PnP ADMM Assignment Report

Introduction

Alternating direction method of multipliers (ADMM) is an algorithm that is used for convex optimization problems. It traditionally breaks a function into small parts which are computed separately. The concept of Plug and Play for the ADMM algorithm was introduced by Venkatakrishnan et al. [1] which essentially allowed forward models to work alongside any selected denoising algorithm (or priors). In this assignment, codes for (a) deconvolution of an image and (b) Compressed sensing of an undersampled image are written, keeping the paper by Stanley H. Chan et al. [2] as reference.

Algorithm and Results

Deconvolution

In this algorithm, the test house image having the shape 128x128 was imported. It was corrupted with two gaussian kernels to create two blurred images (img_1 and img_2). White gaussian noise was added to both these images. The proposed algorithm in [2] consists of three parts. (i) calculate $\hat{x}$, keeping v constant (ii) Calculate $\hat{v}$ keeping x constant (iii) calculate the penalty function u. For the scope of this assignment the mentioned steps are repeated in a loop for 100 interactions. Step (i) of the algorithm has been computed in the frequency domain for simplicity. The equation (1) can be written in the frequency domain by taking a fourier transform as shown in (2) and then taking it's inverse fourier transform to obtain $\hat{x}$

$$\hat{x} = \text{argmin } f(x) + (\rho/2) \,||\, x - \tilde{x} ||^2 \quad \ldots\ldots(1)$$

$$\hat{x} = F^{-1} \left\{ \frac{F(h)^{*}.\, F(y\,) + \rho F(\tilde{x})}{|F(h).F(h)| + \rho} \right\} \quad \ldots\ldots..(2)$$

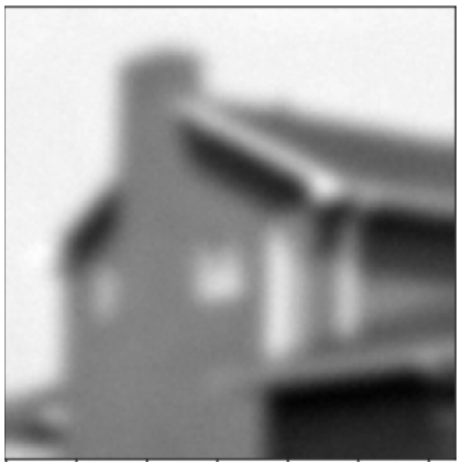
Here F(h) and F(y) are the fourier transform of the gaussian kernel and noisy image respectively. The parameters are set such as: ρ=1, γ=1, σ=14, λ=196 and η has not been considered.

Results

(Image Blurred with kernel 1)

(Resulting Deconvolved Image)

The parameters for set 2 $\rho=1$, γ, $\sigma=14$, $\lambda=196$ and η has not been considered.



(Image Blurred with kernel 2)

(Resulting Deconvolved Image)

From the resulting images it can be observed that the noisy images have become slightly sharp after 100 iterations of the algorithm. However, there seem to be some artifacts introduced around the edges of the image. The PSNR values for both denconvolutions are 29.947dB and 20.70dB respectively

Compressed Sensing

In this algorithm, we take an input test image of 128x128 size. This input image is then flattened into a single column matrix. Two measurement matrices are created having dimensions (4096*32*32) and (8192*32*32 with their standard deviations being $\frac{1}{\sqrt{4096}}$ and $\frac{1}{\sqrt{8192}}$ respectively. Using the measurement matrix and the original flattened image to get a column vector y. The reconstruction of the image is done by referencing (3) provided in [1]

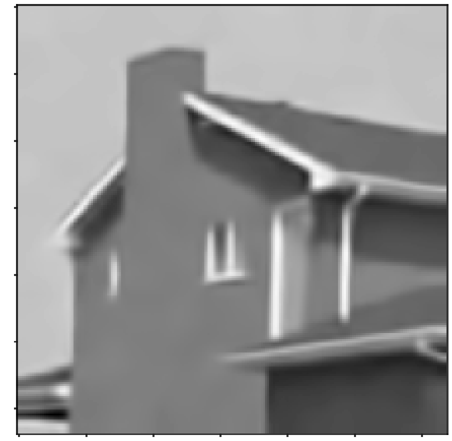
$\hat{x} = (G^T G + \rho I)^{-1} (G^T y + \rho \tilde{x})$.........(3)

Where G is the measurement matrix.

This algorithm also follows the 3 steps mentioned in the deconvolution algorithm, the difference being $\hat{x}$ is obtained from (3). Results are obtained over a period of 100 iterations. The parameters have been set as follows:

$\rho=1$, $\gamma=1$, $\sigma=35$, $\lambda=1225$ and η has not been considered.

Results



(orignal image)

(Compressed sensing reconstruction)

ρ=0.8, γ=1, σ=12, λ=144 and η has not been considered

(orignal image)

(Compressed sensing reconstruction)

From the resulting images it can be observed that the reconstructed images are quite similar to the original image after 100 iterations of the algorithm. However, there seem to be some artifacts introduced around the edges of the image. The PSNR values for both denconvolutions are 30.67dB and 37.381dB respectively

References

[1] Venkatakrishnan, Singanallur & Bouman, Charles & Wohlberg, Brendt. (2013). Plug-and-Play priors for model based reconstruction. 2013 IEEE Global Conference on Signal and Information Processing

[2] Chan, S.H., Wang, X. and Elgendy, O.A. (2016). Plug-and-play ADMM for image restoration: Fixed-point convergence and applications. IEEE Transactions on Computational Imaging, 3(1), pp.84-98.