Assignment 1

AI1110:Probability and Random Variables
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12.13.6.12 Question: Suppose we have four boxes A,B,C and D containing coloured marbles as given in tables folder: One of the boxes has been selected at random and a single marble is drawn from it. If the marble is read, what is the probability that it was drawn from
1) Box A ?
2) Box B ?
3) Box C ?

Answer: 1)$\frac{1}{15}$ 2)$\frac{2}{5}$ 3)$\frac{8}{15}$

Solution:

Events are defined in tables folder.

Probability of chosen box being A given that drawn marble is red is given by: $\Pr(E_1|E)$

Probability of chosen box being B given that drawn marble is red is given by: $\Pr(E_2|E)$

Probability of chosen box being C given that drawn marble is red is given by: $\Pr(E_3|E)$

Here,

$$\Pr(E|E_1) = \frac{1}{10} \tag{1}$$

$$\Pr(E|E_2) = \frac{6}{10} \tag{2}$$

$$\Pr(E|E_3) = \frac{8}{10} \tag{3}$$

$$\Pr(E|E_4) = \frac{0}{10} \tag{4}$$

$$\Pr(E_1) = \Pr(E_2) = \Pr(E_3) = \Pr(E_4) = \frac{1}{4} \tag{5}$$

Now by Bayes' theorem,

$$\Pr(E_1|E) = \frac{\Pr(E|E_1) . \Pr(E_1)}{\Pr(E|E_1) . \Pr(E_1) + \Pr(E|E_2) . \Pr(E_2) + \Pr(E|E_3) . \Pr(E_3) + \Pr(E|E_4) . \Pr(E_4)}$$
$$= \frac{\frac{1}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4} + \frac{6}{10}\cdot\frac{1}{4} + \frac{8}{10}\cdot\frac{1}{4} + \frac{0}{10}\cdot\frac{1}{4}}$$
$$= \frac{\frac{1}{40}}{\frac{15}{40}}$$
$$= \frac{1}{15}$$

$$\Pr(E_2|E) = \frac{\Pr(E|E_2) . \Pr(E_2)}{\Pr(E|E_1) . \Pr(E_1) + \Pr(E|E_2) . \Pr(E_2) + \Pr(E|E_3) . \Pr(E_3) + \Pr(E|E_4) . \Pr(E_4)}$$
$$= \frac{\frac{6}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4} + \frac{6}{10}\cdot\frac{1}{4} + \frac{8}{10}\cdot\frac{1}{4} + \frac{0}{10}\cdot\frac{1}{4}}$$
$$= \frac{\frac{6}{40}}{\frac{15}{40}}$$
$$= \frac{2}{5}$$

$$\Pr(E_3|E) = \frac{\Pr(E|E_3) . \Pr(E_3)}{\Pr(E|E_1) . \Pr(E_1) + \Pr(E|E_2) . \Pr(E_2) + \Pr(E|E_3) . \Pr(E_3) + \Pr(E|E_4) . \Pr(E_4)}$$
$$= \frac{\frac{8}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4} + \frac{6}{10}\cdot\frac{1}{4} + \frac{8}{10}\cdot\frac{1}{4} + \frac{0}{10}\cdot\frac{1}{4}}$$
$$= \frac{\frac{8}{40}}{\frac{15}{40}}$$
$$= \frac{8}{15}$$