

Assignment 1

AI1110: Probability and Random Variables
Indian Institute Of Technology Hyderabad

Name: Pradyumn Kangule
Roll no.: CS22BTECH11048

12.13.6.12 Question: Suppose we have four boxes A,B,C and D containing coloured marbles as given in table below: One of the boxes has been

Box	Marble colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

TABLE 0: Question Table

Here,

$$\Pr(E|E_1) = \frac{1}{10} \quad (1)$$

$$\Pr(E|E_2) = \frac{6}{10} \quad (2)$$

$$\Pr(E|E_3) = \frac{8}{10} \quad (3)$$

$$\Pr(E|E_4) = \frac{0}{10} \quad (4)$$

$$\Pr(E_i) = \frac{1}{4} \quad \forall 1 \leq i \leq 4 \quad (5)$$

selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from

1) Box A ?

2) Box B ?

3) Box C ?

Answer: 1) $\frac{1}{15}$ 2) $\frac{2}{5}$ 3) $\frac{8}{15}$

Solution: Probability of chosen box being A given

As,

$$\Pr(E_1 + E_2 + E_3 + E_4) = 1 \quad (6)$$

$$\Pr(E_i E_j) = 0 \quad \forall 1 \leq i < j \leq 4 \quad (7)$$

We can write,

$$\Pr(E) = \sum_{i=1}^{i=4} \Pr(E E_i) \quad (8)$$

Also,

$$\Pr(E|E_i) = \frac{\Pr(E E_i)}{\Pr(E_i)} \quad (9)$$

$$\implies \Pr(E E_i) = \Pr(E|E_i) \Pr(E_i) \quad (10)$$

Now,

By equation (9)

$$\Pr(E_1|E) = \frac{\Pr(E E_1)}{\Pr(E)} \quad (11)$$

that drawn marble is red is given by: $\Pr(E_1|E)$

Probability of chosen box being B given that drawn marble is red is given by: $\Pr(E_2|E)$

By equation (10)

Probability of chosen box being C given that drawn marble is red is given by: $\Pr(E_3|E)$

$$\Pr(E_1|E) = \frac{\Pr(E|E_1) \Pr(E_1)}{\Pr(E)} \quad (12)$$

By equations (8) and (10)

$$\Pr(E_1|E) = \frac{\Pr(E|E_1) \Pr(E_1)}{\sum_{i=1}^{i=4} (\Pr(E|E_i) \Pr(E_i))} \quad (13)$$

$$= \frac{\frac{1}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + \frac{0}{10} \times \frac{1}{4}} \quad (14)$$

$$= \frac{\frac{1}{40}}{\frac{15}{40}} \quad (15)$$

$$= \frac{1}{15} \quad (16)$$

By equation (9)

$$\Pr(E_2|E) = \frac{\Pr(EE_2)}{\Pr(E)} \quad (17)$$

By equation (10)

$$\Pr(E_2|E) = \frac{\Pr(E|E_2) \Pr(E_2)}{\Pr(E)} \quad (18)$$

By equations (8) and (10)

$$\Pr(E_2|E) = \frac{\Pr(E|E_2) \Pr(E_2)}{\sum_{i=1}^{i=4} (\Pr(E|E_i) \Pr(E_i))} \quad (19)$$

$$= \frac{\frac{6}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + \frac{0}{10} \times \frac{1}{4}} \quad (20)$$

$$= \frac{\frac{6}{40}}{\frac{15}{40}} \quad (21)$$

$$= \frac{2}{5} \quad (22)$$

By equation (9)

$$\Pr(E_3|E) = \frac{\Pr(EE_3)}{\Pr(E)} \quad (23)$$

By equation (10)

$$\Pr(E_3|E) = \frac{\Pr(E|E_3) \Pr(E_3)}{\Pr(E)} \quad (24)$$

By equations (8) and (10)

$$\Pr(E_3|E) = \frac{\Pr(E|E_3) \Pr(E_3)}{\sum_{i=1}^{i=4} (\Pr(E|E_i) \Pr(E_i))} \quad (25)$$

$$= \frac{\frac{8}{10} \times \frac{1}{4}}{\frac{1}{10} \times \frac{1}{4} + \frac{6}{10} \times \frac{1}{4} + \frac{8}{10} \times \frac{1}{4} + \frac{0}{10} \times \frac{1}{4}} \quad (26)$$

$$= \frac{\frac{8}{40}}{\frac{15}{40}} \quad (27)$$

$$= \frac{8}{15} \quad (28)$$