Assignment - 3

Solution of ordinary

Numerical methods

Q1 Define a differential equation. What is ment by solution of a differential equation? verify that

(i) $y = c_1 e^x + c_2 e^x x$ is a solution of the differential $y'' - 2y' + y = 0$

(ii) $y = 2\sqrt{c - x}$ is a solution of the differential equation $y + 1/y = 0$

Ans Define a differential eq.

→ A differential equation is an equation involving independent variable, dependent variable and its one or more derivatives

ex

$$y'' + 4y = 0$$

Here y is dependent variable, y is function of x

⇒ Solution of differential equation

A solution of a differential equation is a specific function that satisfies the equation.

(i) Her differential equation is

$y'' - 2y' + y = 0$ —— ⓐ

let $y(x) = c_1 e^x + c_2 e^x x$ —— ①

$y'(x) = c_1 e^x + c_2 \frac{d}{dx}(x e^x)$

$= c_1 e^x + c_2 \left(e^x x \frac{d}{dx} + e^x \frac{d}{dx} x\right)$

$= c_1 e^x + c_2 (x e^x + e^x)$

$y'(x) = c_1 e^x + c_2 e^x + c_2 x e^x$ —— ②

$y''(x) = c_1 e^x + c_2 e^x + c_2 (e^x + x e^x)$

$y''(x) = c_1 e^x + 2 c_2 e^x + c_2 x e^x$ —— ③

subsitituting ① ② ③ in ⓐ,

$0 = c_1 e^x + 2c_2 e^x + c_2 x e^x - 2(c_1 e^x + c_2 e^x + c_2 x e^x) + c_1 e^x + c_2 x e^x$

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$= 2c_1e^x + 2c_2e^x + 2c_2xe^x$

$- 2c_1e^x - 2c_2e^x - 2c_2xe^x$

$= 0$

$\therefore y = c_1e^x + c_2xe^x$ is a solution of $y'' - 2y' + y = 0$

(ii) Here differential ~~between~~ equation is $y' + \frac{1}{y} = 0$

let $y = 2\sqrt{c-x} = 2(c-x)^{\frac{1}{2}}$ — (1)

$y' = 2 \times \frac{1}{2}(c-x)^{\frac{1}{2}-1} \times \frac{d}{dx}(c-x)$

$= (c-x)^{-\frac{1}{2}} \times -1 = \frac{-1}{\sqrt{c-x}}$ — (2)

(a) $\Rightarrow \frac{-1}{\sqrt{c-x}} + \frac{1}{2\sqrt{c-x}} \neq 0$

$\therefore y = 2\sqrt{c-x}$ is not a solution of $y' + \frac{1}{y} = 0$

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Q2 how is ordinary differential equation different from Partial differential equation? Give one example of each.

Ans.

When dependent variable is a function of only one variable, then all the derivatives involved in the equation are ordinary and the equation is called ordinary differential equation.

- if dependent variable is a function of more than one independent variable. That is, it is a function of several variables, then the equation contains partial derivation with respect to different independent variables and the equation is called Partial Differential Equation

⇒ Example of ordinary

$y'' + 4y = 0$

$y' + \frac{1}{y} = 0$

⇒ Example of partial

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

(Laplace's Eq)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$$

(Poisson's Eq)

Q3 Differentiate between initial value Problem and Boundary value problem. classify following differential equation in initial value problem and Boundary value problem.

Ans.

→ If the condition are specified at a single point these condition are called intial condition and the differential eq combined with conditions is called initial value problem.

→ If the condition are specified at more than one Point these condition are Called Boundary condition and the Differential Eq Combined with conditions is Called Boundary value Problem

(i) $y' = t - y \,,\; y(0) = 1$

→ Initial Value Problem

(ii) $y'' + 9y = e^x \sin x, \; y(0) = 1, \; y(1) = 2$

→ Boundary Value ~~Point~~ problem

(iii) $y'' + y = 0, \; y(0) = 0, \; y'(0) = L$

→ Boundary value ~~Point~~ problem

(iv) $y'' + y' = t^2 + y^2, \; y(0) = 1,$
$y'(0) = 1$

→ Boundary Value ~~point~~ Problem

(V) $y' = t^2 + y^2, \; y(0) = 1.$

→ Initial Value problem

Q4 Determine order and degree of following differential eq.

Ans (i) $y'' + 4y = e^x$

→ order = 2, degree = 1

(ii) $y'' + 4(y')^3 + y^2 = x^3 + x^2$

→ order = 2, degree = 1

(iii) $(y'')^2 + (y')^3 + 3y = 5x$

→ order = 2, degree = 2

(iv) $y' + 2y^2 = x^2$

→ order = 1, degree = 1

Q5 What are the characteristics of single step numerical methods to find solution of first order, first degree IVP?

Ans These are the characteristics of Single step numerical methods.

1. It is direct
2. It is non iterative
3. It is based on Taylor series method.
4. It is used estimating at previous step & function information.
5. Practically, error cannot be estimated.

Sanjana J. Narmadar

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Q6 Differentiate between single step and multistep numerical methods to obtain solution of IVP.

Ans

Single step methods:

→ Direct

→ Not iterative

→ Based on Taylor Series methods

→ Self starting.

→ Practically, accuracy cannot be estimated.

Multistep method:

→ More than one previous step

→ Iterative.

→ Predictor connector formulas.

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→ Not self starting

→ Practically, accuracy can be estimated.