

Assignment 1

AI1110: Probability and Random Variables
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12.13.6.12 Question: Suppose we have four boxes A,B,C and D containing coloured marbles as given in tables folder: One of the boxes has been selected at random and a single marble is drawn from it. If the marble is read, what is the probability that it was drawn from

- 1) Box A ?
- 2) Box B ?
- 3) Box C ?

Answer: 1) $\frac{1}{15}$ 2) $\frac{2}{5}$ 3) $\frac{8}{15}$

Solution:

Events	Definition
E	drawn marble is red
E_1	selected box is A
E_2	selected box is B
E_3	selected box is C
E_4	selected box is D

TABLE 0
EVENTS TABLE

Probability of chosen box being A given that drawn marble is red is given by: $\Pr(E_1|E)$

Probability of chosen box being B given that drawn marble is red is given by: $\Pr(E_2|E)$

Probability of chosen box being C given that drawn marble is red is given by: $\Pr(E_3|E)$

Here,

$$\Pr(E|E_1) = \frac{1}{10} \quad (1)$$

$$\Pr(E|E_2) = \frac{6}{10} \quad (2)$$

$$\Pr(E|E_3) = \frac{8}{10} \quad (3)$$

$$\Pr(E|E_4) = \frac{0}{10} \quad (4)$$

$$\Pr(E_1) = \Pr(E_2) = \Pr(E_3) = \Pr(E_4) = \frac{1}{4} \quad (5)$$

Now by Bayes' theorem,

$$\Pr(E_1|E) = \frac{\Pr(E|E_1) \Pr(E_1)}{\sum_{i=1}^4 (\Pr(E|E_i) \Pr(E_i))} \quad (6)$$

$$= \frac{\frac{1}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + \frac{0}{10} \times \frac{1}{4}} \quad (7)$$

$$= \frac{\frac{1}{40}}{\frac{15}{40}} \quad (8)$$

$$= \frac{1}{15} \quad (9)$$

$$\Pr(E_2|E) = \frac{\Pr(E|E_2) \Pr(E_2)}{\sum_{i=1}^4 (\Pr(E|E_i) \Pr(E_i))} \quad (10)$$

$$= \frac{\frac{6}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + \frac{0}{10} \times \frac{1}{4}} \quad (11)$$

$$= \frac{\frac{6}{40}}{\frac{15}{40}} \quad (12)$$

$$= \frac{2}{5} \quad (13)$$

$$\Pr(E_3|E) = \frac{\Pr(E|E_3) \Pr(E_3)}{\sum_{i=1}^4 (\Pr(E|E_i) \Pr(E_i))} \quad (14)$$

$$= \frac{\frac{8}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + \frac{0}{10} \times \frac{1}{4}} \quad (15)$$

$$= \frac{\frac{8}{40}}{\frac{15}{40}} \quad (16)$$

$$= \frac{8}{15} \quad (17)$$