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Assignment 1

AI1110:Probability and Random Variables Indian Institute Of Technology Hyderabad

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12.13.6.12 Question: Suppose we have four boxes A,B,C and D containing coloured marbles as given in tables folder: One of the boxes has been selected at random and a single marble is drawn from it. If the marble is read, what is the probability that it was drawn from

- 1) Box A?
- 2) Box B?
- 3) Box C?

Answer: $1)\frac{1}{15}$ $2)\frac{2}{5}$ $3)\frac{8}{15}$ **Solution:**

Events	Definition
E	drawn marble is red
E_1	selected box is A
E_2	selected box is B
E_3	selected box is C
E_4	selected box is D

Probability of chosen box being A given that drawn marble is red is given by: $Pr(E_1|E)$

Probability of chosen box being B given that drawn marble is red is given by: $Pr(E_2|E)$

Probability of chosen box being C given that drawn marble is red is given by: $Pr(E_3|E)$

Here,

$$\Pr(E|E_1) = \frac{1}{10} \tag{1}$$

$$\Pr(E|E_2) = \frac{6}{10}$$
 (2)

$$\Pr(E|E_3) = \frac{8}{10} \tag{3}$$

$$\Pr(E|E_4) = \frac{0}{10} \tag{4}$$

$$Pr(E_1) = Pr(E_2) = Pr(E_3) = Pr(E_4) = \frac{1}{4}$$
 (5)

$$\Pr(E_1|E) = \frac{\Pr(E|E_1)\Pr(E_1)}{\sum_{i=1}^{i=4} (\Pr(E|E_i)\Pr(E_i))}$$

$$\frac{1}{10} \times \frac{1}{4}$$
(6)

$$= \frac{\frac{\frac{1}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + \frac{0}{10} \times \frac{1}{4}}$$
 (7)

$$= \frac{40}{\frac{15}{40}}$$

$$= \frac{1}{10}$$
(9)

$$\Pr(E_2|E) = \frac{\Pr(E|E_2)\Pr(E_2)}{\sum_{i=1}^{i=4} (\Pr(E|E_i)\Pr(E_i))}$$
(10)

$$= \frac{\frac{6}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + \frac{0}{10} \times \frac{1}{4}}$$
(11)

$$=\frac{\frac{6}{40}}{\frac{15}{40}}\tag{12}$$

$$=\frac{2}{5}\tag{13}$$

$$\Pr(E_3|E) = \frac{\Pr(E|E_3)\Pr(E_3)}{\sum_{i=1}^{i=4}(\Pr(E|E_i)\Pr(E_i))}$$
(14)

$$= \frac{\frac{8}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + \frac{0}{10} \times \frac{1}{4}}$$
 (15)

$$=\frac{\frac{8}{40}}{\frac{15}{40}}\tag{16}$$

$$=\frac{8}{15}\tag{17}$$

Now by Bayes' theorem,