Uniform probability Distribution

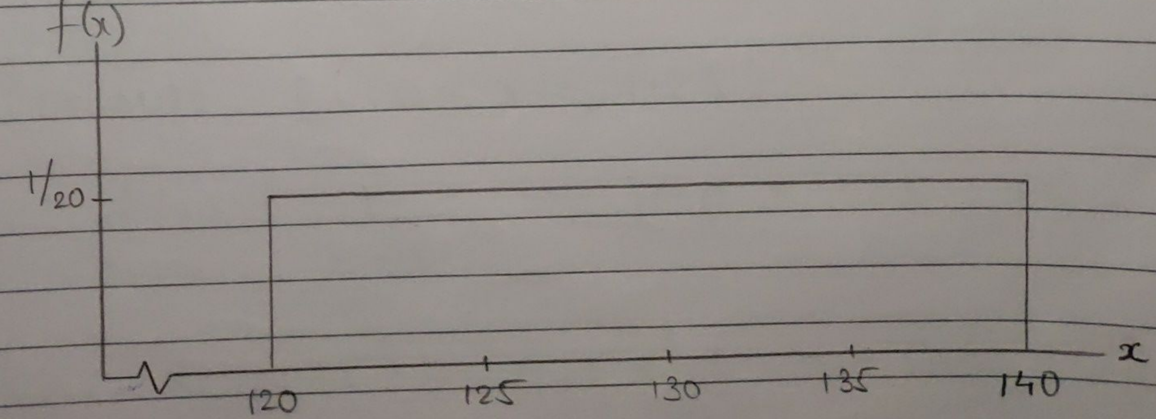
* Uniform probability Distribution:

A continuous probability distribution for which the probability that the random variable will assume a value in any interval is the same for each interval of equal length.

* Uniform probability Density function:

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

* Graph of uniform probability density function:

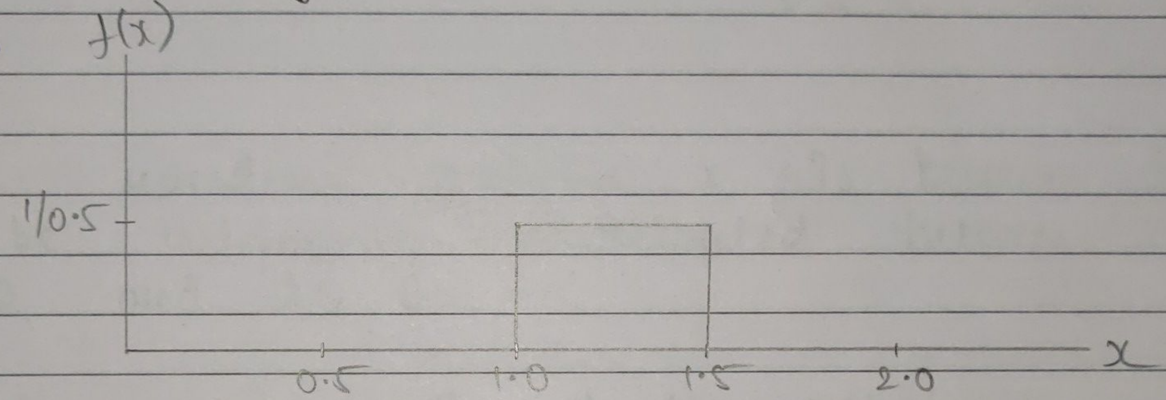


Exercises

(1) The random variable x is known to be uniformly distributed between 1.0 and 1.5.

(a) Show the graph of the probability density function.

→



(b) compute $P(x = 1.25)$

→ $P(x = 1.25) = 0$

The probability of any single point is zero because the area under the curve above any single point is zero.

(c) Compute $P(1.0 \le x \le 1.25)$

→ $f(x) = \frac{1}{b-a}$; $b - a = 1.25 - 1.0$
$= 0.25$

$P(1.0 \le x \le 1.25) = 0.25 \times \frac{1}{0.5} = \underline{\underline{0.5}}$

(d) Compute $P(1.20 \leq x \leq 1.5)$

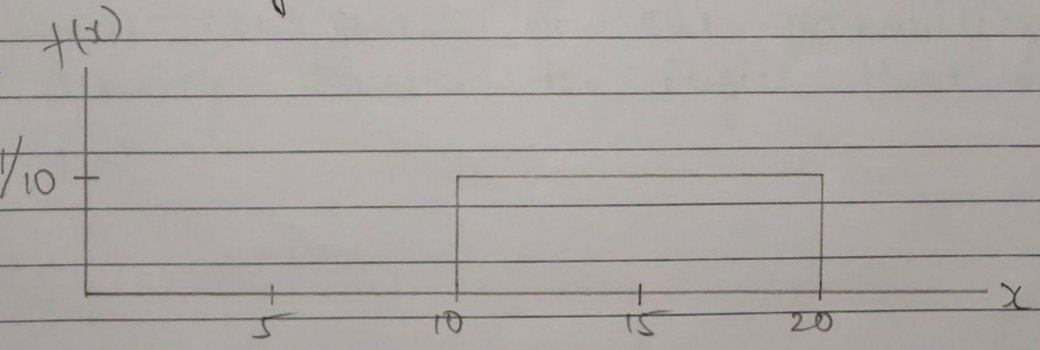
$\rightarrow$ $(b-a) = (1.5 - 1.2) = 0.3$

$$P(1.2 < x < 1.5) = 0.3 \times \frac{1}{0.5} = 0.6$$

2) The random variable x is known to be uniformly distributed between 10 and 20.

(a) Show the graph of the probability density function.

$\rightarrow$



(b) Compute $P(x < 15)$

$\rightarrow$ $(b-a) = (15 - 10) = 5$

$$P(x < 15) = 5 \times \frac{1}{10} = 0.5$$

(c) compute $p(12 \le x \le 18)$

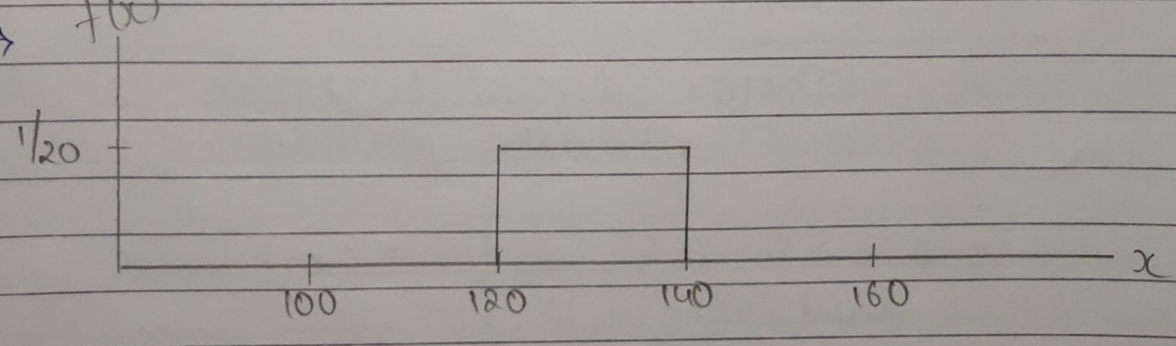
$\rightarrow$ $(b-a) = (18-12) = 6$

$p(12 \le x \le 18) = 6 \times \frac{1}{10} = 0.6$

3) Delta Airlines quotes a flight time of 2 hours, 5 minutes for its flights from Cincinnati to Tampa. Suppose we believe that actual flight times are uniformly distributed between 2 hours and 2 hours, 20 min.

(a) Show the graph of the probability density function for flight time.

$\rightarrow$



(b) What is the probability that the flight will be no more than 5 min late?

$\rightarrow$ Given arrival time is 125 mins
so, 130 min is 5 min late.

$$(b-a) = (130-120) = 10$$

$$P(120 \le x \le 130) = 10 \times \frac{1}{20} = 0.5$$

(c) What is the probability that the flight will be more than 10 mins late?

→ Given arrival time is 125 mins.
So more than 10 mins late is $x > 135$

$$(b-a) = (140-135) = 5$$

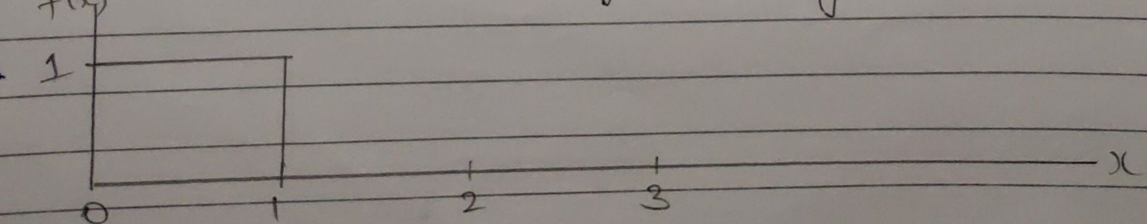
$$P(x > 135) = 5 \times \frac{1}{20} = 0.25$$

(4) x is a continuous random variable with probability density function

$$f(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Graph the probability density function.

→



(b) what is the probability of generating a random numbers between 0.25 and 0.75.

$\rightarrow$ $(b-a) = (0.75 - 0.25) = 0.5$

$P(0.25 \le x \le 0.75) = 0.5 \times 1 = 0.5$

(c) what is the probability of generating a random number with a value less than or equal to 0.3 ?

$\rightarrow$ $P(x \le 0.3) = 0.3 \times 1 = 0.3$

(d) what is the probability of generating a random number with a value greater than 0.60 ?

$\rightarrow$ $(b-a) = (1 - 0.6) = 0.4$

$P(x > 0.6) = 0.4 \times 1 = 0.4$

6. On average, 30 mins television sitcoms have 22 mins of programming (CNBC, Feb 23, 2006). Assume that the probability distribution for minutes of programming can be approximated by a uniform distribution from 18 mins to 26 mins.

- (a) What is the probability a sitcom will have 25 or more minutes of programming?

→ $f(x) = \frac{1}{b-a} = \frac{1}{26-18} = \frac{1}{8}$

$P(x>25) = (b-a) = (26-25) = 1$

$P(x>25) = 1 \times \frac{1}{8} = 0.125$

- (b) What is the probability a sitcom will have between 21 and 25 mins of programming?

→ $(b-a) = (25-21) = 4$

$P(21 \leq x \leq 25) = 4 \times \frac{1}{8} = 0.5$

(c) sitcom will have more than 10 mins of commercials or other programs?

→ $30 - 22 = 8$ mins for other commercials.

$P(x>10) = (10-8) \times \frac{1}{8} = 0.25$