

Poisson Probability Distribution

- The Poisson Probability Distribution is often used to model random arrivals in waiting line situations.
- A discrete random variable is often useful in estimating the number of occurrence over a specified interval of time or space.

Properties of a Poisson Experiment

1. The Probability of an occurrence is the same for any two interval of equal length.
2. The occurrence or non occurrence in any interval is independent of the occurrence or non occurrence in any other interval.

Poisson Probability Function

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

39) Consider a poisson distribution with a mean of two occurrence per time period.

→ Given : $\mu = 2$ per time period.

a) Write the appropriate poisson Probability function.

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

$$= \frac{2^x e^{-2}}{x!}$$

b) What is expected number of occurrences in three time periods?

→ Expected no. of occurrence in three time periods is 6.

c) Write the appropriate poisson probability function to determine the probability of x occurrence in three time periods.

$$\rightarrow f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

$$\mu = 6$$

① Compute the probability of two occurrence in one time period.

→ Given: $\mu = 2$
 $x = 2$

$$f(2) = \frac{2^2 e^{-2}}{2!} = \frac{4(0.1353)}{2} = 0.2706 //$$

② Compute the probability of six occurrence in three time periods.

→ Given: For three time periods $\mu = 6$
 $x = 6$

$$f(6) = \frac{6^6 e^{-6}}{6!} = \frac{46656(0.0025)}{720} = 0.1606$$

③ Compute the probability of five occurrence in two time periods.

→ Given: $\mu = 4$
 $x = 5$

$$f(5) = \frac{4^5 e^{-4}}{5!} = \frac{1024(0.0183)}{120} = 0.1563$$

(40) Phone calls arrive at the rate of 48 per hour at the reservation desk for Regional Airways.

→ Given : $\mu = 48$ per hour.

(a) Compute the probability of receiving three calls in a 5 minute interval of time.

→ for 5-minutes $\mu = 4$

$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \quad x=3 \quad = \frac{4^3 (0.0183)}{3!} = 0.1952$$

(b) Compute the probability of receiving exactly 10 calls in 15 minutes.

→ for 15 minutes $\mu = 12$

$$x = 10$$

$$f(10) = \frac{12^{10} e^{-12}}{10!} = 0.1048 //$$

(c) Suppose no calls

(i) for 5-minutes $\mu = 4$

which means Regional Airways receive 4 calls every 5-minutes

if the agent takes 5-minutes to complete the current calls so 3 callers are

in waiting.

So, 3 callers are waiting every 5-minutes
 So 36 callers are waiting per hour

Probability that none will be waiting
 $x = 0 \quad \mu = 4$

$$P(0) = \frac{4^0 e^{-4}}{0!} = 0.0183$$

② for 3-minute $\mu = 2.4$

probability for personal time without
 being interrupted by a call.

$x = 0$

$$P(0) = \frac{2.4^0 e^{-2.4}}{0!} = 0.0907 //$$

- (43) Airline passengers arrive randomly and independently at the passenger screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.

→ Given : $\mu = 10$ per minute.

a) $\mu = 10$ per minute.
 $x = 0$

$$f(0) = \frac{10^0 e^{-10}}{0!} = 0.0000454$$

b) $\mu = 10$ per minute.

$$f(1) = \frac{10^1 e^{-10}}{1!} = 0.000454$$

c) $f(2) = \frac{10^2 e^{-10}}{2!} = 0.0023$

$$f(3) = \frac{10^3 e^{-10}}{3!} = 0.0076$$

$$\begin{aligned} P(x \leq 3) &= f(0) + f(1) + f(2) + f(3) \\ &= 0.0000454 + 0.000454 + 0.0023 + 0.0076 \end{aligned}$$

$$P(x \leq 3) = 0.0104$$

c) for 15-seconds $\lambda = 2.5$
 $x = 0$

$$f(0) = \frac{2.5^0 \cdot e^{-2.5}}{0!} = 0.0821 //$$

d)

$$P(x \geq 1) = 1 - f(0)$$

$$= 1 - 0.0821$$

$$P(x \geq 1) = 0.9179 //$$