PRADNYA TOPALE
BE COMPS
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Tutorial 5

Q1. Null Hypothesis $H_0 : p = 0.7$

Alternate Hypothesis $H_1 : p \neq 0.7$

Level of significance $\alpha = 0.10$

Test Statistic : Binomial variable X with

$p = p_0 = 0.7$ and $n = 15$.

We have,

$x = 8$ and $np_0 = 15(0.7) = 10.5$

$\therefore P = 2P(X \leq 8 \text{ when } p = 0.7)$

$= 2 \sum_{x=0}^{8} b(x, 15, 0.7)$.

$= 2(0.1311)$

$= 0.2622$

$\therefore P > 0.10$

Thus,

We cannot reject H_0. There is insufficient evidence to doubt the builder's claim.

Q2. Null Hypothesis $H_0 : p = 0.6$

Alternate Hypothesis $H_1 : p > 0.6$

Level of significance $\alpha = 0.05$

We have,

$x = 70$, $n = 100$, $p_0 = 0.6$

Test Statistic $= z = \dfrac{x - np_0}{\sqrt{np_0 q_0}}$.

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$$= \frac{70 - (100)(0.6)}{\sqrt{100(0.6)(0.4)}}$$

$= 2.0412$

$\therefore P(z > 2.04)$

$P = 0.0207$

As $P < \alpha$, we reject Ho and conclude that the new drug is superior to commonly prescribed one.

Q3. Let P_1 be the proportion of Mumbai voters and P_2 be the propagation of surrounding area residential

$\hat{P}_1 = \frac{120}{200} = 0.6$ $\hat{P}_p = \frac{120 + 240}{200 + 500} = 0.514$

$\hat{P}_2 = \frac{240}{500} = 0.48$ $\alpha = 5\% = 0.05$

Null Hypothesis Ho : $P_1 \leq P_2$.

Alternate Hypothesis H_1 : $P_1 > P_2$.

$$z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_p (1 - \hat{P}_p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.6 - 0.48}{\sqrt{0.514(1 - 0.514)\left(\frac{1}{200} + \frac{1}{500}\right)}}$$

$= 2.869$

$P = P(z > 2.869) = 0.0044$

As $P < \alpha$, we reject Ho and conclude that the proportion of Mumbai voters favouring the proposal is higher than the proportion of surrounding area voters.

Q4. a) Null Hypothesis Ho : $p = 0.2$
Alternate Hypothesis H_1 : $p > 0.2$
The critical region is in right tail.

b) Null Hypothesis Ho : $\mu = 3$
Alternate Hypothesis H_1 : $\mu \neq 3$
The critical region is in both tails.

c) Null Hypothesis Ho : $p = 0.15$
Alternate Hypothesis H_1 : $p < 0.15$
The critical region is in left tail.

d) Null Hypothesis Ho : $\mu = 500$.
Alternate Hypothesis H_1 : $\mu > 500$.
The critical region is in right tail.

e) Null Hypothesis Ho : $\mu = 15$
Alternate Hypothesis H_1 : $\mu \neq 15$
The critical region is in both tails.

Q5. Let μ_1 and μ_2 be the population mean 'robustness' of laptops supplied by company A and B resp.

Null Hypothesis $H_0 : \mu_1 = \mu_2$

Alternate Hypothesis $H_1 : \mu_1 \neq \mu_2$

Level of significance $\alpha = 0.05$.

$$\bar{X}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} x_{1i}$$

$$= \frac{9.3 + 8.8 + 6.8 + 8.7 + 8.5 + 6.7 + 8 + 6.5 + 9.2 + 7}{10}$$

$$= 7.95$$

IIIly,

$$\bar{X}_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} x_{2i}$$

$$= \frac{11 + 9.8 + 9.9 + 10.2 + 10.1 + 9.7 + 11 + 11.1 + 10.2 + 9.6}{10}$$

$$= 10.26$$

$$\therefore S_1^2 = \frac{1}{n_1 - 1} \left[\sum_{i=1}^{n_1} x_{1i}^2 - n_1 \bar{x}_1^2 \right]$$

$$= \frac{10.865}{9}$$

$$= 1.207$$

Also –

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$$S_2^2 = \frac{1}{n_2 - 1}\left[\sum_{i=1}^{n} x_{2i}^2 - n_2 \bar{x}_2^2\right]$$

$$= \frac{2.924}{9}$$

$$= 0.325$$

Since sample variances are different, we cannot assume that population variances are equal so we will use the unpooled t-test.

The degree of freedom is calculated as -

$$V = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{S_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{1.207}{10} + \frac{0.325}{10}\right)^2}{\frac{1}{10-1}\left(\frac{1.207}{10}\right)^2 + \frac{1}{10-1}\left(\frac{0.325}{10}\right)^2}$$

$$= 10.3$$

$$V \sim 10.$$

The test statistics used to test these hypothesis -

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

which under null hypothesis follows approximated t- distribution with $v = 10$ degrees of freedom.

Also,

Under the null hypothesis we have $\mu_1 - \mu_2 = 0$ so value of t- test is –

$$T = \frac{7.95 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.9$$

Since the test is two-sided, then the value of test is the doubled area under density curve of t- distribution with 10 degree of freedom, right of the absolute value of test statistic

$|t| = |-5.90| = 5.90$ i.e. the p-value is

$2P(T \geq |t|) = 2P(T \geq 5.90)$

$t_{0.0005}(10) = 4.587$ and since $|t| = 5.90$ is even greater than $P(T \geq 5.90) < 0.0005$

So,

p-value < 0.001

As $p < \alpha$, we can reject the null hypothesis in favour of the alternate hypothesis and conclude that the mean robustness of laptops is not the same for 2 companies.