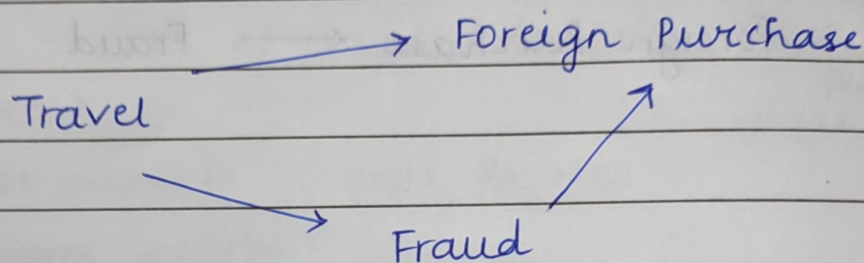


Tutorial 7

Exercise

Network Structure



- i) Travel and fraud can each cause foreign purchase. Travel explains foreign purchase and so is evident against fraud.
- ii) Increased probability of travel makes fraud more likely. Travel can cause fraud.
- iii) Increased probability of foreign purchase makes fraud more likely. Foreign purchase is evidence for fraud.

Conditional Probabilities :

Travel -			Foreign Purchase -			
True	False		Travel	Fraud	True	False
0.05	0.95		True	True	0.90	0.10
			False	True	0.10	0.90
			True	False	0.90	0.10
			False	False	0.01	0.99
Fraud -						
Travel	True	False				
True	0.01	0.99				
False	0.002	0.998				

1. Probability of a fraud if we don't know whether the card holder is traveling or not?

Classify with hidden variables

travel = ?, foreign purchase = true, fraud = ?

$$P(\text{fraud} = \text{true} \mid \text{foreign purchase} = \text{true}) =$$

$$\alpha * [P(\text{fraud} = \text{true} \mid \text{travel}) * P(\text{foreign purchase} = \text{true} \mid \text{travel}, \text{fraud} = \text{true}) * P(\text{travel})]$$

$$= \alpha * [P(\text{fraud} = \text{true} \mid \text{travel} = \text{true}) * P(\text{foreign purchase} = \text{true} \mid \text{travel} = \text{true}, \text{fraud} = \text{true}) * P(\text{travel} = \text{true})$$

$$+ P(\text{fraud} = \text{true} \mid \text{travel} = \text{false}) * P(\text{foreign purchase} = \text{true} \mid \text{travel} = \text{false}, \text{fraud} = \text{true}) * P(\text{travel} = \text{false})]$$

$$= \alpha * [0.01 * 0.90 * 0.05 + 0.002 * 0.10 * 0.95]$$

$$= \alpha * [0.00045 + 0.00019]$$

$$= 0.00064\alpha$$

$$P(\text{fraud} = \text{false} \mid \text{foreign purchase} = \text{true}) =$$

$$\alpha * [P(\text{fraud} = \text{false} \mid \text{travel} = \text{true}) * P(\text{foreign purchase} = \text{true} \mid \text{travel} = \text{true}, \text{fraud} = \text{false}) * P(\text{travel} = \text{true})$$

$$+ P(\text{fraud} = \text{false} \mid \text{travel} = \text{false}) * P(\text{foreign purchase} = \text{true} \mid \text{travel} = \text{false}, \text{fraud} = \text{false}) * P(\text{travel} = \text{false})]$$

$$= \alpha * [0.99 * 0.90 * 0.05 + 0.998 * 0.01 * 0.95]$$

$$= \alpha * [0.04455 + 0.009481]$$

$$= 0.054031\alpha$$

Thus —

$$\lambda = 1$$

$$(0.00064 + 0.054031)$$

$$\lambda = 18.2912$$

$$P(\text{fraud} = \text{true} \mid \text{foreign purchase} = \text{true}) =$$

$$0.00064 (18.2912)$$

$$= 0.01170$$

$$= 1.17\%$$

2. Probability of fraud charges

$$P(\text{fraud} = \text{true} \mid \text{foreign purchase} = \text{true},$$

$$\text{travel} = \text{true}) = \lambda * 0.00045$$

$$P(\text{fraud} = \text{false} \mid \text{foreign purchase} = \text{true},$$

$$\text{travel} = \text{true}) = \lambda * 0.04455$$

Thus,

$$\lambda = 1$$

$$(0.00045 + 0.04455)$$

$$= 22.2222$$

$$P(\text{fraud} = \text{true} \mid \text{foreign purchase} = \text{true} \mid$$

$$\text{travel} = \text{true})$$

$$= \lambda (0.00045)$$

$$= 22.2222 (0.00045)$$

$$= 0.01$$

$$= 1\%$$