COSM

Poisson Probability Distribution.

- The Poisson probability distribution is often used to model random arrivals in waiting line situations.

- A discrete random variable is often useful in estimating the number of occurrences over a specified interval of time or space.

Properties of a Poisson Experiment.

1. The probability of an occurrence is the same for any two interval of equal length.

2. The occurrence or nonoccurrence in any interval is independent of the occurrence or nonoccurrence in any other interval.

Poisson Probability Function.

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

; $f(x)$ = prob of x occurrence in an interval.

μ = expected value or mean no. of occurrence

$e = 2.71828$

39. Consider a Poisson distribution with a mean of two occurrences per time period.

→ Given: $\mu = 2$ per time period.

(a) Write the appropriate Poisson probability function.

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

; $f(x)$ = prob of x occurrences in an interval.

μ = expected value or mean no. of occurrences

$e = 2.71828$.

$$= \frac{2^x e^{-2}}{x!}$$

(b) What is expected number of occurrences in three time periods?

→ Expected no. of occurrences in three time periods is 6.

(c) Write the appropriate Poisson probability function to determine the probability of x occurrences in three time periods.

→ $$f(x) = \frac{6^x e^{-6}}{x!}$$

$\mu = 6$

(d) Compute the probability of two occurrences in one time period.

→ Given: $\mu = 2$
$x = 2$

$$f(2) = \frac{2^2 e^{-2}}{2!} = \frac{4(0.1353)}{2} = \underline{0.2706}$$

(e) Compute the probability of six occurrences in three time periods.

→ Given: For three time periods $\mu = 6$
$x = 6$.

$$f(6) = \frac{6^6 e^{-6}}{6!} = \frac{46,656(0.0025)}{720} = \underline{0.1606}$$

(f) Compute the probability of five occurrences in two time periods.

→ Given: $\mu = 4$
$x = 5$

$$f(5) = \frac{4^5 e^{-4}}{5!} = \frac{1024(0.0183)}{120} = \underline{0.1563}$$

(40) Phone calls arrive at the rate of 48 per hour at the reservation desk for Regional Airways.

→ Given: $\mu = 48$ per hour.

(a) compute the probability of receiving three calls in a 5-minute interval of time.

→ For 5-minutes $\mu = 4$
$x = 3$

$$f(3) = \frac{4^3 e^{-4}}{3!} = \frac{64(0.0183)}{6} = \underline{0.1952}$$

(b) compute the probability of receiving exactly 10 calls in 15 minutes.

→ For 15-minutes $\mu = 12$
$x = 10$

$$f(10) = \frac{12^{10} e^{-12}}{10!} = \underline{0.1048}$$

(c) Suppose no calls are currently on hold. If the agent takes 5 minutes to complete the current call, how many callers do you expect to be waiting by that time? What is the probability that none will be waiting.

→ For 5-minutes $\mu = 4$
which means Regional Airways receive 4 calls every 5-minutes.

If the agent takes 5-minutes to complete the current calls rest 3 callers are in waiting.

So, 3 callers are waiting every 5-minutes
so 36 callers are waiting per hour.

Probability that none will be waiting
$x = 0 \quad \mu = 4$

$$f(0) = \frac{4^0 e^{-4}}{0!} = \underline{0.0183}$$

(d) If no calls are currently being processed, what is the probability that the agent can take 3 minutes for personal time without being interrupted by a call?

→ For 3-minutes $\mu = 2.4$

probability for personal time without being interrupted by a call.

$x = 0$

$$f(0) = \frac{2.4^0 \, e^{-2.4}}{0!}$$

$$= \underline{0.0907}$$

(43) Airline passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.

→ Given : $\mu = 10$ per minute.

(a) Compute the probability of no arrivals in a one-minute period.

→ Given: $\mu = 10$ per minute
$x = 0$

$$f(0) = \frac{10^0 e^{-10}}{0!} = 0.0000454.$$

(b) Compute the probability that three or fewer passengers arrive in a one-minute period.

→ $\mu = 10$ per minute

$$f(1) = \frac{10^1 e^{-10}}{1!} = 0.000454$$

$$f(2) = \frac{10^2 e^{-10}}{2!} = 0.0023$$

$$f(3) = \frac{10^3 e^{-10}}{3!} = 0.0076$$

$$P(x \leq 3) = f(0) + f(1) + f(2) + f(3)$$

$$= 0.0000454 + 0.000454 + 0.0023 + 0.0076$$

$$P(x \leq 3) = \underline{0.0104}$$

(c) compute the probability of no arrivals in a 15-second period

→ For 15-seconds $\mu = 2.5$
$x = 0$

$$f(0) = \frac{2.5^0 e^{-2.5}}{0!} = \underline{0.0821}$$

(d) compute the probability of at least one arrival in a 15-second period.

→ $P(x \geq 1) = 1 - f(0)$

$$= 1 - 0.0821$$

$$P(x \geq 1) = \underline{0.9179}$$