Fall 2024 Final Project

RS-25 Propellant Feed System Analysis

MAE 565: Rocket Propulsion

By

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Submitted to

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Introduction

The information on the RS-25 rocket engine, which is relatively reliable, modern, and robust, has become an essential sign of contemporary rocket propulsion technology. This liquid-fueled rocket engine was first designed for NASA's Space Shuttle program; it has now been adapted for the Space Launch System (SLS) as the foundation for future space missions, such as the Artemis series. The RS-25 is a staged combustion cycle engine that takes the propellant's mass and turns it into thrust. LH2 as a fuel and LO2 as an oxidizer, both cryogenic propellants, are supplied and controlled by a highly integrated feed system.

Primarily, the engine's propellant feed system is a key feature that controls the flow of mass, and both the pressure and temperature of the propellants are significantly required for the engine. This system entails a family of turbopumps, pre-burners, and injectors to realize high efficiency and thrust. Propulsion integration at the thermal management level is best demonstrated by the regenerative cooling system, whereby hydrogen, in a cryogenic state, is used to cool essential components of the engine.

The indicated diagram shows the propellant feed system's principal layout and the fuel, oxidizer, hot gases, and coolant flow paths within the RS-25 engine components. RED sections depict the fuel flow path, blue sections represent the oxygen flow path, and yellow sections represent gaseous fuel-rich combustion products. All this reveals intricate designs that support the high performance of the RS-25.

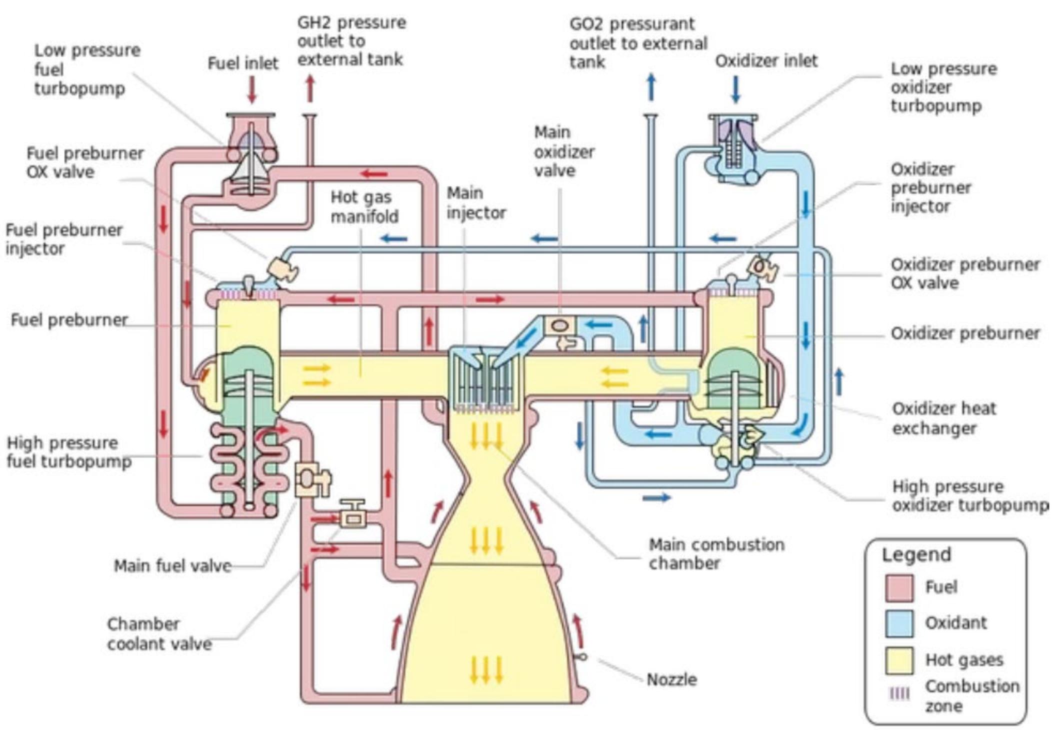

Figure 1: Overall view of the flow paths in the RS-25 propellant feed system.

Problem Statement

The RS-25 engine, used by NASA for space shuttles and the SLS launch vehicle, uses a staged combustion cycle for the propellant feed system. This system involves the utilization of high- and low-pressure turbopumps, pre-burners, and regenerative cooling systems to deliver LH2 and LO2 at high pressures and very low temperatures. However, the RS-25 for propellant feed system gives a record of efficiency and performance; the following are the significant challenges for engineering feasibility of the concept: Flow management and pressure controllers' dynamics during the feed, thermal shock & fatigue of system components, and integration of feed system under operation loads.

Notably, a summary of the propellant flow paths and pressure variations at a 100% rated power level (RPL) is essential in assessing the engine. This implies that a systematic system analysis is required to realize maximum efficiency reduction of risks and improve succeeding designs because of the convoluted flow paths of fuel, oxidizer, and combustion products. This project aims to overcome these challenges by providing a methodical approach to analyzing the RS-25 propellant feed system to define the parameters influencing its work.

Fluid Properties

Thermodynamic and physical properties of the propellants, which include liquid hydrogen (LH2) and liquid oxygen (LO2), play a critical role in the performance of the engine propellant feed system. Steady and variable conditions refer to the state by which the engine works efficiently and is stable during its operations. The given fluid properties are:

Sr. No.	Fluid (Gas and Liquid) Properties	Values	Units
1	LO2 Temperature in Tank	90	K
2	LH2 Temperature in Tank	20	K
3	LO2 Density	1141	kg/m^3
4	LH2 Density	70.8	kg/m^3
5	LO2 Specific heat C_v	1669	J/kg-K
6	LH2 Specific heat C_v	9668	J/kg-K
7	GO2 Specific heat C_p	919.1	J/kg-K
8	GH2 Specific Heat C_p	14340	J/kg-K
9	GH2 Gamma value	1.483	---
10	SSME vertical acceleration (T/M)	14.9	m/s^2

Table 1: Fluid Properties

LO2 and LH2 Tankage

The RS-25 engine propellant storage comprises LO2 and LH2 tanks, which are critical for consistent engine operation and performance. It is usually stored at these specific thermodynamic and physical states, where it can flow through the feed system of the cryogenic engine as required. LO2/LH2 fuel tanks are attached to the system utilizing "oxidizer inlet" and "fuel inlet" points on the flow diagram in Fig 1. To avoid varying flow rates and pressure fluctuations, the LO2 tank uses a pressurization system with GO2 produced by a heat exchanger. The LH2 tank is also pressurized using gaseous hydrogen (GH2) derived from the re-circulation of LH2 through regenerative cooling of thrust chamber walls. Such systems are intended to create the sufficient "pressure head" needed for propellant-free flow into the feed system. The temperature, pressure, and density of LO2 and LH2, which affect their characteristics during engine processes, are shown in the following tables. These properties are relevant to determining feed propellant systems' flow dynamics, pressure distribution, and stability.

Sr. No.	LO2 Tankage Properties	Values	Units
1	GO2 Pressure above the liquid surface	246	kPa
2	LO2 Tankage height above MOV	40	m
3	LO2 Hydrostatic head before MOV	1127	kPa
4	LO2 Total head before MOV	1373	kPa
5	LO2 Pressure after MOV	689	kPa
6	LO2 Temperature after MOV	90	K

Table 2: LO2 Tankage Properties

Sr. No.	LH2 Tankage Properties	Values	Units
1	GH2 Pressure above the liquid surface	225	kPa
2	LH2 Tankage height above fuel valve	27	m
3	LH2 Hydrostatic head before fuel valve	47	kPa
4	LH2 Total head before fuel valve	272	kPa
5	LH2 Pressure after fuel valve	207	kPa
6	LH2 Temperature after fuel valve	20	K

Table 3: LH2 Tankage Properties

Low-Pressure Oxygen Turbopump (LPOTP)

Low-Pressure Oxygen Turbopump (LPOTP) is designed to increase the pressure of the LO2 before feeding into HPOTP, and it is an integral part of the RS-25 engine propellant feed. The LPOTP is a turbopump employing an axial flow, a turbine driving an inducer. This design allows LPOTP to effectively pressurize LO2 beyond this limit while providing a continuous flow rate necessary for combustion. High-pressure LO2 in the LPOTP propels the turbine, which occupies alternating rotor/stator blades, to give the required rotary force to induce the inducer. The inducer then raises the pressure of LO2 to make it easy to pump it into the HPOTP. This pretreatment step guarantees the flow stability of the LO2 into the main combustion chamber. The operational parameters and the key properties of the proposed LPOTP are considered in the following calculations.

Pump Section:

Inlet Conditions:

The Low-Pressure Oxygen Turbopump (LPOTP) deals with the flow of liquid oxygen (LO2), as the problem under study describes. LO2 enters the pump at 698kPa pressure and leaves with the same pressure after the Main Oxidizer Valve (MOV). Likewise, the LO2 enters the pump at a temperature of 90 K, which is the temperature of the flow after the MOVReferring to the spreadsheet:

Sr. No.	Parameters	Values	Units
1	LO2 Pressure at pump inlet	698	kPa
2	LO2 Temperature entering pump	90	K
3	LO2 Mass flow rate through the pump	401	kg/s
4	LO2 Pressure Increase across the pump	2.1	MPa
5	LPOTP Pump efficiency	0.632	---

Calculation for Outlet Conditions:

1) LO2 Pressure exiting pump
   The work performed by the inducer on the liquid oxygen (LO2) increases its pressure by 2.1 MPa. The output pressure P_{t2} can be calculated as:
   P_{t2} = Pressure inlet (P1) + Pressure increase across the pump
   P_{t2} = 698 kPa + 2100 kPa
   P_{t2} = 2789 kPa

2) LO2 Temperature exiting the pump
To derive the exit temperature, we have a formula:

$$(T_2 - T_1) = \left(\frac{1-\eta_p}{\eta_p}\right)\frac{(p_{t2} - p_{t1})}{\rho c_v}$$

Substituting values from the spreadsheet, we get,

$$(T_2 - 90) = \left(\frac{1-0.632}{0.632}\right)\left[\frac{(2789 \times 10^3 - 689 \times 10^3)\ {}^{N}/_{m^3}}{1141 \times 1669\ {}^{J}/_{kg-K}}\right]$$

$T_2 = 91$ K

3) LPOTP Pump power
The formula used for deriving the pump power W $_{actual}$ is

$$|\dot{W}|_{actual} = \dot{m}\frac{1}{\eta_p}\frac{(p_{t2} - p_{t1})}{\rho}$$

$$|\dot{W}|_{actual} = 401 \times \frac{1}{0.632} \times \left[\frac{(2789 \times 10^3 - 689 \times 10^3)}{1141}\right]$$

$$|\dot{W}|_{actual} = 1167.77\ kW \approx 1168\ kW$$

Turbine Section:

The turbine section of the Low-Pressure Oxygen Turbopump (LPOTP) pumps the liquid oxygen (LO2) in line, utilizing the unspent energy in high-pressure LO2 to turn the axial turbine. As mentioned in these sections, high-pressure LO2 is provided, which can convert the kinetic energy of the liquid into the rotational energy around the shaft with an efficiency of 0.644, which in turn is used for the operation of the turbopump.

Inlet Conditions:

Liquid oxygen (LO2) circulates through the pump section, where the turbine section sets the inlet conditions with information from the High-Pressure Oxygen Turbopump (HPOTP).

1) LO2 Temperature entering the turbine: 100 K
2) LO2 Pressure entering the turbine: 29.6 MPa
3) LO2 Mass flow rate through the turbine:
To find the mass flow rate of LO2 through the turbine, the power that is produced ideally by the pump can be worked out as follows:

$$|\dot{W}|_{ideal} = \frac{1}{\rho}(p_{t2} - p_{t1})$$

Where p_{t2} & p_{t1} are calculated to represent the total inlet and outlet pressure, respectively. By substituting the known values into this equation, the ideal work is calculated as follows:

$$|\dot{W}|_{ideal} = \frac{1}{1141}(29.6 \times 10^3 - 2789)$$

$$|\dot{W}|_{ideal} = 23.497\ KNm/kg$$

The isentropic efficiency of the turbine is generally described as η_T, relates the actual work to the ideal work as:

$$\eta_T = \frac{|\dot{W}|_{actual}}{|\dot{W}|_{ideal}}$$

$$|\dot{W}|_{actual} = \eta_T \times \dot{m} \times |\dot{W}|_{ideal}$$

Since now we know that the turbine power equals the pump power, which equals 1168 kW turbine power, the mass flow rate can be calculated:

$$1168\ kW = 0.644 \times \dot{m} \times 23.497$$

$$\dot{m} = 77.2\ kg/s$$

Outlet Conditions:

1) LO2 Pressure exiting the turbine: 2789 kPa
2) LO2 Temperature exiting the turbine: 103K
3) LPOTP Turbine power is 1168 kW

Low-Pressure Fuel Turbopump (LPFTP)

The Low-Pressure Fuel Turbopump (LPFTP) mentioned in the RS-25 engine system is an axial flow turbopump equipped with a four-blade inducer, while a major part of the pump is a two-stage axial turbine. This design helps LPFTP manage the flow of LH2 and guarantee increased pressure to serve the propulsion system's needs. The LH2, with the help of regenerative cooling, gasifies and, by spinning the turbine, gives the required rotations to the inducer for a stable and efficient flow of LH2 to the following stages of high-pressure turbopumps and other feeding systems parts.

Pump Section:

Liquid hydrogen (LH2) passes through the pump section of the LPFTP in the condition that is characterized as liquid according to the problem statement.

Inlet Condition:

1) LH2 Pressure at the pump inlet: 207 kPa
2) LH2 Temperature entering the pump: 20 K
3) LH2 Mass flow rate through the pump: 67.1 kg/s
4) LH2 Pressure Increase across the pump: 1.60 Mpa

Outlet Condition:

1) LH2 Pressure exiting pump.
   The work performed by the inducer on the liquid hydrogen (LH2) increases its pressure by 1.6 MPa. The output pressure P_{t2} can be calculated as:
   $$P_{t2} = P_{t1} + 1.6 \text{ Mpa}$$
   $$P_{t2} = 1807 \text{ kPa}$$
2) LH2 Temperature exiting pump
   To derive the exit temperature, we have a formula:
   $$(T_2 - T_1) = \left(\frac{1-\eta_p}{\eta_p}\right)\frac{(p_{t2} - p_{t1})}{\rho c_v}$$
   As we have LPFTP pump efficiency, it is given as 0.674. We can substitute the remaining values in the above equation.
   $$(T_2 - 20) = \left(\frac{1-0.674}{0.674}\right)\left[\frac{(1807 \times 10^3 - 207 \times 10^3)}{70.8 \times 9668}\right]$$
   $$T_2 = 21.131\ K$$
3) LPFTP Pump efficiency: 0.674 (from spreadsheet)

4) LPFTP Pump power:
   The formula used for deriving the pump power W $_{actual}$ is

$$|\dot{W}|_{actual} = \dot{m}\frac{1}{\eta_p}\frac{(p_{t2} - p_{t1})}{\rho}$$

$$|\dot{W}|_{actual} = 67.1 \times \frac{1}{0.674} \times \left[\frac{(1807 \times 10^3 - 207 \times 10^3)}{70.8}\right]$$

$$|\dot{W}|_{actual} = 2.24\ MW$$

Turbine Section:

The turbine uses energy from high-pressure gaseous hydrogen (GH2) produced from the gasification of liquid hydrogen (LH2) used in the regenerative cooling of thrust chamber walls, as stated in the problem. Therefore, gas flows through the turbine.

Inlet Section:

1) GH2 Pressure at turbine inlet: 32,500 kPa
2) GH2 Temperature at the turbine inlet: 269 K
3) GH2 Mass flow rate through the turbine:
   The mass flow rate through the turbine can be calculated using the formula:

$$|\dot{W}|_{actual} = \dot{m}\eta_T\frac{(p_{t2} - p_{t1})}{\rho}$$

   However, approximately 20.3% of the liquid hydrogen (LH2) is utilized for cooling the thrust chamber.
   Therefore, GH2 mass flow rate $(\dot{m}) = 20.3\% \times 67.1\ kg/s$

$$(\dot{m}) = \frac{20.3}{100} \times 67.1\ kg/s$$

$$(\dot{m}) = 13.6\ kg/s$$

4) LPFTP Turbine efficiency: 0.536

Outlet Condition:

1) GH2 Pressure ratio across turbine = 1.30
2) GH2 Pressure at the turbine outlet:

$$\frac{P_{t2}}{P_{t1}} = 1.3$$

$$P_{t2} = 25{,}000\ kPa$$

3) GH2 Temperature at turbine outlet:
   To calculate the temperature at the turbine outlet, we can use the formula:

$$\frac{T_{t2}}{T_{t1}} = 1 - \eta_T \left[1 - \left(\frac{P_{t2}}{P_{t1}} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$\frac{T_{t2}}{269} = 1 - 0.536 \left[1 - \left(\frac{25000}{32500} \right)^{\frac{1.483-1}{1.483}} \right]$$

$$T_{t2} = 257\ K$$

4) LPFTP Turbine power:
   To calculate turbine power, we can use the formula:

$$\dot{W}_T = \dot{m} \times C_p (T_{t1} - T_{t2})$$

$$\dot{W}_T = 13.6 \times 14340(269 - 257)$$

$$\dot{W}_T = 2.34\ MW$$

High-Pressure Fuel Turbopump (HPFTP):

Low-Pressure Fuel Turbopump (LPFTP) provides liquid hydrogen (LH2) to the spacecraft's system. This element significantly contributes to increasing the LH2 pressure in stages and its delivery to the next high-pressure turbopumps and other feed system components. This step is essential to achieve the needed flow dynamic and operational readiness of the RS-25 engine.

Pump Section:

1) Liquid Hydrogen flows through the pump in the section at the top of the LPFTP and at the bottom of the High-Pressure Fuel Turbopump (HPFTP).
2) LH2 Pressure drop between LPFTP outlet and HPFTP inlet: 398 kPa
3) LH2 Pressure at the pump inlet:
   Inlet pressure = 1807 – 398
   LH2 Pressure at the pump inlet = 1409 kPa
4) LH2 Mass flow rate through pump: 67.1 kg/s
   The LH2 that gets into the High-Pressure Fuel Turbopump (HPFTP) comes from the Low-Pressure Fuel Turbopump (LPFTP). As a result, the mass flow rate of the LPFTP becomes equal to that of the HPFTP.
5) LH2 Pressure exiting pump: 41.7 Mpa
6) LH2 Pressure exiting pump:

$$p_{t2} - p_{t1} = 41.7\ MPa$$

$$p_{t2} = 41.7 + 1409$$

$$p_{t2} = 43109\ kPa$$

7) HPFTP Pump efficiency: 0.758
8) LH2 Temperature entering the pump: 21 K (The value is obtained from the LPFTP)

9) LH2 Temperature exiting the pump:
To derive the exit temperature, we have a formula:

$$(T_2 - T_1) = \left(\frac{1-\eta_p}{\eta_p}\right)\frac{(p_{t2} - p_{t1})}{\rho c_v}$$

$$(T_2 - 21) = \left(\frac{1-0.758}{0.758}\right)\left[\frac{(43109 \times 10^3 - 1409 \times 10^3)}{70.8 \times 9668}\right]$$

$$T_2 = 40.45\ K$$

10) HPFTP Pump Power:
The formula used for deriving the pump power W actual is

$$|\dot{W}|_{actual} = \dot{m}\frac{1}{\eta_p}\frac{(p_{t2} - p_{t1})}{\rho}$$

$$|\dot{W}|_{actual} = 67.1 \times \frac{1}{0.758}\left[\frac{(43109 \times 10^3 - 1409 \times 10^3)}{70.9}\right]$$

$$|\dot{W}|_{actual} = 52.138\ MW \ \approx 52.1\ MW$$

HP LH2 Flow Splits:

In the case of the RS-25 engine, this is an inherent component of the propellant feed system known as regenerative cooling. It employs LH2 developed by the High-Pressure Fuel Turbopump to cover the thrust chamber and the nozzle in effective thermal management and stable operations.

Thrust Chamber cooling:

1) LH2 Percentage from HPFTP going to thrust chamber cooling: 20.3%
2) LH2 Mass flow rate for thrust chamber cooling:
LH2 mass flow rate $(\dot{m}) = 20.3\% \ \times 67.1\ kg/s$

$$(\dot{m}) = \frac{20.3}{100} \times 67.1\ kg/s$$

$$(\dot{m}) = 13.6\ kg/s$$

3) LH2 Temperature entering thrust chamber cooling is 40 K.
The temperature of liquid hydrogen (LH2) entering the thrust chamber cooling system is 40 K. This is because the LH2 exiting the pump section of the HPFTP, at this temperature, directly flows into the thrust chamber cooling system.
4) LH2 Pressure entering thrust chamber cooling: 43109 kPa
The pressure in the LH2 of the thrust chamber cooling system is 43109Kpa. This value, obtained in the previous section for the pump of the HPFTP, corresponds to the exit pressure of LH2 leaving the pump and entering the scenario as the inlet pressure.
5) GH2 Temperature exiting thrust chamber cooling: 269 K
6) GH2 Pressure exiting thrust chamber cooling: 32500 kPa

Expansion Nozzle:

1) LH2 Percentage from HPFTP going to expansion nozzle cooling: 42.4%
2) LH2 mass flow rate for expansion nozzle cooling:

$$= \frac{42.4}{100} \times 67.1 \frac{kg}{s}$$

$$= 28.45 \frac{kg}{s}$$

3) GH2 Temperature entering expansion nozzle cooling: 40 K
   The temperature of gaseous hydrogen (GH2) entering the expansion nozzle cooling system is 40 K, which is the temperature at the exit of the pump section.
4) GH2 Pressure entering expansion nozzle cooling: 43109 kPa
   The pressure of gaseous hydrogen (GH2) at the inlet of the expansion nozzle cooling system is laid down as 43,109 kPa. This pressure is defined by the liquid hydrogen (LH2) pressure at the outlet of the pump section, which is used as the pressure in the cooling system inlet.
5) GH2 Temperature exiting expansion nozzle cooling: 265 K
6) GH2 Pressure exiting expansion nozzle cooling: 35200 kPa

Bypass Flow:

1) LH2 Percentage from HPFTP bypassing thrust chamber and nozzle: 37.3%
2) LH2 mass flow rate bypassing thrust chamber and nozzle:
   LH2 mass flow rate $(\dot{m}) = 37.3\% \times 67.1\ kg/s$

$$(\dot{m}) = \frac{37.3}{100} \times 67.1\ kg/s$$

$$(\dot{m}) = 25 kg/s$$

3) Resulting GH2 pressure after expanding bypass LH2 to GH2: 35200 kPa
4) Resulting GH2 temperature after expanding bypass LH2 to GH2: 28 K
5) GH2 Combined total mass flow rate going to pre-burners:

$$= \dot{m}_{Bypassing} + \dot{m}_{expansion\ nozzle}$$

$$= 28.5 + 25$$

$$= 53.5 \frac{kg}{s}$$

6) GH2 Temperature going to pre-burners:
   We can calculate GH2 temperature as it goes to the pre-burner as:

$$T = \gamma_1 T_1 + \gamma_2 T_2$$

$$T = \frac{25}{53.5} \times 28\ K + \frac{28.5}{53.5} \times 265\ K$$

$$T = 154.25\ K \ \approx 154\ K$$

7) GH2 Pressure going to pre-burners: 35200 kPa
   The flow as given here is 20.3% for the thrust chamber, 42.4 % for the expansion nozzle, and 37.3% is bypassed through a valve that drops pressure to 35.2 MPa, i.e. 35200 kPa.

Pre-burner:

In the RS-25 engine, a part of the combined gaseous hydrogen (GH2) streams is recirculated to the upper part of the High-Pressure Fuel Turbopump (HPFTP); here, it mixes with liquid oxygen (LO2) supplied by the boost pumps of the High-Pressure Oxygen Turbopump (HPOTP). At this point of the design, a pre-burner is included with a total of 264 coaxial injectors of gas-liquid for the atomization of GH2 and LO2. In the HPFTP, the pre-burner combustion chamber measures 26.5 cm in outer diameter and 11.1 cm in length, ensuring proper propellant mixing and combustion.

1) GH2 Percentage of rejoined GH2 flows going to HPFTP pre-burner: 68%
2) GH2 Mass flow rate entering pre-burner:
   LH2 mass flow rate $(\dot{m}) = 68\% \times 53.5\ kg/s$
   $$(\dot{m}) = \frac{68}{100} \times 53.5\ kg/s$$
   $$(\dot{m}) = 36.38\frac{kg}{s} \approx 36.4\frac{kg}{s}$$
3) GH2 Temperature entering pre-burner: 154 K
   In the inlet of the pre-burner, the gaseous hydrogen (GH2) is at 154 K, as described in the previous section.
4) GH2 Pressure entering pre-burner: 35200 kg/s
   This originates from the high-pressure liquid hydrogen (LH2) flow division.
5) Pre-burner O/F mass ratio: 0.970
6) LO2 Mass flow rate entering pre-burner from LO2 boost pump:
   $$\frac{O}{F} = \frac{\dot{m}_{LO2}}{\dot{m}_{GH2}}$$
   $$\dot{m}_{LO2} = \frac{O}{F} \times \dot{m}_{GH2}$$
   $$\dot{m}_{LO2} = 0.970 \times 36.4\frac{kg}{s}$$
   $$\dot{m}_{LO2} = 35.30\frac{kg}{s}$$
7) LO2 Temperature entering pre-burner from LO2 boost pump: 100 K
8) LO2 Pressure entering pre-burner from LO2 boost pump: 50200 kPa
9) Pre-burner product gas mass flux: 71.7 kg/s
   = 36.4 kg/s +35.3 kg/s
   = 71.7 kg/s
10) Pre-burner product gas Y_H2 = 0.446
11) Pre-burner product gas Y_H2O = 0.554
12) Pre-burner product gas temperature = 1117 kPa
13) Pre-burner product gas pressure = 35500 kPa
14) Pre-burner product gas gamma value = 1.35
15) Pre-burner product gas C_p value = 8088 J/kg-K
16) Pre-burner product gas MW value = 3.97 g/mol

Turbine Section:

The primary function of the turbine section is to capture the energy within the combustion product gases produced within the pre-burner.

1) HPFTP Turbine power: 47.2 MW
2) Turbine inlet gas temperature: 1117 K
   The temperature of the hot gases exiting the pre-burner serves as the inlet temperature for the turbine.
3) Turbine inlet gas pressure: 35500 kPa
   The pressure of the hot gas products in the pre-burner serves as the inlet pressure for the turbine.
4) Turbine pressure ratio p_in / p_out: 1.52
5) Turbine outlet gas pressure:

$$\frac{P_{in}}{P_{out}} = 1.52$$

   P_{in} = 35500 kPa

$$P_{out} = \frac{35500}{1.52}$$

$$P_{out} = 23355\ kPa$$

6) Turbine outlet mass flux: 71.7 kg/s
   This is similar to pre-burner product gas mass flux.
7) Turbine isentropic efficiency: 0.770
8) Turbine outlet gas temperature:

$$\frac{T_{t2}}{T_{t1}} = 1 - \eta_T \left[1 - \left(\frac{P_{t2}}{P_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\frac{T_{t2}}{1117} = 1 - 0.77 \left[1 - \left(\frac{23355}{35500}\right)^{\frac{1.35-1}{1.35}}\right]$$

$$T_{t2} = 1029\ K$$

High-Pressure Oxygen Turbopump (HPOTP)

High-pressure oxygen turbopump (HPOTP) pressures and pumps LO2 to the combustion chamber. Together with the Low-Pressure Oxygen Turbopump (LPOTP), it guarantees a stable LO2 supply and demand ratio. The HPOTP raises LO2 pressure through its reciprocal inducer and screw-type impeller in its pump section. The turbine behind this turbopump, through hot gases from a pre-burner, turns the pump, resulting in pressurized LO2 flowing into the combustion chamber for efficient combustion.

Pump Section:

1) The flow through the pump is liquid.
   Liquid oxygen (LO2) leaving the Low-Pressure Oxygen Turbopump (LPOTP) flows into the pump section of the High-Pressure Oxygen Turbopump (HPOTP).
2) LO2 Pressure at the pump inlet: 2.8 Mpa
   The rotating inducer and impeller in the LO2 pump raise the pressure of liquid oxygen (LO2) from 2.8 MPa at the inlet to 29.6 MPa at the outlet.
3) LO2 Mass flow rate through the pump: The mass flow rate is 401 kg/s
4) LO2 Pressure exiting pump: 29.6 Mpa
5) HPOTP Pump efficiency: 0.681
6) LO2 Temperature entering the pump:
   The temperature of LO2 exiting the pump is determined by the exit conditions of the pump section and the turbine section of the LPOTP.
   Let,
   x: HPOTP pump inlet temperature.
   y: HPOTP pump outlet temperature
   z: LPOTP turbine outlet temperature.

$$(T_2 - T_1) = \left(\frac{1-\eta_p}{\eta_p}\right)\frac{(p_{t2} - p_{t1})}{\rho c_v}$$

   Modifying the formula to get the unknown:

$$(y - x) = \left(\frac{1-\eta_p}{\eta_p}\right)\frac{(p_{t2} - p_{t1})}{\rho c_v}$$

$$(y - x) = \left(\frac{1-0.681}{0.681}\right)\frac{(29.6 \times 10^6 - 2.8 \times 10^6)}{1141 \times 1669}$$

$$(y - x) = 6.59$$

   Similarly, we get,

$$(z - y) = 5.012$$

$$(z - x) = 11.602$$

   Using temperature calculation relationship:

$$T = \gamma_1 T_1 + \gamma_2 T_2$$

$$x = \frac{91 \times 401 + z \times 77.2}{77.2 + 401}$$

$$478.2x = 36491 + 77.2z$$

Now, substituting and calculating, we get,

$$z = 104.83$$
$$y = 99.81$$
$$x = 93.23$$

LO2 Temperature entering the pump = 93 K

7) LO2 Temperature exiting the pump: 100 K
8) HPOTP Pump power:

$$|\dot{W}| = \dot{m}\frac{1}{\eta_p}\frac{(p_{t2} - p_{t1})}{\rho}$$

$$|\dot{W}| = 401 \times \frac{1}{0.681}\frac{(29.6 \times 10^6 - 2.8 \times 10^6)}{1141}$$

$$|\dot{W}| = 13.8\ MPa$$

Boost Pump:

1) The flow through the pump is liquid.
   The primary function of the LO2 Boost Pump in the RS 25 engine's propellant feed system is to pressurize the liquid oxygen (LO2) before the Relay Section pumps it into the High-Pressure Oxygen Turbopump (HPOTP).
2) LO2 Pressure entering boost pump: 29.6 Mpa
3) LO2 Temperature entering boost pump: 100 K
4) LO2 Pressure exiting boost pump: 50.2 Mpa
5) Boost pump efficiency: 0.803
6) LO2 Temperature exiting boost pump:

$$(T_2 - T_1) = \left(\frac{1 - \eta_p}{\eta_p}\right)\frac{(p_{t2} - p_{t1})}{\rho c_v}$$

$$(T_2 - 100) = \left(\frac{1 - 0.803}{0.803}\right)\frac{(50.2 \times 10^6 - 29.6 \times 10^6)}{1141 \times 1669}$$

$$T_2 = 100 + 2.65$$

$$T_2 = 102.65\ K \ \approx 103\ K$$

7) LO2 Mass flow rate through boost pump: 48.3 kg/s
8) Boost pump power:

$$|\dot{W}| = \dot{m}\frac{1}{\eta_p}\frac{(p_{t2} - p_{t1})}{\rho}$$

$$|\dot{W}| = 48.3 \times \frac{1}{0.803}\frac{(50.2 \times 10^6 - 29.6 \times 10^6)}{1141}$$

$$|\dot{W}| = 1.085\ MW \ \approx\ 1.1\ MW$$

Pre-burner:

1) GH2 Percentage of rejoined GH2 flows going to HPOTP pre-burner: 32%
2) GH2 Mass flow rate entering pre-burner:
   GH2 mass flow rate $(\dot{m}) = 32\% \times 53.5\ kg/s$
   $$(\dot{m}) = \frac{32}{100} \times 53.5 kg/s$$
   $$(\dot{m}) = 17.12 kg/s$$
3) GH2 Temperature entering pre-burner: 154 K
   The gaseous hydrogen (GH2) originates from the flow split at the pump section of the HPFTP, corresponding to the exit temperature of the pump section.
4) GH2 Pressure entering pre-burner: 35200 kPa
5) Pre-burner O/F mass ratio: 0.668
6) LO2 Mass flow rate entering pre-burner from LO2 boost pump: 11.3 kg/s
7) LO2 Temperature entering pre-burner from LO2 boost pump: 103 K
8) LO2 Pressure entering pre-burner from LO2 boost pump: 50.2 Mpa
9) Pre-burner product gas mass flux:
   $$= \dot{m}_{GH2} + \dot{m}_{LO2}$$
   $$= 17.1\frac{kg}{s} + 11.3\frac{kg}{s}$$
   $$= 28.4\frac{kg}{s}$$

Turbine Section:

1) The flow through the pump is gas.
2) HPOTP Turbine power: $13.8\ MW$
   The power generated by the High-Pressure Oxygen Turbopump (HPOTP) turbine drives both the pump and boost pump sections. The turbine power is equivalent to the energy required by the HPOTP pump, which is 13.8 MW.
3) Turbine inlet gas temperature: 836 K
4) Turbine inlet gas pressure: 34400 kPa
5) Turbine outlet gas pressure: 23355 kPa
6) Turbine pressure ratio p_in / p_out:
   $$\frac{P_{in}}{P_{out}} = \frac{34400}{23355}$$
   $$\frac{P_{in}}{P_{out}} = 1.47$$
7) Turbine outlet mass flux: 28.4 kg/s
8) Turbine outlet gas temperature:
   $$\dot{W}_T = \dot{m} \times C_p(T_{t1} - T_{t2})$$
   $$13.8 \times 10^6 = 28.4 \times 9073(836 - T_{t2})$$
   $$T_{t2} = 782.4\ K$$

9) Turbine isentropic efficiency:

$$\frac{P_{in}}{P_{out}} = 1.47$$

$$\frac{T_{t2}}{T_{t1}} = 1 - \eta_T \left[1 - \left(\frac{P_{t2}}{P_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\frac{T_{t2}}{T_{t1}} = 1 - \eta_T \left[1 - \left(\frac{23355}{34400}\right)^{\frac{1.37-1}{1.37}}\right]$$

$$1 - \frac{782}{836} = \eta_T[0.0988]$$

$$\eta_T = 0.65$$

Main Injectors:

In the RS-25 engine described and illustrated in Fig. 1, the staged combustion cycle is a key characteristic, and it only comprises repeating cycles and sub-cycles of combustion. This cycle employs gaseous hydrogen-rich (GH2-rich) combustion product mixtures for the High-Pressure Oxygen Turbopump (HPOTP) and High-Pressure Fuel Turbopump (HPFTP), indicated in yellow in the figure. These mixtures combine and continue into the principal injector of the fundamental combustor section, distinct from the injectors utilized in the pre-burners of the HPFTP and HPOTP. The "Staged Combustion Cycle" is the term used to describe the process of burning liquid oxygen Lo2 and liquid hydrogen LH2 in steps. It starts in the pre-burners of the HPOTP and HPFTP and is characterized by a partial combustion of the propellants. The last of the combustion steps occurs in the primary combustion chamber, where the GH2-rich combustion products of the pre-burners mix for final burn-up. This staged process also increases the combustion process's velocity, control, and optimization. All these phases add value to the overall energy release and the thrust to be produced, thus creating one highly efficient rocket propulsion system that produces high performance.

GH2 Injectors:

In the staged combustion cycle, the main injectors receive the GH2-rich product from the High-Pressure Fuel Turbopump (HPFTP) with the following specifications:

1) GH2-rich product mass flow rate from HPFTP supplied to injectors: 71.7 kg/s
2) GH2-rich product temperature from HPFTP supplied to injectors: 1029 K
3) GH2-rich product C_p from HPFTP supplied to injectors: 8088 J/kg-K
4) GH2-rich product MW from HPFTP supplied to injectors: 3.97 g/mol
5) GH2-rich product pressure from HPFTP supplied to injectors: 23355 kPa
6) GH2-rich product Y_H2 from HPFTP supplied to injectors: 0.446

7) GH2-rich product Y_H2O from HPFTP supplied to injectors: 0.549

The primary injectors are supplied with GH2 from the Low-Pressure Fuel Turbopump (LPFTP) turbine, featuring the following characteristics:

8) GH2 Mass flow rate from LPFTP turbine supplied to injectors: 13.3 kg/s
9) GH2 Temperature from LPFTP turbine supplied to injectors: 257 K
10) GH2 Pressure from LPFTP turbine supplied to injectors: 25000 kPa
11) GH2 C_p from LPFTP turbine supplied to injectors: 14340 J/kg-K
12) GH2 MW from LPFTP turbine supplied to injectors: 2.02 g/mol

The parameters of GH2 from the LPFTP turbine, going to the primary injectors, are essential drivers of the combustion process in the main combustion chamber. These characteristics considerably affect the efficiency and effectiveness of the rocket engine. The primary injectors of the RS-25 engine obtain a GHC2 dense flow through the fuel side with contributions of High-Pressure Fuel Turbine Pumper (HPFTPP) and Low-Pressure Fuel Turbine Pumper (LPFTPP). This flow's individual and combined characteristics are described below to illustrate its function in enhancing the process of staged combustion and generating thrust.

13) Combined GH2-rich mass flow rate from the fuel side going to injectors:

$$= 13.6\frac{kg}{s}(from\ LPFTP) + 71.6\frac{kg}{s}(from\ HPFTP)$$

$$= 84.9\ kg/s$$

14) Combined GH2-rich temperature from the fuel side going to injectors:

$$\gamma_1 = \frac{\dot{m}_1}{m} = \frac{71.6}{84.9}$$

$$\gamma_1 = 0.843$$

$$\gamma_2 = \frac{\dot{m}_2}{m} = \frac{13.3}{84.9}$$

$$\gamma_2 = 0.156$$

$$C_p = \gamma_1 C_{p_1} + \gamma_2 C_{p_2}$$

$$C_p = 0.843 \times 8088 + 0.159 \times 14340$$

$$C_p = 9055.224\frac{J}{kg.K}$$

$$T = \gamma_1\left(\frac{C_{p_1}}{C_p}\right) + \gamma_2\left(\frac{C_{p_2}}{C_p}\right)$$

$$T = \gamma_1\left(\frac{C_{p_1}}{C_p}\right)T_1 + \gamma_2\left(\frac{C_{p_2}}{C_p}\right)T_2$$

$$T = 0.843\left(\frac{8088}{9055.2}\right) \times 1029 + 0.156\left(\frac{14340}{9055.22}\right)$$

$$T = 837.7K$$

15) Combined GH2-rich pressure from the fuel side going to injectors:

$$\dot{\eta}_1 = \frac{m_1}{mw_1}$$

$$\dot{\eta}_1 = \frac{71.7}{3.97} = 18.0604$$

$$\dot{\eta}_2 = \frac{m_2}{mw_2}$$

$$\dot{\eta}_2 = \frac{13.6}{2.02} = 6.7326$$

$$\eta = \dot{\eta}_1 + \dot{\eta}_2$$

$$\eta = 24.793$$

$$X_1 = \frac{\dot{\eta}_1}{\eta}$$

$$X_1 = \frac{18.0604}{24.793} = 0.7284$$

$$X_2 = \frac{\dot{\eta}_2}{\eta}$$

$$X_2 = \frac{6.7326}{24.793} = 0.25715$$

$$p = X_1 p_1 + X_2 p_2$$

$$p = 23796\ kPa$$

16) Combined Y_H2 in GH2-rich flow from the fuel side going to injectors:

$$Y_{H2} = \frac{0.446 \times 71.7 + 1 \times 13.6}{85.3}$$

$$Y_{H2} = 0.534$$

17) Combined Y_H2O in GH2-rich flow from the fuel side going to injectors:

$$\gamma_{H2} + \gamma_{H2O} = 1$$

$$Y_{H2O} = 0.466$$

18) Combined C_p in GH2-rich flow from the fuel side going to injectors:

$$C_p = Y_1 C_{p_1} + Y_2 C_{p_2}$$

$$C_p = 9055.224 \frac{J}{kg.K}$$

19) Combined MW of GH2-rich flow from the fuel side going to injectors:

$$MW = X_1 MW_1 + X_2 MW_2$$

$$MW = 3.44 \frac{g}{mol}$$

The total of these flows from the fuel side of the RS-25 engine is such that they combine to impinge on and help to create the correct mixture for injection into the main combustion chamber RHS. Due to this fact, this mixture is very vital to most engines to ensure efficient combustion and high engine power output. High-Pressure Oxygen Turbopump (HPOTP) yields the injector's requirements to control composition burning to increase thrust production and engine efficiency.

20) GH2-rich product mass flow rate from HPOTP supplied to injectors: 28.4 kg/s
21) GH2-rich product temperature from HPOTP supplied to injectors: 782 K
22) GH2-rich product pressure from HPFTP supplied to injectors: 23355 kPa
23) GH2-rich product Y_H2 from HPOTP supplied to injectors: 0.549
24) GH2-rich product Y_H2O from HPOTP supplied to injectors: 0.451
25) GH2-rich product C_p from HPOTP supplied to injectors: 9037 J/kg.K
26) GH2-rich product MW from HPOTP supplied to injectors: 3.36 g/mol
27) Total GH2-rich pre-burner product mass flow supplied to injectors:

$$= 28.4\frac{kg}{s} + \frac{85.3kg}{s}$$

$$= 113.7\frac{kg}{s}$$

28) Y_H2 in combined GH2-rich pre-burner flows supplied to injectors:

$$Y_{H2} = \frac{Y_{H2} \times \dot{m}_1 + Y_{H2} \times \dot{m}_2}{\dot{m}_1 + \dot{m}_2}$$

$$Y_{H2} = 0.5377$$

$$Y_{H2O} = \frac{Y_{H2O} \times \dot{m}_1 + Y_{H2O} \times \dot{m}_2}{\dot{m}_1 + \dot{m}_2}$$

$$Y_{H2O} = 0.4622$$

29) C_p of combined GH2-rich pre-burner flows to injectors:

$$\gamma_1 = \frac{\dot{m}_1}{m} = \frac{85.3}{85.3 + 28.4}$$

$$\gamma_1 = 0.7502$$

$$\gamma_2 = \frac{\dot{m}_2}{m} = \frac{28.4}{85.3 + 28.4}$$

$$\gamma_2 = 0.2497$$

$$C_p = \gamma_1 C_{p_1} + \gamma_2 C_{p_2}$$

$$C_p = 9068.97\frac{J}{kg.K}$$

30) Temperature of combined GH2-rich pre-burner flows to injectors:

$$T = \gamma_1 \left(\frac{C_{p_1}}{C_p}\right) T_1 + \gamma_2 \left(\frac{C_{p_2}}{C_p}\right) T_2$$

$$T = 0.7502 \left(\frac{9082}{9068}\right) 835.2 + 0.2497 \left(\frac{9073}{9068}\right) 768$$

$$T = 824\ K$$

31) MW of combined GH2-rich pre-burner flows to injectors:

$$\dot{\eta}_1 = \frac{m_1}{mw_1}$$

$$\dot{\eta}_1 = \frac{85.3}{3.41} = 25.0146$$

$$\dot{\eta}_2 = \frac{m_2}{mw_2}$$

$$\dot{\eta}_2 = \frac{28.4}{3.36} = 8.4523$$

$$\eta = \dot{\eta}_1 + \dot{\eta}_2$$

$$\eta = 33.4669$$

$$X_1 = \frac{\dot{\eta}_1}{\eta}$$

$$X_1 = \frac{25.0146}{33.4669} = 0.7474$$

$$X_2 = \frac{\dot{\eta}_2}{\eta}$$

$$X_2 = \frac{8.4523}{33.4669} = 0.2525$$

$$MW = X_1 MW_1 + X_2 MW_2$$

$$MW = 0.7474 \times 3.41 + 0.2525 \times 3.36$$

$$MW = 3.397\ g/mol$$

32) Pressure of combined GH2-rich pre-burner flows to injectors:

$$p = X_1 p_1 + X_2 p_2$$

$$p = 23684\ kPa$$

33) GH2 Pressure drop across injectors: 16200 kPa

34) GH2 Pressure at the exit of injectors: 23414kPa – 16200kPa

= 7284 kPa

LO2 Injectors:

1) LO2 mass flow rate from HPOTP supplied to injectors:
   From HPOTP = 478.2 kg/s – 48.3 kg/s – 77.2 kg/s
   = 353 kg/s
2) LO2 Temperature from HPOTP supplied to injectors: 100 K
3) LO2 Pressure from HPOTP supplied to injectors: 29.6 MPa
4) LO2 Pressure drop across injectors: 9000 kPa
5) LO2 Pressure at the exit of injectors:
   =29600 kPa – 9000 kPa
   = 20600 kPa

Thrust Chamber:

1) Main combustion chamber (MCC) overall O/F mass ratio:

$$\frac{\mathrm{O}}{\mathrm{F}}\text{Overall} = \frac{LO2}{LH2} = \frac{401\ kg/s}{67.1\ kg/s} = 5.976$$

   overall O/F mass ratio = 5.98
2) Main combustion chamber (MCC) pressure: 20.6 MPa
3) Mass flow rate of combined GH2-rich pre-burner flows entering MCC: 113.7 kg/s
4) Y_H2 in combined GH2-rich pre-burner flows entering MCC: 0.538
5) Y_H2O in combined GH2-rich pre-burner flows entering MCC: 0.462
6) Temperature of combined GH2-rich pre-burner flows entering MCC: 824 K
7) Mass flow rate of LO2 entering MCC: 353 kg/s
8) Resulting O/F mixture fraction (mass ratio) entering MCC:

$$\frac{\dot{m}_{LO2}}{\dot{m}_{H2}} = \frac{353}{113.7} = 3.10$$

9) O2/H2 mass flux ratio entering MCC:

$$\frac{O2}{H2} = \frac{353\ kg/s}{Y_{H2} \times \dot{m}_{H2}}$$

$$\frac{O2}{H2} = \frac{353\,\frac{kg}{s}}{0.538 \times 113.7}$$

$$\frac{O2}{H2} = 5.77$$

10) Combustion product gas pressure in the combustion chamber: 20.6 MPa

11) Combustion product gas mass flux exiting the combustion chamber:

$$= \dot{m}_{GH2} + \dot{m}_{LO2}$$

$$= 113.7\frac{kg}{s} + \frac{353kg}{s}$$

$$= 466.7\frac{kg}{s}$$

12) Combustion chamber cross-sectional area A_C:

$$A = \pi r^2$$

$$A = \pi \times (\frac{45.1 \times 10^{-2}}{2})^2$$

$$A = 0.1597\ m^2$$

13) Throat area A*:

$$A = \pi \times (\frac{26.2 \times 10^{-2}}{2})^2$$

$$A = 0.0539\ m^2$$

14) Ratio of A_C over A*

$$\frac{A_c}{A^*} = \frac{0.16}{0.054} = 0.2962$$

Expansion Nozzle:

1) Nozzle exit diameter: 2.304 m
2) Nozzle exit area A_e:

$$A_e = \pi \times (\frac{2.304}{2})^2$$

$$A_e = 4.17m^2$$

3) Nozzle A_e/A*:

$$\frac{A_e}{A^*} = \frac{4.17}{0.054} = 77.3$$

4) Nozzle isentropic efficiency: 0.97

Frozen Flow:

1) Combustion product gas gamma value entering nozzle: 1.17
2) Combustion product gas MW entering the nozzle: 10.13 g/mol
3) M_e from non-isentropic nozzle flow w/ gamma entering nozzle:

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left\{\left(\frac{2}{\nu+1}\right)\left(1+\frac{\nu-1}{2}M^2{}_e\right)\right\}^{\frac{\nu+1}{2(\nu-1)}}\left\{1+\left(1-\frac{1}{\eta}\right)\left(\frac{\nu-1}{2}\right)M^2{}_e\right\}^{\frac{\nu}{(\nu-1)}}$$

From the given data $\frac{A_e}{A^*} = 77.3$ therefore, we get 4.063.

$$M_e = 4.063$$

4) p_e/p_t2 from non-isentropic nozzle flow w/ gamma entering nozzle:

$$\frac{P_e}{P_{t2}} = \left\{1 - \frac{1}{n}\left[\frac{(\nu-1)M_e^2}{2+(\nu-1)M_e^2}\right]\right\}^{\frac{\nu}{\nu-1}}$$

$$\frac{P_e}{P_{t2}} = 0.00136$$

5) p_e from non-isentropic nozzle flow w/ gamma entering nozzle:

$$\frac{P_e}{P_{t2}} = 0.00136$$

$$P_e = 0.00136 \times 20.6$$

$$P_e = 27.3\ kPa$$

6) T_e from non-isentropic nozzle flow w/ gamma entering nozzle:

$$\frac{T_e}{T_{t2}} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-1}$$

$$T_e = 1867.27\ K$$

7) V_e from non-isentropic nozzle flow w/ gamma entering nozzle:

$$u_e = \left\{ \frac{2\gamma}{\gamma - 1} - \frac{R_u T_{t2}}{M_w} \left[1 - \left(\frac{P_e}{P_{t2}} \right)^{\frac{\gamma - 1}{\gamma}} \right] \right\}^{\frac{1}{2}}$$

$$u_e = 5566.69 \frac{m}{s}$$

Shifting Equilibrium:

1) Combustion product gas gamma value exiting nozzle: 1.109
2) Combustion product gas MW exiting nozzle: 12.62 g/mol
3) M_e from non-isentropic nozzle flow w/ gamma exiting nozzle: 4.213
4) p_e from non-isentropic nozzle flow w/ gamma exiting nozzle: 26.1kPa
5) T_e from non-isentropic nozzle flow w/ gamma exiting nozzle: 2815 K
6) V_e from non-isentropic nozzle flow w/ gamma exiting nozzle:

$$u_e = \left\{ \frac{2\gamma}{\gamma - 1} - \frac{R_u T_{t2}}{M_w} \left[1 - \left(\frac{P_e}{P_{t2}} \right)^{\frac{\gamma - 1}{\gamma}} \right] \right\}^{\frac{1}{2}}$$

$$u_e = 6042 \frac{m}{s}$$

Thrust:

Sea level (SL):

1) Resulting jet thrust (SL):

$$Jet\ thrust = m_p \times V_e$$

$$Jet\ thrust = 6042 \times 466.7$$

$$Jet\ thrust = 2819.8\ kN$$

2) Resulting pressure thrust (SL):

$$(SL) = (P_e - P_\infty)A_c$$

$$(SL) = (26.1 \times 10^3 - 1.0135 \times 10^5) \times 4.16$$

$$(SL) = -314\ kN$$

3) Resulting nominal thrust (SL):
   It is a sum of jet thrust and pressure,

$$= 2819.8 + (-314\ kN)$$

$$= 2505\ kN$$

4) Nozzle divergence thrust loss: 0.80%
5) Resulting divergence-corrected thrust (SL):

$$= \left(1 - \frac{0.8}{100}\right) \times 2505$$

$$= 2485\ kN$$

6) Actual thrust coefficient C_T (SL)

$$C_\tau = \frac{\tau}{A_t P_0} = \frac{2485 \times 10^3}{20.6 \times 10^6 \times 0.054}$$

$$C_\tau = 2.23$$

7) Ideal thrust coefficient (C_T)_ideal (SL):

$$(C_\tau)\_ideal = \frac{2505 \times 10^3}{20.6 \times 10^6 \times 0.054}$$

$$(C_\tau)\_ideal = 2.25$$

8) Resulting nozzle C_T efficiency (SL):

$$\eta_N = \frac{(C_T)_{Actual}}{(C_T)_{Ideal}}$$

$$\eta_N = 0.991$$

9) Specific Impulse I_sp (SL):

$$I_{sp} = \frac{T}{\dot{m} \times g}$$

$$I_{sp} = 543\ s$$

Vacuum (vac):

1) Resulting jet thrust (vac):

$$= m_p \times V_e$$

$$= 2816\ kN$$

2) Resulting pressure thrust (vac):

$$\text{Resulting pressure thrust } (SL) = (P_e - P_\infty)A_c$$

$$\text{Resulting pressure thrust } (SL) = 109\ kN$$

3) Resulting nominal thrust (vac):
It is a sum of jet thrust and pressure,

$$= 2816 + 109$$

$$= 2926\ kN$$

4) Nozzle divergence thrust loss: 0.80%
5) Resulting divergence-corrected Thrust (vac):

$$= \left(1 - \frac{0.8}{100}\right) \times 2926$$

$$= 2903\ kN$$

6) Actual thrust coefficient C_T (vac):

$$C_\tau = \frac{\tau}{A_t P_0}$$

$$C_\tau = 2.61$$

7) Ideal thrust coefficient (C_T)_ideal (vac)

$$(C_\tau)\_ideal = \frac{2926 \times 10^3}{0.054 \times 20.6}$$

$$(C_\tau)\_ideal = 2.63$$

8) Resulting nozzle C_T efficiency (vac):

$$\eta_N = \frac{(C_T)_{Actual}}{(C_T)_{Ideal}}$$

$$\eta_N = 0.99$$

9) Specific Impulse I_sp (vac):

$$I_{sp} = \frac{T}{\dot{m} \times g}$$

$$I_{sp} = 634\ s$$