Assignment - 3

Pradyoth Singenahalli Prabhu - 02071847

Problem 1

The following table provides a probability distribution for the random variable x.

x	f(x)
2	0.2
4	0.3
7	0.4
8	0.1

a) Compute Expected value E(x).

$E(X) = \mu = \sum x \cdot f(x)$

Expected value E(X): `=SUMPRODUCT(A2:A5,B2:B5)` = 5.2

b) Compute Variance and Standard Deviation.

$Var(X) = \sigma 2 = \sum (x - \mu) 2 \cdot f(x)$

Variance: `= SUMPRODUCT(E2:E5,B2:B5)` = 4.56

$\text{Standard Deviation} : \sigma = \sqrt{\text{Variance}}$

Standard Deviation: `=SQRT(I4)` = 2.14

x	f(x)	x * f(x)	x - mean	(x-mean)^2	(x-mean)^2f(x)
2	0.2	0.4	-3.25	10.5625	2.1125
4	0.3	1.2	-1.25	1.5625	0.46875
7	0.4	2.8	1.75	3.0625	1.225
8	0.1	0.8	2.75	7.5625	0.75625
21	1	5.2	0	22.75	4.5625

Solution:

Mean	5.25
Expected Value	5.20
Variance	4.56
Standard Deviation	2.14

Problem 2

Game Description: Bob gives the contestant a free chip just for playing the game. The contestant climbs to the top of the Plinko board (see picture below) and drops the chips one at a time. The pegs send the chip bouncing all over the board until they land in slots representing money amounts at the bottom. The slots are, from left to right; $100, $500, $1000, $0, $10000, $0, $1000, $500, $100.

a) For each of the three middle slots at the top of the board (slots 4, 5, &6), find the probability that a chip starting in each slot results in winning $10,000. (Write Excel function)

The probability that a chip placed in each of the three middle slots at the top of the board (slots 4, 5, and 6) results in winning $10,000 is as follows:

$n = 12, p = 0.5$

For slot 4:

Number of trials (n) = 12 (because there are 12 pegs)

Taking exactly 5 left bounces

P(X=5) `=BINOM.DIST(5,12,0.5,FALSE)` = 0.1934

For slot 5:

Number of trials (n) = 12 (because there are 12 pegs)

Taking exactly 6 left bounces

P(X=6) `=BINOM.DIST(6,12,0.5,FALSE)` = 0.2256

For slot 6:

Number of trials (n) = 12 (because there are 12 pegs)

Taking exactly 7 left bounces

P(X=7) `=BINOM.DIST(7,12,0.5,FALSE)` = 0.1934

b) Compute the expected winnings for a chip dropped in slot 5 and the expected winnings for a chip dropped in slot 4 and 6.

Using slot 4, the expected winnings for a single chip is:

E(X) `=BINOM.DIST(1,12,0.5,FALSE)*100+BINOM.DIST(2,12,0.5,FALSE)*500 +BINOM.DIST(3,12,0.5,FALSE)*1000+BINOM.DIST(4,12,0.5,FALSE)*0 +BINOM.DIST(5,12,0.5,FALSE)*10000+BINOM.DIST(6,12,0.5,FALSE)*0 +BINOM.DIST(7,12,0.5,FALSE)*1000 + BINOM.DIST(8,12,0.5,FALSE)*500 +BINOM.DIST(9,12,0.5,FALSE)*100` = $2254.8

or

$$E(X) = 0 \cdot \frac{355}{1024} + 100 \cdot \frac{58}{1024} + 500 \cdot \frac{140.25}{1024} + 1000 \cdot \frac{253}{1024} + 10000 \cdot \frac{198}{1024} = \$2254.8$$

Using slot 5, the expected winnings for a single chip are computed as

E(X) `=(BINOM.DIST(2,12,0.5,FALSE)+BINOM.DIST(10,12,0.5,FALSE))*100+ (BINOM.DIST(3,12,0.5,FALSE)+BINOM.DIST(9,12,0.5,FALSE))*500+ (BINOM.DIST(4,12,0.5,FALSE)+BINOM.DIST(8,12,0.5,FALSE))* 1000+ (BINOM.DIST(5,12,0.5,FALSE)+BINOM.DIST(7,12,0.5,FALSE))* 0+BINOM.DIST(6,12,0.5,FALSE)*10000` = $2,554.5

or

$$E(X) = 0 \cdot \frac{396}{1024} + 100 \cdot \frac{33}{1024} + 500 \cdot \frac{110}{1024} + 1000 \cdot \frac{247.5}{1024} + 10000 \cdot \frac{231}{1024} = \$2,554.3$$

Using slot 6, the expected winnings for a single chip is

E(X) `=(BINOM.DIST(3,12,0.5,FALSE)+BINOM.DIST(11,12,0.5,FALSE))*100+`
`(BINOM.DIST(4,12,0.5,FALSE)+BINOM.DIST(10,12,0.5,FALSE))*500+`
`(BINOM.DIST(5,12,0.5,FALSE)+BINOM.DIST(9,12,0.5,FALSE))*` *1000+*
`(BINOM.DIST(6,12,0.5,FALSE)+BINOM.DIST(8,12,0.5,FALSE))*` `0+BINOM.DIST(7,12,0.5,FALSE)*10000` = $2,254.8

or

$$E(X) = 0 \cdot \frac{355}{1024} + 100 \cdot \frac{58}{1024} + 500 \cdot \frac{140.25}{1024} + 1000 \cdot \frac{253}{1024} + 10000 \cdot \frac{198}{1024} = \$2,254.8$$

Problem 3

During the period of time that a local university takes phone in registrations, calls come in at the rate of one every two minutes.

a) What is the expected number of calls in one hour?

Given that calls come in at the rate of one in every 2 minutes. it means 0.5 calls per minute so, $0.5 * 60 = 30$

b) What is the probability of three calls in ten minutes? (Write Excel function)

Expected call for ten minutes, $0.5 * 10 = 5$

P (X=3) `=POISSON.DIST(3,5,FALSE)` = 0.14

c) What is the probability of more than 3 calls in ten minutes? (Write Excel function)

P (X > 3) = 1 - P (X ≤ 3) `=1 - POISSON.DIST(3,5,TRUE)` = 1 - 0.27 = 0.73

Probability of three calls in ten minutes	0.14
probability of more than 3 calls in ten minutes	0.73

Problem 4

Axline Computers manufactures personal computers at two plants, one in Texas and the other in Hawaii. The Texas plant has 40 employees; the Hawaii plant has 20. A random sample of 10 employees is to be asked to fill out a benefits questionnaire.

Texas has 40 employees

Hawaii has 20 employees

The chances of Texas is 40/(40+20) = $\frac{40}{60}$

The chances of Hawaii is 20/(40+20) = $\frac{20}{60}$

n = 10

a) What is the probability that none of the employees in the sample work at the plant in Hawaii (to 4 decimals)? (Write Excel function)

n = 10

FALSE: We want to calculate the probability of exactly 0 events, so use FALSE.

P (X = 0) `=HYPGEOM.DIST(0, 10, 20, 60, FALSE)` = 0.0112

b) What is the probability that two or fewer of the employees in the sample works at the plant in Hawaii (to 3 decimals)? (Write Excel function)

n = 10

TRUE: We want to calculate the probability of 2 or fewer, it is cumulative probability so TRUE

P (X≤ 2) `=HYPGEOM.DIST(2,10,20,60,TRUE)` = 0.278

c) What is the probability that 3 or more of the employees in the sample work at the plant in Hawaii (to 3 decimals)? (Write Excel function)

n = 10

TRUE: We want to calculate the probability of 3 or more, it is cumulative probability so TRUE

$$P(X \geq 3) = 1 - P(X < 2)$$

P (X≥3) `=1-HYPGEOM.DIST(2,10,20,60,TRUE)` = 0.722

d) What is the probability that 9 of the employees in the sample work at the plant in Texas (to 3 decimals)? (Write Excel function)

n = 10

FALSE: We want to calculate the probability of 9, it is specific number of events occurring so FALSE

P (X = 9) `=HYPGEOM.DIST(9, 10, 40, 60, FALSE)` = 0.073

Case Study: Whole Foods Market Grows Through Mergers and Acquisitions

Discussion:
1. Whole Foods Market has shown steady growth at a time when traditional supermarkets have been flat. This could be attributed to a growing awareness of and demand for more natural foods. According to a study by Mintel in 2006, 30% of consumers have a high level of concern about the safety of the food they eat. Write the name of the distribution and the related parameters. Suppose we want to test this figure to determine if consumers have changed since then.

a) What probability distribution could be used to study this problem? (1 point)

The **binomial distribution** is a valuable probability distribution for examining whether there has been a change in customers' level of concern about the safety of the food they consume since 2006.

b) Assuming that the 30% figure still holds, what is the probability of randomly sampling 25 consumers and having 12 or more respond that they

have a high level of concern about the safety of the food they eat? (1 point)

p = 0.30; n = 25;

X = 12 or more

TRUE : cumulative

$$P(x \geq 12) = 1 - P(x \leq 11)$$

`=1 - BINOM.DIST(11, 25, 0.3, TRUE)` = 0.04424649

c) What would the expected number be? (1 point)

The expected value (μ) of a binomial distribution is given by the formula:

$$\mu = n * p$$

Where:

n = 25; p = 0.30

$$\mu = 25 * 0.30 = 7.5$$

So, assuming the 30% figure is still accurate, 7.5 individuals should be included in the sample of 25 consumers who are highly concerned about food safety. You would anticipate that, on average, 7–8 consumers would have a high level of concern because you cannot have a fraction of a consumer.

d) If a researcher actually got 12 or more out of 25 to respond that they have a high level of concern about the safety of the food they eat, what might this mean? (1 point)

The presumption that 30% of consumers care deeply about the safety of the food they eat may not be accurate if a researcher actually got 12 or more out of 25 respondents to say they had a high degree of worry about the food they eat.

2. Suppose that, on average, in a Whole Foods Market in Dallas, 3.4 customers want to check out every minute. Based on this figure, store management wants to staff checkout lines such that less than 1% of the time demand for checkout cannot be met.

a) In this case, store management would have to staff for what number of customers? (1.5 points)

Arrival rate is 3.4 per minute

Using poisson distribution

$$P(x < k) = 1 - P(x \geq k)$$

We are aiming for the cumulative probability, P(x < k), to be greater than or equal to 0.99 to ensure that less than 1% of the demand remains unmet. Our goal is to find the smallest value of "k" for which this condition is met.

P(X ≤ 8) `=POISSON.DIST(8,3.4,TRUE)` = 0.9917

i.e. 99.17% there will be 8 or less than 8 customers, so store management would have to staff for 8 customers.

b) Based on the 3.4 customer average per minute, what percentage of the time would the store have 12 or more customers who want to check out in any two-minute period? (1.5 points)

First, let's calculate the average number of customers arriving in a two-minute period based on the given average rate of 3.4 customers per minute:

= 3.4 * 2 = 6.8

Now, we want to find the probability that 12 or more customers arrive in this 2-minute period. We can use the Poisson distribution formula:

$$P(X \geq 12) = 1 - P(X \leq 11)$$

`=1 - POISSON.DIST(11, 6.8, TRUE)` = 0.044825089

= 0.044825089 * 100

= 4.48%

3. Suppose a survey is taken of 30 managers of Whole Foods Market stores and it is determined that 17 are at least 40 years old.

a) If another researcher randomly selects 10 of these 30 managers to interview, what is the probability that 3 or fewer are at least 40 years old? (1.5 points)

Probability of success (p) = $\frac{17}{30}$
Probability of failure (q) = 1 - p = $\frac{13}{30}$
Number of trials (n) = 10

To find the probability of getting 3 or fewer successes, we can calculate the cumulative probability for 0, 1, 2, and 3 successes and then sum them up.

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

P(X ≤ 3) `=HYPGEOM.DIST(3,10,17,30,TRUE)` = 0.045

i.e. 4.5% of the managers are at least 40 years old.

b) Suppose 9 of the 30 surveyed managers are female. What is the probability of randomly selecting 10 managers from the 30 and finding out that 7 of the 10 are female? (1.5 points)

This is indeed a binomial probability problem. The probability of success, which in this case is the probability of selecting a female manager, can be calculated as the ratio of the number of female managers to the total number of managers.

Probability of success (p) = $\frac{9}{30}$
Probability of failure (q) = 1 - p = $\frac{21}{30}$
Number of trials (n) = 10

Now, we want to find the probability of getting exactly 7 successes:

P(X = 7) `=HYPGEOM.DIST(7,10,9,30,FALSE)` = 0.001593609