Analysis Report: Multistage Rocket Optimization

Theory vs. Reality with a Falcon 9 Case Study

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1 Introduction

This report presents an analysis comparing the theoretical performance of an optimized multistage rocket against the real-world performance of a SpaceX Falcon 9. The theoretical model, based on the rocket equation and Lagrange multipliers, seeks to maximize the payload mass fraction (Γ) for a given mission delta-V (ΔV). By contrasting the model's "perfect" solution with a real-world vehicle, we can quantify and understand the critical engineering trade-offs involved in rocket design, such as reusability, reliability, and cost.

2 Analysis: Theoretical Optimum vs. Real-World Performance

Using publicly available data for the Falcon 9 (Block 5), we can directly compare its design and performance against the results from our optimization model.

2.1 Mass Breakdown Comparison

The first point of comparison is the vehicle's mass distribution. We compare the actual mass breakdown of the Falcon 9 to that of a theoretically optimal rocket designed to produce the same total ΔV with the same technology (Isp and structural coefficients), assuming an identical total liftoff mass.

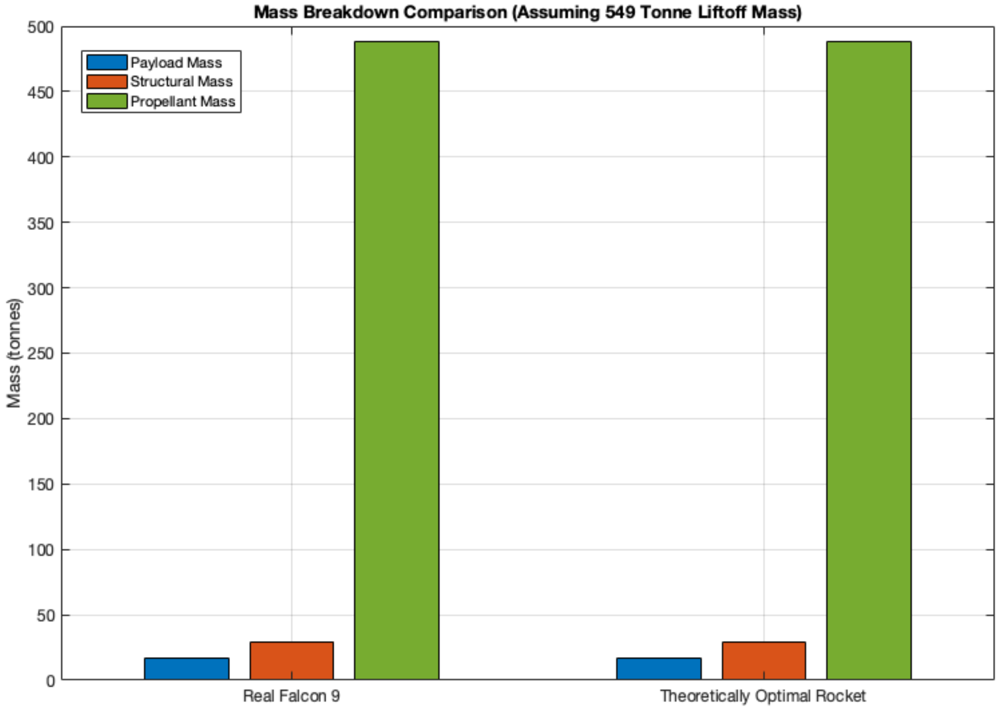

Figure 1: Comparison of mass distribution between the actual Falcon 9 and a theoretically optimal rocket with the same liftoff mass and technology.

Interpretation

Figure 1 reveals a significant difference in design philosophy between the real vehicle and the idealized model.

- **Observation:** The "Theoretically Optimal Rocket" allocates a larger portion of its mass to both **payload** and **propellant**. This is achieved by drastically reducing the **structural mass**.
- **Conclusion:** This result is a direct consequence of the model's single-minded goal: maximize payload. The mathematics concludes that the most efficient way to achieve the target ΔV is to make the vehicle's structure as light as possible, allowing more mass to be dedicated to the fuel that generates velocity and the payload that justifies the mission. This highlights the first major trade-off: our model assumes it's possible to reduce structural mass without consequence, a simplification that ignores the engineering realities of safety, reliability, and reusability.

2.2 Performance Curve Analysis

A more powerful visualization is the performance curve, which plots the maximum achievable payload fraction (Γ) as a function of the total ΔV. This creates a "performance frontier" for the given rocket technology.

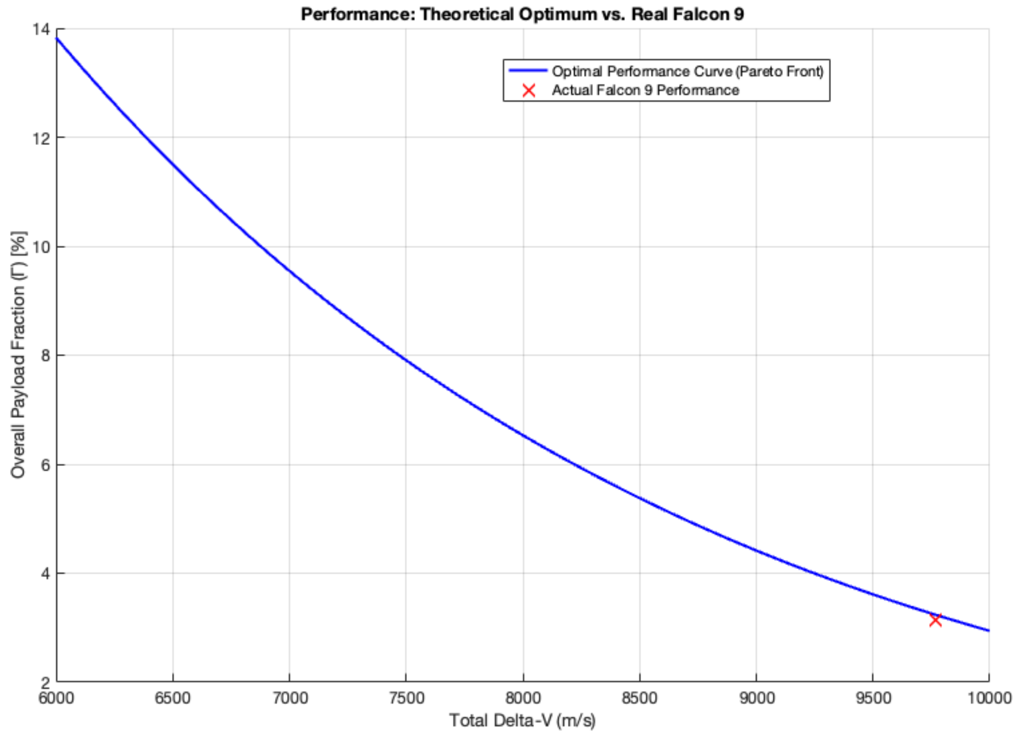

Figure 2: The theoretical performance curve (Pareto Front) for Falcon 9's technology, with its actual operating point shown as a red "X".

Interpretation

Figure 2 provides the most critical insight of this analysis.

- **The Blue Curve (Pareto Front):** This line represents the theoretical performance limit. For any given ΔV, no rocket using this specific Isp and structural technology can achieve a higher payload fraction. It is the absolute "best-case" scenario, derived purely from the physics of the rocket equation.
- **The Red "X" (Falcon 9 Operating Point):** This marks the actual performance of the Falcon 9.
- **The Gap:** The key observation is that the Falcon 9's actual performance lies **below** the optimal curve. This gap does not signify a design flaw; rather, it quantifies the performance "cost" of the real-world engineering trade-offs that our single-objective model ignores. These critical factors include:
  1. **Reusability:** The single most important factor for Falcon 9. The first stage carries extra propellant for boost-back and landing burns. It also carries heavy hardware like landing legs, grid fins, and additional thermal protection, all of which count against the payload-to-orbit capability.
  2. **Safety and Reliability:** Real-world structures are built with safety margins, making them heavier than the theoretical minimum required. This ensures the vehicle can withstand unexpected stresses and fly reliably.

3. **Cost and Manufacturability:** The chosen materials and designs may prioritize ease of manufacturing and lower cost over achieving the absolute lowest possible mass.
4. **Mission Flexibility:** The Falcon 9 is designed to service a wide range of orbits and payloads, meaning it is a versatile "jack of all trades" rather than a vehicle perfectly optimized for one specific mission profile.

3 Overall Conclusion

The optimization model is a powerful tool for establishing a theoretical baseline of performance. It answers the question: "What is the absolute best performance physically possible with this technology?" The gap between this theoretical limit and the performance of a real vehicle like the Falcon 9 provides a quantitative measure of the complex, multi-variable trade-offs that define modern aerospace engineering. In essence, the model shows us what is **possible**, while the real rocket shows us what is **practical and economically viable**.