Normal Probability Distribution

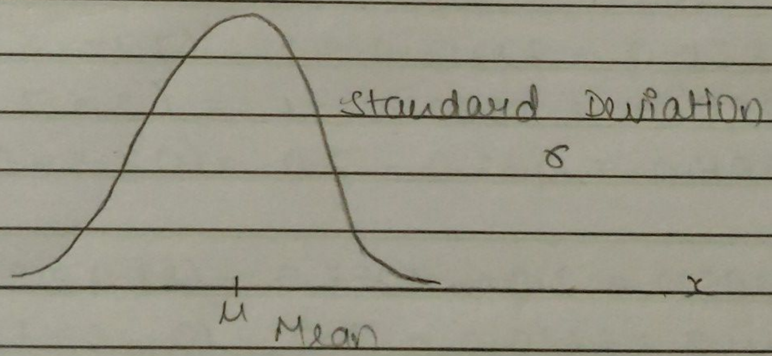
* Normal Probability Distribution:

A continuous probability distribution. Its probability density function is bell-shaped and determined by its mean μ and standard deviation σ.

* Normal Probability Density function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where μ = mean $\pi = 3.14159$
σ = standard deviation $e = 2.71828$



* Standard Normal Probability Distribution:

A random variable that has a normal distribution with a mean of zero and a standard deviation of one is said to have a standard Normal Probability Distribution.

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

* Converting to Standard Normal Random Variable

$$z = \frac{x - \mu}{\sigma}$$

Exercises :

(10) (a) $P(z \leq 1.5) = 0.9332$
(b) $P(z \leq 1) = 0.8413$
(c) $P(1 \leq z \leq 1.5) = 0.9332 - 0.8413 = 0.0919$.
(d) $P(0 < z < 2.5) = 0.9938 - 0.5 = 0.4938$.

(11) (a) $P(z \leq -1.0) = 0.1587$
(b) $P(z \geq -1) = 1 - 0.1587 = 0.8413$
(c) $P(z \geq -1.5) = 1 - 0.0668 = 0.9332$.
(d) $P(-2.5 \leq z) = 1 - 0.0062 = 0.9938$.
(e) $P(-3 < z \leq 0) = 0.5 - 0.0013 = 0.4987$.

(12) (a) $P(0 \leq z \leq 0.83) = 0.7967 - 0.5 = 0.2967$.
(b) $P(-1.57 \leq z \leq 0) = 0.5 - 0.0582 = 0.4418$.
(c) $P(z > 0.44) = 1 - 0.6700 = 0.3300$
(d) $P(z \geq -0.23) = 1 - 0.4090 = 0.5910$
(e) $P(z < 1.20) = 0.8849$
(f) $P(z < -0.71) = 0.2389$.

(13) (a) $P(-1.98 \leq z \leq 0.49) = 0.6879 - 0.0239 = 0.6640$
(b) $P(0.52 \leq z \leq 1.22) = 0.8888 - 0.6985 = 0.1903$
(c) $P(-1.75 \leq z \leq -1.04) = 0.1492 - 0.0401 = 0.1091$

(14) (a) $z = 1.96$

(b) $z = 0.5 + 0.4750 = 0.9750$ so $z = 1.96$

(c) $z = 0.61$

(d) $1 - 0.1314 = 0.8686$ so $z = 1.12$

(e) $z = 0.44$

(F) $1 - 0.3300 = 0.6700$ so $z = 0.44$

(15) (a) $z = -0.80$

(b) $0.9030/2 + 0.5 = 0.9515$ so $z = 1.66$

(c) $0.2052/2 + 0.5 = 0.6026$ so $z = 0.26$

(d) $z = 2.57$

(e) $1 - 0.6915 = 0.3085$ so $z = -0.50$

(16) (a) $1 - 0.01 = 0.9900$ so $z = 2.33$

(b) $1 - 0.025 = 0.9750$ so $z = 1.96$

(c) $1 - 0.05 = 0.9500$ so $z = 1.65$

(d) $1 - 0.10 = 0.9000$ so $z = 1.29$

(20) Given $\mu = 77$ $\sigma = 20$

(a) $x \leq 50$ $\quad z = \dfrac{x - \mu}{\sigma} = \dfrac{50 - 77}{20} = -1.35$

so, $P(z \leq -1.35) = 0.0885$

(b) $x > 100$ $\quad z = \dfrac{100 - 77}{20} = 1.15$

$P(z > 1.15) = 1 - 0.8749 = 0.125$

so, 12.5% of workers spent more than 100 hours.

(c) Given: upper 20% of usage.
So left side of z (i.e. bottom) = 1 − 0.2 = 0.80
also, $\mu = 77$ $\sigma = 20$

$$z = \frac{x - \mu}{\sigma}$$

$$\therefore x = \mu + z\sigma$$
$$= 77 + 0.8(20)$$
$$\underline{x = 93}$$

So, a worker logged on to the internet for 93 hours is considered as heavy user.

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* Normal Approximation of Binomial Prob.

$\mu = np \qquad \sigma = \sqrt{np(1-p)}$

(26) Given $p = 0.20 \quad n = 100$

(a) $\mu = np = (100)(0.20) = 20$

$\sigma = \sqrt{np(1-p)} = \sqrt{16} = 4.$

(b) Yes because $np = 20$ so, binomial probability can be approximated by the normal probability distribution

(c) Given x exactly 24
considering continuity correction factor.

$P(23.5 \le x \le 24.5)$

For $x = 23.5 \quad z = \frac{x - \mu}{\sigma} = \frac{23.5 - 20}{4} = 0.8750$

$P(z \le 0.88) = 0.8106$

For $x = 24.5 \quad z = \frac{24.5 - 20}{4} = 1.1250.$

$P(z \le 1.13) = 0.8708$

So, $P(0.88 \le z \le 1.13) = 0.8708 - 0.8106$
$= 0.0602.$

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27 Given $p = 0.60$ $n = 200$

(a) $\mu = np = (200)(0.6) = 120$

$\sigma = \sqrt{np(1-p)} = \sqrt{48} = 6.9282$

(b) Yes because $\mu = 120$. So, binomial probabilities can be approximated by the normal probability distribution.

(c) Given $x = 100$ to 110.
considering continuity correction factor.

$P(99.5 \leq x \leq 110.5)$

$z = \frac{99.5 - 120}{6.9282} = -2.96$ $P(z \leq -2.96) = 0.0015$

$z = \frac{110.5 - 120}{6.9282} = -1.37$ $P(z \leq -1.37) = 0.0853$

$P(-2.96 \leq z \leq -1.37) = 0.0853 - 0.0015 = 0.0838$

(d) Given $x \geq 130$
considering continuity correction factor

$P(129.5 \leq x) \cancel{\leq 130.5)}$

$z = \frac{129.5 - 120}{6.9282} = 1.37$ $P(z \leq 1.37) = 0.9147$

$\cancel{z = \frac{130.5 - 120}{6.9282}} = \cancel{1.52}$ $\cancel{P(z \leq 1.52)} = \cancel{0.9357}$

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So, $P(x \geq 130) = 1 - 0.9147 = 0.0853$

(28) Given $n = 250$ $p = 0.20$

(a) $\mu = np = 250 \times 0.20 = 50$

So, 50 adults smoke.

$\sigma = \sqrt{np(1-p)} = \sqrt{40} = 6.3246$.

(b) $x < 40$

$\cancel{z = \frac{40-50}{6.3246} = -1.58}$

considering continuity correction factor.

$P(x < 39.5)$

$z = \frac{39.5 - 50}{6.3246} = -1.66$ $P(z \leq -1.66) = 0.0485$

$P(x \leq 39.5) = 0.0485$

(c) $P(55 \leq x \leq 60)$

considering continuity correction factor.

$P(54.5 \leq x \leq 60.5)$

$$z = \frac{54.5 - 50}{6.3246} = 0.71 \quad P(z \le 0.71) = 0.7611$$

$$z = \frac{60.5 - 50}{6.3246} = 1.66 \quad P(z \le 1.66) = 0.9515$$

$$P(54.5 \le z \le 60.5) = 0.9515 - 0.7611$$
$$= \underline{\underline{0.1904}}$$

(d) $x \geq 70$

considering continuity correction factor.

$P(x \leq 69.5)$

$$z = \frac{69.5 - 50}{6.3246} = 3.08 \quad P(z \le 3.08) = 0.9990$$

So, $P(x \geq 69.5) = 1 - 0.9990 = \underline{\underline{0.0010}}$

* Exponential Probability Distribution:

The exponential probability distribution is used for random variables such as the time between arrivals at a car wash, the time required to load a truck, the distance between major defects in a highway and so on.

* Exponential Probability Density function:

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \quad \text{for } x \geq 0$$

* Exponential Distribution Cumulative probability:

$$P(x \leq x_0) = 1 - e^{-x_0/\mu}$$

(32) Given $f(x) = \frac{1}{8} e^{-x/8}$ for $x \geq 0$

(a) $P(x \leq 6) = 1 - e^{-6/8} = 0.5276$

(b) $P(x \leq 4) = 1 - e^{-4/8} = 0.3935$

(c) $P(x \geq 6) = 1 - 0.5276 = 0.4724$

(d) $P(4 \leq x \leq 6) = 0.5276 - 0.3935 = 0.1341$

(33) $f(x) = \frac{1}{3} e^{-x/3}$ for $x \geq 0$.

(a) $P(x \leq x_0) = 1 - e^{-x_0/\mu}$

(b) $P(x \leq 2) = 1 - e^{-2/3} = 0.4866$

(c) $P(x \geq 3) = e^{-3/3} = 0.3679$

(d) $P(x \leq 5) = 1 - e^{-5/3} = 0.8111$

(e) $P(2 \leq x \leq 5) = \cancel{1-e}\ 0.8111 - 0.4866 = 0.3245$

(34) Given $\mu = 12.1$ min

(a) $P(x \leq 10) = 1 - e^{-10/12.1} = 0.5624$

(b) $P(x > 20) = \cancel{1-}e^{-20/12.1} = 0.1915$

(c) $P(10 \leq x \leq 20) = 0.8085 - 0.5624 = 0.2461$

(d) $P(x > 18) = e^{-18/12.1} = 0.2259$

(38) $\mu = 5.5$ per hour

(a) $f(x) = 5.5 e^{-5.5x}$

(b) $5.5/4 = 1.3750$ per 15 mins.
for no-interruptions: $e^{-1.3750} = 0.2528$.

(c) $5.5/6 = 0.9167$ per 10 min.
for interruption: $1 - e^{-0.9167} = 0.6002$