Hypergeometric Probability Distribution

* Hypergeometric Probability Distribution:

It is closely related to the binomial distribution.

The two probability distributions differ in two key ways.

- In hypergeometric distribution, the trials are not independent.
- the probability of success changes from trial-to-trial.

* Hypergeometric probability function:

It is used to compute the probability that in a random selection of n elements, selected without replacement, we obtain x elements labeled success and n-x elements labeled failure.

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

where x = no. of success n = no. of trials
N = no. of elements in population.
r = success in population.

Exercises

(46) Given $N = 10$ and $x = 3$.
Compute the hypergeometric probabilities for values of n and x.

(a) $n = 4$, $x = 1$

$$f(x) = \frac{\binom{x}{x}\binom{N-x}{n-x}}{\binom{N}{n}}$$

$$f(1) = \frac{\binom{3}{1}\binom{10-3}{4-1}}{\binom{10}{4}} = \frac{\left(\frac{3!}{1!2!}\right)\left(\frac{7!}{3!4!}\right)}{\left(\frac{10!}{4!6!}\right)}$$

$$f(1) = 0.50$$

(b) $n = 2$, $x = 2$

$$f(2) = \frac{\binom{3}{2}\binom{7}{0}}{\binom{10}{2}}$$

$$f(2) = 0.067$$

(c) $n = 2$, $x = 0$

$$f(0) = \frac{\binom{3}{0}\binom{7}{2}}{\binom{10}{2}}$$

$$f(0) = 0.4667$$

(d) $n=4$, $x=2$

$$f(2) = \frac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}}$$

$$f(2) = 0.30$$

(e) $n=4$, $x=4$

Given x is greater than n so,

$$f(4)=0$$

(48) Given $N = 10$ Football - 7
Basketball - 3
$n = 3$

(a) What is the probability that exactly two prefer football?

→ $N = 10 \quad n = 3$
$x = 2 \quad r = 7$

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$$f(2) = \frac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}}$$

$$f(2) = 0.5250$$

(b) What is the probability that the majority (either two or three) prefer football?

→ $$f(2) = \frac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}} = 0.5250$$

$$f(3) = \frac{\binom{7}{3}\binom{3}{0}}{\binom{10}{3}} = 0.2917$$

$+ \quad = 0.8167$

(50) Given $N = 60$, $n = 10$.

(a) What is the probability that none of the employees in the sample work at the plant in Hawaii?

→ $N = 60 \quad n = 10$

$x = 0 \quad r = 20$

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$$f(0) = \frac{\binom{20}{0}\binom{40}{10}}{\binom{60}{10}}$$

$$f(0) = 0.0112$$

(b) What is the probability that one of the employees in the sample works at the plant in Hawaii?

→ $N = 60 \quad n = 10$

$x = 1 \quad r = 20$

$$f(1) = \frac{\binom{20}{1}\binom{40}{9}}{\binom{60}{10}}$$

$$f(1) = 0.0725$$

(c) What is the probability that two or more of the employees in the sample work at the plant in Hawaii?

→ for two or more employees, let's exclude them from the total probability i.e. 1.

$$f(x) = 1 - f(0) - f(1)$$

$$f(x) = 1 - 0.0112 - 0.0725$$

$$f(x) = 0.9163$$

(d) What is the probability that nine of the employees in the sample work at the plant in Texas?

→ for Texas: $N = 60$ $n = 10$
$x = 9$ $r = 40$

$$f(9) = \frac{\binom{40}{9}\binom{20}{1}}{\binom{60}{10}}$$

$$f(9) = 0.0725$$