Discrete Assignment

Praful Kesavadas
EE23BTECH11049

Question 11.9.5.15: The pth, qth and rth terms of an AP are a,b,c respectively. Show that

$$(q-r)\,a+(r-p)\,b+(p-q)\,c=0$$

Solution:
The AP has the following parameters

Term	Value	Description
$x(0)$	-	First term
d	-	Common Difference
$x(n)$	$(x(0)+nd)\,u(n)$	General term
$x(p)$	a	pth term
$x(q)$	b	qth term
$x(r)$	c	rth term

TABLE 0
INPUT PARAMETERS

Now,

$$x(0)+pd=a \tag{1}$$
$$x(0)+qd=b \tag{2}$$
$$x(0)+rd=c \tag{3}$$

which can be represented as,

$$x(0)+p.d+a.(-1)=0 \tag{4}$$
$$x(0)+q.d+b.(-1)=0 \tag{5}$$
$$x(0)+r.d+c.(-1)=0 \tag{6}$$

resulting in the matrix equation,

$$\begin{pmatrix}1 & p & a\\ 1 & q & b\\ 1 & r & c\end{pmatrix}\mathbf{x}=\mathbf{0} \tag{7}$$

where,

$$\mathbf{x}=\begin{pmatrix}x(0)\\ d\\ -1\end{pmatrix} \tag{8}$$

solving the equations (1),(2) and (3) by row reducing the matrix in (7),

$$\begin{pmatrix}1 & p & a\\ 1 & q & b\\ 1 & r & c\end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3 - R_1} \begin{pmatrix}1 & p & a\\ 0 & q-p & b-a\\ 0 & r-p & c-a\end{pmatrix} \tag{9}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{q-p}} \begin{pmatrix}1 & p & a\\ 0 & 1 & \frac{b-a}{q-p}\\ 0 & r-p & c-a\end{pmatrix} \tag{10}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - p.R_2} \begin{pmatrix}1 & 0 & a - p.\frac{b-a}{q-p}\\ 0 & 1 & \frac{b-a}{q-p}\\ 0 & r-p & c-a\end{pmatrix} \tag{11}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - (r-p).R_2} \begin{pmatrix}1 & 0 & a - p.\frac{b-a}{q-p}\\ 0 & 1 & \frac{b-a}{q-p}\\ 0 & 0 & (c-a) - \frac{(r-p)(b-a)}{q-p}\end{pmatrix} \tag{12}$$

$$\implies \begin{pmatrix}1 & 0 & \frac{aq-pb}{q-p}\\ 0 & 1 & \frac{b-a}{q-p}\\ 0 & 0 & \frac{a(r-q)+b(p-r)+c(q-p)}{q-p}\end{pmatrix} \tag{13}$$

After row reduction of matrix we get,

$$x(0) = \frac{aq - pb}{q - p} \tag{14}$$

$$d = \frac{b - a}{q - p} \tag{15}$$

$$\frac{a(r-q) + b(p-r) + c(q-p)}{q-p} = 0 \tag{16}$$

$$\therefore (q-r)a + (r-p)b + (p-q)c = 0 \tag{17}$$

$$x(n) \xrightarrow{Z} X(z) \tag{18}$$

$$X(z) = \frac{aq - pb}{(q-p)(1 - z^{-1})} + \frac{(b-a)z^{-1}}{(q-p)(1-z^{-1})^2} \tag{19}$$

$$R.O.C\,(|z| > 1) \tag{20}$$

Hence proved