

Assignment 2

AI1110: Probability and Random Variables
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12.13.4.5 Question: Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(i) number greater than 4

(ii) six appears on at least one die

Solution:

Let X be a random variable denoting the outcome of a die toss so,

$$X = \{1, 2, 3, 4, 5, 6\} \quad (1)$$

(i)

Let the Cumulative Distribution function be:

$$F_X(i) = \Pr(X \leq i) \quad (2)$$

$$= \sum_{n=1}^{n=i} \Pr(X = n) \quad (3)$$

Now,

$$\Pr(X = i) = \frac{1}{6} \quad \forall 1 \leq i \leq 6 \quad (4)$$

$$\therefore F_X(i) = \sum_{n=1}^{n=i} \frac{1}{6} \quad (5)$$

$$\Rightarrow F_X(i) = i/6 \quad (6)$$

Now,

$$\Pr(X > 4) = F_X(6) - F_X(4) \quad (7) \quad \therefore$$

$$\text{by (6)} \quad (8)$$

$$= \frac{6}{6} - \frac{4}{6} \quad (9)$$

$$= 1 - \frac{2}{3} \quad (10)$$

$$= \frac{1}{3} \quad (11)$$

$\therefore \frac{1}{3}$ is the probability of success in this case for a die.

Now,

Let Y be the random variable denoting number of successes.

$$\therefore Y \sim \text{Bin}(n, p) \quad (12)$$

where $n = 2$ and $p = \frac{1}{3}$

$$\therefore \Pr(Y = i) = {}^2C_i (1-p)^{2-i} p^i \quad (13)$$

By (13),

$$\Pr(Y = 0) = {}^2C_0 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^0 \quad (14)$$

$$\Rightarrow \Pr(Y = 0) = \frac{4}{9} \quad (15)$$

$$\Pr(Y = 1) = {}^2C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^1 \quad (16)$$

$$= 2 \times \frac{2}{3} \times \frac{1}{3} \quad (17)$$

$$\Rightarrow \Pr(Y = 1) = \frac{4}{9} \quad (18)$$

$$\Pr(Y = 2) = {}^2C_2 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^2 \quad (19)$$

$$\Rightarrow \Pr(Y = 2) = \frac{1}{9} \quad (20)$$

$$p_Y(i) = \begin{cases} \frac{4}{9}, & i = 0 \\ \frac{4}{9}, & i = 1 \\ \frac{1}{9}, & i = 2 \\ 0, & \text{otherwise} \end{cases}$$

(ii)

Let p denote the probability of success in this case.

$\therefore 1 - p$ is probability of failure i.e., getting no 6 in either of the tosses

$$\therefore 1 - p = \Pr(Y = 0) + \Pr(Y = 1) \Pr(\text{no } 6|Y = 1) + \Pr(Y = 2) \Pr(\text{no } 6|Y = 2)$$

$$(21)$$

$$= \Pr(Y = 0) + \Pr((\text{no } 6)(Y = 1)) + \Pr((\text{no } 6)(Y = 2))$$

$$(22)$$

by (15),(18),(20)

$$(23)$$

$$= \frac{4}{9} + 2 \times \left(\frac{1}{6}\right) \left(\frac{2}{3}\right) + \left(\frac{1}{6}\right)^2$$

$$(24)$$

$$= \frac{4}{9} + \frac{2}{9} + \frac{1}{36}$$

$$(25)$$

$$= \frac{25}{36}$$

$$(26)$$

$$\Rightarrow p = 1 - \frac{25}{36}$$

$$(27)$$

$$\Rightarrow p = \frac{11}{36}$$

$$(28)$$

Let Z be a random variable denoting number of successes.

$$\therefore Z \sim \text{Ber}(p)$$

$$(29)$$

where $p = \frac{11}{36}$ by (28)

\therefore

$$p_Z(i) = \begin{cases} \frac{25}{36}, & i = 0 \\ \frac{11}{36}, & i = 1 \\ 0, & \text{otherwise} \end{cases}$$