Assignment 2

AI1110:Probability and Random Variables
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12.13.4.5 Question: Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as
(i) number greater than 4
(ii) six appears on at least one die
Solution:
Let X be a random variable denoting the outcome of a die toss so,

$$X = \{1, 2, 3, 4, 5, 6\} \tag{1}$$

(i)
Let the Cumulative Distribution function be:

$$F_X(i) = \Pr(X \le i) \tag{2}$$
$$= \sum_{n=1}^{n=i} \Pr(X = n) \tag{3}$$

Now,

$$\Pr(X = i) = \frac{1}{6} \; \forall 1 \le i \le 6 \tag{4}$$
$$\therefore F_X(i) = \sum_{n=1}^{n=i} \frac{1}{6} \tag{5}$$
$$\implies F_X(i) = i/6 \tag{6}$$

Now,

$$\Pr(X > 4) = F_X(6) - F_X(4) \tag{7}$$
$$\text{by (6)} \tag{8}$$
$$= \frac{6}{6} - \frac{4}{6} \tag{9}$$
$$= 1 - \frac{2}{3} \tag{10}$$
$$= \frac{1}{3} \tag{11}$$

$\therefore \frac{1}{3}$ is the probability of success in this case for a die.

Now,
Let Y be the random variable denoting number of successes.

$$\therefore Y \sim Bin(n, p) \tag{12}$$

where $n = 2$ and $p = \frac{1}{3}$

$$\therefore \Pr(Y = i) = {}^2C_i (1 - p)^{2-i} p^i \tag{13}$$

By (13),

$$\Pr(Y = 0) = {}^2C_0 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^0 \tag{14}$$
$$\implies \Pr(Y = 0) = \frac{4}{9} \tag{15}$$
$$\Pr(Y = 1) = {}^2C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^1 \tag{16}$$
$$= 2 \times \frac{2}{3} \times \frac{1}{3} \tag{17}$$
$$\implies \Pr(Y = 1) = \frac{4}{9} \tag{18}$$
$$\Pr(Y = 2) = {}^2C_2 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^2 \tag{19}$$
$$\implies \Pr(Y = 2) = \frac{1}{9} \tag{20}$$

$\therefore$

$$p_Y(i) = \begin{cases} \frac{4}{9}, & i = 0 \\ \frac{4}{9}, & i = 1 \\ \frac{1}{9}, & i = 2 \\ 0, & \text{otherwise} \end{cases}$$

(*ii*)
Let p' be the probability of getting 6 in a die toss. $\therefore$

$$p' = \Pr(X = 6) \tag{21}$$
$$\text{by (4)} \tag{22}$$
$$= \frac{1}{6} \tag{23}$$

Let Y be a random variable denoting the number 6's in die tosses,

$$\therefore Y \sim Bin(n, p') \tag{24}$$
$$\text{where n=2 and } p' = \frac{1}{6} \tag{25}$$

$$\therefore \Pr(Y = i) = {}^2C_i p'^i (1 - p')^{(2-i)} \tag{26}$$
$$\implies \Pr(Y = i) = {}^2C_i \left(\frac{1}{6}\right)^i \left(\frac{5}{6}\right)^{(2-i)} \tag{27}$$

Let the Cumulative Distribution function be:

$$F_Y(i) = \Pr(Y \le i) \tag{28}$$
$$= \sum_{n=0}^{n=i} \Pr(Y = n) \tag{29}$$
$$\therefore by (27) \tag{30}$$
$$F_Y(0) = {}^2C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^2 \tag{31}$$
$$\implies F_Y(0) = \frac{25}{36} \tag{32}$$
$$F_Y(2) = 1 \tag{33}$$
$$\because Y \le 2 \tag{34}$$

Let p denote the probability of success in this case.

$$\therefore p = F_Y(2) - F_Y(0) \tag{35}$$
$$\text{by (32),(33)} \tag{36}$$
$$p = 1 - \frac{25}{36} \tag{37}$$
$$\implies p = \frac{11}{36} \tag{38}$$

Let Z be a random variable denoting number of successes.

$$\therefore Z \sim Ber(p) \tag{39}$$

where $p=\frac{11}{36}$ by (38)
$\therefore$

$$p_Z(i) = \begin{cases} \frac{25}{36}, & i = 0 \\ \frac{11}{36}, & i = 1 \\ 0, & \text{otherwise} \end{cases}$$