Assignment-4

Page No.:
Date:

Ordinary Differential Equation-II

Q1 A ball at 1200k is allowed cool down in air at an ambient temperature of 300k. Assuming heat is lost only due to radiation the differential equation for temperature of the ball is given by $\frac{d\theta}{dt} = -2.2067 \times 10^{-12} (\theta^4 - 81 \times 10^8)$ $\theta(0) = 1200k$

when θ is in k and t in seconds. Find the temperature at $t = 480$ seconds using Runge kutta 2^{nd} order method. Assume a step size of $h = 240$ seconds

Ans Runge kutta formulas of order 2 is $\frac{dy}{dx} = f(x, y(x)), y(x_0) = y_0$

$a \le x \le b$

$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$ with

$k_0 = h f(x_i, y_i)$

$k_1 = h f(x_i + h, y_i + k_0)$ for $i = 0, 1, 2, \ldots n.$

Page No.:
Date:

=> Here we have $\frac{d\theta}{dt} = f(t, \theta(t))$

$= -2.2067 \times 10^{-12} (\theta^4 - 81 \times 10^8)$,

$t_0 = 0$

$\theta_0 = \theta(0) = 1200k$, $0 \le t \le 480$,

$h = 240$ sec.

let $t_1 = 240$, $t_2 = 480$

$\therefore \theta_1 = \theta_0 + \frac{1}{2}(k_0 + k_1)$ — ①

$k_0 = 240 \times f(t_0, \theta_0) = 240 \times f(0, 1200)$

$= 240 \times (-2.2067 \times 10^{-12} \times (1200)^4 - 81 \times 10^8))$

$k_0 = -1093.9033$ — ②

$k_1 = h \times f(t_0 + h, \theta_0 + k_0)$

$= 240 \times f(240, 1200 + (-1093.9033))$

$= 240 \times f(240, 106.104)$

$= 240 \times (-2.2067 \times 10^{-12} \times (106.104)^4 - 81 \times 10^8)$

$k_1 = 4.2227$ —— ②

Substituting eq ①, ② & ③

$\theta_1 = 1200 + \frac{1}{2}(-1093.905 + 4.2227)$

$\theta_1 = 655.1587$ —— ④

→ Now $t_2 = 480$ sec

$\theta_2 = \theta_1 + \frac{1}{2}(k_0 + k_1)$

$k_0 = hf(t_1, \theta_1)$ ①

$= 240 \times f(240, 655.1587)$

$k_0 = -93.2856$

$k_1 = hf(t_1 + h, \theta_1 + k_0)$

$= 240 \times f(480, (655.1587 + (-93.2856))$

$= 240 \times (-2.2007 \times 10^{-12}(561.8731^4 - 81 \times 10^8)$

$k_1 = -48.4948$

$\theta_2 = 655.1587 + \frac{1}{2}(-93.2856 - 48.4948)$

$\theta_2 = 584.2685\ k$ ——— (3)

Date:

Q2 Use Taylor's method of order 2 to approximate the solutions of each of the following Problems.

(a) $\frac{dy}{dx} = y - x^2 + 1$, $y(0) = 0.5$, $h = 0.5$

Ans Here $\frac{dy}{dx} = y' = y - x^2 + 1$, $x_0 = 0$, $y_0 = 0.5$, $0 \le x \le 2$, $h = 0.5$

$\rightarrow$ at $x_1 = 0.5$ By Taylor's method of order 2

$$y_{i+1} \simeq y_i + h y_i' + \frac{h^2}{2!} y_i''$$

Here $y' = y - x^2 + 1$

$$\therefore y'' = \frac{d y'}{dx} = \frac{d}{dx}(y - x^2 + 1) = -2x$$

$$\therefore y_1 = y_0 + h y' + \frac{h^2}{2!} y_0''$$

$$= 0.5 + 0.5(y_0 - x_0^2 + 1) + \frac{(0.5)^2}{2!}(-2x_0)$$

Page No.:
Date:

$= 0.5 + 0.5(0.5 - 0 + 1) + \frac{(0.5)^2}{2!}$

$(-2x_0)$

$y_1 = 1.25$ —— ①

$\rightarrow a + x_2 = 1$

$y_2 = y_1 + hy_1' + \frac{h^2}{2!} y_1''$

$= 1.25 + 0.5(y_1 - x_1^2 + 1) +$

$\frac{(0.5)^2}{2!}(-2x_1)$

$= 1.25 + 0.5(1.25 - (0.5)^2 + 1)$

$+ \frac{0.25}{2}(-2.(0.5))$

$y_2 = 2.125$ —— ②

$\rightarrow a + x_3 = 1.50$

$y_3 = y_2 + hy_2' + \frac{h^2}{2!} y_2''$

$= 2.125 + 0.5\,(y_2 - x_2^2 + 1)\,\frac{(0.5)^2}{2!}\,(-2x_2)$

$= 2.125 + 0.5\,(2.125 - 1^2 + 1) + \frac{0.25}{2}$

$(-2(1))$

$y_3 = 2.9375$ ——— (3)

at $x_4 = 1$

$y_4 = y_3 + h\,y_3' + \frac{h^2}{2!}\,y_3''$

$= 2.9375 + 0.5\,(y_3 - x_3^2 + 1) + \frac{(0.5)^2}{2!}$

$(-2x_3)$

$= 2.9375 + 0.5\,(2.9375 - (1.5)^2 + 1)$

$+ \frac{0.25}{2}\,(-2(1.5))$

$\boxed{y_4 = 3.40625}$

Page No.:
Date:

(b) $\frac{dy}{dt} = \cos 2t + \sin 3t, \quad 0 \le t \le 1,$
$y(0) = 1, \; h = 0.25$

Ans Here $\frac{dy}{dt} = y' = \cos 2t + \sin 3t,$

$y_0 = 1, \; t_0 = 0, \; h = 0.25$

$y' = \cos 2t + \sin 3t$
$y'' = -2 \sin 2t + 3 \cos 3t$

→ at $t_1 = 0.25$, using taylor method of order 2.

$$y_1 = y_0 + h y' + \frac{h^2}{2!} y_0''$$

$$= 1 + 0.25(\cos 0 + \sin 0) + \frac{0.0625}{2}(-2\sin 0 + 3\cos 0)$$

$y_1 = 1.34375$ ——— ①

→ at $t_2 = 0.5$

$$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$$

Page No.:
Date:

$= 1.34373 + 0.25(\cos 0.5 + \sin 0.75)$

$+ \frac{0.0625}{2}(-2\sin 0.5 + 3\cos 0.75)$

$y_2 = 1.7722$ —— (2)

$\rightarrow$ at $t_3 = 0.75$

$y_3 = 1.7722 + (0.21)(\cos 1 + \sin 1.5) + \frac{0.062}{2})(-2\sin 1 + 3\cos(1.5))$

$y_3 = 2.1107$ —— (3)

$a + t_4 = 1$

$y_4 = 2.1107 + (0.25)(\cos 1.5 + \sin(2.25) + \frac{0.0625}{2}(-2\sin 1.5 + 3\cos 2.25)$

$\boxed{y_4 = 2.20167}$

Page No.:
Date:

(i) $y' = \frac{1+t}{1+y} \quad 1 \le t \le 2, \; y(1) = 2$

$h = 0.5$

Ans Here

$t_0 = 1, \; y_0 = 2, \; 1 \le t \le 2,$

$h = 0.5$

$y' = \frac{1+t}{1+y}, \; y'' = \frac{1}{1+y}\frac{d}{dt}(1+t) = \frac{1}{1+y}$

using Taylor method of order

at $t_1 = 1.5$

$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0''$

$= 2 + 0.5\left(\frac{1+t_0}{1+y_0}\right) + \frac{(0.5)^2}{2}\left(\frac{1}{1+y_0}\right)$

$= 2 + 0.5\left(\frac{1+1}{1+2}\right) + \frac{0.25}{2}\left(\frac{1}{1+2}\right)$

$y_1 = 2.375$

$\rightarrow$ at $t_2 = 2$

$$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$$

$$= 2.375 + 0.5 \left(\frac{1 + 1.5}{1 + 2.375} \right) + 0.125$$

$$\left(\frac{1}{1 + 2.375} \right)$$

$$\boxed{y_2 = 2.7824}$$

Page No.:
Date:

(d) $y' = 1 + \frac{y}{t}$, $y(1) = 1$, $h = 0.25$
$1 \le t \le 1.5$

Ans here, $t_0 = 1$, $y_0 = 1$, $h = 0.25$,
$1 \le t' \le 1.5$

$y' = 1 + \frac{y}{t}$, $y'' = y \frac{d}{dt}\left(\frac{1}{t}\right) =$
$y \frac{d}{yt}(t^{-1})$

$\therefore y'' = \frac{-y}{t^2}$

at $t_1 = 1.25$

$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0''$

$= 1 + (0.25)\left(1 + \frac{y_0}{t_0}\right) +$
$\frac{0.25^2}{2}\left(\frac{-y_0}{t_0^2}\right)$

$= 1 + 0.25\left(1 + \frac{1}{1}\right) + \frac{0.0625}{2}$
$\left(\frac{-1}{1}\right)$

$y_1 = 1.46875$ —— (1)

Page No.:
Date:

at $t_2 = 1.50$

$$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$$

$$= 1.46875 + 0.25 \times \left(1 + \frac{1.46875}{1.25}\right)$$

$$+ 0.03125 \left(- \frac{1.46875}{(1.25)^2}\right)$$

$$\boxed{y_2 = 1.983125} \quad \text{(2)}$$

Page No.:
Date:

(e) $y' = xe^{y+x} - 1$, $y(0) = -1$, $h = 0.5$
$0 \leq x \leq 1$

Ans here
$y_0 = -1$, $x_0 = 0$, $h = 0.5$,
$0 \leq x \leq 1$

$y' = xe^{y+x} - 1$, $y'' = x \frac{d}{dx}(e^{y+x})$
$+ e^{y+x} \frac{d}{dx} x$

$y'' = xe^{y+x} + e^{y+x}$

$\rightarrow$ at $x_1 = 0.5$

$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0''$

$= -1 + 0.5(x_0 \times e^{y_0 + x_0} - 1)$
$+ \frac{0.5^2}{2} (x_0 e^{y_0 + x_0} + e^{y_0 + x_0})$

$\therefore y_1 = -1 + 0.5(0 \times e^{-1+0} - 1)$
$+ \frac{0.25}{2} (0 \times e^{-1+0} + e^{-1+0})$

Page No.:
Date:

$y_1 = -1.4540$ —— (1)

$\rightarrow a + x_2 = 1$

$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1''$

$= -1.4540 + 0.5 (x_1 e^{y_1 + x_1} - 1)$

$+ \frac{0.25}{2} (x_1 e^{y_1 + x_1} + e^{y_1 + x_1})$

$\boxed{y_2 = 1.7855}$

Q3 Compare the performance of Euler's method, second order Taylor's method, RK of order 2 and RK order 4 for the following differential eq.

Ans Here $\frac{dy}{dx} = \frac{x}{y}$, $h = 0.5$

$y_0 = 1$, $x_0 = 0$, $0 \le x \le 2$

⇒ with Euler's method

$y_{i+1} = y_i + h\, f(x_i, y_i)$
$y_{i+1} = y_i + 0.5\, (x_i / y_i)$

→ at $x_0 = 0$, $y_0 = 1$
→ at $x_1 = 0.5$
$y_1 = y_0 + 0.5 \left(\frac{x_0}{y_0}\right) =$
$1 + 0.5 \left(\frac{0}{1}\right) = 1$

→ at $x_2 = 1$
$y_2 = y_1 + 0.5 \left(\frac{x_1}{y_1}\right)$
$= 1 + 0.5 \left(\frac{0.5}{1}\right)$
$= 1.25$

→ at $x_3 = 1.5$
$y_3 = y_2 + 0.5 \left(\frac{x_2}{y_2}\right)$
$= 1.25 + 0.5 \left(\frac{1}{1.25}\right)$
$= 1.65$

Page No.:
Date:

$\rightarrow$ at $x_4 = 2$

$$y_4 = y_3 + 0.5\left(\frac{x_3}{y_3}\right)$$
$$= 1.65 + 0.5\left(\frac{1.5}{1.65}\right)$$
$$= 2.1045$$

$\Rightarrow$ Second order Taylor's method.

here $y' = x/y$, $y'' = \frac{1}{y}$, $x_0 = 0$, $y_0 = 1$
$h = 0.5$, $0 \le x \le 2$

$$y_{i+1} = y_i + h y_i' + \frac{h^2}{2!} y_i''$$

$\rightarrow$ at $x_1 = 0.5$

$$y_1 = y_0 + h\left(\frac{x_0}{y_0}\right) + \frac{h^2}{2!}\left(\frac{1}{y}\right)$$
$$= 1 + 0.5\left(\frac{0}{1}\right) + \frac{(0.5)^2}{2!}\left(\frac{1}{1}\right)$$

$$= 1.125$$

$\rightarrow$ at $x_2 = 1$

$$y_2 = 1.125 + 0.5\left(\frac{0.5}{1.125}\right) +$$
$$0.125\left(\frac{1}{1.125}\right)$$
$$= 1.4583$$

$\rightarrow$ at $x_3 = 1.5$

$$y_3 = 1.4583 + 0.5\left(\frac{1}{1.4583}\right)$$
$$+ 0.125\left(\frac{1}{1.4583}\right) = 1.8869$$

$\rightarrow a + x_4 = 2$

$$y_4 = 1.8869 + 0.5\left(\frac{1.50}{1.8869}\right) h$$

$$+ 0.125\left(\frac{1}{1.8883}\right) = 2.3506$$

$\Rightarrow$ (3) RK of order 2

$$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$$
$$k_0 = h f(x_i, y_i)$$
$$k_1 = h f(x_i + h, y_i + k)$$

$\rightarrow a + x_1 = 0.5$

$$y_1 = y_0 + \frac{1}{2}(k_0 + k_1)$$
$$k_0 = 0.5\left(\frac{x_0}{y_0}\right) = 0.5\left(\frac{0}{1}\right) = 0$$
$$k_1 = 0.5 f(x_0 + 0.5, y_0 + 0)$$
$$= 0.5 f(0.5, 1)$$
$$k_1 = 0.5(0.5/1) = 0.25$$
$$y_1 = 1 + \frac{1}{2}(0 + 0.25) = 1.125$$

$\rightarrow a + x_2 = 1$

$$k_0 = 0.5(0.5/1.125) = 0.2$$
$$k_1 = 0.5 f(0.5 + 0.5,$$
$$1.125 + 0.2)$$
$$= 0.5 f(1, 1.325)$$

$k_1 = 0.3774$

$y_2 = 1.125 + \frac{1}{2}(0.2 + 0.3774)$

$= 1.4137$

$\rightarrow$ at $x_3 = 1.5$

$k_0 = 0.5\left(\frac{1}{1.4137}\right)$

$= 0.3532$

$k_1 = 0.5 f(1.5, 1.7674)$

$= 0.4244$

$y_3 = 1.4137 + \frac{1}{2}(0.3537 + 0.4244) = 1.80275$

$\rightarrow$ at $x_4 = 2$

$k_0 = 0.5\left(\frac{1.5}{1.8028}\right) = 0.4162$

$k_1 = 0.5 f(2, 2.1565) = 0.4037$

$y_4 = 1.8028 + \frac{1}{2}(0.4162 + 0.4037)$

$= 2.2428$

$\rightarrow$ Table of comparision

x	Euler	Taylor	RK2	RK4	Exact
0	1	1	1	1	1
0.5	1	1.125	1.125	1.10053	1.1180
1	1.25	1.4588	1.4137	1.3	1.4142
1.5	1.65	1.8869	1.8028	1.8341	1.8028
2	2.1045	2.3806	2.2428	2.3423	2.2361

Date:

Qn. Explain Taylor's method of order n to approximate a solution to differential equation $y'(x) = f(x, y(x))$, $a \le x \le b$, $y(a) = c$. What are its advantages? Why is it seldom used in practice for computer implementation?

Ans: The taylor's method of order n is

$$y(x) = y_i + (x - x_i) y_i' + \frac{(x - x_i)^2}{2!} y_i'' + \frac{(x - x_i)^3}{3!} y_i''' + \dots + \frac{(x - x_i)^n}{n!} y_i^{(n)}$$

using this taylor series to evaluate $y'(x) = f(x, y(x))$, $a \le x \le b$, $y(a) = c$

$$y_{i+1} = y_i + h y_i' + \frac{h^2}{2!} y_i'' + \dots + \frac{h^n}{n!} y_i^{(n)}$$

where $y^{(n)}$ is for n^{th} derivation of $y(x)$ at $x = x_i$

→ Taylor's Series is seldom used in practical for complex implementation due to requirement of derivation of expression of derivatives and evaluation of their values.

→ Taylor's Series is back bone of direct single step formulas. Euler formula and family of Runge kutta formula are derived from Taylor Series method.

Page No.:
Date:

Q5 Repeat question 2 with Runge kutta method of order 2.

Ans. Here $\frac{dy}{dx} = y - x^2 + 1$, $y(0) = 0.5$

$0 \leq x \leq 2$, $h = 0.5$

$\therefore y_0 = 0.5$, $x_0 = 0$

→ RK method of order 2 is

$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$

$k_0 = hf(x_i, y_i)$,

$k_1 = hf(x_i + h, y_i + k_0)$

→ at $x_1 = 0.5$

$y_1 = y_0 + \frac{1}{2}(k_0 + k_1)$

$k_0 = 0.5 + f(x_0, y_0)$

$= 0.5 \times (0.5 - 0^2 + 1)$

$k_0 = 0.75$

$k_1 = 0.5 \times f(x_0 + 0.5, y_0 + 0.75)$

$= 0.5 \times f(0.5, 1.25)$

$k_1 = 0.5 (1.25 - 0.5^2 + 1)$

$= 1$

$y_1 = 0.5 + \frac{1}{2}(0.75 + 1)$

$= 1.375$

Page No.:
Date:

$\rightarrow$ at $x_2 = 1$

$k_0 = 0.5\,(1.375 - (0.5)^2 + 1$

$= 1.0625$

$k_1 = 0.5\, f(0.5 + 0.5,\ 1.375 + 1.0625)$

$= 0.5\, f(1,\ 2.4375)$

$k_1 = 1.2188$

$y_2 = 1.375 + \frac{1}{2}(1.0625 + 1.2188)$

$= 2.5156$

$\rightarrow$ at $x_3 = 1.5$

$k_0 = 0.5\,(2.5156)$

$= 1.2578$

$k_1 = 0.5\,(2.5234)$

$= 1.2617$

$y_3 = 2.5156 + \frac{1}{2}(1.2578 + 1.2617)$

$= 3.7754$

$\rightarrow$ at $x_4 = 2$

$k_0 = 1.2627$

$k_1 = 1.0492$

$y_4 = 4.9163$

Page No.
Date

(b) $\frac{dy}{dt} = \cos 2t + \sin 3t, \; 0 \le t \le 1$
$y(0) = 1, \; h = 0.25$

RK method of order 2 is

$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$

$k_0 = hf(x_i, y_i) = hf(t_i, y_i)$
$k_1 = hf(x_i + h, y_i + k_0)$
$= hf(t_i + h, y_i + k_0)$

→ at point $t_0 = 0, \; y_0 = 1$

→ at point $t_1 = 0.25$

$k_0 = 0.25 f(0, 1) = 0.25(1)$
$= 0.25$
$k_1 = 0.25 f(0 + 0.25, 1 + 0.25)$
$= 0.25 \times (\cos(2 \times 0.25) +$
$\sin(3 \times 0.25))$
$k_1 = 0.3898$
$y_1 = 1 + \frac{1}{2}(0.25 + 0.3898)$
$y_1 = 1.3199$

→ at point $t_2 = 0.50$
$k_0 = 0.25 \, f(0.25, 1.3199)$
$= 0.3898$
$k_1 = 0.25 \times (\cos(2 \times 0.50)$
$+ \sin 3(0.50))$

Page No.:
Date:

$k_1 = 0.3845$
$y_2 = 1.3199 + \frac{1}{2}(0.3898 + 0.3845)$
$= 1.7071$

→ at point $t_3 = 0.75$

$k_0 = 0.25 ((\cos 2(0.50) + \sin 3(0.50))$
$k_0 = 0.3845$
$k_1 = 0.25 \times (\cos(2 \times 0.75) + \sin(3 \times 0.75))$
$k_1 = 0.2122$
$y_3 = 1.7071 + \frac{1}{2}(0.3845 + 0.2122)$
$= 2.0055$

→ at point $t_4 = 1$

$k_0 = 0.25 \times (\cos(2 \times 0.75) + \sin(3 \times 0.75))$
$k_0 = 0.2122$
$k_1 = 0.25 \times (\cos(2 \times 1) + \sin(3 \times 1))$
$k_1 = -0.0688$
$y_4 = 2.0055 + \frac{1}{2}(0.2122 - 0.0688)$
$y_4 = 2.0772$

Date:

(c) $y' = \frac{1+t}{1+y}, \quad 1 \le t \le 2, \quad y(1) = 0$

$h = 0.5$

RK method of 2

Ans

$y_{i+1} = y_i + \frac{1}{2}(k_0 + k_1)$

$k_0 = hf(t_i, y_i)$

$k_1 = hf(t_i + h, y_i + k_0)$

at point $t_0 = 1$, $y_0 = 2$

→ at point $t_1 = 1.5$

$k_0 = 0.5 f(1, 2) = 0.5 \times \left(\frac{1+1}{1+2}\right) = 0.3$

$k_1 = 0.5 \times f(1 + 0.5, 2 + 0.3)$
$= 0.378$

$y_1 = 2 + \frac{1}{2}(0.3 + 0.378)$
$= 2.339$

→ at point $t_2 = 2$

$k_0 = 0.5 \, f(1.5, 2.339)$
$= 0.3744$

$$k_1 = 0.5\, f(2,\ 2.339 + 0.3744)$$
$$= 0.4039$$

$$y_2 = 2.339 + \frac{1}{2}(0.3744 + 0.4039)$$

$$\boxed{y_2 = 2.7282}$$

Page No.:
Date:

Q.6. Repeat question 2 with Runge kutta method of order 4.

Ans. RK method of order 4 is

$$y_{i+1} = y_i + \frac{1}{6}(k_0 + 2k_1 + 2k_2 + k_3)$$

$k_0 = hf(x_i, y_i)$
$k_1 = hf(x_i + \frac{h}{2}, y_i + \frac{k_0}{2})$
$k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$
$k_3 = hf(x_i + h, y_i + k_2)$

→ here $f(x, y) = y - x^2 + 1$, $y(0) = 0.5$
$0 \le x \le 2$, $h = 0.5$

→ at point $x_0 = 0$, $y_0 = 0.5$

→ At point $x_1 = 0.5$

$k_0 = 0.5\, f(0, 0.5) = 0.75$
$k_1 = 0.5\, f(0 + \frac{0.5}{2}, 0.5 + \frac{0.75}{2})$
$= 0.90625$

$k_2 = 0.5\, f(0.25, 0.5 + \frac{0.90625}{2})$

$= 0.9453$

$k_3 = 0.5\, f(0.5,\ 0.5 + 0.9453)$
$= 1.09765$

$y_1 = 0.5 + \frac{1}{6}(0.93 + 2(0.90625) + 2 \times (0.9453) + 1.09765)$

$\boxed{y_1 = 1.4251}$

→ At $x_2 = 1$

$k_0 = 0.5\, f(0.5, 1.4251)$
$= 1.08755$
$k_1 = 0.5\, f(0.75, 1.9689)$
$= 1.2082$
$k_2 = 0.5\, f(0.75, 2.0267)$
$= 1.2321$
$k_3 = 0.5\, f(1, 2.6572)$
$= 1.3286$
$y_2 = 1.4251 + \frac{1}{6}(1.08755 + 2(1.2032) + 2(1.2321) + 1.3286)$

$\boxed{y_2 = 2.6396}$

Date:

$\rightarrow$ At point $x_3 = 1.5$

$k_0 = 0.5 f(1, 2.6396)$
$= 1.3198$
$k_1 = 0.5 f(1.25, 3.2995)$
$= 1.3685$
$k_2 = 0.5 f(1.25, 3.32385)$
$= 1.3807$
$k_3 = 0.5 f(1.5, 4.0203)$
$= 1.3852$
$y_3 = 2.6396 + \frac{1}{6}(1.3198 + 2(1.3685) + 2(1.3807) + 1.3852)$
$y_3 = 4.0068$

$\rightarrow$ At point $x_4 = 2$

$k_0 = 0.5 f(1.5, 4.0068)$
$= 1.3784$
$k_1 = 0.5 f(1.75, 4.696)$
$= 1.31675$
$k_2 = 0.5 f(1.75, 4.6652)$
$= 1.3014$
$k_3 = 0.5 f(2, 5.3082)$
$= 1.1541$

$y_4 = 4.0068 + \frac{1}{6}(1.3784 + 2(1.3168) + 2(1.3014) + 1.1541)$

Page No.:
Date:

$\boxed{y_4 = 5.3016}$

(b) $\frac{dy}{dt} = \cos 2t + \sin 3t, 0 \le t \le 1$
$y(0) = 1, h = 0.25$

→ At point $t_0 = 0, y_0 = 1$

→ At Point $t_1 = 0.25$

$k_0 = 0.25 f(0,1) = 0.25$
$k_1 = 0.25 f(0.125, 1.125) = 0.3338$
$k_2 = 0.25 f(0.125, 1.1669) = 0.3338$
$k_3 = 0.25 f(0.25, 1.3338) = 0.3842$

→ At Point $t_2 = 0.5$

$k_0 = 0.25 f(0.25, 1.3292)$
$= 0.3898$
$k_1 = 0.25 f(0.375, 1.5241)$
$= 0.4085$
$k_2 = 0.25 f(0.375, 1.5334)$
$= 0.4083$
$k_3 = 0.5 f(0.50, 1.7377)$
$= 0.3845$
$y_2 = 1.3292 + \frac{1}{6}(0.3898 + 2(0.4085) + 2(0.4085) + 0.3845$

$\boxed{y_2 = 1.7306}$

At Point $t_3 = 0.75$

$\rightarrow$ $k_0 = 0.25 f(0.503, 1.7306)$
$= 0.3845$

$k_1 = 0.25 f(0.625, 1.92225)$
$= 0.3174$

$k_2 = 0.25 f(0.625, 1.8892)$
$= 0.3174$

$k_3 = 0.25 f(0.75, 2.048)$
$= 0.2122$

$y_3 = 1.7306 + \frac{1}{6}(0.3854 + 2(0.3174) + 2(0.3174) + 0.2122)$

$\boxed{y_3 = 2.0418}$

$\rightarrow$ At point $t_4 = 1$

$k_0 = 0.25 f(0.75, 2.0418) = 0.2122$
$k_1 = 0.25 f(0.875, 2.1479) = 0.0789$
$k_2 = 0.25 f(0.875, 2.0813) = 0.0789$
$k_3 = 0.25 f(1, 2.1207) = -0.0688$

$y_4 = 2.0418 + \frac{1}{6}(0.2122 + 4(0.0789) + (-0.0686))$

$\boxed{y_4 = 2.1183}$ //

Page No.:
Date:

(c) $y' = \frac{1+t}{1+y}$, $1 \le t \le 2$, $y(1) = 2$

$h = 0.5$

Ans At $t_0 = 1$, $y_0 = 2$

→ At $t_1 = 1.5$

$k_0 = 0.5f(1,2) = 0.3333$
$k_1 = 0.5f(1.25, 2.1667) = 0.3553$
$k_2 = 0.5f(1.25, 2.1777) = 0.3540$
$k_3 = 0.5f(1.5, 2.354) = 0.3727$

$y_1 = 2 + \frac{1}{6}(0.3333 + 2(0.3553) + 2(0.3540) + 0.3727)$

$y_1 = 2.3541$

→ At $t_2 = 2$

$k_0 = 0.5f(1.5, 2.3541) = 0.3727$
$k_1 = 0.5f(1.75, 2.5405) = 0.38836$
$k_2 = 0.5f(1.75, 2.5483) = 0.3875$
$k_3 = 0.5f(2, 2.7416) = 0.40090$

$y_2 = 2.3541 + \frac{1}{6}(0.3727 + 2(0.3883) + 2(0.3875) + 0.4009)$

$y_2 = 2.7417$

(d) $y' = 1 + \frac{y}{t}$, $y(1) = 1$, $1 \le t \le 1.5$,

$h = 0.25$

Ans. At $t_0 = 1$, $y_0 = 1$

$\rightarrow$ At $t_1 = 1.25$

$k_0 = 0.25\, f(1, 1) = 0.5$

$k_1 = 0.25\, f(1.125) = 0.5278$

$k_2 = 0.25\, f(1.125, 12639) = 0.5309$

$k_3 = 0.25\, f(1.25, 1.5309) = 0.5562$

$y_1 = 1 + \frac{1}{6}(0.5 + 2(0.5278) + 2(0.5309)$
$+ 0.5562)$

$y_1 = 1.5289$

$\rightarrow$ At $t_2 = 1.5$

$k_0 = 0.25\, f(1.25, 1.5289) = 0.5558$

$k_1 = 0.25\, f(1.375, 1.8068) = 0.5785$

$k_2 = 0.25\, f(1.375, 1.8182) = 0.5806$

$k_3 = 0.25\, f(1.5, 2.1095) = 0.6016$

$y_2 = 1.5289 + \frac{1}{6}(0.5558 + 2(0.5785)$
$+ 2(0.5806) + 0.6016)$

$y_2 = 2.1082$

Q7 Give answer to the following in one - two sentence.

(a) What is the advantage of Rk method over Taylor series methods of the same order?

Ans Rk method and Taylor series method of the same order are equally efficient but Rk methods having advantages of no requirement of deriving the expressions of derivation.

(b) What is the order of the local truncation error in Rk method of order 2 and order 4! What is the order of global truncation error in these methods?

Ans In Rk method of order 2 the order of local truncation error is $O(h^3)$ and order of global truncation error is $O(h^2)$

→ In Rk method of order 2 of local truncation error is $O(h^5)$

Date:

and order of global truncation error is $O(h^4)$.

(c) Explain the effect of reducing step size in arriving at approximate solution of differential equation at a point.

Ans Reduction in step size yields better results, increases accuracy.

(d) What is the basic difference in method of deriving formula for ~~sing~~ single step methods and multistep method?

Ans Taylor's method is used to derive formula of single step methods while Derivation of multistep methods is based on numerical integration of interpolating polynomial fitted at previous step points.

(e) What is the main drawback of multistep method?

Ans Multistep methods are not self

starting.

Q) What are the advantages of predictor corretor methods over RK method?

Ans In predictor corrector methods only 1/2/3 function evaluation per step, depending of ~~mem~~ number of times corrector formula is applied.

→ when in RK methods 4 function evaluation evalotation per step required.

Page No.
Date
YOUVA

Q8. Consider the IVP

$y' = y - t^2 + 1$, $0 \le t \le 2$, $y(0) = 0.5$, taking $h = 0.2$, exact solution is
$y(t) = (t+1)^2 - 0.5e^t$

(a) use Milne simpson predictor corrector formula to estimate values at $y(0.8)$ & $y(1.0)$

Ans Milne simpson Predictor formula is

$y_{i+1} = y_{i-3} + \frac{4h}{3}(2f_i - f_{i-1} + 2f_{i-2})$

→ Corrector formula is
$y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i+1} + 4f_i + f_{i-1})$

now $y' = y - t^2 + 1$, $0 \le t \le 2$,
$y(0) = 0.5$, $h = 0.2$

∴ $t_0 = 0$, $t_1 = 0.2$, $t_2 = 0.4$
$t_3 = 0.6$, $t_4 = 0.8$, $t_5 = 1$

→ First we have to estimate values at $t_4 = 0.8$

Page No.:
Date:

exact solution is $y(t) = (t+1)^2 - 0.5e^t$

so by exact solution

$t_0 = 0, y_0 = 0.5$

$t_1 = 0.2, y_1 = 0.8293$

$t_2 = 0.4, y_2 = 1.2141$

$t_3 = 0.6, y_3 = 1.6489$

$t_4 = 0.8, y_4 = 2.1272$

Now by milne simpson predictor formula.

$y_4 = y_0 + 4 \times \frac{0.2}{3} (2f_3 - f_2 + 2f_1)$

$= 0.5 + 0.2667 (2f(t_3, y_3) - (t_2, y_2) + 2(t_1, y_1))$

$= 0.5 + 0.2667 (2(1.6489 - (0.6)^2 + 1) - (1.2141 - (0.4)^2 + 1) + 2(0.8293 - (0.2)^2 + 1)$

Page No.:
Date:

$y_4 = 0.5 + 0.2667(2 \times 2.2889 - 2.0541 + 2 \times 1.7893)$

$y_4 = 2.1275$

→ Now by corrector formula

$y_4 = y_2 + \frac{0.2}{3}(f_4 + 4f_3 + f_2)$

$= 1.2141 + 0.0667(f(0.8, 2.1275) + 4f(0.6, 1.6499) + f(0.4, 1.2141))$

$y_4 = 2.1277$

Again using corrector formula

$y_4 = 1.2141 + 0.0667(f(0.8, 2.1277) + 4f(0.6, 1.6489) + f(0.4, 1.2141))$

$\boxed{y_4 = 2.1277}$

Now to estimate values at $y(1.0)$ i.e. at $t_5 = 1.0$

By Milne Simpson Predictor formula

Page No.:
Date:

$y_5 = y_1 + \frac{4\times 0.2}{3}(2f_4 - f_3 + 2f_2)$

$= 0.8293 + 0.2667(2f(0.8, 2.1272) - f(0.6, 1.6489) + 2f(0.4, 1.2141))$

$y_5 = 2.5878$

Now using corrector formula

$y_5 = y_3 + \frac{0.2}{3}(f_5 + 4f_4 + f_3)$

$= 1.6489 + 0.0667(f(1, 2.5878) + 4f(0.8, 2.1272) + f(0.6, 1.6489))$

$y_5 = 2.6378$

Again,

$y_5 = 1.6489 + 0.0667(f(1, 2.6378) + 4f(0.8, 2.1272) + f(0.6, 1.6489))$

$y_5 = 2.6411$

Again

$\boxed{y_5 = 2.6413}$

Page No.:
Date:

(3) Use Adams Moulton for steps predictor corrector method to arrive at approximate solution at $y(0.8)$ & $y(1.0)$.

Ans Adams Moulton predictor formula.

$$y_{i+1} = y_i + \frac{h}{24}(55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3})$$

→ Corrector formula,

$$y_{i+1} = y_i + \frac{h}{24}(9f(x_{i+1}, y_{i+1}) + 19f_i - 5f_i + f_{i-2})$$

→ We have $y' = y - (t)^2 + 1$,
$0 \le t \le 2$, $y(0) = 0.5$, $h = 0.2$

→ Now By exact solution,

$t_0 = 0$, $y_0 = 0.5$
$t_1 = 0.2$, $y_1 = 0.8293$
$t_2 = 0.4$, $y_2 = 1.2141$
$t_3 = 0.6$, $y_3 = 1.6489$
$t_4 = 0.8$, $y_4 = 2.1272$

Page No.:
Date:

we have to find $y(0.8)$
i.e at $t_4 = 0.8$

$$y_4 = y_3 + \frac{0.2}{24}(55f_3 - 59f_2 + 37f_1 - 9f_0)$$

$y_4 = 2.1273$

→ By Corrector formula

$$y_4 = y_3 + \frac{0.2}{24}(9f(0.8, 2.1273) + 19f_3 - 5f_2 + f_1)$$

$y_4 = 2.1272$

Now to find $y(1.0)$
i.e at $t_5 = 1.0$

By Predictor method.

$$y_5 = y_4 + \frac{0.2}{24}(55f_4 - 59f_3 + 37f_2 - 9f_1)$$

$y_5 = 2.6409$.

Page No.:
Date:

By Corrector method.

$$y_5 = y_4 + \frac{0.2}{24}\left(9f(1.0, 2.6409) + 19f_4 - 5f_3 + f_2\right)$$

$$\boxed{y_5 = 2.64082}$$

(c) Compare the result obtain in (a), (b) from the exact solution by calculation error.

Ans

Milne simpson	Exact	Error
y(0.8) 2.1277	2.1272	0.0005
y(1.0) 2.6413	2.6409	0.0323

Adams-moulton	Exact	Error
y(0.8) 2.1272	2.1272	0.0
y(1.0) 2.6408	2.6409	0.0001