Assignment 1

AI1110:Probability and Random Variables
Indian Institute Of Technology Hyderabad

Name: Pradyumn Kangule
Roll no.: CS22BTECH11048

12.13.6.12 Question: Suppose we have four boxes A,B,C and D containing coloured marbles as given in table below:

Box	Marble colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

Table 1: Question Table

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from
1) Box A ?
2) Box B ?
3) Box C ?

Answer: 1) $\frac{1}{15}$ 2) $\frac{2}{5}$ 3) $\frac{8}{15}$
Solution:

Events	Definition
E	drawn marble is red
E_1	selected box is A
E_2	selected box is B
E_3	selected box is C
E_4	selected box is D

Table 2: Events Table

Probability of chosen box being A given that drawn marble is red is given by: $\Pr(E_1|E)$

Probability of chosen box being B given that drawn marble is red is given by: $\Pr(E_2|E)$

Probability of chosen box being C given that drawn marble is red is given by: $\Pr(E_3|E)$

Here,

$$\Pr(E|E_1) = \frac{1}{10} \tag{1}$$
$$\Pr(E|E_2) = \frac{6}{10} \tag{2}$$
$$\Pr(E|E_3) = \frac{8}{10} \tag{3}$$
$$\Pr(E|E_4) = \frac{0}{10} \tag{4}$$
$$\Pr(E_1) = \Pr(E_2) = \Pr(E_3) = \Pr(E_4) = \frac{1}{4} \tag{5}$$

As,

$$\Pr(E_1 \cup E_2 \cup E_3 \cup E_4) = 1 \tag{6}$$
$$\Pr\left(E_i E_j\right) = 0 \forall 1 \le i < j \le 4 \tag{7}$$

We can write,

$$\Pr(E) = \sum_{i=1}^{i=4} \Pr(EE_i) \tag{8}$$
$$Also, \tag{9}$$
$$\Pr(E/E_i) = \frac{\Pr(EE_i)}{\Pr(E_i)} \tag{10}$$
$$\implies \Pr(EE_i) = \Pr(E/E_i)\Pr(E_i) \tag{11}$$

Now,

$$\textit{By equation}\,(10) \tag{12}$$

$$\Pr(E_1|E) = \frac{\Pr(EE_1)}{\Pr(E)} \tag{13}$$

$$\textit{By equation}\,(11) \tag{14}$$

$$\Pr(E_1|E) = \frac{\Pr(E/E_1)\Pr(E_1)}{\Pr(E)} \tag{15}$$

$$\textit{By equations}\,(8)\ \textit{and}\,(11) \tag{16}$$

$$\Pr(E_1|E) = \frac{\Pr(E|E_1)\Pr(E_1)}{\sum_{i=1}^{i=4}(\Pr(E|E_i)\Pr(E_i))} \tag{17}$$

$$= \frac{\frac{1}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{6}{10}\times\frac{1}{4}+\frac{8}{10}\times\frac{1}{4}+\frac{0}{10}\times\frac{1}{4}} \tag{18}$$

$$= \frac{\frac{1}{40}}{\frac{15}{40}} \tag{19}$$

$$= \frac{1}{15} \tag{20}$$

$$\textit{By equation}\,(10) \tag{21}$$

$$\Pr(E_2|E) = \frac{\Pr(EE_2)}{\Pr(E)} \tag{22}$$

$$\textit{By equation}\,(11) \tag{23}$$

$$\Pr(E_2|E) = \frac{\Pr(E/E_2)\Pr(E_2)}{\Pr(E)} \tag{24}$$

$$\textit{By equations}\,(8)\ \textit{and}\,(11) \tag{25}$$

$$\Pr(E_2|E) = \frac{\Pr(E|E_2)\Pr(E_2)}{\sum_{i=1}^{i=4}(\Pr(E|E_i)\Pr(E_i))} \tag{26}$$

$$= \frac{\frac{6}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{6}{10}\times\frac{1}{4}+\frac{8}{10}\times\frac{1}{4}+\frac{0}{10}\times\frac{1}{4}} \tag{27}$$

$$= \frac{\frac{6}{40}}{\frac{15}{40}} \tag{28}$$

$$= \frac{2}{5} \tag{29}$$

$$\textit{By equation}\,(10) \tag{30}$$

$$\Pr(E_3|E) = \frac{\Pr(EE_3)}{\Pr(E)} \tag{31}$$

$$\textit{By equation}\,(11) \tag{32}$$

$$\Pr(E_3|E) = \frac{\Pr(E/E_3)\Pr(E_3)}{\Pr(E)} \tag{33}$$

$$\textit{By equations}\,(8)\ \textit{and}\,(11) \tag{34}$$

$$\Pr(E_3|E) = \frac{\Pr(E|E_3)\Pr(E_3)}{\sum_{i=1}^{i=4}(\Pr(E|E_i)\Pr(E_i))} \tag{35}$$

$$= \frac{\frac{8}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{6}{10}\times\frac{1}{4}+\frac{8}{10}\times\frac{1}{4}+\frac{0}{10}\times\frac{1}{4}} \tag{36}$$

$$= \frac{\frac{8}{40}}{\frac{15}{40}} \tag{37}$$

$$= \frac{8}{15} \tag{38}$$