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Tutorial 6

Q1. a) $P(H) = \lambda$ —— given

$P(\text{Head at } (k+1) \text{ th toss})$

Since $P(H) = \lambda$, $P(T) = 1 - P(H)$
$= 1 - \lambda$

$P(\text{Tails in the first } k \text{ tosses}) = (1-\lambda)^k$.

Following that,

first head at $(k+1) = (1-\lambda)^k (\lambda)$.

b) Let M be the number of tosses required to get the first head

Also,

Let $S = E[M]$

Given that,

Tosses are independent and expectations is additive —

$S = \lambda(1) + (1-\lambda)(S+1)$

$= \lambda + S + 1 - \lambda S - \lambda$

$\lambda S = 1$

$S = \dfrac{1}{\lambda}$

Q2. X is a random variable

a) $Var(X) = E[(X - E[X])^2]$ — given.

To prove: $Var(X) = E[X^2] - E[X]^2$.

Proof: From given,

$$Var(X) = E[(X - E[X])^2]$$

$$= E[(X^2 - 2 \cdot X \cdot E[X] + E[X]^2)]$$

$$= E[X^2] - 2 \cdot E[X \cdot E[X]] + E[X]^2$$

$$= E[X^2] - 2E[X]^2 + E[X]^2$$

$$Var(X) = E[X^2] - E[X]^2$$

Hence proved.

b) $E[X] = 0$ and $E[X^2] = 1$ — given.

$\therefore Var(X) = E[X^2] - E[X]^2$ — From (a).

$= 1 - 0^2$ — given

$Var(X) = 1$

If $Y = a + bX$ —

$Var(Y) = E[Y^2] - E[Y]^2$

$\therefore E[Y^2] = E[(a + bX)^2]$

$= E[(a^2 + 2abX + b^2X^2)]$

$= E[a^2] + 2 \cdot E[abX] + E(b^2X^2)$

$= a^2 + 2ab E[X] + b^2 E[X^2]$

$= a^2 + 0 + b^2(1)$

$= a^2 + b^2$

$\therefore E[Y]^2 = E[a + bX]^2$

$= E[a] + E[bX]$

$= a + bE[X] = a + 0 = a$

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$\therefore E[Y]^2 = a^2$

$\therefore Var(Y) = E[Y^2] - E[Y]^2$

$= a^2 + b^2 - a^2$

$= b^2$

Q3. a) $P(W') = 0.99$

$P(W) = P(W, H) + P(W, \sim H)$

$= P(W|H) P(H) + P(W|\sim H) P(\sim H)$

$= 0.99 \times 10^{-5} + (1 - 0.99999)(1 - 10^{-5})$

$\approx 1.99 \times 10^{-5}$

b) $P(H|W) = \dfrac{P(H, W)}{P(W)} = \dfrac{P(W|H) P(H)}{P(W)}$

$= \dfrac{0.99 \times 10^{-5}}{0.99 \times 10^{-5} + (1 - 0.99999)(1 - 10^{-5})}$

≈ 0.497

Ex. 1