MATH 473/MTH 573 Assignment # 1

Due on September 27, 2022 (Tuesday)

Instruction:
1. For questions solved by hand, please show middle steps. A simple final answer without necessary justification will receive no credit.

2. For questions involving coding, please include all the MATLAB functions that you defined in the MATLAB editor window, all the commands you typed in the MATLAB main window, and all the **required** numerical results. Please do NOT show intermediate outputs that are not required!
3. Please submit your solution as a single .pdf file on MyCourses. Homework late for more than 3 days will not be accepted.

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1. **[14 pts]** Given

$$A = \begin{pmatrix} 1 & 3 & -2 \\ -4 & 0 & -1 \\ 2 & -2 & 0 \end{pmatrix} \qquad \text{and} \qquad \vec{v} = \begin{pmatrix} -2 \\ 1+i \\ 1 \end{pmatrix},$$

find the values of the following expressions.

(**a**) $A\vec{v}$, (**b**) $\|\vec{v}\|_1$, (**c**) $\|\vec{v}\|_2$, (**d**) $\|\vec{v}\|_\infty$, (**e**) $\|A\|_1$, (**f**) $\|A\|_\infty$, (**g**) $\|A\|_F$

2. **[7 pts]** Given the linear system

$$\begin{aligned} 2x_1 - 6\alpha x_2 &= 3, \\ 3\alpha x_1 - x_2 &= \frac{3}{2}. \end{aligned}$$

(a) Find value(s) of α for which the system has no solutions.
(b) Find value(s) of α for which the system has an infinite number of solutions.
(c) Assuming a unique solution exists for a given α, find the solution in terms of α.

3. **[9 pts]** **(a)** Write a MATLAB function to implement Gaussian Elimination without pivoting.

(b) Use your MATLAB function from part **(a)** to solve the following linear systems

$$A\vec{x} = \vec{b}, \quad \text{where } A = (a_{ij}), \quad \vec{b} = (b_j)$$

with

$$a_{jj} = 5,\ a_{j,j-1} = 1,\ a_{j,j+1} = 1,\ b_{50} = 1,$$

and $a_{ij} = b_j = 0$, otherwise, with $1 \leq i, j \leq 100$. Plot the solution vector $\vec{x} = (x_j)$ as a function of j.

4. (Bonus problem) [3 pts]: Write a MATLAB function for Gaussian Elimination with partial pivoting and use it to solve the following linear system

$$A\vec{x} = \vec{b}, \quad \text{where } A = (a_{ij}), \quad \vec{b} = (b_j)$$

with

$$a_{jj} = \mathbf{1},\ a_{j,j-1} = 1,\ a_{j,j+1} = 1,\ b_{50} = 1,$$

and $a_{ij} = b_j = 0$, otherwise, with $1 \leq i, j \leq 100$. Plot the solution vector $\vec{x} = (x_j)$ as a function of j.