Assignment 1
AI1110:Probability and Random Variables
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12.13.6.12 Question: Suppose we have four boxes A,B,C and D containing coloured marbles as given in tables folder: One of the boxes has been selected at random and a single marble is drawn from it. If the marble is read, what is the probability that it was drawn from
1) Box A ?
2) Box B ?
3) Box C ?

Answer: 1)$\frac{1}{15}$ 2)$\frac{2}{5}$ 3)$\frac{8}{15}$

Solution:

Events	Definition
E	drawn marble is red
E_1	selected box is A
E_2	selected box is B
E_3	selected box is C
E_4	selected box is D

Probability of chosen box being A given that drawn marble is red is given by: $\Pr(E_1|E)$

Probability of chosen box being B given that drawn marble is red is given by: $\Pr(E_2|E)$

Probability of chosen box being C given that drawn marble is red is given by: $\Pr(E_3|E)$

Here,

$$\Pr(E|E_1) = \frac{1}{10} \tag{1}$$

$$\Pr(E|E_2) = \frac{6}{10} \tag{2}$$

$$\Pr(E|E_3) = \frac{8}{10} \tag{3}$$

$$\Pr(E|E_4) = \frac{0}{10} \tag{4}$$

$$\Pr(E_1) = \Pr(E_2) = \Pr(E_3) = \Pr(E_4) = \frac{1}{4} \tag{5}$$

Now by Bayes' theorem,

$$
\begin{aligned}
\Pr(E_1|E) &= \frac{\Pr(E|E_1).\Pr(E_1)}{\sum_{i=1}^{i=4}(\Pr(E|E_i).\Pr(E_i))} \\
&= \frac{\frac{1}{10}.\frac{1}{4}}{\frac{1}{10}.\frac{1}{4} + \frac{6}{10}.\frac{1}{4} + \frac{8}{10}.\frac{1}{4} + \frac{0}{10}.\frac{1}{4}} \\
&= \frac{\frac{1}{40}}{\frac{15}{40}} \\
&= \frac{1}{15}
\end{aligned}
$$

$$
\begin{aligned}
\Pr(E_2|E) &= \frac{\Pr(E|E_2).\Pr(E_2)}{\sum_{i=1}^{i=4}(\Pr(E|E_i).\Pr(E_i))} \\
&= \frac{\frac{6}{10}.\frac{1}{4}}{\frac{1}{10}.\frac{1}{4} + \frac{6}{10}.\frac{1}{4} + \frac{8}{10}.\frac{1}{4} + \frac{0}{10}.\frac{1}{4}} \\
&= \frac{\frac{6}{40}}{\frac{15}{40}} \\
&= \frac{2}{5}
\end{aligned}
$$

$$
\begin{aligned}
\Pr(E_3|E) &= \frac{\Pr(E|E_3).\Pr(E_3)}{\sum_{i=1}^{i=4}(\Pr(E|E_i).\Pr(E_i))} \\
&= \frac{\frac{8}{10}.\frac{1}{4}}{\frac{1}{10}.\frac{1}{4} + \frac{6}{10}.\frac{1}{4} + \frac{8}{10}.\frac{1}{4} + \frac{0}{10}.\frac{1}{4}} \\
&= \frac{\frac{8}{40}}{\frac{15}{40}} \\
&= \frac{8}{15}
\end{aligned}
$$