MATH 473/573 Assignment # 3

Due on November 15, 2022 (Tuesday)

Instruction:

1. For questions solved by hand, please show middle steps. A simple final answer without necessary justification will receive no credit.
2. For questions involving coding, please include all the MATLAB functions that you defined in the MATLAB editor window, all the commands you typed in the MATLAB main window, and all the **required** numerical results. Please do NOT show intermediate outputs that are not required!
3. Please submit your solution as a single .pdf file on MyCourses. Homework late for more than 3 days will not be accepted.

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1. [**6 pts**] Consider the matrices

$$A_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{pmatrix}.$$

(a) Find the orthogonal projection onto range(A_1).
(b) Find the orthogonal projection P onto range(A_2)

2. [**11 pts**] **(a)** Write a MATLAB function $[\mathsf{Q},\mathsf{R}] = \mathsf{mgs}(\mathsf{A})$ that computes a reduced QR factorization $A = \hat{Q}\hat{R}$ of an $m \times n$ matrix A with $m \geq n$ using the modified Gram-Schmidt orthogonalization algorithm that we discussed in class. The output variables are a matrix $Q \in \mathbb{C}^{m\times n}$ with orthonormal columns and a triangular matrix $R \in \mathbb{C}^{n\times n}$.
(b) Use your code in part **(a)** to compute the reduced QR factorization of the matrix A_2 in problem 1, and then use it to solve the least squares problem $A_2\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (1, 1, 1)^T$. Output the matrices Q, R and the least squares solution $\mathbf{x}$.

3. [**13 pts**] **(a)** Write a MATLAB function $[\mathsf{V},\mathsf{R}] = \mathsf{house}(\mathsf{A})$ that computes an implicit representation of a full QR factorization $A = QR$ of an $m \times n$ matrix A with $m \geq n$ using the Householder reflection algorithm that we discussed in class. The output variables are a lower triangular matrix $V \in \mathbb{C}^{m\times n}$ whose columns in the lower triangular part are the vectors v_k defining the successive Householder reflections, and a triangular matrix $R \in \mathbb{C}^{m\times n}$.

(b) Write a MATLAB function Q = formQ(V) that takes the matrix V produced by house as input and generates a corresponding $m \times n$ orthogonal matrix Q.
(c) Use your code in part **(a)** **(b)** to compute the full QR factorization of the matrix A_2 in problem 1. Output the matrices V, R and Q.

4. (Bonus problem) [**5 pts**] Suppose $m = 50, n = 10$. We want to use a polynomial of degree $n - 1$ (with n coefficients) for the least squares fitting of the function $\cos(4t)$ on m equally spaced grid points from 0 to 1. The least squares problem is in the form of $A\mathbf{x} = \mathbf{b}$, where A is the $m \times n$ Vandermonde matrix and $\mathbf{b}$ is the column vector containing the values of the function $\cos(4t)$ evaluated on the m grid points.

Now form the matrix A and the vector $\mathbf{b}$. Then calculate and print (to sixteen-digit precision) the least squares coefficient vector x by the following methods:
(a) Formation and solution of the normal equations, using MATLAB's \,
(b) QR factorization computed by mgs (modified Gram-Schmidt, problem 2),
(c) QR factorization computed by house (Householder triangularization, problem 3),
(d) x = A\b in MATLAB (also based on QR factorization),

The calculations above will produce four lists of ten coefficients. In each list, marked with red pen the digits that appear to be wrong (affected by rounding error). Comment on what differences you observes.