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Date:
Page:

Tutorial 4

1. $x := \sum_i w_i s_i$ —— given

We know that –

$$Var(x) = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

$$= \langle (\sum_i w_i s_i)^2 \rangle - \langle \sum_i w_i s_i \rangle^2 \text{ — From given}$$

$$= \langle (\sum_i w_i s_i)^2 \rangle - \left(\sum_i w_i \langle s_i \rangle \right)^2$$

$$= \left\langle \left(\sum_i w_i s_i \right) \left(\sum_j w_j s_j \right) \right\rangle - \left(\sum_i w_i \langle s_i \rangle \right) \left(\sum_j w_j \langle s_j \rangle \right)$$

$$= \left\langle \sum_{i,j} w_i w_j s_i s_j \right\rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_{i,j} w_i w_j \langle s_i s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_i w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right) +$$

$$\sum_{i,j : i \neq j} w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$$

$$= \sum_i w_i^2 \left(\langle s_i s_j \rangle - \langle s_i \rangle^2\right) + \sum_{i,j : j \neq i} w_i w_j \left(\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle\right)$$

s_i and s_j are statistically independent for $i \neq j \Rightarrow \langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle = 0$.

Also,

$Var(s_i) = 1$

$\therefore Var(x) = \sum_i w_i^2$.

To guarantee that mixture has unit variance

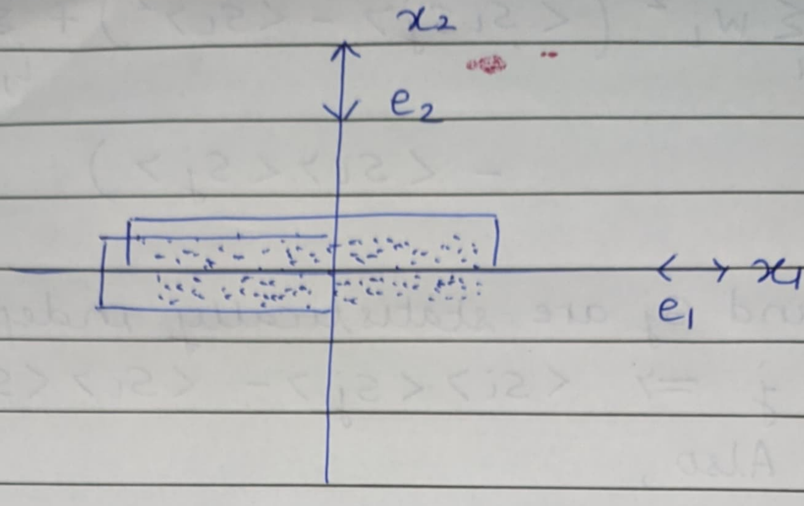
$Var(x) = 1$

$\therefore \sum_i w_i^2 = 1$

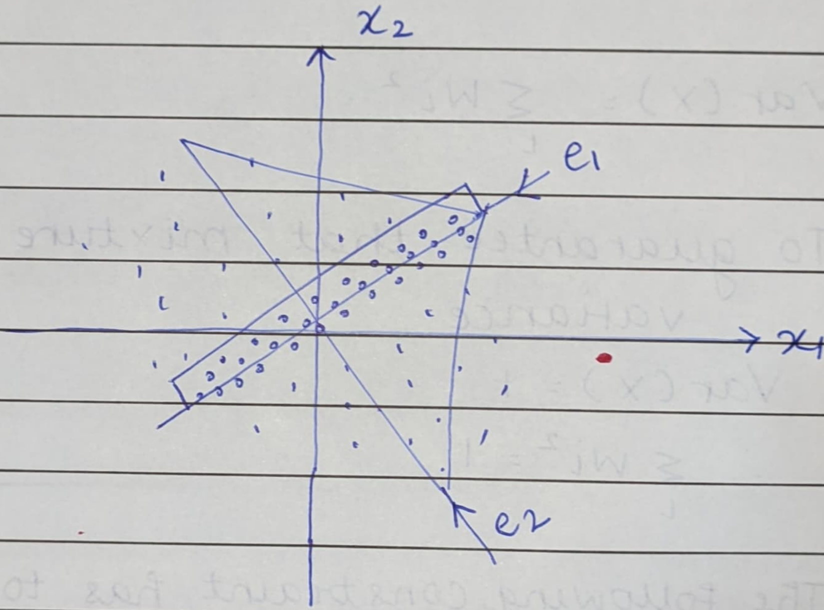
$\therefore$ The following constraint has to be imposed on the weights w_i for the mixture to have unit variance

$$\sum_i w_i^2 = 1$$

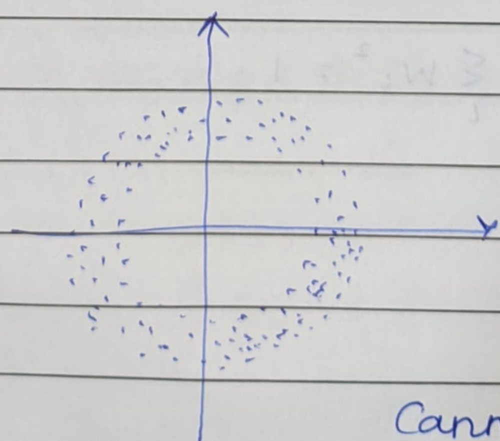
2. a)



b)



c)



Cannot be separated into ~~?~~ independent components