

B.TECH/AEIE/CSE/7TH SEM/MECH 4130/2022
ECOLOGY AND ENVIRONMENTAL ENGINEERING

Time Allotted : 3 hrs

(MECH 4130)

Full Marks: 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and

any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group - A

(Multiple Choice Type Questions)

1. Choose the correct alternative for the following:

$10 \times 1 = 10$

(i) $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ is equal to

- (a) 0
- (b) 1
- (c) -1
- (d) ∞ .

Solution

- (b) 1

(ii) If $f(x) = x^2$, then $f'(2)$ is equal to

- (a) 2
- (b) 3
- (c) 4
- (d) 5.

Solution

- (c) 4

(iii) The derivative of a constant function is

- (a) 1
- (b) the constant itself
- (c) 0
- (d) undefined.

Solution

- (c) 0

(iv) $\lim_{x \rightarrow a} x =$

- (a) 0
- (b) 1
- (c) a
- (d) 2a.

Solution

- (c) a
- (v) The derivative of $\sin x$ is
- (a) $\cos x$
 - (b) $-\cos x$
 - (c) $\sin x$
 - (d) $-\sin x$.

Solution

- (a) $\cos x$
- (vi) $\lim_{x \rightarrow 2}(x + 2) =$
- (a) 2
 - (b) 0
 - (c) 4
 - (d) 1.

Solution

- (c) 4
- (vii) The derivative of x^n is
- (a) nx^{n-1}
 - (b) x^{n-1}
 - (c) nx^{n+1}
 - (d) x^{n+1} .

Solution

- (a) nx^{n-1}
- (viii) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} =$
- (a) 0
 - (b) 1
 - (c) $\frac{1}{2}$
 - (d) 2.

Solution

- (c) $\frac{1}{2}$
- (ix) The derivative of $\cos x$ is
- (a) $\sin x$
 - (b) $-\sin x$
 - (c) $\cos x$
 - (d) $-\cos x$.

Solution(b) $-\sin x$

(x) $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1} =$

(a) 0

(b) 1

(c) 2

(d) 3.

Solution

(c) 2

Group - B

2. (a) Define limit of a function. Evaluate $\lim_{x \rightarrow 2}(x^2 + 2x + 1)$. [(CO1)(Remember/LOCQ)] $(1 + 2 + 3) + 6 = 12$

Solution

Limit of a function: Let f be a function defined on an open interval containing c (except possibly at c itself). The limit of $f(x)$ as x approaches c is L , written as $\lim_{x \rightarrow c} f(x) = L$, if for every $\epsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

$$\lim_{x \rightarrow 2}(x^2 + 2x + 1) = (2)^2 + 2(2) + 1 = 4 + 4 + 1 = 9$$

- (b) Find the derivative of $f(x) = x^2$ using the first principle. [(C01)(Analyse/IOCQ)]

Solution

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2-x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2+2xh+h^2-x^2}{h} = \lim_{h \rightarrow 0} \frac{2xh+h^2}{h} = \\ \lim_{h \rightarrow 0}(2x+h) &= 2x \end{aligned}$$

3. (a) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$. [(CO3)(Evaluate/HOCQ)] $6 + (1 + 1 + 1 + 3) = 12$

Solution

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \cdot 2 = 2 \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = 2 \cdot 1 = 2$$

- (b) Find the derivative of $f(x) = \cos x$ using the first principle. [(CO4)(Remember/LOCQ)]

Solution

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} = \\ &= \lim_{h \rightarrow 0} \frac{\cos x (\cos h - 1) - \sin x \sin h}{h} = \lim_{h \rightarrow 0} \cos x \frac{(\cos h - 1)}{h} - \sin x \frac{\sin h}{h} = \cos x \cdot 0 - \sin x \cdot 1 = \\ &= -\sin x \end{aligned}$$

Group - C

4. (a) Find the derivative of $f(x) = \tan x$. [(CO4)(Remember/LOCQ)] (1 + 2 + 3) + (1 + 2 + 3) = 12

Solution

$$f(x) = \tan x = \frac{\sin x}{\cos x} \quad f'(x) = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

- (b) Evaluate $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$. [(CO4)(Analyse/IOCQ)]

Solution

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{x - 1} = \lim_{x \rightarrow 1} (x^2 + x + 1) = 1^2 + 1 + 1 = 3$$

5. (a) State the product rule of differentiation. Find the derivative of $f(x) = x \sin x$. [(CO5)(Analyse/IOCQ)] (2 + 2 + 2) + (4 + 2) = 12

Solution

Product Rule: If $f(x) = u(x)v(x)$, then $f'(x) = u'(x)v(x) + u(x)v'(x)$.

$$f(x) = x \sin x \quad f'(x) = (1)(\sin x) + (x)(\cos x) = \sin x + x \cos x$$

- (b) Evaluate $\lim_{x \rightarrow 0} \frac{\tan x}{x}$. [(CO6)(Remember/LOCQ)]

Solution

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x \cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos x} = 1 \cdot \frac{1}{1} = 1$$

Group - D

6. (a) Find the derivative of $f(x) = \frac{1}{x}$. [(CO4)(Remember/LOCQ)] (2 + 4) + (1 + 2 + 3) = 12

Solution

$$f(x) = x^{-1} \quad f'(x) = -1 \cdot x^{-2} = -\frac{1}{x^2}$$

(b) State the quotient rule of differentiation. If $f(x) = \frac{x}{x+1}$, find $f'(x)$. [(CO5)(Analyze/IOCQ)]

Solution

Quotient Rule: If $f(x) = \frac{u(x)}{v(x)}$, then $f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}$. $f(x) = \frac{x}{x+1}$ $f'(x) = \frac{(1)(x+1) - (x)(1)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$

7. (a) Evaluate $\lim_{x \rightarrow 2} \frac{x^2-4}{x-2}$. [(CO4)(Remember/LOCQ)] $6 + (2 + 4) = 12$

Solution

$$\lim_{x \rightarrow 2} \frac{x^2-4}{x-2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2} (x+2) = 2+2 = 4$$

(b) Find the derivative of $f(x) = 5x^3 + 2x^2 - x + 7$. [(CO5)(Remember/LOCQ)]

Solution

$$f(x) = 5x^3 + 2x^2 - x + 7 \quad f'(x) = 15x^2 + 4x - 1$$

Group - E

8. (a) Find the derivative of $f(x) = \sin(x^2)$. [(CO2)(Remember/LOCQ)] $6 + (1 + 1 + 1 + 1 + 2) = 12$

Solution

$$f(x) = \sin(x^2) \quad f'(x) = \cos(x^2) \cdot 2x = 2x \cos(x^2)$$

(b) Evaluate $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2}$. [(CO2)(Understand/LOCQ)]

Solution

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^2 = (1)^2 = 1$$

9. (a) If $y = x^2 \cos x$, find $\frac{dy}{dx}$. [(CO2)(Remember/LOCQ)] $6 + 6 = 12$

Solution

$$y = x^2 \cos x \quad \frac{dy}{dx} = (2x)(\cos x) + (x^2)(-\sin x) = 2x \cos x - x^2 \sin x$$

(b) Find the derivative of $f(x) = \frac{\sin x}{x}$. [(CO6)(Understand/LOCQ)]

Solution

$$f(x) = \frac{\sin x}{x} \quad f'(x) = \frac{(\cos x)(x) - (\sin x)(1)}{x^2} = \frac{x \cos x - \sin x}{x^2}$$

LOCQ & 68.75

Cognition Level IOCQ & 18.75

HOCQ & 12.5

Course Outcome (CO):

After the completion of the course students will be able to

CO 1: Identify the current and emerging environmental engineering issues

CO 2: Learn ethical and societal responsibilities and to act accordingly

CO 3: Assess the impact of human activities on the environment

CO 4: Interpret the various types of pollutants and its probable remedies

___CO 5: Formulate and construct solutions to minimize and mitigate environmental impacts

CO 6: Analyze and practice the profession of environmental engineering in the public and/or private sector.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question _____