Assigment

Open type Integration formula & Gauss Quadrature formulas

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Q1 Write Gauss Legedre quarature formula for estimating $\int_{-1}^{1} f(x)\,dx$ for $n=1,2$ and 3 ($n=$ number of abscissas

Ans Gauss Legedre quadrature formula for estimating $\int_{-1}^{1} f(x)\,dx$

for $n=1$ is

$$\int_{-1}^{1} f(x)\,dx \cong 2f(0)$$

for $n=2$

$$\int_{-1}^{1} f(x)\,dx \cong f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

for $n=3$

$$\int_{-1}^{1} f(x)\,dx \cong \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0)$$

$$+ \frac{5}{9} + \left(\sqrt{\frac{3}{5}}\right)$$

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Q2 Approximate $\int_{}^{1} e^{x} \cos x \, dx$ using Gauss Legendre quadrature formula

(i) taking $n=2$

Ans The Gauss Legendre quadrature formula for $\int_{-1}^{} f(x)\,dx$ for $n=2$ is

$$\int_{-1}^{1} f(x)\,dx \simeq f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$\int_{-1}^{1} e^{x} \cos x \, dx \simeq e^{\frac{-1}{\sqrt{3}}} \cos\left(\frac{-1}{\sqrt{3}}\right) + e^{\frac{1}{\sqrt{3}}} \cos\left(\frac{1}{\sqrt{3}}\right)$$

$$= 1.96297$$

(ii) for $n=3$

$$\int_{-1}^{1} f(x)\,dx \simeq \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)$$

$$\int_{-1}^{1} e^{x} \cos x \, dx = \frac{5}{9} \times e^{-\sqrt{\frac{3}{5}}} \cos\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} e^{0} \cos 0 + \frac{5}{9} e^{\sqrt{\frac{3}{5}}} \times \cos\left(\sqrt{\frac{3}{5}}\right)$$

$$= 1.93339$$

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Q3 Show, how an integral $\int_a^b f(x)\,dx$ over an arbitrary $[a,b]$ can be transformed into integral over $[-1,1]$.

Ans To transform $\int_a^b f(x).\,dx$ to. $\int_{-1}^{1} g(t)\,dt$

we use change of variables from x to t define by: $x = \frac{b-a}{2}t + \frac{b+a}{2}$

- on putting $x = a$ and solving for t we obtain; $a = \frac{b-a}{2} t + \frac{b+a}{2}$

which on solving gives value of t as -1. similarly putting $x = b$, we obtain, $b = \frac{b-a}{2}t + \frac{b+a}{2}$ which on solving gives

$t = 1$. Thus limits of integration

change from a to b into -1 to 1 now also $dx = \frac{b-a}{2} dt$

$$\int_a^b f(x)\,dx = \frac{b-a}{2} \int_{-1}^{1} f\left(\frac{b-a}{2} t + \frac{b+a}{2}\right) dt$$

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Q4 Write open Newton Cotes integral formulas for

(a) $n = 0$ (midpoint rule)
(b) $n = 1$

And show midpoint rule is same as Gauss quadrature one point formula

Ans open Newton Cotes integral formula for

(1) $n = 0$

midpoint rule, $n = 0$, $h = \frac{b-a}{2}$

let $a = x_{-1}$, $b = x_1$

$$\int_{x_{-1}}^{x_1} f(x)\,dx = 2h\,f(x_0) + \frac{1}{3} h^3 f^2(\xi)$$

Here last term is error term.

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(b) $n=1$; $h = \frac{(b-a)}{3}$

$a = x_{-1}$, $b = x_2$

$$\int_{x_{-1}}^{x_2} f(x)\,dx = \frac{3h}{2}\left[f(x_0) + f(x_1)\right] + \frac{3h^3}{4} f''(\xi)$$

Here last term is error term

→ The gauss quadrature one point formula is $\int_a^b f(x) = w_1 f(x_1)$

∴ It gives exact resut, when $f(x)$ of degree $\leq 2n-1 = 1$

Now let $f(x) = 1$ & $f(x) = x$ one by one

for $f(x) = 1$, $\int_a^b 1\,dx = w_1$

$\therefore w_1 = b - a$ —— (1)

for $f(x) = x$, $\int_a^b x\,dx = w_1 x_1$

$$\Rightarrow \left[\frac{x^2}{2}\right]_a^b = w_1 x_1$$

$$\Rightarrow w_1 x_1 = \frac{1}{2}(b^2 - a^2) \quad \text{--- (2)}$$

putting (1) in (2), $x_1 = \frac{1}{2}(b+a)$

$\therefore$ The resulting Gausian quadrature formula is

$$\int_a^b f(x)\,dx \simeq (b-a)\, f\left(\frac{a+b}{2}\right)$$

which is same as midpoint rule.

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Q5 Consider $\int_1^3 (x^6 - x^2 \sin 2x)\,dx$.
Exact value is 317.3442466
Estimate value of integral using

① simple trapezoidal rule.

Ans.

let $f(x) = (x^6 - x^2 \sin 2x)\,dx$

$n = 1,\ h = \frac{3-1}{1} = 2$

$\int_1^3 f(x)\,dx = \frac{h}{2}[f(3) + f(1)]$

$= \frac{2}{2}[731.5147 + 0.0907]$

$\int_1^3 f(x)\,dx = 731.6054$

② open Newton-Cotes formula
for $n=0,\ n=1$

for $n=0,\ h = \frac{3-1}{2} = \frac{2}{2} = 1$

$f(x) = (x^6 - x^2 \sin 2x)$

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$$\therefore \int_1^3 f(x)\,dx = 2h\left(f\left(\frac{3+1}{2}\right)\right)$$
$$= 2f(2)$$
$$= 2(67.0272)$$
$$= 134.0544$$

for $n=1$, $h = \frac{b-a}{3} = \frac{2}{3}$

$$\int_1^3 f(x)\,dx = \frac{3h}{2}\left[f\left(1+\frac{2}{3}\right) + f\left(1+2\left(\frac{2}{3}\right)\right)\right]$$
$$= 1\left[f\left(\frac{5}{3}\right) + f\left(\frac{7}{3}\right)\right]$$
$$= 1\left[21.9623 + 166.8228\right]$$
$$= 188.7856$$

(c) Simpson's $\frac{1}{3}$ rule.

Ans $\int_1^3 f(x)\,dx = \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$

Here $f(x) = (x^6 - x^2 \sin 2x)$

$n = 2, \; h = \frac{3-1}{n} = \frac{2}{2} = 1$

$\int_1^3 f(x)\,dx = \frac{1}{3}\left[f(1) + 4f(2) + f(3)\right]$

$= \frac{1}{3}\left[0.0907 + 4(67.0272) + 731.3147\right]$

$= \frac{1}{3}\left[999.7142\right]$

$\int_1^3 f(x)\,dx = 333.2381$

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(d) Gauss Quadrature two point formula.

we know that the formula is

$$\int_{-1}^{1} f(x)\,dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) \quad -(a)$$

our integral is $\int_{1}^{3} (x^6 - x^2 \sin 2x)\,dx$

- we shall need to transform the given interval to

$\int_{-1}^{1} g(t)\,dt$ through below

equation.

$$\int_{a}^{b} f(x)\,dx = \frac{(b-a)}{2} \int_{-1}^{1} f\left(\frac{b-a}{2}t + \frac{b+a}{2}\right) dt$$

=> Here $a=1$, $b=3$, $f(x) = x^6 - x^2 \sin 2x$

substitude $x = \frac{3-1}{2} t + \frac{3+1}{2}$

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$\Rightarrow x = t + 2$

$$\therefore \int_1^3 f(x)\,dx = \left(\frac{3-1}{2}\right) \int_{-1}^{1} f(t+2)\,dt \quad \text{--- (1)}$$

$$= 1 \times \int_{-1}^{1} \left[(t+2)^6 - (t+2)^2 \sin 2 (t+2) \right] dt.$$

(e.g)

(a) $\Rightarrow \int_1^3 f(x)\,dx =$

$$1 \times \left[\left(\left(-\frac{1}{\sqrt{3}}\right)+2\right)^6 - \left(-\frac{1}{\sqrt{3}}+2\right)^2 \sin 2\left(2+\left(-\frac{1}{\sqrt{3}}\right)\right) + \left(\frac{1}{\sqrt{3}}+2\right)^6 - \left(\frac{1}{\sqrt{3}}+2\right)^2 \sin 2\left(2+\frac{1}{\sqrt{3}}\right) \right]$$

$= 8.2906 - 0.5909 + 293.1168 + 6.0035$

$$\int_1^3 f(x)\,dx = 306.82$$

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(c) Gauss Quadrature three point formula.

3 point formula is

$$\int_{-1}^{1} f(x)\,dx \simeq \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right) \quad \text{--- (b)}$$

By transforming the given interval to $\int_{-1}^{1} g(t)\,dt$ we have eq. (1)

$$\Rightarrow \int_{1}^{3} f(x)\,dx = \left(\frac{3-1}{2}\right) \int_{-1}^{1} f(t+2)\,dt$$

$$= 1 \times \int_{-1}^{1} (t+2)^6 - (t+2)^2 \sin 2(t+2)\,dt$$

e.g.

(b) $\Rightarrow \int_{1}^{3} f(x)\,dx = \Big[\frac{5}{9}\Big[\left(-\sqrt{\frac{3}{5}}+2\right)^6 - \left(-\sqrt{\frac{3}{5}}+2\right)^2 \sin 2\left(-\sqrt{\frac{3}{5}}+2\right)\Big] + \frac{8}{9}\Big[(0+2)^6 - (0+2)^2 \sin 2(0+2)\Big] + \frac{5}{9}\Big[\left(\sqrt{\frac{3}{5}}+2\right)^6 - \left(\sqrt{\frac{3}{5}}+2\right)^2 \sin 2\left(\sqrt{\frac{3}{5}}+2\right)\Big]\Big]$

$$= \frac{5}{9}(2.4292) + \frac{8}{9}(67.0272) + \frac{5}{9}(461.4028)$$

$$= 317.2642$$

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Q6. List the Legendre polynomials $P_n(x)$ for $n=0,1,2,3$ and find their roots. Verify that each $P_n(x)$ is

(a) Roots are distinct
(b) Roots lie within interval $(-1,1)$
(c) Roots are symmetric with respect to origin.

Ans

Legendre Polynomials for $n=0,1,2,3$

→ for $n=0$, $P_0(x)=1$

There is no root for $P_0(x)$

→ for $n=1$, $P_1(x)=x$

then $\int_{-1}^{1} f(x)\,dx = w_1 f(x_1)$

it should give exact result when $f(x)$ is of degree 1

∴ let $f(x)=1$.

∴ $\int_{-1}^{1} 1\,dx = w_1 \Rightarrow w_1 = [x]_{-1}^{1} = 2$ ①

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$\therefore$ let $f(1) = x$

$$\therefore \int_{-1}^{1} x\,dx = w_1 x_1$$

$$\Rightarrow \left[\frac{x^2}{2}\right]_{-1}^{1} = w_1 x_1$$

$$\therefore w_1 x_1 = \frac{1}{2} - \frac{1}{2} = 0$$

e.g

(1) $\Rightarrow$

$$2x_1 = 0 \Rightarrow x_1 = 0$$

Roots of $P_1(x)$ is $x_1 = 0$ which is distinct, $-1 < 0 < 1$ and on origin

$\rightarrow$ for $n = 2$, $P_2(x) = \left(\frac{3x^2 - 1}{2}\right)$

Here $\int_{-1}^{1} f(x)\,dx = w_1 f(x_1) + w_2 f(x_2)$

let $f(x) = 1 \Rightarrow w_1 + w_2 = \int_{-1}^{1} 1\,dx = [x]_{-1}^{1} = 2$

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$f(x) = x \Rightarrow w_1 x_1 + w_2 x_2 = \int_{-1}^{1} x\,dx =$

$\left[\frac{x^2}{2}\right]_{-1}^{1} = \left[\frac{1}{2} - \frac{1}{2}\right] = 0$

$f(x) = x^2 \Rightarrow w_1 x_1^2 + w_2 x_2^2 = \int_{-1}^{1} x^2 dx =$

$\left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{2}{3}$

$f(x) = x^3 \Rightarrow w_1 x_1^3 + w_3 x_2^3 = \int_{-1}^{1} x^3 dx =$

$\left[\frac{x^4}{4}\right]_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0$

By solving above equation, we get the roots

$x_1 = \frac{1}{\sqrt{3}}$ and $x_2 = \frac{1}{\sqrt{3}}$

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$\rightarrow$ for $n = 3$, $P_3(x) = \dfrac{(5x^3 - 3x)}{3}$

here $\int_{-1}^{1} f(x)\,dx = w_1 f(x_1) + w_2 f(x_2)$
$+ w_3 f(x_3))$

let

$f(x) = 1 \Rightarrow w_1 + w_2 + w_3 = 2$

$f(x) = x \Rightarrow w_1 x_1 + w_2 x_2 + w_3 x_3 = 0$

$f(x) = x^2 \Rightarrow w_1 x_1^2 + w_2 x_2^2 + w_3 x_3^2 = \dfrac{2}{3}$

$f(x) = x^3 \Rightarrow w_1 x_1^3 + w_2 x_2^3 + w_3 x_3^3 = 0$

$f(x) = x^4 \Rightarrow w_1 x_1^4 + w_2 x_2^4 + w_3 x_3^4 = \dfrac{2}{5}$

$f(x) = x^5 \Rightarrow w_1 x_1^5 + w_2 x_2^5 + w_3 x_3^5 = 0$

$\therefore$ solving above equation, roots x_1, x_2, x_3 are

$x_1 = -\sqrt{\dfrac{3}{5}}$, $x_2 = 0$, $x_3 = \sqrt{\dfrac{3}{5}}$

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→ Here all are distinct, $\forall i = 1, 2, 3,$

$-1 < x_i < 1$ and roots are symmetric with respect to origin.

→ For $n = 2$ roots $x_1 = -\frac{1}{\sqrt{3}}$

and $x_2 = \frac{1}{\sqrt{3}}$ are also distinct, both are between -1 to 1 and both are symmetric.

Q7 ~~calculati~~ calculate $\int_{3}^{3.5} \frac{x}{\sqrt{x^2-4}} dx$ using

(i) 2 point Gauss quadrature formula.

Ans We need to transform the given integral to $\int_{-1}^{1} g(t)\,dt$

The two point Gaussian quadrature formula is

$$\int_{-1}^{1} f(x)\,dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) \quad -(1)$$

wkt

$$\int_{a}^{b} f(x)\,dx = \frac{(b-a)}{2}\int_{-1}^{1} f\left(\frac{b-a}{2}t + \frac{b+a}{2}\right) dt$$

here $a=3$, $b=3.5$, $f(x) = \frac{x}{\sqrt{x^2-4}}$

$\therefore x = \frac{3.5-3}{2} t + \frac{3.5+3}{2}$

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$$= \frac{0.5}{2} + \frac{6.5}{2}$$

$$\int_{3}^{3.5} f(x)\,dx = \frac{(3.5-3)}{2}\int_{-1}^{1} f\left(\frac{0.5}{2}t + \frac{6.5}{2}\right)dt \quad — ①$$

$$= 0.25 \int_{-1}^{1} f(0.25\,t + 3.25)\,dt$$

$$= 0.25 \int_{-1}^{1} \frac{(0.25\,t + 3.25)}{\sqrt{(0.25\,t + 3.2)^2 - 4}}\,dt$$

$$= 0.25 \left[\frac{\left(0.25 \times \left(-\frac{1}{\sqrt{3}}\right) + 3.25\right)}{\sqrt{\left(0.25 \times -\frac{1}{\sqrt{3}} + 3.2\right)^2 - 4}} \right.$$

$$\left. + \frac{\left(0.25 \times \frac{1}{\sqrt{3}} + 3.25\right)}{\sqrt{\left(0.25 \times \frac{1}{\sqrt{3}} + 3.25\right)^2 - 4}} \right]$$

$$= 0.25\,[1.3071 + 1.2377]$$

$$= 0.63619$$

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(2) 3 Point Gauss quadrature formula.

$$\int_{-1}^{1} f(x)\,dx \cong \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right) \quad \text{--- (2)}$$

By tranforming given interval to $\int_{-1}^{1} g(t)\,dt$ we have

eg. (1) $\Rightarrow \int_{3}^{3.5} f(x)\,dx =$

$$\left(\frac{3.5-3}{2}\right) \int_{-1}^{1} (0.25\,t + 3.25)\,dt$$

$$= 0.25 \int_{-1}^{1} (0.25t + 3.25)\,dt$$

$$= 0.25 \int_{-1}^{1} \frac{(0.25\,t + 3.25)}{\sqrt{(0.25\,t + 3.21)^2 - 4}}\,dt$$

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e9.

(2) $\Rightarrow \int_{3}^{3.5} f(x)\,dx =$

$$0.25\left[\frac{5}{9}\frac{\left(0.25 \times -\sqrt{\frac{3}{5}} + 3.25\right)}{\sqrt{\left(0.25 \times \left(-\sqrt{\frac{3}{5}}\right) + 3.25\right)^2 - 4}} + \frac{8}{9}\frac{\left(0.25 \times 0 + 3.25\right)}{\sqrt{\left(0.25\right)0 + 3.25)^2 - 4}} + \frac{5}{9}\frac{\left(0.25 \times \sqrt{\frac{3}{5}} + 3.25\right)}{\sqrt{\left(0.25 \times \sqrt{\frac{3}{5}} + 3.25\right)^2 - 4}}\right]$$

$= 0.25\left[0.7347 + 1.1277 + 0.6825\right]$

$= 0.6362$ //

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Q8 Determin constants a, b, c, d that will produce a quadrature formula.

$$\int_{-1}^{1} f(x)\,dx = a f(-1) + b f(1) + c f'(-1) + d f'(1)$$

That has degree of precision 3.

Ans. Using method of undetermined coefficient.

let $f(x) = 1$

$\therefore f(-1) = 1,\ f(1) = 1;\ f'(-1) = \frac{d}{dx}(1) = 0,\ f'(1) = \frac{d}{dx}(1) = 0$

$\therefore \int_{-1}^{1} 1\,dx = [x]_{-1}^{1} = 2 = a.1 + b.1 + c.0 + d.0$ ——— ①

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→ let $f(x) = x$

$\therefore f(-1) = -1,\ f(1) = 1,$

$f'(-1) = \frac{d}{dx}(x) = 1,\ f'(1) = \frac{d}{dx}(x) = 1$

$\therefore \int_{-1}^{1} x\,dx = \left(\frac{x^2}{2}\right)_{-1}^{1} = 0 = a.-1 + b.1 +$

$c.1 + d.1$ ——— (2)

→ let $f(x) = x^2$

$\therefore f(-1) = 1,\ f(1) = 1,\ f'(-1) = \frac{d}{dx}(x^2)$

$= -2,\ f'(1) = \frac{d}{dx}(x^2) = 2x = 2$

$\therefore \int_{-1}^{1} x^2\,dx = \left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{2}{3} = a.1 + b.1 +$

$c.-2 + d.2$ ——— (3)

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$\rightarrow$ let $f(x) = x^3$

$\therefore f(-1) = -1,\ f(1) = 1,\ f'(-1) = \frac{d}{dx}(x^3)$

$= 3x^2 = 3(-1)^2 = 3,$

$f'(1) = 3(1)^2 = 3$

$\therefore \int_{-1}^{1} x^3 dx = \left[\frac{x^4}{4}\right]_{-1}^{1} = 0 = a.-1$

$+ b.1 + c.3 + d.3$ —— (4)

$\therefore$ we have 4 eqvation

$a + b = 2$ —— (1)
$-a + b + c + d = 0$ —— (2)
$a + b - 2c + 2d = \frac{2}{3}$ —— (3)
$-a + b + 3c + 3d = 0$ —— (4)

(1), (3) $\Rightarrow 2 - 2c + 2d = \frac{2}{3}$

$\therefore 2(1 - c + d) = \frac{2}{3}$
$\therefore 1 - c + d = \frac{1}{3}$

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$\therefore -c + d = \frac{1}{3} - 1$

$\therefore -c + d = \frac{1}{3} - 1$

$\therefore -c + d = \frac{-2}{3}$

$\therefore c - d = \frac{2}{3}$

$\therefore c = \frac{2}{3} + d$ ______ (5)

(1) $\Rightarrow a = 2 - b$

$\therefore$ (2) $\Rightarrow -(2-b) + b + c + d = 0$

$-2 + 2b + c + d = 0$

$2b + c + d = 2$

(5) $\Rightarrow 2b + \frac{2}{3} + d + d = 2$

$\therefore 2b + \frac{2}{3} + 2d = 2$

$b + \frac{1}{3} + d = 1$

$\therefore b + d = 1 - \frac{1}{3}$

$\therefore b + d = \frac{2}{3}$ ______ (6)

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$$④ \Rightarrow -(2-b) + b + 3\left(\frac{2}{3} + d\right) + 3d = 0$$

$$\therefore -2 + b + b + 2 + 3d + 3d = 0$$

$$\therefore -2 + 2b + 6d + 2 = 0$$

$$\therefore -1 + b + 3d + 1 = 0$$

$$\therefore b + 3d = 0 \quad \text{—— } ⑦$$

$$⑥ - ⑤ \rightarrow \quad b + d = \frac{2}{3}$$

$$b + 3d = 0$$

$$-2d = \frac{2}{3}$$

$$d = -\frac{1}{3} \quad \text{—— } ⑧$$

$$⑤ \Rightarrow \quad c = \frac{2}{3} + d = \frac{2}{3} - \frac{1}{3}$$

$$c = \frac{1}{3} \quad \text{—— } ⑨$$

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⑥ $\Rightarrow b+d = \frac{2}{3}$

$\therefore b - \frac{1}{3} = \frac{2}{3}$

$\therefore b = \frac{2}{3} + \frac{1}{3} = \frac{3}{3}$

$\therefore b = 1$ ——— ⑩

① $\Rightarrow a+b = 2$

$a + 1 = 2$

$\therefore a = 1$ ——— ⑪

$$\boxed{\begin{aligned} &\therefore a = 1 \\ &\therefore b = 1 \\ &\therefore c = \frac{1}{3} \\ &\therefore d = -\frac{1}{3} \end{aligned}}$$

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Q9 Derive mid Point formula using method of Undetermined Coefficient

Ans let $n=1$ in method of undetermined coefficients

$$\therefore \int_a^b f(x)\,dx \simeq w_1 f(x_1)$$

It gives exact result when $f(x)$ is of degree $\leq 2\times1-1=1$

$\therefore$ let $f(x)=1$ & $f(x)=x$

for

$f(x)=1$, $\int_a^b 1\,dx = w_1 \Rightarrow$

$w_1 = b-a$ ——— ①

for $f(x)=x$, $\int_a^b x\,dx = w_1 x_1$

$\Rightarrow w_1 x_1 = \frac{1}{2}(b^2-a^2)$ ——— ②

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①, ② $\Rightarrow x_1 = \frac{1}{2}(b+a)$

$\therefore$ The Resulting Gaussian quadrature formula is

$$\int_a^b f(x)\,dx \simeq (b-a)\, f\left(\frac{a+b}{2}\right)$$

which is same as midpoint rule.