ASSIGNMENT – 4

1) $\vec{x} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ $\quad F\vec{x} = (\sqrt{3}, 0, 0)^T = \begin{pmatrix} \sqrt{3} \\ 0 \\ 0 \end{pmatrix}$

Finding $\vec{v} = F\vec{x} - \vec{x} = \begin{pmatrix} \sqrt{3} \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.732 \\ -1 \\ -1 \end{pmatrix}$

Matrix $F = \left(I - 2 \dfrac{\vec{v}\,\vec{v}^*}{\vec{v}^* \vec{v}} \right)$

Finding, $\vec{v}\,\vec{v}^* = \begin{pmatrix} 2.732 \\ -1 \\ -1 \end{pmatrix} (2.732 \quad -1 \quad -1)$

$$\vec{v}\,\vec{v}^* = \begin{pmatrix} 7.464 & -2.732 & -2.732 \\ -2.732 & 1 & 1 \\ -2.732 & 1 & 1 \end{pmatrix}$$

Finding, $\vec{v}^* \vec{v} = (2.732 \quad -1 \quad -1) \begin{pmatrix} 2.732 \\ -1 \\ -1 \end{pmatrix} = 9.464$

Finding $\dfrac{2\vec{v}\,\vec{v}^*}{\vec{v}^* \vec{v}} = \dfrac{2}{9.464} \begin{pmatrix} 7.464 & -2.732 & -2.732 \\ -2.732 & 1 & 1 \\ -2.732 & 1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1.577 & -0.577 & -0.577 \\ -0.577 & 0.211 & 0.211 \\ -0.577 & 0.211 & 0.211 \end{pmatrix}$$

$$\therefore \quad F = I - 2\frac{\vec{u}\,\vec{v}^{*}}{\vec{v}^{*}\vec{v}}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1.577 & -0.577 & -0.577 \\ -0.577 & 0.211 & 0.211 \\ -0.577 & 0.211 & 0.211 \end{pmatrix}$$

$$F = \begin{pmatrix} 0.577 & 0.577 & 0.577 \\ 0.577 & 0.788 & -0.211 \\ 0.577 & -0.211 & 0.788 \end{pmatrix}$$

2) Given data points $\{(0,1), (1,2), (2,2), (3,3)\}$

$$P(x) = C_0 + C_1 x + \cdots + C_{n-1} x^{n-1} \quad \text{for } n \leq m$$

$$\left.\begin{aligned} P_1(x_1) &= C_0 + C_1 x_1 \\ P_2(x_2) &= C_0 + C_1 x_2 \\ P_3(x_3) &= C_0 + C_1 x_3 \\ P_4(x_4) &= C_0 + C_1 x_4 \end{aligned}\right\} = \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} C_0 \\ C_1 \end{bmatrix}}_{x} \simeq \underbrace{\begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}}_{b}$$

$$\begin{bmatrix} C_0 \\ C_0 + C_1 \\ C_0 + 2C_1 \\ C_0 + 3C_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}$$

$$A^* A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix}$$

$$A^* B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 15 \end{bmatrix}$$

$$A^* A \hat{x} = A^* b$$

$$\begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = \begin{bmatrix} 8 \\ 15 \end{bmatrix}$$

$\hat{x}$ is given by $\hat{x} = (A^* A)^{-1} (A^* b)$

$$\Rightarrow \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 14 & -6 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} 8 \\ 15 \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 22 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = \begin{bmatrix} 11/10 \\ 3/5 \end{bmatrix}$$

Problem 3a

```
function H = Hessenberg(A)
    [m, ~] = size(A);

    for k = 1:m - 2
        x = A(k + 1:m, k);
        e = zeros(size(x));
        e(1) = 1;

        if sign(x(1)) == 0
            v = norm(x) * e + x;
        else
            v = sign(x(1)) * norm(x) * e + x;
        end

        v = v / norm(v);

        A(k + 1:m, k:m) = A(k + 1:m, k:m) - 2 * v * (v.' * A(k + 1:m, k:m));

        A(1:m, k + 1:m) = A(1:m, k + 1:m) - 2 * (A(1:m, k + 1:m) * v) * v.';
    end

    H = A

end
```

```
Not enough input arguments.

Error in Hessenberg (line 2)
    [m, ~] = size(A);
```

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Problem 3b

```
A = hilb(4);

Hessenberg(A);
```

```
H =

  Columns 1 through 3

   1.000000000000000  -0.650854139658888   0.000000000000000
  -0.650854139658888   0.650585480093677   0.063911879959868
  -0.000000000000000   0.063911879959868   0.025320143416558
  -0.000000000000000   0.000000000000000  -0.001165208041306

  Column 4

   0.000000000000000
  -0.000000000000000
  -0.001165208041306
   0.000284852680241
```

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Problem 4a

```
function lambda = Rayleigh(A,v0,tol,N)
v=zeros(size(v0,1),(N+1));

v(:,1) = v0;
lambda(1) = v(:,1).'*A*v(:,1);

I = eye(size(A,1));

for k=1:N
        temp = A - (lambda(k) * I);
        w = temp\v(:,k);
        v(:,k+1) = w/norm(w);
        lambda(k+1) = v(:,k+1).'*A*v(:,k+1);

        if abs(lambda(k+1) - lambda(k)) < tol
            break;
        end

end

lambda
```

```
Not enough input arguments.

Error in Rayleigh (line 2)
v=zeros(size(v0,1),(N+1));
```

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Problem 4b

```
format long

A = [1 1 1;1 2 1;1 1 3];
v0 = [1; 0; 0];
tol = 10^(-10);

Rayleigh(A,v0,tol,50);
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate.
RCOND =  2.749333e-18.

lambda =

  Columns 1 through 3

   1.000000000000000   0.500000000000000   0.330508474576271

  Columns 4 through 6

   0.324869268764669   0.324869129433354   0.324869129433354
```

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