Assignment - 4

Pradyoth Singenahalli Prabhu - 02071847

Problem - 1

Given a standard normal random variable (z values) or probabilities, compute the following probabilities or z values, as applicable.

a) P(0 ≤ z ≤ 0.83) (Write Excel function)

P (0 ≤ z ≤ 0.83) `=NORM.S.DIST(0.83,TRUE)-NORM.S.DIST(0,TRUE)` = 0.29673061

b) P(−1.57 ≤ z < 0) (Write Excel function)

P(−1.57 ≤ z < 0) `=NORM.S.DIST(0,TRUE)-NORM.S.DIST(-1.57,TRUE)` = 0.44179244

c) P(z > 0.44) (Write Excel function)

P(z > 0.44) `=1 - NORM.S.DIST(0.44, TRUE)` = 0.32996855

d) The area to the left of z is 0.975 (Write Excel function)

`=NORM.S.INV(0.975)` = 1.95996398

e) The area to the right of z is 0.35 (Write Excel function)

`=NORM.S.INV(0.65)` = 0.38532047

Problem - 2

The average annual amount American households spend for daily transportation is $6312 (Money, August 2001). Assume that the amount spent is normally distributed.

a) Suppose you learn that 5% of American households spend less than $1000 for daily transportation. What is the standard deviation of the amount spent? (Write Excel function)

Finding `z` for 5%:

z `=NORM.S.INV(0.05)` = -1.6448536

$z = \frac{(x - \mu)}{\sigma}$

$\sigma = \frac{(x - \mu)}{z}$

$\sigma = \frac{(1000 - 6312)}{= NORM.S.INV(0.05)}$

$\sigma = 3229.46669$

b) What is the probability that a household spends between $4000 and $6000? (Write Excel function)

Probability: `=NORM.DIST(6000,6312,F3,TRUE)-NORM.DIST(4000,6312,F3,TRUE)` = 0.22449373

Here 'F3' cell means standard deviation.

c) What is the range of spending for the 3% of households with the highest daily transportation cost? (Write Excel function)

Range: `=6312 + NORM.INV(0.97, 0, 1) * F3` = $12385.9603

Problem - 3

Conde Nast Traveler publishes a Gold List of the top hotels all over the world. The Broad-moor Hotel in Colorado Springs contains 700 rooms and is on the 2004 Gold List (Conde Nast Traveler, January 2004). Suppose Broadmoor's marketing group forecasts a mean demand of 670 rooms for the coming weekend. Assume that demand for the upcoming week-end is normally distributed with a standard deviation of 30.

a) What is the probability all the hotel's rooms will be rented? (Write Excel function)

Probability: `=NORM.DIST(700, 670, 30, TRUE)` = 0.84134475

b) What is the probability 50 or more rooms will not be rented? (Write Excel function)

To calculate the probability that 50 or more rooms will not be rented, we can use the following Excel function:

Probability: `=1 - NORMDIST(650, 670, 30, TRUE)` = 0.74750746

c) Would you recommend the hotel consider offering a promotion to increase demand? What considerations would be important?

Whether or not the Broad-moor Hotel should offer a promotion to increase demand depends on a number of factors, including the current occupancy rate, demand for rooms in the area, cost of the promotion, and impact on future bookings.

In the case of the Broad-moor Hotel, the hotel is expecting a mean demand of 670 rooms, which is below the hotel's capacity of 700 rooms. This suggests that the hotel may have some rooms available, so offering a promotion may be a good way to increase demand.

However, the hotel also needs to consider the cost of the promotion and the impact on future bookings. If the promotion is too expensive or if it leads to lower rates in the future, it may not be worth it.

Overall, the decision of whether or not to offer a promotion is a complex one that should be made on a case-by-case basis.

Here are some additional considerations for the Broad-moor Hotel:

- **Target market:** Who is the hotel's target market for this promotion? Is it families, couples, business travelers, or another group? The hotel should tailor the promotion to its target market.
- **Promotion type:** What type of promotion should the hotel offer? A discount on the room rate, a free upgrade, or a bonus amenity are all options. The hotel should choose a promotion that is attractive to its target market and that will provide a good value.
- **Promotion timing:** When should the hotel run the promotion? If the hotel is expecting low occupancy during a certain time period, it could offer a promotion to attract more guests during that time. The hotel could also offer a promotion for a limited time to create a sense of urgency.

The Broad-moor Hotel should carefully consider all of these factors before making a decision about whether or not to offer a promotion.

Problem - 4

Suppose interarrival times at a hospital emergency room during weekday are exponentially distributed, with an average interarrival time of 10 minutes. If the arrivals are Poisson

distributed, what would the average number of arrivals per hour be? What is the probability that less than 5 minutes will elapse between any two arrivals? (Write Excel function)

Average number of arrivals per hour:

The determination of the average number of arrivals per hour involves leveraging the interrelation between the exponential distribution and the Poisson distribution. According to the characteristics of an exponentially distributed variable with a rate parameter `λ`, both the mean and the variance can be expressed as `1/λ`. Thus, when considering an interarrival time of 10 minutes, the corresponding rate parameter (λ) equals `1/10` arrivals per minute. To ascertain the average number of arrivals per hour, the rate is converted to arrivals per hour.

$$\text{Average arrivals per hour} = \frac{60}{\text{interval time}}$$

For the probability that less than 5 minutes will elapse between any two arrivals, we find the cumulative distribution function (CDF) for the exponential distribution with the given rate parameter.

In Excel, the function for the average number of arrivals per hour would be:

`=60/10` = 6 arrivals per hour

Probability that less than 5 minutes will elapse between any two arrivals

The probability that less than 5 minutes will elapse between any two arrivals is given by the cumulative distribution function of the exponential distribution with parameter `λ` `=0.1` evaluated at `t` `=5`. This can be calculated using the following Excel function:

`=EXPONDIST(5, 0.1, TRUE)` = 0.39346934

The result of the function is 0.39346934, which means that there is a **39.35%** probability that less than 5 minutes will elapse between any two arrivals.

Case Study: Specialty Toys

Specialty Toys, Inc., sells a variety of new and innovative children's toys. Management learned that the preholiday season is the best time to introduce a new toy, because many families use this time to look for new ideas for December holiday gifts. When Specialty discovers a new toy with good market potential, it chooses an October market entry date. In order to get toys in its stores by October, Specialty places one-time orders with its manufacturers in June or July of each year. Demand for children's toys can be highly volatile. If a new toy catches on, a sense of shortage in the marketplace often increases

the demand to high levels and large profits can be realized. However, new toys can also flop, leaving Specialty stuck with high levels of inventory that must be sold at reduced prices. The most important question the company faces is deciding how many units of a new toy should be purchased to meet anticipated sales demand. If too few are purchased, sales will be lost; if too many are purchased, profits will be reduced because of low prices realized in clearance sales. For the coming season, Specialty plans to introduce a new product called Weather Teddy. This variation of a talking teddy bear is made by a company in Taiwan. When a child presses Teddy's hand, the bear begins to talk. A built- in barometer selects one of five responses that predict the weather conditions. The responses range from "It looks to be a very nice day! Have fun." to "I think it may rain today. Don't forget your umbrella." Tests with the product show that, even though it is not a perfect weather predictor, its predictions are surprisingly good. Several of Specialty's managers claimed Teddy gave predictions of the weather that were as good as many local television weather forecasters. As with other products, Specialty faces the decision of how many Weather Teddy units to order for the coming holiday season. Members of the management team suggested order quantities of 15 000, 18 000, 24 000, or 28 000 units. The wide range of order quantities suggested indicating considerable disagreement concerning the market potential. The product management team asks you for an analysis of the stock-out probabilities for various order quantities, an estimate of the profit potential, and to help make an order quantity recommendation. Specialty expects to sell Weather Teddy for $24 based on a cost of $16 per unit. If inventory remains after the holiday season, Specialty will sell all surplus inventory for $5 per unit. After reviewing the sales history of similar products, Specialty's senior sales forecaster predicted an expected demand of 20 000 units with a 0.95 probability that demand would be between 10 000 units and 30 000 units.

Managerial Report

Prepare a managerial report that addresses the following issues and recommends an order quantity for the Weather Teddy product.

1. Use the sales forecaster's prediction to describe a normal probability distribution that can be used to approximate the

demand distribution. Sketch the distribution and show its mean and standard deviation. (Write Excel function)

As stated in the problem, the senior sales forecaster has predicted an expected demand of 20,000 units for the Weather Teddy. Therefore, the mean of the demand distribution is 20,000 units, as given in the problem description.

Therefore, $\mu = 20000$

z-value `=NORM.S.INV(0.95)` = 1.64485363

$\sigma = \frac{(x-\mu)}{z}$

σ `=(30000-20000)/P3` = 6079.56832

2. Compute the probability of a stock-out for the order quantities suggested by members of the management team. (Write Excel function)

For 15000 orders, `=1-NORM.DIST(15000,P2,P4,TRUE)` = 0.79458299

For 18000 orders, `=1-NORM.DIST(18000,P2,P4,TRUE)` = 0.62891109

For 24000 orders, `=1-NORM.DIST(24000,P2,P4,TRUE)` = 0.25528788

For 28000 orders, `=1-NORM.DIST(28000,P2,P4,TRUE)` = 0.09410667

Orders	Probability
15000	0.79458299
18000	0.62891109
24000	0.25528788
28000	0.09410667

3. Compute the projected profit for the order quantities suggested by the management team under three scenarios: worst case in which sales = 10000 units, most likely case in which sales = 20000 units, and best case in which sales = 30000 units.

Given data:
Selling price per unit (P12) = $24

Selling price per unit of surplus (P14) = $5

Cost per unit (P13)= $16

$$\text{Revenue} = (\text{selling price per unit} \times \text{quantity sold}) + (\text{surplus price per unit} \times \text{surplus inventory})$$
$$\text{Cost} = \text{cost per unit} \times \text{order quantity}$$
$$\text{Profit} = \text{Revenue} - \text{Cost}$$

Order quantities suggested: 15,000, 18,000, 24,000, and 28,000 units

Scenario 1: Worst Case Sales = 10,000 units
Scenario 2: Most Likely Case Sales = 20,000 units
Scenario 3: Best Case Sales = 30,000 units

Computing the projected profit for each order quantity under the three scenarios.

1. For an order quantity of **15,000** units:

   Scenario 1: Profit `=(P12 * 10000)+(P14 * 5000)-(P13 * 15000)` = $25000
   Scenario 2: Profit `=(P12 * 20000)-(P13 * 15000)` = $240000
   Scenario 3: Profit `=(P12 * 30000)-(P13 * 15000)` = $480000

2. For an order quantity of **18,000** units:

   Scenario 1: Profit `=(P12 * 10000)+(P14 * 8000)-(P13 * 18000)` = $-8000
   Scenario 2: Profit `=(P12 * 20000)-(P13 * 18000)` = $192000
   Scenario 3: Profit `=(P12 * 30000) - (P13 * 18000)` = $432000

3. For an order quantity of **24,000** units:

   Scenario 1: Profit `=(P12 * 10000) + (P14 * 14000) - (P13 * 24000)` = $-74000
   Scenario 2: Profit `=(P12 * 20000) + (P14 * 4000) - (P13 * 24000)` = $116000
   Scenario 3: Profit `=(P12 * 30000) - (P13 * 24000)` = $336000

4. For an order quantity of **28,000** units:

   Scenario 1: Profit `=(P12 * 10000) + (P14 * 18000) - (P13 * 28000)` = $-118000
   Scenario 2: Profit `=(P12 * 20000) + (P14 * 8000) - (P13 * 28000)` = $72000
   Scenario 3: Profit `=(P12 * 30000) - (P13 * 28000)` = $272000

4. One of Specialty's managers felt that the profit potential was so great that the order quantity should have a 70% chance of meeting demand and only a 30% chance of any stock-outs. What quantity would be ordered under this policy, and what is the projected profit under the three sales scenarios? (Write Excel function)

Calculating the order quantity `=NORM.INV(0.7,P2,P4)` = 23188.12874 ≈ **23188**

Best-case scenario (30,000 units sold):

=MIN(order quantity, 30000) * (selling price - cost per unit) + MAX(0, (MIN(order quantity, 30000) - 30000)) * (surplus selling price - cost per unit)

`=MIN(23188, 30000) * (24 - 16) + MAX(0, (MIN(23188, 30000) - 30000)) * (5 - 16)`

= **185504**

Expected case (20,000 units sold):

=MIN(order quantity, 20000) * (selling price - cost per unit) + MAX(0, (MIN(order quantity, 20000) - 20000)) * (surplus selling price - cost per unit)

`=MIN(23188, 20000) * (24 - 16) + MAX(0, (MIN(23188, 20000) - 20000)) * (5 - 16)`

= **160000**

Worst-case scenario (10,000 units sold):

=MIN(order quantity, 10000) * (selling price - cost per unit) + MAX(0, (MIN(order quantity, 10000) - 10000)) * (surplus selling price - cost per unit)

`=MIN(23188, 10000) * (24 - 16) + MAX(0, (MIN(23188, 10000) - 10000)) * (5 - 16)`

= **80000**

5. Provide your own recommendation for an order quantity and note the associated profit projections. Provide a rationale for your recommendation.

Considering the data and calculations, I recommend ordering 24,000 units. While the calculated optimal order quantity is approximately 23,188 units, rounding it off to 24,000 units would provide a buffer against any potential unexpected increase in demand, and it aligns well with the management's concerns regarding stock-outs (Best-Case Scenario Profit: $336,000).

This decision is based on the fact that ordering 24,000 units strikes a balance between meeting potential demand while minimizing the risk of surplus inventory that might need to be sold at a reduced price. It also aligns with the company's goal of maximizing profit potential, as the projected profits for the 24,000 unit order are quite substantial across all three sales scenarios, particularly when compared to the profits from the other suggested order quantities.

Additionally, the projected profits for this order quantity are notably higher across all three sales scenarios compared to the other suggested order quantities, demonstrating the potential for maximizing profit potential.