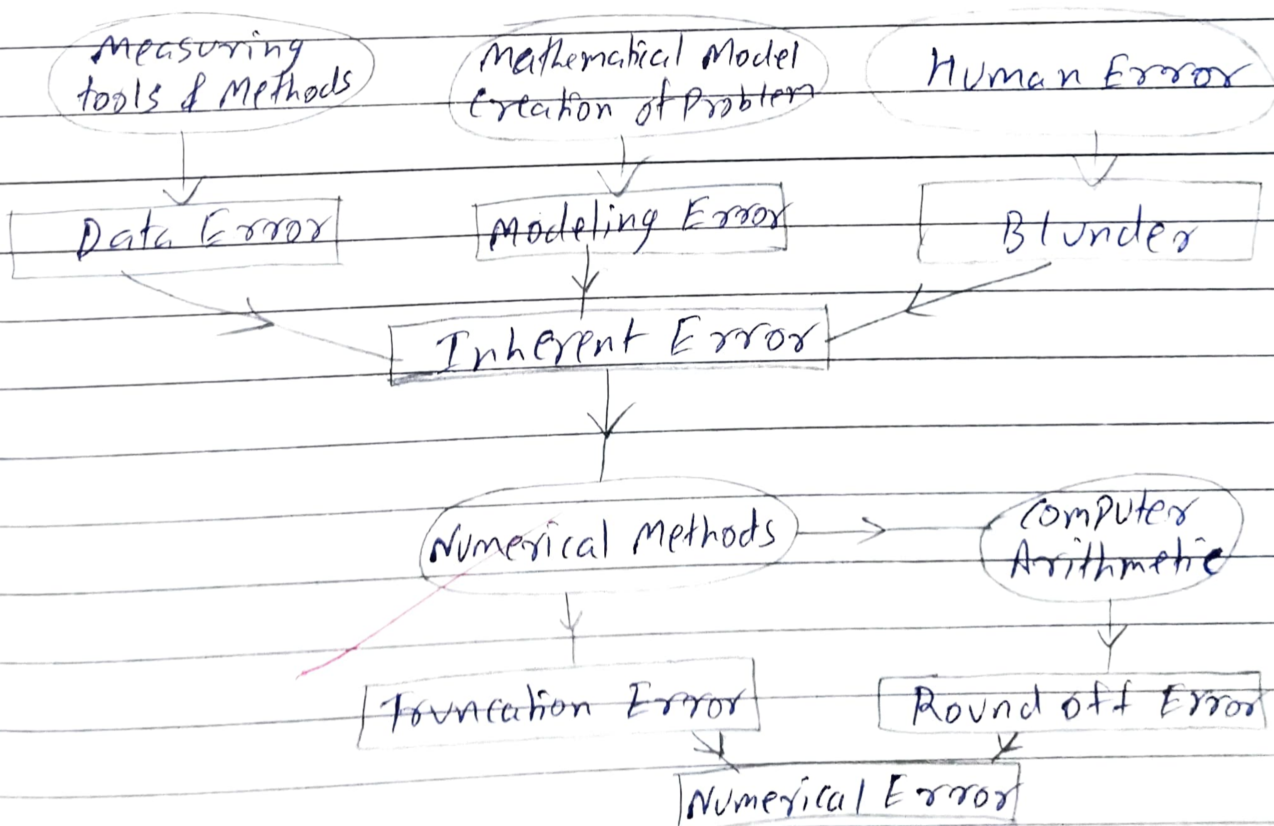
Explain the different types of errors that occur during computation.

Ans:-

There are two types of errors that occur during computation

(1) Truncation Error
(2) Round off Error

These two errors are being inherited by the numerical analyst are called Inherent Errors.



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Truncation Error

Truncation error is an error in implementation of numerical approximation method occurring due to ~~tra~~ truncations a process involving infinite number of steps to ~~of~~ finite numbers of steps, like

(i) Limiting infites step/series to finite number of terms

(ii) Limiting infinite number of iterations to finite number of iteration $(f(x)=0)$

(iii) Taking finite step size instead of infinitesimal step size.

Round off error

Round off error occur due to finite Precision in a computer. A number may not always have a finite representation

$\frac{1}{3} = 0.3333....$

$\sqrt{2} = 1.4142135623...$

$e = 2.718281828 45.....;$

Whereas, 2, 10, 9.32 have finite representation in decimal system. Moreover, a number having finite representation in one number system may not have finite representation in another number system, for example.

$(1.1)_{10} = (1.00011\,0011\,0011\,00.....)_2$

One can never represent 1.1 exactly in binary system. So, let us understand first how a number is stored in a computer.

* Total Numerical error.

Total Numerical Error = Truncation error + Round of error.

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The only advisable way to reduce round off errors is to increase number of significant digits. The round off error increase due to either subtractive cancellation or due to increase in the number of computations in an analysis. The truncation error can be reduced by decreasing step size. So, determining appropriate step size is essential in order to balance truncation and round off error to minimize the total error.

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Explain the Bisection method graphically.

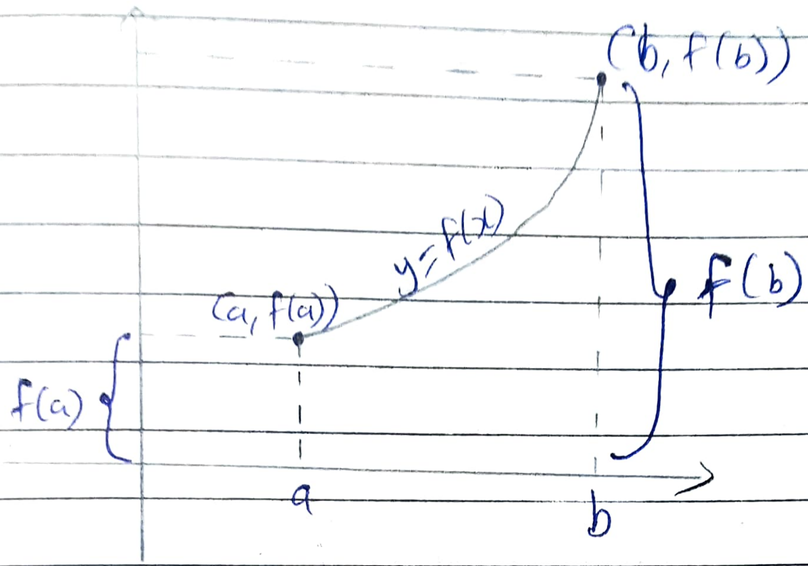
Bisection Method:

principle on which Bisection method is based ~~on~~ is as follows:

If $f(x)$ is continuous in a closed interval $[a,b]$ and $f(a), f(b)$ are of opposite signs, then the equation $f(x) = 0$ will have at least one real root between a & b. This is based on intermediate value theorem.

Let us understand it graphically in lay person terms:

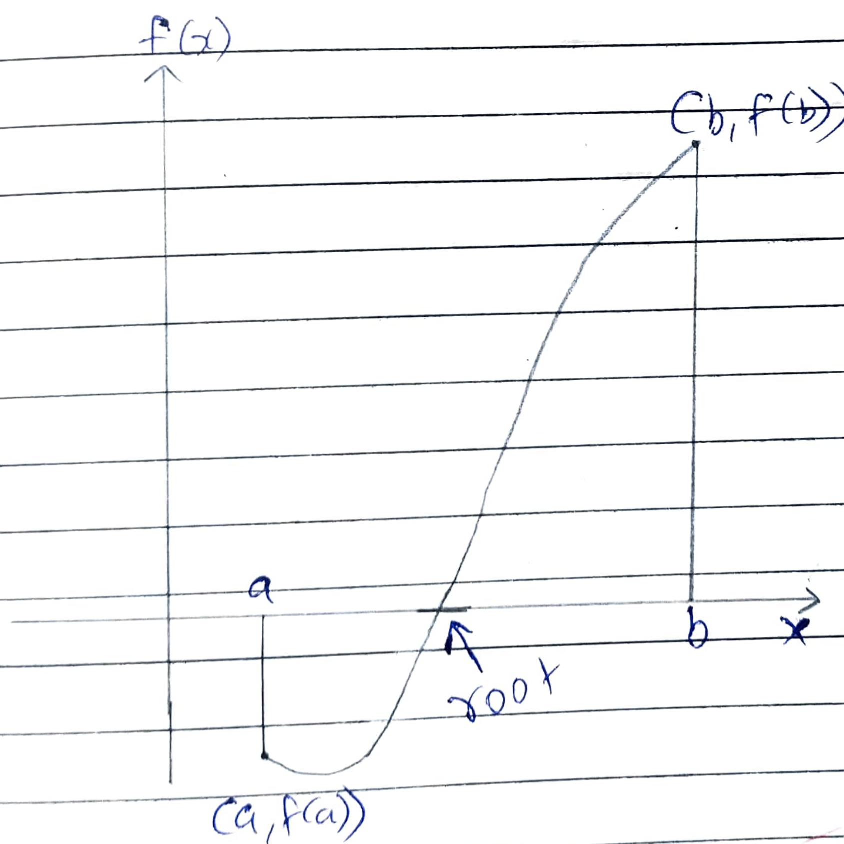
Since $f(x)$ is continuous on $[a,b]$, the graph of $y = f(x)$ for $a \leq x < b$ shall have no break. That is, it can be drawn from $(a, f(a))$ to $(b, f(b))$ without any jump.



No intermediate value can be skipped. Value of $f(x)$ completely occupy the range from $f(a)$ to $f(b)$.

As $f(a)$ and $f(b)$ are of opposite sign, function has to be zero for some $a \le x \le b$. Moreover if $f(a) \cdot f(b) < 0$ then one end of the graph is below x-axis and other end is above x axis. $f(x)$ being continuous, the graph of $y = f(x)$ has to cross x axis in going from one side of x axis to the other. In lay person term: It is like, if the stream of rive is flowing outside your home and you are permitted to walk only by taking infinite small continuous steps, that is you are not permitted to lift your legs and jump, you cannot get inside your home from outside without making your feet wet and vice versa.

Without loss of generality, let $f(a) < 0$. Thus $f(b) \geq 0$.



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Explain the advantages and disadvantages of Bisection Method.

Advantages of Bisection Method

1. It is very simple
2. Once initial a, b are known, number of iterations needed to be performed to achieved desired accuracy can be predetermined.
3. Reliable
4. ~~Gau~~ Guarantees Convergence.
5. Function needs to be only continuous.
6. Only one function evaluation per iteration.
7. Calculation for making guess of root for the next iteration is very easy simply $c = \frac{a+b}{2}$.

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Disadvantages of Bisection Method

① Not self starting. To begin with, requires two initial guesses at which function must of opposite sign. Thus, one has to evaluate function at subinterval points to get such 'a' and 'b'.

② slowest method.

③ It doesnot take into accout the ~~natural~~ nature of function to make next guess of the root.

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Find the root of the following using Bisection method.

(a) $f(x) = x^3 - x - 1 = 0$

a	b	f(a)	f(b)	c	f(c)
1	2	-1.000000	5.000000	1.500000	0.875000
1	1.500000	-1.000000	0.875000	1.250000	-0.296875
1.250000	1.500000	-0.296875	0.875000	1.375000	0.224609
1.250000	1.375000	-0.296875	0.224609	1.312500	-0.051514
1.312500	1.375000	-0.051514	0.224609	1.343750	0.082611
1.312500	1.343750	-0.051514	0.082611	1.328125	0.014576
1.312500	1.328125	-0.051514	0.014576	1.320313	-0.018711
1.320313	1.328125	-0.018711	0.014576	1.324219	-0.002128
1.324219	1.328125	-0.002128	0.014576	1.326172	0.006209
1.324219	1.326172	-0.002128	0.006209	1.325195	0.002037
1.324219	1.325195	-0.002128	0.002037	1.324707	-0.000047
1.324707	1.325195	-0.000047	0.002037	1.324951	0.000995
1.324707	1.324951	-0.000047	0.000995	1.324829	0.000474
1.324707	1.324829	-0.000047	0.000474	1.324768	0.000214
1.324707	1.324768	-0.000047	0.000214	1.324738	0.000084
1.324707	1.324738	-0.000047	0.000084	1.324722	0.000018
1.324707	1.324722	-0.000047	0.000018	1.324715	-0.000014
1.324715	1.324722	-0.000014	0.000018	1.324718	0.000002

Root = 1.324718

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(b) $xe^x = 1$

$\therefore xe^x - 1 = 0$

$f(x) = xe^x - 1 = 0$

a	b	f(a)	f(b)	c	f(c)
0	1	-1.000000	1.718282	0.500000	-0.175639
0.500000	1.000000	-0.175639	1.718282	0.750000	0.587750
0.500000	0.750000	-0.175639	0.587750	0.625000	0.167654
0.500000	0.625000	-0.175639	0.167654	0.562500	-0.012782
0.562500	0.625000	-0.012782	0.167654	0.593750	0.075142
0.562500	0.593750	-0.012782	0.075142	0.578125	0.030619
0.562500	0.578125	-0.012782	0.030619	0.570313	0.008780
0.562500	0.570313	-0.012782	0.008780	0.566406	-0.002035
0.566406	0.570313	-0.002035	0.008780	0.568359	0.00364
0.566406	0.568359	-0.002035	0.003364	0.567383	0.000662
0.566406	0.567383	-0.002035	0.000662	0.566895	-0.000687
0.566895	0.567383	-0.000687	0.000662	0.567139	-0.000013
0.567139	0.567383	-0.000013	0.000662	0.567261	0.000325
0.567139	0.567261	-0.000013	0.000325	0.567200	0.000156
0.567139	0.567200	-0.000013	0.000156	0.567169	0.000072
0.567139	0.567169	-0.000013	0.000072	0.567154	0.000029
0.567139	0.567154	-0.000013	0.000029	0.567146	0.000008
0.567139	0.567146	-0.000013	0.000008	0.567142	-0.000002
0.567142	0.567146	-0.000002	0.000008	0.567144	0.000003
0.567142	0.567144	-0.000002	0.000003	0.567143	0.000000

Root = 0.567143

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How many iterations do you need in Bisection method to get the root if you start with a=1 and b=2 and the tolerance is 10^{-4}?

a = 1 , b = 2

if $f(x) = x^3 - x - 1$

a	f(a)	b	f(b)	c	f(c)
1.000000	-1.000000	2.000000	5.000000	1.500000	0.875000
1.000000	-1.000000	1.500000	0.875000	1.250000	-0.296875
1.250000	-0.296875	1.500000	0.875000	1.375000	0.224609
1.250000	-0.296875	1.375000	0.224609	1.312500	-0.051514
1.312500	-0.051514	1.375000	0.224609	1.343750	0.082611
1.312500	-0.051514	1.343750	0.082611	1.328125	0.014576
1.312500	-0.051514	1.328125	0.014576	1.320313	-0.018711
1.320313	-0.018711	1.328125	0.014576	1.324219	-0.002128
1.324219	-0.002128	1.328125	0.014576	1.326172	0.006209
1.324219	-0.002128	1.326172	0.006209	1.325195	0.002037
1.324219	-0.002128	1.325195	0.002037	1.324707	-0.000047

Root = 1.324707

Total iteration = 11

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Explain False position or Regula falsi method graphically.

In Regula falsi method, to take into consideration the function value at 'a' and 'b', straight line is drawn, joining (a, f(a)) and (b, f(b))

-> The point, where it cuts x-axis, is the new estimate of the root.

-> Mathematically, estimate of formula for c can be derived as follows:-

$$c = \frac{a.f(b) - b.f(a)}{f(b) - f(a)}$$

-> If f(c) = 0, c is the root otherwise, if f(a). f(c) > 0, c takes the role of 'a' otherwise c takes the role of 'b'