Hypothesis Testing

Q9 Given $H_0: \mu \geq 20$ (lower-Tail)
$H_a: \mu < 20$

$n = 50$ mean $= 19.4$ $\sigma = 2$

(a) $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$ $\qquad \sigma_{\bar{x}} = \sigma / \sqrt{n}$

$= 2 / 7.0711$

$\sigma_{\bar{x}} = 0.2828$

$z = \dfrac{19.4 - 20}{0.2828}$

$\boxed{z = -2.12}$

$\therefore P(z \geq -2.12) = 0.0170$

(b) ~~Given~~ α = p-value = ?

$P(z \geq -2.12) = 0.0170$

(c) Given $\alpha = 0.05$

To accept the Testing p-value should be

p-value $\leq \alpha$

$0.0170 \not\leq \alpha$

So, the Testing is Rejected.

(d) Reject H_0 if p-value $< \alpha$, so Rejected.

(10) Given $H_0: \mu \leq 25$ (Upper)
$H_a: \mu > 25$

$n = 40$ $\bar{x} = 26.4$ $\sigma = 6$ $\sigma_{\bar{x}} = \sigma / \sqrt{n}$
$= 6 / \sqrt{40}$
$\sigma_{\bar{x}} = 0.9487$

(a) $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{26.4 - 25}{0.9487}$

$z = 1.4757$. consider 1.48

(b) p-value.

$P(z \geq 1.48) = 1 - 0.9306 = 0.0694.$

p-value $= 0.0694.$

(c) Given $\alpha = 0.01$

p-value $\leq \alpha$ Accepted

So, p-value > 0.01 So donot reject H_0

(d) Reject H_0 if p-value $\leq \alpha$ but p-value > 0.01
so do not reject H_0.

(11) Given $H_0: \mu = 15$
$H_a: \mu \neq 15$

$n = 50$ $\bar{x} = 14.15$ $\sigma = 3$ $\sigma_{\bar{x}} = \sigma / \sqrt{n}$
$= 3 / \sqrt{50}$
$\sigma_{\bar{x}} = 0.4243$

(a) $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{14.15 - 15}{0.4243}$

$\boxed{z = -2}$

(b) p-value $P(z > -2)$

For $\mu = 15$ double p-value.

$2\,P(z > -2) = 2\,(0.0228)$

p-value $= 0.0456$

(c) $\alpha = 0.05$ p-value $< \alpha$ So, Reject.

(d) As p-value $< \alpha$, H_0 is Rejected.

(12) Given $H_0: \mu \geq 80$ (Lower-Tail)
$H_a: \mu < 80$

$n = 100 \quad \sigma = 12 \quad \alpha = 0.01$

(a) $\bar{x} = 78.5 \qquad \sigma_{\bar{x}} = \sigma / \sqrt{n} = 12 / \sqrt{100}$

$\sigma_{\bar{x}} = 1.2$

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{78.5 - 80}{1.2}$$

$$\boxed{z = -1.25}$$

p-value $= P(z \geq -1.25) = 0.1056$.
Here p-value > 0.01 so not rejected.

(b) $\bar{x} = 77 \qquad z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{77 - 80}{1.2} = -2.5$

$$\boxed{z = -2.5}$$

p-value $= P(z \geq -2.5) = 0.0062$
p-value ≤ 0.01, so H_0 is Rejected.

(c) $\bar{x} = 75.5 \quad z = \frac{75.5 - 80}{1.2} = -3.75$

p-value $= P(z \geq -3.75) \approx 0$ H_0 Rejected.

(d) $\bar{x} = 81 \quad z = \frac{81 - 80}{1.2} = 0.83$

$\boxed{z = 0.83}$ p-value $= P(z \geq 0.83) = 0.7967$
H_0 Not Rejected

(13) $H_0: \mu \leq 50$ (Upper-Tail)
$H_a: \mu > 50$

$n = 60 \quad \sigma = 8 \quad \sigma_{\bar{x}} = \sigma / \sqrt{n} = 8/\sqrt{60} = 1.0328$
$\alpha = 0.05$

(a) $\bar{x} = 52.5 \quad z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{52.5 - 50}{1.0328}$
$= 2.42$

$\boxed{z = 2.42}$

p-value: $P(z > 2.42) = 1 - 0.9922$
$= 0.0078$
p-value ≤ 0.05 so H_0 Rejected.

(b) $\bar{x} = 51 \quad z = \dfrac{51 - 50}{1.0328} = 0.9682$

$\boxed{z = 0.97}$

p-value: $P(z > 0.97) = 1 - 0.8340$
p-value $= 0.1660$
p-value > 0.05 so, H_0 is not Rejected.

(c) $\bar{x} = 51.8 \quad z = \dfrac{51.8 - 50}{1.0328} = 1.7428$

$\boxed{z = 1.74}$

p-value: $P(z > 1.74) = 1 - 0.9591 = 0.0409$
p-value $\leq \alpha$ so, H_0 is Rejected.

(14) Given $H_0 : \mu = 22$
$H_a : \mu \neq 22$.

$n = 75$ $\sigma = 10$ $\alpha = 0.01$ $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{75} = 1.1547$
$\mu_0 = 22$

(a) $\bar{x} = 23$ $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{23 - 22}{1.1547}$

$$\boxed{z = 0.87}$$

For $\mu_0 = 22$ p-value $= 2P(z \geq 0.87) = 2(0.1922)$
p-value $= 0.3844$

p-value > 0.01 so H_0 is not Rejected.

(b) $\bar{x} = 25.1$ $z = \dfrac{25.1 - 22}{1.1547} = 2.68$

p-value $= 2P(z > 2.68) = 2(0.0037)$
$= 0.0074$
p-value ≤ 0.01 so H_0 is Rejected.

(c) $\bar{x} = 20$ $z = \dfrac{20 - 22}{1.1547} = -1.73$

p-value $= 2P(z \geq -1.73) = 2(0.0418)$
$= 0.0836$
p-value > 0.01 so H_0 is not Rejected.

vision

type 2 diabetes mellitus.

Ghar Mefah. 2011;13:258-267.