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Hypothesis Testing

Q9. Given $H_0 : \mu \geq 20$
$H_a : \mu < 20$

$n = 50 \quad \mu = 19.4 \quad \sigma = 2$

(a) $Z = \dfrac{\bar{X} - \mu}{\sigma_{\bar{X}}}$ $\quad \sigma_{\bar{X}} = \sigma / \sqrt{n}$

$= 2 / 7.0711$

$Z = \dfrac{19.4 - 20}{0.2828}$ $\quad \sigma_{\bar{X}} = 0.2828$

$Z = -2.12$

$\therefore P(Z \geq -2.12) = 0.0170$

(b) p-value = ?

$P(Z \geq -2.12) = 0.0170$

(c) Given $\alpha = 0.05$
To accept the testing p-value should be
p-value $< \alpha$
$0.0170 \leq \alpha$
So, the testing is Rejected.

(d) Reject H_0 if p-value $< \alpha$, so Rejected

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(b) Given $H_0 : \mu \leq 25$
$H_a : \mu > 25$

$n = 40 \quad \bar{x} = 26.4 \quad \sigma = 6$

(a) $Z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{26.4 - 25}{0.9487}$

$Z = 1.4757$ //

(b) P-Value

$P(Z \geq 1.48) = 1 - 0.9306 = 0.0694$ //

(c) Given $\alpha = 0.01$
P-value $\leq \alpha$ Accepted.

P-value > 0.01 so donot reject H_0

(d) Reject H_0 if p-value $\leq \alpha$ but p-value > 0.01 so do not Reject H_0.

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(11) Given $H_0 : \mu = 15$
$H_a : \mu \neq 15$

$n = 50 \quad \bar{x} = 14.15 \quad \sigma = 3 \quad \sigma_{\bar{x}} = \sigma/\sqrt{n}$
$= 3/\sqrt{50}$
$\sigma_{\bar{x}} = 0.4243$

(a) $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{14.15 - 15}{0.4243} = -2$

(b) p-value $P(Z \geq -2)$

for $\mu = 15$ double p-value
$2\,P(z \geq -2) = 2(0.0228)$
p-value $= 0.0456$

(c) $\alpha = 0.05$ p-value $\leq \alpha$ so, Reject.

(d) As p-value $\leq \alpha$, H_0 is Rejected.

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(12) Given $H_0 : \mu \geq 80$
$H_a : \mu < 80$

$n = 100 \quad \sigma = 12 \quad \alpha = 0.01$

(a) $\bar{x} = 78.5 \quad \sigma_{\bar{x}} = 1.2$

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{78.5 - 80}{1.2} = -1.25 //$$

P-value $= P(z \geq -1.25) = 0.1056$

Here P-value > 0.01 so not rejected.

(b) $\bar{x} = 77 \quad z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{77 - 80}{1.2} = -2.5$

P-value $= P(z \geq -2.5) = 0.0062$

P-value ≤ 0.01, so H_0 is Rejected

(c) $\bar{x} = 75.5 \quad z = \frac{75.5 - 80}{1.2} = -3.75$

P-value $= P(z \geq -3.75) = 0$ H_0 Rejected.

(d) $\bar{x} = 81 \quad z = \frac{81 - 80}{1.2} = 0.83$

P-value $= P(z \geq 0.83) = 0.7967 //$

H_0 Not Rejected

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(13) $H_0 = \mu \leq 50$
$H_a = \mu > 50$

$n = 60 \quad \sigma = 8, \quad \sigma_{\bar{x}} = 1.0328$
$\alpha = 0.05$

(a) $\bar{x} = 52.5 \quad z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{52.5 - 50}{1.0328}$

$= 2.42$

p-Value : $P(z \geq 2.42) = 1 - 0.9922$
$= 0.0078$

p-Value ≤ 0.05, so, H_0 Rejected.

(b) $\bar{x} = 51 \quad z = \frac{51 - 50}{1.0328} = 0.9682$

$z = 0.97$
p-value : $P(z \geq 0.97) = 1 - 0.8340$
p-value $= 0.1660$
p-value > 0.05, so, H_0 is not Rejected

(c) $\bar{x} = 51.8 \quad z = \frac{51.8 - 50}{1.0328} = 1.7428$

P-Value : $P(z \geq 1.74) = 1 - 0.9591 = 0.0409$
p-value $\leq \alpha$
so, H_0 is Rejected.

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(14) Given $H_0 : \mu \geq 22$
$H_a : \mu \neq 22$

$n = 75$, $\sigma = 10$ $\alpha = 0.01$ $\sigma_{\bar{x}} = 1.1547$

(a) $\bar{x} = 23$ $z = \frac{23 - 22}{1.1547} = 0.87$ //

for $\mu_0 = 22$ p-value $= 2P(z \geq 0.87) =$
$= 2P(0.1922)$
$= 0.3844$

p-value > 0.01 so, H_0 is not Rejected.

(b) $\bar{x} = 25.1$ $z = \frac{25.1 - 22}{1.1547} = 2.68$ //

p-value $= 2P(z > 2.68) = 2(0.0037)$
$= 0.0074$

p-value ≤ 0.01 so H_0 is Rejected.

(c) $\bar{x} = 20$ $z = \frac{20 - 22}{1.1547} = -1.73$

p-value $= 2P(z \geq 1.73) = 2(0.0418)$
$= 0.0836$

p-value ≥ 0.01 so H_0 is not Rejected.