Pradip Karmakar

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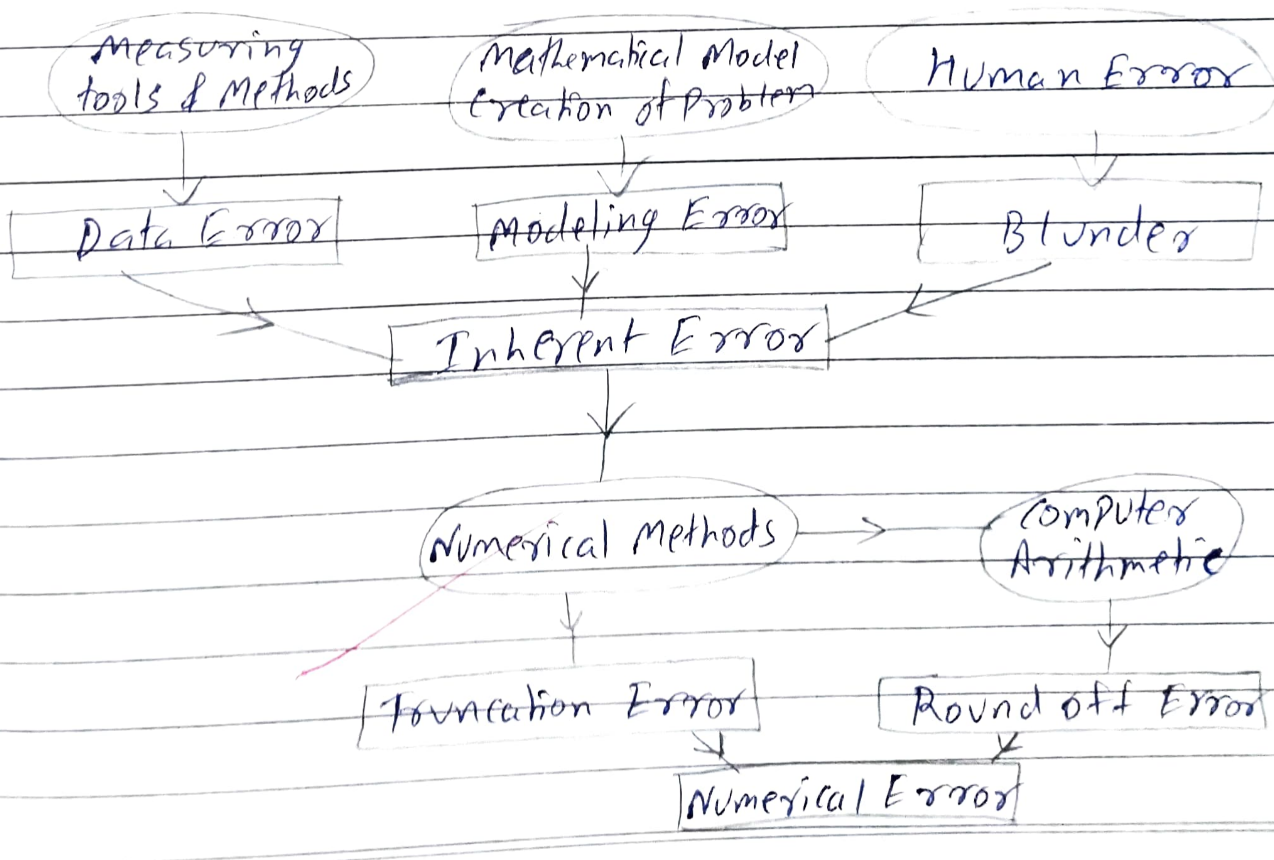
Explain the different types of errors that occur during computation.

There are two types of errors that occur during computation

1. Truncation Error
2. Round off Error

These two errors are being inherited by the numerical analyst are called Inherent Errors.



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Truncation Error

Truncation error is an error in implementation of numerical approximation method occurring due to ~~tru~~ truncations a process involving infinite number of steps to ~~of~~ finite numbers of steps, like

(i) Limiting infites step/series to finite number of terms

(ii) Limiting infinite number of iterations to finite number of iteration $(f(x)=0)$

(iii) Taking finite step size instead of infinitesimal step size.

Round off error

Round off error occur due to finite Precision in a computer. A number may not always have a finite representation

$\frac{1}{3} = 0.3333\ldots$

$\sqrt{2} = 1.4142135623\ldots$

$e = 2.718281828459\ldots;$

Whereas, 2, 10, 9.32 have finite representation in decimal system. Moreover, a number having finite representation in one number system may not have finite representation in another number system, for example.

$(1.1)_{10} = (1.00011001100110 0\ldots)_2$

One can never represent 1.1 exactly in binary system. So, let us understand first how a number is stored in a computer.

* Total Numerical error.

Total Numerical Error = Truncation error + Round of error.

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The only advisable way to reduce round off errors is to increase number of significant digits. The round off error increase due to either subtractive cancellation or due to increase in the number of computations in an analysis. The truncation error can be reduced by decreasing step size. So, determining appropriate step size is essential in order to balance truncation and round off error to minimize the total error.

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Explain the Bisection method graphically.

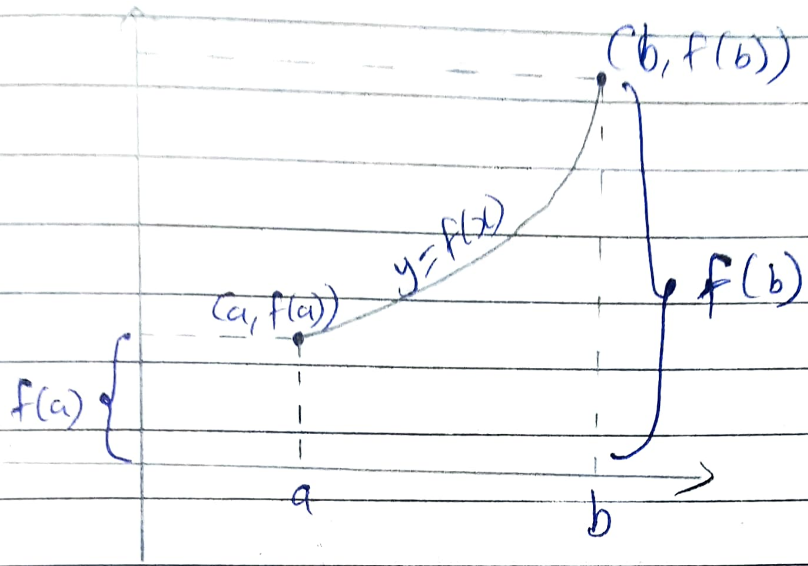
Bisection Method:

principle on which Bisection method is based ~~on~~ is as follows:

If $f(x)$ is continuous in a closed interval $[a,b]$ and $f(a), f(b)$ are of opposite signs, then the equation $f(x)=0$ will have at least one real root between a & b. This is based on intermediate value theorem.

Let us understand it graphically in lay person terms:

Since $f(x)$ is continuous on $[a,b]$, the graph of $y=f(x)$ for $a \le x \le b$ shall have no break. That is, it can be drawn from $(a, f(a))$ to $(b, f(b))$ without any jump.



No intermediate value can be skipped. Value of $f(x)$ completely occupy the range from $f(a)$ to $f(b)$.

As $f(a)$ and $f(b)$ are of opposite sign, function has to be zero for some $a \leq x \leq b$. Moreover if $f(a) \cdot f(b) < 0$ then one end of the graph is below x-axis and other end is above x axis. $f(x)$ being continuous, the graph of $y = f(x)$ has to cross x axis in going from one side of x axis to the other. In lay person term: It is like, if the stream of rive is flowing outside your home and you are permitted to walk only by taking infinite small continuous steps, that is you are not permitted to lift your legs and jump, you

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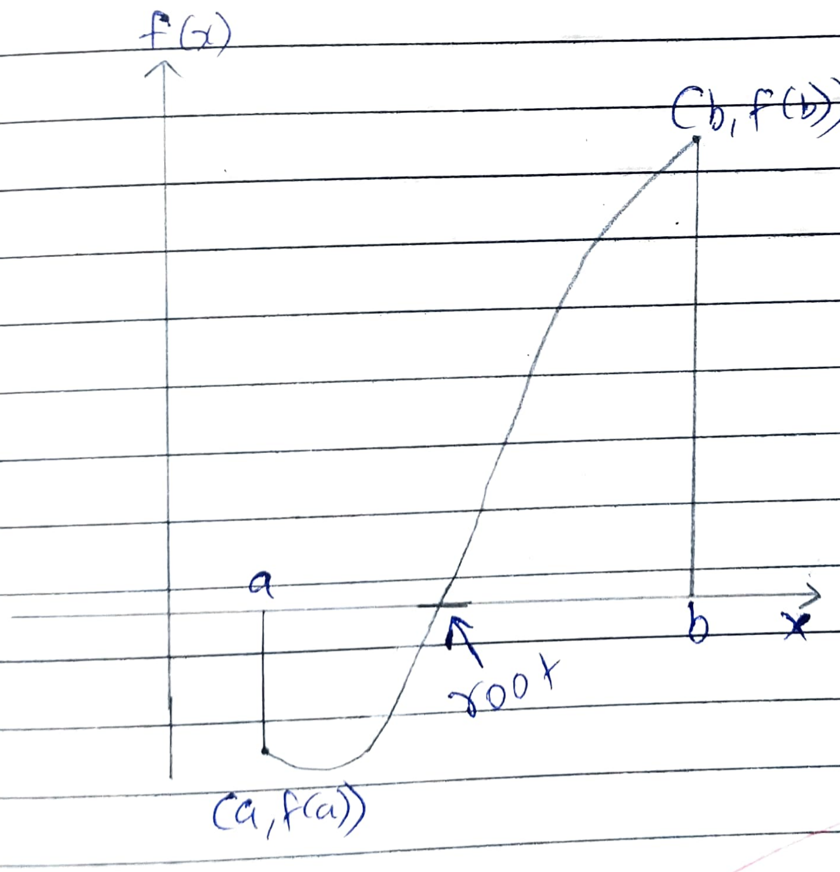
cannot get inside your home from outside without making your feet wet and vice versa.

Without loss of generality, let $f(a) < 0$. Thus $f(b) > 0$.



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Explain the advantages and disadvantages of Bisection Method.

Advantages of Bisection Method

1. It is very simple
2. Once initial a, b are known, number of iterations needed to be performed to achieved desired accuracy can be predetermined.
3. Reliable
4. ~~Gau~~ Guarantees convergence.
5. Function needs to be only continuous.
6. Only one function evaluation per iteration.
7. Calculation for making guess of root for the next iteration is very easy simply $c = \frac{a+b}{2}$.

Disadvantages of Bisection Method

① Not self starting. To begin with, requires two initial guesses at which function must of opposite sign. Thus, one has to evaluate function at subinterval points to get such 'a' and 'b'.

② slowest method.

③ It doesnot take into accout the ~~natural~~ nature of function to make next guess of the root.

Find the root of the following using Bisection method.

(a) $f(x) = x^3 - x - 1 = 0$

a	b	f(a)	f(b)	c	f(c)
1	2	-1.000000	5.000000	1.500000	0.875000
1	1.500000	-1.000000	0.875000	1.250000	-0.296875
1.250000	1.500000	-0.296875	0.875000	1.375000	0.224609
1.250000	1.375000	-0.296875	0.224609	1.312500	-0.051514
1.312500	1.375000	-0.051514	0.224609	1.343750	0.082611
1.312500	1.343750	-0.051514	0.082611	1.328125	0.014576
1.312500	1.328125	-0.051514	0.014576	1.320313	-0.018711
1.320313	1.328125	-0.018711	0.014576	1.324219	-0.002128
1.324219	1.328125	-0.002128	0.014576	1.326172	0.006209
1.324219	1.326172	-0.002128	0.006209	1.325195	0.002037
1.324219	1.325195	-0.002128	0.002037	1.324707	-0.000047
1.324707	1.325195	-0.000047	0.002037	1.324951	0.000995
1.324707	1.324951	-0.000047	0.000995	1.324829	0.000474
1.324707	1.324829	-0.000047	0.000474	1.324768	0.000214
1.324707	1.324768	-0.000047	0.000214	1.324738	0.000084
1.324707	1.324738	-0.000047	0.000084	1.324722	0.000018
1.324707	1.324722	-0.000047	0.000018	1.324715	-0.000014
1.324715	1.324722	-0.000014	0.000018	1.324718	0.000002

Root = 1.324718

(b) $xe^x = 1$

$\therefore xe^x - 1 = 0$

$f(x) = xe^x - 1 = 0$

a	b	f(a)	f(b)	c	f(c)
0	1	-1.000000	1.718282	0.500000	-0.175639
0.500000	1.000000	-0.175639	1.718282	0.750000	0.587750
0.500000	0.750000	-0.175639	0.587750	0.625000	0.167654
0.500000	0.625000	-0.175639	0.167654	0.562500	-0.012782
0.562500	0.625000	-0.012782	0.167654	0.593750	0.075142
0.562500	0.593750	-0.012782	0.075142	0.578125	0.030619
0.562500	0.578125	-0.012782	0.030619	0.570313	0.008780
0.562500	0.570313	-0.012782	0.008780	0.566406	-0.002035
0.566406	0.570313	-0.002035	0.008780	0.568359	0.00364
0.566406	0.568359	-0.002035	0.003364	0.567383	0.000662
0.566406	0.567383	-0.002035	0.000662	0.566895	-0.000687
0.566895	0.567383	-0.000687	0.000662	0.567139	-0.000013
0.567139	0.567383	-0.000013	0.000662	0.567261	0.000325
0.567139	0.567261	-0.000013	0.000325	0.567200	0.000156
0.567139	0.567200	-0.000013	0.000156	0.567169	0.000072
0.567139	0.567169	-0.000013	0.000072	0.567154	0.000029
0.567139	0.567154	-0.000013	0.000029	0.567146	0.000008
0.567139	0.567146	-0.000013	0.000008	0.567142	-0.000002
0.567142	0.567146	-0.000002	0.000008	0.567144	0.000003
0.567142	0.567144	-0.000002	0.000003	0.567143	0.000000

Root = 0.567143

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How many iterations do you need in Bisection method to get the root if you start with a=1 and b=2 and the tolerance is 10^{-4}?

$a = 1, b = 2$

if $f(x) = x^3 - x - 1$

a	f(a)	b	f(b)	c	f(c)
1.000000	-1.000000	2.000000	5.000000	1.500000	0.875000
1.000000	-1.000000	1.500000	0.875000	1.250000	-0.296875
1.250000	-0.296875	1.500000	0.875000	1.375000	0.224609
1.250000	-0.296875	1.375000	0.224609	1.312500	-0.051514
1.312500	-0.051514	1.375000	0.224609	1.343750	0.082611
1.312500	-0.051514	1.343750	0.082611	1.328125	0.014576
1.312500	-0.051514	1.328125	0.014576	1.320313	-0.018711
1.320313	-0.018711	1.328125	0.014576	1.324219	-0.002128
1.324219	-0.002128	1.328125	0.014576	1.326172	0.006209
1.324219	-0.002128	1.326172	0.006209	1.325195	0.002037
1.324219	-0.002128	1.325195	0.002037	1.324707	-0.000047

Root = 1.324707

Total iteration = 11

Explain False position or Regula falsi method graphically.

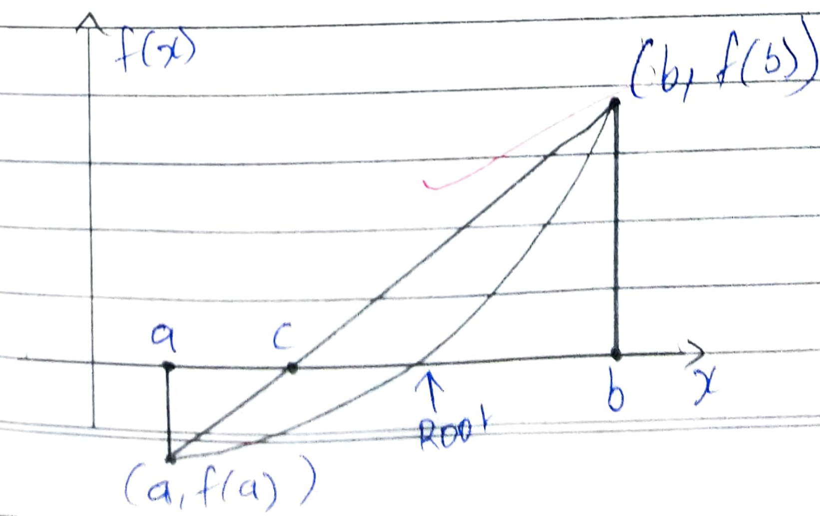
In Regula falsi method, to take into consideration the function value at 'a' and 'b' straight line is drawn, joining $(a, f(a))$ and $(b, f(b))$

→ The point, where it cuts x-axis, is the new estimate of the root.

→ Mathematically, estimate of formula for c can be derived as follows:-

$$c = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

→ If $f(c) = 0$, c is the root otherwise, if $f(a) \cdot f(c) > 0$, c takes the role of 'a' otherwise c takes the role of 'b'



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Q1 What is the need of numerical Integration? Explain different situation where such need arises.

Ans In integration, whenever anti derivative is known indefinite integral is known. The difficulty arieses in computing definite integral in many situation owns to different reasons.

→ Situation 1:

For evaluating $\int_a^b f(x)dx$, anti derivative of f exists but cannot be represented in a form of standard functions. For example, consider

i) $\int_0^1 e^{x^e} dx$ f(2) , $\int^2 \frac{dx}{\log x}$

One may say, anti derivative of e^{x^1} is $\int_0^x e^{t^2} dt$, but is not of no use to us. Both these cited integrals have precise numerical values. For all practical purposes, anti-derivation cannot be used to calculate the value F and thus of definite integral cannot be determined.

Many such examples are encountered in scientific, mathematical & statistical and engineering problems at. ① The error function.

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt,$$ used in calculating error bounds in many computations is of great importance in statistics. ② whereas $\int_2^x \frac{dt}{\ln(t)}$ is logarithm function, which gives approximately number of primes less than or equal to x.

③ In electric field theory, it is proved that the magnetic field induced by a current flowing in a circular loop of wire has intensity.

$$H(x) = \frac{4 I r}{r^2 - x^2} \int_0^{\frac{\pi}{2}} \left(1 - \left(\frac{x}{r}\right)^2 \sin(\theta)\right)^{\frac{1}{2}} d\theta$$

- where I is the current, r is the radius of the loop, x is the distance from center to the point, where the magnetic intensity is being computed ($0 \le x \le r$). If x, r and I are given, the integral occurring in the equation is called elliptic integral and cannot be expressed in terms of standard form.

- All three integrals are ferequently occuring integrals and we shall learn numerical methods called methods of numerical integration. Using these methods, we would be able to evaluate these integrals to as many decimal places as desired.

⇒ <u>Situation 2</u> :-

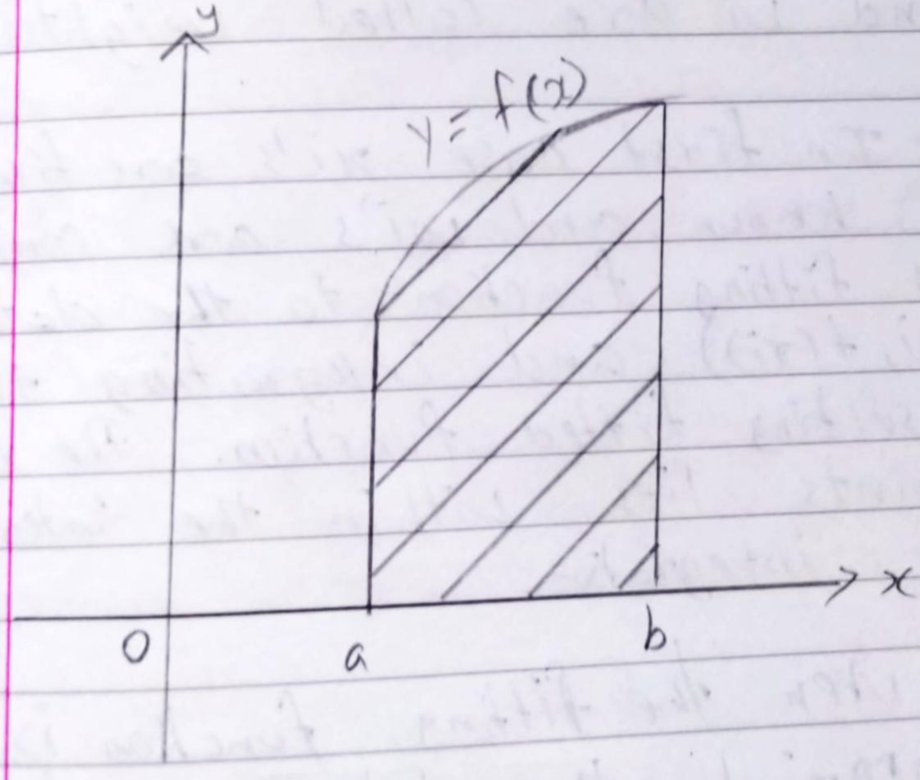
Anti-derivative may exist known but may be complicated expression to compute. In these cases, rather than evaluate anti-derivative, it is preferred to use numerical integration formula to evaluate the integral. A typical situation could be where anti-derivative expression is in the form of an infinite series or infinite product and its evaluation requires numerical techniques.

=> <u>Situation 3 :-</u>

Expression of function to be integrated is not available functions are defined with the help of discreate data. For example, speed of an object may have been measured at different time intervals and distance needs to be computed or integrated may have been obtained by sampling, so values are available at certain points only.

Q2. Give geometrical interpretation of $\int_a^b f(x)\,dx$, with $f(x) >, 0$ and $\leq x \leq b$.

Ans The definite integral $\int_a^b f(x)\,dx$ of a non-negative integral $f(x)$, on a closed interval $[a,b]$ area under the graph of f.



Q3 Differentiate between:
(a) Newton cotes Integration formulas and Gauss Quadrative formulas.
(b) Closed type and open type formulas.

Ans.

(a) Here x_i are called abscissas and w_i are called weights.

- In first case x_i's are fixed, that is known and w_i's are computed by fitting function to the data $(x_i, f(x_i))$ and integrating the resulting fitted function. The x_i's points lies within the interval $[a,b]$ of integration.

- When the fitting function is a polynomial, the integration formulas so obtained are called Newton cotes Integration formulas

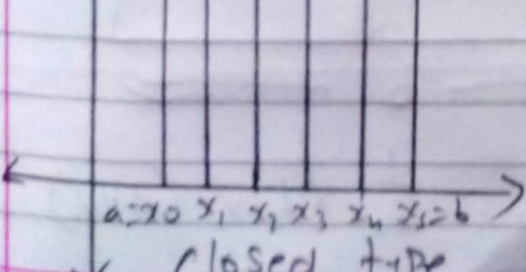
- On the other hand, if x_i's and w_i's both are assumed to be unknown and they are computed, the formulas so obtained are called Gauss Quadrature formulas

Integration formulas	
x_i's fixed w_i's unknown Newton Cotes Integration formulas	x_i's unknown w_i's unknown Gauss quadrature formula.

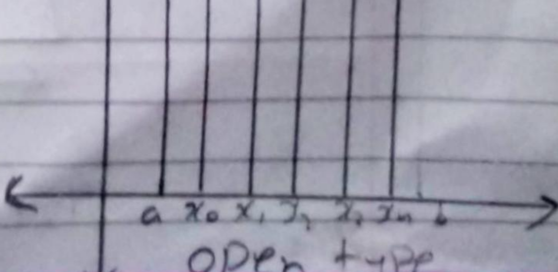
(b) Closed type and open type formulas.

- If $x_0 = a$ and $x_n = b$, i.e., $a = x_0 \le x_1 \le x_2 \le \dots \le x_{n-1} \le x_n = b$ then integration formula is called closed type integration formula.

- If $a \le x_0 < x_1 \le x_2 \le x_{n-1} \le x_n \le b$ then the integration formula is called open type integration formula.



closed type

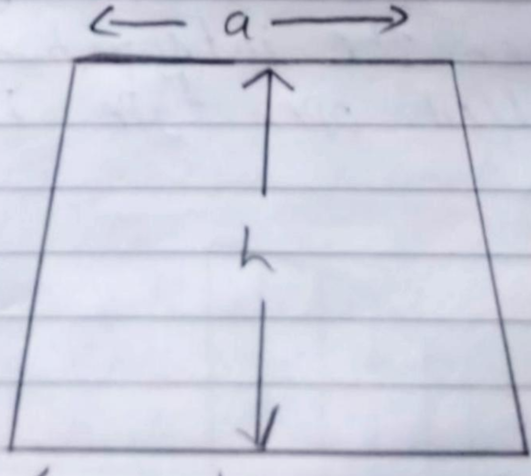


open type

Q4 Explain trapezoidal rule for estimating $\int_a^b f(x)\,dx$. Explain it graphically also.

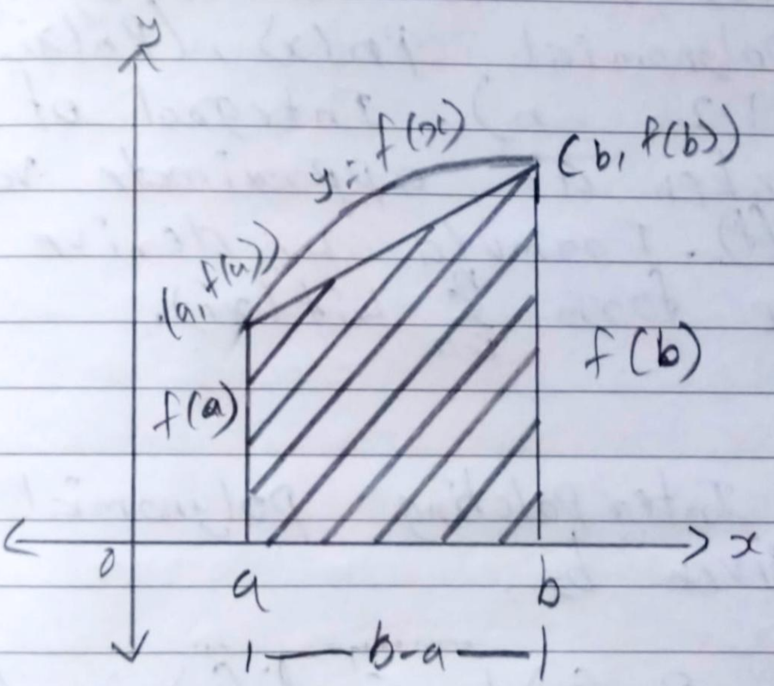
Ans By trapezoidal rule. $\int_a^b f(x)\,dx = \frac{\Delta x}{2}\left[f(a) + f(b)\right]$

→ This rule is called trapezoidal rule as it gives area of trapezium formed from joining the points $(a, f(a))$ and $(b, f(b))$ by a straight line and the vertical lines $x=a$ and $x=b$.
We know that area of trapezium = height × (average of parallel lines)



$$\text{Area} = \frac{1}{2} \times h \times (a+b)$$

→ So if we join the ~~pon~~ points $(a, f(a))$ and $(b, f(b))$, it forms a trapezium, with parallel side length as $f(a)$ and $f(b)$. height of trapezium is $(b-a)$, hence from the following graph, area of trapezium is $\frac{b-a}{2}[f(a)+f(b)] = \frac{\Delta x}{x}[f(a)+f(b)]$



Q5 Explain basic Principle (No Derivation) in deriving Newton Cote's integration formula.

Ans The basic principle in obtaining Newton cote's integration formula is to fix absissas $x_0, x_1, x_2 \ldots x_n \in [a,b]$ beforehand and integral $f(x)$ is approximated by interpolating polynomial $P_n(x)$, $(P_n(x_i) = y_i, \forall i = 1, 2 \ldots n)$. Integral of $P_n(x)$ is taken as approximate value of $I(f)$. Formula we derive, is of the form $\sum_{i=0}^{n} w_i f(x_i)$.

- Interpolating polynomial $P_n(x)$ is given by.

$$P_n(x) = \sum_{i=0}^{n} l_i(x) y_i \quad , (y_i = f(x_i))$$

where, $l_i(x) = \prod_{j=0, j \neq i} \frac{(x - x_j)}{x_i - x_j}$

$\forall i = 0, 1, \ldots, n.$

Thus,

$$\int_a^b P_n(x)\,dx = \int_a^b \left(\sum_{i=0}^{n} l_i(x) y_i\right) dx$$

$$= \sum_{i=0}^{n} \left(\int_a^b l_i(x)\,dx\right) y_i$$

$$= \sum_{i=0}^{n} w_i y_i \,;$$

giving $w_i = \int_a^b l_i(x)\,dx$

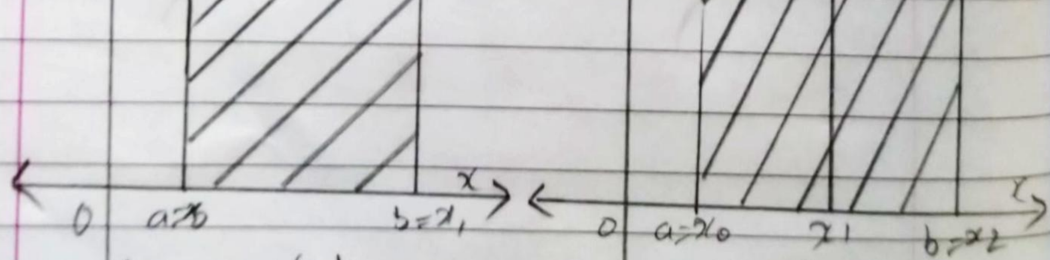
Q6. Write the composite form of trapezoidal rule for $\int_a^b f(x)\,dx$.

Ans.

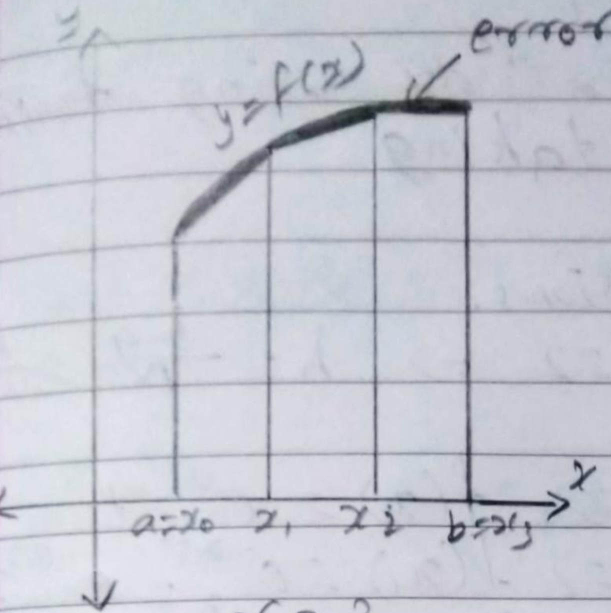
If $[a,b]$ is divided into N equal subdivisions, the composite form of trapezoidal rule for $\int_a^b f(x)\,dx$ is,

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_N)\right]$$

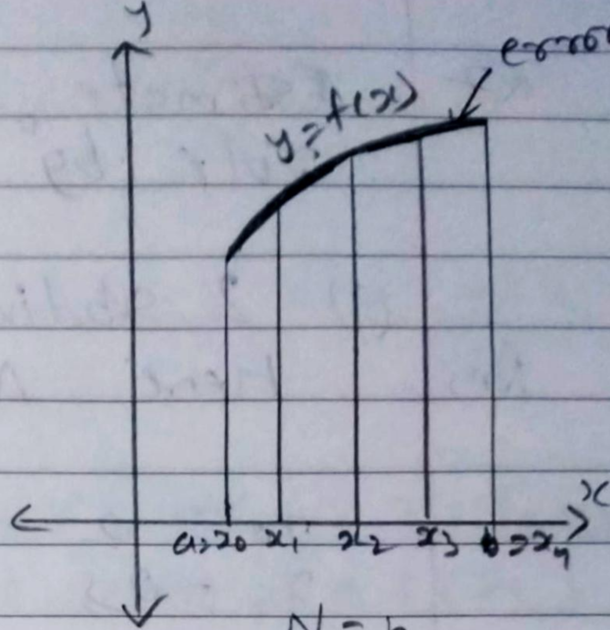
→ The following figures shows as Number of segments of intervals are increasing, i.e. use of composite form reduces ~~fraction error~~ truncation error.



N = 1 N = 2



$N = 3$



$N = n$

Q7 Estimate $\int_0^1 e^{-x^2} dx$ using trapezoidal rule by taking

(i) 2 subdivisions.

Ans Here $N=2 \Rightarrow h = \frac{b-a}{N} = \frac{1-0}{2} = 0.5$

$x_0 = 0 \Rightarrow f(x_0) = e^0 = 1$
$x_1 = 0.5 \Rightarrow f(x_1) = e^{-0.5^2} = e^{-0.25} = 0.7788$
$x_2 = 1 \Rightarrow f(x_2) = e^{-1^2} = e^{-1} = 0.3679$

$$\int_0^1 = e^{-x^2} dx = \int_0^1 f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2f(x_1) + f(x_2)\right]$$

$$= \frac{0.5}{2}\left[1 + 2(0.7788) + 0.3679\right]$$

$$= 0.25[2.9255]$$

$$\int_0^1 e^{-x^2} dx = 0.7313$$

(ii) 4 subdivisions:

$\therefore N = 4 \Rightarrow h = \frac{1}{4} = 0.25$

$x_0 = 0 \Rightarrow f(x) = e^{0^2} = 1$
$x_1 = 0.25 \Rightarrow f(x_1) = e^{-0.25^2} = 0.9394$
$x_2 = 0.50 = f(x_2) = e^{-0.50^2} = 0.7788$
$x_3 = 0.75 = f(x_3) = e^{-0.75^2} = 0.5698$

$x_4 = 1 \quad = f(x_4) = e^{-1^2} = 0.3679$

$$\int_0^1 f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{3} f(x_i) + f(x_4)\right]$$

$$\int_0^1 e^{-x^2}\,dx = \frac{0.25}{2}\left[1 + 2(0.9394 + 0.7788 + 0.5698) + 0.3679\right]$$

$$= 0.7430$$

(iii) 6 subdivision.

Here $N = 6$, $h = \frac{1-0}{6} = 0.1667$

$x_0 = 0 \Rightarrow f(x_0) = 1$

$x_1 = 0.1667 \Rightarrow f(x_1) = 0.9726$

$x_2 = 0.3334 \Rightarrow f(x_2) = 0.8948$

$x_3 = 0.5001 \Rightarrow f(x_3) = 0.7788$

$x_4 = 0.6668 \Rightarrow f(x_4) = 0.6412$

$x_5 = 0.8335 \Rightarrow f(x_5) = 0.4994$

$x_6 = 1 \Rightarrow f(x_6) = 0.3679$

$$\int_0^1 f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum_{1}^{5} f(x_i) + f(x_6)\right]$$

$$= \frac{0.1667}{2}\left[1 + 2(0.9726 + 0.8948 + 0.7788 + 0.6412 + 0.4994) + 0.3679\right]$$

$$= \boxed{0.7452}$$

Q8 Calculate approximate integral value of $\int f(x)\,dx$ when $f(x)$ is (a) x^2 using (i) Trapezoidal rule.

Ans $\int_a^b f(x)\,dx = \frac{h}{2}[f(a)+f(b)]$

Here $n=1$

$h = \frac{b-a}{n}$

$= \frac{2-0}{1}$

$= 2$

$\therefore \int_0^2 x^2\,dx = \frac{2}{2}[f(0)+f(2)]$

$= 1 \cdot [\quad] \; [1+2.236]$

$= 3.2361$

(ii) Simpson's $\frac{1}{3}$ rule.

$\rightarrow \int_a^b f(x)\,dx = \frac{h}{3}[f(a)+4f(a+h) + f(a+2h)]$

Here $n = 2$

$$h = \frac{b-a}{n}$$

$$= \frac{2-0}{2}$$

$$= 1$$

$$\int_0^2 x^2\,dx = \frac{1}{3}\left[f(0) + 4(f(0+2)) + f(0+2)\right]$$

$$= \frac{1}{3}\left[1 + 4(1.4142) + 2.8361\right]$$

$$= \frac{1}{3}\left[8.8929\right]$$

$$= 2.9643$$

(b) x^4

(i) Trapezoidal Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x) + f(b)\right]$$

Here $n = 1$, $h = \frac{2}{1} = 2$

$$\int_0^2 x^4 \, dx = \frac{2}{2}[f(0) + f(2)]$$

$$= 1[0 + 16]$$

$$\boxed{= 16}$$

(ii) Simpson's $\frac{1}{3}$ rule. $n = 2, h = 1$

$$\int_0^2 x^4 \, dx = \frac{1}{3}[0 + 4 + 16]$$

$$\boxed{= 6.6667}$$

(iii) Simpson's $\frac{3}{8}$ rule.

$$\int_a^b f(x) \, dx = \frac{1}{4}[f(0) + 3f(\tfrac{2}{3}) + 3f(2\tfrac{2}{3}) + f(3'\tfrac{2}{3})]$$

$$= \frac{1}{4}[0 + 3 \times \frac{16}{81} + 3 \times \frac{256}{81} + 1296]$$

$$= \frac{1}{4}[\frac{16}{27} + \frac{256}{27} + 16]$$

$$= \frac{16}{4}[\frac{1}{27} + \frac{16}{27} + 1]$$

$$= 4[\frac{44}{27}]$$

$$\boxed{= 6.5185}$$

⓪ $1/(x+1)$

(i) Trapazoidal, $n=1$, $h=2$

$$\int_0^2 1/(x+1)\,dx = \frac{2}{2}[f(0)+f(2)]$$

$$= 1\,[1+0.3333]$$

$$= 1.3333$$

(ii) Simpson's $\frac{1}{3}$ rule $n=2$, $h=1$

$$\int_0^2 1/(x+1)\,dx = \frac{h}{3}[f(0)+4f(0+1)+f(0+2)]$$

$$= \frac{1}{3}[1+4\times\frac{1}{2}+0.3333]$$

$$= \frac{1}{3}[3.3333]$$

$$= 1.1111$$

(iii) Simpson's $\frac{3}{8}$ rule. $n=3$ $h=\frac{2}{3}$

$$\int_0^2 1/(x+1)\,dx = \frac{3h}{8}[f(0)+3f(0+\frac{h}{3})+3f(0+2.\frac{2}{3}) + f(0+3.\frac{2}{3})]$$

$$= \frac{1}{4}[1+3\times(\frac{3}{5})+3\times(\frac{3}{7})+\frac{1}{3}]$$

$$= \frac{1}{4}[1+1.8+1.2858+0.3333]$$

$$= 1.1048$$

(d) $\sqrt{1+x^2}$

(i) Trapezoidal rule $n=1, h=2$

$$\int_0^2 \sqrt{1+x^2}\,dx = \frac{h}{2}\left[f(0)+f(2)\right]$$

$$= \frac{2}{2}\left[0+2.2361\right]$$

$$= 2.2361$$

(ii) Simpson's $\frac{1}{3}$ rule $n=2, h=1$

$$\int_0^2 \sqrt{1+x^2}\,dx = \frac{h}{3}\left[f(0)+4f(0+1)+f(0+2)\right]$$

$$= \frac{1}{3}\left[0+4\times 1.4142+2.2361\right]$$

$$= \frac{1}{3}\left[7.8929\right]$$

$$= 2.63097$$

(iii) Simpson's $\frac{3}{8}$ rule $n=3 \quad h=\frac{2}{3}$

$$\int_0^2 \sqrt{1+x^2} = \frac{3h}{8}\left[f(0)+3f\left(0+\frac{2}{3}\right)+3f\left(0+2\left(\frac{2}{3}\right)\right)+f\left(0+3\left(\frac{2}{3}\right)\right)\right]$$

$$= \frac{1}{4}\left[0+3\times 1.2019+3\times 1.6667+2.236\right]$$

$= \frac{1}{4} [10.8419]$

$= 2.7105$ //

(e) Sin x

(i) Trapezoidal $n=1, h=2$

$\int_0^2 \sin x \, dx = \frac{h}{2} [f(0) + f(2)]$

$\int_0^2 \sin x \, dx = 1 [0 + 0.9093]$

$= 0.9093$ //

(ii) Simpson's $\frac{1}{3}$ rule. $n=2$ $h=1$

$\int_0^2 \sin x \, dx = \frac{h}{3} [f(0) + 4(f(0+1)) + f(0+2)]$

$= \frac{1}{3} [0 + 4 \times 0.8415 + 0.9093]$

$= \frac{1}{3} [4.2753]$

$= 1.4251$ //

(iii) Simpson's $\frac{3}{8}$ rule. $n=3$ $h=\frac{2}{3}$

$\int_0^2 \sin x \, dx$:

$= \frac{3h}{8}\left[f(0) + 3\left(f\left(0+\frac{2}{3}\right) + 3f\left(0+2\left(\frac{2}{3}\right)\right) + f\left(0+3\left(\frac{2}{3}\right)\right)\right]$

$= \frac{1}{4}\left[0 + 3(0.6184 + 3(0.9720) + 0.9093\right)$

$= \frac{1}{4}[5.6805]$

$\boxed{= 1.4201}$

(f) e^x

(i) Trapezoidal rule.

$\int_0^2 e^x \, dx = \frac{h}{2}\left[f(0) + f(2)\right]$

$= 1[1 + 7.3891]$

$\boxed{= 8.3891}$

(iii) simpson's $\frac{3}{8}$ rule $n=3$ $h=\frac{2}{3}$

$${}_0\int^2 e^x dx = \frac{3h}{8}\left[f(0)+3f\left(0+\frac{2}{3}\right)+3f\left(0+2\left(\frac{2}{3}\right)\right)+f\left(0+3\left(\frac{2}{3}\right)\right)\right]$$

$$=\frac{1}{4}[1+3(1.9477)+3(3.7937)+7.3891]$$

$$=\frac{1}{4}(25.6133)$$

$$I=6.4033$$

(ii) simpson $\frac{1}{3}$ rule $n=2$ $h=1$

$${}_0\int^2 e^x dx = \frac{h}{3}\left[f(0)+4f(0+1)+f(0+2)\right]$$

$$=\frac{1}{3}[1+4(2.7183)+7.3891]$$

$$=\frac{1}{3}[19.2623]$$

$$I=6.4208$$

Q9. Given function f at the following values.

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	3.12014	4.42569	6.04241	8.03014	10.46072

approximate $\int_{1.8}^{0.6} f(x)\,dx$ using

ⓐ Trapezoidal Rule ⓑ Simpson's $\frac{1}{3}$ Rule

Ans: $N = 4$, $h = \frac{2.6 - 1.8}{4} = 0.2$

ⓐ Trapezoidal Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{N=4} f(x_i) + f(x_N)\right]$$

$$\int_{1.8}^{2.6} f(x)\,dx = \frac{0.2}{2}\left[3.12014 + 2(4.42569 + 6.04241 + 8.03014) + 10.46075\right]$$

$$= 0.1\,[50.58337]$$

$$= 5.05834$$

(b) Simpson's $\frac{1}{3}$ rule.

$$\int_{1.8}^{2.6} f(x)\,dx = \frac{0.2}{3}\left[3.12014 + 4(4.42569 + 8.03014) + 3(6.04241) + 10.46675\right]$$

$$= 0.06667\,[81.53744]$$

$$\int_{1.8}^{2.6} f(x)\,dx = 5.4361$$

Q10 Given the form of closed type Newton Cote's integration formula as

$$\int_a^b f(x)\,dx \simeq nh \sum_{i=0}^{n} c_i^n f(x_i) \text{ with}$$

$$h = \frac{b-a}{n}, \quad x_i = x_0 + ih, \quad i = 0,1,2 \ldots n$$

State two properties of Newton Cote's coefficient c_i^n.

Ans. c_i^n have two important properties.

① $\sum_{i=0}^{n} c_i^n = 1$

② $c_i^n = c_{n-i}^n$

Q11 Derive.

Ans (a) Trapezoidal Rule

We have the closed type Newton Cote's integration formula

$$\int_a^b f(x)\,dx \cong nh \sum_{i=0}^{n} c_i^n f(x_i)$$

with $h = \frac{b-a}{n}$, $x_i = x_0 + ih$,

$i = 0, 1, 2, \ldots n$

let $n = 1$

$$\int_a^b f(x)\,dx = 1h \sum_{i=0}^{1} c_i^n f(x_i)$$

$$= h c_0^1 f(x_0) + h c_1^1 f(x_1)$$

By two Properties of Newton Cote's Coefficients.

$c_0^1 = c_1^1$ and $c_0^1 + c_1^1 = 1$ gives

$c_0^1 = c_1^1 = \frac{1}{2}$

$$\Rightarrow \int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right],$$ which

is trapezoidal formula.

(b) simpson's $\frac{1}{3}$ Rule

$${}_a\int^b f(x)\,dx \cong nh \sum_{i=0}^{n} c_i^n f(x_i)$$

with $h = \frac{b-a}{2}$,

$x_i = x_0 + ih$,

$i = 0, 1, 2 \ldots n$

let $n = 2$

$${}_a\int^b f(x)\,dx \cong 2h \sum_{i=0}^{2} c_i^2 f(x_i) \quad \text{—} (1)$$

Here $c_0^2 = c_2^2$ and $c_0^2 + c_1^2 + c_2^2$
$= 1$

By evaluating c_2^2

we know that $c_i^n = \frac{(1)^{n-i}}{ni!\,(n-i)!}$

$$\int_0^n \frac{t(t-1)(t-n)}{(t-1)}\,dt$$

$$C_2^2 = \frac{(-1)^{2-2}}{2\,2!\,(2-2)!}\int_0^2 \frac{t(t-1)(t-2)}{(t-2)}\,dt$$

$$C_2^2 = \frac{1}{4}\int_0^2 t(t-1)\,dt$$

$$= \frac{1}{4}\int_0^2 (t^2 - t)\,dt$$

$$= \frac{1}{4}\left[\frac{t^3}{3} - \frac{t^2}{2}\right]_0^2$$

$$= \frac{1}{4}\left[\frac{8}{3} - \frac{4}{2}\right]$$

$$= \frac{1}{4}\left[\frac{8}{3} - 2\right)$$

$$= \frac{1}{4}\left[\frac{8-6}{3}\right]$$

$$= \frac{1}{{}_2 4}\left[\frac{2^1}{3}\right]$$

$$= \frac{1}{6}$$

$$\therefore C_2^2 = \frac{1}{6} \Rightarrow C_0^2 = C_2^2 = \frac{1}{6}$$

and $c_1^2 = 1 - (c_0^2 + c_1^2)$

$= 1 - \frac{2}{6} = \frac{4}{6}$

$\therefore$ ① $\Rightarrow \int_a^b f(x)\,dx \simeq 2h\, c_0^2 f(x_0)$

$+ 2h\, c_1^2 f(x_1) + 2h\, c_2^2 f(x_2)$

$\simeq \frac{h}{3} f(x_0) + \frac{4h}{3} f(x_1) + \frac{h}{3} f(x_2)$

$\Rightarrow \int_a^b f(x)\,dx \simeq \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$

Which can also be expressed as

$\int_a^b f(x)\,dx = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

or

$\int_a^b f(x)\,dx \simeq \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$

This is Simpson's $\frac{1}{3}$ Rule

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Q12 Estimate $\int_0^4 e^x \, dx$

Ans

(a) Simpson's $\frac{1}{3}$ rule

$$\int_a^b f(x)\,dx = \frac{h}{3}[f(a) + 4f(a+h) + f(a+2h)]$$

here $n = 2, h = 2$

$$\int_0^4 e^x\,dx = \frac{2}{3}[f(0) + 4f(0+2) + f(0+2(2))]$$

$$= \frac{2}{3}[1 + 4(7.38906) + 54.59815]$$

$$= \frac{2}{3}[85.15439]$$

$$= 56.769593$$

(b) Composite Simpson's $\frac{1}{3}$ rule with $N = 2$

Ans

Here $N = 2, h = 2$

$x_0 = 0 \quad = 1$
$x_1 = 2 \quad = 7.38906$
$x_2 = 4 \quad = 54.59815$

$$\int_0^4 e^x dx = \frac{h}{3}\left[f(x_0) + 2(f(x_1)) + f(x_2)\right]$$

$$= \frac{2}{3}\left[1 + 4(7.38906) + 54.59815\right]$$

$$= \frac{2}{3}\left[85.15439\right]$$

$$= 56.769593$$

(d) Calculate error in above

error in (a) simpson's $\frac{1}{3}$ Rule

error = True value - Approximate value

$$= 53.59815 - 56.769593$$

$$= -3.171443$$

error in (b) composite Simpson's Rule with $N=2$

$\therefore$ error $= 53.59815 - 96.769593$

$= -3.171443$

error in (c) composite simpson's rule with $N=4$

error $= 53.59813 - 53.86385$

$= -0.2657$

Q13 Write psedocode for following integration formula to obtain results to desired accuracy

(a) Simpson's $\frac{1}{3}$ Rule

Ans pseudocode for simpson's $\frac{1}{3}$ Rule.

step 1 : a, b, f(x), n = max no of iteration as inputs.

step 2 : h = (b - a) / (2 * N)

Step 3 : Do
for i = 1 to N - 1
x(2 * i) = a + h * 2 * i;
x(2 * i + 1) = a + h * (2 * i + 1)
Sum = sum + f(x(2 * i)).
End do.

step 4 : Sum = 2 * Sum + 4 * Sum1 + f(a) + f(b)

Step 5 : output : Sum = h * Sum / 3
Print sum

step 6 : if $|sum - old\ sum| < \epsilon$;
output : Ans = sum ;
Exit
Else

step 7 : k = k+1, N = 2 * N ;
old sum = sum;
if $k \leq M$, goto step 3

step 8 : Else :

print does not give desired accuracy in N itteration.

(b) Trapezoidal Rule

~~step 1 : Input N, a, b, f(x), max~~

step 1 : Input N, a, b, f(x),
M = max itteration.

Step 2 : sum = 0 ; k = 0 ;
N = 1 ; old sum = 0 ;
h = (b - a) / N ;

step 3:

Do
for i = 1 to N-1
x = a + h * i
sum += 2 * f(x)
end do

step 4: sum += f(a) + f(b)

step 5 : sum = (h/2) * sum;
print sum;

step 6 : if |sum - oldsum| < ε;
output : The estimated the integral is sum;
Exit;

step 7 : Else
k = k+1; N = 2*N;
old sum = sum;
if k ≤ m goto step 3

step 8 : Else
Print does not give desired accuracy in 'n' itteration

Q14

Time	0	6	12	18	24	30	36	42	48
speed	124	134	148	156	147	133	121	109	99

Time	54	60	66	72	78	84
speed	85	78	89	104	116	123

how long is the track

$\int_0^{84} f(x)\,dx$

$\therefore$ By simpsons $\frac{1}{3}$ formula

here $N = 14$ $h = \frac{84}{14} = 6$

$\therefore \int_0^{84} f(x)\,dx =$

$\frac{h}{3}[f(x_0) + 4[f(x_1) + f(x_3) + f(x_5) + f(x$
$+ f(x_9) + f(x_{11}) + f(x_{13})] + 2[f(x$
$+ f(x_4) + f(x_6) + f(x_8) + f(x_{10}) +$
$f(x_{12})] + f(x_{14})]$

$= \frac{6}{3}[124 + 4[134 + 156 + 133 + 109 + 85$
$89 + 116] + 2[148 + 147 + 121 + 9$
$78 + 104] + 123]$

$= 2\,[4929] = 9858$ feet

Q15 Approximate $\int_0^2 x^2 \ln(x^2+1)\,dx$ using $h = 0.25$ use

(1) composite trapezoidal Rule

Ans Here $h = 0.25$, $N = \frac{b-a}{0.25} = \frac{2}{0.25} = 8$

$$\int_0^2 x^2 \ln(x^2+1)\,dx = \frac{h}{2}\left[f(x_0 + 2[f(x_1) + f(x_2) + f(x_3 + f(x_4) + f(x_5) + f(x_6) + f(x_7)] + f(x_8)\right]$$

here

$x_0 = 0 \Rightarrow f(x_0) = 0$

$x_1 = 0.25 \Rightarrow f(x_1) = 0.00379$

$x_2 = 0.50 \Rightarrow f(x_2) = 0.05579$

$x_3 = 0.75 \Rightarrow f(x_3) = 0.25104$

$x_4 = 1 \Rightarrow f(x_4) = 0.6931$

$x_5 = 1.25 \Rightarrow f(x_5) = 1.47029$

$x_6 = 1.50 \Rightarrow f(x_6) = 2.65197$

$x_7 = 1.75 \Rightarrow f(x_7) = 4.29301$

$x_8 = 2 \Rightarrow f(x_8) = 6.43775$

$\int_0^2 x^2 \ln(x^2+1)dx$

$= \frac{0.25}{2}[0 + 2[0.00379 + 0.05579$

$+ 0.25104 + 0.69315 +$
$1.47029 + 2.65197 +$
$4.29301] + 6.43775]$

$= 0.125[25.27583]$

$\boxed{= 3.1595}$

(2) Composite simpson's 1/3 rule.

here $h = 0.25$, $n = 8$

$= \frac{0.25}{3}[0 + 4(0.00379 + 0.25104 +$
$1.47025 + 4.29301) + 2(0.0557$
$+ 0.69315 + 2.65197) + 6.437$

$= 0.083334[37.31209]$

$= 3.10921$

(3) Composite Midpoint Rule.

$$\text{Midpont} = \int_a^b f(x)\,dx = (b-a)\, f\left(\frac{a+b}{2}\right)$$

$$\therefore \int_0^2 x^2 \ln(x^2+1)\,dx = \int_0^{0.25} x^2 \ln(x^2+1)\,dx + \int_{0.25}^{0.50} x^2 \ln(x^2+1)\,dx + \dots + \int_{0.75}^{2} x^2 \ln(x^2+1)\,dx$$

$$\therefore \int_0^{2.25} x^2 \ln(x^2+1)\,dx = (0.25-0)\, f\left(\frac{0.25+0}{2}\right)$$

$$= 0.25\,(f(0.125))$$

$$= 0.25\,(0.00024)$$

$$= 0.00006$$

$$\therefore \int_{0.25}^{0.50} x^2 \ln(x^2+1)\,dx = 0.25\, f(0.375)$$

$$= 0.25\,(0.01858)$$

$$= 0.00463$$

$$\int_{0.50}^{0.75} x^2 \ln(x^2+1)\,dx = 0.25 \times f(0.625)$$

$$= 0.25\,(0.12881)$$

$$= 0.03220$$

$$\int_{0.75}^{1} x^2 \ln(x^2+1)\,dx = 0.25 \times f(0.875)$$

$$= 0.25\,(0.43526)$$

$$= 0.10882$$

$$\int_{1}^{1.25} x^2 \ln(x^2+1)\,dx = 0.25 \times f(1.125)$$

$$= 0.25 \times 1.03504$$

$$= 0.25877$$

$$\int_{1.25}^{1.50} x^2 \ln(x^2+1)\,dx = 0.25 \times f(1.375)$$

$$= 0.25\,(2.00685)$$

$$= 0.50171$$

$$\int_{1.50}^{1.75} x^2 \ln(x^2+1)\,dx = 0.25\,(f(1.625))$$

$$= 0.25\,(3.41210)$$

$$= 0.85302$$

$$\int_{1.75}^{2} x^2 \ln(x^2+1)\,dx = 0.25 \times f(1.875)$$

$$= 0.25(5.29996)$$

$$= 1.32499$$

$$\int_{0}^{2} x^2 \ln(x^2+1)\,dx = 0.00006 + 0.00463$$
$$+ 0.03220 + 0.10882$$
$$+ 0.25877 + 0.50171$$
$$+ 0.85302 + 1.32499$$

$$\int_{0}^{2} x^2 \ln(x^2+1)\,dx = 3.0842$$

Q16 Approximate $\int_0^2 x^2 e^{-x^2} dx$ using

$h = 0.25$

(a) Composite trapezoidal Rule.

Ans $h = 0.25$, $N = 8$

$x_0 = 0 \quad = 0$
$x_1 = 0.25 \quad = 0.05871$
$x_2 = 0.50 \quad = 0.1947$
$x_3 = 0.75 \quad = 0.3205$
$x_4 = 1 \quad = 0.36788$
$x_5 = 1.25 \quad = 0.3275$
$x_6 = 1.50 \quad = 0.23715$
$x_7 = 1.75 \quad = 0.14324$
$x_8 = 2 \quad = 0.07326$

$$= \frac{0.2}{2}[0 + 2(0.05871 + 0.1947 + 0.3205 + 0.36788 + 0.3275 + 0.23715 + 0.14324) + 0.07326]$$

$= 0.42158$

(b) Composite simpson $\frac{1}{3}$ rule

$h = 0.25$, $N = 8$

$$\int_0^2 x^2 e^{-x^2} dx =$$

$$= \frac{0.25}{3} [0 + 4(0.05871 + 0.32051 + 0.32275 + 0.14324) + 2(0.1947 + 0.367887 + 0.23715) + 0.07326]$$

$$= 0.083 [5.07252]$$

$$\boxed{= 0.42102}$$

(c) Composite midpoint Rule.

$$\int_0^{0.25} x^2 e^{-x^2} dx = 0.25 \times f(0.125)$$

$$= 0.25 \times 0.01538$$

$$= 0.00385.$$

Like that

$$\int_{0.25}^{0.50} x^2 e^{-x^2} dx = 0.03054$$

$$\int_{0.50}^{0.75} x^2 e^{-x^2} dx = 0.06608$$

$$\int_{0.75}^{1} x^2 e^{-x^2} dx = 0.08901$$

$$\int_{1}^{1.25} x^2 e^{-x^2} dx = 0.08925$$

$$\int_{1.25}^{1.50} x^2 e^{-x^2} dx = 0.07136$$

$$\int_{1.50}^{1.75} x^2 e^{-x^2} dx = 0.04708$$

$$\int_{1.75}^{2} x^2 e^{-x^2} dx = 0.02613$$

$$\int_{0}^{2} x^2 e^{-x^2} dx = 0.00325 + 0.03054$$
$$+ 0.06608 + 0.08901 + 0.08925$$
$$+ 0.07136 + 0.04708 + 0.02613$$

$$\therefore \int_{0}^{2} x^2 e^{-x^2} dx = \boxed{0.4233}$$

Assigment

Open type Integration formula & Gauss Quadrature formulas

Q1 Write Gauss Legedre quarature formula for estimating $\int_{-1}^{1} f(x)\,dx$ for $n = 1, 2$ and 3 (n = number of abscissas

Ans Gauss Legedre quadrature formula for estimating $\int_{-1}^{1} f(x)\,dx$

for $n = 1$ is

$$\int_{-1}^{1} f(x)\,dx \cong 2f(0)$$

for $n = 2$

$$\int_{-1}^{1} f(x)\,dx \cong f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

for $n = 3$

$$\int_{-1}^{1} f(x)\,dx \cong \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} + \left(\sqrt{\frac{3}{5}}\right)$$

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Q2 Approximate $\int^{1} e^{x} \cos x \, dx$ using Gauss Legendre quadrature formula

(i) taking $n=2$

Ans The Gauss Legendre quadrature formula for $\int_{-1}^{1} f(x)\,dx$ for $n=2$ is

$$\int_{-1}^{1} f(x)\,dx \simeq f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$\int_{-1}^{1} e^{x} \cos x \, dx \simeq e^{\frac{-1}{\sqrt{3}}} \cos\left(\frac{-1}{\sqrt{3}}\right) + e^{\frac{1}{\sqrt{3}}} \cos\left(\frac{1}{\sqrt{3}}\right)$$

$$= 1.96297$$

(ii) for $n=3$

$$\int_{-1}^{1} f(x)\,dx \simeq \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)$$

$$\int_{-1}^{1} e^{x} \cos x \, dx = \frac{5}{9} \times e^{-\sqrt{\frac{3}{5}}} \cos\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} e^{0} \cos 0 + \frac{5}{9} e^{\sqrt{\frac{3}{5}}} \times \cos\left(\sqrt{\frac{3}{5}}\right)$$

$$= 1.93339$$

Q3 Show, how an integral $\int_a^b f(x)dx$ over an arbitrary $[a,b]$ can be transformed into integral over $[-1,1]$.

Ans To transform $\int_a^b f(x).dx$ to. $\int_{-1}^{1} g(t)dt$

we use change of variables from x to t define by: $x = \frac{b-a}{2}t + \frac{b+a}{2}$

- on putting $x = a$ and solving for t we obtain; $a = \frac{b-a}{2} t + \frac{b+a}{2}$

which on solving gives value of t as -1. similarly putting $x = b$, we obtain, $b = \frac{b-a}{2}t + \frac{b+a}{2}$ which on solving gives

$t = 1$. Thus limits of integration

change from a to b into: -1 to 1 now also $dx = \frac{b-a}{2} dt$

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b-a}{2}t + \frac{b+a}{2}\right)dt$$

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Q4 Write open Newton Cotes integral formulas for

(a) $n = 0$ (midpoint rule)
(b) $n = 1$

And show midpoint rule is same as Gauss quadrature one point formula

Ans open Newton Cotes integral formula for

(1) $n = 0$

midpoint rule, $n = 0$, $h = \frac{b-a}{2}$

let $a = x_{-1}$, $b = x_1$

$$\int_{x_{-1}}^{x_1} f(x)\,dx = 2h\,f(x_0) + \frac{1}{3} h^3 f^2(\xi)$$

Here last term is error term.

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(b) $n=1$, $h = \frac{(b-a)}{3}$

$a = x_{-1}$, $b = x_2$

$$\int_{x_{-1}}^{x_2} f(x)\,dx = \frac{3h}{2}[f(x_0) + f(x_1)] + \frac{3h^3}{4} f''(e)$$

Here last term is error term

→ The gauss quadrature one point formula is $\int_a^b f(x) = w_1 f(x_1)$

∴ It gives exact resut, when $f(x)$ of degree $\leq 2n-1 = 1$

Now let $f(x) = 1$ & $f(x) = x$ one by one

for $f(x) = 1$, $\int_a^b 1\,dx = w_1$

∴ $w_1 = b - a$ —— (1)

for $f(x) = x$, $\int_a^b x\,dx = w_1 x_1$

$\Rightarrow \left[\frac{x^2}{2}\right]_a^b = w_1 x_1$

$\Rightarrow w_1 x_1 = \frac{1}{2}(b^2 - a^2)$ — ②

putting ① in ②, $x_1 = \frac{1}{2}(b+a)$

∴ The resulting Gausian quadrature formula is

$$\int_a^b f(x)\,dx \simeq (b-a)\, f\left(\frac{a+b}{2}\right)$$

which is same as midpoint rule.

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Q5 Consider $\int_1^3 (x^6 - x^2 \sin 2x)\,dx$.
Exact value is 317.3442466
Estimate value of integral using

① simple trapezoidal rule.

Ans.

let $f(x) = (x^6 - x^2 \sin 2x)\,dx$

$n = 1, \quad h = \frac{3-1}{1} = 2$

$\int_1^3 f(x)\,dx = \frac{h}{2}[f(3) + f(1)]$

$= \frac{2}{2}[731.5147 + 0.0907]$

$\int_1^3 f(x)\,dx = 731.6054$

② open Newton-cotes formula
for $n=0, n=1$

for $n=0, \quad h = \frac{3-1}{2} = \frac{2}{2} = 1$

$f(x) = (x^6 - x^2 \sin 2x)$

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$$\therefore \int_1^3 f(x)\,dx = 2h\left[f\left(\frac{3+1}{2}\right)\right]$$

$$= 2f(2)$$

$$= 2(67.0272)$$

$$= 134.0544$$

for $n=1$, $h = \frac{b-a}{3} = \frac{2}{3}$

$$\int_1^3 f(x)\,dx = \frac{3h}{2}\left[f\left(1+\frac{2}{3}\right) + f\left(1+2\left(\frac{2}{3}\right)\right)\right]$$

$$= 1\left[f\left(\frac{5}{3}\right) + f\left(\frac{7}{3}\right)\right]$$

$$= 1\left[21.9623 + 166.8228\right]$$

$$= 188.7856$$

(c) Simpson's $\frac{1}{3}$ rule.

Ans $\int_1^3 f(x)\,dx = \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$

Here $f(x) = (x^6 - x^2 \sin 2x)$

$n = 2, \quad h = \frac{3-1}{n} = \frac{2}{2} = 1$

$\int_1^3 f(x)\,dx = \frac{1}{3}\left[f(1) + 4f(2) + f(3)\right]$

$= \frac{1}{3}\left[0.0907 + 4(67.0272) + 731.3147\right]$

$= \frac{1}{3}\left[999.7142\right]$

$\int_1^3 f(x)\,dx = 333.2381$

(d) Gauss Quadrature two point formula.

We know that the formula is

$$\int_{-1}^{1} f(x)\,dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) \quad \text{---(a)}$$

our integral is $\int_{1}^{3} (x^6 - x^2 \sin 2x)\,dx$

- We shall need to transform the given interval to

$\int_{-1}^{1} g(t)\,dt$ through below

equation.

$$\int_{a}^{b} f(x)\,dx = \frac{(b-a)}{2} \int_{-1}^{1} f\left(\frac{b-a}{2}t + \frac{b+a}{2}\right) dt$$

=> Here $a=1$, $b=3$, $f(x) = x^6 - x^2 \sin 2x$

substitude $x = \frac{3-1}{2} t + \frac{3+1}{2}$

$\Rightarrow x = t + 2$

$$\therefore \int_1^3 f(x)\,dx = \left(\frac{3-1}{2}\right)\int_{-1}^{1} f(t+2)\,dt \quad \text{---} ①$$

$$= \cancel{\theta}\,1 \times \int_{-1}^{1} \left[(t+2)^6 - (t+2)^2 \sin 2 (t+2)\right] dt.$$

e.g.

ⓐ $\Rightarrow \int_1^3 f(x)\,dx =$

$$1 \times \left[\left(\left(-\frac{1}{\sqrt{3}}\right)+2\right)^6 - \left(-\frac{1}{\sqrt{3}}+2\right)^2 \sin 2\left(2+\left(-\frac{1}{\sqrt{3}}\right)\right) + \left(\frac{1}{\sqrt{3}}+2\right)^6 - \left(\frac{1}{\sqrt{3}}+2\right)^2 \sin 2\left(2+\frac{1}{\sqrt{3}}\right)\right]$$

$= 8.2906 - 0.5909 + 293.1168 + 6.0035$

$$\int_1^3 f(x)\,dx = 306.82$$

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(c) Gauss Quadrature three point formula.

3 point formula is

$$\int_{-1}^{1} f(x)\,dx \simeq \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right) \quad \text{---} \quad (6)$$

By transforming the given interval to $\int_{-1}^{1} g(t)\,dt$ we have eq. (1)

$$\Rightarrow \int_{1}^{3} f(x)\,dx = \left(\frac{3-1}{2}\right) \int_{-1}^{1} f(t+2)\,dt$$

$$= 1 \times \int_{-1}^{1} (t+2)^6 - (t+2)^2 \sin 2(t+2)\,dt$$

e.g.

$$(6) \Rightarrow \int_{1}^{3} f(x)\,dx = \left[\frac{5}{9}\left[\left(-\sqrt{\frac{3}{5}}+2\right)^6 - \left(-\sqrt{\frac{3}{5}}+2\right)^2 \sin 2\left(-\sqrt{\frac{3}{5}}+2\right)\right] + \frac{8}{9}\left[(0+2)^6 - (0+2)^2 \sin 2(0+2)\right] + \frac{5}{9}\left[\left(\sqrt{\frac{3}{5}}+2\right)^6 - \left(\sqrt{\frac{3}{5}}+2\right)^2 \sin 2\left(\sqrt{\frac{3}{5}}+2\right)\right]\right]$$

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$$= \frac{5}{9}(2.4292) + \frac{8}{9}(67.0272) + \frac{5}{9}(461.4028)$$

$$= 317.2642$$

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Q6. List the Legendre polynomials $P_n(x)$ for $n=0,1,2,3$ and find their roots. Verify that each $P_n(x)$ is

(a) Roots are distinct
(b) Roots lie within interval $(-1,1)$
(c) Roots are symmetric with respect to origin.

Ans

Legendre Polynomials for $n=0,1,2,3$

→ for $n=0$, $P_0(x)=1$

There is no root for $P_0(x)$

→ for $n=1$, $P_1(x)=x$

then $\int_{-1}^{1} f(x)\,dx = w_1 f(x_1)$

it should give exact result when $f(x)$ is of degree 1

∴ let $f(x)=1$.

∴ $\int_{-1}^{1} 1\,dx = w_1 \Rightarrow w_1 = [x]_{-1}^{1} = 2$

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$\therefore$ let $f(1) = x$

$$\therefore \int_{-1}^{1} x\,dx = w_1 x_1$$

$$\Rightarrow \left[\frac{x^2}{2}\right]_{-1}^{1} = w_1 x_1$$

$$\therefore w_1 x_1 = \frac{1}{2} - \frac{1}{2} = 0$$

e.g

(1) $\Rightarrow$

$$2x_1 = 0 \Rightarrow x_1 = 0$$

Roots of $P_1(x)$ is $x_1 = 0$ which is distinct, $-1 < 0 < 1$ and on origin

$\rightarrow$ for $n = 2$, $P_2(x) = \left(\frac{3x^2 - 1}{2}\right)$

Here $\int_{-1}^{1} f(x)\,dx = w_1 f(x_1) + w_2 f(x_2)$

let $f(x) = 1 \Rightarrow w_1 + w_2 = \int_{-1}^{1} 1\,dx = [x]_{-1}^{1} = 2$

$$f(x) = x \Rightarrow w_1 x_1 + w_2 x_2 = \int_{-1}^{1} x \, dx =$$

$$\left[\frac{x^2}{2}\right]_{-1}^{1} = \left[\frac{1}{2} - \frac{1}{2}\right] = 0$$

$$f(x) = x^2 \Rightarrow w_1 x_1^2 + w_2 x_2^2 = \int_{-1}^{1} x^2 \, dx =$$

$$\left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{2}{3}$$

$$f(x) = x^3 \Rightarrow w_1 x_1^3 + w_3 x_2^3 = \int_{-1}^{1} x^3 \, dx =$$

$$\left[\frac{x^4}{4}\right]_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0$$

By solving above equation, we get the roots

$$x_1 = \frac{1}{\sqrt{3}} \text{ and } x_2 = \frac{1}{\sqrt{3}}$$

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$\rightarrow$ for $n=3$, $P_3(x) = \dfrac{(5x^3 - 3x)}{3}$

here $\int_{-1}^{1} f(x)\,dx = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)$

let

$f(x) = 1 \Rightarrow w_1 + w_2 + w_3 = 2$

$f(x) = x \Rightarrow w_1 x_1 + w_2 x_2 + w_3 x_3 = 0$

$f(x) = x^2 \Rightarrow w_1 x_1^2 + w_2 x_2^2 + w_3 x_3^2 = \dfrac{2}{3}$

$f(x) = x^3 \Rightarrow w_1 x_1^3 + w_2 x_2^3 + w_3 x_3^3 = 0$

$f(x) = x^4 \Rightarrow w_1 x_1^4 + w_2 x_2^4 + w_3 x_3^4 = \dfrac{2}{5}$

$f(x) = x^5 \Rightarrow w_1 x_1^5 + w_2 x_2^5 + w_3 x_3^5 = 0$

$\therefore$ solving above equation, roots x_1, x_2, x_3 are

$x_1 = -\sqrt{\dfrac{3}{5}}$, $x_2 = 0$, $x_3 = \sqrt{\dfrac{3}{5}}$

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→ Here all are distinct, $\forall i = 1, 2, 3,$

$-1 < x_i < 1$ and roots are symmetric with respect to origin.

→ For $n = 2$ roots $x_1 = -\frac{1}{\sqrt{3}}$

and $x_2 = \frac{1}{\sqrt{3}}$ are also distinct, both are between -1 to 1 and both are symmetric.

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Q7 ~~Calculate~~ calculate $\int_{3}^{3.5} \frac{x}{\sqrt{x^2-4}} dx$ using

(1) 2 point Gauss quadrature formula.

Ans We need to transform the given integral to $\int_{-1}^{1} g(t) dt$

The two point Gaussian quadrature formula is

$$\int_{-1}^{1} f(x) dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) \quad — (1)$$

Wkt $\int_{a}^{b} f(x) dx = \frac{(b-a)}{2} \int_{-1}^{1} f\left(\frac{b-a}{2} t + \frac{b+a}{2}\right) dt$

here $a = 3$, $b = 3.5$, $f(x) = \frac{x}{\sqrt{x^2-4}}$

$\therefore x = \frac{3.5-3}{2} t + \frac{3.5+3}{2}$

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$$= \frac{0.5}{2} + \frac{6.5}{2}$$

$$\int_3^{3.5} f(x)\,dx = \frac{(3.5-3)}{2} \int_{-1}^{1} f\left(\frac{0.5}{2}t + \frac{6.5}{2}\right) dt \quad \text{— (1)}$$

$$= 0.25 \int_{-1}^{1} f(0.25\,t + 3.25)\,dt$$

$$= 0.25 \int_{-1}^{1} \frac{(0.25\,t + 3.25)}{\sqrt{(0.25\,t + 3.25)^2 - 4}}\,dt$$

$$= 0.25 \left[\frac{\left(0.25 \times \left(-\frac{1}{\sqrt{3}}\right) + 3.25\right)}{\sqrt{\left(0.25 \times -\frac{1}{\sqrt{3}} + 3.25\right)^2 - 4}} + \frac{\left(0.25 \times \frac{1}{\sqrt{3}} + 3.25\right)}{\sqrt{\left(0.25 \times \frac{1}{\sqrt{3}} + 3.25\right)^2 - 4}} \right]$$

$$= 0.25\,[1.3071 + 1.2377]$$

$$= 0.63619$$

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② 3 Point Gauss quadrature formula.

$$\int_{-1}^{1} f(x)\,dx \cong \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right) \quad \text{—— ②}$$

By tranforming given interval to $\int_{-1}^{1} g(t)\,dt$ we have

eg. ① $\Rightarrow \int_{3}^{3.5} f(x)\,dx =$

$$\left(\frac{3.5-3}{2}\right) \int_{-1}^{1} (0.25\,t + 3.25)\,dt$$

$$= 0.25 \int_{-1}^{1} (0.25t + 3.25)\,dt$$

$$= 0.25 \int_{-1}^{1} \frac{(0.25\,t + 3.25)}{\sqrt{(0.25\,t + 3.21)^2 - 4}}\,dt$$

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e4.

(2) $\Rightarrow \int_{3}^{3.5} f(x)\,dx =$

$$0.25\left[\frac{5}{9}\frac{(0.25\times-\sqrt{\frac{3}{5}}+3.25)}{\sqrt{(0.25\times(-\sqrt{\frac{3}{5}})+3.25)^2-4}}\right.$$

$$+\frac{8}{9}\frac{(0.25\times 0+3.25)}{\sqrt{(0.25)0+3.25)^2-4}}$$

$$\left.+\frac{5}{9}\frac{(0.25\times\sqrt{\frac{3}{5}}+3.25)}{\sqrt{(0.25\times\sqrt{\frac{3}{5}}+3.25)^2-4}}\right]$$

$= 0.25\,[0.7347+1.1277+0.6825]$

$= 0.6362$ //

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Q8 Determin Constants a, b, c, d that will produce a quadrature formula.

$$\int_{-1}^{1} f(x)\,dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

That has degree of Precision 3.

Ans.

Using method of undetermined coefficient.

let $f(x) = 1$

$\therefore f(-1) = 1;\ f(1) = 1;\ f'(-1) = \frac{d}{dx}(1) = 0,\ f'(1) = \frac{d}{dx}(1) = 0$

$\therefore \int_{-1}^{1} 1\,dx = [x]_{-1}^{1} = 2 = a.1 + b.1 + c.0 + d.0$ ——— ①

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$\rightarrow$ let $f(x) = x$

$\therefore f(-1) = -1,\ f(1) = 1,$

$f'(-1) = \frac{d}{dx}(x) = 1,\ f'(1) = \frac{d}{dx}(x) = 1$

$\therefore \int_{-1}^{1} x\,dx = \left(\frac{x^2}{2}\right)_{-1}^{1} = 0 = a.-1 + b.1 +$

$c.1 + d.1$ ——— (2)

$\rightarrow$ let $f(x) = x^2$

$\therefore f(-1) = 1,\ f(1) = 1,\ f'(-1) = \frac{d}{dx}(2x)$

$= -2,\ f'(1) = \frac{d}{dx}(2x) = 2$

$\therefore \int_{-1}^{1} x^2\,dx = \left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{2}{3} = a.1 + b.1 +$

$c.-2 + d.2$ ——— (3)

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$\rightarrow$ let $f(x) = x^3$

$\therefore f(-1) = -1, f(1) = 1, f'(-1) = \frac{d}{dx}(x^3)$

$= 3x^2 = 3(-1)^2 = 3,$

$f'(1) = 3(1)^2 = 3$

$\therefore \int_{-1}^{1} x^3 dx = \left[\frac{x^4}{4}\right]_{-1}^{1} = 0 = a \cdot -1$

$+ b \cdot 1 + c \cdot 3 + d \cdot 3$ ———— (4)

$\therefore$ we have 4 equation

$a + b = 2$ ———— (1)

$-a + b + c + d = 0$ ———— (2)

$a + b - 2c + 2d = \frac{2}{3}$ ———— (3)

$-a + b + 3c + 3d = 0$ ———— (4)

(1), (3) $\Rightarrow 2 - 2c + 2d = \frac{2}{3}$

$\therefore 2(1 - c + d) = \frac{2}{3}$

$\therefore 1 - c + d = \frac{1}{3}$

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$\therefore -c + d = \frac{1}{3} - 1$

$\therefore -c + d = \frac{1}{3} - 1$

$\therefore -c + d = \frac{-2}{3}$

$\therefore c - d = \frac{2}{3}$

$\therefore c = \frac{2}{3} + d$ ——— (5)

(1) $\Rightarrow a = 2 - b$

$\therefore$ (2) $\Rightarrow -(2-b) + b + c + d = 0$

$-2 + 2b + c + d = 0$

$2b + c + d = 2$

(5) $\Rightarrow 2b + \frac{2}{3} + d + d = 2$

$\therefore 2b + \frac{2}{3} + 2d = 2$

$b + \frac{1}{3} + d = 1$

$\therefore b + d = 1 - \frac{1}{3}$

$\therefore b + d = \frac{2}{3}$ ——— (6)

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(4) $\Rightarrow$ $-(2-b)+b+3\left(\frac{2}{3}+d\right)+3d=0$

$\therefore -2+b+b+2+3d+3d=0$

$\therefore -2+2b+6d+2=0$

$\therefore -1+b+3d+1=0$

$\therefore b+3d=0$ —— (7)

(6) − (5) $\rightarrow$ $b+d=\frac{2}{3}$

$b+3d=0$

$-2d=\frac{2}{3}$

$d=-\frac{1}{3}$ —— (8)

(5) $\Rightarrow$ $c=\frac{2}{3}+d=\frac{2}{3}-\frac{1}{3}$

$c=\frac{1}{3}$ —— (9)

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(8) $\Rightarrow$ $b+d = \frac{2}{3}$

$\therefore b - \frac{1}{3} = \frac{2}{3}$

$\therefore b = \frac{2}{3} + \frac{1}{3} = \frac{3}{3}$

$\therefore b = 1$ —— (10)

(1) $\Rightarrow$ $a+b = 2$

$a+1 = 2$

$\therefore a = 1$ —— (11)

$$\boxed{\begin{aligned} \therefore a &= 1 \\ \therefore b &= 1 \\ \therefore c &= \frac{1}{3} \\ \therefore d &= -\frac{1}{3} \end{aligned}}$$

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Q9 Derive mid point formula using method of undetermined coefficient

Ans let $n=1$ in method of undetermined coefficients

$$\therefore \int_a^b f(x)\,dx \simeq w_1 f(x_1)$$

It gives exact result when $f(x)$ is of degree $\leq 2\times1-1=1$

$\therefore$ let $f(x)=1$ & $f(x)=x$

for $f(x)=1$, $\int_a^b 1\,dx = w_1 \Rightarrow$

$$w_1 = b-a \quad \text{--- (1)}$$

for $f(x)=x$, $\int_a^b x\,dx = w_1 x_1$

$$\Rightarrow w_1 x_1 = \frac{1}{2}(b^2-a^2) \quad \text{--- (2)}$$

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(1), (2) $\Rightarrow$ $x_1 = \frac{1}{2}(b+a)$

$\therefore$ The Resulting Gaussian quadrature formula is

$$\int_a^b f(x)\,dx \cong (b-a)\, f\left(\frac{a+b}{2}\right)$$

which is same as midpoint rule.

Assignment-3
Solution of ordinary
Numerical methods

Q1 Define a differential equation. What is ment by solution of a differential equation? verify that

(i) $y = c_1 e^x + c_2 e^x x$ is a solution of the differential $y'' - 2y' + y = 0$

(ii) $y = 2\sqrt{c-x}$ is a solution of the differential equation $y + 1/y = 0$

Ans Define a differential eq.
→ A differential equation is an equation involving independent variable, dependent variable and its one or more derivatives

ex

$$y'' + 4y = 0$$

Here y is dependent variable, y is function of x

⇒ Solution of differential equation

A solution of a differential equation is a specific function that satisfies the equation.

(i) Her differential equation is

$y'' - 2y' + y = 0$ —— ⓐ

let $y(x) = c_1 e^x + c_2 e^x x$ —— ①

$y'(x) = c_1 e^x + c_2 \frac{d}{dx}(xe^x)$

$= c_1 e^x + c_2 \left(e^x x \frac{d}{dx} + e^x \frac{d}{dx} x\right)$

$= c_1 e^x + c_2 (xe^x + e^x)$

$y'(x) = c_1 e^x + c_2 e^x + c_2 x e^x$ —— ②

$y''(x) = c_1 e^x + c_2 e^x + c_2 (e^x + xe^x)$

$y''(x) = c_1 e^x + 2c_2 e^x + c_2 x e^x$ —— ③

subsituting ① ② ③ in ⓐ.

$0 = c_1 e^x + 2c_2 e^x + c_2 x e^x - 2(c_1 e^x +$

$c_2 e^x + c_2 x e^x) + c_1 e^x + c_2 x e^x$

$= 2c_1e^x + 2c_2e^x + 2c_2xe^x$
$- 2c_1e^x - 2c_2e^x - 2c_2xe^x$
$= 0$

$\therefore y = c_1e^x + c_2xe^x$ is a solution of $y'' - 2y' + y = 0$

(ii) Here differential ~~between~~ equation is $y' + \frac{1}{y} = 0$

let $y = 2\sqrt{c-x} = 2(c-x)^{\frac{1}{2}}$ — (1)

$y' = 2 \times \frac{1}{2}(c-x)^{\frac{1}{2}-1} \times \frac{d}{dx}(c-x)$

$= (c-x)^{-\frac{1}{2}} \times -1 = \frac{-1}{\sqrt{c-x}}$ — (2)

(2) $\Rightarrow \frac{-1}{\sqrt{c-x}} + \frac{1}{2\sqrt{c-x}} \neq 0$

$\therefore y = 2\sqrt{c-x}$ is not a solution of $y' + \frac{1}{y} = 0$

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Q2 how is ordinary differential equation different from Partial differential equation? Give one example of each.

Ans.

when dependent variable is a function of only one variable, Then all the derivatives involved in the equation are ordinary and the equation is called ordinary differential equation.

- if dependent variable is a function of more than one independent variable. That is, it is a function of several variables, then the equation contains partial derivation with respect to different independent variables and the equation is called Partial Differential Equation

⇒ Example of ordinary

$$y'' + 4y = 0$$

$$y' + \frac{1}{y} = 0$$

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⇒ Example of partial

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

(Laplace's Eq)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$$

(Poisson's Eq)

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Q3 Differentiate between initial value Problem and Boundary value problem. classify following differential equation in initial value problem and Boundary value problem.

Ans.

→ If the condition are specified at a single point these condition are called intial condition and the differential eq combines with conditions is called initial value problem.

→ If the condition are specified at more than one Point these condition are called Boundary condition and the Differential Eq combined with conditions is called Boundary value Problem

(i) $y' = t - y, \; y(0) = 1$

→ Initial Value Problem

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(ii) $y'' + 9y = e^{x}\sin x,\ y(0) = 1,\ y(1) = 2$

→ Boundary Value ~~Point~~ Problem

(iii) $y'' + y = 0,\ y(0) = 0,\ y'(0) = 2$

→ Boundary Value ~~Point~~ Problem

(iv) $y'' + y' = t^2 + y^2,\ y(0) = 1,$
$y'(0) = 1$

→ Boundary Value ~~Point~~ Problem

(v) $y' = t^2 + y^2,\ y(0) = 1$

→ Initial Value Problem

Q4 Determine order and degree of following differential eq.

Ans (i) $y'' + 4y = e^x$

→ order = 2, degree = 1

(ii) $y'' + 4(y')^3 + y^2 = x^3 + x^2$

→ order = 2, degree = 1

(iii) $(y'')^2 + (y')^3 + 3y = 5x$

→ order = 2, degree = 2

(iv) $y' + 2y^2 = x^2$

→ order = 1, degree = 1

Q5 What are the characteristics of single step numerical methods to find solution of first order, first degree IVP?

Ans These are the characteristics of single step numerical methods.

1. It is direct
2. It is non iterative
3. It is based on Taylor series method.
4. It is used estimating at previous step & function information.
5. Practically, error cannot be estimated.

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Q6 Differentiate between single step and multistep numerical methods to obtain solution of IVP.

Ans

Single step methods:

→ Direct

→ Not iterative

→ Based on Taylor Series methods

→ Self starting.

→ Practically, accuracy cannot be estimated.

Multistep Method:

→ More than one Previous step

→ Iterative.

→ Predictor Corrector formulas.

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→ Not self starting

→ Practically, accuracy can be estimated.

Assignment-4

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Ordinary Differential Equation-II

Q1 A ball at 1200k is allowed cool down in air at an ambient temperature of 300k. Assuming heat is lost only due to radiation the differential equation for temperature of the ball is given by $\frac{d\theta}{dt} = -2.2067 \times 10^{-12} (\theta^4 - 81 \times 10^8)$ $\theta(0) = 1200k$

When θ is in k and t in seconds. Find the temperature at $t = 480$ seconds using Runge kutta 2nd order method. Assume a step size of $h = 240$ seconds

Ans Runge kutta formulas of order 2 is $\frac{dy}{dx} = f(x, y(x))$, $y(x_0) = y_0$

$a \le x \le b$

$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$ with

$k_0 = h f(x_i, y_i)$

$k_1 = h f(x_i + h, y_i + k_0)$ for $i = 0, 1, 2, \ldots n$.

$\Rightarrow$ Here we have $\frac{d\theta}{dt} = f(t, \theta(t))$

$= -2.2067 \times 10^{-12} (\theta^4 - 81 \times 10^8)$,

$t_0 = 0$

$\theta_0 = \theta(0) = 1200k$, $0 \le t \le 480$,

$h = 240$ sec.

let $t_1 = 240$, $t_2 = 480$

$\therefore \theta_1 = \theta_0 + \frac{1}{2}(k_0 + k_1)$ — ①

$k_0 = 240 \times f(t_0, \theta_0) = 240 \times f(0, 1200)$

$= 240 \times (-2.2067 \times 10^{-12} \times (1200)^4 - 81 \times 10^8))$

$k_0 = -1093.9033$ — ②

$k_1 = h \times f(t_0 + h, \theta_0 + k_0)$

$= 240 \times f(240, 1200 + (-1093.9033))$

$= 240 \times f(240, 106.104)$

$= 240 \times (-2.2067 \times 10^{-12} \times (106.104)^4 - 81 \times 10^8)$

$k_1 = 4.2227$ —— (2)

Substituting eq (1), (2) & (3)

$\theta_1 = 1200 + \frac{1}{2}(-1093.905 + 4.2227)$

$\theta_1 = 655.1587$ —— (4)

→ Now $t_2 = 480$ sec

$\theta_2 = \theta_1 + \frac{1}{2}(k_0 + k_1)$

$k_0 = h f(t_1, \theta_1)$ (1)

$= 240 \times f(240, 655.1587)$

$k_0 = -93.2856$

$k_1 = h f(t_1 + h, \theta_1 + k_0)$

$= 240 \times f(480, (655.1587 + (-93.2856))$

$= 240 \times (-2.2007 \times 10^{-12} (561.873)^4 - 81 \times 10^{8})$

$k_1 = -48.4948$

$\theta_2 = 655.1587 + \frac{1}{2}(-93.2856 - 48.4948)$

$\theta_2 = 584.2685\ k$ ——— (3)

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Q2 Use Taylor's method of order 2 to approximate the solutions of each of the following problems.

(a) $\frac{dy}{dx} = y - x^2 + 1$, $y(0) = 0.5$, $h = 0.5$

Ans Here $\frac{dy}{dx} = y' = y - x^2 + 1$, $x_0 = 0$,
$y_0 = 0.5$, $0 \le x \le 2$, $h = 0.5$

→ at $x_1 = 0.5$ By Taylor's method of order 2

$$y_{i+1} \cong y_i + h y_i' + \frac{h^2}{2!} y_i''$$

Here $y' = y - x^2 + 1$

$$\therefore y'' = \frac{d y_i}{dx} = \frac{d}{dx}(y - x^2 + 1) = -2x$$

$$\therefore y_1 = y_0 + h y' + \frac{h^2}{2!} y_0''$$

$$= 0.5 + 0.5(y_0 - x_0^2 + 1) +$$

$$\frac{(0.5)^2}{2!}(-2x_0)$$

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$= 0.5 + 0.5(0.5 - 0 + 1) + \frac{(0.5)^2}{2!}$

$(-2x_0)$

$y_1 = 1.25$ —— ①

$\Rightarrow a + x_2 = 1$

$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$

$= 1.25 + 0.5(y_1 - x_1^2 + 1) +$

$\frac{(0.5)^2}{2!} (-2x_1)$

$= 1.25 + 0.5(1.25 - (0.5)^2 + 1)$

$+ \frac{0.25}{2} (-2.(0.5))$

$y_2 = 2.125$ —— ②

$\rightarrow a + x_3 = 1.50$

$y_3 = y_2 + h y_2' + \frac{h^2}{2!} y_2''$

$= 2.125 + 0.5\,(y_2 - x_2^2 + 1)\,\frac{(0.5)^2}{2!}(-2x_2)$

$= 2.125 + 0.5\,(2.125 - 1^2 + 1) + \frac{0.25}{2}$
$(-2(1))$

$y_3 = 2.9375$ ——— 3

at $x_4 = 1$

$y_4 = y_3 + h y_3' + \frac{h^2}{2!} y_3''$

$= 2.9375 + 0.5\,(y_3 - x_3^2 + 1) + \frac{(0.5)^2}{2!}$
$(-2x_3)$

$= 2.9375 + 0.5\,(2.9375 - (1.5)^2 + 1)$
$+ \frac{0.25}{2}(-2(1.5))$

$\boxed{y_4 = 3.40625}$

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(b) $\frac{dy}{dt} = \cos 2t + \sin 3t$, $0 \le t \le 1$,
$y(0) = 1$, $h = 0.25$

Ans. Here $\frac{dy}{dt} = y' = \cos 2t + \sin 3t$,

$y_0 = 1$, $t_0 = 0$, $h = 0.25$

$y' = \cos 2t + \sin 3t$
$y'' = -2 \sin 2t + 3 \cos 3t$

→ at $t_1 = 0.25$, using taylor method of order 2.

$y_1 = y_0 + h y' + \frac{h^2}{2!} y_0''$

$= 1 + 0.25(\cos 0 + \sin 0) + \frac{0.0625}{2}(-2 \sin 0 + 3 \cos 0)$

$y_1 = 1.34375$ —— (1)

→ at $t_2 = 0.5$

$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$

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$= 1.34373 + 0.25(\cos 0.5 + \sin 0.75)$

$+ \frac{0.0625}{2}(-2\sin 0.5 + 3\cos 0.75)$

$y_2 = 1.7722$ —— (2)

$\rightarrow$ a + $t_3 = 0.75$

$y_3 = 1.7722 + (0.21)(\cos 1 + \sin 1.5) + \frac{0.062}{2})(-2\sin 1 +$

$3\cos(1.5))$

$y_3 = 2.1107$ —— (3)

$a + t_4 = 1$

$y_4 = 2.1107 + (0.25)(\cos 1.5 +$
$\sin(2.25) + \frac{0.0625}{2}(-2\sin 1.5 +$

$3\cos 2.25)$

$\boxed{y_4 = 2.20167}$

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(c) $y' = \frac{1+t}{1+y}$ $1 \le t \le 2$, $y(1) = 2$
$h = 0.5$

Ans Here
$t_0 = 1$, $y_0 = 2$, $1 \le t \le 2$,
$h = 0.5$

$y' = \frac{1+t}{1+y}$, $y'' = \frac{1}{1+y}\frac{d}{dt}(1+t) = \frac{1}{1+y}$

using Taylor method of order
at $t_1 = 1.5$

$$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0''$$

$$= 2 + 0.5\left(\frac{1+t_0}{1+y_0}\right) + \frac{(0.5)^2}{2}\left(\frac{1}{1+y_0}\right)$$

$$= 2 + 0.5\left(\frac{1+1}{1+2}\right) + \frac{0.25}{2}\left(\frac{1}{1+2}\right)$$

$y_1 = 2.375$

$\rightarrow$ at $t_2 = 2$

$$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$$

$$= 2.375 + 0.5\left(\frac{1+1.5}{1+2.375}\right) + 0.125$$

$$\left(\frac{1}{1+2.375}\right)$$

$$\boxed{y_2 = 2.7824}$$

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(d) $y' = 1 + \frac{y}{t}$, $y(1) = 1$, $h = 0.25$
$1 \le t \le 1.5$

Ans here, $t_0 = 1$, $y_0 = 1$, $h = 0.25$,
$1 \le t' \le 1.5$

$y' = 1 + \frac{y}{t}$, $y'' = y \frac{d}{dt}\left(\frac{1}{t}\right) =$
$y \frac{d}{yt}(t^{-1})$

$\therefore y'' = \frac{-y}{t^2}$

at $t_1 = 1.25$

$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0''$

$= 1 + (0.25)\left(1 + \frac{y_0}{t_0}\right) +$
$\frac{0.25^2}{2} \left(\frac{-y_0}{t_0^2}\right)$

$= 1 + 0.25\left(1 + \frac{1}{1}\right) + \frac{0.0625}{2}$
$\left(\frac{-1}{1}\right)$

$y_1 = 1.46875$ ——— (1)

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$a + t_2 = 1.50$

$$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$$

$$= 1.46875 + 0.25 \times \left(1 + \frac{1.46875}{1.25}\right)$$

$$+ 0.03125 \left(- \frac{1.46875}{(1.25)^2}\right)$$

$$\boxed{y_2 = 1.983125} \quad \text{——} \quad (2)$$

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(e) $y' = xe^{y+x} - 1$, $y(0) = -1$, $h = 0.5$
$0 \le x \le 1$

Ans here
$y_0 = -1$, $x_0 = 0$, $h = 0.5$,
$0 \le x \le 1$

$y' = xe^{y+x} - 1$, $y'' = x\frac{d}{dx}(e^{y+x})$
$+ e^{y+x}\frac{d}{dx}x$

$y'' = xe^{y+x} + e^{y+x}$

$\rightarrow$ at $x_1 = 0.5$

$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0''$

$= -1 + 0.5(x_0 \times e^{y_0+x_0} - 1)$
$+ \frac{0.5^2}{2}(x_0 e^{y_0+x_0} + e^{y_0+x_0})$

$\therefore y_1 = -1 + 0.5(0 \times e^{-1+0} - 1)$
$+ \frac{0.25}{2}(0 \times e^{-1+0} + e^{-1+0})$

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$y_1 = -1.4540$ —— (1)

$\rightarrow a + x_2 = 1$

$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$

$= -1.4540 + 0.5 (x_1 e^{y_1 + x_1} - 1)$

$+ \frac{0.25}{2} (x_1 e^{y_1 + x_1} + e^{y_1 + x_1})$

$\boxed{y_2 = 1.7855}$

Q3 Compare the Performance of Euler's method, second order Taylor's method, RK of order 2 and RK order 4 for the following differential eq.

Ans Here $\frac{dy}{dx} = \frac{x}{y}$, $h = 0.5$

$y_0 = 1$, $x_0 = 0$, $0 \le x \le 2$

$\Rightarrow$ with Euler's method

$y_{i+1} = y_i + h\, f(x_i, y_i)$

$y_{i+1} = y_i + 0.5\, (x_i / y_i)$

$\rightarrow$ at $x_0 = 0$, $y_0 = 1$

$\rightarrow$ at $x_1 = 0.5$

$y_1 = y_0 + 0.5\left(\frac{x_0}{y_0}\right) =$

$1 + 0.5\left(\frac{0}{1}\right) = 1$

$\rightarrow$ at $x_2 = 1$

$y_2 = y_1 + 0.5\left(\frac{x_1}{y_1}\right)$

$= 1 + 0.5\left(\frac{0.5}{1}\right)$

$= 1.25$

$\rightarrow$ at $x_3 = 1.5$

$y_3 = y_2 + 0.5\left(\frac{x_2}{y_2}\right)$

$= 1.25 + 0.5\left(\frac{1}{1.25}\right)$

$= 1.65$

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$\rightarrow$ at $x_4 = 2$

$$y_4 = y_3 + 0.5\left(\frac{x_3}{y_3}\right)$$
$$= 1.65 + 0.5\left(\frac{1.5}{1.65}\right)$$
$$= 2.1045$$

$\Rightarrow$ Second order Taylor's method.

here $y' = x/y$, $y'' = \frac{1}{y}$, $x_0 = 0$, $y_0 = 1$
$h = 0.5$, $0 \le x \le 2$

$$y_{i+1} = y_i + h y_i' + \frac{h^2}{2!} y_i''$$

$\rightarrow$ at $x_1 = 0.5$

$$y_1 = y_0 + h\left(\frac{x_0}{y_0}\right) + \frac{h^2}{2!}\left(\frac{1}{y}\right)$$
$$= 1 + 0.5\left(\frac{0}{1}\right) + \frac{(0.5)^2}{2!}\left(\frac{1}{1}\right)$$

$$= 1.125$$

$\rightarrow$ at $x_2 = 1$

$$y_2 = 1.125 + 0.5\left(\frac{0.5}{1.125}\right) + 0.125\left(\frac{1}{1.125}\right)$$
$$= 1.4583$$

$\rightarrow$ at $x_3 = 1.5$

$$y_3 = 1.4583 + 0.5\left(\frac{1}{1.4583}\right) + 0.125\left(\frac{1}{1.4583}\right) = 1.8869$$

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→ $a + x_4 = 2$

$y_4 = 1.8869 + 0.5\left(\frac{1.50}{1.8869}\right)^h + 0.125\left(\frac{1}{1.8883}\right) = 2.3506$

⇒ (3) Rk of order 2

$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$
$k_0 = h f(x_i, y_i)$
$k_1 = h f(x_i + h, y_i + k)$

→ $a + x_1 = 0.5$

$y_1 = y_0 + \frac{1}{2}(k_0 + k_1)$
$k_0 = 0.5\left(\frac{x_0}{y_0}\right) = 0.5\left(\frac{0}{1}\right) = 0$
$k_1 = 0.5 f(x_0 + 0.5, y_0 + 0)$
$= 0.5 f(0.5, 1)$
$k_1 = 0.5(0.5/1) = 0.25$
$y_1 = 1 + \frac{1}{2}(0 + 0.25) = 1.125$

→ $a + x_2 = 1$
$k_0 = 0.5(0.5/1.125) = 0.2$
$k_1 = 0.5 f(0.5 + 0.5,$
$1.125 + 0.2)$
$= 0.5 f(1, 1.325)$

$k_1 = 0.3774$

$y_2 = 1.125 + \frac{1}{2}(0.2 + 0.3774)$

$= 1.4137$

→ at $x_3 = 1.5$

$k_0 = 0.5\left(\frac{1}{1.4137}\right)$

$= 0.3532$

$k_1 = 0.5 f(1.5, 1.7674)$

$= 0.4244$

$y_3 = 1.4137 + \frac{1}{2}(0.3532 + 0.4244) = 1.80275$

→ at $x_4 = 2$

$k_0 = 0.5\left(\frac{1.5}{1.8028}\right) = 0.4162$

$k_1 = 0.5 f(2, 2.1565) = 0.4037$

$y_4 = 1.8028 + \frac{1}{2}(0.4162 + 0.4037)$

$= 2.2428$

→ Table of comparision

x	Euler	Taylor	RK2	RK4	Exact
0	1	1	1	1	1
0.5	1	1.125	1.125	1.10053	1.1180
1	1.25	1.4588	1.4137	1.3	1.4142
1.5	1.65	1.8864	1.8028	1.8341	1.8028
2	2.1045	2.3806	2.2428	2.3423	2.2361

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Qn. Explain Taylor's method of order n to approximate a solution to differential equation $y'(x) = f(x, y(x))$ $a \le x \le b$, $y(a) = c$. What are its advantages? Why is it seldom used in practice for computer implementation?

Ans The taylor's method of order n is

$$y(x) = y_i + (x - x_i) y_i' + \frac{(x-x_i)^2}{2!} y_i'' + \frac{(x-x_i)^3}{3!} y_i''' + \dots + \frac{(x-x_i)^n}{n!} y_i^{(n)}$$

using this taylor series to evaluate $y'(x) = f(x, y(x))$, $a \le x \le b$, $y(a) = c$

$$y_{i+1} = y_i + h y_i' + \frac{h^2}{2!} y_i'' + \dots + \frac{h^n}{n!} y_i^{(n)}$$

where $y^{(n)}$ is for n^{th} derivation of $y(x)$ at $x = x_i$

→ Taylor's Series is seldom used in practical for complex implementation due to requirement of derivation of expression of derivatives and evaluation of their values.

→ Taylor's Series is back bone of direct single step formulas. Euler formula and family of Runge Kutta formula are derived from Taylor Series method.

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Q5 Repeat question 2 with Runge kutta method of order 2.

Ans. Here $\frac{dy}{dx} = y - x^2 + 1$, $y(0) = 0.5$

$0 \le x \le 2$, $h = 0.5$

$\therefore y_0 = 0.5$, $x_0 = 0$

→ RK method of order 2 is

$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$

$k_0 = hf(x_i, y_i)$,

$k_1 = hf(x_i + h, y_i + k_0)$

→ at $x_1 = 0.5$

$y_1 = y_0 + \frac{1}{2}(k_0 + k_1)$

$k_0 = 0.5 + f(x_0, y_0)$

$= 0.5 \times (0.5 - 0^2 + 1)$

$k_0 = 0.75$

$k_1 = 0.5 \times f(x_0 + 0.5, y_0 + 0.75)$

$= 0.5 \times f(0.5, 1.25)$

$k_1 = 0.5(1.25 - 0.5^2 + 1)$

$= 1$

$y_1 = 0.5 + \frac{1}{2}(0.75 + 1)$

$= 1.375$

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→ at $x_2 = 1$

$k_0 = 0.5\,(1.375 - (0.5)^2 + 1$

$= 1.0625$

$k_1 = 0.5\, f(0.5 + 0.5,\ 1.375 + 1.0625)$

$= 0.5\, f(1,\ 2.4375)$

$k_1 = 1.2188$

$y_2 = 1.375 + \frac{1}{2}(1.0625 + 1.2188)$

$= 2.5156$

→ at $x_3 = 1.5$

$k_0 = 0.5\,(2.5156)$

$= 1.2578$

$k_1 = 0.5\,(2.5234)$

$= 1.2617$

$y_3 = 2.5156 + \frac{1}{2}(1.2578 + 1.2617)$

$= 3.7754$

→ at $x_4 = 2$

$k_0 = 1.2627$

$k_1 = 1.0792$

$y_4 = 4.9163$

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(b) $\frac{dy}{dt} = \cos 2t + \sin 3t, \; 0 \le t \le 1$
$y(0) = 1, \; h = 0.25$

RK method of order 2 B

$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$

$k_0 = hf(x_i, y_i) = hf(t_i, y_i)$
$k_1 = hf(x_i + h, y_i + k_0)$
$= hf(t_i + h, y_i + k_0)$

→ at point $t_0 = 0, \; y_0 = 1$

→ at point $t_1 = 0.25$

$k_0 = 0.25 f(0, 1) = 0.25(1)$
$= 0.25$
$k_1 = 0.25 f(0 + 0.25, 1 + 0.25)$
$= 0.25 \times (\cos(2 \times 0.25) + \sin(3 \times 0.25))$
$k_1 = 0.3898$
$y_1 = 1 + \frac{1}{2}(0.25 + 0.3898)$
$y_1 = 1.3199$

→ at point $t_2 = 0.50$
$k_0 = 0.25 f(0.25, 1.3199)$
$= 0.3898$
$k_1 = 0.25 \times (\cos(2 \times 0.50) + \sin 3(0.50))$

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$k_1 = 0.3845$

$y_2 = 1.3199 + \frac{1}{2}(0.3898 + 0.3845)$

$= 1.7071$

→ at point $t_3 = 0.75$

$k_0 = 0.25\,(\cos 2(0.50) + \sin 3(0.50))$

$k_0 = 0.3845$

$k_1 = 0.25 \times (\cos(2 \times 0.75) + \sin(3 \times 0.75))$

$k_1 = 0.2122$

$y_3 = 1.7071 + \frac{1}{2}(0.3845 + 0.2122)$

$= 2.0055$

→ at point $t_4 = 1$

$k_0 = 0.25 \times (\cos(2 \times 0.75) + \sin(3 \times 0.75))$

$k_0 = 0.2122$

$k_1 = 0.25 \times (\cos(2 \times 1) + \sin(3 \times 1))$

$k_1 = -0.0688$

$y_4 = 2.0055 + \frac{1}{2}(0.2122 - 0.0688)$

$y_4 = 2.0772$

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(1) $y' = \frac{1+t}{1+y}$, $1 \le t \le 2$, $y(1) = 0$

$h = 0.5$

RK method of 2

Ans

$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$

$k_0 = hf(t_i, y_i)$

$k_1 = hf(t_i + h, y_i + k_0)$

at point $t_0 = 1$, $y_0 = 2$

→ at point $t_1 = 1.5$

$k_0 = 0.5\, f(1, 2) = 0.5 \times \left(\frac{1+1}{1+2}\right) = 0.3$

$k_1 = 0.5 \times f(1 + 0.5, 2 + 0.3)$
$= 0.378$

$y_1 = 2 + \frac{1}{2}(0.3 + 0.378)$
$= 2.339$

→ at point $t_2 = 2$

$k_0 = 0.5\, f(1.5, 2.339)$
$= 0.3744$

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$k_1 = 0.5\, f(2,\ 2.339 + 0.3744)$
$= 0.4039$

$y_2 = 2.339 + \frac{1}{2}(0.3744 + 0.4039)$

$\boxed{y_2 = 2.7282}$

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Q. Repeat question 2 with Runge kutta method of order 4.

Ans. RK method of order 4 is

$$y_{i+1} = y_i + \frac{1}{6}(k_0 + 2k_1 + 2k_2 + k_3)$$

$k_0 = hf(x_i, y_i)$
$k_1 = hf(x_i + \frac{h}{2}, y_i + \frac{k_0}{2})$
$k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$
$k_3 = hf(x_i + h, y_i + k_2)$

→ here $f(x, y) = y - x^2 + 1$, $y(0) = 0.5$
$0 \leq x \leq 2$, $h = 0.5$

→ at point $x_0 = 0$, $y_0 = 0.5$

→ At point $x_1 = 0.5$

$k_0 = 0.5\, f(0, 0.5) = 0.75$
$k_1 = 0.5\, f(0 + \frac{0.5}{2}, 0.5 + \frac{0.75}{2})$
$= 0.90625$

$k_2 = 0.5\, f(0.25, 0.5 + \frac{0.90625}{2})$

$= 0.9453$

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$k_3 = 0.5\, f(0.5,\ 0.5 + 0.9453)$
$= 1.09765$

$y_1 = 0.5 + \frac{1}{6}(0.93 + 2(0.90625) + 2\times(0.9453) + 1.09765)$

$\boxed{y_1 = 1.4251}$

→ At $x_2 = 1$

$k_0 = 0.5\, f(0.5, 1.4251)$
$= 1.08755$
$k_1 = 0.5\, f(0.75, 1.9689)$
$= 1.2082$
$k_2 = 0.5\, f(0.75, 2.0267)$
$= 1.2321$
$k_3 = 0.5\, f(1, 2.6572)$
$= 1.3286$
$y_2 = 1.4251 + \frac{1}{6}(1.08755 + 2(1.2032) + 2(1.2321) + 1.3286)$

$\boxed{y_2 = 2.6396}$

→ At point $x_3 = 1.5$

$k_0 = 0.5 f(1, 2.6396)$
$= 1.3198$
$k_1 = 0.5 f(1.25, 3.2995)$
$= 1.3685$
$k_2 = 0.5 f(1.25, 3.32385)$
$= 1.3807$
$k_3 = 0.5 f(1.5, 4.0203)$
$= 1.3852$
$y_3 = 2.6396 + \frac{1}{6}(1.3198 + 2(1.3685) + 2(1.3807) + 1.3852)$
$y_3 = 4.0068$

→ At point $x_4 = 2$

$k_0 = 0.5 f(1.5, 4.0068)$
$= 1.3784$
$k_1 = 0.5 f(1.75, 4.696)$
$= 1.31675$
$k_2 = 0.5 f(1.75, 4.6652)$
$= 1.3014$
$k_3 = 0.5 f(2, 5.3082)$
$= 1.1541$

$y_4 = 4.0068 + \frac{1}{6}(1.3784 + 2(1.3168) + 2(1.3014) + 1.1541)$

$\boxed{y_4 = 5.3016}$

(b) $\frac{dy}{dt} = \cos 2t + \sin 3t, \; 0 \le t \le 1$
$y(0) = 1, \; h = 0.25$

→ At point $t_0 = 0, \; y_0 = 1$

→ At point $t_1 = 0.25$

$k_0 = 0.25 f(0, 1) = 0.25$
$k_1 = 0.25 f(0.125, 1.125) = 0.3338$
$k_2 = 0.25 f(0.125, 1.1669) = 0.3338$
$k_3 = 0.25 f(0.25, 1.3338) = 0.3894$

→ At point $t_2 = 0.5$

$k_0 = 0.25 f(0.25, 1.3292)$
$= 0.3898$
$k_1 = 0.25 f(0.375, 1.5241)$
$= 0.4085$
$k_2 = 0.25 f(0.375, 1.5334)$
$= 0.4083$
$k_3 = 0.5 f(0.50, 1.7377)$
$= 0.3845$
$y_2 = 1.3292 + \frac{1}{6}(0.3898 + 2(0.4085) + 2(0.4085) + 0.3845$

$\boxed{y_2 = 1.7306}$

At Point $t_3 = 0.75$

$k_0 = 0.25 f(0.50, 1.7306)$
$= 0.3845$

$k_1 = 0.25 f(0.625, 1.92225)$
$= 0.3174$

$k_2 = 0.25 f(0.625, 1.8892)$
$= 0.3174$

$k_3 = 0.25 f(0.75, 2.048)$
$= 0.2122$

$y_3 = 1.7306 + \frac{1}{6}(0.3854 + 2(0.3174) + 2(0.3174) + 0.2122)$

$\boxed{y_3 = 2.0418}$

At point $t_4 = 1$

$k_0 = 0.25 f(0.75, 2.0418) = 0.2122$
$k_1 = 0.25 f(0.875, 2.1479) = 0.0789$
$k_2 = 0.25 f(0.875, 2.0813) = 0.0789$
$k_3 = 0.25 f(1, 2.1207) = -0.0688$

$y_4 = 2.0418 + \frac{1}{6}(0.2122 + 4(0.0789) + (-0.0686))$

$\boxed{y_4 = 2.1183}$ //

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(c) $y' = \frac{1+t}{1+y}$, $1 \le t \le 2$, $y(1) = 2$

$h = 0.5$

Ans At $t_0 = 1$, $y_0 = 2$

→ At $t_1 = 1.5$

$k_0 = 0.5 f(1, 2) = 0.3333$

$k_1 = 0.5 f(1.25, 2.1667) = 0.3553$

$k_2 = 0.5 f(1.25, 2.1777) = 0.3540$

$k_3 = 0.5 f(1.5, 2.354) = 0.3727$

$y_1 = 2 + \frac{1}{6}(0.3333 + 2(0.3553) + 2(0.3540) + 0.3727)$

$y_1 = 2.3541$

→ At $t_2 = 2$

$k_0 = 0.5 f(1.5, 2.3541) = 0.3727$

$k_1 = 0.5 f(1.75, 2.5405) = 0.38836$

$k_2 = 0.5 f(1.75, 2.5483) = 0.3875$

$k_3 = 0.5 f(2, 2.7416) = 0.40090$

$y_2 = 2.3541 + \frac{1}{6}(0.3727 + 2(0.3883) + 2(0.3875) + 0.4009)$

$y_2 = 2.7417$

(d) $y' = 1 + \frac{y}{t}$, $y(1) = 1$, $1 \leq t \leq 1.5$,

$h = 0.25$

Ans At $t_0 = 1$, $y_0 = 1$

→ At $t_1 = 1.25$

$k_0 = 0.25\, f(1, 1) = 0.5$

$k_1 = 0.25\, f(1.125) = 0.5278$

$k_2 = 0.25\, f(1.125, 1.2639) = 0.5309$

$k_3 = 0.25\, f(1.25, 1.5309) = 0.5562$

$y_1 = 1 + \frac{1}{6}(0.5 + 2(0.5278) + 2(0.5309)$
$+ 0.5562)$

$y_1 = 1.5289$

→ At $t_2 = 1.5$

$k_0 = 0.25\, f(1.25, 1.5289) = 0.5558$

$k_1 = 0.25\, f(1.375, 1.8068) = 0.5785$

$k_2 = 0.25\, f(1.375, 1.8182) = 0.5806$

$k_3 = 0.25\, f(1.5, 2.1095) = 0.6016$

$y_2 = 1.5289 + \frac{1}{8}(0.5558 + 2(0.5785)$
$+ 2(0.5806) + 0.6016)$

$y_2 = 2.1082$

Q7 Give answer to the following in one - two sentence.

(a) What is the advantage of Rk method over Taylor series methods of the same order?

Ans Rk method and Taylor series method of the same order are equally efficient but Rk methods having advantages of no requirement of deriving the expressions of derivation.

(b) What is the order of the local truncation error in Rk method of order 2 and order 4! what is the order of global truncation error in these methods?

Ans In Rk method of order 2 the order of local truncation error is $O(h^3)$ and order of global truncation error is $O(h^2)$

→ In Rk method of order 2 o local truncation error is $O(h^5)$

and order of global truncation error is $O(h^4)$.

(c) Explain the effect of reducing step size in arriving at approximate solution of differential equation at a point.

Ans Reduction in step size yields better results, increases accuracy.

(d) What is the basic difference in method of deriving formula for ~~sing~~ single step methods and multistep method?

Ans Taylor's method is used to derive formula of single step methods while Derivation of multistep methods is based on numerical integration of interpolating polynomial fitted at previous step points.

(e) What is the main draw back of multistep method?

Ans Multistep methods are not self

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starting.

Q) What are the advantages of predictor corrector methods over RK method?

Ans In predictor corrector method only 1/2/3 function evaluation per step, depending of ~~...~~ number of times corrector formula is applied.

→ When in RK methods 4 function evaluation per step required.

Q8. Consider the IVP

$y' = y - t^2 + 1$, $0 \le t \le 2$, $y(0) = 0.5$, taking $h = 0.2$, exact solution is

$y(t) = (t+1)^2 - 0.5e^t$

(a) Use Milne Simpson predictor corrector formula to estimate values at $y(0.8)$ & $y(1.0)$

Ans. Milne Simpson predictor formula is

$y_{i+1} = y_{i-3} + \frac{4h}{3}(2f_i - f_{i-1} + 2f_{i-2})$

→ Corrector formula is

$y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i+1} + 4f_i + f_{i-1})$

now $y' = y - t^2 + 1$, $0 \le t \le 2$,

$y(0) = 0.5$, $h = 0.2$

∴ $t_0 = 0$, $t_1 = 0.2$, $t_2 = 0.4$

$t_3 = 0.6$, $t_4 = 0.8$, $t_5 = 1$

→ First we have to estimate values at $t_4 = 0.8$

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exact solution is $y(t) = (t+1)^2 - 0.5e^t$

so by exact solution

$t_0 = 0, \; y_0 = 0.5$

$t_1 = 0.2, \; y_1 = 0.8293$

$t_2 = 0.4, \; y_2 = 1.2141$

$t_3 = 0.6, \; y_3 = 1.6489$

$t_4 = 0.8, \; y_4 = 2.1272$

Now by milne simpson predictor formula.

$y_4 = y_0 + 4 \times \frac{0.2}{3}(2f_3 - f_2 + 2f_1)$

$= 0.5 + 0.2667(2f(t_3, y_3) - (t_2, y_2) + 2(t_1, y_1))$

$= 0.5 + 0.2667(2(1.6489 - (0.6)^2 + 1) - (1.2141 - (0.4)^2 + 1) + 2(0.8293 - (0.2)^2 + 1)$

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$y_4 = 0.5 + 0.2667(2 \times 2.2889 - 2.0541 + 2 \times 1.7893)$

$y_4 = 2.1275$

→ Now by corrector formula

$y_4 = y_2 + \frac{0.2}{3}(f_4 + 4f_3 + f_2)$

$= 1.2141 + 0.0667(f(0.8, 2.1275) + 4f(0.6, 1.6489) + f(0.4, 1.2141))$

$y_4 = 2.1277$

Again using corrector formula

$y_4 = 1.2141 + 0.0667(f(0.8, 2.1277) + 4f(0.6, 1.6489) + f(0.4, 1.2141))$

$\boxed{y_4 = 2.1277}$

Now to estimate values at $y(1.0)$ i.e. at $t_5 = 1.0$

By Milne Simpson Predictor formula

$$y_5 = y_1 + \frac{4 \times 0.2}{3}(2f_4 - f_3 + 2f_2)$$

$$= 0.8293 + 0.2667(2f(0.8, 2.1272) - f(0.6, 1.6489) + 2f(0.4, 1.2141))$$

$$y_5 = 2.5878$$

Now using corrector formula

$$y_5 = y_3 + \frac{0.2}{3}(f_5 + 4f_4 + f_3)$$

$$= 1.6489 + 0.0667(f(1, 2.5878) + 4f(0.8, 2.1272) + (0.6, 1.6489))$$

$$y_5 = 2.6378$$

Again,

$$y_5 = 1.6489 + 0.0667(f(1, 2.6378) + 4f(0.8, 2.1272) + f(0.6, 1.6489))$$

$$y_5 = 2.6411$$

Again

$$\boxed{y_5 = 2.6413}$$

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(b) Use Adams Moulton for steps predictor corrector method to arrive at approximate solution at y(0.8) & y(1.0).

Ans Adams moulton predictor formula.

$$y_{i+1} = y_i + \frac{h}{24}(55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3})$$

→ Corrector formula,

$$y_{i+1} = y_i + \frac{h}{24}(9f(x_{i+1}, y_{i+1}) + 19f_i - 5f_i + f_{i-2})$$

→ we have $y' = y - (t)^2 + 1$,
$0 \le t \le 2$, $y(0) = 0.5$, $h = 0.2$

→ Now By exact solution,

$t_0 = 0$, $y_0 = 0.5$
$t_1 = 0.2$, $y_1 = 0.8293$
$t_2 = 0.4$, $y_2 = 1.2141$
$t_3 = 0.6$, $y_3 = 1.6489$
$t_4 = 0.8$, $y_4 = 2.1272$

we have to find $y(0.8)$
i.e at $t_4 = 0.8$

$$y_4 = y_3 + \frac{0.2}{24}(55f_3 - 59f_2 + 37f_1 - 9f_0)$$

$y_4 = 2.1273$

→ By Corrector formula

$$y_4 = y_3 + \frac{0.2}{24}(9f(0.8, 2.1273) + 19f_3 - 5f_2 + f_1)$$

$y_4 = 2.1272$

Now to find $y(1.0)$
i.e at $t_5 = 1.0$

By Predictor method.

$$y_5 = y_4 + \frac{0.2}{24}(55f_4 - 59f_3 + 37f_2 - 9f_1)$$

$y_5 = 2.6409$

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By Corrector method.

$$y_5 = y_4 + \frac{0.2}{24}\left(9f(1.0, 2.6409) + 19f_4 - 5f_3 + f_2\right)$$

$$\boxed{y_5 = 2.64082}$$

(c) Compare the result obtain in (a), (b) from the exact solution by calculation error.

Ans

Milne simpson	Exact	Error
y(0.8) 2.1277	2.1272	0.0005
y(1.0) 2.6413	2.6409	0.0323

Adams-moulton	Exact	Error
y(0.8) 2.1272	2.1272	0.0
y(1.0) 2.6408	2.6409	0.0001

Assignment-5
Numerical Differential

Q1 Values for $f(x) = xe^x$ are given below

x	1.8	1.9	2.0	2.1	2.2
$f(x)$	10.889365	12.703199	14.778122	17.148957	19.855030

Use all applicable three point formulas and five point formulas to approximate $f'(2.0)$

Ans Three point endpoint formula

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)]$$

Here $h = 0.1$

So

$$f'[2.0] = \frac{1}{2 \times (0.1)}[-3f(2.0) + 4f(2.1) - f(2.2)]$$

$$= 5[-3 \times (14.778122) + 4 \times (17.148957) - (19.855030)]$$

$$f'[2.0] = 22.03216$$

→ Three point midpoint formula

$$f'(x_0) = \frac{1}{2h}(-f(x_0-h) + f(x_0+h))$$

$$f'(2.0) = 5[-f(1.9) + f(2.1)]$$

$= 5[-12.703199 + 17.148957]$

$f'(2.0) = 22.22879$

→ Three point endpoint formula

$$f'(x_0) = \frac{1}{2h}[f(x_0-2h) - 4f(x_0-h) + 3f(x_0)]$$

$f'(2.0) = 22.054675$

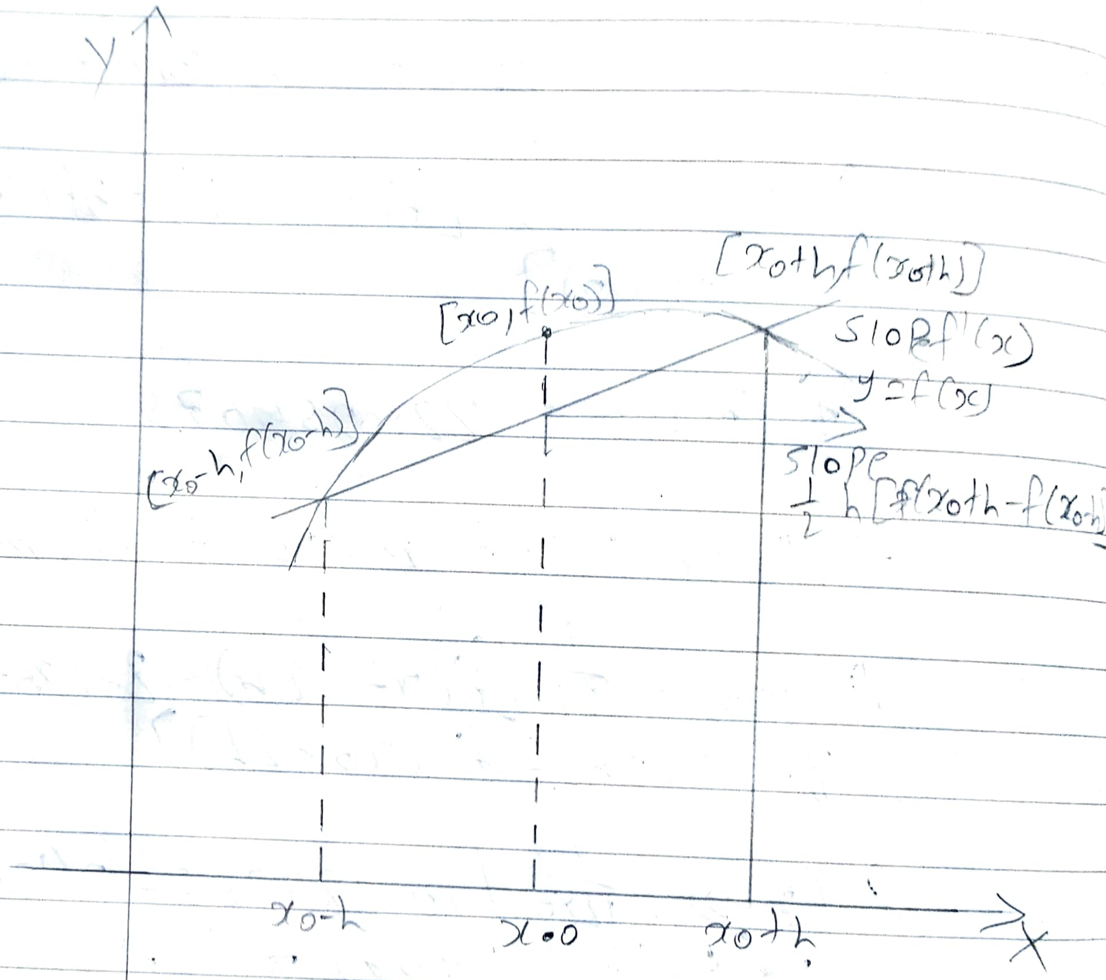
→ Five point midpoint formula.

$$f'(x_0) = \frac{1}{12h}[f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)]$$

$$f'(2.0) = \frac{1}{12\times0.1}[10.889365 - 8(12.703199) + \cancel{f}\, 8(17.148937) - 19.855030]$$

$f'(2.0) = 22.1689992$

Q2 Give Graphical representation of three point midpoint formula.



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Q3 Explain how h too small is not advantageous in numerical differentiation.

Ans In each of the numerical Differential formulas, h occurs in denominator, As h is made smaller and smaller to increase accuracy, division by small number causes round off errors to increase. Moreover in numerator also, difference of almost equal values occurs, which also contributes to round off errors. As a result, beyond a certain value, h can not be reduced further, as round off errors. As a result beyond a certain value, h can not be reduced further, as round off errors start dominating and accuracy cannot be improved further.

Q5. In a circuit with impressed voltage $E(t)$ and inductane L, kirchhoff's first law gives the relationship

$$E(t) = L\frac{di}{dt} + Ri,$$

where R is the resistance in the circuit and i is the current. Suppose we measure the current for several values of t and obtain.

t	1.00	1.01	1.02	1.03	1.04
i	3.10	3.12	3.14	3.18	3.24

where t is measured in seconds, i is in ameres, the inductance L is a constant 0.98 hennenies and the resistance is 0.142 ohms. Approximate the voltage $E(t)$ when $t = 1.00, 1.01, 1.03, 1.04$.

Ans Here

$$E(t) = L\frac{di}{dt} + Ri \quad \text{where } L = 0.98$$

$$h = 0.01, \quad R = 0.142$$

To find $\varepsilon(t)$ at $t = 1.00$ we need to find $\frac{di}{dt}$ at 1.00

By three point end point formula,

$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)]$

$\therefore f'(1.00) = \frac{1}{2 \times 0.01}[-3(3.10) + 4(3.12) - 3.14]$

$= 50 \times (0.04) = 2$

So, $\varepsilon(1.00) = 0.98 \times 2 + 0.142 \times (3.10)$

$\varepsilon(1.00) = 2.4002$ — ①

→ now at $t = 1.01$

By three point midpoint formula,

$f'(x_0) = \frac{1}{2h}[-f(x_0-h) + f(x_0+h)]$

$\therefore f(1.01) = 50[-3.10 + 3.14]$

$f(1.01) = 2$

So, $\varepsilon(1.01) = 0.98 \times 2 + 0.142 \times (3.12)$

$= 2.40304$ ← ②

→ now at $t = 1.02$
By five point midpoint formula,

$f'(1.0) = \frac{1}{12 \times h}[f(x_0 - 2h) - 8f(x_0 - h) + 8f(x_0 + h) - f(x_0 + 2h)]$

$f'(1.02) = \frac{1}{0.12}[3.10 - 8(3.12) + 8(3.18) - (3.24)]$

$f'(1.02) = 2.83$

So

$E(1.02) = 0.98 \times 2.83 + 0.142 \times (3.14)$

$E(1.02) = 3.21928$ —— (3)

→ now at $t = 1.03$
By three point midpoint formula,

$f'(1.03) = \frac{1}{2 \times 0.01}[-3.14 + 3.24]$

$= 5$

So

$E(1.03) = 0.98 \times 5 + 0.142 \times 3.18$

$= 5.35156$ —— (4)

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→ now at $t = 1.04$
By three point endpoint formula,

$$f'(x_0) = \frac{1}{2h}\left[f(x_0 - 2h) - 4f(x_0 - h) + 3f(x_0)\right]$$

$$f'(1.04) = 50\left[3.14 - 4(3.18) + (3.24)\right]$$

$$= 7$$

so

$$E(1.04) = 0.98 \times 7 + 0.142 \times 3.24$$
$$E(1.04) = 7.32008 \text{ ---- } (5)$$

****

NAME : Pradip S Karmakar

ROLL NO : 10

CLASS : MCA-II

SUBJECT : Computer Oriented Numerical Methods (CONM)

****

1. Apply Bisection method to solve the algebraic equation

```
#include<stdio.h>
#include<conio.h>
#include<math.h>

double fun(double x)
{
	double funx;
	funx=x*log(x)-1.2;
	return funx;
}

void content(double a,double b)
{
	double c;
	int no=0;

	printf("Enter Value of a:");
	scanf("%lf",&a);
	printf("\nEnter Value of b:");
	scanf("%lf",&b);
	//currunt - previos < epsilon
//epsilon have to be in macro
```

```
//function evaluation (divide by zero error) in the function
	while((fun(a)*fun(b))>0)
	{
		printf("\nInvalid input!");


		printf("\nEnter Value of a:");
		scanf("%lf",&a);
		printf("\nEnter Value of b:");
		scanf("%lf",&b);
	}
		c=(a+b)/2;


		printf("\nNO\tA\t\tf(A)\t\tB\t\tf(B)\t\tC\t\tf(C)\n");
		printf("\n----------------------------------------------------------------------------------------------------
---");
		while(fabs(fun(c))>0.0000005)
		{
			fun(c);
			if((fun(a)>0 && fun(c)>0) || (fun(a)<0 && fun(c)<0))
			{
				a=c;
			}
			if((fun(b)<0 && fun(c)<0) || (fun(b)>0 && fun(c)>0))
			{
				b=c;
			}
			c=(a+b)/2;

	printf("\n%d\t%lf\t%lf\t%lf\t%lf\t%lf\t%lf\n",no++,a,fun(a),b,fun(b),c,fun(c));
		}


}
```

```
void main()
{
	double a=0,b=0;

	content(a,b);

	getch();
}
```

****

OUTPUT:

****

```
Enter Value of a:1

Enter Value of b:2

NO   A          f(A)       B          f(B)       C          f(C)

----------------------------------------------------------------------------------------------------
0    1.500000   -0.591802  2.000000   0.186294   1.750000   -0.220672

1    1.750000   -0.220672  2.000000   0.186294   1.875000   -0.021359

2    1.875000   -0.021359  2.000000   0.186294   1.937500   0.081460

3    1.875000   -0.021359  1.937500   0.081460   1.906250   0.029794

4    1.875000   -0.021359  1.906250   0.029794   1.890625   0.004153

5    1.875000   -0.021359  1.890625   0.004153   1.882813   -0.008619
```

6	1.882813	-0.008619	1.890625	0.004153	1.886719	-0.002237
7	1.886719	-0.002237	1.890625	0.004153	1.888672	0.000957
8	1.886719	-0.002237	1.888672	0.000957	1.887695	-0.000640
9	1.887695	-0.000640	1.888672	0.000957	1.888184	0.000158
10	1.887695	-0.000640	1.888184	0.000158	1.887939	-0.000241
11	1.887939	-0.000241	1.888184	0.000158	1.888062	-0.000041
12	1.888062	-0.000041	1.888184	0.000158	1.888123	0.000059
13	1.888062	-0.000041	1.888123	0.000059	1.888092	0.000009
14	1.888062	-0.000041	1.888092	0.000009	1.888077	-0.000016
15	1.888077	-0.000016	1.888092	0.000009	1.888084	-0.000004
16	1.888084	-0.000004	1.888092	0.000009	1.888088	0.000002
17	1.888084	-0.000004	1.888088	0.000002	1.888086	-0.000001
18	1.888086	-0.000001	1.888088	0.000002	1.888087	0.000001
19	1.888086	-0.000001	1.888087	0.000001	1.888087	0.000000

****

NAME : Pradip S Karmakar

ROLL NO : 10

CLASS : MCA-II

SUBJECT : Computer Oriented Numerical Methods (CONM)

****

2. Apply Bisection method to solve the algebraic equation

```
#include<stdio.h>

#include<conio.h>

#include<math.h>


double fun(double x)

{

	double funx;

	funx=(x*x*x)-x-1;

	return funx;

}


void content(double a,double b)

{

	double c;

	int no=0;


	printf("Enter Value of a:");

	scanf("%lf",&a);

	printf("\nEnter Value of b:");

	scanf("%lf",&b);


	while((fun(a)*fun(b))>0)
```

```
	{
		printf("\nInvalid input!");

		printf("\nEnter Value of a:");
		scanf("%lf",&a);
		printf("\nEnter Value of b:");
		scanf("%lf",&b);
	}
		c=(a+b)/2;

		printf("\nNO\tA\t\tf(A)\t\tB\t\tf(B)\t\tC\t\tf(C)\n");
		printf("\n-----------------------------------------------------------------------------------------------------
---");
		while(fabs(fun(c))>0.0000005)
		{
			fun(c);
			if((fun(a)>0 && fun(c)>0) || (fun(a)<0 && fun(c)<0))
			{
				a=c;
			}
			if((fun(b)<0 && fun(c)<0) || (fun(b)>0 && fun(c)>0))
			{
				b=c;
			}
			c=(a+b)/2;

	printf("\n%d\t%lf\t%lf\t%lf\t%lf\t%lf\t%lf\n",no++,a,fun(a),b,fun(b),c,fun(c));
		}
}
void main()
{
	double a=0,b=0;
```

```
	content(a,b);

	getch();
}
```

****

OUTPUT:

****

```
Enter Value of a:0

Enter Value of b:1

Invalid input!
Enter Value of a:1

Enter Value of b:2

NO   A        f(A)       B        f(B)       C        f(C)

-----------------------------------------------------------------------------------------------------
0   1.000000   -1.000000   1.500000   0.875000   1.250000   -0.296875

1   1.250000   -0.296875   1.500000   0.875000   1.375000   0.224609

2   1.250000   -0.296875   1.375000   0.224609   1.312500   -0.051514

3   1.312500   -0.051514   1.375000   0.224609   1.343750   0.082611

4   1.312500   -0.051514   1.343750   0.082611   1.328125   0.014576
```

5	1.312500	-0.051514	1.328125	0.014576	1.320313	-0.018711
6	1.320313	-0.018711	1.328125	0.014576	1.324219	-0.002128
7	1.324219	-0.002128	1.328125	0.014576	1.326172	0.006209
8	1.324219	-0.002128	1.326172	0.006209	1.325195	0.002037
9	1.324219	-0.002128	1.325195	0.002037	1.324707	-0.000047
10	1.324707	-0.000047	1.325195	0.002037	1.324951	0.000995
11	1.324707	-0.000047	1.324951	0.000995	1.324829	0.000474
12	1.324707	-0.000047	1.324829	0.000474	1.324768	0.000214
13	1.324707	-0.000047	1.324768	0.000214	1.324738	0.000084
14	1.324707	-0.000047	1.324738	0.000084	1.324722	0.000018
15	1.324707	-0.000047	1.324722	0.000018	1.324715	-0.000014
16	1.324715	-0.000014	1.324722	0.000018	1.324718	0.000002
17	1.324715	-0.000014	1.324718	0.000002	1.324717	-0.000006
18	1.324717	-0.000006	1.324718	0.000002	1.324718	-0.000002
19	1.324718	-0.000002	1.324718	0.000002	1.324718	0.000000

****

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CLASS   : MCA-II

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****

3. Apply False Position method to solve the algebraic equation

```
#include<stdio.h>

#include<conio.h>

#include<math.h>


double fun(double x)

{

	double funx;

	funx=x*log10(x)-1.2;

	return funx;

}


void content(double a,double b)

{

	double c;

	int no=0;


	printf("Enter Value of a:");

	scanf("%lf",&a);

	printf("\nEnter Value of b:");

	scanf("%lf",&b);
```

```
	while((fun(a)*fun(b))>0)
	{
		printf("\nInvalid input!\n");

		printf("\nEnter Value of a:");
		scanf("%lf",&a);
		printf("\nEnter Value of b:");
		scanf("%lf",&b);
	}
		c=((b*(fun(a)))-(a*(fun(b))))/((fun(a))-(fun(b)));

		printf("\nNO\tA\t\tf(A)\t\tB\t\tf(B)\t\tC\t\tf(C)\n");
		printf("\n---------------------------------------------------------------------------------------------------
---");
		while(fabs(fun(c))>0.0000005)
		{
	printf("\n%d\t%lf\t%lf\t%lf\t%lf\t%lf\t%lf\n",++no,a,fun(a),b,fun(b),c,fun(c));
			fun(c);
			if((fun(a)>0 && fun(c)>0) || (fun(a)<0 && fun(c)<0))
			{
				a=c;
			}
			if((fun(b)<0 && fun(c)<0) || (fun(b)>0 && fun(c)>0))
			{
				b=c;
			}
			c=((b*(fun(a)))-(a*(fun(b))))/((fun(a))-(fun(b)));
		}
		printf("\n%d\t%lf\t%lf\t%lf\t%lf\t%lf\t%lf\n",++no,a,fun(a),b,fun(b),c,fun(c));
}
void main()
```

```
{
            double a=0,b=0;

            content(a,b);

            getch();
}
```

****

OUTPUT:

****

Enter Value of a:2

Enter Value of b:3

NO	A	f(A)	B	f(B)	C	f(C)
1	2.000000	-0.597940	3.000000	0.231364	2.721014	-0.017091
2	2.721014	-0.017091	3.000000	0.231364	2.740206	-0.000384
3	2.740206	-0.000384	3.000000	0.231364	2.740636	-0.000009
4	2.740636	-0.000009	3.000000	0.231364	2.740646	-0.000000

****

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****

4. Apply Secant method to solve the algebraic equation

```
#include<stdio.h>
#include<conio.h>
#include<math.h>

double fun(double x)
{
        double funx;
        funx=(3*x)-cos(x)-1;
        return funx;
}

void content(double a,double b)
{
        double c;
        int no=0;

        printf("Enter Value of a:");
        scanf("%lf",&a);
        printf("\nEnter Value of b:");
        scanf("%lf",&b);

                c=((a*(fun(b)))-(b*(fun(a))))/((fun(b))-(fun(a)));
```

```
		printf("\nNO\tA\t\tf(A)\t\tB\t\tf(B)\t\tC\t\tf(C)\n");
		printf("\n----------------------------------------------------------------------------------------------------
---");
		while(fabs(fun(c))>0.0000005)
		{
			fun(c);
	printf("\n%d\t%lf\t%lf\t%lf\t%lf\t%lf\t%lf\n",++no,a,fun(a),b,fun(b),c,fun(c));

			a = b;
			b = c;

			c=((a*(fun(b)))-(b*(fun(a))))/((fun(b))-(fun(a)));
		}
		printf("\n%d\t%lf\t%lf\t%lf\t%lf\t%lf\t%lf\n",++no,a,fun(a),b,fun(b),c,fun(c));
}
void main()
{
	double a=0,b=0;

	content(a,b);

	getch();
}
```

****

OUTPUT:

****

Enter Value of a:0

Enter Value of b:1

NO	A	f(A)	B	f(B)	C	f(C)
1	0.000000	-2.000000	1.000000	1.459698	0.578085	-0.103255
2	1.000000	1.459698	0.578085	-0.103255	0.605959	-0.004081
3	0.578085	-0.103255	0.605959	-0.004081	0.607106	0.000014
4	0.605959	-0.004081	0.607106	0.000014	0.607102	-0.000000

****

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****

5. Apply Newton Raphson method to solve the algebraic equation

```
#include<stdio.h>

#include<conio.h>

#include<math.h>

double fun(double x)

{

	double funx;

	funx = (x*x)-5;

	return funx;

}

double derfun(double x)

{

	double funx;

	funx = 2*x;

	return funx;

}

void cntnt(double a)

{

	double c;

	int no=0;
```

```c
	printf("Enter Value:");
	scanf("%lf",&a);

	c = a-((fun(a))/(derfun(a)));
	printf("\nNO\tXn\t\tf(Xn)\t\tf'(Xn)\t\tXn+1\n");
	printf("\n----------------------------------------------------------------------------------------------------");

	while(fabs(fun(a))>0.0000005)
	{
		printf("\n%d\t%lf\t%lf\t%lf\t%lf\n",++no,a,fun(a),derfun(a),c);
		a = c;

		c = a-((fun(a))/(derfun(a)));
	}
	printf("\n%d\t%lf\t%lf\t%lf\t%lf\n",++no,a,fun(a),derfun(a),c);
}

void main()
{
	double a=0;

	cntnt(a);

	getch();
}
```

****

OUTPUT:

****

Enter Value:1

NO	Xn	f(Xn)	f'(Xn)	Xn+1
1	1.000000	-4.000000	2.000000	3.000000
2	3.000000	4.000000	6.000000	2.333333
3	2.333333	0.444444	4.666667	2.238095
4	2.238095	0.009070	4.476190	2.236069
5	2.236069	0.000004	4.472138	2.236068
6	2.236068	0.000000	4.472136	2.236068

****

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****

```
#include<stdio.h>

#include<conio.h>

#include<math.h>

#define epsilone 0.0000005


double fun(double input)

{

	double funx;


	funx = (1+cos(input))/3;


	return funx;

}


void content(double input)

{

	double x;

	int num=1;


	printf("Enter value of X:");

	scanf("%lf",&x);


	printf("\nNO\tX\t\tf(X)\n");

	printf("\n------------------------------------");
```

```
	while(  fabs(x-(fun(x))) > epsilone )
	{
		if(num > 40)
		{
			break;
		}

		else
		{
			printf("\n%d\t%lf\t%lf\n",num,x,fun(x));
			num++;

			x = fun(x);
		}
	}
}

void main()
{
	double input=0;

	content(input);

	getch();
}
```

****

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SUBJECT : CONM

****

Q(1): Evaluate Integral of (e^x^2)*sin x dx from 0 to 1 using Trapezoidal rule correct to 3 decimal places

****

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
#define epsilon 0.0005

void trapezoidal(double,double,int);

double f(double x)
{
	return (exp(x*x)*sin(x));
}

void main()
{
	int N=2;
	double a,b;
	a=0;
	b=1;
	trapezoidal(a,b,N);
	getch();
}
```

```
void trapezoidal(double a,double b,int N)

{

	int i,limit=20,k=1;

	double sum=0,old_sum=0,h;

	printf("=====================================Trapezoidal
Rule===================================\n\n");

	printf("\nSr No\t\t|\tN\t\t|\th\t\t\t|\tIntegral\n");

	printf("______________________________________________________________________
_____________________");

	while(k<=limit)

	{

		sum=0;

		h=(b-a)/N;

		for(i=1;i<N;i++)

		{

			sum+=2*f(a+i*h);

		}

		sum+=(f(a)+f(b));

		sum *=h/2;

		printf("\n%d\t\t|\t%d\t\t|\t%lf\t\t|\t%lf",k,N,h,sum);

		if(fabs(sum-old_sum)<epsilon)

		{

			printf("\n\n-->The Estimate of the Integral is %lf",sum);

			break;

		}

		N*=2;

		k++;

		old_sum=sum;

	}

}
```

****

output:

===================================Trapezoidal
Rule=================================

Sr No	N	h	Integral
1	2	0.500000	0.879636
2	4	0.250000	0.804736
3	8	0.125000	0.785295
4	16	0.062500	0.780386
5	32	0.031250	0.779156
6	64	0.015625	0.778848

-->The Estimate of the Integral is 0.778848

****

****

Q(2): Evaluate the integral:

integral of dx/(1+x) from 0 to 1

Using

(i) Simpson's 1/3 Rule correct to six decimal places

(ii) Simpson's 3/8 rule correct to six decimal places

****

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
#define epsilon 0.0000005

void simpsons1_3(double,double,int);
void simpsons3_8(double,double,int);
double f(double x)
{
        return (1/(1+x));
}

void main()
{
        int N=2;
        double a,b;
        a=0;
        b=2;
        simpsons1_3(a,b,N);
        simpsons3_8(a,b,N);
        getch();
}
```

```
void simpsons1_3(double a,double b,int N)
{
	printf("====================================Simpsons 1/3 Rule==================================\n\n");
	int i,limit=20,k=1;
	double sum=0,old_sum=0,h;
	printf("\nSr No\t\t|\tN\t\t|\th\t\t\t|\tIntegral\n");
	printf("______________________________________________________________________________________________________");
	while(k<=limit)
	{
		sum=0;
		h=(b-a)/N;
		for(i=1;i<N;i++)
		{
			if(i%2==0)
				sum+=2*f(a+i*h);
			else
				sum+=4*f(a+i*h);
		}
		sum+=(f(a)+f(b));
		sum *=h/3;
		printf("\n%d\t\t|\t%d\t\t|\t%lf\t\t|\t%lf",k,N,h,sum);
		if(fabs(sum-old_sum)<epsilon)
		{
			printf("\n\n-->The Estimate of the Integral Using simpsons1/3 Rule is %lf",sum);
			break;
		}
		N*=2;
		k++;
```

```
		old_sum=sum;
	}
	printf("\n\n");
}

void simpsons3_8(double a,double b,int N)
{
	printf("=====================================Simpsons 3/8 
Rule==================================\n\n");
	int i,limit=20,k=1;
	double sum=0,old_sum=0,h;
	printf("\nSr No\t\t|\tN\t\t|\th\t\t\t|\tIntegral\n");
	printf("___________________________________________________________________________
_______________________");
	while(k<=limit)
	{
		sum=0;
		h=(b-a)/N;
		for(i=1;i<N;i++)
		{
			if(i%3==0)
				sum+=2*f(a+i*h);
			else
				sum+=3*f(a+i*h);
		}
		sum+=(f(a)+f(b));
		sum *=3*h/8;
		printf("\n%d\t\t|\t%d\t\t|\t%lf\t\t|\t%lf",k,N,h,sum);
		if(fabs(sum-old_sum)<epsilon)
		{
			printf("\n\n-->The Estimate of the Integral Using simpsons3/8 Rule is 
%lf",sum);
```

```
                    break;
                }
                N*=2;
                k++;
                old_sum=sum;
        }
}
```

****

output:

```
===================================Simpsons 1/3
Rule===================================



Sr No      |    N       |    h            |    Integral
______________________________________________________________________________
1        |    2       |    1.000000         |    1.111111
2        |    4       |    0.500000         |    1.100000
3        |    8       |    0.250000         |    1.098725
4        |    16      |    0.125000         |    1.098620
5        |    32      |    0.062500         |    1.098613
6        |    64      |    0.031250         |    1.098612

-->The Estimate of the Integral Using simpsons1/3 Rule is 1.098612

===================================Simpsons 3/8
Rule===================================



Sr No      |    N       |    h            |    Integral
______________________________________________________________________________
```

1	2	1.000000	1.062500
2	4	0.500000	1.056250
3	8	0.250000	1.087541
4	16	0.125000	1.087999
5	32	0.062500	1.095955
6	64	0.031250	1.095995
7	128	0.015625	1.097958
8	256	0.007813	1.097960
9	512	0.003906	1.098449
10	1024	0.001953	1.098449

-->The Estimate of the Integral Using simpsons3/8 Rule is 1.098449

****

****

Q(3): A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in feet/second by the entries in the following table.

Time 0 6 12 18 24 30 36 42 48 54 60 66 72 78 84

Speed 124 134 148 156 147 133 121 109 99 85 78 89 104 116 123

How long is the track?

Use (i) Trapezoidal Rule (ii) Simpson's 1/3 rule (iii) Simpson's3/8 rule

****

```
#include<stdio.h>
#include<conio.h>
#include<math.h>

void simpsons1_3(double,double,int);
void simpsons3_8(double,double,int);
void trapezoidal(double,double,int);

double f(int x)
{
	switch(x)
	{
		case 0:return 124;
		case 6:return 134;
		case 12:return 148;
		case 18:return 156;
		case 24:return 147;
		case 30:return 133;
		case 36:return 121;
		case 42:return 109;
		case 48:return 99;
```

```
		case 54:return 85;
		case 60:return 78;
		case 66:return 89;
		case 72:return 104;
		case 78:return 116;
		case 84:return 123;
	}
}
void main()
{
	int N=14;
	double a,b;
	a=0;
	b=84;
	trapezoidal(a,b,N);
	simpsons1_3(a,b,N);
	simpsons3_8(a,b,N);
	getch();
}

void trapezoidal(double a,double b,int N)
{
	int i;
	double sum=0,h;
	sum=0;
	h=(b-a)/N;
	for(i=1;i<N;i++)
	{
		sum+=2*f(a+i*h);
	}
	sum+=(f(a)+f(b));
```

```
        sum *=h/2;
        printf("\n-->Length of track using Trapezoidal Rule=%0.2lf Feet",sum);
}

void simpsons1_3(double a,double b,int N)
{
        int i;
        double sum=0,h;
                sum=0;
                h=(b-a)/N;
                for(i=1;i<N;i++)
                {
                        if(i%2==0)
                                sum+=2*f(a+i*h);
                        else
                                sum+=4*f(a+i*h);
                }
                sum+=(f(a)+f(b));
                sum *=h/3;
                printf("\n-->Length of track using Simpsons 1/3 Rule=%0.2lf Feet",sum);
}

void simpsons3_8(double a,double b,int N)
{
        int i;
        double sum=0,h;
        sum=0;
        h=(b-a)/N;
        for(i=1;i<N;i++)
        {
                if(i%3==0)
```

```
                sum+=2*f(a+i*h);
            else
                sum+=3*f(a+i*h);
        }
        sum+=(f(a)+f(b));
        sum *=3*h/8;
        printf("\n-->Length of track using Simpsons 3/8 Rule=%0.2lf Feet",sum);
}
```

****

output:

```
-->Length of track using Trapezoidal Rule=9855.00 Feet
-->Length of track using Simpsons 1/3 Rule=9858.00 Feet
-->Length of track using Simpsons 3/8 Rule=9760.50 Feet
```

****

****

Q(4): Write a program to solve the differential equation dy/dx=(y-x)/(y+x), where y(0) = 1, using

(i) Euler's method

(ii) Runge - Kutta second order method

in the interval 0 to 1 using step-size 0.1 Tabulate your results

****

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
void euler(double,double,double,int);
void runge_kutta_2(double,double,double,int);

double f(double x,double y)
{
	return ((y-x)/(y+x));
}
void main()
{
	int limit;
	double xi,yi,h;
	xi=0;
	yi=1;
	h=0.1;
	limit=1;
	euler(xi,yi,h,limit);
	runge_kutta_2(xi,yi,h,limit);
	getch();
}

void euler(double xi,double yi,double h,int limit)
```

```
{
	double yi_1;
	yi_1=yi;
	printf("==============EULER METHOD=============\n\n");
	printf("\nx\t\t|\tSolution\n");
	printf("____________________________________\n");
	while(xi<=limit)
	{
		yi=yi_1;
		printf("\n%0.2lf\t\t|\t%lf",xi,yi);
		yi_1=yi + h* f(xi,yi);
		xi+=h;
	}
	printf("\n\n-->Solution With Eulers method= %lf\n\n",yi);
}

void runge_kutta_2(double xi,double yi,double h,int limit)
{
	double yi_1,k0,k1;
	yi_1=yi;
	printf("========RUNGE-KUTTA SECOND ORDER METHOD========\n\n");
	printf("\nx\t\t|\tSolution\n");
	printf("____________________________________");
	while(xi<=limit)
	{
		yi=yi_1;
		printf("\n%0.2lf\t\t|\t%lf",xi,yi);
		k0=h*f(xi,yi);
		k1=h*f(xi+h,yi+k0);
		yi_1=yi + (0.5)*(k0+k1);
		xi+=h;
```

```
    }
    printf("\n\n-->Solution With RUNGE-KUTTA SECOND ORDER METHOD= %lf",yi);
}
```

****

output:

```
==============EULER METHOD=============


x          |     Solution
____________________________________


0.00       |     1.000000
0.10       |     1.100000
0.20       |     1.183333
0.30       |     1.254418
0.40       |     1.315818
0.50       |     1.369193
0.60       |     1.415694
0.70       |     1.456161
0.80       |     1.491231
0.90       |     1.521399
1.00       |     1.547062

-->Solution With Eulers method= 1.547062

========RUNGE-KUTTA SECOND ORDER METHOD========


x          |     Solution
____________________________________
```

```
0.00      |     1.000000
0.10      |     1.091667
0.20      |     1.168728
0.30      |     1.234629
0.40      |     1.291489
0.50      |     1.340729
0.60      |     1.383361
0.70      |     1.420135
0.80      |     1.451627
0.90      |     1.478291
1.00      |     1.500491

-->Solution With RUNGE-KUTTA SECOND ORDER METHOD= 1.500491
********************************************************************************
```

****

Q(5): Find the solution of differential equation, for the range 0 <= t <= 1 dy/dt = t + (y)^(1/2)

with y(0) = 1, taking step size h = 0.2 using Runge-Kutta method of order 4

****

```
#include<stdio.h>
#include<conio.h>
#include<math.h>

void runge_kutta_4(double,double,double,int);

double f(double t,double y)
{
        return (t+sqrt(y));
}
void main()
{
        int limit;
        double ti,yi,h;
        ti=0;
        yi=1;
        h=0.2;
        limit=1;
        runge_kutta_4(ti,yi,h,limit);
        getch();
}
void runge_kutta_4(double ti,double yi,double h,int limit)
{
        double yi_1,k0,k1,k2,k3;
        yi_1=yi;
        printf("========RUNGE-KUTTA FORTH ORDER METHOD========\n\n");
```

```
	printf("\nt\t\t|\tSolution\n");
	printf("___________________________________");
	while(ti<=limit)
	{
		yi=yi_1;
		printf("\n%0.2lf\t\t|\t%lf",ti,yi);
		k0=h*f(ti,yi);
		k1=h*f(ti+(h/2),yi+(k0/2));
		k2=h*f(ti+(h/2),yi+(k1/2));
		k3=h*f(ti+h,yi+k2);
		yi_1=yi + (k0+2*k1+2*k2+k3)/6;
		ti+=h;
	}
	printf("\n\n-->Solution With RUNGE-KUTTA FORTH ORDER METHOD= %lf",yi);
}
```

****

output:

```
========RUNGE-KUTTA FORTH ORDER METHOD========


t		|	Solution
___________________________________
0.00		|	1.000000
0.20		|	1.230632
0.40		|	1.524809
0.60		|	1.885413
0.80		|	2.314716
1.00		|	2.814506
-->Solution With RUNGE-KUTTA FORTH ORDER METHOD= 2.814506
```

****

****

Q(6): Find the solution of differential equation dy/dt = 1/2 (t+y), for y (2.0) given

y(0) = 2

y(0.5) = 2.636

y(1.0) = 3.595

and y(1.5) = 4.968 , use h = 0.5

using (i) Milne-Simpson's predictor corrector method

(ii) Adam-Bashforth-Moulton's predictor-corrector method

****

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
#define epsilon 0.00005
void milne_simpsone_predictor_corrector(double[],double[],double);
void adam_bashforth_moultons_predictor_corrector(double[],double[],double);

double f(double t,double y)
{
	return ((t+y)/2);
}
void main()
{
	double h,y[10],t[10];
	h=0.5;
	y[0]=2;
	y[1]=2.636;
	y[2]=3.595;
	y[3]=4.968;
	t[0]=0;
	t[1]=0.5;
```

```
        t[2]=1.0;

        t[3]=1.5;

        t[4]=2.0;

        milne_simpsone_predictor_corrector(y,t,h);

        adam_bashforth_moultons_predictor_corrector(y,t,h);

        getch();

}

void milne_simpsone_predictor_corrector(double y[],double t[],double h)

{

        double yi_old=0;

        int i;

        i=3;

        printf("========milne_simpsone_predictor_corrector METHOD========\n\n");

        //predictor Method

        y[i+1]=y[i-3]+(4*h)*(2*f(t[i],y[i])-f(t[i-1],y[i-1])+2*f(t[i-2],y[i-2]))/3;

        printf("Using Predictor Formula y4 =%lf",y[i+1]);



        //Corrector formula

        while(fabs(yi_old-y[i+1])>epsilon)

        {

                yi_old=y[i+1];

                y[i+1]=y[i-1] + (h/3) *(f(t[i+1],y[i+1])+ 4* f(t[i],y[i])+f(t[i-1],y[i-1]));

                printf("\n-->Using Corrector Formula y4=%lf",y[i+1]);

        }

        printf("\n\n---->Solution With milne_simpsone_predictor_corrector METHOD=
%lf\n\n",y[i+1]);

}


void adam_bashforth_moultons_predictor_corrector(double y[],double t[],double h)

{
```

```
        double yi_old=0;
        int i;
        i=3;
        printf("========adam_bashforth_moultons_predictor_corrector METHOD========\n\n");
        //predictor Method
        y[i+1]=y[i]+(h/24)*(55*f(t[i],y[i])-59*f(t[i-1],y[i-1])+37*f(t[i-2],y[i-2])-9*f(t[i-3],y[i-3]));
        printf("Using Predictor Formula y4 =%lf",y[i+1]);



        //Corrector formula
        while(fabs(yi_old-y[i+1])>epsilon)
        {
                yi_old=y[i+1];
                y[i+1]=y[i] + (h/24) *(9*f(t[i+1],y[i+1])+ 19 * f(t[i],y[i])-5*f(t[i-1],y[i-1])+f(t[i-2],y[i-2]));
                printf("\n-->Using Corrector Formula y4=%lf",y[i+1]);
        }
        printf("\n\n---->Solution With adam_bashforth_moultons_predictor_corrector METHOD=
%lf",y[i+1]);
}
```

****

output:

========milne_simpsone_predictor_corrector METHOD========

Using Predictor Formula y4 =6.871000

-->Using Corrector Formula y4=6.873167

-->Using Corrector Formula y4=6.873347

-->Using Corrector Formula y4=6.873362

---->Solution With milne_simpsone_predictor_corrector METHOD= 6.873362

========adam_bashforth_moultons_predictor_corrector METHOD========

Using Predictor Formula y4 =6.870781

-->Using Corrector Formula y4=6.873104

-->Using Corrector Formula y4=6.873322

-->Using Corrector Formula y4=6.873343

---->Solution With adam_bashforth_moultons_predictor_corrector METHOD= 6.873343

****

****

Q(7): Use Adam-Bashforth-Moulton's predictor-corrector method to obtain the solution of the equation dy/dx= 1-xy/x^2 at x = 1.4, where y(1) = 1.

Compute y(1.1), y(1.2) and y(1.3) using Runge-Kutta second order method.

Tabulate the results obtained thus.

****

```
#include<stdio.h>

#include<conio.h>

#include<math.h>

#define epsilon 0.00005


void adam_bashforth_moultons_predictor_corrector(double[],double[],double);

double runge_kutta_2(double,double,double,double);

double f(double x,double y)

{

	return ((1-x*y)/(x*x));

}

void main()

{

	double h,y[10],x[10];

	h=0.1;

	x[0]=1;

	x[1]=1.1;

	x[2]=1.2;

	x[3]=1.3;

	x[4]=1.4;


	y[0]=1;

	y[1]=runge_kutta_2(x[0],y[0],h,1.2);

	y[2]=runge_kutta_2(x[0],y[0],h,1.3);
```

```
	y[3]=runge_kutta_2(x[0],y[0],h,1.4);

	printf("\n=========================By Runge-Kutta second order method\n");
	printf("y(1.1)=%lf\ny(1.2)=%lf\ny(1.3)=%lf\n\n",y[1],y[2],y[3]);
	adam_bashforth_moultons_predictor_corrector(y,x,h);
	getch();
}

void adam_bashforth_moultons_predictor_corrector(double y[],double x[],double h)
{
	double yi_old=0;
	int i;
	i=3;
	printf("========adam_bashforth_moultons_predictor_corrector METHOD========\n\n");
	//predictor Method
	y[i+1]=y[i]+(h/24)*(55*f(x[i],y[i])-59*f(x[i-1],y[i-1])+37*f(x[i-2],y[i-2])-9*f(x[i-3],y[i-3]));
	printf("Using Predictor Formula y(1.4) =%lf",y[i+1]);



	//Corrector formula
	while(fabs(yi_old-y[i+1])>epsilon)
	{
		yi_old=y[i+1];
		y[i+1]=y[i] + (h/24) *(9*f(x[i+1],y[i+1])+ 19 * f(x[i],y[i])-5*f(x[i-1],y[i-1])+f(x[i-2],y[i-
2]));
		printf("\n-->Using Corrector Formula y(1.4)=%lf",y[i+1]);
	}
	printf("\n\n---->Solution With adam_bashforth_moultons_predictor_corrector METHOD=
%lf",y[i+1]);
}

double runge_kutta_2(double xi,double yi,double h,double limit)
```

```
{
	double yi_1,k0,k1;
	yi_1=yi;
	while(xi<limit)
	{
		yi=yi_1;
		k0=h*f(xi,yi);
		k1=h*f(xi+h,yi+k0);
		yi_1=yi + (0.5)*(k0+k1);
		xi+=h;
	}
	return yi;
}
```

```
*********************************************************************************
output:
========================By Runge-Kutta second order method
y(1.1)=0.995868
y(1.2)=0.985480
y(1.3)=0.971311

========adam_bashforth_moultons_predictor_corrector METHOD========

Using Predictor Formula y(1.4) =0.954695
-->Using Corrector Formula y(1.4)=0.954878
-->Using Corrector Formula y(1.4)=0.954873
---->Solution With adam_bashforth_moultons_predictor_corrector METHOD= 0.954873
*********************************************************************************
```

****

Q(8): Use Milne Simpson predictor corrector method to obtain the solution of the equation dy/dx= 1-xy/x^2 at x = 1.4, where y(1) = 1.

Compute y(1.1), y(1.2) and y(1.3) using Runge-Kutta fourth order method.

Tabulate the results obtained thus.

****

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
#define epsilon 0.00005

void milne_simpsone_predictor_corrector(double[],double[],double);
double runge_kutta_4(double,double,double,double);
double f(double x,double y)
{
	return ((1-x*y)/(x*x));
}
void main()
{
	double h,y[10],x[10];
	h=0.1;
	x[0]=1;
	x[1]=1.1;
	x[2]=1.2;
	x[3]=1.3;
	x[4]=1.4;

	y[0]=1;
	y[1]=runge_kutta_4(x[0],y[0],h,1.2);
	y[2]=runge_kutta_4(x[0],y[0],h,1.3);
```

```
        y[3]=runge_kutta_4(x[0],y[0],h,1.4);


        printf("\n========================By Runge-Kutta Forth order method\n");
        printf("y(1.1)=%lf\ny(1.2)=%lf\ny(1.3)=%lf\n\n",y[1],y[2],y[3]);
        milne_simpsone_predictor_corrector(y,x,h);
        getch();
}
void milne_simpsone_predictor_corrector(double y[],double x[],double h)
{
        double yi_old=0;
        int i;
        i=3;
        printf("========milne_simpsone_predictor_corrector METHOD========\n\n");
        //predictor Method
        y[i+1]=y[i-3]+(4*h)*(2*f(x[i],y[i])-f(x[i-1],y[i-1])+2*f(x[i-2],y[i-2]))/3;
        printf("Using Predictor Formula y(1.4)=%lf",y[i+1]);



        //Corrector formula
        while(fabs(yi_old-y[i+1])>epsilon)
        {
                yi_old=y[i+1];
                y[i+1]=y[i-1] + (h/3) *(f(x[i+1],y[i+1])+ 4* f(x[i],y[i])+f(x[i-1],y[i-1]));
                printf("\n-->Using Corrector Formula y(1.4)=%lf",y[i+1]);
        }
        printf("\n\n---->Solution With milne_simpsone_predictor_corrector METHOD=
%lf\n\n",y[i+1]);
}
double runge_kutta_4(double xi,double yi,double h,double limit)
{
        double yi_1,k0,k1,k2,k3;
```

```
        yi_1=yi;
        while(xi<limit)
        {
                yi=yi_1;
                k0=h*f(xi,yi);
                k1=h*f(xi+(h/2),yi+(k0/2));
                k2=h*f(xi+(h/2),yi+(k1/2));
                k3=h*f(xi+h,yi+k2);
                yi_1=yi + (k0+2*k1+2*k2+k3)/6;
                xi+=h;
        }
        return yi;
}
```

****

output:

```
========================By Runge-Kutta Forth order method
y(1.1)=0.995737
y(1.2)=0.985268
y(1.3)=0.971050

========milne_simpsone_predictor_corrector METHOD========

Using Predictor Formula y(1.4)=0.954478
-->Using Corrector Formula y(1.4)=0.954629
-->Using Corrector Formula y(1.4)=0.954626
---->Solution With milne_simpsone_predictor_corrector METHOD= 0.954626
```

****

****

Q(9): From the following table estimate y'(1.1) and y'(1.2) using 3 point formulas and 5 point formulas

x 1.0 1.05 1.10 1.15 1.20 1.25 1.30

y 1.0 1.0247 1.0488 1.0724 1.0954 1.1180 1.1402

****

```
#include<stdio.h>

#include<conio.h>

#include<math.h>


void _3point_formulas(double[],double[],double);

void _5point_formulas(double[],double[],double);


void main()

{

	double x[10],y[10],h=0.5;

	x[0]=1.0;

	x[1]=1.05;

	x[2]=1.10;

	x[3]=1.15;

	x[4]=1.20;

	x[5]=1.25;

	x[6]=1.30;

	y[0]=1.0;

	y[1]=1.0247;

	y[2]=1.0488;

	y[3]=1.0724;

	y[4]=1.0954;

	y[5]=1.1180;

	y[6]=1.1402;
```

```
	_3point_formulas(x,y,h);
	_5point_formulas(x,y,h);
	getch();
}

void _3point_formulas(double x[],double y[],double h)
{

	double x0=x[2],ans;
	int i=2;
	//Endpoint formula
	printf("\n===============3 Pont End Point Formula===============\n");
	ans=(1/(2*h)) * (-3 * y[i] + 4*y[i+1]-y[i+2]);
	printf("\n--->y(1.1)'=%lf",ans);
	i=4;
	ans=(1/(2*h)) * (-3 * y[i] + 4*y[i+1]-y[i+2]);
	printf("\n--->y(1.2)'=%lf",ans);
	//Midpoint Formula
	i=2;
	printf("\n===============3 Pont Mid Point Formula===============\n");
	ans=(1/(2*h)) * (-y[i-1] + y[i+1]);
	printf("\n--->y(1.1)'=%lf",ans);
	i=4;
	ans=(1/(2*h)) * (-y[i-1] + y[i+1]);
	printf("\n--->y(1.2)'=%lf",ans);
	//Endpoint formula
	printf("\n===============3 Pont End Point Formula===============\n");
	i=2;
	ans=(1/(2*h)) * (y[i-2] - 4*y[i-1]+3*y[i]);
	printf("\n--->y(1.1)'=%lf",ans);
```

```
	i=4;
	ans=(1/(2*h)) * (y[i-2] - 4*y[i-1]+3*y[i]);
	printf("\n--->y(1.2)'=%lf",ans);
}


void _5point_formulas(double x[],double y[],double h)
{

	double x0=x[2],ans;
	int i=2;
	//Endpoint formula
	printf("\n\n\n===============5 Pont End Point Formula===============\n");
	ans=(1/(12*h)) * ( -25*y[i] +48*y[i+1]-36* y[i+2]+16*y[i+3]-3*y[i+4]);
	printf("\n--->y(1.1)'=%lf",ans);

	//Midpoint Formula
	i=2;
	printf("\n===============5 Pont Mid Point Formula===============\n");
	ans=(1/(12*h)) * ( y[i-2] - 8*y[i-1]+8* y[i+1]-y[i+2]);
	printf("\n--->y(1.1)'=%lf",ans);
	i=4;
	ans=(1/(12*h)) * ( y[i-2] - 8*y[i-1]+8* y[i+1]-y[i+2]);
	printf("\n--->y(1.2)'=%lf",ans);
}
```

****

output:

```
===============3 Pont End Point Formula===============
```

--->y(1.1)'=0.047800

--->y(1.2)'=0.045600

===============3 Pont Mid Point Formula===============

--->y(1.1)'=0.047700

--->y(1.2)'=0.045600

===============3 Pont End Point Formula===============

--->y(1.1)'=0.047600

--->y(1.2)'=0.045400

===============5 Pont End Point Formula===============

--->y(1.1)'=0.048033

===============5 Pont Mid Point Formula===============

--->y(1.1)'=0.047700

--->y(1.2)'=0.045567

****