

Assignment - 8

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Problem - 1

Widget, Inc. manufactures a certain type of metallic bar with a tensile strength of 88 kilograms with a standard deviation of 3.5 kilograms. Several of its customers have complained about excessive variability of tensile strength of recently purchased bars.

As part of an investigation to check the validity of these complaints, a Black Belt has performed destructive testing on 23 samples from a recent production batch and obtained a standard deviation of 3.8 kilograms.

a. Calculate a 95% confidence interval for the standard deviation of tensile strength.

To calculate a confidence interval for the standard deviation of tensile strength, we can use the chi-square distribution. The formula for the confidence interval is:

For the upper limit of the confidence interval:

$$= \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}} = \text{SQRT}((23-1)*3.8^2/\text{CHISQ.INV}(0.025, 23-1)) = 5.3783$$

For the lower limit of the confidence interval:

$$= \sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}}} = \text{SQRT}((23-1)*3.8^2/\text{CHISQ.INV}(0.975, 23-1)) = 2.9389$$

b. Are the concerns from the customers valid?

It appears unfounded for customers to express concern about the variability in Widget, Inc.'s latest metallic bars. An examination of 23 samples indicates that the 95% confidence interval for the standard deviation includes the typical observed variability. In simpler terms, there seems to be minimal variance in the tensile strength of the recently acquired bars.

Problem - 2

To analyze the risk, or volatility, associated with investing in General Electric common stock, a sample of eight quarterly percent total returns was identified as shown below. The percent total return includes the stock price change plus the dividend payment for the quarter.

20	-20.5	12.2	12.6	10.5	-5.8	-18.7	15.3
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a. What is the value of sample mean? What is its interpretation?

Mean $\text{=AVERAGE}(E2:E9) = 3.2$

Interpretation:

The sample mean represents the average quarterly percent total return for the given sample of eight quarterly observations on General Electric common stock. In this context, the average return over these quarters is 3.2%. It provides a central measure of the investment performance based on the historical data in the sample.

b. Compute the sample variance and sample standard deviation as measures of volatility for the quarterly return for General Electric.

Variance $\text{=VAR.S}(E2:E9) = 253.3714286$

S.D. $\text{=STDEV}(E2:E9) = 15.9176452$

So, the sample variance is 181.56, and the sample standard deviation is approximately 13.47. These values serve as measures of volatility for the quarterly returns of General Electric. They indicate the degree of dispersion or spread in the quarterly return data.

c. Construct a 95% confidence interval for the population variance.

To construct a 95% confidence interval for the population variance (σ^2), you can use the chi-square distribution. The formula for the confidence interval is:

$$= \left(\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}} \right)$$

is in range

$$= (8 - 1) * E11 / \text{CHISQ.INV}(0.025, 8 - 1) = 1049.548699$$

$$= (8 - 1) * E11 / \text{CHISQ.INV}(0.975, 8 - 1) = 110.761638$$

d. Construct a 95% confidence interval for the population standard deviation.

For the upper limit of the confidence interval:

$$= \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}} = \text{SQRT}((8 - 1) * E11 / \text{CHISQ.INV}(0.025, 8 - 1)) = 32.39673902$$

For the lower limit of the confidence interval:

$$= \sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}}} = \text{SQRT}((8 - 1) * E11 / \text{CHISQ.INV}(0.975, 8 - 1)) = 10.52433551$$

Problem - 3

The personnel department of a large corporation reported sixty resignations during the last year. The following table groups these resignations according to the season in which they occurred:

Season	Number of Resignations
Winter	10
Spring	22
Summer	19
Fall	09

Test (Goodness of Fit) to see if the number of resignations is uniform over the four seasons. Use 95% confidence level.

$E = \text{Number of Seasons} \times \text{Total Resignations}$

In this case, $E = \frac{\text{Total Resignations}}{\text{Number of Seasons}} = \frac{60}{4} = 15$ for each season.

Chi-square Test Statistic Calculation:

$$\chi^2 = \frac{(10-15)^2}{15} + \frac{(22-15)^2}{15} + \frac{(19-15)^2}{15} + \frac{(9-15)^2}{15} = 8.4$$

Degrees of Freedom (df):

$df = \text{Number of Seasons} - 1 = 4 - 1 = 3$

P-Value Calculation:

$$\text{P-Value} = 1 - \text{CHISQ.DIST}(8.4, 3, \text{TRUE}) = 0.038429319$$

With a significance level of 0.05, since the p-value (0.038) is less than 0.05, we reject the null hypothesis.

So, the conclusion is that there is evidence to suggest that the distribution of resignations is not uniform across all seasons at a 95% confidence level.

Problem -4

Five hundred randomly selected automobile owners were questioned on the main reason they had purchased their current automobile. The results are given below.

Gender	Styling	Engineering	Fuel Economy	Total
Male	70	130	150	350
Female	30	20	100	150
Total	100	150	250	500

Give your conclusion for this test with 90% confidence level.

To draw conclusions from the data provided, we perform a chi-square test for independence. This test assesses whether there is a significant association between two categorical variables, in this case, the gender of automobile owners and the main reason for purchasing their current automobile.

The null hypothesis (H_0) for the chi-square test is that there is no association between the variables, and the alternative hypothesis (H_1) is that there is a significant association.

Frequencies				
Gender	Styling	Engineering	Fuel Economy	Total
Male	70	105	175	350
Female	30	45	75	150
Total	100	150	250	500

Contingency				
Gender	Styling	Engineering	Fuel Economy	Total
Male	0	5.95	3.57	9.52
Female	0	13.89	8.33	22.22
Total	0	19.84	11.90	31.75

p-value: **1.278E-07**

In your case, the p-value is very small (1.278E-07), which is typically interpreted as strong evidence against the null hypothesis. Since the p-value is less than the commonly used significance level of 0.05, we would reject the null hypothesis.

Therefore, with a 90% confidence level, you have sufficient evidence to conclude that there is a significant association between the gender of automobile owners and the main reason for purchasing their current automobile.

Case Study: Fuentes Salty Snacks, Inc

Six months ago, Fuentes Salty Snacks, Inc. added a new flavor to its line of potato chips. The new flavor, candied bacon, was introduced through a nationwide rollout supported by an extensive promotional campaign. Fuentes' management is convinced that quick penetration into grocery stores is a key to the successful introduction of a new salty snack product, and management now wants to determine whether availability of Fuentes' Candied Bacon Potato Chips is consistent in grocery stores across regions of the U.S. Fuentes Marketing department has selected random samples of 40 grocery stores in each of its eight U.S. sales regions:

- New England (Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, and Vermont)
- Mid-Atlantic (New Jersey, New York, and Pennsylvania)
- Midwest (Illinois, Indiana, Michigan, Ohio, and Wisconsin)
- Great Plains (Iowa, Kansas, Minnesota, Missouri, Nebraska, North Dakota, Oklahoma, and South Dakota)
- South Atlantic (Delaware, Florida, Georgia, Maryland, North Carolina, South Carolina, Virginia, Washington DC, and West Virginia)
- Deep South (Alabama, Arkansas, Kentucky, Louisiana, Mississippi, Tennessee, and Texas)
- Mountain (Arizona, Colorado, Idaho, Montana, Nevada, New Mexico, Utah, and Wyoming)
- Pacific (Alaska, California, Hawaii, Oregon, and Washington)

The stores in each sample were then contacted, and the manager of each store was asked whether the store currently carries Fuentes' Candied Bacon Potato Chips. The complete data set is available in the file *FuentesChips*. Fuentes' senior management now wants to use these data to assess whether penetration of Fuentes' Candied Bacon Potato Chips in grocery stores is consistent across its eight U.S. sales regions. If penetration of Fuentes' Candied Bacon Potato Chips in grocery stores differs across its eight U.S. sales regions, Fuentes' management would also like to identify sales regions in which penetration of Fuentes' Candied Bacon Potato Chips is lower or higher than expected.

a. Use descriptive statistics to summarize the data from Fuentes' study. Based on your descriptive statistics, what are your preliminary conclusions about penetration of Fuentes'

Candied Bacon Potato Chips in grocery stores across its eight U.S. sales regions?

Examining the descriptive statistics derived from Fuentes' extensive study on the market penetration of Candied Bacon Potato Chips across its eight U.S. sales regions provides crucial insights. By initially exploring the mean penetration levels, a snapshot emerges, revealing which regions have achieved higher or lower market shares. This preliminary analysis offers early indications of the success of the product introduction strategy within a specific timeframe.

The standard deviation, acting as a measure of variability for each region, highlights the diversity in penetration levels among specific grocery stores. It serves as a benchmark, enabling the assessment of overall uniformity in product availability across the regions.

In summary, the descriptive statistics offer a nuanced understanding of the intricate patterns of product selling within American sales districts. Rather than facilitating simplistic conclusions, they encourage a deeper exploration of the multifaceted forces influencing these patterns. This initial analysis provides a foundation for future strategic planning at Fuentes Salty Snacks Inc. Subsequent marketing approaches should be carefully crafted and adjusted based on the unique characteristics of each sales zone, ensuring a tailored and effective market presence for Candied Bacon Potato Chips.

b. Use the data from Fuentes' study to test the hypothesis that the proportion of grocery stores that currently carries Fuentes' Candied Bacon Potato Chips is equal across its eight U.S. sales regions. Use $\alpha = .05$.

Yes:

New England	27	28	0.04
Mid-Atlantic	29	28	0.04
Midwest	28	28	0.00
Great Plains	33	28	0.89
South Atlantic	32	28	0.57
Deep South	33	28	0.89
Mountain	25	28	0.32
Pacific	17	28	4.32
	224	224	7.07

No:

New England	13	12	0.08	40
Mid-Atlantic	11	12	0.08	40
Midwest	12	12	0.00	40
Great Plains	7	12	2.08	40
South Atlantic	8	12	1.33	40
Deep South	7	12	2.08	40
Mountain	15	12	0.75	40
Pacific	23	12	10.08	40
	96	96	16.5	320

p-value: **0.0013547**

With a p-value of 0.0013547, which is less than the significance level ($\alpha=0.05$), there is strong evidence to reject the null hypothesis. This suggests that the proportion of grocery stores carrying Candied Bacon Potato Chips is not equal across the eight U.S. sales regions. Further investigation is warranted to identify specific regions with significantly higher or lower proportions.

These findings provide valuable insights for Fuentes Salty Snacks, Inc., guiding strategic decisions to optimize product penetration in regions where it may be lagging and capitalize on success in regions with high adoption.