Discrete Assignment

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Question 11.9.5.15: The pth, qth and rth terms of an AP are a,b,c respectively. Show that

$$(q - r)\,a + (r - p)\,b + (p - q)\,c = 0$$

Solution:

The AP has the following parameters

Term	Value	Description
$x(0)$	-	First term
d	-	Common Difference
$x(n)$	$x(0) + nd$	General term
$x(p)$	a	pth term
$x(q)$	b	qth term
$x(r)$	c	rth term

TABLE 0

INPUT PARAMETERS

Now,

$$x(0) + pd = a \tag{1}$$
$$x(0) + qd = b \tag{2}$$
$$x(0) + rd = c \tag{3}$$

which can be represented as,

$$x(0) + p.d + a.(-1) = 0 \tag{4}$$
$$x(0) + q.d + b.(-1) = 0 \tag{5}$$
$$x(0) + r.d + c.(-1) = 0 \tag{6}$$

resulting in the matrix equation,

$$\begin{pmatrix} 1 & p & a \\ 1 & q & b \\ 1 & r & c \end{pmatrix} \mathbf{x} = \mathbf{0} \tag{7}$$

where,

$$\mathbf{x} = \begin{pmatrix} x(0) \\ d \\ -1 \end{pmatrix} \tag{8}$$

solving the equations (1),(2) and (3) by row reducing the matrix in (7),

$$\begin{pmatrix} 1 & p & a \\ 1 & q & b \\ 1 & r & c \end{pmatrix} \xrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & p & a \\ 0 & q-p & b-a \\ 0 & r-p & c-a \end{pmatrix} \tag{9}$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{q-p}} \begin{pmatrix} 1 & p & a \\ 0 & 1 & \frac{b-a}{q-p} \\ 0 & r-p & c-a \end{pmatrix} \tag{10}$$

$$\xrightarrow{R_1 \leftarrow R_1 - p.R_2} \begin{pmatrix} 1 & 0 & a - p.\frac{b-a}{q-p} \\ 0 & 1 & \frac{b-a}{q-p} \\ 0 & r-p & c-a \end{pmatrix} \tag{11}$$

$$\xrightarrow{R_3 \leftarrow R_3 - (r-p).R_2} \begin{pmatrix} 1 & 0 & a - p.\frac{b-a}{q-p} \\ 0 & 1 & \frac{b-a}{q-p} \\ 0 & 0 & (c-a) - \frac{(r-p)(b-a)}{q-p} \end{pmatrix} \tag{12}$$

$$\implies \begin{pmatrix} 1 & 0 & \frac{aq-pb}{q-p} \\ 0 & 1 & \frac{b-a}{q-p} \\ 0 & 0 & \frac{a(r-q)+b(p-r)+c(q-p)}{q-p} \end{pmatrix} \tag{13}$$

After row reduction of matrix we get,

$$x(0) = \frac{aq - pb}{q - p} \tag{14}$$

$$d = \frac{b - a}{q - p} \tag{15}$$

$$\frac{a(r-q) + b(p-r) + c(q-p)}{q-p} = 0 \tag{16}$$

$$\therefore (q-r)a + (r-p)b + (p-q)c = 0 \tag{17}$$

Hence proved