
Practical Linear Algebra for Data Science

*From Core Concepts to Applications
Using Python*

Mike X Cohen

Beijing • Boston • Farnham • Sebastopol • Tokyo

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Practical Linear Algebra for Data Science

by Mike X Cohen

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