Problem 1(i)

```
function R = cholesky_factorization(A)
n = size(A,1);

R = triu(A);

for k = 1:n
    for i = (k+1):n
        m = R(k,i)/R(k,k);
        R(i,i:n) = R(i,i:n) - m * R(k,i:n);
    end

    R(k,k:n) = R(k,k:n)/sqrt(R(k,k));
end

end
```

```
Not enough input arguments.

Error in cholesky_factorization (line 2)
n = size(A,1);
```

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Problem 1(ii)

```
function X = backward_substitution(R,b)
n = size(R,1);

X = zeros(1,n);

X(n) = b(n)/R(n,n);

for i = n-1:-1:1
    sum_ = 0;

    for j=i+1:n
        sum_ = sum_ + R(i,j)*X(j);
    end

    X(i) = (b(i) - sum_)/R(i,i);

end
```

```
Not enough input arguments.

Error in backward_substitution (line 2)
n = size(R,1);
```

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Problem 1(ii)

```
function Y = forward_substitution(R,b)
n=size(R,1);

Y(1) = b(1)/R(1,1);

for i=2:n
    sum = 0;
    for j=1:i-1
        sum = sum + R(i,j)*Y(j);
    end
    Y(i) = (b(i)-sum)/R(i,i);
end
```

```
Not enough input arguments.

Error in forward_substitution (line 2)
n=size(R,1);
```

Problem 1(iii)

```
A = [4 1 1 1;1 3 -1 1;1 -1 2 0;1 1 0 2];
b = [0.65;0.05;0;0.5];

R = cholesky_factorization(A);

Y = forward_substitution(R.',b);
X = backward_substitution(R,Y);

disp(X)
```

```
    0.2000   -0.2000   -0.2000    0.2500
```

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Problem 2(i)

2(i)

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \tilde{b} = \begin{bmatrix} 1+\varepsilon \\ 1-\varepsilon \end{bmatrix}$$

$$x - \tilde{x} = A^{-1}b - A^{-1}\tilde{b}$$

Finding A^{-1},

$$A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{1-(-1)}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

So $$A^{-1}b = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5-0.5 \\ 0.5+0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A^{-1}\tilde{b} = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 1+\varepsilon \\ 1-\varepsilon \end{bmatrix} = \begin{bmatrix} 0.5(1+\varepsilon) - 0.5(1-\varepsilon) \\ 0.5(1+\varepsilon) + 0.5(1-\varepsilon) \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{0.5} + 0.5\varepsilon - \cancel{0.5} + 0.5\varepsilon \\ 0.5 + \cancel{0.5\varepsilon} + 0.5 - \cancel{0.5\varepsilon} \end{bmatrix} = \begin{bmatrix} \varepsilon \\ 1 \end{bmatrix}$$

So $$x - \tilde{x} = A^{-1}b - A^{-1}\tilde{b}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} \varepsilon \\ 1 \end{bmatrix}$$

$$x - \tilde{x} = \begin{bmatrix} -\varepsilon \\ 0 \end{bmatrix}$$

Problem 2(i)

```
e = 10^-10;

A = [1 1; -1 1];
b = [1; 1];

cond_A = cond(A);
inv_A = inv(A);

b_err = [1+e; 1-e];

x = A\b;

x_err = A\b_err;

x_diff = x - x_err;

fprintf("\ncond A: %d", cond_A)
fprintf("\ninv A %d", inv_A)
fprintf("\nX difference: %d\n", x_diff);
```

```
cond A: 1.000000e+00
inv A 5.000000e-01
inv A 5.000000e-01
inv A -5.000000e-01
inv A 5.000000e-01
X difference: -1.000000e-10

X difference: 0
```

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Comment:
Magnitude of error in b = e = 10^-10 and x = e = 10^-10

Problem 2(ii)

ii)
$$A = \begin{bmatrix} -1+\varepsilon & 1 \\ -1 & 1 \end{bmatrix}$$

So
$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 1 & -1 \\ 1 & -1+\varepsilon \end{bmatrix} = \frac{1}{(-1+\varepsilon)-(-1)} \begin{bmatrix} 1 & -1 \\ 1 & -1+\varepsilon \end{bmatrix}$$

$$A^{-1} = \frac{1}{\varepsilon} \begin{bmatrix} 1 & -1 \\ 1 & -1+\varepsilon \end{bmatrix} = \begin{bmatrix} 1/\varepsilon & -1/\varepsilon \\ 1/\varepsilon & 1-1/\varepsilon \end{bmatrix}$$

finding
$$A^{-1}b = \begin{bmatrix} 1/\varepsilon - 1/\varepsilon \\ \cancel{1/\varepsilon} + 1 - \cancel{1/\varepsilon} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

finding
$$A^{-1}\tilde{b} = \begin{bmatrix} 1/\varepsilon & -1/\varepsilon \\ 1/\varepsilon & 1-1/\varepsilon \end{bmatrix} \begin{bmatrix} 1+\varepsilon \\ 1-\varepsilon \end{bmatrix} = \begin{bmatrix} 1/\varepsilon(1+\varepsilon) - 1/\varepsilon(1-\varepsilon) \\ 1/\varepsilon(1+\varepsilon) + (1-1/\varepsilon)(1-\varepsilon) \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \cancel{1/\varepsilon} - \cancel{1/\varepsilon} + 1 \\ \cancel{1/\varepsilon} + 1 + 1 - \varepsilon - \cancel{1/\varepsilon} + 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3-\varepsilon \end{bmatrix}$$

$$x - \tilde{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -2 \\ 3-\varepsilon \end{bmatrix} = \begin{bmatrix} -2 \\ 1-3-\varepsilon \end{bmatrix} = \begin{bmatrix} -2 \\ -2+\varepsilon \end{bmatrix}$$

Problem 2(ii)

```
e = 10^-10;

A = [-1+e 1; -1 1];
b = [1; 1];

cond_A = cond(A);
inv_A = inv(A);

b_err = [1+e; 1-e];

x = A\b;

x_err = A\b_err;

x_diff = x - x_err;

fprintf("\ncond A: %d", cond_A)
fprintf("\ninv A %d", inv_A)
fprintf("\nX difference: %d\n", x_diff);
```

```
cond A: 4.000001e+10
inv A 9.999999e+09
inv A 9.999999e+09
inv A -9.999999e+09
inv A -9.999999e+09
X difference: -2.000000e+00

X difference: -2
```

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Comment:
Magnitude of error in b = e = 10^-10 and x = 10^0