Hypothesis Testing

Objectives:

- Differentiate between null and alternate hypotheses
- Describe the two types of errors in sampling
- Interpret the confidence level, significance level, and power of a test
- Explain the types of hypothesis tests

Hypothesis

It is an assertion or a statement about:

- The state of nature
- The true value of an unknown population parameter

Each hypothesis:

- Implies its contradiction or alternative
- Is either true or false
- Can be rejected on the basis of trial testimony and evidence and sample data

Examples:

- The accused is innocent.
- $\mu = 100$

Null Hypothesis

- Is the first step in hypothesis testing
- Is usually a hypothesis of "no difference"
- Is denoted by H0
- Is performed for a possible rejection under a true assumption
- Always refers to a specified value of the population parameter, such as μ

Example:

The population mean is 100.

Or

$H0:\mu = 100$

This hypothesis:

- Commonly represents the status quo situation
- Is held to be true until a test results in its rejection
- Is accepted as "true" or rejected as "false" based on a consideration of a test statistic

Alternate Hypothesis

This hypothesis is:

- Complementary to null hypothesis
- Denoted by H1
- Used to decide whether to employ a single-tailed test or two-tailed test

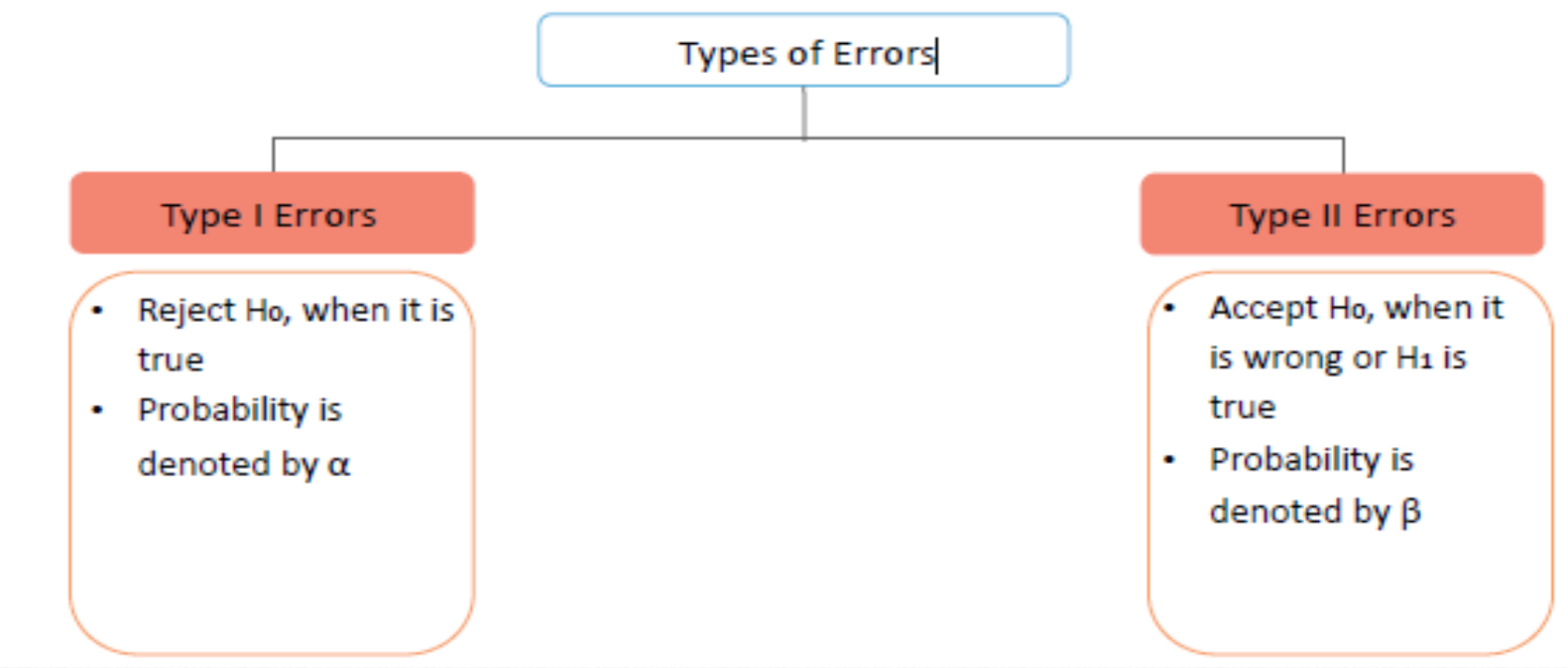
Example:

For H0:$\mu = 100$, the alternative hypothesis could be:

- H1:$\mu \neq 100$
- H1:$\mu > 100$ (right tailed)
- H1:$\mu < 100$ (left tailed)

Types of Errors

The errors in statistical decisions are of two types:



In practice, a Type I error means rejecting a lot when it is good (producer's risk) and Type II error means accepting a lot when it is bad (consumer's risk).

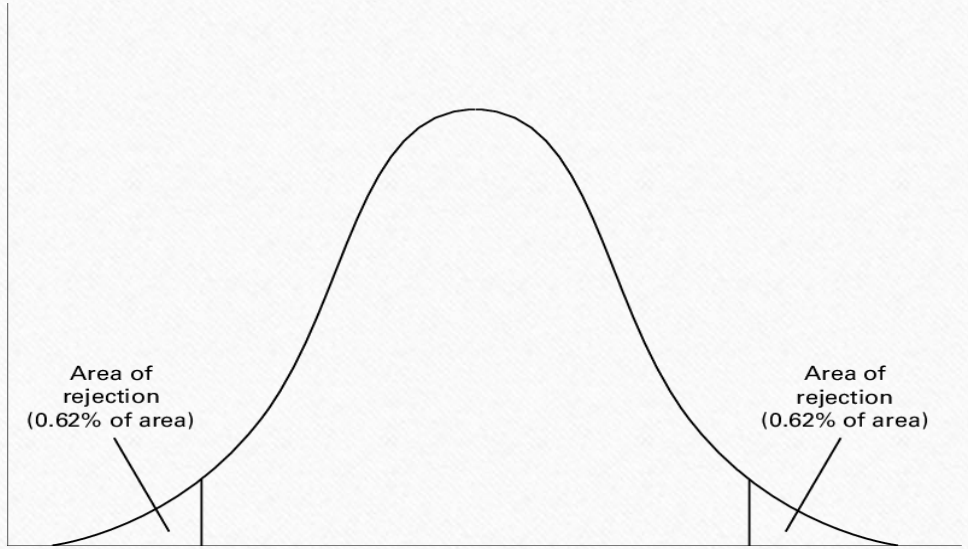
Contingency Table

It lists the possible outcomes of a statistical hypothesis test, as depicted below:

	State of Nature	
Decision	H_0 True	H_0 False
Accept H_0	Correct	Type II Error (β)
Reject H_0	Type I Error (α)	Correct

Critical Region:

The sampling distribution of a test statistic has two regions—a region of rejection (critical region) and a region of acceptance. The critical region amounts to rejection of H0,corresponding to the test statistic t in the sample space S.



Level of Significance

It is:

- The probability of a Type I error (α), that is, a random value of statistic t belongs to the critical region
- Usually set at 5% or 1% when employed in hypothesis testing

Important Points:

- If α = 0.05 and you reject H0, then there is a 5% probability that you have rejected H0 when it is true.
- The desired level of significance depends on the amount of risk you want to take in rejecting H0when it is true.

Confidence Coefficient

It:

- Is the complement of the probability of a Type I error (1-α)
- Yields confidence level, when multiplied by 100%
- Represents the probability of concluding that a specific value of parameter being tested under H0 is possible when, in fact, it is true

β Risk

It is:

- The probability of committing a Type II error
- Depends on the difference between the hypothesized and actual values of the population parameter
- Inversely proportional to α

Power of Test

It is:

- The complement of the probability of a Type II error (1-β)
- The probability of rejecting H0,when it is false
- Required to be as powerful as possible for all critical regions of the same size

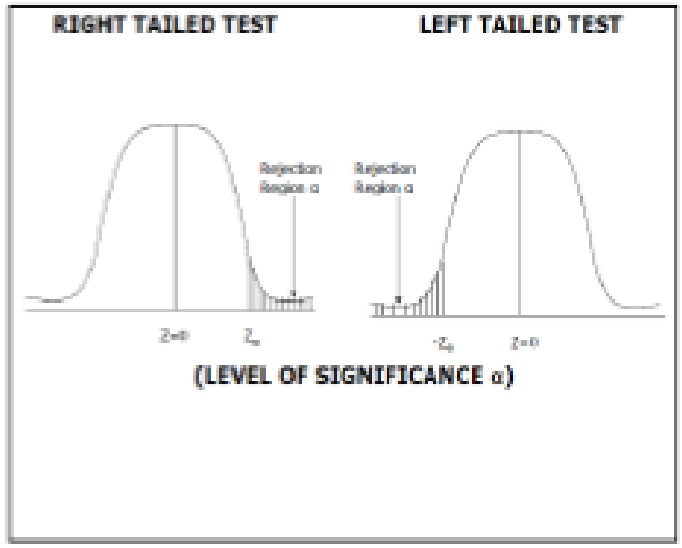
Types of Statistical Hypothesis Tests

In a test, critical region is an area under the probability curve of a sampling distribution of a test statistic. There are two types of statistical hypothesis tests:

One tailed	Two tailed
• In this test, H_1 is one tailed (left tailed or right tailed). • In a right-tailed test, critical region lies in the right tail of a sampling distribution, while for a left-tailed test, it lies in the left tail of distribution.	• In this test, H_1 is two tailed. • Critical region is the area lying in both tails of the probability curve of the test statistic.

The graphs below show a comparison of rejection regions in two types of tailed tests:

Single-Tailed Test



Two-Tailed Test