Assignment 1

AI1110:Probability and Random Variables

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12.13.6.12 Question: Suppose we have four boxes A,B,C and D containing coloured marbles as given below:

Box	Marble colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is read, what is the probability that it was drawn from
1) Box A ?
2) Box B ?
3) Box C ?

Answer: 1)$\frac{1}{15}$ 2)$\frac{2}{5}$ 3)$\frac{8}{15}$

Solution:

Let E be the event that the drawn marble is red.

Let E_1 be the event that the selected box is A.

Let E_2 be the event that the selected box is B.

Let E_3 be the event that the selected box is C.

Let E_4 be the event that the selected box is D.

Probability of chosen box being A given that drawn marble is red is given by: $\Pr(E_1|E)$

Probability of chosen box being B given that drawn marble is red is given by: $\Pr(E_2|E)$

Probability of chosen box being C given that drawn marble is red is given by: $\Pr(E_3|E)$

Here,

$$\Pr(E|E_1) = \frac{1}{10} \tag{1}$$
$$\Pr(E|E_2) = \frac{6}{10} \tag{2}$$
$$\Pr(E|E_3) = \frac{8}{10} \tag{3}$$
$$\Pr(E|E_4) = \frac{0}{10} \tag{4}$$
$$\Pr(E_1) = \Pr(E_2) = \Pr(E_3) = \Pr(E_4) = \frac{1}{4} \tag{5}$$

Now by Bayes' theorem,

$$\Pr(E_1|E) = \frac{\Pr(E|E_1).\Pr(E_1)}{\Pr(E|E_1).\Pr(E_1)+\Pr(E|E_2).\Pr(E_2)+\Pr(E|E_3).\Pr(E_3)+\Pr(E|E_4).\Pr(E_4)}$$
$$= \frac{\frac{1}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4}+\frac{6}{10}\cdot\frac{1}{4}+\frac{8}{10}\cdot\frac{1}{4}+\frac{0}{10}\cdot\frac{1}{4}}$$
$$= \frac{\frac{1}{40}}{\frac{15}{40}}$$
$$= \frac{1}{15}$$

$$\Pr(E_2|E) = \frac{\Pr(E|E_2).\Pr(E_2)}{\Pr(E|E_1).\Pr(E_1)+\Pr(E|E_2).\Pr(E_2)+\Pr(E|E_3).\Pr(E_3)+\Pr(E|E_4).\Pr(E_4)}$$
$$= \frac{\frac{6}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4}+\frac{6}{10}\cdot\frac{1}{4}+\frac{8}{10}\cdot\frac{1}{4}+\frac{0}{10}\cdot\frac{1}{4}}$$
$$= \frac{\frac{6}{40}}{\frac{15}{40}}$$
$$= \frac{2}{5}$$

$$\Pr(E_3|E) = \frac{\Pr(E|E_3).\Pr(E_3)}{\Pr(E|E_1).\Pr(E_1)+\Pr(E|E_2).\Pr(E_2)+\Pr(E|E_3).\Pr(E_3)+\Pr(E|E_4).\Pr(E_4)}$$
$$= \frac{\frac{8}{10}\cdot\frac{1}{4}}{\frac{1}{10}\cdot\frac{1}{4}+\frac{6}{10}\cdot\frac{1}{4}+\frac{8}{10}\cdot\frac{1}{4}+\frac{0}{10}\cdot\frac{1}{4}}$$
$$= \frac{\frac{8}{40}}{\frac{15}{40}}$$
$$= \frac{8}{15}$$