

# Discrete Assignment

## EE1205 Signals and Systems

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**Question 11.9.1.2:** Write the first five terms of the sequence whose  $n^{th}$  terms  $x(n) = \frac{n}{n+1}$

**Solution:** Given the terms of the sequence are  $x(n) = \frac{n}{n+1}$  where  $n = 0, 1, 2, 3, 4, \dots$

$$x(0) = 0$$

$$x(1) = \frac{1}{2}$$

$$x(2) = \frac{2}{3}$$

$$x(3) = \frac{3}{4}$$

$$x(4) = \frac{4}{5}$$

Now,

$$x(n) = \begin{cases} 0 & \text{if } n = 0 \\ u(n) - \frac{u(n)}{n+1} & \text{if } n > 0 \\ \text{not defined} & \text{if } n < 0 \end{cases}$$

Z-transform is defined as,

$$x(n) \xleftrightarrow{z} X(z)$$

$$X(z) = \sum_{i=-\infty}^{\infty} x(n).z^{-n} \quad (1)$$

Here, Z-transform

$$X(z) = \sum_{i=1}^{\infty} x(n).z^{-n} \quad (2)$$

$$= \sum_{i=1}^{\infty} \frac{n}{n+1}.z^{-n} \quad (3)$$

On solving,

$$\begin{aligned} X(z) &= \frac{z^{-1}}{2} + \frac{2z^{-2}}{3} + \frac{3z^{-3}}{4} + \dots \\ &= z^{-1} - \frac{z^{-1}}{2} + z^{-2} - \frac{z^{-2}}{3} + \dots \\ &= 1 + z^{-1} + z^{-2} + \dots + \left[-z^0 - \frac{z^{-1}}{2} - \frac{z^{-2}}{3} - \dots\right] \end{aligned}$$

Applying the formula for logarithmic expansion and sum to infinite terms of a GP,

$$X(z) = \frac{1}{1-z^{-1}} + z \log(1-z^{-1}) \quad (4)$$

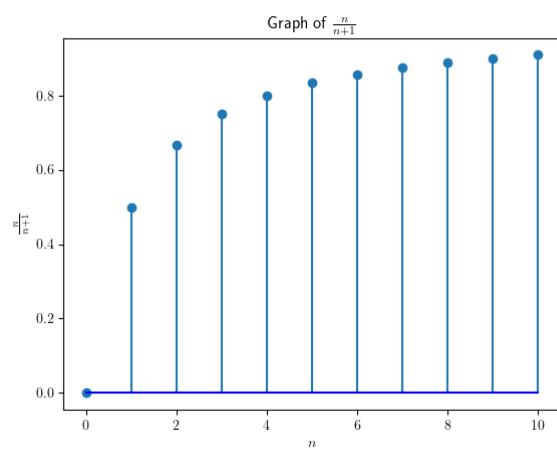


Figure 1: Sequence plot generated from Python script