Assignment 2

AI1110:Probability and Random Variables
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12.13.4.5 Question: Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as
(i) number greater than 4
(ii) six appears on at least one die

Solution:
Let X be a random variable denoting the outcome of a die toss so,

$$X = \{1, 2, 3, 4, 5, 6\} \tag{1}$$

(i)
Let the Cumulative Distribution function be:

$$F_X(i) = \Pr(X \le i) \tag{2}$$

$$= \sum_{n=1}^{n=i} \Pr(X = n) \tag{3}$$

Now,

$$\Pr(X = i) = \frac{1}{6} \ \forall 1 \le i \le 6 \tag{4}$$

$$\therefore F_X(i) = \sum_{n=1}^{n=i} \frac{1}{6} \tag{5}$$

$$\implies F_X(i) = i/6 \tag{6}$$

Now,

$$\Pr(X > 4) = F_X(6) - F_X(4) \tag{7}$$

$$\text{by (6)} \tag{8}$$

$$= \frac{6}{6} - \frac{4}{6} \tag{9}$$

$$= 1 - \frac{2}{3} \tag{10}$$

$$= \frac{1}{3} \tag{11}$$

$\therefore \frac{1}{3}$ is the probability of success in this case for a die.
Now,

Let Y be the random variable denoting number of successes.

$$\therefore Y \sim Bin(n, p) \tag{12}$$

where n=2 and p = $\frac{1}{3}$

$$\therefore \Pr(Y = i) = \binom{2}{i}(1-p)^{2-i}p^i \tag{13}$$

By (13),

$$\Pr(Y = 0) = \binom{2}{0}(\frac{2}{3})^2(\frac{1}{3})^0 \tag{14}$$

$$= \frac{4}{9} \tag{15}$$

$$\Pr(Y = 1) = \binom{2}{1}(\frac{2}{3})^1(\frac{1}{3})^1 \tag{16}$$

$$= 2 \times \frac{2}{3} \times \frac{1}{3} \tag{17}$$

$$= \frac{4}{9} \tag{18}$$

$$\Pr(Y = 2) = \binom{2}{2}(\frac{2}{3})^0(\frac{1}{3})^2 \tag{19}$$

$$= \frac{1}{9} \tag{20}$$

$\therefore$

$$\Pr(Y = i) = \begin{cases} \frac{4}{9}, & i = 0 \\ \frac{4}{9}, & i = 1 \\ \frac{1}{9}, & i = 2 \\ 0, & \text{otherwise} \end{cases}$$

(ii)

Let p denote the probability of success in this case.
$\therefore$ 1-p is probability of failure i.e., getting no 6 in either of the tosses

$$\therefore 1 - p = F_X(5)F_X(5) \tag{21}$$

$$\text{by (6)} \tag{22}$$

$$= \frac{5}{6} \times \frac{5}{6} \tag{23}$$

$$= \frac{25}{36} \tag{24}$$

$$\implies p = 1 - \frac{25}{36} \tag{25}$$

$$\implies p = \frac{11}{36} \tag{26}$$

Let Z be a random variable denoting number of successes.

$$\therefore Z \sim Ber(p) \tag{27}$$

where p=$\frac{11}{36}$ by (26)
$\therefore$

$$\Pr(Z = i) = \begin{cases} \frac{25}{36}, & i = 0 \\ \frac{11}{36}, & i = 1 \\ 0, & \text{otherwise} \end{cases}$$