Assignment 3

AI1110:Probability and Random Variables
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11.16.3.7 Question: A fair coin is tossed four times, and a person win Rs 1 for each head and lose Rs 1.50 for each tail that turns up. Calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution:

Let X be a random variable denoting the number of heads obtained after four coin tosses.

$$\therefore X \sim Bin(n, p)$$

where $p = \frac{1}{2}$

$$\therefore \Pr(X = i) = {}^nC_i \left(\frac{1}{2}\right)^n \tag{1}$$

Let A be a random variable denoting the amount money a person can have.

$$\therefore \text{ by question} \tag{2}$$
$$A = X - (4 - X) \times 1.5 \tag{3}$$
$$\implies A = 2.5 \times X - 6 \tag{4}$$

$\therefore A$ takes $n + 1$ different values.

By (1),(4)

$$\Pr(X = i) = \Pr(A = 2.5i - 6) = {}^nC_i \left(\frac{1}{2}\right)^n \tag{5}$$

$$\therefore p_A(2.5k - 6) = \begin{cases} {}^nC_k \left(\frac{1}{2}\right)^n, & \forall\ 0 \le k \le n + 1 \\ 0, & \text{otherwise} \end{cases}$$

For $n = 4$ by (5)

$$\Pr(X = 0) = \Pr(A = -6) \qquad = \frac{1}{16} \tag{6}$$

$$\Pr(X = 1) = \Pr(A = -3) \qquad = \frac{1}{4} \tag{7}$$

$$\Pr(X = 2) = \Pr(A = -1) \qquad = \frac{3}{8} \tag{8}$$

$$\Pr(X = 3) = \Pr(A = 1.5) \qquad = \frac{1}{4} \tag{9}$$

$$\Pr(X = 4) = \Pr(A = 4) \qquad = \frac{1}{16} \tag{10}$$

$$\therefore p_A(k) = \begin{cases} \frac{1}{16}, & k = -6 \\ \frac{1}{4}, & k = -3 \\ \frac{3}{8}, & k = -1 \\ \frac{1}{4}, & k = 1.5 \\ \frac{1}{16}, & k = 4 \\ 0, & \text{otherwise} \end{cases}$$