MATH 473/MTH 573 Assignment # 2

Due on October 13, 2022 (Thusday)

Instruction:

1. For questions solved by hand, please show middle steps. A simple final answer without necessary justification will receive no credit.

2. For questions involving coding, please include all the MATLAB functions that you defined in the MATLAB editor window, all the commands you typed in the MATLAB main window, and all the **required** numerical results. Please do NOT show intermediate outputs that are not required!

3. Please submit your solution as a single .pdf file on MyCourses. Homework late for more than 3 days will not be accepted.

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1. [**16 pts**] **(i)** Write a MATLAB function to implement the Cholesky factorization $A = R^T R$.
(ii) Write MATLAB functions to implement the forward substitution and the backward substitution for solving triangular systems.
(iii) Use your MATLAB functions from part **(i)** and part **(ii)** to solve the following linear system

$$\begin{aligned} 4x_1 + x_2 \quad &+x_3 + x_4 &&= 0.65, \\ x_1 + 3x_2 \ &-x_3 + x_4 &&= 0.05, \\ x_1 - x_2 \ &+2x_3 &&= 0, \\ x_1 + x_2 \quad &\quad + 2x_4 &&= 0.5. \end{aligned}$$

2. [**14 pts**] **(i)** Consider the linear system $Ax = b$, where

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

The solution to the linear system $Ax = b$ is $x = A^{-1}b$. If you make a small mistake in b and instead use

$$\tilde{b} = \begin{pmatrix} 1+\epsilon \\ 1-\epsilon \end{pmatrix}$$

as the new right hand side, then the solution to the linear system $A\tilde{x} = \tilde{b}$ is $\tilde{x} = A^{-1}\tilde{b}$. First, compute by hand to get an explicit expression of the error

$$x - \tilde{x} := A^{-1}b - A^{-1}\tilde{b}.$$

Next, set $\epsilon = 10^{-10}$ and use Matlab's built-in functions **cond** and **inv** to compute the condition number of A and the error $x - \tilde{x} := A^{-1}b - A^{-1}\tilde{b}$. Comment on the magnitude of the error of x compared to the error of b.

(ii) Now we consider a different matrix A, which is

$$A = \begin{pmatrix} -1+\epsilon & 1 \\ -1 & 1 \end{pmatrix}.$$

The vectors b and $\tilde{b}$ are the same as in part (i). First, compute the error $x - \tilde{x} = A^{-1}b - A^{-1}\tilde{b}$ by hand. Next, set $\epsilon = 10^{-10}$ and use Matlab's built-in functions **cond** and **inv** to compute the condition number of A and the error $A^{-1}b - A^{-1}\tilde{b}$. Comment on the magnitude of the error of x compared to the error of b.