Assignment 1

AI1110:Probability and Random Variables
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12.13.6.12 Question: Suppose we have four boxes A,B,C and D containing coloured marbles as given below:

Box	Marble colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is read, what is the probability that it was drawn from
1) Box A ?
2) Box B ?
3) Box C ?

Answer: 1)$\frac{1}{15}$ 2)$\frac{2}{5}$ 3)$\frac{8}{15}$

Solution:
Let E be the event that the drawn marble is red.
Let E_1 be the event that the selected box is A.
Let E_2 be the event that the selected box is B.
Let E_3 be the event that the selected box is C.
Let E_4 be the event that the selected box is D.

Probability of chosen box being A given that drawn marble is red is given by: P($E_1|E$)
Probability of chosen box being B given that drawn marble is red is given by: P($E_2|E$)
Probability of chosen box being C given that drawn marble is red is given by: P($E_3|E$)

Here,
$P(E|E_1) = \frac{1}{10}$ $P(E|E_2) = \frac{6}{10}$

$P(E|E_3) = \frac{8}{10}$ $P(E|E_4) = \frac{0}{10}$

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

Now by Bayes' theorem,

$$P(E_1|E) = \frac{P(E|E_1).P(E_1)}{P(E|E_1).P(E_1)+P(E|E_2).P(E_2)+P(E|E_3).P(E_3)+P(E|E_4).P(E_4)}$$
$$= \frac{\frac{1}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4}+\frac{6}{10}\cdot\frac{1}{4}+\frac{8}{10}\cdot\frac{1}{4}+\frac{0}{10}\cdot\frac{1}{4}}$$
$$= \frac{\frac{1}{40}}{\frac{15}{40}}$$
$$= \frac{1}{15}$$

$$P(E_2|E) = \frac{P(E|E_2).P(E_2)}{P(E|E_1).P(E_1)+P(E|E_2).P(E_2)+P(E|E_3).P(E_3)+P(E|E_4).P(E_4)}$$
$$= \frac{\frac{6}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4}+\frac{6}{10}\cdot\frac{1}{4}+\frac{8}{10}\cdot\frac{1}{4}+\frac{0}{10}\cdot\frac{1}{4}}$$
$$= \frac{\frac{6}{40}}{\frac{15}{40}}$$
$$= \frac{6}{15}$$
$$= \frac{2}{5}$$

$$P(E_3|E) = \frac{P(E|E_3).P(E_3)}{P(E|E_1).P(E_1)+P(E|E_2).P(E_2)+P(E|E_3).P(E_3)+P(E|E_4).P(E_4)}$$
$$= \frac{\frac{8}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4}+\frac{6}{10}\cdot\frac{1}{4}+\frac{8}{10}\cdot\frac{1}{4}+\frac{0}{10}\cdot\frac{1}{4}}$$
$$= \frac{\frac{8}{40}}{\frac{15}{40}}$$
$$= \frac{8}{15}$$