8. This question involves the use of simple linear regression on the Auto data set.

```
In [ ]: import matplotlib.pyplot as plt
        import numpy as np
        import pandas as pd
        import seaborn as sns
        from statsmodels.api import OLS, add_constant
        from statsmodels.graphics.gofplots import ProbPlot
```

Importing data

```
In [ ]: auto = pd.read_csv("https://static1.squarespace.com/static/5ff2adbe3fe4fe33c
```

```
In [ ]: auto_1 = auto.copy()
```

Removing symbol '?'

```
In [ ]: auto = auto[auto.horsepower != '?']
```

```
In [ ]: auto = auto.drop(['cylinders','displacement','weight','acceleration','year',
```

Converting data-type 'object' to 'int64'

```
In [ ]: auto["horsepower"] = pd.to_numeric(auto["horsepower"])
        auto.dtypes
```

```
Out[ ]: mpg            float64
        horsepower       int64
        dtype: object
```

8(a) Use the lm() function to perform a simple linear regression with mpg as the response and horsepower as the predictor. Use the summary() function to print the results. Comment on the output. For example:

In [ ]:
```
def run_model(x, y, model_fit=OLS):
    '''
    accepts x and y and returns fitted model
    '''
    return model_fit(y, add_constant(x)).fit()
```

In [ ]:
```
X=auto['horsepower']
y=auto['mpg']
model_fit = run_model(x=X, y=y, model_fit=OLS)
print(model_fit.summary())
```

```
                            OLS Regression Results
==============================================================================
===
Dep. Variable:                    mpg   R-squared:                       0.
606
Model:                            OLS   Adj. R-squared:                  0.
605
Method:                 Least Squares   F-statistic:                     59
9.7
Date:                Sun, 02 Oct 2022   Prob (F-statistic):           7.03e
-81
Time:                        22:07:19   Log-Likelihood:                -117
8.7
No. Observations:                 392   AIC:                             23
61.
Df Residuals:                     390   BIC:                             23
69.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
===
                 coef    std err          t      P>|t|      [0.025      0.9
75]
------------------------------------------------------------------------------
---
const         39.9359      0.717     55.660      0.000      38.525      41.
347
horsepower    -0.1578      0.006    -24.489      0.000      -0.171      -0.
145
==============================================================================
===
Omnibus:                       16.432   Durbin-Watson:                   0.
920
Prob(Omnibus):                  0.000   Jarque-Bera (JB):                17.
305
Skew:                           0.492   Prob(JB):                     0.000
175
Kurtosis:                       3.299   Cond. No.                         3
22.
==============================================================================
===

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is corr
ectly specified.
```

Observation:

1. We can see that 'R-squared' is 0.606. So model is correct for ~60% of the time. So next I am looking for outliers.

So the equation of the linear regression line from the model is: **Y_Predict= 39.9359 - 0.1578*X**

i. Is there a relationship between the predictor and the response?

Yes, the relationship between the predictor variables and the response variable was statistically significant, because the F-statistic: 599.7 which is grater than 1 and probability value which less. This tells us we reject the null hypothesis and conclude there is statiscally relationship between "horsepower" and "mpg".

ii. How strong is the relationship between the predictor and the response?

Here value of the R-squared is 0.606, so 60% variability is explained by the model.

iii. Is the relationship between the predictor and the response positive or negative?

Here is we have negetive coefficient, which means we have negetive relationship.

iv. What is the predicted mpg associated with a horsepower of 98? What are the associated 95 % confidence and prediction intervals?

In [ ]:
```
import scipy.stats as st

#Predicting for horsepower 98
y_pred=39.9359-0.1578*98
print(y_pred)

#95% confidence interval
st.t.interval(confidence=0.95, df=len(auto['mpg'])-1, loc=np.mean(auto['mpg'
```

```
24.4715
```

Out[ ]:
```
(22.670877187773137, 24.22095954692074)
```

Answer:

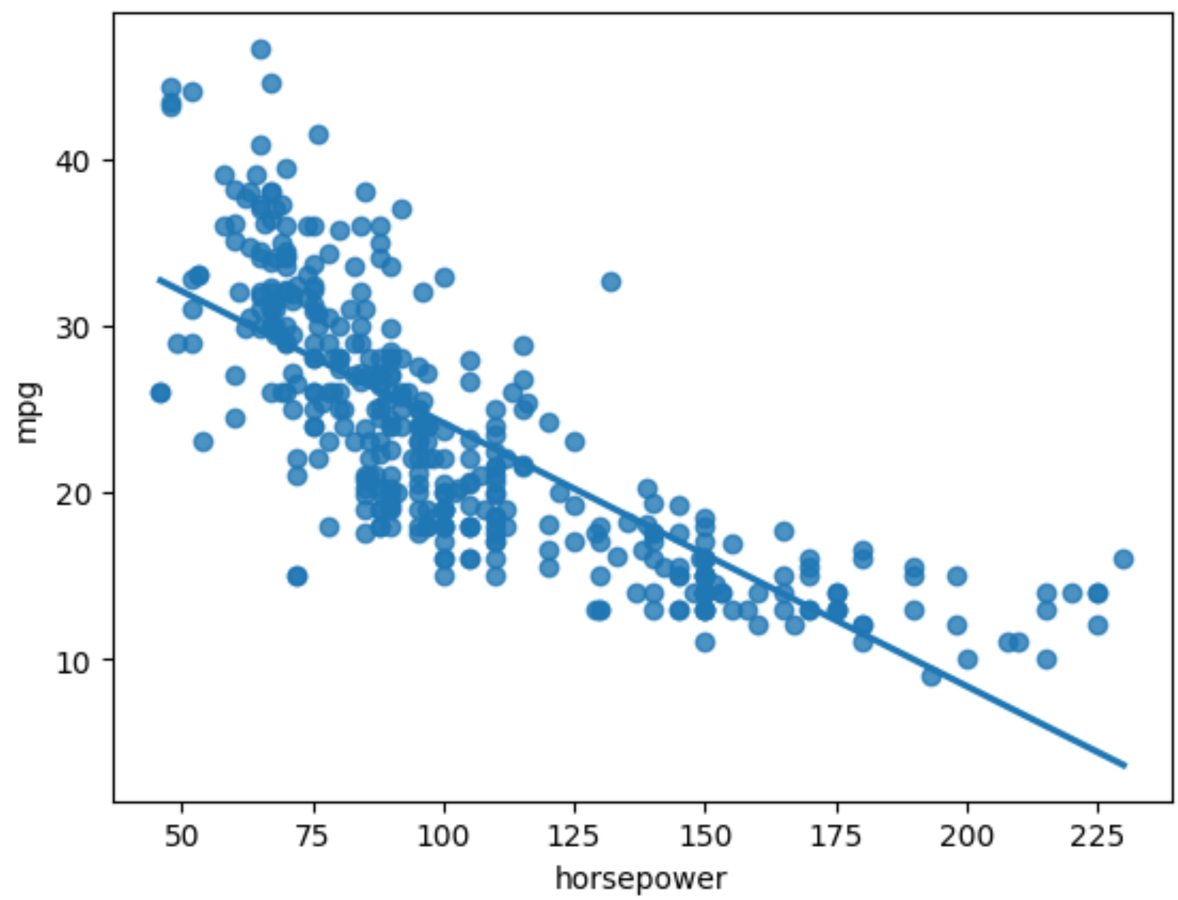
1. Predicted horsepower value of 98 is 24.47.

2. we can say With 95% confidence that this model's mileage value lies between 22.67 and 24.22.

(b) Plot the response and the predictor. Use the abline() function to display the least squares regression line.

In [ ]:
```
sns.regplot(x='horsepower', y='mpg', data=auto, ci=None)
```

Out[ ]: <AxesSubplot: xlabel='horsepower', ylabel='mpg'>



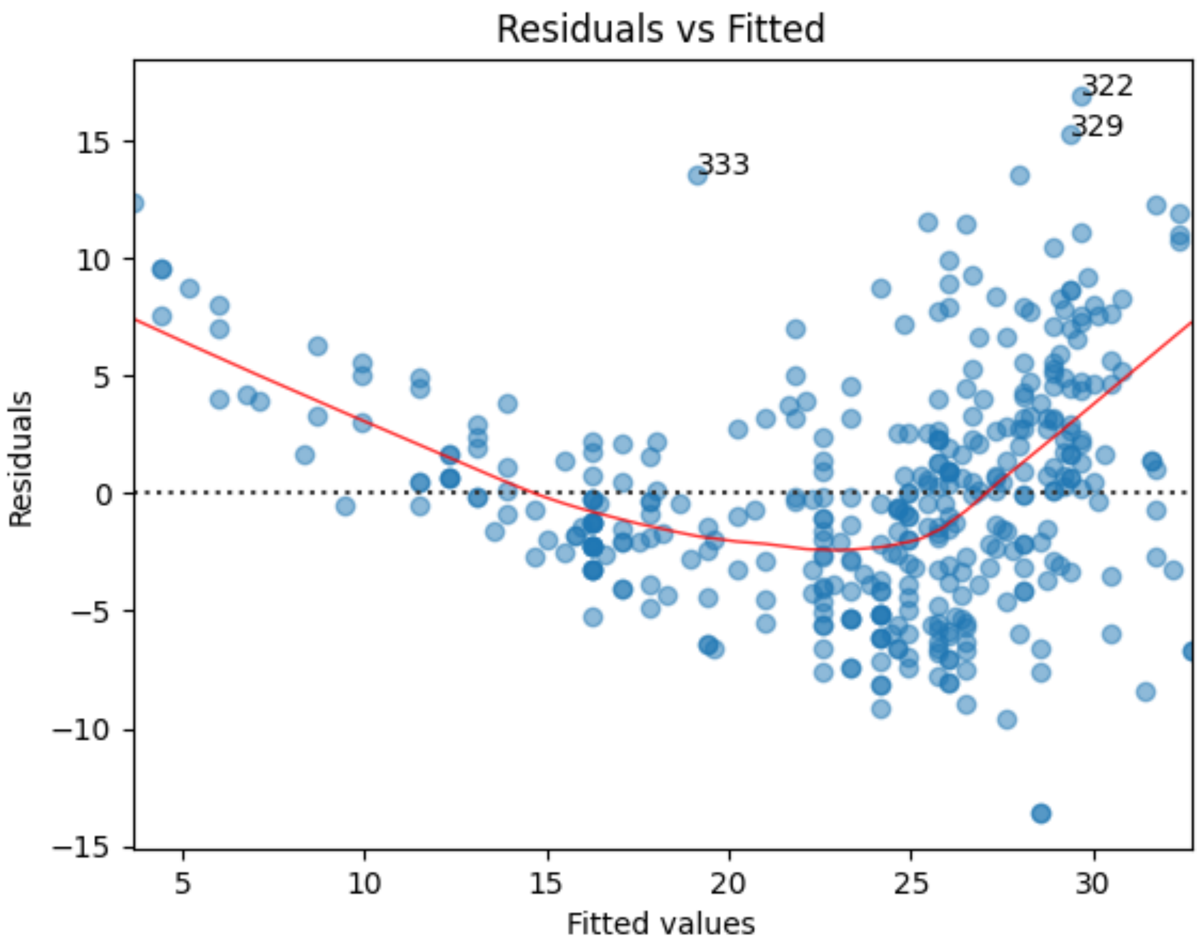
(c) Use the plot() function to produce diagnostic plots of the least squares regression fit. Comment on any problems you see with the fit.

Reference: https://robert-alvarez.github.io/2018-06-04-diagnostic_plots/

In [ ]:
```
# create dataframe from X, y for easier plot handling
dataframe = pd.concat([X, y], axis=1)

# model values
model_fitted_y = model_fit.fittedvalues
# model residuals
model_residuals = model_fit.resid
# normalized residuals
model_norm_residuals = model_fit.get_influence().resid_studentized_internal
# absolute squared normalized residuals
model_norm_residuals_abs_sqrt = np.sqrt(np.abs(model_norm_residuals))
# absolute residuals
model_abs_resid = np.abs(model_residuals)
# leverage, from statsmodels internals
model_leverage = model_fit.get_influence().hat_matrix_diag
# cook's distance, from statsmodels internals
model_cooks = model_fit.get_influence().cooks_distance[0]
```

In [ ]:
```
def _residplot(x, y, model_fitted_y):
    plot = plt.figure()
    plot.axes[0] = sns.residplot(x=model_fitted_y, y=y,
                                 lowess=True,
                                 scatter_kws={'alpha': 0.5},
                                 line_kws={'color': 'red', 'lw': 1, 'al

    plot.axes[0].set_title('Residuals vs Fitted')
    plot.axes[0].set_xlabel('Fitted values')
    plot.axes[0].set_ylabel('Residuals')

    # annotations
    abs_resid = model_abs_resid.sort_values(ascending=False)
    abs_resid_top_3 = abs_resid[:3]
    for i in abs_resid_top_3.index:
        plot.axes[0].annotate(i,
                              xy=(model_fitted_y[i],
                                  model_residuals[i]))

    return True


_ = _residplot(X, y, model_fitted_y)
```

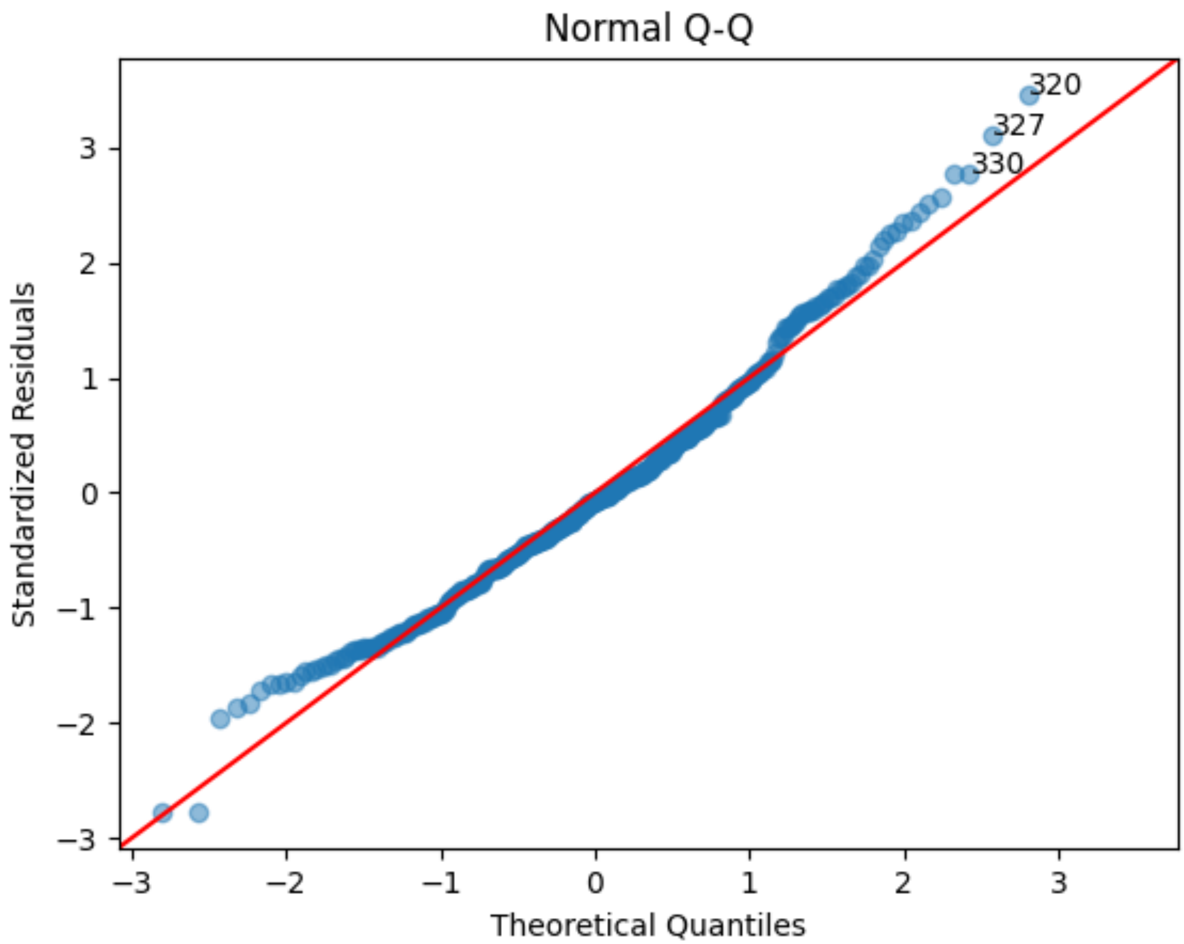
```
/var/folders/qd/78cm3bf52xs_khrj7wkd_myc0000gn/T/ipykernel_31738/765372950.
py:14: FutureWarning: The behavior of `series[i:j]` with an integer-dtype i
ndex is deprecated. In a future version, this will be treated as *label-bas
ed* indexing, consistent with e.g. `series[i]` lookups. To retain the old b
ehavior, use `series.iloc[i:j]`. To get the future behavior, use `series.lo
c[i:j]`.
  abs_resid_top_3 = abs_resid[:3]
```



Observations: In the above plot, residuals shows a U-shaped pattern, so data might be non-linear.

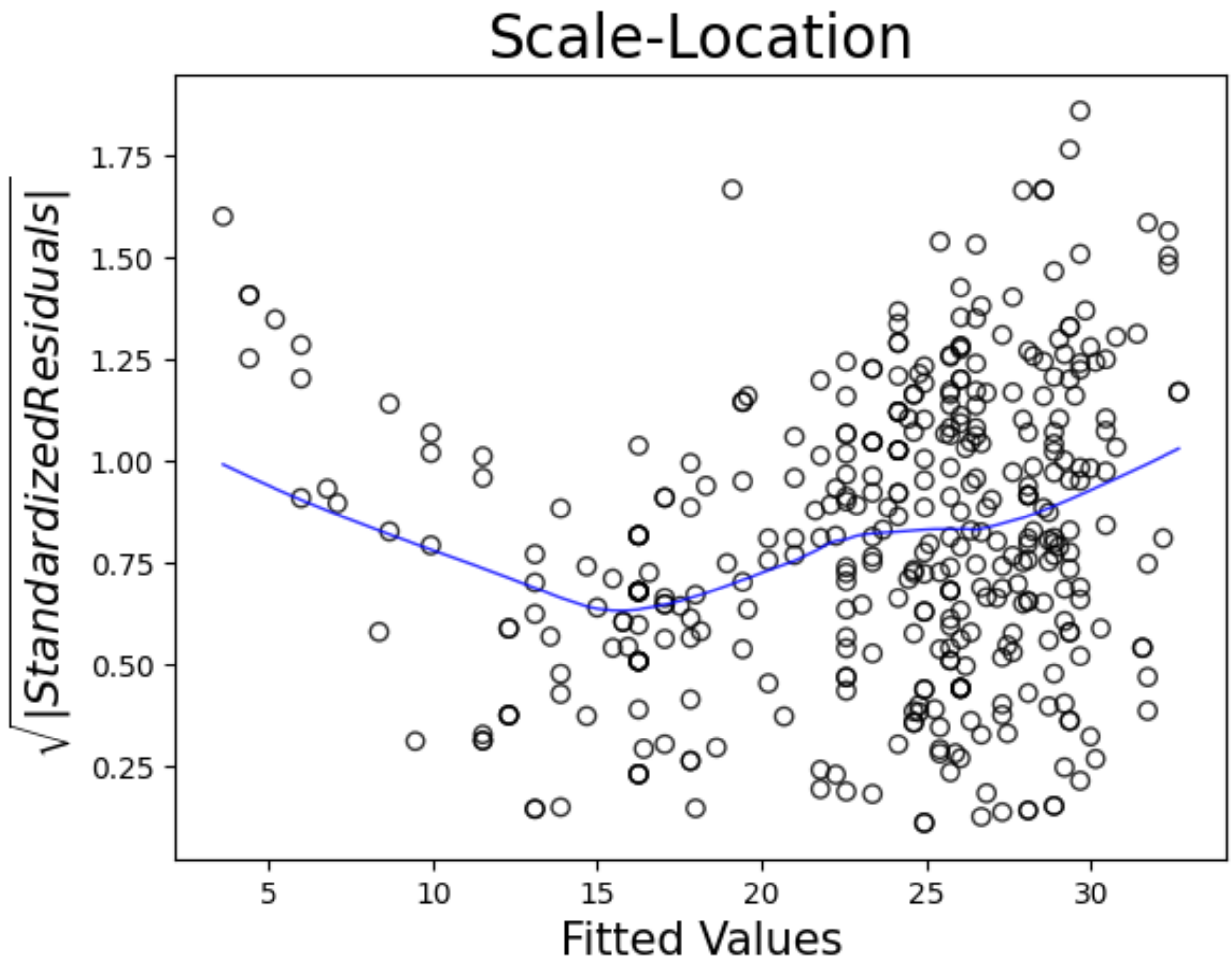
In [ ]:
```python
def _normal_qq_plot(model_norm_residuals):
    QQ = ProbPlot(model_norm_residuals)
    plot = QQ.qqplot(line='45', alpha=0.5, color='#4C72B0', lw=1)
    plot.axes[0].set_title('Normal Q-Q')
    plot.axes[0].set_xlabel('Theoretical Quantiles')
    plot.axes[0].set_ylabel('Standardized Residuals')

    # annotations
    abs_norm_resid = np.flip(np.argsort(np.abs(model_norm_residuals)), 0)
    abs_norm_resid_top_3 = abs_norm_resid[:3]

    for r, i in enumerate(abs_norm_resid_top_3):
        plot.axes[0].annotate(
            i,
            xy=(np.flip(QQ.theoretical_quantiles, 0)[r],
                model_norm_residuals[i])
        )

    return True, abs_norm_resid_top_3


_, abs_norm_resid_top_3 = _normal_qq_plot(model_norm_residuals)
```

```
/Volumes/work/MTH522/project/block_2/.venv/lib/python3.9/site-packages/stat
smodels/graphics/gofplots.py:1045: UserWarning: color is redundantly define
d by the 'color' keyword argument and the fmt string "b" (-> color=(0.0, 0.
0, 1.0, 1)). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



Observation: The above plot that residual plot is not normally distributed as all othe data points do not lie on the red color line

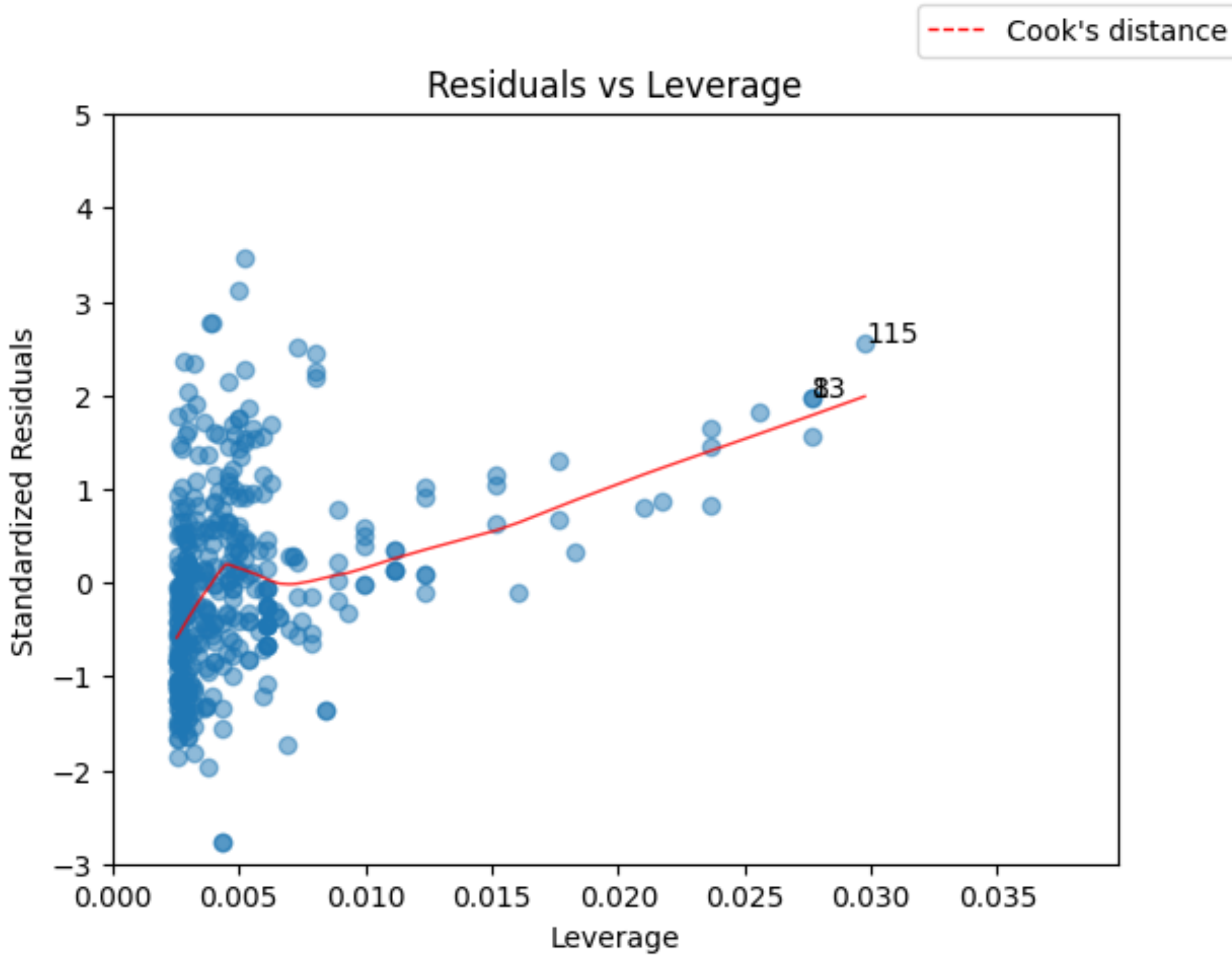
In [ ]:
```
def _scale_location_plot(abs_norm_resid_top_3):
    sns.regplot(x=model_fit.fittedvalues, y=model_norm_residuals_abs_sqrt, s
                ci=False,
                lowess=True,
                line_kws={'color': 'blue', 'lw': 1, 'alpha': 0.8},
                scatter_kws={'facecolors': 'none', 'edgecolors': 'black'})

    plt.title('Scale-Location', fontsize=20)
    plt.xlabel('Fitted Values', fontsize=15)
    plt.ylabel('$\sqrt{|Standardized Residuals|}$', fontsize=15)

    return True


_ = _scale_location_plot(abs_norm_resid_top_3)
```



Observations:

1. We can come to the conclusion that there are no outliers. As the data is standardized values should be between[-3,3]. The above data is between 0 to 2 which is accetable.

In [ ]:
```
def graph(formula, x_range, label=None):
    """
    Helper function for plotting cook's distance lines
    """
    x = x_range
    y = formula(x)
    plt.plot(x, y, label=label, lw=1, ls='--', color='red')


def _residuals_vs_leverage_plot():
    plot = plt.figure()
    plt.scatter(model_leverage, model_norm_residuals, alpha=0.5)
    sns.regplot(x=model_leverage, y=model_norm_residuals,
                scatter=False,
                ci=False,
                lowess=True,
                line_kws={'color': 'red', 'lw': 1, 'alpha': 0.8})
    plot.axes[0].set_xlim(0, max(model_leverage)+0.01)
    plot.axes[0].set_ylim(-3, 5)
    plot.axes[0].set_title('Residuals vs Leverage')
    plot.axes[0].set_xlabel('Leverage')
    plot.axes[0].set_ylabel('Standardized Residuals')

    # annotations
    leverage_top_3 = np.flip(np.argsort(model_cooks), 0)[:3]
    for i in leverage_top_3:
        plot.axes[0].annotate(i,
                                   xy=(model_leverage[i],
                                       model_norm_residuals[i])
                                   )

    p = len(model_fit.params)  # number of model parameters
    graph(lambda x: np.sqrt((0.5 * p * (1 - x)) / x),
          np.linspace(0.001, max(model_leverage), 50),
          'Cook\'s distance')  # 0.5 line
    graph(lambda x: np.sqrt((1 * p * (1 - x)) / x),
          np.linspace(0.001, max(model_leverage), 50))  # 1 line
    plot.legend(loc='upper right')

    return True


_ = _residuals_vs_leverage_plot()
```



Observation:

1. From the above graph we can say that, there is no higher leverage points. The points above cooks distance are conidered high leverage points.