

NCERT Discrete - 11.9.1.2

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Question : 11.9.1.2:

Write the first five terms of the sequence whose n^{th} terms $a_n = \frac{n}{n+1}$

Solution:

Term	Value	Description
$x(n)$	$\frac{n+1}{n+2}u(n)$	General term

TABLE 0

INPUT PARAMETERS

Here, Z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \quad (1)$$

$$= \sum_{n=-\infty}^{\infty} \frac{n+1}{n+2} \cdot u(n) \cdot z^{-n} \quad (2)$$

$$= \sum_{n=-\infty}^{\infty} u(n) \cdot z^{-n} - \frac{1}{n+2} u(n) \cdot z^{-n} \quad (3)$$

Now,

$$u(n) \xleftrightarrow{Z} \frac{1}{1-z^{-1}}, \quad |z| > 1 \quad (4)$$

$$\begin{aligned} \sum_{n=-\infty}^{\infty} -\frac{1}{n+2} u(n) \cdot z^{-n} &= -\frac{1}{2} - \frac{z^{-1}}{3} - \frac{z^{-2}}{4} \dots \\ &= z^2 \left[-z^{-1} - \frac{z^{-2}}{2} - \frac{z^{-3}}{3} \dots \right] + z \\ &= z + z^2 \log(1 - z^{-1}) \end{aligned}$$

$$\frac{-1}{n+2} \cdot u(n) \xleftrightarrow{Z} z + z^2 \log(1 - z^{-1}), \quad |z| > 1 \quad (5)$$

$$X(z) = \frac{1}{1-z^{-1}} + z + z^2 \log(1 - z^{-1}), \quad |z| > 1 \quad (6)$$

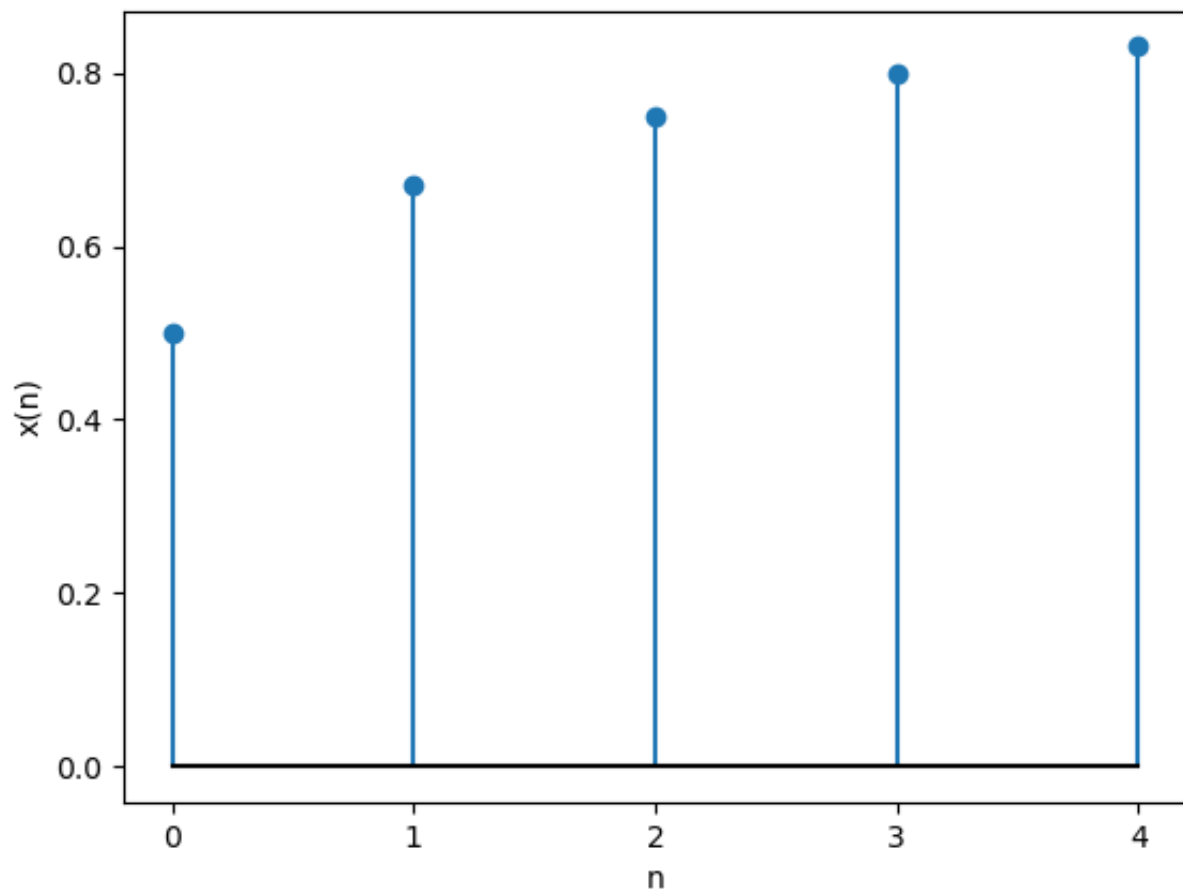


Fig. 0. Stem plot for $x(n]$