

Discrete Assignment

Praful Kesavadas
EE23BTECH11049

Question 11.9.5.15: The p th, q th and r th terms of an AP are a, b, c respectively. Show that

$$(q - r)a + (r - p)b + (p - q)c = 0$$

Solution:

The AP has the following parameters

| Term | Value | Description |
|--------|-------------|-------------------|
| $x(0)$ | - | First term |
| d | - | Common Difference |
| $x(n)$ | $x(0) + nd$ | General term |
| $x(p)$ | a | p th term |
| $x(q)$ | b | q th term |
| $x(r)$ | c | r th term |

TABLE 0
INPUT PARAMETERS

Now,

$$x(0) + pd = a \quad (1)$$

$$x(0) + qd = b \quad (2)$$

$$x(0) + rd = c \quad (3)$$

$$(4)$$

which can be represented as,

$$x(0) + p.d + a.(-1) = 0 \quad (5)$$

$$x(0) + q.d + b.(-1) = 0 \quad (6)$$

$$x(0) + r.d + c.(-1) = 0 \quad (7)$$

resulting in the matrix equation,

$$\begin{pmatrix} 1 & p & a \\ 1 & q & b \\ 1 & r & c \end{pmatrix} \mathbf{x} = 0 \quad (8)$$

where,

$$\mathbf{x} = \begin{pmatrix} x(0) \\ d \\ -1 \end{pmatrix} \quad (9)$$

solving the equations (1),(2) and (3) by row reducing the matrix in (8),

$$\begin{pmatrix} 1 & p & a \\ 1 & q & b \\ 1 & r & c \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & p & a \\ 0 & q-p & b-a \\ 0 & r-p & c-a \end{pmatrix} \quad (10)$$

$$\xleftrightarrow[R_2 \leftarrow \frac{R_2}{q-p}]{R_2 \leftarrow \frac{R_2}{q-p}} \begin{pmatrix} 1 & p & a \\ 0 & 1 & \frac{b-a}{q-p} \\ 0 & r-p & c-a \end{pmatrix} \quad (11)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - p.R_2} \begin{pmatrix} 1 & 0 & a - p \cdot \frac{b-a}{q-p} \\ 0 & 1 & \frac{b-a}{q-p} \\ 0 & r-p & c-a \end{pmatrix} \quad (12)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - (r-p).R_2} \begin{pmatrix} 1 & 0 & a - p \cdot \frac{b-a}{q-p} \\ 0 & 1 & \frac{b-a}{q-p} \\ 0 & 0 & (c-a) - \frac{(r-p)(b-a)}{q-p} \end{pmatrix} \quad (13)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{aq-pb}{q-p} \\ 0 & 1 & \frac{b-a}{q-p} \\ 0 & 0 & \frac{a(r-q)+b(p-r)+c(q-p)}{q-p} \end{pmatrix} \quad (14)$$

After row reduction of matrix we get,

$$x(0) = \frac{aq - pb}{q - p} \quad (15)$$

$$d = \frac{b - a}{q - p} \quad (16)$$

$$\frac{a(r-q) + b(p-r) + c(q-p)}{q-p} = 0 \quad (17)$$

$$\therefore (q-r)a + (r-p)b + (p-q)c = 0 = 0 \quad (18)$$

Hence proved