Assignment 2

AI1110:Probability and Random Variables
Indian Institute Of Technology Hyderabad

Name: Pradyumn Kangule
Roll no.: CS22BTECH11048

12.13.4.5 Question: Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as
(i) number greater than 4
(ii) six appears on at least one die
Solution:
(i)
Let X be a random variable denoting the outcome of a die toss so,

$$X = \{1, 2, 3, 4, 5, 6\} \tag{1}$$

Let the Cumulative Distribution function be:

$$F_X(i) = \Pr(X \le i) \tag{2}$$
$$= \sum_{n=1}^{n=i} \Pr(X = n) \tag{3}$$

Now,

$$\Pr(X = i) = \frac{1}{6} \ \forall 1 \le i \le 6 \tag{4}$$
$$\therefore F_X(i) = \sum_{n=1}^{n=i} \frac{1}{6} \tag{5}$$
$$\implies F_X(i) = i/6 \tag{6}$$

Now,

$$\Pr(X > 4) = F_X(6) - F_X(4) \tag{7}$$
$$\text{by (6)} \tag{8}$$
$$= \frac{6}{6} - \frac{4}{6} \tag{9}$$
$$= 1 - \frac{2}{3} \tag{10}$$
$$= \frac{1}{3} \tag{11}$$

$\therefore \frac{1}{3}$ is the probability of success in this case for a die.

Now,
Let Y be the random variable denoting number of successes.

$$\therefore Y \sim Bin(n, p) \tag{12}$$

where $n = 2$ and $p = \frac{1}{3}$

$$\therefore \Pr(Y = i) = {}^2C_i (1 - p)^{2-i} p^i \tag{13}$$

By (13),

$$\Pr(Y = 0) = {}^2C_0 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^0 \tag{14}$$
$$\implies \Pr(Y = 0) = \frac{4}{9} \tag{15}$$
$$\Pr(Y = 1) = {}^2C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^1 \tag{16}$$
$$= 2 \times \frac{2}{3} \times \frac{1}{3} \tag{17}$$
$$\implies \Pr(Y = 1) = \frac{4}{9} \tag{18}$$
$$\Pr(Y = 2) = {}^2C_2 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^2 \tag{19}$$
$$\implies \Pr(Y = 2) = \frac{1}{9} \tag{20}$$

$\therefore$

$$p_Y(i) = \begin{cases} \frac{4}{9}, & i = 0 \\ \frac{4}{9}, & i = 1 \\ \frac{1}{9}, & i = 2 \\ 0, & \text{otherwise} \end{cases}$$

(ii)
Let Y be a random variable denoting the outcome of a die toss so,

$$Y = \{1, 2, 3, 4, 5, 6\} \tag{21}$$

Let the Cumulative Distribution function be:

$$F_Y(i) = \Pr(Y \le i) \tag{22}$$
$$= \sum_{n=1}^{n=i} \Pr(Y = n) \tag{23}$$

Now,

$$\Pr(Y = i) = \frac{1}{6} \ \forall 1 \le i \le 6 \tag{24}$$
$$\therefore F_Y(i) = \sum_{n=1}^{n=i} \frac{1}{6} \tag{25}$$
$$\implies F_Y(i) = i/6 \tag{26}$$

Let p denote the probability of success in this case. $\therefore$ 1 - p is probability of failure i.e., getting no 6 in either of the tosses

$$\therefore 1 - p = F_Y(5) F_Y(5) \tag{27}$$
$$\text{by (26)} \tag{28}$$
$$= \frac{5}{6} \times \frac{5}{6} \tag{29}$$
$$= \frac{25}{36} \tag{30}$$
$$\implies p = 1 - \frac{25}{36} \tag{31}$$
$$\implies p = \frac{11}{36} \tag{32}$$

Let Z be a random variable denoting number of successes.

$$\therefore Z \sim Ber(p) \tag{33}$$

where p=$\frac{11}{36}$ by (32)
$\therefore$

$$p_Z(i) = \begin{cases} \frac{25}{36}, & i = 0 \\ \frac{11}{36}, & i = 1 \\ 0, & \text{otherwise} \end{cases}$$