1 (a).

$A = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

→ $\det(sI - A + BK) = 0$

$$sI - A + BK = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times 1} \begin{bmatrix} K_1 & K_2 \end{bmatrix}_{1\times 2}$$

$$= \begin{bmatrix} s+1 & -1 \\ 0 & s-2 \end{bmatrix} + \begin{bmatrix} K_1 & K_2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} s+1+K_1 & -1+K_2 \\ 0 & s-2 \end{bmatrix}$$

→ Poles of the CL system is given by: $\det(sI - A + BK) = 0$

$$0 = \begin{vmatrix} s+1+K_1 & -1+K_2 \\ 0 & s-2 \end{vmatrix}$$

$$= (s-2)(s+1+K_1) - 0$$

$$\Rightarrow (s-2)(s+1+K_1) = 0$$

Here,

$s = -K_1 - 1$

$s = 2$

↑

This shows that at least one of the closed loop poles is in the right-half plane.

(2).

$$\frac{Y(s)}{U(s)} = \frac{10}{(s+1)(s+2)(s+3)}$$

(a). => sol^n:

$$\frac{Y(s)}{U(s)} = \frac{10}{(s+1)(s+2)(s+3)} = \frac{10}{(s^2+2s+s+2)(s+3)} = \frac{10}{(s^2+3s+2)(s+3)}$$

$$= \frac{10}{s^3+3s^2+3s^2+9s+2s+6}$$

$$= \frac{10}{s^3+6s^2+11s+6}$$

In time domain,

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 10u$$

$$\underbrace{\dddot{y}}_{\dot{x}_3} = -6\underbrace{\ddot{y}}_{x_3 \to \dot{x}_2} - 11\underbrace{\dot{y}}_{x_2 \to \dot{x}_1} - 6\underbrace{y}_{x_1} + 10u$$

$y = x_1$

$\dot{y} = \dot{x}_1 = x_2$

$\ddot{y} = \dot{x}_2 = x_3$

State variable :

$$\dot{x}_3 = -6x_3 - 11x_2 - 6x_1 + 10u$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_1 = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]\, u$$

(2) (c) ⇒ Soln:

Here, The system is completely state controllable from part (b).
The desired characteristic equation is: for poles to be placed at $s=-2\pm 2\sqrt{3}j$ & -10

$$(s+10)\,(s+2-2\sqrt{3}j)\,(s+2+2\sqrt{3}j)=0$$

$$(s+10)\,(s^2+2s+\cancel{2\sqrt{3}js}+2s+4+\cancel{4\sqrt{3}j}-\cancel{2\sqrt{3}js}-\cancel{4\sqrt{3}j}+12)=0$$

$$(s+10)\,(s^2+4s+16)=0$$

$$s^3+4s^2+16s+10s^2+40s+160=0$$

$$s^3+14s^2+56s+160=0$$

Here, $A=\begin{bmatrix}0&1&0\\0&0&1\\-6&-11&-6\end{bmatrix}$, $B=\begin{bmatrix}0\\0\\10\end{bmatrix}$

$$BK=\begin{bmatrix}0\\0\\10\end{bmatrix}\begin{bmatrix}K_1 & K_2 & K_3\end{bmatrix}$$

Now,

$$\det(sI-A+BK)$$

then,

$$\det\left(\begin{bmatrix}s&0&0\\0&s&0\\0&0&s\end{bmatrix}-\begin{bmatrix}0&1&0\\0&0&1\\-6&-11&-6\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\10K_1&10K_2&10K_3\end{bmatrix}\right)$$

or,

$$\det\left(\begin{bmatrix}s&-1&0\\0&s&-1\\10K_1+6&10K_2+11&s+10K_3+6\end{bmatrix}\right)$$

$$= s\begin{vmatrix}s&-1\\10K_2+11&s+10K_3+6\end{vmatrix}-(-1)\begin{vmatrix}0&\cancel{10K_3+6}\ -1\\10K_1+6&s+10K_3+6\end{vmatrix}+0.$$

$$= s\left(s(s+10K_3+6)+10K_2+11\right)+(10K_1+6) = \underbrace{s^3+(10K_3+6)s^2+(10K_2+11)s+(10K_1+6)}_{\text{Comparing it with}}$$

$$= s\left(s(s+10K_3+6)\right.\cancel{+(10K_2+11)s+10K_1+6}$$

$$\cancel{= s^2(s+10K_3+6)+(10K_2+11)}$$

$$s^3+14s^2+56s+160$$

$10k_3 + 6 = 14 \rightarrow k_3 = 0.8$

$10k_2 + 11 = 56 \rightarrow k_2 = 4.5$

$10k_1 + 6 = 160 \rightarrow k_1 = 15.4$

$\therefore u = -Kx$

$= -k_1 x_1 - k_2 x_2 - k_3 x_3$

$$\boxed{u = -15.4x_1 - 4.5x_2 - 0.8x_3}$$

(4) (b).

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -6 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$s = -10$, $s = -10$ & $s = -15$,

$$\left| sI - (A - LC) \right| = (s+10)(s+10)(s+15)$$
$$= (s+10)^2 (s+15)$$
$$= s^3 + 35s^2 + 400s + 1500$$

$$|sI - A + LC| = \det(sI - A + LC)$$

$$sI - A + LC = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -6 & 0 \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ +5 & 6 & s \end{bmatrix} + \begin{bmatrix} L_1 & 0 & 0 \\ L_2 & 0 & 0 \\ L_3 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} s+L_1 & -1 & 0 \\ L_2 & s & -1 \\ +5+L_3 & 6 & s \end{bmatrix}$$

$$|sI - A + LC| = \begin{vmatrix} s+L_1 & -1 & 0 \\ L_2 & s & -1 \\ 5+L_3 & 6 & s \end{vmatrix} = (s+L_1)(s^2+6) + (sL_2 + 5 + L_3) + 0$$

$$= s^3 + 6s + L_1 s^2 + 6L_1 + sL_2 + 5 + L_3$$
$$= s^3 + L_1 s^2 + 6s + sL_2 + 6L_1 + 5 + L_3$$
$$= s^3 + L_1 s^2 + s(6 + L_2) + 6L_1 + L_3 + 5$$

comparing this with characteristic eqn, we get

$$s^3 + 35s^2 + 400s + 1500 = s^3 + L_1 s^2 + s(6 + L_2) + 6L_1 + L_3 + 5$$

$$L_1 = 35, \quad 6 + L_2 = 400$$
$$L_2 = 394$$

$$6L_1 + L_3 + 5 = 1500$$
$$6 \cdot 35 + L_3 = 1495$$
$$L_3 = 1285$$

$$\therefore L = \begin{bmatrix} 35 \\ 394 \\ 1285 \end{bmatrix}$$