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Assignment - 1

Central Tendency

Q1 Consider the numbers 2, 3, 4, 5, 5

(a) Compute the mean, median, & mode.

(i) mean : $\bar{X} = \frac{\Sigma x_i}{n} = \frac{2+3+4+5+5}{5} = \frac{19}{5}$

mean = 3.8

(ii) median : $\left(\frac{n+1}{2}\right)^{th}$ term

$\therefore$ median = $\left(\frac{6}{2}\right)^{th}$ term = 3^{rd} term

$\therefore$ Median = 4 //

(iii) Mode : value that occurs with greatest frequency

$\therefore$ Mode = 5

(b) If the numbers represented codes for the color of T-shirts ordered from catalog which average(s) would make sense?

→ Mode.

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(c) If the numbers represented one-way mileages for trails to different lakes, which average(s) would make sense.

→ Mean, Median & mode.

(d) Suppose the numbers represent survey response from 1 to 5, with 1 - disagree strongly, 2 - disagree, 3 - agree, 4 - agree strongly, 5 - agree very strongly. which average make sense?

→ Mode & Median

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Q2 Consider a data set of 15 distinct measurement with mean A and median B.

(a) If the highest number were increased, what would be the effect on the median and mean? Explain.

→ As the highest number is increased the value of Σx_i will also increase. So, the mean would increase while. The median would remain the same.

(b) If the higest number were decreased to a value still larger than B, what would be the effect on the median and mean?

→ As the highest number is decreased the value of Σx_i will also decreased so the mean would decrease while median would remain the same.

(c) If the highest number were decreased to a value smaller than B, what would be the effect on the median & mean?

→ As the highest number is decreased to a value smaller than B, Both mean and median would decrease.

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Q3 Consider the data set 2, 2, 3, 6, 10

(a) Compute the mode, Median & mean.

→ mode : value that occurs with greatest ~~fo~~frequency.

mode = 2

→ median : $\left(\frac{n+1}{2}\right)^{th}$ term $= \left(\frac{6}{2}\right)^{th}$ term

$= 3^{rd}$ term

median = 3

→ mean : $\bar{x} = \frac{\Sigma x_i}{n} = \frac{2+2+3+6+10}{5}$

$\therefore \bar{x} = \frac{23}{6} = 4.6$

(b) Multiply each data value by 5 compute the mode, median and mean.

→ New data values : 10, 10, 15, 30, 50

Mode = 10

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Median $= \left(\frac{6}{2}\right)^{th}$ term $= 3^{rd}$ term

$= 15$

Mean $= \bar{x} = 23$

(c) Compare the results of part (a) & (b)

→ In part (a) mode = 2
After multiplying each data value by 5
in part (b) Mode = 10

In part (a) Median = 3
After multiplying each data value by 5
in part (b) Median = 15

In part (a) Mean = 4.6
After multiplying each data value by 5
in part (b) Mean = 23

- In General when each data valle in a set is multiply by same constant, the result of mean, median & mode is same if multiplied by the same constant.

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(d) Suppose you have information about average heights of a random sample of airplane passangers. The mode is 70 inches, the median is 68 inches and the mean is 71 inches. To convert the data into cms. multiply each data ~~ent~~ value by 2.54. What are the values of the mode, median and mean in cms?

→ So, the value of mode, median & mean in centimeters are.

Mode $= 70 \times 2.54 = 177.8$ cms
Median $= 68 \times 2.54 = 172.72$ cms
Mean $= 71 \times 2.54 = 180.34$ cms.

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Q4 Consider the data set 2, 2, 3, 6, 10

(a) Compute the mean, median & mode.

mean $= \frac{2+2+3+6+10}{5} = \frac{23}{5} = 4.6$ //

Median $= \left(\frac{6}{2}\right)^{th}$ term $= 3^{rd}$ term $= 3$ //

mode = Greatest frequency data = 2 //

(b) Add 5 to each data value. And compute Mean, median & mode.

New data : 7, 7, 8, 11, 15

Mean $= \frac{7+7+8+11+15}{5} = \frac{48}{5} = 9.6$ //

Median $= \left(\frac{n+1}{2}\right) = \left(\frac{6}{2}\right) = 3^{rd}$ term $= 8$ //

Mode = Greatest frequency data = 7 //

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(c) compare the result of part (a) & (b)

→ In part (a) mean = 4.6
After adding 5 to each data value
in part (b) mean = 9.6

→ In part (a) median = 3
After adding 5 to each data value
in part (b) median = 8

→ In part (a) mode = 2
After adding 5 to each data value
in part (b) mode = 7

In General when the same constant is added to each data value, the result of mean, median and mode is also same if added by the same constant.

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Q5 Environmental Studies:

The following data are taken from a study conducted by the National park system, of which Death valley is a unit.

The ground temperatures (8F) were taken from May to Nov in the vicinity of furnance creek.

146, 152, 168, 174, 180, 178, 179, 180, 178, 178, 168, 165, 152, 144.

Compute the mean median & mode for these ground temperatures.

→ Mean $= \frac{\Sigma x_i}{n} = \frac{2342}{14} = 167.28$ //

Median = (For even) $\left(\frac{\left(\frac{n}{2}\right) + \left(\frac{n+1}{2}\right)}{2} \right)^{th}$ term

$\therefore \frac{7^{th} \text{ term} + 8^{th} \text{ term}}{2} = \frac{168 + 174}{2}$

$\therefore 171$ //

Mode $= 178$ //

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Q6 The following information is from a random sample of winter wolf packs in region of Alaska, minnesto, michigan, wisCovsin, canada & Finland

winter pack size : 13, 10, 7, 5, 7, 7, 2, 4, 3, 2, 3, 15, 4, 4, 2, 8, 7, 8

→ Mean : $\bar{x} = \frac{\Sigma x_i}{n} = \frac{111}{18} = 6.16$

→ Median : $\frac{9^{th}\text{ term} + 10^{th}\text{ term}}{2} = \frac{5+7}{2}$

$\therefore \frac{12}{2} = 6$

Median = 6

→ Mode = 7

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Q7 upper canyon: 2, 3, 1, 1, 3, 4, 6, 9, 3, 1, 3

lower canyon: 8, 1, 1, 0, 6, 7, 2, 14, 3, 0, 1, 13, 2, 1

(a) compute mean, median & mode for upper

-> Mean: $\frac{\Sigma xi}{n} = \frac{36}{11} = 3.27$ //

-> Median $= \left(\frac{n+1}{2}\right)^{th}$ term $= \frac{12}{2} = 6^{th}$ term

median $= 4$

-> Mode $= 3$

(b) compute mean, median & mode for Lower

-> Mean $= \frac{\Sigma xi}{n} = \frac{59}{14} = 4.21$

-> median $= \frac{\left(\frac{n}{2}\right) + \left(\frac{n+1}{2}\right)}{2} = \frac{7^{th}\text{ term} + 8^{th}\text{ term}}{2}$

$= \frac{2+14}{2} = 8$ //

-> Mode $= 1$ //

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(c) Compare the results of part (a) & (b).

→ The mean of upper canyon : 3.27
The mean of lower canyon : 4.21

→ The median of upper canyon : 4
The median of lower canyon : 8

→ The mode of upper canyon = 3
The mode of lower canyon = 1

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Q8 Costs in dollars per day in Island of mavi

89, 50, 68, 60, 375, 55, 500, 71, 40, 350, 60, 50
250, 45, 45, 125, 235, 65, 60, 130

(a) Compute the mean, median & mode for the data.

→ Mean : $\bar{x} = \frac{\sum x_i}{n} = \frac{2723}{20} = 136.15$ //

→ median: $\frac{\left(\frac{n}{2}\right) + \left(\frac{n+1}{2}\right)}{2} = \frac{10^{th}\text{ term} + 11^{th}\text{ term}}{2}$

$= \frac{65 + 68}{2} = 66.5$ //

→ Mode : 60 //

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Assignment - 2

Variation

Q1

x :	11	0	36	21	31	23	24	-11	-11	-21
y :	10	-2	29	14	22	18	14	-2	-3	10

(a) compute Σx, Σx^2, Σy and Σy^2

→

x	y	x^2	y^2
11	10	121	100
0	-2	0	4
36	29	1296	841
21	14	441	196
31	22	961	484
23	18	529	324
24	14	576	196
-11	-2	121	4
-11	-3	121	9
-21	10	441	100
103	90	4607	2258

So, $\Sigma x = 103$ $\qquad \Sigma x^2 = 4607$

$\Sigma y = 90$ $\qquad \Sigma y^2 = 2258$

(b) Use the results of part (a) to compute the sample mean, variance and standard deviation for x and for y.

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→ Mean, Variance and standard for x :

x	$\bar{x}$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
11	10.3	0.7	0.49
0	10.3	-10.3	106.09
36	10.3	25.7	660.49
21	10.3	10.7	114.49
31	10.3	20.7	428.49
23	10.3	12.7	161.29
24	10.3	13.7	187.69
-11	10.3	-21.3	453.69
-11	10.3	-21.3	453.69
-21	10.3	-31.3	979.69
103		0	3546.10

Mean : $\bar{x} = \frac{103}{10} = 10.3$

Variance : $s^2 = \frac{3546.10}{9} = 394.01$

Standard deviation : $s = \sqrt{394.01}$

$= 19.85$ //

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Mean, variance and standard deviation for y

y	$\bar{x}$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
10	9	1	1
-2	9	-11	121
29	9	20	400
14	9	5	25
22	9	13	169
18	9	9	81
14	9	5	85
-2	9	-11	121
-3	9	-12	144
-10	9	-19	361
90		0	1448

$$\text{Mean} = \bar{x} = \frac{90}{10} = 9$$

$$\text{variance} = s^2 = \frac{1448}{9} = 160.89$$

$$\text{standard deviation} = \sqrt{160.89}$$

$$= 12.68$$

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(c) Compute a 75% Chebyshev interval around the mean for x values and also for y values. Use the intervals to compare the two ~~finds~~ funds.

→ We know that 75% of the observation here values within two standard deviations of the mean.

So, $\bar{x} \pm 2s = 10.3 - 2(19.85)$
$= -29.4$
and $10.3 + 2(19.85)$
$= 50$

So, for x it is -29.4 to 50

for y $\bar{x} \pm 2s = 9 - 2(12.68)$
$= -16.36$
and $9 + 2(12.68)$
$= 34.36$

So, for y it is -16.36 to 34.36

Vanguard Balanced index (y) has a smaller spread than Vanguard total stock index (x)

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(d) Compute the coefficient of variation for each funds. Use the coefficient of variation to compare the two funds. If S represents risks and $\bar{x}$ represents expected return, then $S/\bar{x}$ can be thought of as a measure of risk per unit of expected return. In this case, why is smaller CV better? Explain

→ Co-efficient of variation

$$= \frac{SD}{\bar{X}} \times 100\%$$

For x : $CV = \frac{19.85}{10.3} \times 100\%$

$= 192.7\%$

For y = $CV = \frac{12.68}{9} \times 100\%$

$= 140.9\%$

If S represents risks and $\bar{x}$ represents expected return, then as the $S/\bar{x}$ is lesser, lower is the risk
so, a smaller CV is better because it means a lower risk

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Q2 Let X be a random variable representing time to failure (in hours) at 90% breaking strength.

0.54	1.80	1.52	2.05	1.03	1.18	0.80	1.33	1.29
1.11	3.34	1.54	0.08	0.12	0.60	0.72	0.92	1.05
1.43	3.03	1.81	2.17	0.63	0.56	0.03	0.09	0.18
0.34	1.57	1.45	1.52	0.19	1.55	0.02	0.07	0.65
0.40	0.24	1.57	1.45	1.60	1.80	4.69	0.08	7.89
1.58	1.64	0.03	0.23	0.72				

(a) Find the range

→ Range = Maximum Value − Min Value
= 7.89 − 0.02
= 7.87

(b) Compute Mean, Variance and SD

→ Mean: $\bar{x} = \frac{\Sigma x}{n} = \frac{62.11}{50} = 1.24$

→ Variance: $s^2 = \frac{89}{50} = 1.78$

→ SD : $s = \sqrt{1.78} = 1.33$

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(d) Compute the coefficient of variation. What does this number say about time to failure? Why does a small CV indicate more consistent data, whereas a larger CV indicate less consistent data? Explain.

→ Coefficient of variation

$$= \frac{SD}{Mean} \times 100\%$$

$$= \frac{1.33}{1.24} \times 100\% = 107\%$$

$CV = 107\%$

The SD of the time to failure is just slightly larger than the average time.

A smaller CV indicates more consistent data because the vale of s in the numerator is smaller

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Q3 Given :-

pax world Balanced:

Mean = 9.58%

SD (s) = 14.05%

Vanguard Balanced Index:

Mean = 9.02%

SD (s) = 12.50%

(a) Compute the coefficient of variation for each fund.

→ for pax world Balanced:

$$\text{Coefficient of variation (CV)} = \frac{SD}{\bar{x}} \times 100\%$$

$$CV = \frac{14.05}{9.58} \times 100\%$$

$$= 146.7\%$$

→ For Vanguard Balanced Index:

$$CV = \frac{12.50}{9.02} \times 100\%$$

$$CV = 138.6\%$$

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here we are using the coefficient of variation to represent risk per unit of return because coefficient of variation indicates how large is the SD relative to the Mean.

From this point of view, the risk per unit of return of vanguard Balanced Index ~~ox~~ is slightly lesser than that of Pax world Balanced.

So, the fund of vanguard Balanced Index appears to be better.

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Qn Given average number of physician visits by males per year.

Mean = 2.2

Coefficient of Variation = 1.5%

Determine standard of Deviation

$\rightarrow$ $CV = \frac{s}{\bar{x}} \times 10\%$

So, $s = 0.015 \times 2.2 = 0.033$

So, the standard deviation of the annual number of visits to physicians made by males is 0.033

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COSM Chapter-4

Probability

① steps outcomes

steps	outcomes
1	3
2	2
3	4

how many experimental outcomes exists for the entire experiment?

→ Total experimental outcomes = 3 x 2 x 4
= 24 outcomes

② how many ways can three items be selected from a group of six items?

→ $^6C_3 = \frac{6!}{3!(6-3)!} = \frac{6 \times 5 \times 4}{3 \times 2} = 20$

There are 20 ways to select 3 items from a group of 6 items

Given A, B, C, D, E & F are 6 ~~item~~ items

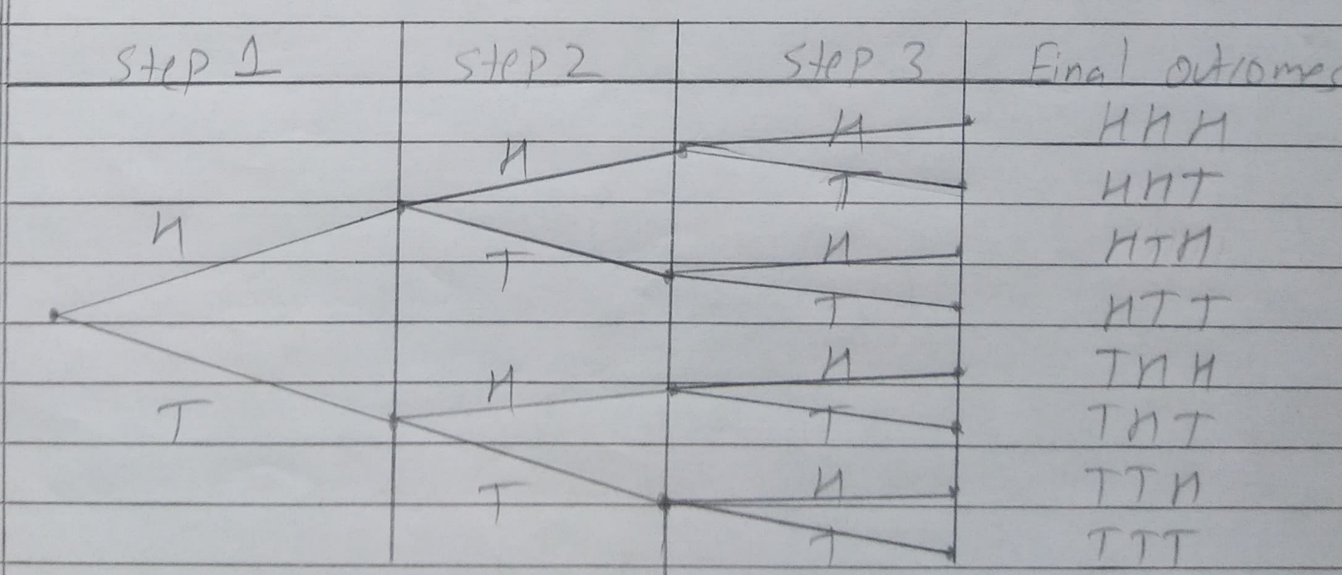
Combinations : { (ABC), (ACD), (ADE), (AEF), (ABD), (ABE), (ABE), (AB, F), (ACE), (ACF), (AD,F), (BCD), (BDE), (BEF), (BCF), (BCF), (BDF), (CDE), (CDF), (DEF) (CEF) }

③ How many permutations of 3 items can be selected from a group of 6

$\rightarrow \quad {}^6P_3 = \frac{6!}{(6-3)!} = 6 \times 5 \times 4 = 120$

There are 120 Permutation of 3 items can be selected from a group of 6

④ Tree Diagram



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(b) Experimental outcomes
{
(HHH), (HHT), (HTH) (HTT)
(THT), (THT (TTH) (TTT) }

(c) Probability for each experiment outcome is $\frac{1}{8}$.

(5) Outcomes : E_1, E_2, E_3, E_4, E_5

Assigning Probabilities :

Probability for E_1 to occur $= \frac{1}{5} = 0.2$
similarly for E_2, E_3, E_4 and $E_5 = 0.2$

that satisfies equation 4.3 which says 'each experimental outcomes is always between 0 and 1

ie. $0 \leq p(E_i) \leq 1$ for all i

Also, adding probality of each experiments outcomes gives $0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1$

that satisfies equation 4.4 which says 'sum of probabilities of all the experimental outcomes is equal to 1.0'
ie. $P(E_1) + P(E_2) + \ldots P(E_n) = 1$//

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(a) Experiment outcomes

Experiment	outcomes
E1	20
E2	13
E3	17
	50

Assigning Probabilities:

$E_1 = 20/50 = 0.4$

$E_2 = 13/50 = 0.26$

$E_3 = 17/50 = 0.34$

here, 'relative frequency method' is used as the data are available to the estimate the proportion of occurance.

(b) Given subjectively assigned probability.

$P(E_1) = 0.10$ $P(E_2) = 0.15$

$P(E_3) = 0.40$ $P(E_4 = 0.20$

The subjectively assigned probabilities are not Valid because it must satisfy two basic requirement of equation (4.3) and (4.4)

$P(E_1) + P(E_2) + P(E_3) + P(E_4) = 0.85$

As the sum of all probability for the experimental outcomes is not 1, it is not valid.

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Discrete / Continuous Random variable.

Q1 Define random variable write Probability distribution?

→ Random variable : A random variable provides a means for describing experimental outcomes using numeric values.

- A random variable is a numerical description of the outcome of an experiment.

- A random variable can be a 'Discrete random variable' or a 'continuous random variable'.

Q2 Give example of Discrete random variable & Continuous random variable.

→ Discrete random variable:

(i) Number of customer who place an order
ie. 0, 1, 2, 3, 4, 5.

(ii) number of defective radios out of 50 radios.
ie. 0, 1, 2, 3, ... 49, 50.

(iii) Number of customer in a resturant
ie. 0, 1, 2, 3. ....

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(iv) Gender of the Customer
ie. 0 if male.
1 if female.

Continuous random variable:

(i) Time of customer arrivals in a bank in minutes
ie. $x \geq 0$

(ii) height of students in a class in inches.

ie. $x \geq 0$

(iii) Percentage of project complete after six months.
ie. $0 \leq x \leq 100$

(iv) weight a shipment of goods in pounds
ie. $0 \geq 0$

Q 3 Discrete or continuous & Range

→ ① Tossing a coin with outcome as no. of heads.

→ Discrete random variable
Range: 0, 1 0 - Not head
1 - head.

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② Time between 2 consequitive flight arrive.

→ Continuous random variable.
Range $x \geq 0$.

③ Tossing 2 coins with outcomes as no of tails.
→ Discrete random variable.
Range : 0, 1, 2

0 - HH
1 - HT, TH
2 - TT

④ Distance between.

→ Continous random variable
Range: $x \geq 0$

⑤ outcomes of foot ball match
→ Discrete random variable
Range : 0, 1 0 for lose 1 for win

⑥ No. of Person selected in interview
→ Discrete random variable.
Range : 0, 1, 2 . . . .

⑦ weight of shipment
→ continuous random variable
Rang $x \geq 0$

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Binomial probability Distribution

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

(21) Given sucess : 23% $\therefore p = 0.23$

$x = 2$ $n = 6$ $(1-p) = 0.77$

(a) $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$f(2) = \binom{6}{2} p^2 (1-p)^4 = \binom{6}{2} (0.23)^2 (0.77)^4$

$= (15)(0.0529)(0.3515)$

$f(2) = 0.2789$

(b) $p(x \geq 2) = f(2) + f(3) + f(4) + f(5) + f(6)$

$$\left.\begin{aligned} f(2) &= 0.2789 \\ f(3) &= 0.1111 \\ f(4) &= 0.0249 \\ f(5) &= 0.0030 \\ f(6) &= 0.0001 \end{aligned}\right\} = 0.4180 //$$

(c) $n = 10$ $x = 0$

$f(0) = \binom{10}{0} (0.23)^0 (0.77)^{10}$

$f(0) = 0.0733 //$

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(29) Given $p = 30\% = 0.30$ $(1-p) = 0.70$

(a) $n = 10$ $x = 3$

$$f(3) = \binom{10}{3}(0.30)^3(0.70)^7$$

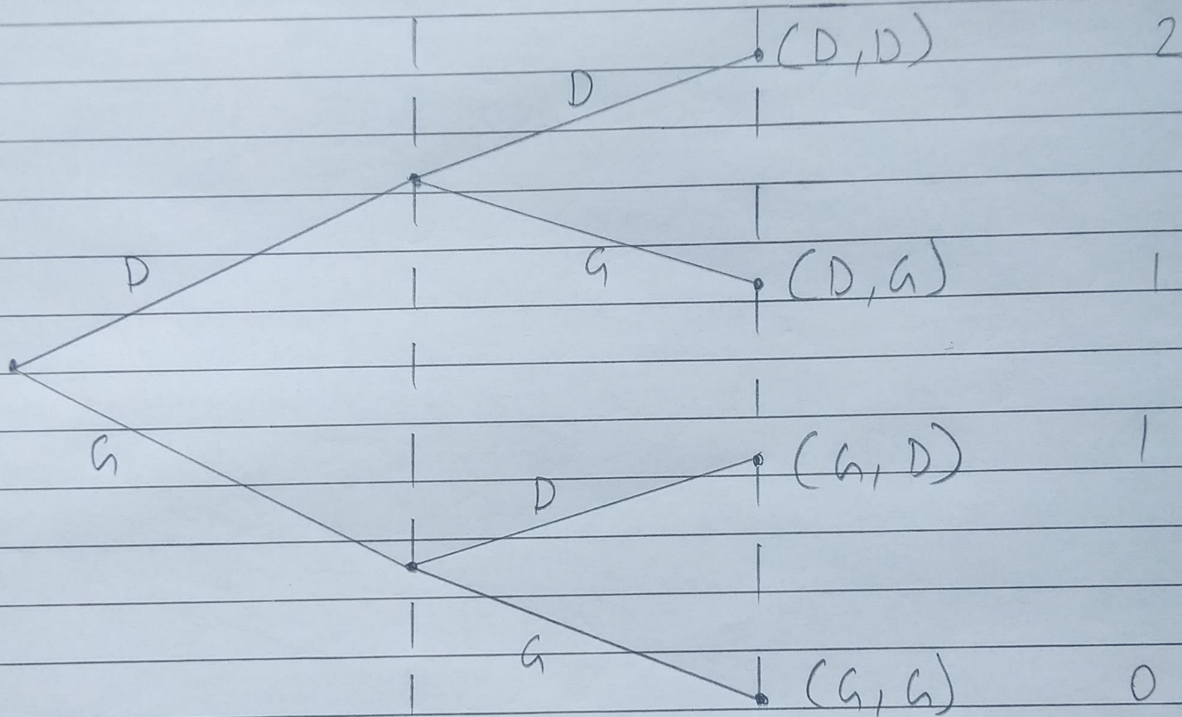
$$f(3) = 0.2668$$

(b) $P(x \geq 3) = f(3) + f(4) + f(5) \dots f(10).$

$$\left.\begin{aligned} f(3) &= 0.2668 \\ f(4) &= 0.2001 \\ f(5) &= 0.1029 \\ f(6) &= 0.0368 \\ f(7) &= 0.0090 \\ f(8) &= 0.0014 \\ f(9) &= 0.0001 \\ f(10) &= 0.0000 \end{aligned}\right\} = 0.6171.$$

(30) (a) Probability of a defective part being produced must be 0.03 for each part selected. parts must be selected independently.

(b) 1st part | 2nd part | outcome | No of defective



(c) Two outcomes with exactly one defective found.

(d) $P(\text{no defects}) = \binom{2}{0}(0.03)^0(0.97)^2 = 0.9409$

$P(1 \text{ defects}) = \binom{2}{1}(0.03)^1(0.97)^1 = 0.0582$

$P(2 \text{ defects}) = \binom{2}{2}(0.03)^2(0.97)^0 = 0.0009$

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Poisson Probability distribution

- The poisson Probability distribution is often used to model random arrivals in waiting line situations.

- A discrete random ~~o~~ variable is often useful in estimating the number of occurance over a specified interval of time or space.

Properties of a Poisson Experiment

1. The Probability of an occurence is the same for any two interval of equal length

2. The occurence or non occurrence in any interval is independent of the occurrance or non occurence in any other interval

Poisson probability function

$$f(x) = \frac{\mu^{x} e^{-\mu}}{x!}$$

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(34) Consider a poission distribution with a mean of two occurrence per time period.

→ Given : $\mu = 2$ per time period.

(a) Write the appropriate Poission Probability function.

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

$$= \frac{2^x e^{-2}}{x!}$$

(b) What is expected number of occurences in three time periods?

→ Expected no. of occurence in three time periods is 6.

(c) Write the appropriate Poisson probability function to determine the probability of x occurence in three time periods.

→ $$f(x) = \frac{6^x e^{-6}}{x!}$$

$\mu = 6$

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(d) Compute the probability of two occurance in one time period.

→ Given: $\mu = 2$

$x = 2$

$$f(2) = \frac{2^2 e^{-2}}{2!} = \frac{4(0.1353)}{2} = 0.2706$$

(e) Compute the probability of six occurance in three time periods.

→ Given: For Three time periods $\mu = 6$

$x = 6$

$$f(6) = \frac{6^6 e^{-6}}{6!} = \frac{46656(0.0025)}{720} = 0.1606$$

(f) Compute the probability of five occurance in two time periods.

→ Given: $\mu = 4$

$x = 5$

$$f(5) = \frac{4^5 e^{-4}}{5!} = \frac{1024(0.0183)}{120} = 0.1563$$

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(40) Phone Calls arrive at the rate of 48 per hour at the reservation desk for Regional Airways.

→ Given : $\mu = 48$ per hour.

(a) Compute the probability of receiving three calls in a 5 minute interval of time.

→ for 5-minutes $\mu = 4$

$x = 3$

$$f(3) = \frac{4^3 e^{-4}}{3!} = \frac{64(0.0183)}{6} = 0.1952$$

(b) Compute the probability of receiving exactly 10 calls in 15 minutes.

→ for 15 minutes $\mu = 12$

$x = 10$

$$f(10) = \frac{12^{10} e^{-12}}{10!} = 0.1048$$

~~(c) Suppose no calls a~~

(c) for 5-minutes $\mu = 4$
which means Regional Airways receive 4 calls every 5-minutes

if the agent takes 5-minutes to complete the current calls ~~rests~~ 3 callers are

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in waiting.

So, 3 callers are waiting every 5-minutes
So 36 callers are waiting per hour

Probability that none will be waiting

$x = 0 \quad \mu = 4$

$$f(0) = \frac{4^0 e^{-4}}{0!} = 0.0183$$

(d) for 3-minute $\mu = 2.4$

Probability for personal time without being interupted by a call.

$x = 0$

$$f(0) = \frac{2.4^0 e^{-2.4}}{0!} = 0.0907$$ //

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(43) Airline passengers arrive randomly and independently at the passenger screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.

→ Given : $\mu = 10$ per minute.

(a) $\mu = 10$ per minute.
$x = 0$

$$f(0) = \frac{10^0 e^{-10}}{0!} = 0.0000454$$

(b) $\mu = 10$ per minute.

$$f(1) = \frac{10^1 e^{-10}}{1!} = 0.000454$$

$$f(2) = \frac{10^2 e^{-10}}{2!} = 0.0023$$

$$f(3) = \frac{10^3 e^{-10}}{3!} = 0.0076$$

$$P(x \le 3) = f(0) + f(1) + f(2) + f(3)$$
$$= 0.0000454 + 0.000454 + 0.0023 + 0.0076$$
$$P(x \le 3) = 0.0104$$

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(c) for 15 - seconds $\mu = 2.5$

$x = 0$

$$f(0) = \frac{2.5^{0} \; e^{-2.5}}{0!} = 0.0821$$

(d)

$$P(x \geq 1) = 1 - f(0)$$

$$= 1 - 0.0821$$

$$P(x \geq 1) = 0.9179$$

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Hypergeometric probability Distribution

It is closely related to the binomial distribution

The two probability distributions differ in two key ways.

- In hypergeometric distribution, the trials are not independent
- the probability of sucess changes from trial to trial.

Hypergeometric probability function:

It is used to compute the probability that is a random selection of n elements, selected without replacement, we obtain x elements labeled success and $n-x$ elements labeled failure.

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

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(10) Given $N = 10$ and $r = 3$

Compute value of n and x.

(a) $n = 4$, $x = 1$

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$$f(1) = \frac{\binom{3}{1}\binom{10-3}{4-1}}{\binom{10}{4}} = \frac{\left(\frac{3!}{1!\,2!}\right)\left(\frac{7!}{3!\,4!}\right)}{\left(\frac{10!}{4!\,6!}\right)}$$

$f(1) = 0.50$

(b) $n = 2$, $x = 2$

$$f(2) = \frac{\binom{3}{2}\binom{7}{0}}{\binom{10}{2}}$$

$f(2) = 0.067$

(c) $n = 2$, $x = 0$

$$f(0) = \frac{\binom{3}{0}\binom{7}{2}}{\binom{10}{2}}$$

$f(0) = 0.4667$

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(d) $n=4, x=2$

$$f(2) = \frac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}}$$

$$f(2) = 0.30$$

(e) $n=4, x=4$

Given x is greater than 4 so,

$f(4) = 0$

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(48) Given $N = 10$ $\quad$ Football − 7

$n = 3$ $\quad$ Basketball − 3

(a)

$N = 10 \quad n = 3$

$x = 2 \quad r = 7$

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$$f(2) = \frac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}}$$

$$f(2) = 0.5250$$

(b)

$$f(2) = \frac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}} = 0.5250$$

+

$$f(3) = \frac{\binom{7}{3}\binom{3}{0}}{\binom{10}{3}} = 0.2917$$

$= 0.8167$

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(50) Given $N=60$, $n=10$

(a) $N=60$, $n=10$
$x=0$, $r=20$

$$f(0)=\frac{\binom{20}{0}\binom{40}{10}}{\binom{60}{10}}$$

$f(0)=0.0112$ //

(b) $N=60$, $n=10$
$x=1$, $r=20$

$$f(1)=\frac{\binom{20}{1}\binom{40}{9}}{\binom{60}{10}}$$

$f(1)=0.0725$ //

(c) for two or more employee
let's exclude them from the total probability i.e. 1

$f(x)=1-f(0)-f(1)$
$=1-0.0112-0.0725$
$=0.9163$ //

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ⓓ for texas : $N = 60$ $n = 10$
$x = 9$ $r = 40$

$$f(9) = \frac{\binom{40}{9}\binom{20}{1}}{\binom{60}{10}}$$

$$f(9) = 0.0725$$

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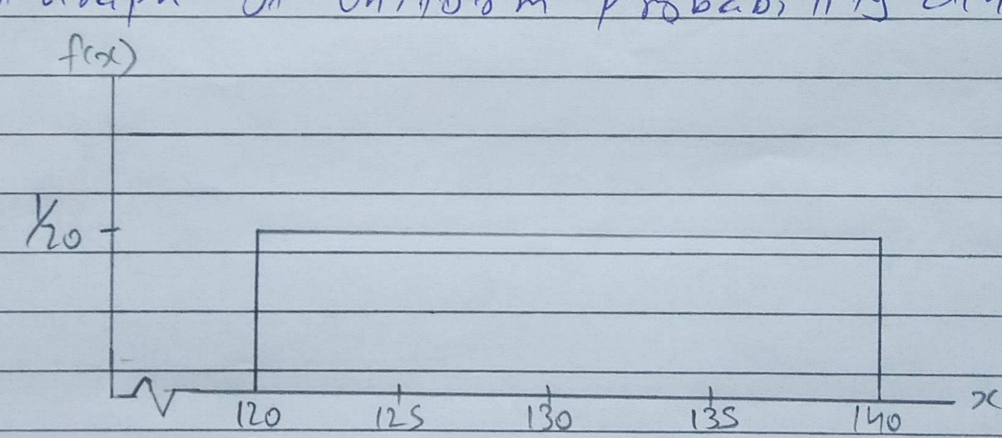
Uniform Probability Distribution

A continuous probability distribution for which the probability that the random variable will assume a value in any interval is the same for each interval of equal length.

* Uniform Probability Density function:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

* Graph of uniform probability density function



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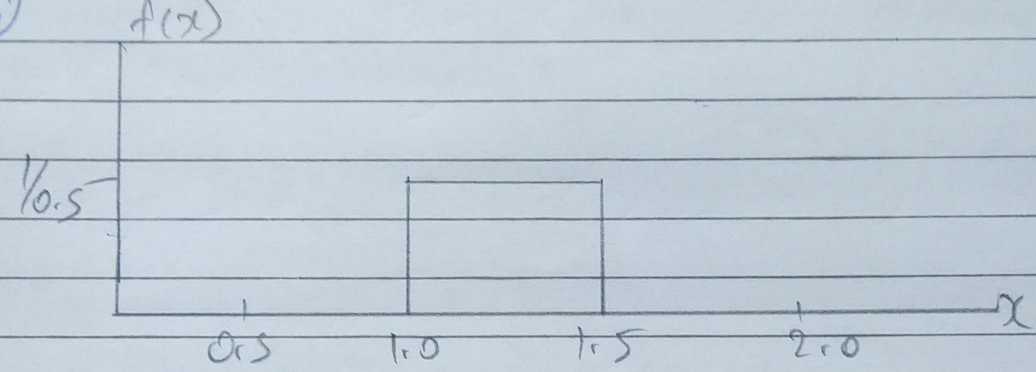
① The random variable x is known to be uniformly distributed between 1.0 and 1.5

ⓐ



ⓑ

$P(x = 1.25) = 0$

The probability of and single Point is zero because the area under the curve above any single point is zero

ⓒ $f(x) = \frac{1}{b-a}$; $b - a = 1.25 - 1.0$
$= 0.25$

$P(1.0 \le x \le 1.25) = 0.25 \times \frac{1}{0.5} = 0.5$ //

ⓓ $(b-a) = (1.5 - 1.2) = 0.3$

$P(1.2 \le x \le 1.5) = 0.3 \times \frac{1}{0.5} = 0.6$ //

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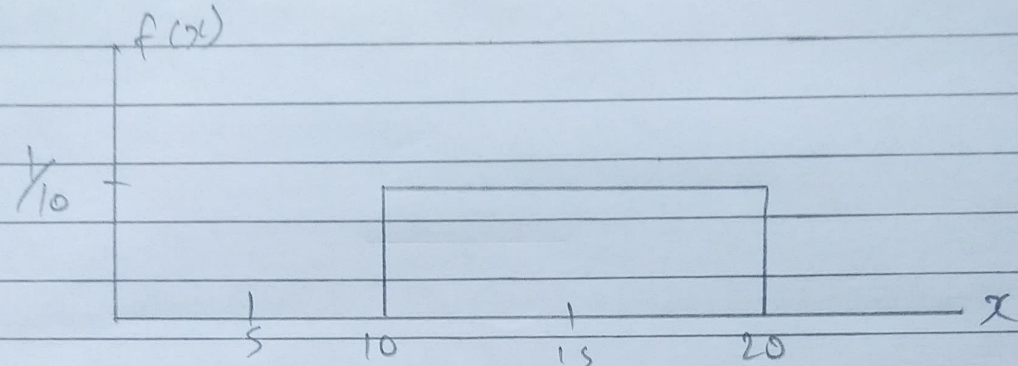
(2) The random variable, x is known to be uniformly distributed between 10 and 20.

(a)



(b) $(b-a) = (15-10) = 5$

$P(x \leq 15) = 5 \times \frac{1}{10} = 0.5$

(c) $(b-a) = (18-12) = 6$

$P(12 \leq x \leq 18) = 6 \times \frac{1}{10} = 0.6$

(3) Delta Airlines quotes a flight time of 2 hours, 5 minutes for its flights from Cincinnati to Taupa. suppose we believe that actual flight times are uniformly distributed between 2 hours and 2 hour 20 min.

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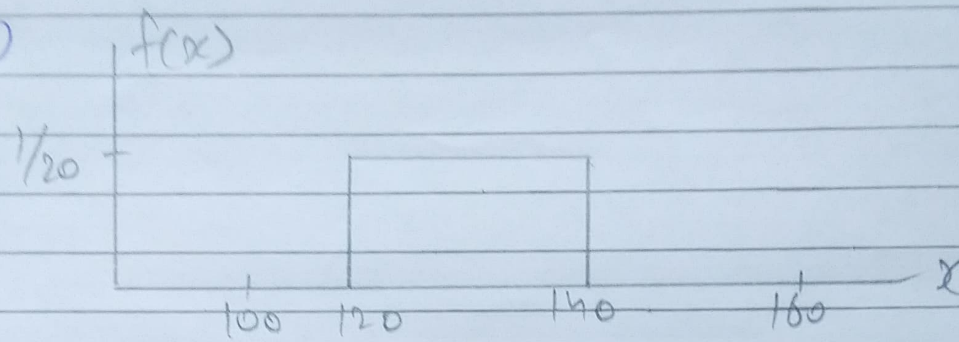
(a)



(b) $(b-a) = (130 - 120) = 10$

$P(120 \leq x \leq 130) = 10 \times \frac{1}{20} = 0.5$ //

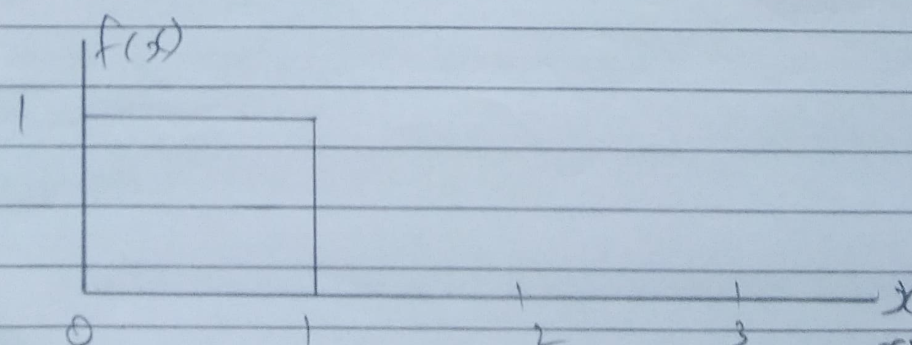
(c) $(b-a) = (140 - 135) = 5$

$P(x > 135) = 5 \times \frac{1}{20} = 0.25$ //

(4) x is a continuous random variable with probability density function.

$$f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a)



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(b) $(b-a)$ $(0.75 - 0.25) = 0.5$

$P(0.25 \leq x \leq 0.75) = 0.5 \times 1 = 0.5$ //

(c) $P(x \leq 0.3) = 0.3 \times 1 = 0.3$ //

(d) $(b-a) = (1 - 0.6) = 0.4$ //

$P(x > 0.6) = 0.4 \times 1 = 0.4$ //

(6) On Average 30 mins television sitcom have 22 mins of programming (CNBC, feb 23 2006). Assume that the probability distribution for minutes of programming can be approximated by a uniform distribution from 18 mins to 26 mins.

(a) $f(x) = \frac{1}{b-a} = \frac{1}{26-18} = \frac{1}{8}$

$P(x > 25) = (b-a) = (26-25) = 1$

$P(x > 25) = 1 \times \frac{1}{8} = 0.125$ //

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(b) $(b-a) = (25-21) = 4$

$P(21 \leq x \leq 25) = 4 \times \frac{1}{8} = 0.5$ //

(c) $30 - 22 = 8$ mins for other commercials

$P(x > 10) = (10-8) \times \frac{1}{8} = 0.25$ //

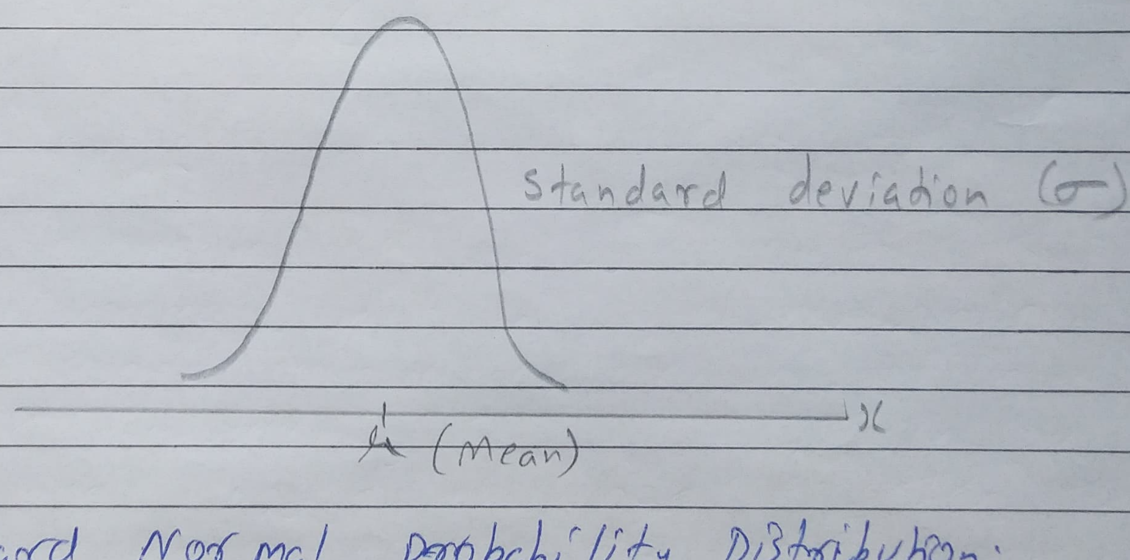
Normal Probability Distribution

A continuous probability distribution its probability density function is bell-shaped and determined by its mean and SD.

* Normal probability Density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where μ = mean $\qquad \pi = 3.14159$
σ = SD $\qquad e = 2.71828$



* Standard Normal probability Distribution:

A random variable that has a normal distribution with a mean of zero and a S.D of one is said to have a standard Normal probability Distribution

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

Converting to standard Normal Random variable

$$z = \frac{x - \mu}{\sigma}$$

Exercises :

(10) (a) $P(z \le 1.5) = 0.9332$
(b) $P(z \le 1) = 0.8413$
(c) $P(1 \le z \le 1.5) = 0.9332 - 0.8413 = 0.0919$.
(d) $P(0 \le z \le 2.5) = 0.9938 - 0.5 = 0.4938$.

(11) (a) $P(z \le -1.0) = 0.1587$
(b) $P(z \geq -1) = 1 - 0.1587 = 0.8413$
(c) $P(z \geq -1.5) = 1 - 0.0668 = 0.9332$.
(d) $P(-2.5 \le z) = 1 - 0.0062 = 0.9938$
(e) $P(-3 < z \le 0) = 0.5 - 0.0013 = 0.4987$

(12) (a) $P(0 \le z \le 0.83) = 0.7967 - 0.5 = 0.2967$
(b) $P(-1.57 \le z \le 0) = 0.5 - 0.0582 = 0.4418$
(c) $P(z > 0.44) = 1 - 0.6700 = 0.3300$
(d) $P(z \geq -0.23) = 1 - 0.4090 = 0.5910$
(e) $P(z < 1.20) = 0.8849$
(f) $P(z < -0.71) = 0.2389$

(13) (a) $P(-1.98 \le z < 0.49) = 0.6879 - 0.0239 = 0.6640$
(b) $P(0.52 \le z \le 1.22) = 0.8888 - 0.6985 = 0.1903$
(c) $P(-1.75 \le z \le -1.04) = 0.1492 - 0.0401 = 0.1091$

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(14) (a) $z = 1.96$
(b) $z = 0.5 + 0.4750 = 0.9750$ so $z = 1.96$
(c) $z = 0.61$
(d) $1 - 0.1314 = 0.8686$ so $z = 1.12$
(e) $z = 0.44$
(f) $1 - 0.3300 = 0.6700$ so $z = 0.44$

(15) (a) $z = -0.80$
(b) $0.9030/2 + 0.5 = 0.9515$ so $z = 1.66$
(c) $0.2052/2 + 0.5 = 0.6026$ so $z = 0.26$
(d) $z = 2.56$
(e) $1 - 0.6915 = 0.3085$ so $z = -0.50$

(16) (a) $1 - 0.01 = 0.9900$ so $z = 2.33$
(b) $1 - 0.025 = 0.9750$ so $z = 1.96$
(c) $1 - 0.05 = 0.9500$ so $z = 1.65$
(d) $1 - 0.10 = 0.9000$ so $z = 1.29$

(20) Given $\mu = 77$ $\sigma = 20$

(a) $x \leq 50$ $z = \frac{x - \mu}{\sigma} = \frac{50 - 77}{20} = -1.35$

so $P(z \leq -1.35) = 0.0855$

(b) $x > 100$ $z = \frac{100 - 77}{20} = 1.15$

$P(z > 1.15) = 1 - 0.8749 = 0.125$

so, 12.5% of workers spent more than 100 hours.

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(c) Given: upper 20% of usage

so left side of z (ie. bottom) = 1 − 0.2 = 0.80

also, $\mu = 77$ $\sigma = 20$

$$z = \frac{x - \mu}{\sigma}$$

$$\therefore x = \mu + z\sigma$$

$$= 77 + 0.8(20)$$

$$x = 93$$

So, a worker logged on to the internet for 93 hours is considered as heavy user.

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* Normal Approximation of Binomial probability

$\mu = np \qquad \sigma = \sqrt{np(1-p)}$

(28) Given $p = 0.20 \qquad n = 100$

(a) $\mu = np = (100)(0.20) = 20$

$\sigma = \sqrt{np(1-p)} = \sqrt{16} = 4$

(b) Yes because $np = 20$ so, binomial probability can be approximated by the normal probability distribution

(c) Given x exactly 24
considering continuity correction factor

$P(23.5 \le x \le 24.5)$

For $x = 23.5 \quad z = \frac{x-\mu}{\sigma} = \frac{23.5-20}{4} = 0.8750$

$P(z \le 0.88) = 0.8106$

For $x = 24.5 \quad z = \frac{x-\mu}{\sigma} = \frac{24.5-20}{4} = 1.1250$

$P(z \le 1.13) = 0.8708$

So, $P(0.88 \le z \le 1.13) = 0.8708 - 0.8106$
$= 0.0602$

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(27) Given $p = 0.60$ $n = 200$

(a) $\mu = np = (200)(0.6) = 120$

$\sigma = \sqrt{np(1-p)} = \sqrt{48} = 6.92 82$

(b) Yes because $\mu = 120$ so, binomial Probability can be approximated by the normal Probability distribution.

(c) Given $x = 100$ to 110.

Considering continuity correction factor.

$P(99.5 \leq x \leq 110.5)$

$z = \frac{99.5 - 120}{6.9282} = -2.96$ $P(z \leq -2.96) = 0.0015$

$z = \frac{110.5 - 120}{6.9282} = -1.37$ $P(z \leq -1.37) = 0.0853$

$P(-2.96 \leq z \leq -1.37) = 0.0853 - 0.0015 = 0.0838$

(d) Given $x \geq 130$

Considering continuity correction factor

$P(129.5 \leq x)$

$z = \frac{129.5 - 120}{6.9282} = 1.37$ $P(z \leq 1.37) = 0.9147$

So $P(x \geq 130) = 1 - 0.9147 = 0.0853$

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(28) Given $n = 250$ $P = 0.20$

(a) $\mu = nP = 250 \times 0.20 = 50$

So, 50 adults smoke.

$\sigma = \sqrt{nP(1-P)} = \sqrt{40} = 6.3246$

(b) $x < 40$

considering continuity correction factor

$P(x < 39.5)$

$z = \frac{39.5 - 50}{6.3246} = -1.66$ $P(z \le 1.66) = 0.0485$

$P(x \le 39.5) = 0.0485$

(c) $P(55 \le x \le 60)$

considering continuity correction factor

$P(54.5 < x \le 60.5)$

$z = \frac{54.5 - 50}{6.3246} = 0.71$ $P(z \le 0.71) = 0.7611$

$z = \frac{60.5 - 50}{6.3248} = 1.66$ $P(z \le 1.66) = 0.9575$

$P(54.5 \le z \le 60.5) = 0.9575 - 0.7811$
$= 0.1904$

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(d) $x \geq 70$

considering continuity correction factor
$P(x \leq 69.5)$

$z = \frac{69.5 - 50}{6.3248} = 3.08$ $P(z \leq 3.08) = 0.9990$

so, $P(x \geq 69.5) = 1 - 0.9990 = 0.0010$

* Exponential Probability Distribution

The exponential probability distribution is used for random variables such as the time between arrivals at a car wash. The time required to load a truck. The distance between major defects in a highway and so on.

* Exponential probability Density function.

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \text{ for } x \geq 0$$

* Exponential Distribution Cumulative Probality

$$P(x \leq x_0) = 1e^{-x_0/\mu}$$

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(32) Given $f(x) = \frac{1}{8} e^{-x/8}$ for $x \geq 0$

(a) $P(X \leq 6) = 1 - e^{-6/8} = 0.5276$
(b) $P(X \leq 4) = 1 - e^{-4/8} = 0.3935$
(c) $P(X \geq 6) = 1 - 0.5276 = 0.4724$
(d) $P(4 \leq X \leq 6) = 0.5276 - 0.3935 = 0.1341$

(33) $f(x) = \frac{1}{3} e^{-x/3}$ for $x \geq 0$
(a) $P(x \leq x_0) = 1 - e^{-x_0/\mu}$
(b) $P(x \leq 2) = 1 - e^{-2/3} = 0.4866$
(c) $P(x \geq 3) = e^{-3/3} = 0.3679$
(d) $P(x \leq 5) = 1 - e^{-5/3} = 0.8111$
(e) $P(2 \leq x \leq 5) = 0.8111 - 0.4866 = 0.3245$

(34) Given $\mu = 12.1$ min
(a) $P(x \leq 10) = 1 - e^{-10/12.1} = 0.5624$
(b) $P(x > 20) = 1 - e^{-20/12.1} = 0.1915$
(c) $P(10 \leq x \leq 20) = 0.8085 - 0.5624 = 0.2461$
(d) $P(x > 18) = e^{-18/12.1} = 0.2259$

(38) $\mu = 5.5$ Per hour

(a) $f(x) = 5.5 e^{-5.5x}$
(b) $5.5/4 = 1.3750$ per 15 min
for no-interruptions : $e^{-1.3750} = 0.2528$

(c) $5.5/6 = 0.9167$ Per 10 min
for interruption : $1 - e^{-0.9167} = 0.6002$

Hypothesis Testing

Q9. Given $H_0 : \mu \geq 20$
$H_a : \mu < 20$

$n = 50 \quad \mu = 19.4 \quad \sigma = 2$

(a) $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}}$ $\quad \sigma_{\bar{x}} = \sigma / \sqrt{n}$

$= 2 / 7.0711$

$z = \dfrac{19.4 - 20}{0.2828}$ $\quad \sigma_{\bar{x}} = 0.2828$

$z = -2.12$

$\therefore P(z \geq -2.12) = 0.0170$

(b) p-value = ?

$P(z \geq -2.12) = 0.0170$

(c) Given $\alpha = 0.05$
To accept the testing p-value should be
p-value $< \alpha$
$0.0170 \leq \alpha$
So, the Testing is Rejected.

(d) Reject H_0 if p-value $< \alpha$, So Rejected

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(b) Given $H_0 : \mu \leq 25$
$H_a : \mu > 25$

$n = 40 \quad \bar{x} = 26.4 \quad \sigma = 6$

(a) $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{26.4 - 25}{0.9487}$

$z = 1.4757$

(b) p-value

$P(z \geq 1.48) = 1 - 0.9306 = 0.0694$

(c) Given $\alpha = 0.01$
p-value $\leq \alpha$ Accepted.

p-value > 0.01 so donot reject H_0

(d) Reject H_0 if p-value $\leq \alpha$ but p-value > 0.01 so do not Reject H_0.

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(11) Given $H_0 : \mu = 15$
$H_a : \mu \neq 15$

$n = 50 \quad \bar{x} = 14.15 \quad \sigma = 3 \quad \sigma_{\bar{x}} = \sigma / \sqrt{n}$
$= 3/\sqrt{50}$
$\sigma_{\bar{x}} = 0.4243$

(a) $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{14.15 - 15}{0.4243} = -2$

(b) p-value $P(Z \geq -2)$

for $\mu = 15$ double p-value
$2\,P(Z \geq -2) = 2(0.0228)$
p-value $= 0.0456$

(c) $\alpha = 0.05$ p-value $\leq \alpha$ so, Reject.

(d) As p-value $\leq \alpha$, H_0 is Rejected.

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(12) Given $H_0 : \mu \geq 80$
$H_a : \mu < 80$

$n = 100$ $\sigma = 12$ $\alpha = 0.01$

(a) $\bar{x} = 78.5$ $\sigma_{\bar{x}} = 1.2$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{78.5 - 80}{1.2} = 1.25$$

$p\text{-value} = P(z \geq -1.25) = 0.1056$

here $p\text{-value} > 0.01$ so not rejected.

(b) $\bar{x} = 77$ $z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{77 - 80}{1.2} = -2.5$

$p\text{-value} = P(z \geq -2.5) = 0.0062$

$p\text{-value} \leq 0.01$, so H_0 is Rejected

(c) $\bar{x} = 75.5$ $z = \frac{75.5 - 80}{1.2} = -3.75$

$p\text{-value} = P(z \geq -3.75) = 0$ H_0 Rejected.

(d) $\bar{x} = 81$ $z = \frac{81 - 80}{1.2} = 0.83$

$p\text{-value} = P(z \geq 0.83) = 0.7967$

H_0 Not Rejected

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(13) $H_0 = \mu \leq 50$

$H_a = \mu > 50$

$n = 60 \quad \sigma = 8, \quad \sigma_{\bar{x}} = 1.0328$

$\alpha = 0.05$

(a) $\bar{x} = 52.5 \quad z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{52.5 - 50}{1.0328}$

$= 2.42$

p-Value : $P(z \geq 2.42) = 1 - 0.9922$

$= 0.0078$

p-Value ≤ 0.05, so, H_0 Rejected.

(b) $\bar{x} = 51 \quad z = \frac{51 - 50}{1.0328} = 0.9682$

$z = 0.97$

p-Value : $P(z \geq 0.97) = 1 - 0.8340$

p-Value $= 0.1660$

p-Value > 0.05, so, H_0 is not Rejected

(c) $\bar{x} = 51.8 \quad z = \frac{51.8 - 50}{1.0328} = 1.7428$

p-Value : $P(z \geq 1.74) = 1 - 0.9591 = 0.0409$

p-Value $\leq \alpha$

so, H_0 is Rejected.

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(14) Given $H_0 : \mu = 22$
$H_a = \mu \neq 22$

$n = 75$, $\sigma = 10$ $\alpha = 0.01$ $\sigma \bar{x} = 1.1547$

(a) $\bar{x} = 23$ $z = \frac{23 - 22}{1.1547} = 0.87$

for $\mu_0 = 22$ p-value $= 2P(z \geq 0.87) =$
$= 2(0.1922)$
$= 0.3844$

p-value > 0.01 so, H_0 is not Rejected.

(b) $\bar{x} = 25.1$ $z = \frac{25.1 - 22}{1.1547} = 2.68$
p-value $= 2P(z > 2.68) = 2(0.0037)$
$= 0.0074$
p-value ≤ 0.01 so H_0 is Rejected.

(c) $\bar{x} = 20$ $z = \frac{20 - 22}{1.1547} = -1.73$

p-value $= 2P(z \geq 1.73) = 2(0.0418)$
$= 0.0836$
p-value ≥ 0.01 so H_0 is not Rejected.