6. We continue to consider the use of a logistic regression model to predict the probability of default using income and balance on the Default data set. In particular, we will now compute estimates for the standard errors of the income and balance logistic regression co- efficients in two different ways: (1) using the bootstrap, and (2) using the standard formula for computing the standard errors in the glm() function. Do not forget to set a random seed before beginning your analysis.

```
In [13]:
          import numpy as np
          import pandas as pd
          import statsmodels.api as sm
          from sklearn.utils import resample
 In [2]: df = pd.read_csv('/Volumes/work/sem_1/MTH522/data/Default.csv')
          df.head()
             default student
 Out[2]:
                                balance
                                              income
          0
                            729.526495
                No
                        Nο
                                        44361.625074
          1
                No
                        Yes
                              817.180407
                                         12106.134700
          2
                        No 1073.549164
                                         31767.138947
                No
          3
                            529.250605 35704.493935
                No
                        No
          4
                No
                        No
                            785.655883 38463.495879
          df['default'] = np.where(df['default'] == 'Yes', 1, 0)
 In [4]:
          df
```

Out[4]:		default	student	balance	income
	0	0	No	729.526495	44361.625074
	1	0	Yes	817.180407	12106.134700
	2	0	No	1073.549164	31767.138947
	3	0	No	529.250605	35704.493935
	4	0	No	785.655883	38463.495879
	•••	•••	•••		
	9995	0	No	711.555020	52992.378914
	9996	0	No	757.962918	19660.721768
	9997	0	No	845.411989	58636.156984
	9998	0	No	1569.009053	36669.112365
	9999	0	Yes	200.922183	16862.952321

10000 rows × 4 columns

(a) Using the summary() and glm() functions, determine the estimated standard errors for the coefficients associated with income and balance in a multiple logistic regression model that uses both predictors.

```
In [7]: np.random.seed(17)
    x = df.iloc[:, 2:]
    X = sm.add_constant(x)
    Y = df.iloc[:, :1]

model = sm.Logit(Y, X).fit()
model.summary()
```

Optimization terminated successfully.

Current function value: 0.078948

Iterations 10

Out[7]: **Dep.**

Logit Regression Results

Dep. Variable:		de	No. Observations:			10000	
Model:			Logit	Df Residuals:			9997
Method:			MLE	Df Model:			2
Date:		Sun, 30 Oct 2022			Pseudo	0.4594	
Time:		07:5	52:46	Log-Likelihood:			-789.48
converged:		True			ı	-1460.3	
Covariance Type:		nonrobust			LLR p	4.541e-292	
	coef	std err		z	P> z	[0.025	0.975]
const	-11.5405	0.435	-26.5	44	0.000	-12.393	-10.688
balance	0.0056	0.000	24.8	35	0.000	0.005	0.006
income	2.081e-05	4.99e-06	4.1	74	0.000	1.1e-05	3.06e-05

Possibly complete quasi-separation: A fraction 0.14 of observations can be perfectly predicted. This might indicate that there is complete quasi-separation. In this case some parameters will not be identified.

Obsci vation.

- 1. The standard error from the model that is generated is 2.27×10^{-4} for "balance" and 4.985×10^{-6} for "income".
- (b) Write a function, boot.fn(), that takes as input the Default data set as well as an index of the observations, and that outputs the coefficient estimates for income and balance in the multiple logistic regression model.

```
In [11]: def boot_fn(data, obs):
    X = sm.add_constant(data.iloc[:obs, 2:])
    Y = data.iloc[:obs, :1]

    model = sm.Logit(Y, X).fit()

    return model.bse['income'], model.bse['balance']
```

(c) Use the boot() function together with your boot.fn() function to estimate the standard errors of the logistic regression coefficients for income and balance.

```
In [14]: np.random.seed(312)
         income = []
         balance = []
         for i in range(1000):
             sample = resample(df)
             inc, bal = boot_fn(sample, 7000)
             income.append(inc)
             balance.append(bal)
         Optimization terminated successfully.
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                  Iterations 10
         Optimization terminated successfully.
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 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.071681

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.086505

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.076233

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.081788

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.076768

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.078113

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.077561

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.077215

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.075894

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.077273

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.077159

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.076378

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.073847

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.083524

 Iterations 10
- Optimization terminated successfully.

 Current function value: 0.084330

```
Iterations 10
Optimization terminated successfully.
Current function value: 0.083152
Iterations 10
```

```
In [15]: df_error = pd.DataFrame(income, columns=['Income'])
    df_error['Balance'] = balance
    df_error
```

Out[15]:		Income	Balance
	0	0.000006	0.000294
	1	0.000005	0.000238
	2	0.000006	0.000252
	3	0.000006	0.000251
	4	0.000006	0.000270
	•••		
	995	0.000006	0.000276
	996	0.000006	0.000267
	997	0.000006	0.000255
	998	0.000006	0.000261
	999	0.000006	0.000263

1000 rows × 2 columns

```
In [17]: print('Standard error of mean for income: ', np.mean(df_error['Income']))
    print('Standard error of mean for balance: ', np.mean(df_error['Balance']))
```

Standard error of mean for income: 5.986813530229936e-06 Standard error of mean for balance: 0.00027423674240413466

(d) Comment on the estimated standard errors obtained using the glm() function and using your bootstrap function.

Observation:

1. Standard error mean after doing bootstrap is almost similar to result what we have got earlier.

```
In []:
```