

Department of Computer Science

Gujarat University



Certificate

Roll No: 10

Seat No: _____

This is to certify that Mr./Ms. Pradip S Karmakar student of MCA Semester – III has duly completed his/her term work for the semester ending in December 2020, in the subject of Computer Oriented Statistical Methods (COSM) towards partial fulfillment of his/her Degree of Masters in Computer Applications.

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Roll No.: - 10 **Exam Seat No.: - _____**

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Assignment - I

Central Tendency

Q1 Consider the numbers 2, 3, 4, 5, 5

(a) Compute the mean, median, & mode.

$$(i) \text{ Mean : } \bar{x} = \frac{\sum xi}{n} = \frac{2+3+4+5+5}{5} = \frac{19}{5}$$

$$\text{Mean} = 3.8$$

(ii) median : $\left(\frac{n+1}{2}\right)^{\text{th}}$ term

$$\therefore \text{Median} = \left(\frac{5}{2}\right)^{\text{th}} \text{ term} = 3^{\text{rd}} \text{ term}$$

$$\therefore \text{Median} = 4$$

(iii) Mode : Value that occurs with greatest frequency

$$\therefore \text{Mode} = 5$$

(b) If the numbers represented codes for the color of T-shirts ordered from catalog which average(s) could make sense?

→ Mode.

② If the numbers represented one-way mileages for trails to different lakes, which average(s) would make sense.

→ Mean, Median & mode.

③ Suppose the numbers represent survey response from 1 to 5, with 1 - disagree strongly, 2 - disagree, 3 - agree, 4 - agree strongly, 5 - agree very strongly. Which average make sense?

→ Mode & Median

Q2 Consider a data set of 15 distinct measurement with mean A and median B.

① If the highest number were increased, what would be the effect on the median and mean? Explain.

→ As the highest number is increased the value of Σx_i will also increase. So, the mean would increase while the median would remain the same.

② If the highest number were decreased to a value still larger than B, what would be the effect on the median and mean?

→ As the highest number is decreased the value of Σx_i will also decrease so the mean would decrease while median would remain the same.

③ If the highest number were decreased to a value smaller than B, what would be the effect on the median & mean?

→ As the highest number is decreased to a value smaller than B, Both mean and median would decrease.

Q3 Consider the data set 2, 2, 3, 6, 10

(i) Compute the mode, median & mean.

→ mode : value that occurs with greatest frequency.

$$\text{mode} = 2\text{ff}$$

$$\rightarrow \text{median} : \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = \left(\frac{6}{2}\right)^{\text{th}} \text{ term}$$

$$= 3^{\text{rd}} \text{ term}$$

$$\text{median} = 3\text{ff}$$

$$\rightarrow \text{mean} : \bar{x} = \frac{\sum xi}{n} = \frac{2+2+3+6+10}{5}$$

$$\therefore \bar{x} = \frac{23}{5} = 4.6\text{ff}$$

(ii) Multiply each data value by 5 compute the mode, median and mean.

→ new data values : 10, 10, 15, 30, 50

$$\text{Mode} = 10\text{ff}$$

$$\text{Median} = \left(\frac{6}{2}\right)^{\text{th}} \text{ term} = 3^{\text{rd}} \text{ term}$$

$$= 15 //$$

$$\text{Mean} = \bar{x} = 23$$

① Compare the results of part (a) & (b)

→ In part (a) mode 22

After multiplying each data value by 5
in part (b) Mode 220

In part (a) Median = 3

After multiplying each data value by 5
in part (b) Median = 15

In part (a) Mean = 16

After multiplying each data value by 5
in part (b) Mean = 23

- In General when each data value in a set is multiply by same constant, the result of mean, median & mode is same if multiplied by the same constant.

④

Suppose you have information about average heights of a random sample of airplane passengers. The mode is 70 inches, the median is 68 inches and the mean is 71 inches. To convert the data into cms. multiply each data ~~not~~ value by 2.54. What are the values of the mode, median and mean in cms?

→ So, the value of mode, median & mean in centimeters are.

$$\text{Mode} = 70 \times 2.54 = 177.8 \text{ cms}$$

$$\text{Median} = 68 \times 2.54 = 172.72 \text{ cms}$$

$$\text{Mean} = 71 \times 2.54 = 180.34 \text{ cms.}$$

Ques. Consider the data set 2, 2, 3, 6, 10

(a) Compute the mean, median & mode.

$$\text{Mean} = \frac{2+2+3+6+10}{5} = \frac{23}{5} = 4.6 //$$

$$\text{Median} = \left(\frac{6}{2}\right)^{\text{th}} \text{ term} = 3^{\text{rd}} \text{ term} = 3 //$$

mode = Greatest frequency data = 2 //

(b) Add 5 to each data value. And compute Mean, median & mode.

New data : 7, 7, 8, 11, 15

$$\text{Mean} = \frac{7+7+8+11+15}{5} = \frac{48}{5} = 9.6 //$$

$$\text{Median} = \left(\frac{n+1}{2}\right) = \left(\frac{6}{2}\right) = 3^{\text{rd}} \text{ term} = 8 //$$

Mode = Greatest frequency data = 7 //

⑥ compare the result of part (a) & (b)

→ In part (a) mean = 6.6

After adding 5 to each data value
in part (b) mean = 9.6

→ In part (a) median = 3

After adding 5 to each data value
in part (b) median = 8

→ In part (a) mode = 2

After adding 5 to each data value
in part (b) mode = 2

In General when the same constant is
added to each data value, the result
of mean, median and mode is also
same if added by the same constant.

Q5 Environmental Studies:

The following data are taken from a study conducted by the National Park System, of which Death Valley is a unit.

The ground temperatures ($^{\circ}\text{F}$) were taken from May to Nov in the vicinity of Furnace Creek.

146, 152, 168, 174, 180, 178, 179, 180, 178, 178, 178,
168, 165, 152, 144.

Compute the mean median & mode for these ground temperatures.

$$\Rightarrow \text{Mean} = \frac{\sum x_i}{n} = \frac{2342}{14} = 167.28\text{/\textdegree}$$

$$\text{Median} = (\text{For even}) \left(\frac{\left(\frac{n}{2}\right)}{2} + \frac{(n+1)}{2} \right)^{\text{th term}}$$

$$\therefore \frac{7^{\text{th term}} + 8^{\text{th term}}}{2} = \frac{168 + 174}{2}$$

$$\therefore 171\text{/\textdegree}$$

$$\text{Mode} = 178\text{/\textdegree}$$

Q6 The following information is from a random sample of winter wolf packs in region of Alaska, Minnesota, Michigan, Wisconsin, Canada & Finland

winter Pack size : 13, 10, 7, 5, 7, 7, 2, 4, 3, 2, 3, 15, 4, 4, 2, 8, 7, 8

$$\rightarrow \text{Mean} : \bar{X} = \frac{\sum x_i}{n} = \frac{111}{18} = 6.16$$

$$\rightarrow \text{Median} : \frac{9^{\text{th}} \text{ term} + 10^{\text{th}} \text{ term}}{2} = \frac{5+7}{2}$$

$$\therefore \frac{12}{2} = 6 \cancel{1}$$

$$\text{Median} = 6 \cancel{1}$$

$$\rightarrow \text{Mode} = 7 \cancel{1}$$

Q7

Upper Canyon: 2, 3, 1, 1, 3, 4, 6, 9, 3, 1, 3

Lower Canyon: 8, 1, 1, 0, 6, 7, 2, 14, 3, 0, 1, 13, 2, 1

@ Compute mean, median & mode for upper

$$\rightarrow \text{Mean} : \frac{\sum x_i}{n} = \frac{36}{11} = 3.2711$$

$$\rightarrow \text{Median} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term} = \frac{12}{2} = 6^{\text{th}} \text{ term}$$

$$\text{median} = 4$$

$$\rightarrow \text{Mode} = 3$$

@ Compute mean, median & mode for Lower

$$\rightarrow \text{Mean} = \frac{\sum x_i}{n} = \frac{59}{11} = 5.36$$

$$\rightarrow \text{median} = \frac{\left(\frac{n}{2} \right) + \left(\frac{n+1}{2} \right)}{2} = \frac{7^{\text{th}} \text{ term} + 8^{\text{th}} \text{ term}}{2}$$

$$= \frac{2+14}{2} = 811$$

$$\rightarrow \text{Mode} = 1$$

(c) Compare the results of part (a) & (b).

→ The mean of upper canyon = 3.22
The mean of lower canyon = 4.21

→ The median of upper canyon = 4
The median of lower canyon = 8

→ The mode of upper canyon = 3
The mode of lower canyon = 1

Q8 Costs in dollars per day in Island of Maui

89, 50, 68, 60, 375, 55, 500, 71, 40, 350, 60, 50
250, 45, 45, 125, 235, 65, 60, 130

(a) Compute the mean, median & mode for the data.

$$\rightarrow \text{Mean} : \bar{x} = \frac{\sum xi}{n} = \frac{2723}{20} = 136.15 //$$

$$\rightarrow \text{median} : \left(\frac{n}{2} \right) + \left(\frac{n+1}{2} \right) = \frac{10^{\text{th}} \text{ term} + 11^{\text{th}} \text{ term}}{2}$$

$$= \frac{65 + 68}{2} = 66.5 //$$

$$\rightarrow \text{Mode} : 60 //$$

Assignment - 2

Variation

Q1

$$\begin{array}{cccccccccc} x : & 11 & 0 & 36 & 21 & 31 & 23 & 24 & -11 & -11 & -21 \\ y : & 10 & -2 & 29 & 14 & 22 & 18 & 14 & -2 & -3 & 10 \end{array}$$

(a) compute $\sum x$, $\sum x^2$, $\sum y$ and $\sum y^2$

\rightarrow	x	y	x^2	y^2
	11	10	121	100
	0	-2	0	4
	36	29	1296	841
	21	14	441	196
	31	22	961	484
	23	18	529	324
	24	14	576	196
	-11	-2	121	4
	-11	-3	121	9
	-21	10	441	100
	103	90	4607	2258

$$\text{So, } \sum x = 103 \quad \sum x^2 = 4607$$

$$\sum y = 90 \quad \sum y^2 = 2258$$

(b) Use the results of part (a) to compute the sample mean, variance and standard deviation for x and for y.

→ Mean, Variance and standard for x :

x	\bar{x}	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
11	10.3	0.7	0.49
0	10.3	-10.3	106.09
36	10.3	25.7	660.49
21	10.3	10.7	114.49
31	10.3	20.7	428.49
23	10.3	12.7	161.29
24	10.3	13.7	187.69
-11	10.3	-21.3	453.69
-11	10.3	-21.3	453.69
-21	10.3	-31.3	979.69
<u>103</u>		<u>0</u>	<u>3546.10</u>

$$\text{Mean : } \bar{x} = \frac{103}{10} = 10.3$$

$$\text{Variance : } s^2 = \frac{3546.10}{9} = 394.01$$

$$\text{Standard deviation : } s = \sqrt{394.01}$$

$$= 19.851$$

Mean, Variance and standard deviation for

y	\bar{x}	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
12	9	1	1
-2	9	-11	121
29	9	20	400
14	9	5	25
22	9	13	169
18	9	9	81
14	9	5	25
-2	9	-11	121
-3	9	-12	144
<u>-10</u>	<u>9</u>	<u>-19</u>	<u>361</u>
<u>90</u>		<u>0</u>	<u>1448</u>

$$\text{Mean} = \bar{x} = \frac{90}{10} = 9$$

$$\text{Variance} = s^2 = \frac{1448}{9} = 160.89$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{160.89} \\ &= 12.68\end{aligned}$$

① Compute a 75% Chebyshev interval around the mean for x values and also for y values. Use the intervals to compare the two funds.

→ We know that 75% of the observation here values within two standard deviations of the mean.

$$\text{So, } \bar{x} \pm 2s = 10.3 - 2(19.85) \\ = -29.4 \\ \text{and } 10.3 + 2(19.85) \\ = 50$$

So, for x it is -29.4 to 50

$$\text{for } y \quad \bar{x} \pm 2s = 9 - 2(12.68) \\ = -16.36 \\ \text{and } 9 + 2(12.68) \\ = 34.36$$

So, for y it is -16.36 to 34.36

Vanguard Balanced index (y) has a smaller spread than Vanguard total stock index (x)

(d) Compute the coefficient of variation for each funds. Use the coefficient of variation to compare the two funds. If S represents risks and \bar{x} represents expected return, then S/\bar{x} can be thought of as a measure of risk per unit of expected return. In this case, why is smaller CV better? Explain.

\rightarrow Co-efficient of Variation

$$= \frac{SD}{\bar{x}} \times 100\%$$

$$\text{For } x: CV = \frac{19.85}{10.3} \times 100\%$$

$$= 192.7\%$$

$$\text{For } y: CV = \frac{12.68}{9} \times 100\% \\ = 140.9\%$$

If S represents risks and \bar{x} represents expected return, then as the S/\bar{x} is lesser, lower is the risk so, a smaller CV is better because it means a lower risk.

Q2

Let X be a random variable representing time to failure (in hours) at 90% breaking strength.

0.54	1.80	1.52	2.05	1.03	1.18	0.80	1.33	1.29
1.11	3.34	1.54	0.08	0.12	0.60	0.72	0.92	1.05
1.43	3.03	1.81	2.17	0.83	0.56	0.03	0.9	0.18
0.34	1.57	1.45	1.52	0.19	1.55	0.02	0.07	0.65
0.40	0.24	1.57	1.45	1.60	1.80	4.69	0.08	7.89
1.58	1.64	0.03	0.23	0.72				

(a) Find the range

$$\begin{aligned}\rightarrow \text{Range} &= \text{Maximum Value} - \text{Min Value} \\ &= 7.89 - 0.02 \\ &= 7.87\end{aligned}$$

(b) Compute Mean, Variance and SD

$$\rightarrow \text{Mean: } \bar{x} = \frac{\sum x}{n} = \frac{62.11}{50} = 1.244$$

$$\rightarrow \text{Variance: } s^2 = \frac{89}{50} = 1.78$$

$$\rightarrow \text{SD: } s = \sqrt{1.78} = 1.334$$

(d) Compute the coefficient of variation. What does this number say about time to failure? Why does a small CV indicate more consistent data, whereas a larger CV indicate less consistent data? Explain.

→ Coefficient of Variation

$$= \frac{SD}{\text{Mean}} \times 100\%$$

$$= \frac{1.33}{1.25} \times 100\% = 107\%$$

$$CV = 107\%$$

The SD of the time to failure is just slightly larger than the average time.

A smaller CV indicates more consistent data because the value of S in the numerator is smaller

Q3

Given :-

Pax World Balanced:

Mean = 9.58%

SD (%) = 16.05%

Vanguard Balanced Index:

Mean = 9.02%

SD (%) = 12.50%

@ Compute the coefficient of variation for each fund.

→ for Pax World Balanced:

$$\text{Coefficient of variation (CV)} = \frac{\text{SD}}{\bar{x}} \times 100\%$$

$$CV = \frac{16.05}{9.58} \times 100\%$$

$$= 166.7\%$$

→ For Vanguard Balanced Index:

$$CV = \frac{12.50}{9.02} \times 100\%$$

$$CV = 138.6\%$$

here we are using the coefficient of variation to represent risk per unit of return because coefficient of variation indicates how large is the SD relative to the Mean.

From this point of view, the risk per unit of return of Vanguard Balanced Index ~~**~~ is slightly lesser than that of Pax World Balanced.

So, the fund of Vanguard Balanced Index appears to be better.

In Given average number of physician visits by males per year.

$$\text{Mean} = 2.2$$

$$\text{Coefficient of Variation} = 1.5\%$$

Determine Standard of Deviation

$$\rightarrow CV = \frac{s}{\bar{x}} \times 100$$

$$\text{So, } s = 0.015 \times 2.2 = 0.033$$

So, the standard deviation of the annual number of visits to physicians made by males is 0.033

COSM Chapter - 5

Probability

①	Steps	Outcomes
1		3
2		2
3		5

How many experimental outcomes exists for the entire experiment?

$$\rightarrow \text{Total experimental outcomes} = 3 \times 2 \times 5 \\ = 25 \text{ Outcomes}$$

② How many ways can three items be selected from a group of six items?

$$\rightarrow {}^6 C_3 = \frac{6!}{3!(6-3)!} = \frac{6 \times 5 \times 4}{3 \times 2} = 20$$

There are 20 ways to select 3 items from a group of 6 items

Given A, B, C, D, E & F are 6 items

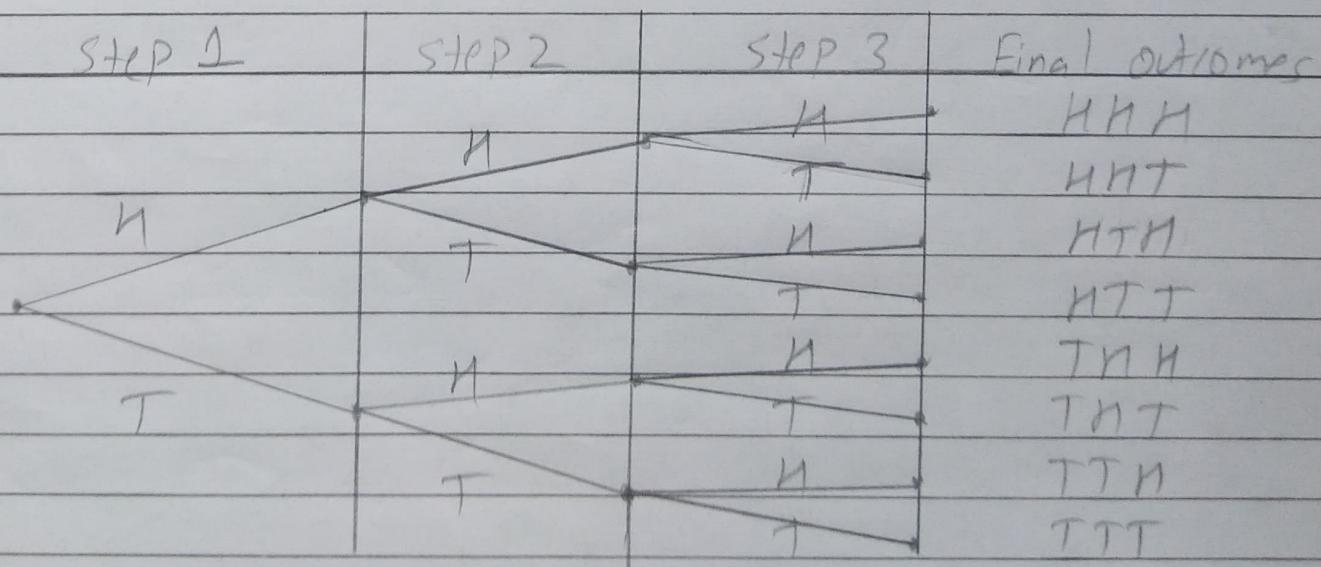
Combinations : { (ABC), (ACD), (ADE), (AEF), (ABD),
 (ABE), (ABE), (DB, F), (ACE), (AC(F)),
 (AD,F), (B(D), (BDE), (BEF), (BCF),
 (BLF), (BDF), (CDE), (CDF), (DEF)
 (CEF) }

- ③ How many permutations of 3 items can be selected from a group of 6

$$\rightarrow 6P_3 = \frac{6!}{(6-3)!} = 6 \times 5 \times 4 = 120$$

There are 120 Permutation of 3 items can be selected from a group of 6

④ Tree Diagram



(b) Experimental outcomes

Ω

(hhh), (HHT), (HTH), (HTT)

, (THH), (THT), (TTH), (TTT) }

① Probability for each experiment outcome is $\frac{1}{8}$.

⑤ Outcomes : E_1, E_2, E_3, E_n, E_S

Assigning Probabilities :

Probability for E_1 to occur = $\frac{1}{5} = 0.2$
Similarly for E_2, E_3, E_n and $E_S = 0.2$

that satisfies equation h.3 which says
each experimental outcomes is always
between 0 and 1

i.e. $0 \leq P(E_i) \leq 1$ for all ;

Also, adding Probability of each experiments
outcomes gives $0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1$

that satisfies equation h.4 which says
sum of probabilities of all, the experimental
outcomes is equal to 1.0

i.e. $P(E_1) + P(E_2) + \dots + P(E_n) = 1$

①	Experiment outcomes
.	E ₁ 20
.	E ₂ 13
.	E ₃ 17
.	<hr/> 50

Assigning Probabilities:

$$E_1 = 20/50 = 0.4$$

$$E_2 = 13/50 = 0.26$$

$$E_3 = 17/50 = 0.34$$

here, 'relative frequency method' is used as the data are available to the estimate the proportion of occurrence.

② Given Subjectively assigned probability.

$$P(E_1) = 0.10 \quad P(E_2) = 0.15$$

$$P(E_3) = 0.40 \quad P(E_4) = 0.20$$

The subjectively assigned probabilities are not valid because it must satisfy two basic requirement of equation (4.3) and (4.4)

$$P(E_1) + P(E_2) + P(E_3) + P(E_4) = 0.81$$

As the sum of all probability for the experimental outcomes is not 1, it is not valid.

Discrete | Continuous Random variable.

Q1 Define random variable with probability distribution?

→ Random variable: A random variable provides a means for describing experimental outcomes using numeric values.

- A random variable is a numerical description of the outcome of an experiment.

- A random variable can be a 'discrete random variable' or a 'continuous random variable'.

Q2 Give example of Discrete random variable & Continuous random variable.

→ Discrete random variable:

(i) Number of customers who place an order
i.e. 0, 1, 2, 3, 4, 5.

(ii) number of defective radios out of 50 radios.

i.e. 0, 1, 2, 3, ..., 49, 50.

(iii) Number of customer in a restaurant
i.e. 0, 1, 2, 3, ...

(iv) Gender of the customer
 i.e. 0 if male.
 1 if female.

Continuous random variable:

(i) Time of customer arrivals in a bank in minutes

$$\text{i.e. } x \geq 0$$

(ii) Height of students in a class in inches.

$$\text{i.e. } x \geq 0$$

(iii) Percentage of project complete after six months.

$$\text{i.e. } 0 \leq x \leq 100$$

(iv) Weight a shipment of goods in pounds

$$\text{i.e. } x \geq 0$$

Q3 Discrete or continuous & Range

→ (i) Tossing a coin with outcome as no. of heads.

→ Discrete random variable

Range: 0, 1

0 - not head
 1 - head.

② Time between 2 consecutive flights arrive.

→ Continuous random variable.
Range: $x \geq 0$.

③ Tossing 2 coins with outcomes as no of tails.

→ Discrete random variable.

Range: 0, 1, 2 0 - HH

1 - HT, TH

2 - TT

④ Distance between.

→ Continuous random variable

Range: $x \geq 0$

⑤ Outcomes of football match

→ Discrete random variable

Range: 0, 1 0 for lose 1 for win

⑥ No. of person selected in interview

→ Discrete random variable.

Range: 0, 1, 2, ...,

⑦ weight of shipment

→ Continuous random variable

Range: $x \geq 0$

Binomial probability Distribution

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

(21) Given success : 23% $\therefore P = 0.23$
 $x = 2 \quad n = 6 \quad (1-P) = 0.77$

a) $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$$f(2) = \binom{6}{2} p^2 (1-p)^4 = \binom{6}{2} (0.23)^2 (0.77)^4$$

$$= (15)(0.0529)(0.3515)$$

$$f(2) = 0.2789$$

b) $P(X \geq 2) = f(2) + f(3) + f(4) + f(5) + f(6)$

$$\left. \begin{array}{l} f(2) = 0.2789 \\ f(3) = 0.1111 \\ f(4) = 0.0249 \\ f(5) = 0.0030 \\ f(6) = 0.0001 \end{array} \right\} = 0.4180$$

c) $n = 10 \quad x = 0$

$$f(0) = \binom{10}{0} (0.23)^0 (0.77)^6$$

$$f(0) = 0.0733$$

(29) Given $P = 30\% = 0.30$ $(1 - P) = 0.70$

(a) $n = 10 \quad x = 3$

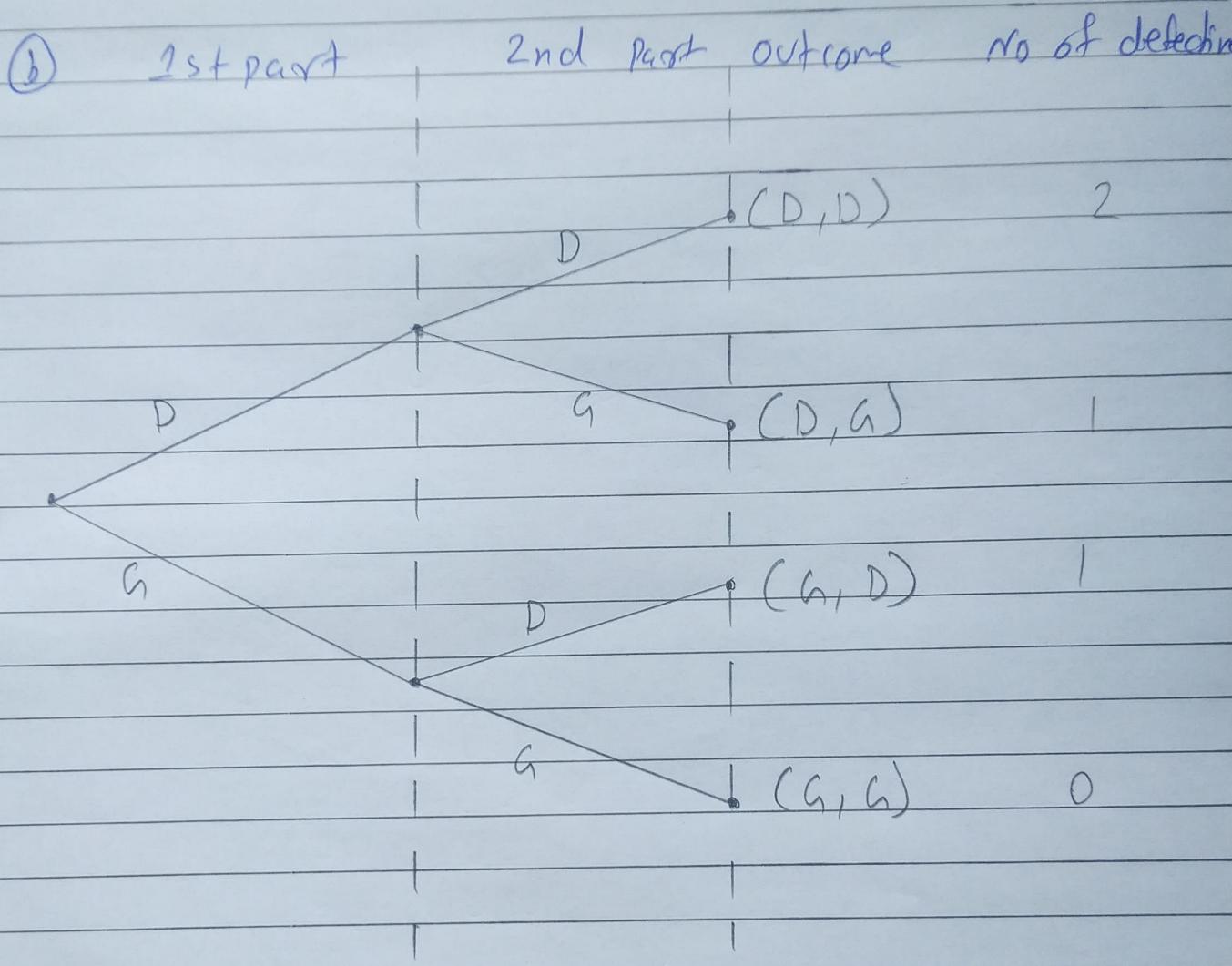
$$f(3) = \binom{10}{3} (0.30)^3 (0.70)^7$$

$$f(3) = 0.2668 \cancel{\quad}$$

(b) $P(x \geq 3) = f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10)$

$$\left. \begin{array}{l} f(3) = 0.2668 \\ f(4) = 0.2001 \\ f(5) = 0.1029 \\ f(6) = 0.0368 \\ f(7) = 0.0090 \\ f(8) = 0.0014 \\ f(9) = 0.0001 \\ f(10) = 0.0000 \end{array} \right\} = 0.6171.$$

(30) (a) Probability of a defective part being produced must be 0.03 for each part selected. Parts must be selected independently.



(c) Two outcomes with exactly one defective found.

$$\text{P}(0 \text{ defects}) = \binom{2}{0} (0.03)^0 (0.97)^2 = 0.9409$$

$$\text{P}(1 \text{ defect}) = \binom{2}{1} (0.03)^1 (0.97)^1 = 0.0582$$

$$\text{P}(2 \text{ defects}) = \binom{2}{2} (0.03)^2 (0.97)^0 = 0.0009$$

Poisson Probability distribution

- The Poisson probability distribution is often used to model random arrivals in waiting line situations.
- A discrete random variable is often useful in estimating the number of occurrence over a specified interval of time or space.

Properties of a Poisson Experiment

1. The Probability of an occurrence is the same for any two intervals of equal length.
2. The occurrence or non occurrence in any interval is independent of the occurrence or non occurrence in any other interval.

Poisson probability function

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

(39)

Consider a Poisson distribution with a mean of two occurrence per time period.

→ Given : $\lambda = 2$ per time period.

(a) Write the appropriate Poisson Probability function.

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \frac{2^x e^{-2}}{x!}$$

(b) What is expected number of occurrences in three time periods?

→ Expected no. of occurrence in three time periods is 6.

(c) Write the appropriate Poisson probability function to determine the probability of x occurrence in three time periods.

$$\rightarrow f(x) = \frac{6^x e^{-6}}{x!}$$

$$\lambda = 6$$

① Compute the probability of two occurrence in one time period.

→ Given : $\lambda = 2$
 $x = 2$

$$f(2) = \frac{2^2 e^{-2}}{2!} = \frac{4(0.1353)}{2} = 0.2706\%$$

② Compute the probability of six occurrence in three time periods.

→ Given : For three time periods $\lambda = 6$
 $x = 6$

$$f(6) = \frac{6^6 e^{-6}}{6!} = \frac{6656(0.0025)}{720} = 0.1606$$

③ Compute the probability of five occurrence in two time periods.

→ Given : $\lambda = 4$
 $x = 5$

$$f(5) = \frac{4^5 e^{-4}}{5!} = \frac{1024(0.0183)}{120} = 0.1563$$

(b) Phone calls arrive at the rate of 48 per hour at the reservation desk for Regional Airways.

→ Given : $\lambda = 48$ per hour.

(a) Compute the probability of receiving three calls in a 5 minute interval of time.

→ for 5-minutes $\lambda = 4$

$$f(3) = \frac{4^3 e^{-4}}{3!} = \frac{64(0.0183)}{6} = 0.1952$$

(b) Compute the probability of receiving exactly 10 calls in 15 minutes.

→ for 15 minutes $\lambda = 12$

$$x = 10$$

$$f(10) = \frac{12^{10} e^{-12}}{10!} = 0.1048 //$$

(c) Suppose no calls a

for 5-minutes $\lambda = 4$

which means Regional Airways receive 4 calls every 5-minutes

if the agent takes 5-minutes to complete the current calls next 3 callers are

in waiting.

So , 3 callers are waiting every 5-minutes
 So 36 callers are waiting per hour

Probability that none will be waiting

$$x = 0 \quad \mu = 5$$

$$f(0) = \frac{5^0 e^{-5}}{0!} = 0.0183$$

② for 3-minute $\mu = 2.4$

probability for personal time without
 being interrupted by a call.

$$x = 0 \\ f(0) = \frac{2.4^0 e^{-2.4}}{0!} = 0.09071$$

(43)

Airline passengers arrive randomly and independently at the passenger screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.

→ Given : $\mu = 10$ per minute.

$$\text{a) } \mu = 10 \text{ per min.}$$

$$x \geq 0$$

$$f(0) = \frac{10^0 e^{-10}}{0!} = 0.0000454$$

$$\text{b) } \mu = 10 \text{ per min.}$$

$$f(1) = \frac{10^1 e^{-10}}{1!} = 0.000454$$

$$f(2) = \frac{10^2 e^{-10}}{2!} = 0.0023$$

$$f(3) = \frac{10^3 e^{-10}}{3!} = 0.0076$$

$$\begin{aligned} P(x \leq 3) &= f(0) + f(1) + f(2) + f(3) \\ &= 0.0000454 + 0.000454 + 0.0023 + 0.0076 \end{aligned}$$

$$P(x \leq 3) = 0.0104$$

c) for 15 - seconds $\mu = 2.5$
 $x \geq 0$

$$f(0) = \frac{2.5^0 e^{-2.5}}{0!} = 0.082\%$$

d)

$$P(x \geq 1) = 1 - f(0)$$

$$= 1 - 0.082$$

$$P(x \geq 1) = 0.9179\%$$

Hypergeometric probability distribution

It is closely related to the binomial distribution.

The two probability distributions differ in two key ways.

- In hypergeometric distribution, the trials are not independent.
- the probability of success changes from trial to trial.

Hypergeometric probability function:

It is used to compute the probability that in a random selection of n elements, selected without replacement, we obtain x elements labeled success and $n-x$ elements labeled failure.

$$f(x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}$$

(M) Given $N=10$ and $r=3$

Compute value of n and x .

① $n=4, x=1$

$$f(x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}$$

$$f(1) = \frac{\binom{3}{1} \binom{10-3}{4-1}}{\binom{10}{4}} = \frac{\binom{3!}{1!2!} \binom{7!}{3!4!}}{\binom{10!}{6!4!}}$$

$$f(1) = 0.50//$$

② $n=2, x=2$

$$f(2) = \frac{\binom{3}{2} \binom{7}{0}}{\binom{10}{2}}$$

$$f(2) = 0.067//$$

③ $n=2, x=0$

$$f(0) = \frac{\binom{3}{0} \binom{7}{2}}{\binom{10}{2}}$$

$$f(0) = 0.4607//$$

④ $n=4, x=2$

$$P(2) = \frac{\binom{3}{2} \binom{2}{2}}{\binom{10}{4}}$$

$$P(2) = 0.30/\cancel{1}$$

⑤ $n=4, x=5$

Given x is greater than 4 so,
 $P(4) = 0$

(48)

Given $N = 10$
 $n = 3$ Football - 2
Basketball - 3

a)

$$N = 10 \quad n = 3$$

$$x = 2 \quad r = 2$$

$$P(x) = \frac{\binom{N}{x} \binom{N-x}{n}}{\binom{N}{n}}$$

$$P(2) = \frac{\binom{7}{2} \binom{3}{1}}{\binom{10}{3}}$$

$$P(2) = 0.5250\%$$

b)

$$P(2) = \frac{\binom{7}{2} \binom{3}{1}}{\binom{10}{3}} = 0.5250\%$$

$$P(3) = \frac{\binom{7}{3} \binom{3}{0}}{\binom{10}{3}} = 0.2917\% \quad + \quad 0.8167\%$$

(a) Given $N=60$, $n=10$

(a) $N=60$, $n=10$
 $x \geq 0$, $r \geq 20$

$$f(0) = \frac{\binom{20}{0} \binom{40}{10}}{\binom{60}{10}}$$

$$f(0) = 0.0112 //$$

(b) $N=60$, $n=10$
 $x \geq 1$, $r \geq 20$

$$f(1) = \frac{\binom{20}{1} \binom{40}{9}}{\binom{60}{10}}$$

$$f(1) = 0.0725 //$$

(c) for two or more employee
let's exclude them from the total
probability i.e. 2

$$\begin{aligned} f(x) &= 1 - f(0) - f(1) \\ &= 1 - 0.0112 - 0.0725 \\ &= 0.9163 // \end{aligned}$$

④ for Texas : $N = 60$ $n = 10$
 $\lambda = 9$ $r = 40$

$$f(9) = \frac{\frac{40}{9}}{\frac{60}{10}}$$

$$f(9) = 0.0722\text{ff}$$

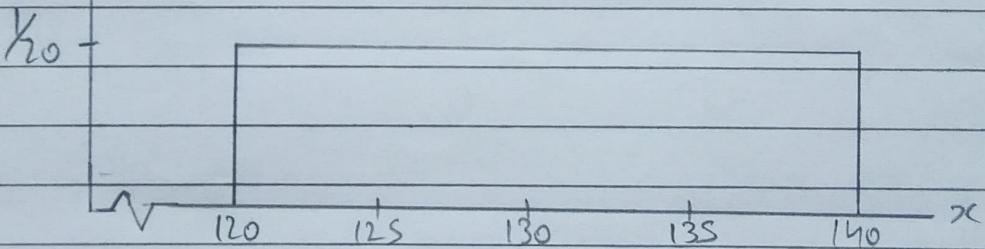
Uniform probability Distribution

A continuous probability distribution for which the probability that the random variable will assume a value in any interval is the same for each interval of equal length.

* Uniform Probability Density function:

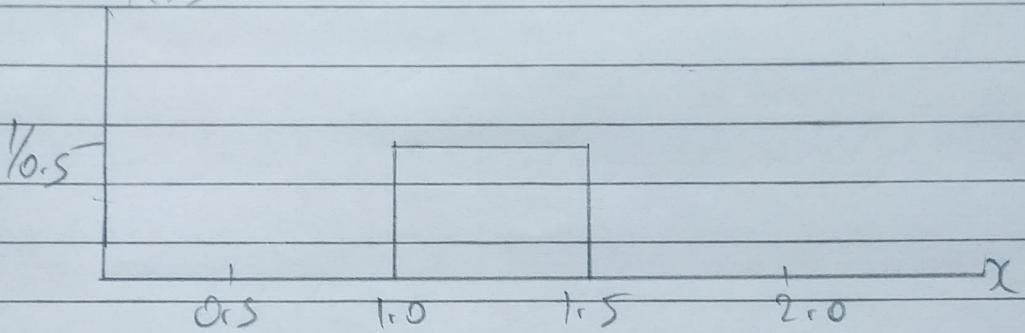
$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

* Graph of uniform probability density function.



① The random variable x is known to be uniformly distributed between 1.0 and 1.5

② $f(x)$



③

$$P(x = 1.25) = 0$$

The probability of a single point is zero because the area under the curve above any single point is zero.

$$\textcircled{1} \quad f(x) = \frac{1}{b-a} \quad ; \quad b-a = 1.25 - 1.0 \\ = 0.25$$

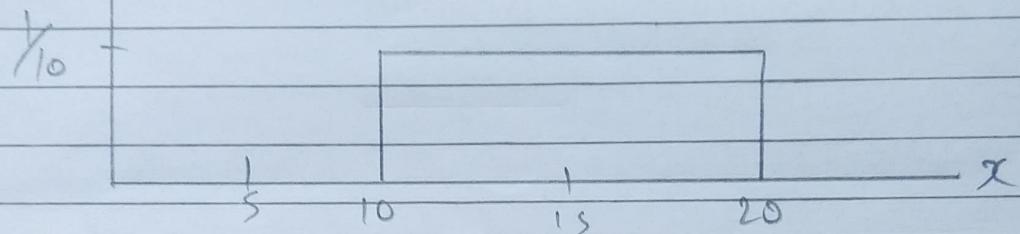
$$P(1.0 \leq x \leq 1.25) = 0.25 \times \frac{1}{0.25} = 0.5 //$$

$$\textcircled{2} \quad (b-a) = (1.5 - 1.2) = 0.3$$

$$P(1.2 \leq x \leq 1.5) = 0.3 \times \frac{1}{0.3} = 0.6 //$$

② The random variable, x is known to be uniformly distributed between 10 and 20.

$$\textcircled{a} \quad f(x)$$



$$\textcircled{b} \quad (b-a) = (15-10) = 5$$

$$P(x \leq 15) = 5 \times \frac{1}{10} = 0.51$$

$$\textcircled{c} \quad (b-a) = (18-12) = 6$$

$$P(12 \leq x \leq 18) = 6 \times \frac{1}{10} = 0.61$$

③ Delta Airlines quotes a flight time of 2 hours, 5 minutes for its flights from Cincinnati to Tampa. Suppose we believe that actual flight times are uniformly distributed between 2 hours and 2 hours 20 min.

(a) $f(x)$ $\frac{1}{20}$

100 120

140

160

 x

$$(b) (b-a) = (130 - 120) = 10$$

$$P(120 \leq x \leq 130) = 10 \times \frac{1}{20} = 0.5\%$$

$$(c) (b-a) = (140 - 130) = 5$$

$$P(x > 135) = 5 \times \frac{1}{20} = 0.25\%$$

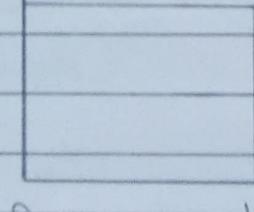
(d) x is a continuous random variable with probability density function.

$$f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(e)

 $f(x)$

1



1

2

3

 x

$$\textcircled{b} \quad (b-a) (0.75 - 0.25) = 0.5$$

$$P(0.25 \leq x \leq 0.75) = 0.5 \times 1 = 0.5 //$$

$$\textcircled{c} \quad P(x \leq 0.3) = 0.3 \times 1 = 0.3 //$$

$$\textcircled{d} \quad (b-a) = (1 - 0.5) = 0.5 //$$

$$P(x > 0.6) = 0.4 \times 1 = 0.4 //$$

\textcircled{e} On Average 30 mins television sitcom have 22 mins of programming (CNBC, Feb 23 2006). Assume that the probability distribution for minutes of programming can be approximated by a uniform distribution from 18 mins to 26 mins.

$$\textcircled{a} \quad f(x) = \frac{1}{b-a} = \frac{1}{26-18} = \frac{1}{8}$$

$$P(x > 25) = (b-a) = (26-25) = 1$$

$$P(x > 25) = 1 \times \frac{1}{8} = 0.125 //$$

$$\textcircled{b} \quad (b-a) = (2s-2l) = 4$$

$$P(2l \leq x \leq 2s) = h \times \frac{1}{8} = 0.5 //$$

\textcircled{c} \quad 30 - 22 = 8 \text{ mins for other commercials}

$$P(x > 10) = (10-8) \times \frac{1}{8} = 0.25 //$$

Normal probability Distribution

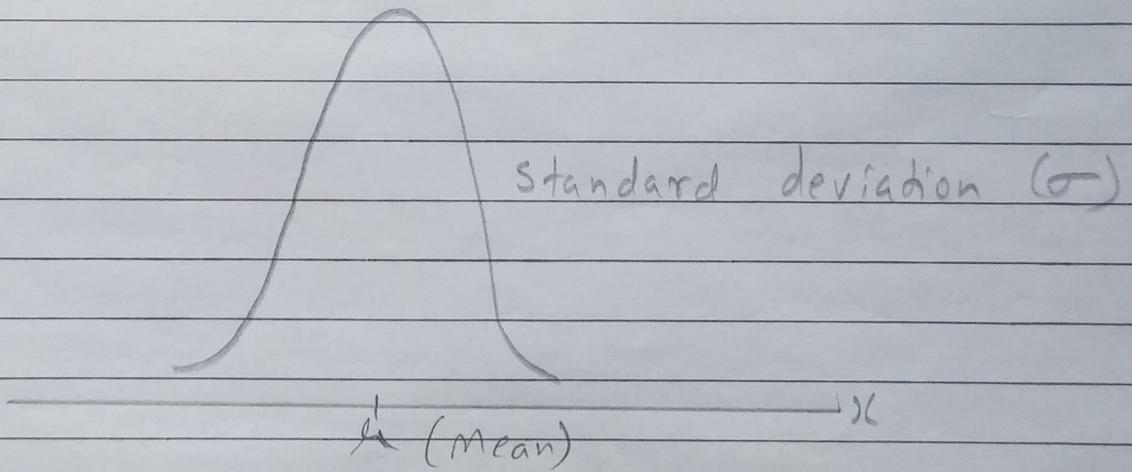
A continuous probability distribution its probability density function is bell-shaped and determined by its mean and SD.

* Normal probability Density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where μ = mean
 σ = SD

$\pi = 3.14159$
 $e = 2.71828$



* Standard Normal probability Distribution:

A random variable that ~~has~~ has a normal distribution with a mean of zero and a S.D of one is said to have a standard Normal probability Distribution

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

Converting to standard Normal Random Variable

$$Z = \frac{X - \mu}{\sigma}$$

Exercises:

- (10) (a) $P(Z \leq 1.5) = 0.9332$
 (b) $P(Z \leq 1) = 0.8413$
 (c) $P(1 \leq Z \leq 1.5) = 0.9332 - 0.8413 = 0.0919.$
 (d) $P(0 \leq Z \leq 2.5) = 0.9938 - 0.5 = 0.4938.$

- (11) (a) $P(Z \leq -1.0) = 0.1587$
 (b) $P(Z \geq -1) = 1 - 0.1587 = 0.8413$
 (c) $P(Z \geq -1.5) = 1 - 0.0668 = 0.9332.$
 (d) $P(-2.5 \leq Z) = 1 - 0.0062 = 0.9938$
 (e) $P(-3 < Z \leq 0) = 0.5 - 0.0013 = 0.4987$

- (12) (a) $P(0 \leq Z \leq 0.83) = 0.7967 - 0.5 = 0.2987$
 (b) $P(-1.57 \leq Z \leq 0) = 0.5 - 0.0582 = 0.4418$
 (c) $P(Z > 0.44) = 1 - 0.6700 = 0.3300$
 (d) $P(Z \geq -0.23) = 1 - 0.4090 = 0.5910$
 (e) $P(Z < 1.20) = 0.8849$
 (f) $P(Z \leq -0.71) = 0.2389$

- (13) (a) $P(-1.98 \leq Z \leq 0.49) = 0.6879 - 0.0239 = 0.6640$
 (b) $P(0.52 \leq Z \leq 1.22) = 0.8888 - 0.8985 = 0.1903$
 (c) $P(-1.75 \leq Z \leq -1.04) = 0.1492 - 0.0401 = 0.1091$

(14)

$$(a) Z = 1.98$$

$$(b) Z = 0.5 + 0.4750 = 0.9750 \text{ so } Z = 1.96$$

$$(c) Z = 0.61$$

$$(d) 1 - 0.1314 = 0.8686 \text{ so } Z = 1.12$$

$$(e) Z = 0.44$$

$$(f) 1 - 0.3300 = 0.6700 \text{ so } Z = 0.44$$

(15)

$$(a) Z = -0.80$$

$$(b) 0.9030/2 + 0.5 = 0.9515 \text{ so } Z = 1.66$$

$$(c) 0.2051/2 + 0.5 = 0.6026 \text{ so } Z = 0.26$$

$$(d) Z = 2.56$$

$$(e) 1 - 0.6915 = 0.3085 \text{ so } Z = -0.50$$

(16)

$$(a) 1 - 0.01 = 0.9900 \text{ so } Z = 2.33$$

$$(b) 1 - 0.025 = 0.9750 \text{ so } Z = 1.96$$

$$(c) 1 - 0.05 = 0.9500 \text{ so } Z = 1.65$$

$$(d) 1 - 0.10 = 0.9000 \text{ so } Z = 1.29$$

(17)

$$\text{Given } \mu = 77 \quad \sigma = 20$$

$$(a) X \leq 50 \quad Z = \frac{x - \mu}{\sigma} = \frac{50 - 77}{20} = -1.35$$

$$\text{so } P(Z \leq -1.35) = 0.0855$$

$$(b) X > 100 \quad Z = \frac{100 - 77}{22} = 1.15$$

$$P(Z \geq 1.15) = 1 - 0.8749 = 0.125$$

so, 12.5% of workers spent more than 100 hours.

(c) Given: Upper 20% of usage

so left side of 2 (i.e. bottom) = $1 - 0.2 = 0.80$
also, $\mu = 77$ $\sigma = 20$

$$2 = \frac{x - \mu}{\sigma}$$

$$\therefore x = \mu + 2\sigma \\ = 77 + 0.8(20)$$

$$x = 93.6$$

So, a worker logged on to the internet for 93 hours is considered as heavy user.

* Normal Approximation of Binomial probability

$$\mu = np \quad \sigma = \sqrt{np(1-p)}$$

(20) Given $P = 0.20$ $n = 100$

$$(a) \mu = np = (100)(0.20) = 20$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{16} = 4$$

(b) Yes because $np = 20$ so, binomial probability can be approximated by the normal probability distribution

(c) Given x exactly 24
considering continuity correction factor

$$P(23.5 \leq x \leq 24.5)$$

$$\text{For } x = 23.5 \quad z = \frac{x - \mu}{\sigma} = \frac{23.5 - 20}{4} = 0.875$$

$$P(z \leq 0.875) = 0.8106$$

$$\text{For } x = 24.5 \quad z = \frac{x - \mu}{\sigma} = \frac{24.5 - 20}{4} = 1.125$$

$$P(z \leq 1.125) = 0.8708$$

$$\text{So, } P(0.875 \leq z \leq 1.125) = 0.8708 - 0.8106 \\ = 0.0602$$

(28) Given $P = 0.60$ $n = 200$

$$(a) \mu = np = (200)(0.6) = 120$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{48} = 6.9282$$

(b) Yes because $\mu = 120$ so, binomial probability can be approximated by the normal probability distribution.

(c) Given $x = 100$ to 110 ,

considering continuity correction factor.

$$P(99.5 \leq x \leq 110.5)$$

$$Z = \frac{99.5 - 120}{6.9282} = -2.96$$

$$P(Z \leq -2.96) = 0.0015$$

$$Z = \frac{110.5 - 120}{6.9282} = -1.37$$

$$P(Z \leq -1.37) = 0.0853$$

$$P(-2.96 \leq Z \leq -1.37) = 0.0853 - 0.0015 = 0.0838 //$$

(d) Given $x \geq 130$

considering continuity correction factor

$$P(129.5 \leq x)$$

$$Z = \frac{129.5 - 120}{6.9282} = 1.37$$

$$P(Z \leq 1.37) = 0.9147$$

$$\text{So } P(x \geq 130) = 1 - 0.9147 = 0.0853 //$$

(28) Given $n = 250$ $P = 0.20$

$$(a) \mu = np = 250 \times 0.20 = 50$$

so, 50 adults smoke,

$$\sigma = \sqrt{np(1-p)} = \sqrt{50} = 6.3246$$

(b) $X < 50$

considering continuity correction factor

$$P(X < 39.5)$$

$$Z = \frac{39.5 - 50}{6.3246} = -1.66 \quad P(Z \leq -1.66) = 0.0488$$

$$P(X \leq 39.5) = 0.0488 //$$

(c) $P(55 \leq X \leq 60)$

considering continuity correction factor

$$P(54.5 \leq X \leq 60.5)$$

$$Z = \frac{54.5 - 50}{6.3246} = 0.71 \quad P(Z \leq 0.71) = 0.7611$$

$$Z = \frac{60.5 - 50}{6.3246} = 1.66 \quad P(Z \leq 1.66) = 0.9525$$

$$P(54.5 \leq Z \leq 60.5) = 0.9525 - 0.7611 \\ = 0.1914 //$$

$$\textcircled{d} \quad X \geq 70$$

considering continuity correction factor
 $P(X \leq 69.5)$

$$Z = \frac{69.5 - 50}{6.3248} = 3.08 \quad P(Z \leq 3.08) = 0.9990$$

$$\text{so, } P(X \geq 69.5) = 1 - 0.9990 = 0.0010$$

* Exponential Probability Distribution

The exponential probability distribution is used for random variables such as the time between arrivals at a car wash, the time required to load a truck, the distance between major defects in a highway and so on.

* Exponential probability density function.

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \text{ for } x \geq 0$$

* Exponential Distribution Cumulative Probability

$$P(X \leq x_n) = 1 - e^{-x_n/\mu}$$

(32) Given $f(x) = \frac{1}{8} e^{-x/8}$ for $x \geq 0$

$$(a) P(X \leq 8) = 1 - e^{-8/8} = 0.5296 \text{ //}$$

$$(b) P(X \leq 4) = 1 - e^{-4/8} = 0.393 \text{ //}$$

$$(c) P(X \geq 8) = 1 - 0.5296 = 0.4704 \text{ //}$$

$$(d) P(4 \leq X \leq 8) = 0.5296 - 0.393 = 0.136 \text{ //}$$

(33) $f(x) = \frac{1}{3} e^{-x/3}$ for $x \geq 0$

$$(a) P(X \leq 2) = 1 - e^{-2/3}$$

$$(b) P(X \leq 2) = 1 - e^{-2/3} = 0.4866$$

$$(c) P(X \geq 3) = e^{-3/3} = 0.3679$$

$$(d) P(X \leq 5) = 1 - e^{-5/3} = 0.8111$$

$$(e) P(2 \leq X \leq 5) = 0.8111 - 0.4866 = 0.3245$$

(34) Given $\mu = 2.1 \text{ min}$

$$(a) P(X \leq 10) = 1 - e^{-10/2.1} = 0.5624$$

$$(b) P(X > 20) = 1 - e^{-20/2.1} = 0.1915$$

$$(c) P(10 \leq X \leq 20) = 0.8085 - 0.5624 = 0.2461$$

$$(d) P(X > 18) = e^{-18/2.1} = 0.2259$$

(35) $\mu = 3.5 \text{ per hour}$

$$(a) f(x) = 3.5 e^{-3.5 x}$$

$$(b) 3.5/4 = 1.3750 \text{ per } 15 \text{ min}$$

$$\text{for no-interruptions : } e^{-1.3750} = 0.2528$$

$$(c) 3.5/6 = 0.9167 \text{ per } 10 \text{ min}$$

$$\text{for interruption : } 1 - e^{-0.9167} = 0.6002 \text{ //}$$

Hypothesis Testing

Q9. Given $H_0 : \mu \geq 20$
 $H_a : \mu < 20$

$$n = 50 \quad \mu = 19.4 \quad \sigma = 2$$

$$\textcircled{a} \quad Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \quad \sigma_{\bar{X}} = \sigma / \sqrt{n}$$

$$= 2 / 7.0711$$

$$Z = \frac{19.4 - 20}{0.2828} \quad \sigma_{\bar{X}} = 0.2828$$

$$Z = -2.12$$

$$\therefore P(Z \geq -2.12) = 0.0170$$

$$\textcircled{b} \quad p\text{-Value} = ?$$

$$P(Z \geq -2.12) = 0.0170$$

$$\textcircled{c} \quad \text{Given } \alpha = 0.05$$

To accept the testing p-value should be
 $p\text{-value} < \alpha$

$$0.0170 \leq \alpha$$

so, the testing is Rejected.

\textcircled{d} Reject H_0 if $p\text{-value} < \alpha$, so Rejected

(b) Given $H_0 : \mu \leq 25$
 $H_A : \mu > 25$

$$n = 20 \quad \bar{x} = 26.4 \quad \sigma = 2.6$$

$$(i) Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{26.4 - 25}{2.6/\sqrt{20}}$$

$$Z = 1.4757$$

(b) P-value

$$P(Z \geq 1.48) = 1 - 0.9300 = 0.0697$$

(c) Given $\alpha = 0.01$
 P-value $\leq \alpha$ Accepted.

P-value > 0.01 so do not reject H_0

(d) Reject H_0 if P-value $\leq \alpha$ but P-value
 > 0.01 so do not Reject H_0 .

⑪ Given $H_0 : \mu = 15$
 $H_a : \mu \neq 15$

$$n = 50 \quad \bar{x} = 14.15 \quad \sigma = 3 \quad \delta \bar{x} = \sigma / \sqrt{n} \\ \approx 3 / \sqrt{50} \\ \delta \bar{x} = 0.4243$$

a) $Z = \frac{\bar{x} - \mu}{\sigma \bar{x}} = \frac{14.15 - 15}{0.4243} \approx -2$

b) P-value $P(Z \geq -2)$

For $\mu = 15$ double p-value

$$2 P(Z \geq -2) = 2(0.0228)$$

$$\text{P-value} = 0.0456$$

c) $\alpha = 0.05$ p-value $\leq \alpha$ so, Reject.

d) As p-value $\leq \alpha$, H_0 is Rejected.

(P)

Given $H_0 : \mu \geq 80$
 $H_a : \mu < 80$

$$n = 100 \quad \sigma = 12 \quad X = 0.01$$

$$\textcircled{a} \quad \bar{X} = 78.5 \quad \sigma_{\bar{X}} = 1.2$$

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{78.5 - 80}{1.2} = -1.25 \text{ //}$$

P-Value = $P(Z \geq -1.25) = 0.1056$
~~Here~~ P-Value > 0.01 so not rejected.

$$\textcircled{b} \quad \bar{X} = 77 \quad Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{77 - 80}{1.2} = -2.5$$

$$\text{P-Value} = P(Z \geq -2.5) = 0.0062$$

P-Value ≤ 0.01 , so H_0 is Rejected

$$\textcircled{c} \quad \bar{X} = 75.5 \quad Z = \frac{75.5 - 80}{1.2} = -3.75$$

$$\text{P-Value} = P(Z \geq -3.75) = 0 \text{ No Rejected.}$$

$$\textcircled{d} \quad \bar{X} = 81 \quad Z = \frac{81 - 80}{1.2} = 0.83$$

$$\text{P-Value} = P(Z \geq 0.83) = 0.7971$$

H_0 Not Rejected

$$(13) H_0 : \mu \leq 50$$

$$H_a : \mu > 50$$

$$n = 60, \sigma = 8, \sigma_{\bar{x}} = 1.0328$$

$$x = 0.05$$

$$(a) \bar{x} = 52.5 \quad z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{52.5 - 50}{1.0328}$$

$$= 2.42$$

$$\text{P-Value} : P(Z \geq 2.42) = 1 - 0.9922 \\ = 0.0078$$

P-Value ≤ 0.05 , so, H_0 Rejected.

$$(b) \bar{x} = 51 \quad z = \frac{51 - 50}{1.0328} = 0.9682$$

$$z = 0.97$$

$$\text{P-Value} : P(Z \geq 0.97) = 1 - 0.8340$$

$$\text{P-Value} = 0.1660$$

P-Value > 0.05 , so, H_0 is not Rejected.

$$(c) \bar{x} = 51.8 \quad z = \frac{51.8 - 50}{1.0328} = 1.7428$$

$$\text{P-Value} : P(Z \geq 1.74) = 1 - 0.9591 = 0.0409$$

$$\text{P-Value} \leq x$$

so, H_0 is Rejected.

④ Given $H_0 : \mu \geq 22$
 $H_a : \mu \neq 22$

$$n=75, \sigma = 10, \alpha = 0.01, \sigma \bar{x} = 1.1547$$

$$\textcircled{a} \quad \bar{x} = 23 - 22 = \frac{23 - 22}{1.1547} = 0.8711$$

$$\text{for } H_0 = 22 \quad \text{P-value} = 2 P(Z \geq 0.87) = \\ = 2 P(Z \geq 0.1922) \\ = 0.3894$$

P-value ≥ 0.01 so H_0 is not rejected.

$$\textcircled{b} \quad \bar{x} = 25.1 - 22 = \frac{25.1 - 22}{1.1547} = 2.6811$$

$$\text{P-value} = 2 P(Z \geq 2.68) = 2(0.0037) \\ = 0.0074$$

P-value ≤ 0.01 so H_0 is rejected.

$$\textcircled{c} \quad \bar{x} = 20 - 22 = \frac{20 - 22}{1.1547} = -1.73$$

$$\text{P-value} = 2 P(Z \geq 1.73) = 2(0.0418) \\ = 0.0836$$

P-value ≥ 0.01 so H_0 is not rejected.