Sheet 5: Entanglement and Schmidt decomposition

In this exercise, we will consider entanglement properties. We will make use of the simple spin-1/2 exact diagonalization code from Problem Sheet 2. If you have not written an exact diagonalization code at this point, you can use the very basic code provided in the moodle.

Problem 1 Entanglement properties

We go back to the transverse field Ising model that we have considered before. You can use your basis and Hamiltonian representation from Problem Sheet 2, the basis should ideally use the binary representation.

a) We consider open boundary conditions and an even number of sites L. Calculate the ground state of the TFIM for your choice of parameter g/J.

b) We now want to split the system into left (L) and right (R) half, to get a representation of the form

$$|\psi\rangle = \sum_{a=0}^{2^{L/2}-1} \sum_{b=0}^{2^{L/2}-1} \psi_{a,b} |a\rangle_L |b\rangle_R. \tag{1}$$

$|a\rangle_L$ labels the basis states in L and $|b\rangle_R$ the basis states in R. Both L and R have $L/2$ sites, which gives a binary representation between 0 and $2^{L/2}-1$. You can interpret $\psi_{a,b}$ as a matrix with $2^{L/2}$ columns and $2^{L/2}$ rows, indexed by a and b. This gives overall 2^L entries. It may be useful to write out a few basis states on paper, split them into L and R, and get the corresponding binary representations to better understand what you need to do.

c) A singular value decomposition of the matrix $\psi_{a,b}$ now gives the Schmidt decomposition discussed in the lecture,

$$|\psi\rangle = \sum_{\alpha} \lambda_\alpha |\alpha\rangle_L |\alpha\rangle_R. \tag{2}$$

d) Plot the Schmidt values in a meaningful way. What do you observe? Was this expected? What happens if you do the same for different Hamiltonian parameters? What is the system size dependence of this behavior?

e) What happens if, instead of the ground state, you consider an eigenstate of your Hamiltonian in the middle of the spectrum?

f) What happens if you arrange your sites in a two-dimensional geometry (i.e. change the couplings in the Hamiltonian to a 2D system)? You still can proceed along the same lines, you only have to choose the cut in a smart way!

g) How do the Schmidt values for a random state look like?