7. In Sections 5.3.2 and 5.3.3, we saw that the cv.glm() function can be used in order to compute the LOOCV test error estimate. Alterna- tively, one could compute those quantities using just the glm() and predict.glm() functions, and a for loop. You will now take this ap- proach in order to compute the LOOCV error for a simple logistic regression model on the Weekly data set. Recall that in the context of classification problems, the LOOCV error is given in (5.4).

Importing Libraries

```
In [1]:  import numpy as np
         import pandas as pd
         import statsmodels.api as sm
         from sklearn.linear_model import LogisticRegression
         from sklearn.metrics import accuracy_score
         from sklearn.model_selection import LeaveOneOut, cross_val_score
```

Importing dataset

```
In [2]:  df_weekly = pd.read_csv("/Volumes/work/sem_1/MTH522/data/Weekly.csv")
         df_weekly.head()
```

Out[2]:

	Year	Lag1	Lag2	Lag3	Lag4	Lag5	Volume	Today	Direction
0	1990	0.816	1.572	-3.936	-0.229	-3.484	0.154976	-0.270	Down
1	1990	-0.270	0.816	1.572	-3.936	-0.229	0.148574	-2.576	Down
2	1990	-2.576	-0.270	0.816	1.572	-3.936	0.159837	3.514	Up
3	1990	3.514	-2.576	-0.270	0.816	1.572	0.161630	0.712	Up
4	1990	0.712	3.514	-2.576	-0.270	0.816	0.153728	1.178	Up

(a) Fit a logistic regression model that predicts Direction using Lag1 and Lag2.

```
In [3]:   df_weekly['Direction'] = np.where(df_weekly['Direction'] == 'Up', 1, 0)
```

```
In [4]:   def get_xy(df):
              X = sm.add_constant(df.iloc[:, 1:3])
              y = df.iloc[:, -1]
              return X, y
```

```
In [5]:   X, y = get_xy(df_weekly)
```

```
In [6]:   model = sm.Logit(y, X).fit()
          model.summary()
```

```
Optimization terminated successfully.
         Current function value: 0.683297
         Iterations 4
```

Out[6]:

Logit Regression Results

Dep. Variable:	Direction	No. Observations:	1089
Model:	Logit	Df Residuals:	1086
Method:	MLE	Df Model:	2
Date:	Sat, 29 Oct 2022	Pseudo R-squ.:	0.005335
Time:	21:43:02	Log-Likelihood:	-744.11
converged:	True	LL-Null:	-748.10
Covariance Type:	nonrobust	LLR p-value:	0.01848

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.2212	0.061	3.599	0.000	0.101	0.342
Lag1	-0.0387	0.026	-1.477	0.140	-0.090	0.013
Lag2	0.0602	0.027	2.270	0.023	0.008	0.112

```
In [7]:   print('Coefficient of Lag1 is ', model.params.values[1])
          print('Coefficient of Lag2 is ', model.params.values[2])
```

```
Coefficient of Lag1 is  -0.03872222247358749
Coefficient of Lag2 is   0.060248302768291226
```

Observation:

1. From summary we can see that `lag2` is significant in predicting direction because it's cofficient is less than 0.05

(b) Fit a logistic regression model that predicts Direction using Lag1 and Lag2 using all but the first observation.

```
In [8]: def get_x1y1(df):
            X = sm.add_constant(df.iloc[1:, 1:3])
            y = df.iloc[1:, -1]
            return X, y
```

```
In [9]: X1, y1 = get_x1y1(df_weekly)

        model_2 = sm.Logit(y1, X1).fit()
        model_2.summary()
```

```
Optimization terminated successfully.
        Current function value: 0.683147
        Iterations 4
```

Out[9]:

Logit Regression Results

Dep. Variable:	Direction	**No. Observations:**	1088
Model:	Logit	**Df Residuals:**	1085
Method:	MLE	**Df Model:**	2
Date:	Sat, 29 Oct 2022	**Pseudo R-squ.:**	0.005387
Time:	21:43:02	**Log-Likelihood:**	-743.26
converged:	True	**LL-Null:**	-747.29
Covariance Type:	nonrobust	**LLR p-value:**	0.01785

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.2232	0.061	3.630	0.000	0.103	0.344
Lag1	-0.0384	0.026	-1.466	0.143	-0.090	0.013
Lag2	0.0608	0.027	2.291	0.022	0.009	0.113

Observations:

1. We can see that coefficient almost remains the same even after removing the 1st row.
2. There is some difference in p-value, it is decresed from 0.01848 to 0.01785.

(c) Use the model from (b) to predict the direction of the first obser- vation. You can do this by predicting that the first observation will go up if P(Direction="Up"|Lag1,Lag2) > 0.5. Was this observation correctly classified?

In [10]:
```python
X3 = df_weekly.iloc[:1, 1:3]
x2 = sm.add_constant(X3, has_constant='add')
model_2.predict(x2)
```

Out[10]:
```
0    0.571392
dtype: float64
```

Observations:

1. Above predicted value is 0.571392, so prediction is value belongs to class `up`.
2. But actually data belongs to class `down`, so our model prediced wrong.

In [11]:
```python
df_weekly.shape
```

Out[11]: (1089, 9)

(d) Write a for loop from i=1 to i=n,wheren is the numberof observations in the data set, that performs each of the following steps:

i. Fit a logistic regression model using all but the ith obser- vation to predict Direction using Lag1 and Lag2.

ii. Compute the posterior probability of the market moving up for the ith observation.

iii. Use the posterior probability for the ith observation in order to predict whether or not the market moves up.

iv. Determine whether or not an error was made in predicting the direction for the ith observation. If an error was made, then indicate this as a 1, and otherwise indicate it as a 0.

In [12]:
```python
error = 0  # Initializing error to zero
n = df_weekly.shape[0]
y_pred = np.empty(n)
scores = np.empty(n)

for i in range(n):
    x_temp = df_weekly.loc[df_weekly.index != i, ["Lag1", "Lag2"]]
    X_new = sm.add_constant(x_temp)
    y_new = df_weekly.loc[df_weekly.index != i, "Direction"]

    mls = sm.Logit(y_new, X_new).fit()
    y_pred[i] = mls.predict(sm.add_constant(df_weekly.loc[i, ["Lag1", "La

    y_pred[i] = np.where(y_pred[i] > 0.5, int(1), 0)

    if df_weekly['Direction'][i] != y_pred[i]:
        error += 1
```

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```
In [13]:  print('Total number of errors: ', error)

          Total number of errors:   490
```

```
In [14]:  df_new = pd.DataFrame(list(y_pred), columns=['Predicted'])
          df_new['Actual'] = df_weekly['Direction'].astype(float)
          df_new.head()
```

Out[14]:

	Predicted	Actual
0	1.0	0.0
1	1.0	0.0
2	1.0	1.0
3	0.0	1.0
4	1.0	1.0

(e) Take the average of the n numbers obtained in (d)iv in order to obtain the LOOCV estimate for the test error. Comment on the results.

```
In [15]:  correct = 0

          for i in range(len(df_new)):
              if df_new['Predicted'][i] == df_new['Actual'][i]:
                  correct += 1

          average = correct / len(df_new)
          print('Average of all the accuracy: ', round(average * 100, 3))
          print('Error generated: ', round((1 - average) * 100, 3))
```

```
          Average of all the accuracy:   55.005
          Error generated:   44.995
```

```
In [16]:  X = df_weekly[["Lag1", "Lag2"]]
          y = df_weekly["Direction"]

          clf = LogisticRegression(penalty="none", solver="lbfgs")
          loo = LeaveOneOut()
          cv_scores = cross_val_score(clf, X, y, cv=loo)
```

```
In [17]:  print('Average accuracy: {0}%'.format(round(cv_scores.mean() * 100)))
          print('Test error using cross validation score: {0}%'.format(round((1 - c
```

```
          Average accuracy: 55%
          Test error using cross validation score: 45%
```

Observation:

1. Test error's LOOCV is ~0.45, which is constant with the result of computing the error using cross validation score and Leave One Out splitter.
2. From this we can conclude that, logistic regressing model uses lag1 and lag2 to predict direction performed better that guessing at random.
3. We also got approximately same error rate using a basic technique of forecasting weekly market gains.

In [ ]: