

Assignment 1

AI1110: Probability and Random Variables
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12.13.6.12 Question: Suppose we have four boxes A,B,C and D containing coloured marbles as given in tables folder: One of the boxes has been selected at random and a single marble is drawn from it. If the marble is read, what is the probability that it was drawn from

- 1) Box A ?
- 2) Box B ?
- 3) Box C ?

Answer: 1) $\frac{1}{15}$ 2) $\frac{2}{5}$ 3) $\frac{8}{15}$

Solution:

Probability of chosen box being A given that drawn marble is red is given by: $\Pr(E_1|E)$

Probability of chosen box being B given that drawn marble is red is given by: $\Pr(E_2|E)$

Probability of chosen box being C given that drawn marble is red is given by: $\Pr(E_3|E)$ Here,

$$\Pr(E|E_1) = \frac{1}{10} \quad (1)$$

$$\Pr(E|E_2) = \frac{6}{10} \quad (2)$$

$$\Pr(E|E_3) = \frac{8}{10} \quad (3)$$

$$\Pr(E|E_4) = \frac{0}{10} \quad (4)$$

$$\Pr(E_1) = \Pr(E_2) = \Pr(E_3) = \Pr(E_4) = \frac{1}{4} \quad (5)$$

Now by Bayes' theorem,

$$\begin{aligned} \Pr(E_1|E) &= \frac{\Pr(E|E_1) \cdot \Pr(E_1)}{\Pr(E|E_1) \cdot \Pr(E_1) + \Pr(E|E_2) \cdot \Pr(E_2) + \Pr(E|E_3) \cdot \Pr(E_3) + \Pr(E|E_4) \cdot \Pr(E_4)} \\ &= \frac{\frac{1}{10} \cdot \frac{1}{4}}{\frac{1}{10} \cdot \frac{1}{4} + \frac{6}{10} \cdot \frac{1}{4} + \frac{8}{10} \cdot \frac{1}{4} + \frac{0}{10} \cdot \frac{1}{4}} \\ &= \frac{\frac{1}{40}}{\frac{15}{40}} \\ &= \frac{1}{15} \end{aligned}$$

$$\begin{aligned} \Pr(E_2|E) &= \frac{\Pr(E|E_2) \cdot \Pr(E_2)}{\Pr(E|E_1) \cdot \Pr(E_1) + \Pr(E|E_2) \cdot \Pr(E_2) + \Pr(E|E_3) \cdot \Pr(E_3) + \Pr(E|E_4) \cdot \Pr(E_4)} \\ &= \frac{\frac{6}{10} \cdot \frac{1}{4}}{\frac{1}{10} \cdot \frac{1}{4} + \frac{6}{10} \cdot \frac{1}{4} + \frac{8}{10} \cdot \frac{1}{4} + \frac{0}{10} \cdot \frac{1}{4}} \\ &= \frac{\frac{6}{40}}{\frac{15}{40}} \\ &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} \Pr(E_3|E) &= \frac{\Pr(E|E_3) \cdot \Pr(E_3)}{\Pr(E|E_1) \cdot \Pr(E_1) + \Pr(E|E_2) \cdot \Pr(E_2) + \Pr(E|E_3) \cdot \Pr(E_3) + \Pr(E|E_4) \cdot \Pr(E_4)} \\ &= \frac{\frac{8}{10} \cdot \frac{1}{4}}{\frac{1}{10} \cdot \frac{1}{4} + \frac{6}{10} \cdot \frac{1}{4} + \frac{8}{10} \cdot \frac{1}{4} + \frac{0}{10} \cdot \frac{1}{4}} \\ &= \frac{\frac{8}{40}}{\frac{15}{40}} \\ &= \frac{8}{15} \end{aligned}$$