Problem 1

1) Given

$$A = \begin{pmatrix} 1 & 3 & -2 \\ -4 & 0 & -1 \\ 2 & -2 & 0 \end{pmatrix} \quad \text{and} \quad \vec{v} = \begin{pmatrix} -2 \\ 1+i \\ 1 \end{pmatrix}$$

a)

$$a\vec{v} = \begin{pmatrix} -2+3+3i-2 \\ 8+0-1 \\ -4-2-2i+0 \end{pmatrix} = \begin{pmatrix} -1+3i \\ 7 \\ -6-2i \end{pmatrix}$$

b)

$$\|\vec{v}\|_1 = |-2| + |1+i| + |1|$$
$$= 2 + \sqrt{2} + 1$$
$$= 3 + \sqrt{2}$$

$$|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$$

c)

$$\|\vec{v}\|_2 = \left(|-2|^2 + |1+i|^2 + |1|^2\right)^{1/2}$$
$$= \left(4 + (\sqrt{2})^2 + 1\right)^{1/2}$$
$$= (4+2+1)^{1/2}$$
$$= (7)^{1/2}$$
$$= \sqrt{7}$$

d)

$$\|\vec{v}\|_\infty = \max(2, \sqrt{2}, 1)$$
$$= 2$$

e) $\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{m} |a_{ij}|$ (max Column Sum)

$= \max(1+4+2,\ 3+0+2,\ 2+1+0)$

$= \max(7, 5, 3)$

$= \underline{\underline{7}}$

f) $\|A\|_\infty = \max_{1 \le i \le m} \sum_{j=1}^{n} |a_{ij}|$ (max row sum)

$= \max(1+3+2,\ 4+0+1,\ 2+2+0)$

$= \max(6, 5, 4)$

$= \underline{\underline{6}}$

g) $\|A\|_F = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2 \right)^{1/2}$

$= (1+16+4+9+0+4+4+1+0)^{1/2}$

$= \underline{\underline{\sqrt{39}}}$

Problem 2

a) Given

$$2x_1 - 6\alpha x_2 = 3$$
$$3\alpha x_1 - x_2 = \frac{3}{2}$$

$$3\alpha x_1 - x_2 = \frac{3}{2} \quad (E_1)$$

$$2x_1 - 6\alpha x_2 = 3 \quad (E_2)$$

$$m_2 = \frac{2}{3\alpha}$$

$$\boxed{E_2 - m_2 E_1 \to E_2} \Rightarrow \left[\begin{array}{cc|c} 3\alpha x_1 & -x_2 & \frac{3}{2} \\ 0 & \left(-6\alpha + \frac{2}{3\alpha}\right)x_2 & 3 - 1/\alpha \end{array}\right]$$

$$\Rightarrow 2x_1 - \frac{2}{\cancel{3\alpha}} \times \cancel{3\alpha} \cancel{x_1} - 6\alpha x_2 + \frac{2}{3\alpha} x_2 = 3 - \frac{\cancel{2}}{3\alpha} \frac{\cancel{3}}{\cancel{2}}$$

c)

$$\Rightarrow 0 + \left(-6\alpha + \frac{2}{3\alpha}\right) x_2 = 3 - \frac{1}{\alpha}$$

$$\left(\frac{-18\alpha^2 + 2}{3\cancel{\alpha}}\right) x_2 = \frac{3\alpha - 1}{\cancel{\alpha}}$$

$$\left(2 - 18\alpha^2\right) x_2 = 9\alpha - 3$$

$$x_2 = \left(\frac{9\alpha - 3}{2 - 18\alpha^2}\right)$$

$$= \frac{3}{2} \frac{(3\alpha - 1)}{(1^2 - (3\alpha)^2)}$$

$$= \frac{3}{2} \frac{\cancel{(1-3\alpha)}}{(1+3\alpha)\cancel{(1-3\alpha)}}$$

$$\boxed{x_2 = -\frac{3}{2}\left(\frac{1}{1+3\alpha}\right)}$$

Backward Substitution

$$x_1 = \frac{3+6\alpha x_2}{2}$$

$$= \frac{3 + \cancel{6}^{3}\alpha\left(\frac{-3}{\cancel{2}} \times \frac{1}{(1+3\alpha)}\right)}{2}$$

$$= \frac{3}{2}\left(1 - \frac{3\alpha}{(1+3\alpha)}\right)$$

$$= \frac{3}{2}\left(\frac{1+\cancel{3\alpha}-\cancel{3\alpha}}{1+3\alpha}\right)$$

$$\boxed{x_1 = \frac{3}{2(1+3\alpha)}}$$

For x_1 and x_2 System has unique solutions.

a) For the equation to have no solution

equation

$$-6\alpha + \frac{2}{3\alpha} = 0$$

$$-18\alpha^2 + 2 = 0$$

$$18\alpha^2 = 2$$

$$\alpha^2 = \frac{1}{9}$$

$$\alpha = \pm\sqrt{\frac{1}{9}}$$

$$\boxed{\alpha = \pm\frac{1}{3}}$$

So $\alpha = \frac{1}{3}$ or $-\frac{1}{3}$

$\Rightarrow$ Substituting $\alpha = -\frac{1}{3}$

$$\Rightarrow \quad -6\alpha + \frac{2}{3\alpha} = 3 - \frac{1}{\alpha}$$

$$\Rightarrow \quad -6\left(-\frac{1}{3}\right) + \frac{2}{3(-1/3)} = 3 - \frac{1}{(-1/3)}$$

$$\Rightarrow \quad 2 - 2 = 3 - (-3)$$

$$0 = 6 \qquad LHS \neq RHS$$

$\Rightarrow$ So for $\alpha = -\frac{1}{3}$, equation has no solution

b) $\Rightarrow$ Substituting $\alpha = +\frac{1}{3}$

$$\Rightarrow \quad -6\alpha + \frac{2}{3\alpha} = 3 - \frac{1}{\alpha}$$

$$\Rightarrow \quad -6\left(\frac{1}{3}\right) + \frac{2}{3}\left(\frac{1}{1/3}\right) = 3 - \frac{1}{(1/3)}$$

$$\Rightarrow \quad -2 + 2 = 3 - 3$$

$$\Rightarrow \quad 0 = 0 \qquad LHS = RHS$$

$\Rightarrow$ So with $\alpha = \frac{1}{3}$, equation has infinite solutions

Problem 3a - Gaussian Elimination

```
% Gaussian Elimination without pivoting

function X=myGE(A)
n=size(A,1);

% Elimination
for k = 1:(n-1)
    for i = (k+1):n
        m = A(i,k)/A(k,k);
        A(i,:) = A(i,:) - m*A(k,:);
    end
end

% Backward Substitution
X = zeros(1,n);

X(n) = A(n,n+1)/A(n,n);

for i = n-1:-1:1
    sum_ = 0;
    for j=i+1:n
        sum_ = sum_ + A(i,j)*X(j);
    end
    X(i) = (A(i,n+1) - sum_)/A(i,i);
end
```

```
Not enough input arguments.

Error in myGE (line 4)
n=size(A,1);
```

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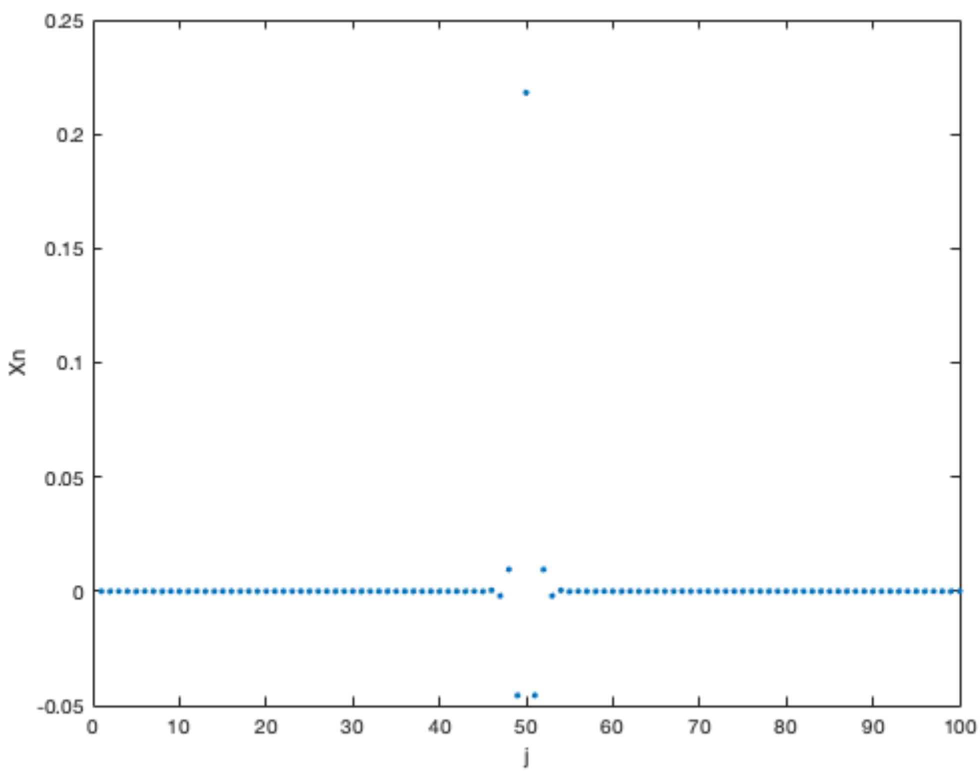
Problem 3b

```
% Generating Matrix A and vector b
n=100;
v = (ones(n,1))*5;
A = diag(v);

A = A + diag(ones(n-1,1),1) + diag(ones(n-1,1),-1);

b = zeros(100,1);
b(50,1) = 1;

A = [A, b];

% Calling function
X=myGE(A);

% Plot
plot(X,'.')
xlabel('j')
ylabel('Xn')
```



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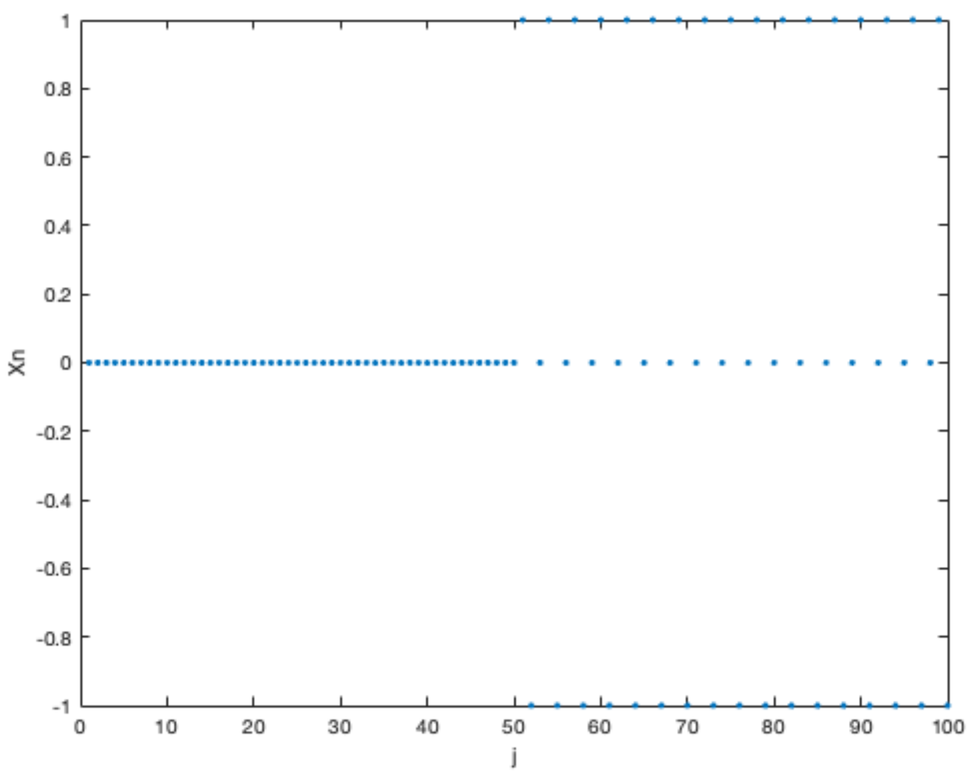
Problem 4a - Gaussian Elimination without pivoting

```
% Gaussian Elimination with partial pivoting

function X=myGEPP(A)
n = size(A,1);

% Elimination
for k = 1:(n-1)

    % Partial Pivoting
    for p = (k+1):n
        if (abs(A(k,k)) < abs(A(p,k)))
            A([k p],:) = A([p k],:);
        end
    end

    for i = (k+1):n
        mi = A(i,k)/A(k,k);
        A(i,:) = A(i,:) - mi*A(k,:);
    end
end

% Backward Substitution
X = zeros(1,n);

X(n) = A(n,n+1)/A(n,n);

for i = n-1:-1:1
    sum_ = 0;
    for j=i+1:n
        sum_ = sum_ + A(i,j)*X(j);
    end
    X(i) = (A(i,n+1) - sum_)/A(i,i);
end
```

```
Not enough input arguments.

Error in myGEPP (line 4)
n = size(A,1);
```

Problem 4b

```
% Generating Matrix A and vector b
n=100;
v = ones(n,1);
A = diag(v);

A = A + diag(ones(n-1,1),1) + diag(ones(n-1,1),-1);

b = zeros(100,1);
b(50,1) = 1;

A = [A, b];

% Calling function
X=myGEPP(A);

% Plot
plot(X, '.')
xlabel('j')
ylabel('Xn')
```



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