Assignment 2

AI1110:Probability and Random Variables
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12.13.4.5 Question: Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

i) number greater than 4
ii) six appears on at least one die

Solution:
Let X be a random variable denoting the outcome of a die toss so,

$$X = \{1, 2, 3, 4, 5, 6\} \tag{1}$$

i) Let the Cumulative Distribution function be:

$$F_X(i) = \Pr(X \leq i) \tag{2}$$
$$= \sum_{n=1}^{n=i} \Pr(X = n) \tag{3}$$

Now,

$$\Pr(X = i) = \frac{1}{6} \ \forall 1 \leq i \leq 6 \tag{4}$$
$$\therefore F_X(i) = \sum_{n=1}^{n=i} \frac{1}{6} \tag{5}$$
$$\implies F_X(i) = i/6 \tag{6}$$

Now,

$$\Pr(X > 4) = F_X(6) - F_X(4) \tag{7}$$
$$\text{by (6)} \tag{8}$$
$$= \frac{6}{6} - \frac{4}{6} \tag{9}$$
$$= \frac{1}{3} \tag{10}$$

$\therefore \frac{1}{3}$ is the probability of success in this case for a die.

Now,

Let Y be the random variable denoting number of successes.

$$\therefore Y \sim Bin(n, p) \tag{11}$$
$$\text{where } n = 2 \text{ and } p = \frac{1}{3}$$

$$\therefore \Pr(Y = i) = {}^2C_i (1 - p)^{2-i} p^i \tag{12}$$

By (12),

$$p_Y(k) = \begin{cases} \frac{4}{9}, & k = 0 \\ \frac{4}{9}, & k = 1 \\ \frac{1}{9}, & k = 2 \\ 0, & \text{otherwise} \end{cases}$$

ii) Let p' be the probability of getting 6 in a die toss.

$$\therefore p' = \Pr(X = 6) \tag{13}$$
$$\text{by (4)} \tag{14}$$
$$= \frac{1}{6} \tag{15}$$

Let Y be a random variable denoting the number 6's in die tosses,

$$\therefore Y \sim Bin(n, p') \tag{16}$$
$$\text{where } n = 2 \text{ and } p' = \frac{1}{6} \tag{17}$$

$$\therefore \Pr(Y = i) = {}^2C_i p'^i (1 - p')^{(2-i)} \tag{18}$$
$$\implies \Pr(Y = i) = {}^2C_i \left(\frac{1}{6}\right)^i \left(\frac{5}{6}\right)^{(2-i)} \tag{19}$$

Let the Cumulative Distribution function be:

$$F_Y(i) = \Pr(Y \le i) \tag{20}$$

$$\implies F_Y(i) = \sum_{n=0}^{n=i} \Pr(Y = n) \tag{21}$$

Let p denote the probability of success in this case.

$$\therefore p = F_Y(2) - F_Y(0) \tag{22}$$

$$\text{by (19),(21)} \tag{23}$$

$$= \sum_{n=0}^{n=2} \Pr(Y = n) - \sum_{n=0}^{n=0} \Pr(Y = n) \tag{24}$$

$$= {}^2C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^0 + {}^2C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^1 \tag{25}$$

$$\implies p = \frac{11}{36} \tag{26}$$

Let Z be a random variable denoting number of successes.

$$\therefore Z \sim Ber(p) \tag{27}$$

$$\text{where } p = \frac{11}{36} \text{ by (26)} \tag{28}$$

$$\therefore \tag{29}$$

$$p_Z(k) = \begin{cases} \frac{25}{36}, & k = 0 \\ \frac{11}{36}, & k = 1 \\ 0, & \text{otherwise} \end{cases}$$