

Assignment 2

AI1110: Probability and Random Variables
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12.13.4.5 Question: Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

- i) number greater than 4
- ii) six appears on at least one die

Solution:

Let X be a random variable denoting the outcome of a die toss so,

$$X = \{1, 2, 3, 4, 5, 6\} \quad (1)$$

i) Let the Cumulative Distribution function be:

$$F_X(i) = \Pr(X \leq i) \quad (2)$$

$$= \sum_{n=1}^{n=i} \Pr(X = n) \quad (3)$$

Now,

$$\Pr(X = i) = \frac{1}{6} \quad \forall 1 \leq i \leq 6 \quad (4)$$

$$\therefore F_X(i) = \sum_{n=1}^{n=i} \frac{1}{6} \quad (5)$$

$$\Rightarrow F_X(i) = i/6 \quad (6)$$

Now,

$$\Pr(X > 4) = F_X(6) - F_X(4) \quad (7)$$

$$\text{by (6)} \quad (8)$$

$$= \frac{6}{6} - \frac{4}{6} \quad (9)$$

$$= \frac{1}{3} \quad (10)$$

$\therefore \frac{1}{3}$ is the probability of success in this case for a die.

Now,

Let Y be the random variable denoting number of successes.

$$\therefore Y \sim \text{Bin}(n, p) \quad (11)$$

where $n = 2$ and $p = \frac{1}{3}$

$$\therefore \Pr(Y = i) = {}^2C_i (1 - p)^{2-i} p^i \quad (12)$$

By (12),

$$p_Y(k) = \begin{cases} \frac{4}{9}, & k = 0 \\ \frac{4}{9}, & k = 1 \\ \frac{1}{9}, & k = 2 \\ 0, & \text{otherwise} \end{cases}$$

ii) Let p' be the probability of getting 6 in a die toss.

\therefore

$$p' = \Pr(X = 6) \quad (13)$$

$$\text{by (4)} \quad (14)$$

$$= \frac{1}{6} \quad (15)$$

Let Y be a random variable denoting the number 6's in die tosses,

$$\therefore Y \sim \text{Bin}(n, p') \quad (16)$$

$$\text{where } n = 2 \text{ and } p' = \frac{1}{6} \quad (17)$$

$$\therefore \Pr(Y = i) = {}^2C_i p'^i (1 - p')^{2-i} \quad (18)$$

$$\Rightarrow \Pr(Y = i) = {}^2C_i \left(\frac{1}{6}\right)^i \left(\frac{5}{6}\right)^{2-i} \quad (19)$$

Let the Cumulative Distribution function be:

$$F_Y(i) = \Pr(Y \leq i) \quad (20)$$

$$\Rightarrow F_Y(i) = \sum_{n=0}^{n=i} \Pr(Y = n) \quad (21)$$

Let p denote the probability of success in this case.

$$\therefore p = F_Y(2) - F_Y(0) \quad (22)$$

$$\text{by (19),(21)} \quad (23)$$

$$= \sum_{n=0}^{n=2} \Pr(Y = n) - \sum_{n=0}^{n=0} \Pr(Y = n) \quad (24)$$

$$= {}^2C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^0 + {}^2C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^1 \quad (25)$$

$$\Rightarrow p = \frac{11}{36} \quad (26)$$

Let Z be a random variable denoting number of successes.

$$\therefore Z \sim \text{Ber}(p) \quad (27)$$

$$\text{where } p = \frac{11}{36} \text{ by (26)} \quad (28)$$

$$\therefore \quad (29)$$

$$p_Z(k) = \begin{cases} \frac{25}{36}, & k = 0 \\ \frac{11}{36}, & k = 1 \\ 0, & \text{otherwise} \end{cases}$$