Analysis Of Variance (ANOVA)

Hypothesis Testing In ANOVA (Partitioning Of Variance In The Anova)

Null Hypothesis H_0 : $\mu_1 = \mu_2 = \mu_3 \cdots \mu_K$

Alternate Hypothesis H_1 : Atleast one of the sample mean is not equal

$$\mu_1 \neq \mu_2 \neq \mu_3 \cdots \neq \mu_K$$

Test Statistics

$$F = \frac{\text{Variance between Samples}}{\text{Variance within Samples}}$$

H_0 $\bar{x}_1 = \bar{x}_2 = \bar{x}_3$

H_1 Atleast one sample mean is not equal

σ^2

Variance between Samples

Variance within Samples

x_1	x_2	x_3
1	6	5
2	7	6
4	3	3
5	2	2
3	1	4
$\bar{x}_1 = 3$	$\bar{x}_2 = 19/5$	$\bar{x}_3 = 4$

One Way ANOVA

One factor with atleast 2 levels, levels are independent

① Doctors want to test a new medication which reduces headache. They split the participant into 3 condition [15mg, 30mg, 45mg]. Later on the doctor ask the patient to rate the headache between [1-10]. Are there any differences between the 3 conditions using alpha = 0.05?

Ans)

15 mg	30mg	45mg
9	7	4
8	6	3
7	6	2
8	7	3
8	8	4
9	7	3
8	6	2

① Define Null and Alternate Hypothesis?

H_0 : $\mu_{15} = \mu_{30} = \mu_{45}$

H_1 : not all μ are equal

② Significance $\alpha = 0.05$ C.I = 95

③ Calculate Degree of freedom

$N = 21 \quad a = 3 \quad n = 7$

$df_{between} = a - 1 = 3 - 1 = 2$

$df_{within} = N - a = 21 - 3 = 18$

$df_{total} = N - 1 = 20$

$df_1 \quad df_2$

$(2, 18)$

⇓

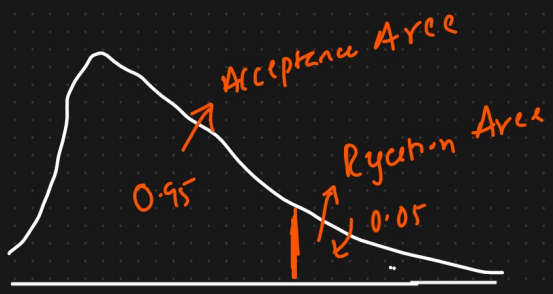
F table

$\alpha = 0.05$

⇓

Critical value

④ Decision Boundary



CV

3.5546

Decision Rule

If F is greater than 3.5546, reject the Null Hypothesis

⑤ Calculate F Test Statistics

$$F = \frac{\text{Variance between Samples}}{\text{Variance within Samples}}$$

	SS	df	MS	F
Between	98.67			
Within	10.29			
Total	108.96			

15 mg	30mg	45mg
9	7	4
8	6	3
7	6	2
8	7	3
8	8	4
9	7	3
8	6	2

① $SS_{between} = \frac{\sum (\sum a_i)^2}{n} - \frac{T^2}{N}$

15mg : 9+8+7+8+8+9+8 = 57

30mg : 7+6+6+7+8+7+6 = 47

45mg : 4+3+2+3+4+3+2 = 21

$$= \frac{57^2+47^2+21^2}{7} - \left[\frac{57^2+47^2+21^2}{21}\right]$$

$$= \boxed{98.67}$$

② $SS_{within} = \sum y^2 - \frac{\sum (\sum a_i)^2}{n}$

$\sum y^2 = 9^2+8^2+7^2+8^2+8^2+\ldots$

$= 853$

$$= 853 - \left[\frac{57^2+47^2+21^2}{7}\right]$$

$$= \boxed{10.29}$$

	SS	df	MS	F
Between	98.67	2	49.34	
Within	10.29	18	0.54	
Total	108.96	20		

$$F_{test} = \frac{MS_{Between}}{MS_{within}}$$

$$F = \frac{\text{Variance between Samples}}{\text{Variance within Samples}}$$

$$F = \frac{49.34}{0.54} = 86.56$$

If F is greater than 3.5546, Reject the H_0

$86.56 > 3.5546$ Reject the H_0