GATE 2023 EC

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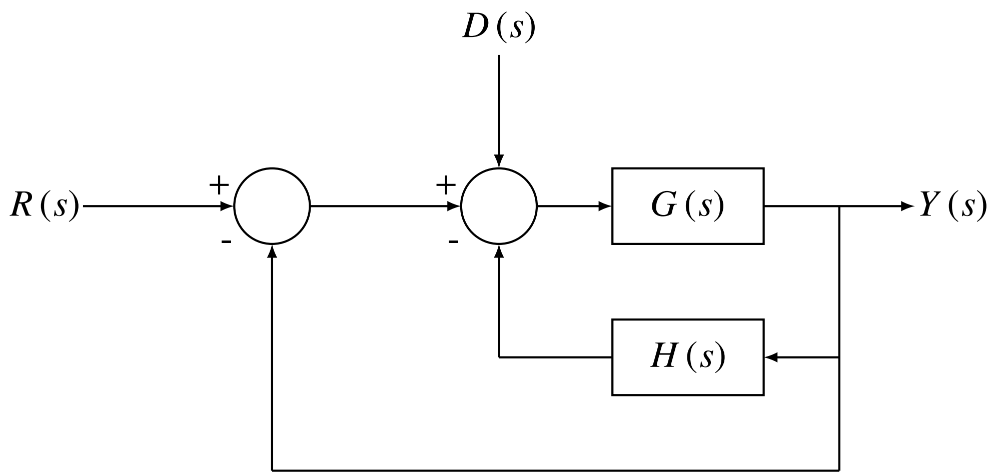
Question: 42 In the following block diagram, $R(s)$ and $D(s)$ are two inputs. The output Y(s) is expressed as $Y(s) = G_1(s)R(s) + G_2(s)D(s)$
$G_1(s)$ and $G_2(s)$ are given by



a) $G_1(s) = \frac{G(s)}{1+G(s)+G(s)H(s)}$ and $G_2(s) = \frac{G(s)}{1+G(s)+G(s)H(s)}$

b) $G_1(s) = \frac{G(s)}{1+G(s)+H(s)}$ and $G_2(s) = \frac{G(s)}{1+G(s)+H(s)}$

c) $G_1(s) = \frac{G(s)}{1+G(s)+H(s)}$ and $G_2(s) = \frac{G(s)}{1+G(s)+G(s)H(s)}$

d) $G_1(s) = \frac{G(s)}{1+G(s)+G(s)H(s)}$ and $G_2(s) = \frac{G(s)}{1+G(s)+H(s)}$

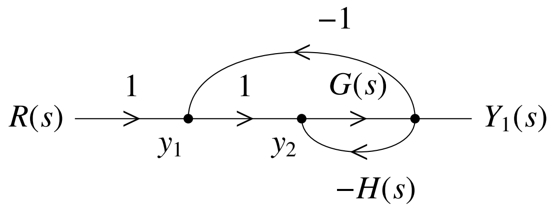
Solution:
By superposition principle, let

$$\begin{aligned} Y(s) &= Y_1(s) + Y_2(s) \\ \text{where } Y_1(s) &= \text{output considering only } R(s) \\ Y_2(s) &= \text{Output considering only } D(s) \end{aligned}$$

When only R(s) is present:

By Mason's gain formula,

$$\frac{Y_1(s)}{R(s)} = \sum_{k=1}^{n} \frac{P_k.\Delta_k}{\Delta} \tag{1}$$



where,

$$
\begin{aligned}
n &= \text{Number of forward paths} \\
\Delta_k &= \text{ Associated path factor} \\
P_k &= \text{ Path gain of the } k^{th} \text{ forward path}
\end{aligned}
$$

Here,

$$n = 1\,(\text{i.e path } R - y_1 - y_2 - y_3 - Y_1) \tag{2}$$
$$P_1 = G\,(s) \tag{3}$$
$$\Delta_1 = 1\,(\text{As no isolated node is present}) \tag{4}$$
$$\Delta = 1 + G\,(s) + G\,(s)\,H\,(s) \tag{5}$$

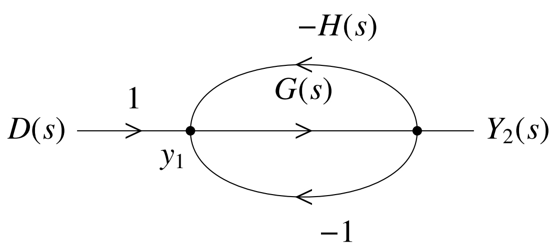
Substituting in (1),

$$\frac{Y_1\,(s)}{R\,(s)} = \frac{G\,(s)}{1 + G\,(s) + G\,(s)\,H\,(s)} \tag{6}$$
$$Y_1\,(s) = \left[\frac{G\,(s)}{1 + G\,(s) + G\,(s)\,H\,(s)}\right] R\,(s) \tag{7}$$

Hence,

$$G_1\,(s) = \frac{G\,(s)}{1 + G\,(s) + G\,(s)\,H\,(s)} \tag{8}$$

When only $D\,(s)$ is present:



Here,

$$n = 1\,(\text{i.e path } D - y_1 - y_2 - Y_2) \tag{9}$$
$$P_1 = G\,(s) \tag{10}$$
$$\Delta_1 = 1 \tag{11}$$
$$\Delta = 1 + G\,(s) + G\,(s)\,H\,(s) \tag{12}$$

Substituting in (1),

$$\frac{Y_2(s)}{D(s)} = \frac{G(s)}{1 + G(s) + G(s)H(s)} \tag{13}$$

$$Y_2(s) = \left[\frac{G(s)}{1 + G(s) + G(s)H(s)}\right] D(s) \tag{14}$$

Hence,

$$G_2(s) = \frac{G(s)}{1 + G(s) + G(s)H(s)} \tag{15}$$

Option (a) is correct