Assignment 3

AI1110:Probability and Random Variables
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11.16.3.7 Question: A fair coin is tossed four times, and a person win Rs 1 for each head and lose Rs 1.50 for each tail that turns up. Calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution:

Parameter	Value	Definition
n	{0,1,2....}	Number of tosses
A	{-1.5n,2.5-1.5n,..n}	Total Amount a person can have
X	$\{0, 1, 2, ...n\}$	Number of heads after n tosses

TABLE : Definitions

$$\therefore X \sim Bin(n, p) \tag{1}$$

where $p = \frac{1}{2}$

$$\therefore \Pr(X = i) = {}^nC_i \left(\frac{1}{2}\right)^n \tag{2}$$

$$\text{By question} \tag{3}$$
$$A = X - (n - X) \times 1.5 \tag{4}$$
$$\implies A = 2.5X - 1.5n \tag{5}$$

$\therefore A$ takes $n + 1$ different values.

By (2),(5)

$$\Pr(X = i) = \Pr(A = 2.5i - 1.5n) = {}^nC_i \left(\frac{1}{2}\right)^n \tag{6}$$

Let $F_X(k)$ denote the cumulative distribution function of X:

$$\therefore F_X(k) = \Pr(X \le k) = \sum_{i=0}^{i=k} {}^nC_i \left(\frac{1}{2}\right)^n \tag{7}$$

Let $F_A(k)$ denote the cumulative distribution function of A:

$$F_A(k) = \Pr(A \le k) \tag{8}$$
$$= \Pr(2.5X - 1.5n \le k) \tag{9}$$
$$= \Pr\left(X \le \frac{k + 1.5n}{2.5}\right) \tag{10}$$
$$= F_X\left(\frac{k + 1.5n}{2.5}\right) \tag{11}$$
$$\therefore \text{by (7)} \tag{12}$$
$$= \sum_{i=0}^{i=\lfloor\frac{k+1.5n}{2.5}\rfloor} {}^nC_i \left(\frac{1}{2}\right)^n \tag{13}$$
$$(\because X \text{ takes only integer values}) \tag{14}$$

$$\therefore p_A(k) = \begin{cases} {}^nC_{\lfloor\frac{k+1.5n}{2.5}\rfloor}\left(\frac{1}{2}\right)^n, & \lfloor\frac{k+1.5n}{2.5}\rfloor > \lfloor\frac{k+1.5n-1}{2.5}\rfloor \\ 0, & \text{otherwise} \end{cases}$$

$\therefore$ For $n = 4$

$$p_A(k) = \begin{cases} \frac{1}{16}, & k = -6 \\ \frac{1}{4}, & k = -3 \\ \frac{3}{8}. & k = -1 \\ \frac{1}{4}, & k = 1.5 \\ \frac{1}{16}, & k = 4 \\ 0, & \text{otherwise} \end{cases}$$