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Q1 What is the need of numerical Integration? Explain different situation where such need arises.

Ans In integration, whenever anti derivative is known indefinite integral is known. The difficulty arieses in computing definite integral in many situation owns to different reasons.

=> Situation 1:

For evaluating $\int_a^b f(x)dx$, anti derivative of f exists but cannot be represented ~~of~~ in a form of standard functions. For example, consider

i) $\int_0^1 e^{x^e} dx$ f(2), $\int^2 \frac{dx}{\log x}$

One may say, anti derivative of e^{x^1} is $\int_0^x$ ~~or~~ $e^{t^2} dt$, but is not of no use to us. Both these cited integrals have precise numerical values. For all practical purposes, anti-derivation cannot be used to calculate the value F and thus of definite integral cannot be determined.

Many such examples are encountered in scientific, mathematical & statistical and engineering problems at. ① The error function.

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt,$$ used in calculating error bounds in many computations is of great importance in statistics. ② whereas $\int_2^x \frac{dt}{\ln(t)}$ is logarithm function, which gives approximately number of primes less than or equal to x.

③ In electric field theory, it is proved that the magnetic field induced by a current flowing in a circular loop of wire has intensity.

$$H(x) = \frac{4Ir}{r^2 - x^2} \int_0^{\frac{\pi}{2}} \left(1 - \left(\frac{x}{r}\right)^2 \sin(\theta)^2\right)^{\frac{1}{2}} d\theta$$

- where I is the current, r is the radius of the loop, x is the distance from center to the point, wher the magnetic intensity is being computed $(0 \leq x \leq r)$. If x, r and I are given, the integral occurring in the equation is called elliptic integral and cannot be expressed in terms of standard form.

- All three integrals are frequently occuring integrals and we shall learn numerical methods called methods of numerical integration. Using these methods, we would be able to evaluate these integrals to as many decimal places as desired.

⇒ <u>Situation 2 :-</u>

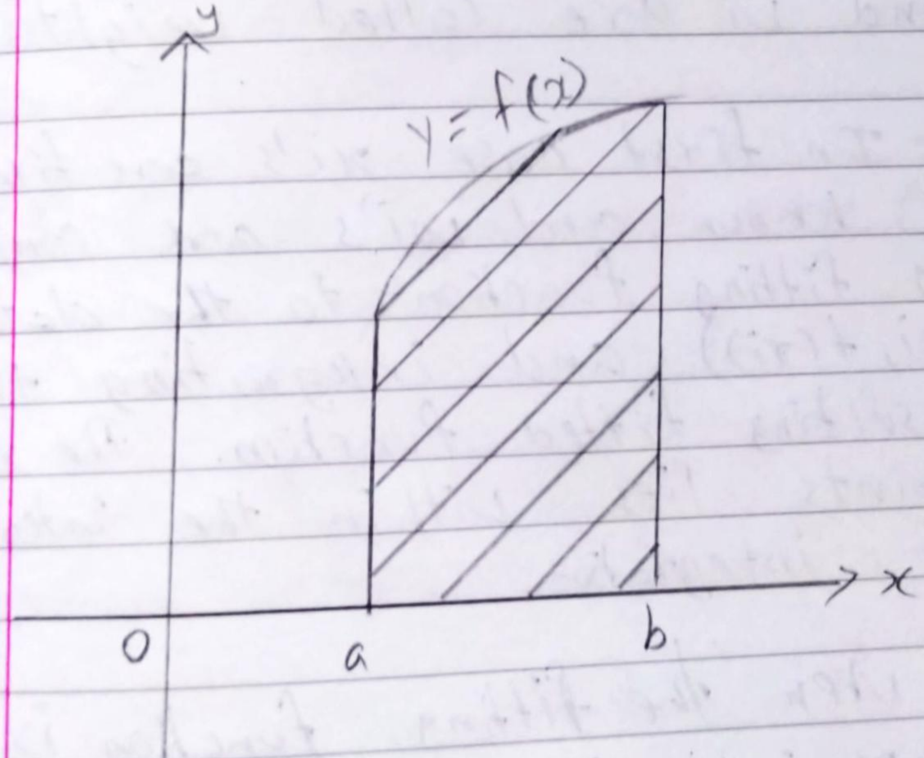
Anti-derivative may exist known but may be complicated expression to compute. In these cases, rather than evaluate anti-derivative, it is preferred to use numerical integration formula to evaluate the integral. A typical situation could be where anti-derivative expression is in the form of an infinite series or infinite product and its evaluation requires numerical techniques.

=> Situation 3 :-

Expression of function to be integrated is not available functions are defined with the help of discreate data. For example, speed of an object may have been measured at different time intervals and distance needs to be computed or integrated may have been obtained by sampling, so values are available at certain points only.

Q2 Give geometrical interpretation of $\int_a^b f(x)\,dx$, with $f(x) >, 0$ and $\leq x \leq b$.

Ans The definite integral $\int_a^b f(x)\,dx$ of a non-negative integral $f(x)$, on a closed interval $[a,b]$ area under the graph of f.



Q3 Differentiate between:
(a) Newton cotes Integration formulas and Gauss Quadrative formulas.
(b) Closed type and open type formulas.

Ans.

(a) Here xi are called abscissas and wi are called weights.

- In first case xi's are fixed, that is known and wi's are computed by fitting function to the data $(xi, f(xi))$ and integrating the resulting fitted function. The xi's points lies within the interval $[a,b]$ of integration.

- When the fitting function is a polynomial, the integration formulas so obtained are called Newton cotes Integration formulas

- On the other hand, if xi's and wi's both are assumed to be Unknown and they are computed, the formulas so obtained are called Gauss Quadrature formulas

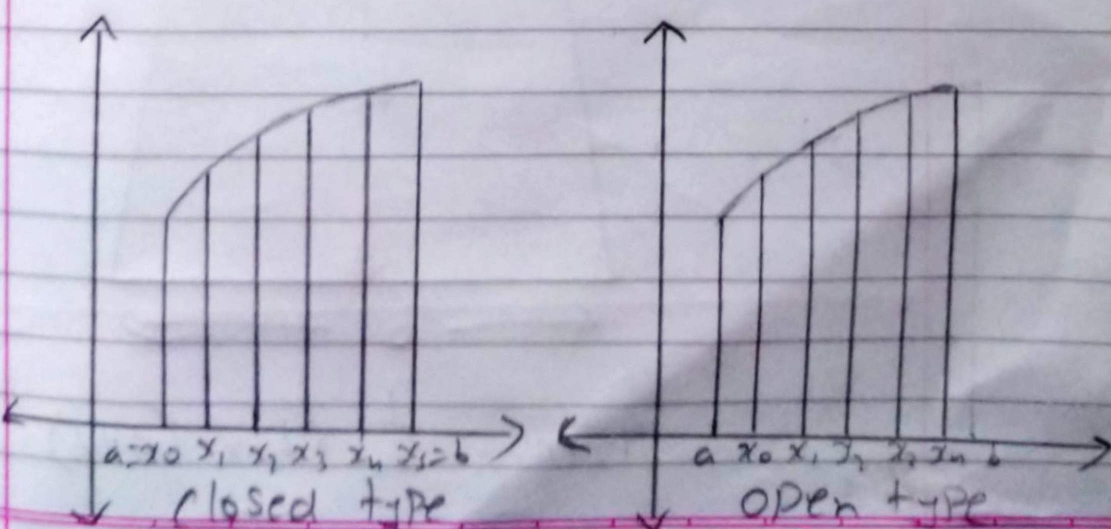
Integration formulas

x_i's fixed w_i's unknown Newton Cotes Integration formular	x_i's unknown w_i's unknown Gauss Quadrature formula.

(b) Closed type and open type formulas.

- If $x_0 = a$ and $x_n = b$, i.e, $a = x_0 \le x_1 \le x_2 \le \dots \le x_{n-1} \le x_n = b$ then integration formula is called closed type integration formula.

- If $a \le x_0 \le x_1 \le x_2 \le x_{n-1} \le x_n \le b$ then the integration formula is called open type integration formula.

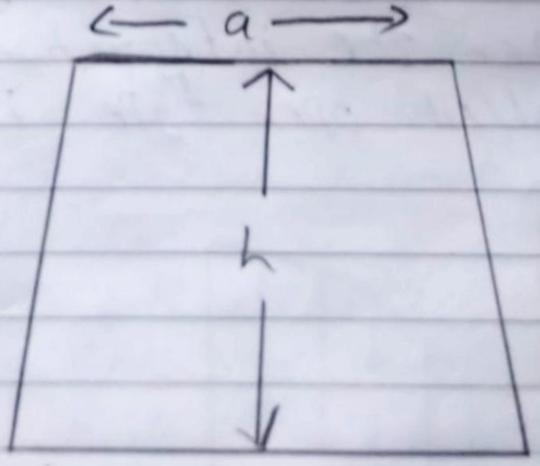


closed type   open type

Q4 Explain trapezoidal rule for estimating $\int_a^b f(x)\,dx$. Explain it graphically also.

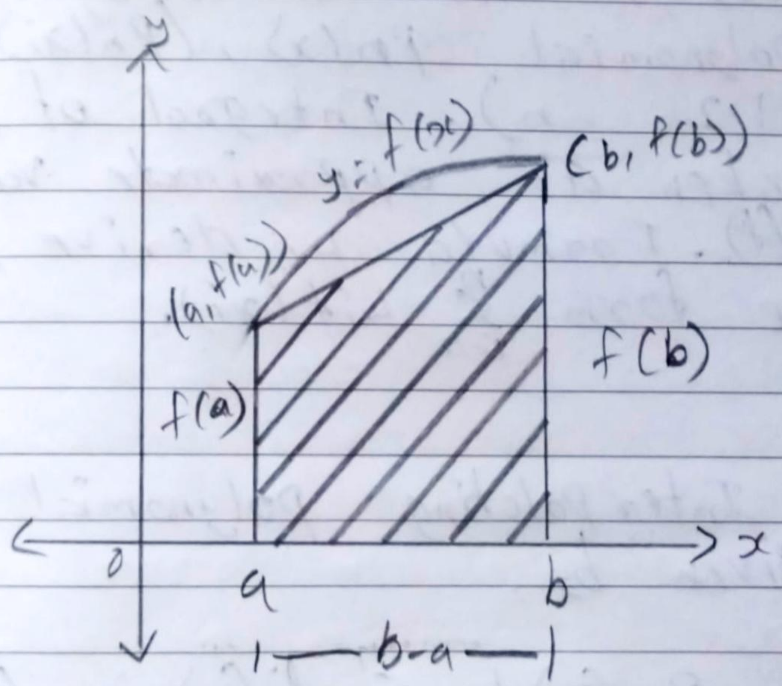
Ans By trapezoidal rule. $\int_a^b f(x)\,dx = \frac{\Delta x}{2}[f(a) + f(b)]$

→ This rule is called trapezoidal rule as it gives area of trapezium formed from joining the points $(a, f(a))$ and $(b, f(b))$ by a straight line and the vertical lines $x=a$ and $x=b$. We know that area of trapezium = height × (average of parellel lines)



$$\text{Area} = \frac{1}{2} \times h \times (a+b)$$

→ So if we join the ~~poi~~ points $(a, f(a))$ and $(b, f(b))$, it forms a trapezium, with parallel side length as $f(a)$ and $f(b)$. Height of trapezium is $(b-a)$, hence from the following graph, area of trapezium is $\frac{b-a}{2}[f(a)+f(b)] = \frac{\Delta x}{2}[f(a)+f(b)]$



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Q5 Explain basic Principle (No Derivation) in deriving Newton Cote's integration formula.

Ans The basic principle in obtaining Newton Cote's integration formula is to fix abscissas $x_0, x_1, x_2 \ldots x_n \in [a, b]$ beforehand and integral $f(x)$ is approximated by interpolating polynomial $P_n(x)$, ($P_n(x_i) = y_i$, $\forall i = 1, 2 \ldots n$). Integral of $P_n(x)$ is taken as approximate value of $I(f)$. Formula we derive, is of the form $\sum_{i=0}^{n} w_i f(x_i)$.

- Interpolating polynomial $P_n(x)$ is given by.

$$P_n(x) = \sum_{i=0}^{n} l_i(x) y_i \quad , (y_i = f(x_i))$$

where, $l_i(x) = \prod_{j=0, j \neq i}^{n} \frac{(x - x_j)}{x_i - x_j}$

$\forall i = 0, 1, \ldots, n.$

Thus,

$$\int_a^b P_n(x)\,dx = \int_a^b \left(\sum_{i=0}^{n} l_i(x) y_i\right) dx$$

$$= \sum_{i=0}^{n} \left(\int_a^b l_i(x)\,dx\right) y_i$$

$$= \sum_{i=0}^{n} w_i y_i \,;$$

giving $w_i = \int_a^b l_i(x)\,dx$

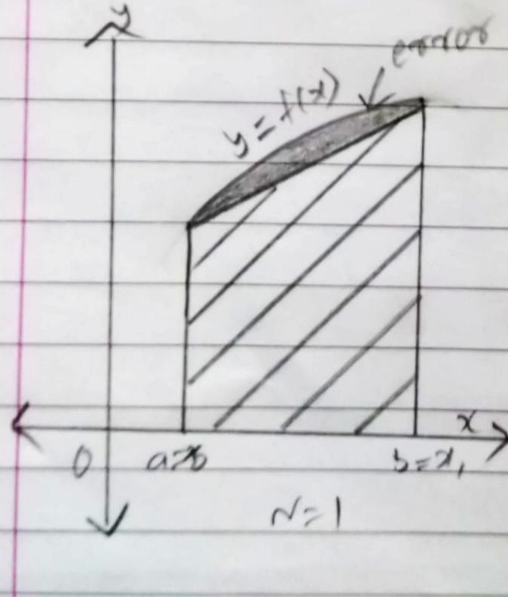
Q6. Write the composite form of trapezoidal rule for $\int_a^b f(x)\,dx$.

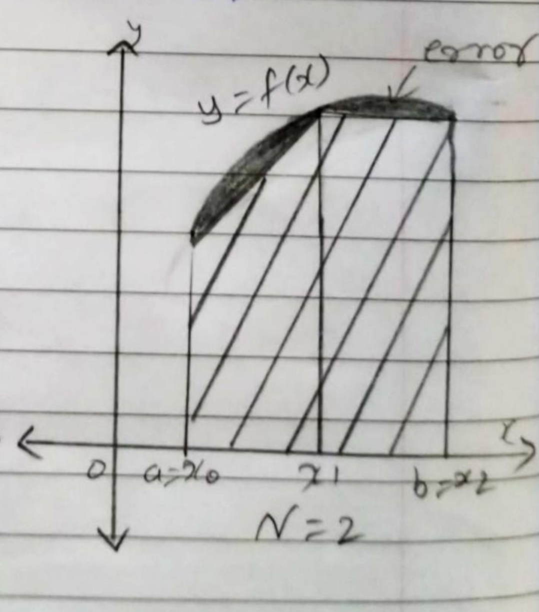
Ans.

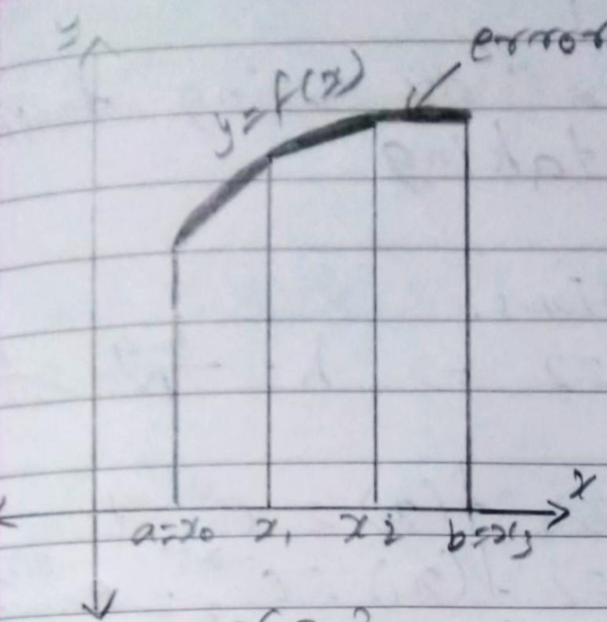
If $[a,b]$ is divided into N equal subdivisions, the composite form of trapezoidal rule for $\int_a^b f(x)\,dx$ is,

$$\int_a^b f(x)\,dx \simeq \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_N)\right]$$

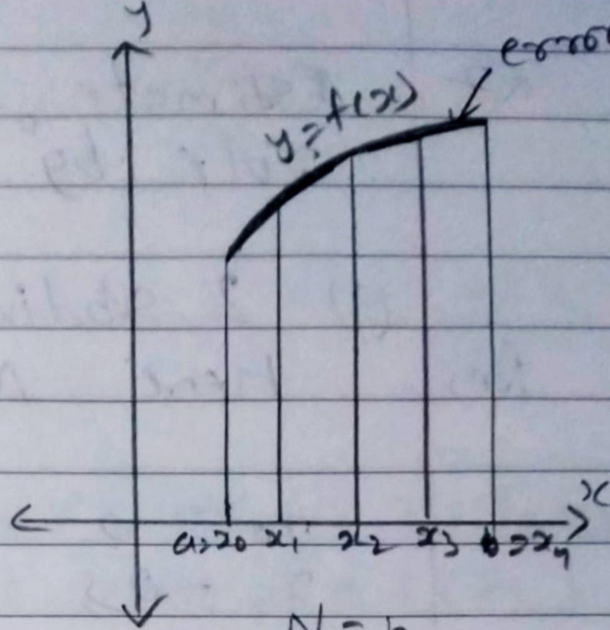
→ The following figures shows as Number of segments of intervals are increasing, i.e. use of composite form reduces ~~fraction error~~ truncation error.







N = 3



N = n

Q7 Estimate $\int_0^1 e^{-x^2} dx$ using trapezoidal rule by taking

(i) 2 subdivisions.

Ans Here $N=2 \Rightarrow h = \frac{b-a}{N} = \frac{1-0}{2} = 0.5$

$x_0 = 0 \quad \Rightarrow f(x_0) = e^0 = 1$
$x_1 = 0.5 \quad \Rightarrow f(x_1) = e^{-0.5^2} = e^{-0.25} = 0.7788$
$x_2 = 1 \quad \Rightarrow f(x_2) = e^{-1^2} = e^{-1} = 0.3679$

$$\int_0^1 = e^{-x^2} dx = \int_0^1 f(x)\, dx = \frac{h}{2}\left[f(x_0) + 2f(x_1) + f(x_2)\right]$$

$$= \frac{0.5}{2}\left[1 + 2(0.7788) + 0.3679\right]$$

$$= 0.25[2.9255]$$

$$\int_0^1 e^{-x^2} dx = 0.7313$$

(ii) 4 subdivisions:

$\therefore N = 4 \Rightarrow h = \frac{1}{4} = 0.25$

$x_0 = 0 \quad \Rightarrow f(x) = e^{0^2} = 1$
$x_1 = 0.25 \quad \Rightarrow f(x_1) = e^{-0.25^2} = 0.9394$
$x_2 = 0.50 \quad = f(x_2) = e^{-0.50^2} = 0.7788$
$x_3 = 0.75 \quad = f(x_3) = e^{-0.75^2} = 0.5698$

$x_4 = 1 \quad = f(x_4) = e^{-1^2} = 0.3679$

$$\int_0^1 f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{3} f(x_i) + f(x_4)\right]$$

$$\int_0^1 e^{-x^2}\,dx = \frac{0.25}{2}\left[1 + 2(0.9394 + 0.7788 + 0.5698) + 0.3679\right]$$

$$= 0.7430$$

(iii) 6 subdivision.

Here $N = 6$, $h = \frac{1-0}{6} = 0.1667$

$x_0 = 0 \quad \Rightarrow f(x_0) = 1$

$x_1 = 0.1667 \quad \Rightarrow f(x_1) = 0.9726$

$x_2 = 0.3334 \quad \Rightarrow f(x_2) = 0.8948$

$x_3 = 0.5001 \quad \Rightarrow f(x_3) = 0.7788$

$x_4 = 0.6668 \quad \Rightarrow f(x_4) = 0.6412$

$x_5 = 0.8335 \quad \Rightarrow f(x_5) = 0.4994$

$x_6 = 1 \quad \Rightarrow f(x_6) = 0.3679$

$$\int_0^1 f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum_{1}^{5} f(x) + f(x_6)\right]$$

$$= \frac{0.1667}{2}\left[1 + 2(0.9726 + 0.8948 + 0.7788 + 0.6412 + 0.4994) + 0.3679\right]$$

$$= \boxed{0.7452}$$

Q8 Calculate approximate integral value of $\int f(x)\,dx$ when $f(x)$ is

(a) x^2 using (i) Trapezoidal rule.

Ans $\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + f(b)\right]$

Here $n = 1$

$h = \frac{b-a}{n}$

$= \frac{2-0}{1}$

$= 2$

$\therefore \int_0^2 x^2\,dx = \frac{2}{2}\left[f(0) + f(2)\right]$

$= 1 \cdot [1 + 2.2()]$

$= 3.2361$

(ii) Simpson's $\frac{1}{3}$ rule.

$\rightarrow \int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$

Here $n = 2$

$$h = \frac{b-a}{n}$$

$$= \frac{2-0}{2}$$

$$= 1$$

$$\int_0^2 x^2\,dx = \frac{1}{3}\left[f(0) + 4(f(0+2)) + f(0+2)\right]$$

$$= \frac{1}{3}\left[1 + 4(1.4142) + 2.8361\right]$$

$$= \frac{1}{3}\left[8.8929\right]$$

$$= 2.9643$$

(b) x^4

(i) Trapezoidal Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x) + f(b)\right]$$

Here $n = 1$, $h = \frac{2}{1} = 2$

$$\int_0^2 x^4\,dx = \frac{2}{2}[f(0)+f(2)]$$

$$= 1[0+16]$$

$$\boxed{=16}$$

(ii) Simpson's $\frac{1}{3}$ rule. $n=2, h=1$

$$\int_0^2 x^4\,dx = \frac{1}{3}[0+4+16]$$

$$\boxed{=6.6667}$$

(iii) Simpson's $\frac{3}{8}$ rule.

$$\int_a^b f(x)\,dx = \frac{1}{4}[f(0)+3f(\tfrac{2}{3})+3f(2\tfrac{2}{3})+f(3'\tfrac{2}{3})]$$

$$= \frac{1}{4}[0+3\times\frac{16}{81}+3\times\frac{256}{81}+1296]$$

$$= \frac{1}{4}[\frac{16}{27}+\frac{256}{27}+16]$$

$$= \frac{16}{4}[\frac{1}{27}+\frac{16}{27}+1]$$

$$= 4[\frac{44}{27}]$$

$$\boxed{=6.5185}$$

ⓔ $1(x+1)$

(i) Trapazoidal, $n=1$, $h=2$

$$\int_0^2 1(x+1)\,dx = \frac{2}{2}[f(0)+f(2)]$$

$$= 1\,[1+0.3333]$$

$$= 1.3333$$

(ii) Simpson's $\frac{1}{3}$ rule $n=2$, $h=1$

$$\int_0^2 1/(x+1)\,dx = \frac{h}{3}[f(0)+4f(0+1)+f(0+2)]$$

$$= \frac{1}{3}[1+4\times\frac{1}{2}+0.3333]$$

$$= \frac{1}{3}[3.3333]$$

$$= 1.1111$$

(iii) Simpson's $\frac{3}{8}$ rule. $n=3$ $h=\frac{2}{3}$

$$\int_0^2 1/(x+1)\,dx = \frac{3h}{8}[f(0)+3f(0+\frac{h}{3})+3f(0+2.\frac{2}{3}) + f(0+3.\frac{2}{3})]$$

$$= \frac{1}{4}[1+3\times(\frac{3}{5})+3\times(\frac{3}{7})+\frac{1}{3}]$$

$$= \frac{1}{4}[1+1.8+1.2858+0.3333]$$

$$= 1.1048$$

(d) $\sqrt{1+x^2}$

(i) Trapezoidal rule $n=1, h=2$

$$\int_0^2 \sqrt{1+x^2}\,dx = \frac{h}{2}[f(0)+f(2)]$$

$$= \frac{2}{2}[0+2.2361]$$

$$= 2.2361$$

(ii) Simpson's $\frac{1}{3}$ rule $n=2, h=1$

$$\int_0^2 \sqrt{1+x^2}\,dx = \frac{h}{3}[f(0)+4f(0+1)+f(0+2)]$$

$$= \frac{1}{3}[0+4\times 1.4142+2.2361]$$

$$= \frac{1}{3}[7.8929]$$

$$= 2.63097$$

(iii) Simpson's $\frac{3}{8}$ rule $n=3 \quad h=\frac{2}{3}$

$$\int_0^2 \sqrt{1+x^2} = \frac{3h}{8}[f(0)+3f(0+\tfrac{2}{3})+3f(0+2(\tfrac{2}{3}))+f(0+3(\tfrac{2}{3}))]$$

$$= \frac{1}{4}[0+3\times 1.2019+3\times 1.6667+2.236]$$

$= \frac{1}{4}[10.8419]$

$= 2.7105$

(e) $\sin x$

(i) Trapezoidal $n=1, h=2$

$\int_0^2 \sin x \, dx = \frac{h}{2}[f(0) + f(2)]$

$\int_0^2 \sin x \, dx = 1[0 + 0.9093]$

$= 0.9093$

(ii) Simpson's $\frac{1}{3}$ rule. $n=2$ $h=1$

$\int_0^2 \sin x \, dx = \frac{h}{3}[f(0) + 4(f(0+1)) + f(0+2)]$

$= \frac{1}{3}[0 + 4 \times 0.8415 + 0.9093]$

$= \frac{1}{3}[4.2753]$

$= 1.4251$

(iii) Simpson's $\frac{3}{8}$ rule. $n=3$ $h=\frac{2}{3}$

$\int_0^2 \sin x \, dx$:

$= \frac{3h}{8}\left[f(0)+3\left(f\left(0+\frac{2}{3}\right)+3f\left(0+2\left(\frac{2}{3}\right)\right)+ f\left(0+3\left(\frac{2}{3}\right)\right)\right]$

$= \frac{1}{4}\left[0+3(0.6184+3(0.9720)+0.9093)\right]$

$= \frac{1}{4}[5.6805]$

$\boxed{= 1.4201}$

(f) e^x

(i) Trapezoidal rule.

$\int_0^2 e^x \, dx = \frac{h}{2}\left[f(0)+f(2)\right]$

$= 1\left[1+7.3891\right]$

$\boxed{= 8.3891}$

(iii) simpson's $\frac{3}{8}$ rule $n=3$ $h=\frac{2}{3}$

$$\int_0^2 e^x\,dx = \frac{3h}{8}\left[f(0)+3f\left(0+\frac{2}{3}\right)+3f\left(0+2\left(\frac{2}{3}\right)\right)+f\left(0+3\left(\frac{2}{3}\right)\right)\right]$$

$$=\frac{1}{4}\left[1+3(1.9477+3(3.7937)+7.3891\right]$$

$$=\frac{1}{4}(25.6138)$$

$$\boxed{I = 6.4033}$$

(ii) simpson $\frac{1}{3}$ rule $n=2$ $h=1$

$$\int_0^2 e^x\,dx = \frac{h}{3}\left[f(0)+4f(0+1)+f(0+2)\right]$$

$$=\frac{1}{3}\left[1+4(2.7183)+7.3891\right]$$

$$=\frac{1}{3}\left[19.2623\right]$$

$$\boxed{I = 6.4208}$$

Q9. Given function f at the following values.

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	3.12014	4.42569	6.04241	8.03014	10.46075

approximate $\int_{1.8}^{0.6} f(x)\,dx$ using

(a) Trapezoidal Rule (b) Simpson's $\frac{1}{3}$ Rule

Ans: $N = 4$, $h = \dfrac{2.6 - 1.8}{4} = 0.2$

(a) Trapezoidal Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{N=4} f(x_i) + f(x_N)\right]$$

$$\int_{1.8}^{2.6} f(x)\,dx = \frac{0.2}{2}\left[3.12014 + 2(4.42569 + 6.04241 + 8.03014) + 10.46075\right]$$

$$= 0.1\,[50.58337]$$

$$= 5.05834$$

(b) Simpson's $\frac{1}{3}$ rule.

$$\int_{1.8}^{2.6} f(x)\,dx = \frac{0.2}{3}\left[3.12014 + 4(4.42569 + 8.03014) + 3(6.04241) + 10.46675\right]$$

$$= 0.06667\left[81.53744\right]$$

$$\int_{1.8}^{2.6} f(x)\,dx = 5.4361$$

Q10 Given the form of closed type Newton Cote's integration formula as

$$\int_a^b f(x)\,dx \simeq nh \sum_{i=0}^{n} c_i^n f(x_i) \text{ with}$$

$$h = \frac{b-a}{n}, \quad x_i = x_0 + ih, \quad i = 0,1,2 \ldots n$$

State two properties of Newton Cote's coefficient c_i^n.

Ans. c_i^n have two important properties.

① $\sum_{i=0}^{n} c_i^n = 1$

② $c_i^n = c_{n-i}^n$

Q11 Derive.

Ans (a) Trapezoidal Rule

We have the closed type Newton Cote's integration formula

$$\int_a^b f(x)\,dx \cong nh \sum_{i=0}^{n} c_i^n f(x_i)$$

with $h = \frac{b-a}{n}$, $x_i = x_0 + ih$,

$i = 0, 1, 2, \ldots n$

let $n = 1$

$$\int_a^b f(x)\,dx = 1h \sum_{i=0}^{1} c_i^n f(x_i)$$

$$= h c_0^1 f(x_0) + h c_1^1 f(x_1)$$

By two Properties of Newton Cote's Coefficients.

$c_0^1 = c_1^1$ and $c_0^1 + c_1^1 = 1$ gives

$c_0^1 = c_1^1 = \frac{1}{2}$

$\Rightarrow \int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right]$, which is trapezoidal formula.

(b) simpson's $\frac{1}{3}$ Rule

$$\int_a^b f(x)\,dx \cong nh \sum_{i=0}^{n} c_i^n f(x_i)$$

with $h = \frac{b-a}{2}$,

$x_i = x_0 + ih$,

$i = 0, 1, 2 \ldots n$

let $n = 2$

$$\int_a^b f(x)\,dx \cong 2h \sum_{i=0}^{2} c_i^2 f(x_i) \quad \text{--- (1)}$$

Here $c_0^2 = c_2^2$ and $c_0^2 + c_1^2 + c_2^2 = 1$

By evaluating c_2^2

we know that $c_i^n = \frac{(1)^{n-i}}{ni!\,(n-i)!}$

$$\int_0^n \frac{t(t-1)(t-n)}{(t-1)}\,dt$$

$$c_2^2 = \frac{(-1)^{2-2}}{2\,2!\,(2-2)!}\int_0^2 \frac{t(t-1)(t-2)}{(t-2)}\,dt$$

$$c_2^2 = \cancel{(-1)^2}\,\frac{1}{4}\int_0^2 t(t-1)\,dt$$

$$= \frac{1}{4}\int_0^2 (t^2-t)\,dt$$

$$= \frac{1}{4}\left[\frac{t^3}{3}-\frac{t^2}{2}\right]_0^2$$

$$= \frac{1}{4}\left[\frac{8}{3}-\frac{4}{2}\right]$$

$$= \frac{1}{4}\left[\frac{8}{3}-2\right)$$

$$= \frac{1}{4}\left[\frac{8-6}{3}\right]$$

$$= \frac{1}{\cancel{4}_2}\left[\frac{\cancel{2}^1}{3}\right]$$

$$= \frac{1}{6}$$

$$\therefore\ c_2^2 = \frac{1}{6} \Rightarrow c_0^2 = c_2^2 = \frac{1}{6}$$

and $c_1^2 = 1 - (c_0^2 + c_1^2)$

$= 1 - \frac{2}{6} = \frac{4}{6}$

$\therefore (1) \Rightarrow \int_a^b f(x)\,dx \simeq 2h\, c_0^2 f(x_0)$

$+ 2h\, c_1^2 f(x_1) + 2h\, c_2^2 f(x_2)$

$\simeq \frac{h}{3} f(x_0) + \frac{4h}{3} f(x_1) + \frac{h}{3} f(x_2)$

$\Rightarrow \int_a^b f(x)\,dx \simeq \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$

Which can also be expressed as

$\int_a^b f(x)\,dx = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

or

$\int_a^b f(x)\,dx \simeq \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$

This is Simpson's $\frac{1}{3}$ Rule

Q12 Estimate $\int_0^4 e^x \, dx$

Ans

(a) Simpson's $\frac{1}{3}$ rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$$

here $n = 2, h = 2$

$$\int_0^4 e^x\,dx = \frac{2}{3}\left[f(0) + 4f(0+2) + f(0+2(2))\right]$$

$$= \frac{2}{3}\left[1 + 4(7.38906) + 54.59815\right]$$

$$= \frac{2}{3}\left[85.15439\right]$$

$$= 56.769593$$

(b) Composite Simpson's $\frac{1}{3}$ rule with $N = 2$

Ans

here $N = 2$, $h = 2$

$x_0 = 0 \qquad = 1$
$x_1 = 2 \qquad = 7.38906$
$x_2 = 4 \qquad = 54.59815$

$$\int_0^4 e^x dx = \frac{h}{3}[f(x_0) + 2(f(x_1)) + f(x_2)]$$

$$= \frac{2}{3}[1 + 4(7.38906) + 54.59815]$$

$$= \frac{2}{3}[85.15439]$$

$$= 56.769593$$

(d) Calculate error in above

error in (c) simpson's $\frac{1}{3}$ Rule

Error = True value - Approximate value

$$= 53.59815 - 56.769593$$

$$= -3.171443$$

error in (b) composite Simpson's Rule with $N=2$

$\therefore$ error $= 53.59815 - 56.769593$

$= -3.171443$

error in (c) composite simpson's rule with $N=4$

error $= 53.59813 - 53.86385$

$= -0.2657$

Q13 Write psedocode for following integration formula to obtain results to desired accuracy

(a) Simpson's $\frac{1}{3}$ Rule

Ans pseudocode for simpson's $\frac{1}{3}$ Rule.

step 1: a, b, f(x), n = max no of iteration as inputs.

step 2: h = (b - a) / (2 * N)

Step 3: Do
for i = 1 to N - 1
x(2 * i) = a + h * 2 * i;
x(2 * i + 1) = a + h * (2 * i + 1)
Sum = sum + f(x(2 * i)).
End do.

step 4: Sum = 2 * Sum + 4 * Sum1 + f(a) + f(b)

Step 5: Output: Sum = h * Sum / 3
Print sum

step 6 : if $|sum - old\ sum| < \epsilon$;
output : Ans = sum ;
Exit
Else

step 7 : $k = k+1$, $N = 2*N$;
old sum = sum;
if $k \leq M$, goto step 3

step 8 : Else:

print does not give desired accuracy in N itteration.

(b) Trapezoidal Rule

~~step 1 : Input N, a, b, f(x), max~~

step 1: Input N, a, b, f(x),
M = max itteration.

step 2: sum = 0 ; K = 0 ;
N = 1 ; old sum = 0;
$h = (b-a)/N$;

step 3:

Do
for i=1 to N-1
  x = a + h * i
  sum += 2 * f(x)
end do

step 4: sum += f(a) + f(b)

step 5: sum = (h/2) * sum;
print sum:

step 6: if |sum - oldsum| < ε;
output : The estimated the integral is sum;
Exit;

step 7: Else
k = k+1; N = 2*N;
old sum = sum;
if k ≤ m goto step3

step 8: Else
Print does not give desired accuracy in 'n' itteration

Q14

Time	0	6	12	18	24	30	36	42	48
speed	124	134	148	156	147	133	121	109	99

Time	54	60	66	72	78	84
speed	85	78	89	104	116	123

how long is the track

$\int_0^{84} f(x)\,dx$

$\therefore$ By simpsons $\frac{1}{3}$ formula

here $N = 14$ $h = \frac{84}{14} = 6$

$\therefore \int_0^{84} f(x)\,dx =$

$\frac{h}{3}\,[f(x_0) + 4[f(x_1) + f(x_3) + f(x_5) + f(x_7) + f(x_9) + f(x_{11}) + f(x_{13})] + 2[f(x_2) + f(x_4) + f(x_6) + f(x_8) + f(x_{10}) + f(x_{12})] + f(x_{14})]$

$= \frac{6}{3}[124 + 4[134 + 156 + 133 + 109 + 85 + 89 + 116] + 2[148 + 147 + 121 + 99 + 78 + 104] + 123]$

$= 2\,[4929] = 9858$ feet

Q15 Approximate $\int_0^2 x^2 \ln(x^2+1)\,dx$ using $h = 0.25$ use

(1) Composite trapezoidal Rule

Ans Here $h = 0.25$, $n = \frac{b-a}{0.25} = \frac{2}{0.25} = 8$

$$\int_0^2 x^2 \ln(x^2+1)\,dx = \frac{h}{2}\left[f(x_0) + 2[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7)] + f(x_8)\right]$$

here

$x_0 = 0 \Rightarrow f(x_0) = 0$

$x_1 = 0.25 \Rightarrow f(x_1) = 0.00379$

$x_2 = 0.50 \Rightarrow f(x_2) = 0.05579$

$x_3 = 0.75 \Rightarrow f(x_3) = 0.25104$

$x_4 = 1 \Rightarrow f(x_4) = 0.6931$

$x_5 = 1.25 \Rightarrow f(x_5) = 1.47029$

$x_6 = 1.50 \Rightarrow f(x_6) = 2.65197$

$x_7 = 1.75 \Rightarrow f(x_7) = 4.29301$

$x_8 = 2 \Rightarrow f(x_8) = 6.43775$

$$\int_0^2 x^2 \ln(x^2+1)\,dx$$

$$= \frac{0.25}{2}\left[0 + 2[0.00379 + 0.05579 + 0.25104 + 0.69315 + 1.47029 + 2.65197 + 4.29301] + 6.43775\right]$$

$$= 0.125[25.27583]$$

$$\boxed{= 3.1595}$$

② Composite Simpson's $\frac{1}{3}$ rule.

here $h = 0.25$, $n = 8$

$$= \frac{0.25}{3}\left[0 + 4(0.00379 + 0.25104 + 1.47025 + 4.29301) + 2(0.0557 + 0.69315 + 2.65197) + 6.437\right]$$

$$= 0.083334[37.31209]$$

$$= 3.1092$$

(3) Composite Midpoint Rule.

$$\text{Midpont} = \int_a^b f(x)\,dx = (b-a)\, f\left(\frac{a+b}{2}\right)$$

$$\therefore \int_0^2 x^2 \ln(x^2+1)\,dx = \int_0^{0.25} x^2 \ln(x^2+1)\,dx + \int_{0.25}^{0.50} x^2 \ln(x^2+1)\,dx + \dots + \int_{0.75}^{2} x^2 \ln(x^2+1)\,dx$$

$$\therefore \int_0^{2.25} x^2 \ln(x^2+1)\,dx = (0.25-0)\, f\left(\frac{0.25+0}{2}\right)$$

$$= 0.25\,(f(0.125))$$

$$= 0.25\,(0.00024)$$

$$= 0.00006$$

$$\therefore \int_{0.25}^{0.50} x^2 \ln(x^2+1)\,dx = 0.25\, f(0.3752)$$

$$= 0.25\,(0.01858)$$

$$= 0.00463$$

$$\int_{0.50}^{0.75} x^2 \ln(x^2+1)\,dx = 0.25 \times f(0.625)$$

$$= 0.25(0.12881)$$

$$= 0.03220$$

$$\int_{0.75}^{1} x^2 \ln(x^2+1)\,dx = 0.25 \times f(0.875)$$

$$= 0.25(0.43526)$$

$$= 0.10882$$

$$\int_{1}^{1.25} x^2 \ln(x^2+1)\,dx = 0.25 \times f(1.125)$$

$$= 0.25 \times 1.03504$$

$$= 0.25877$$

$$\int_{1.25}^{1.50} x^2 \ln(x^2+1)\,dx = 0.25 \times f(1.375)$$

$$= 0.25(2.00685)$$

$$= 0.50171$$

$$\int_{1.50}^{1.75} x^2 \ln(x^2+1)\,dx = 0.25(f(1.625))$$

$$= 0.25(3.41210)$$

$$= 0.85302$$

$$\int_{1.75}^{2} x^2 \ln(x^2+1)\,dx = 0.25 \times f(1.875)$$

$$= 0.25(5.29996)$$

$$= 1.32499$$

$$\int_{0}^{2} x^2 \ln(x^2+1)\,dx = 0.00006 + 0.00463 + 0.03220 + 0.10882 + 0.25877 + 0.50171 + 0.85302 + 1.32499$$

$$\int_{0}^{2} x^2 \ln(x^2+1)\,dx = 3.0842$$

Q16 Approximate $\int_0^2 x^2 e^{-x^2} dx$ using

$h = 0.25$

(a) Composite trapezoidal Rule.

Ans $h = 0.25$, $N = 8$

$x_0 = 0 \quad = 0$
$x_1 = 0.25 \quad = 0.05871$
$x_2 = 0.50 \quad = 0.1947$
$x_3 = 0.75 \quad = 0.3205$
$x_4 = 1 \quad = 0.36788$
$x_5 = 1.25 \quad = 0.3275$
$x_6 = 1.50 \quad = 0.23715$
$x_7 = 1.75 \quad = 0.14324$
$x_8 = 2 \quad = 0.07326$

$= \frac{0.2}{2} [0 + 2(0.05871 + 0.1947 + 0.3205 + 0.36788 + 0.3275 + 0.23715 + 0.14324) + 0.07326]$

$= 0.42158$

(b) Composite simpson $\frac{1}{3}$ rule

$h = 0.25, N = 8$

$\int_0^2 x^2 e^{-x^2} dx =$

$= \frac{0.25}{3} [0 + 4(0.05871 + 0.32051 + 0.32775 + 0.14324) + 2(0.1947 + 0.36788 + 0.23715) + 0.07326]$

$= 0.083 [5.07252]$

$\boxed{= 0.42102}$

(c) Composite midpoint Rule.

$\int_0^{0.25} x^2 e^{-x^2} dx = 0.25 \times f(0.125)$

$= 0.25 \times 0.01538$

$= 0.00385.$

Like that

$$\int_{0.25}^{0.50} x^2 e^{-x^2} dx = 0.03054$$

$$\int_{0.50}^{0.75} x^2 e^{-x^2} dx = 0.06608$$

$$\int_{0.75}^{1} x^2 e^{-x^2} dx = 0.08901$$

$$\int_{1}^{1.25} x^2 e^{-x^2} dx = 0.08925$$

$$\int_{1.25}^{1.50} x^2 e^{-x^2} dx = 0.07136$$

$$\int_{1.50}^{1.75} x^2 e^{-x^2} dx = 0.04708$$

$$\int_{1.75}^{2} x^2 e^{-x^2} dx = 0.02613$$

$$\int_{0}^{2} x^2 e^{-x^2} dx = 0.00325 + 0.03054 + 0.06608 + 0.08901 + 0.08925 + 0.07136 + 0.04708 + 0.02613$$

$$\therefore \int_{0}^{2} x^2 e^{-x^2} dx = \boxed{0.4233}$$