LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Fakultät für Physik im WS 2025/26
Advanced computational physics for many-body systems
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# Sheet 3: Exact diagonalization Part 2: Lanczos methods

In this exercise, we will implement Lanczos methods to calculate ground states and dynamics for our spin-$1/2$ systems.

**Problem 1** Lanczos method for ground states

We start by considering the transverse field Ising model in one dimension, given by the Hamiltonian

$$H = -J \sum_{j=0}^{L-1} \sigma_j^z \sigma_{j+1}^z - g \sum_{j=0}^{L+1} \sigma_j^x. \tag{1}$$

You can use your basis and Hamiltonian representation from Problem Sheet 2 (either with or without symmetries, as you prefer). Our goal is now to implement the Lanczos algorithm to find the ground state.

a) First, we need to generate the basis for our Krylov subspace. Generate a random state in the Hilbert space. Subsequently, construct the basis states $|\phi_m\rangle$ using the algorithm discussed in the lecture to obtain a orthonormal basis of the Krylov space.

b) Construct and diagonalize the tridiagonal matrix $T$. Plot its lowest lying eigenvalue $E_0^{(m)}$ while you add more and more states to your Krylov basis (i.e. make the subspace larger). What do you observe?

c) From the groundstate of $T$, calculate the ground state $|\psi_0\rangle$ of $H$. Compare the result to the ground state obtained on problem set 2.

**Problem 2** Lanczos method for dynamics

We now want to consider dynamical quantities. Hopefully you have written your Krylov basis construction above in a flexible way, such that you can easily re-use that code. We want to calculate the time evolution of our system after applying a spin flip operator $\hat{S}_i^+$ to the ground state $|\psi_0\rangle$ of the TFIM.

a) Find the ground state $|\psi_0\rangle$ for your choice of $g/J$. Apply the spin flip operator to obtain the first state of your Lanczos basis.

b) Construct the Lanczos basis and get the tridiagonal matrix $T$.

c) Calculate the time evolved state $|\psi(t)\rangle$. Check for small systems against the exact solution. If you see deviations, what could they be due to?

d) Calculate the time dependence of the local magnetization. What do you expect to happen?