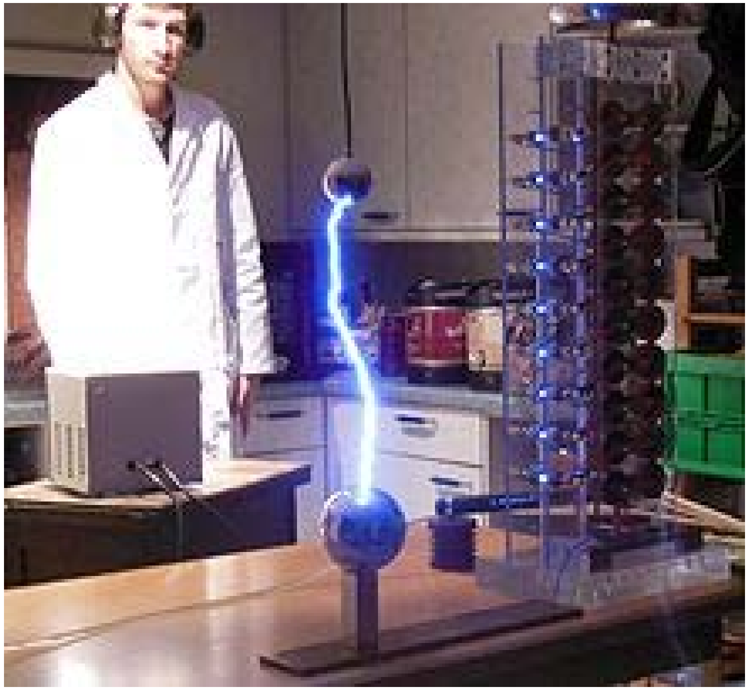
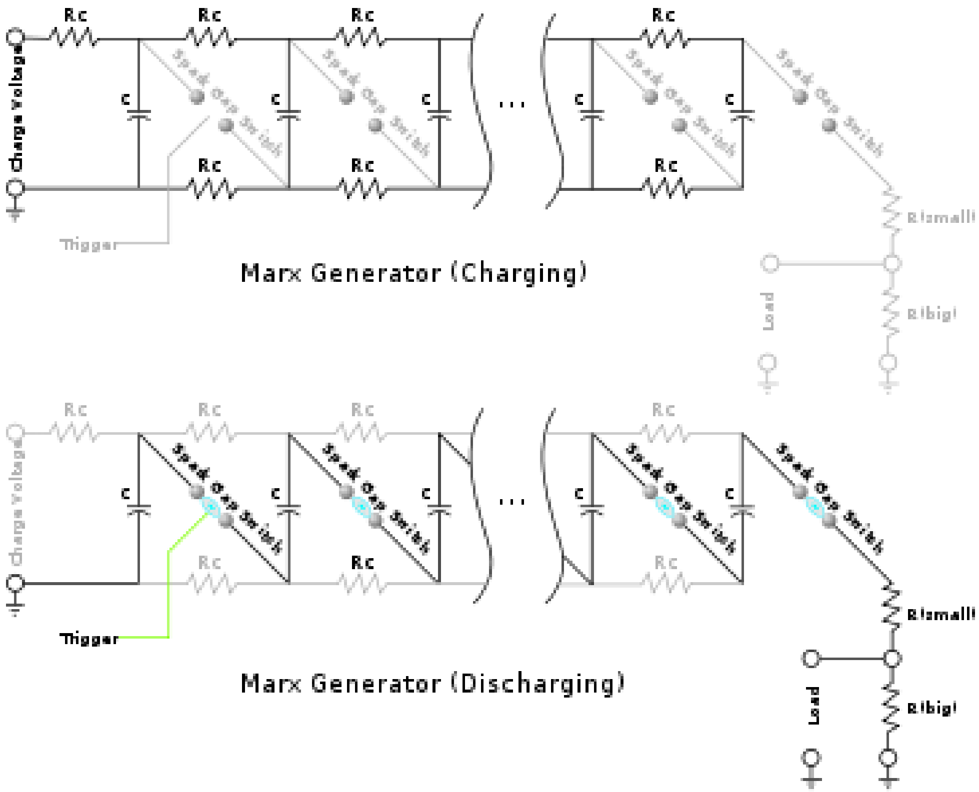
Introduction

- The simplest and most widely used high-voltage impulse generator is the device Erwin Marx introduced in 1925 for testing high-voltage components and equipment for the emerging power industry.
- The basic operation of a Marx generator is simple: Capacitors are charged in parallel through high impedances and discharged in series, multiplying the voltage.



Operational principles of simple Marxes

- A Marx generator is a voltage-multiplying circuit that charges a number of capacitors in parallel and discharges them in series.
- The process of transforming from a parallel circuit to a series one is known as "erecting the Marx."



$$V_M = NV_0$$

$$C_M = C_0/N$$

- What is a role of resistors?
  - Current limiting
  - Ground path
  - Isolation during discharge
  - Could be replaced with inductors

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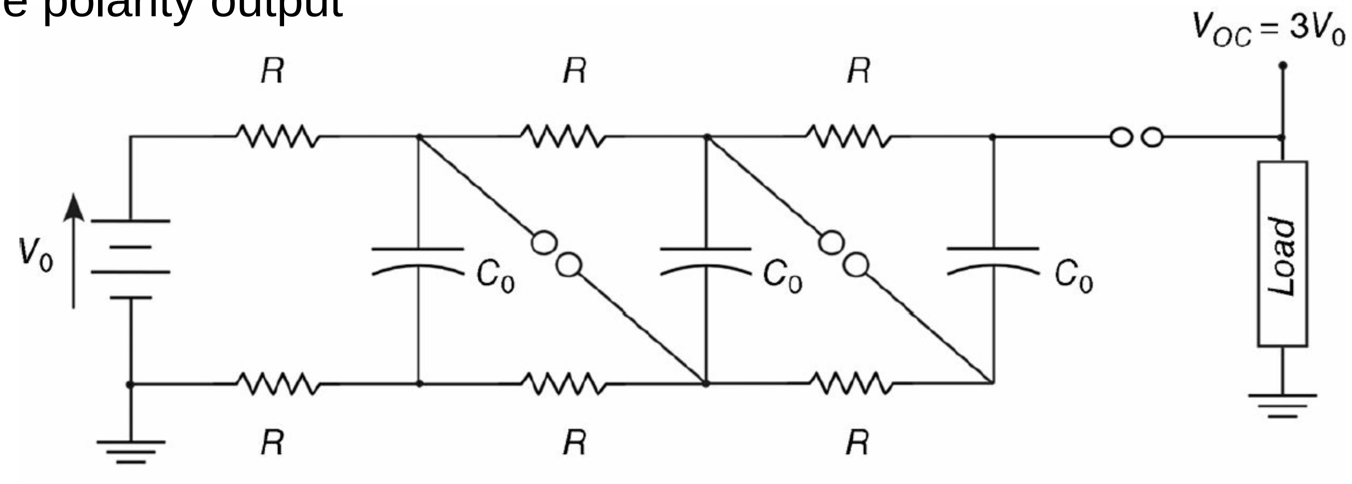
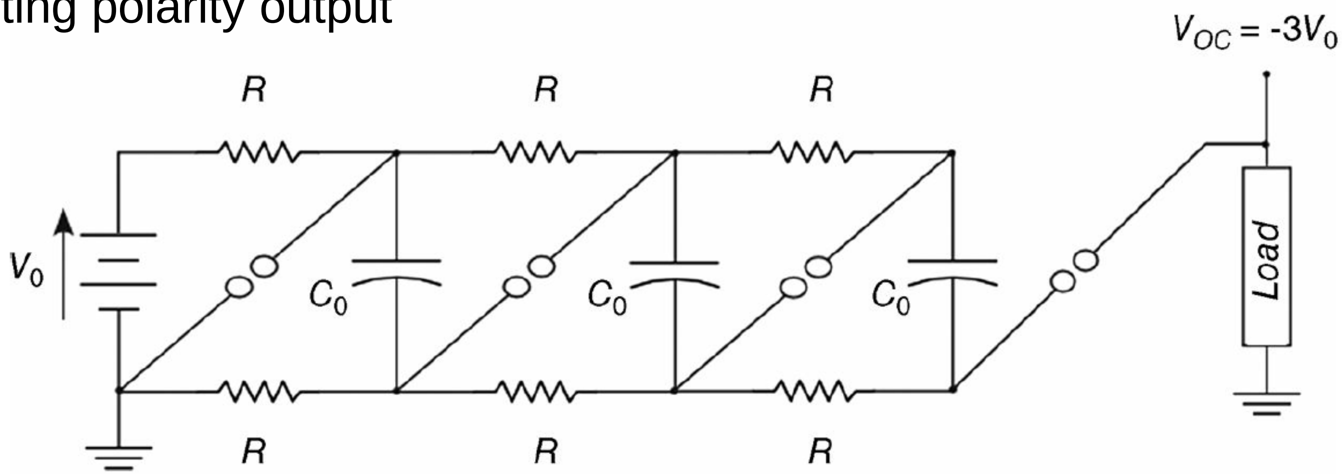
Ladder-type Marx generator

- Same polarity output



(a)

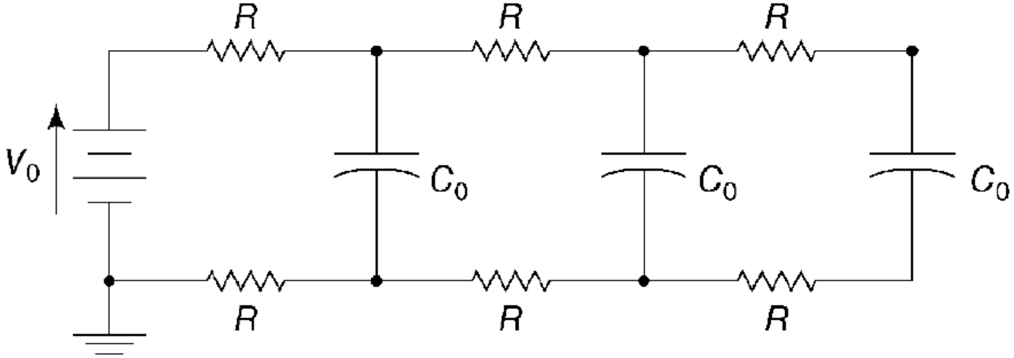
- Inverting polarity output



(b)

Marx charge cycle

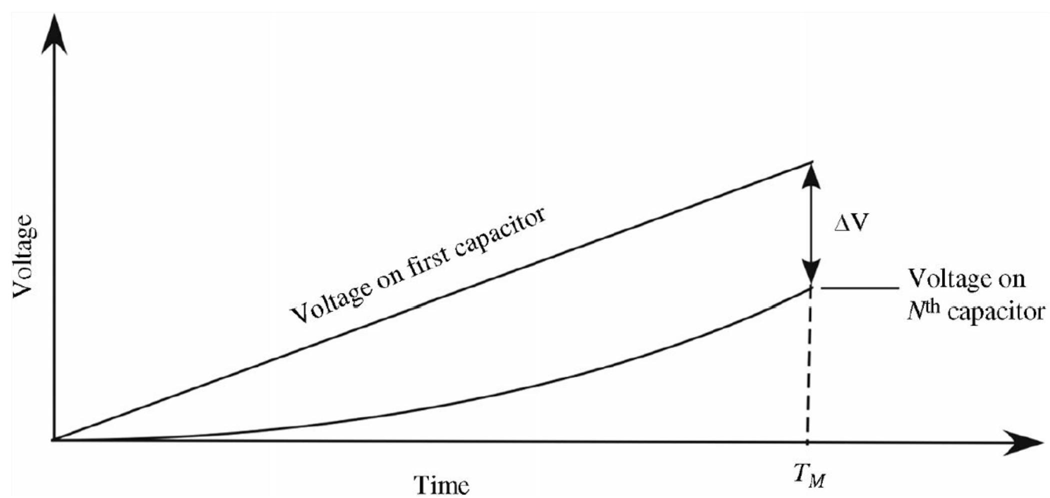
- During the charge cycle, the Marx charges a number of stages, N, each with capacitance C_0 to a voltage V_0, through a chain of charging resistors R.



$$E_{stored} = \frac{1}{2}(NC_0)V_0^2$$

- The time to charge the nth stage with a DC source is given approximately by

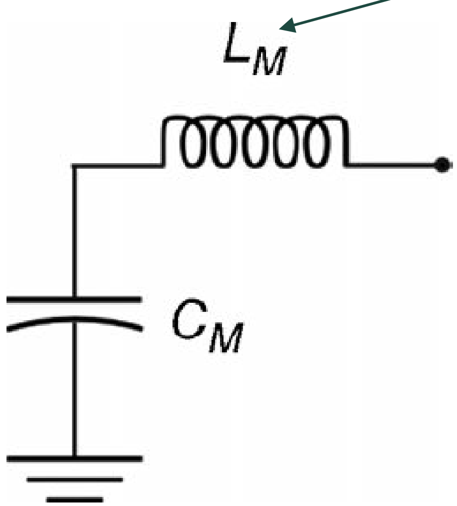
$$\tau_{ch} = n^2 RC_0$$



Marx erection

- The Marx erection is the process of sequentially closing the switches to reconfigure the capacitors from the parallel charging circuit to the series discharge circuit.
- Marx erection is initiated when a spark gap fires resulting in an increased voltage across the remaining stages.
- The equivalent circuit of an ideal Marx generator

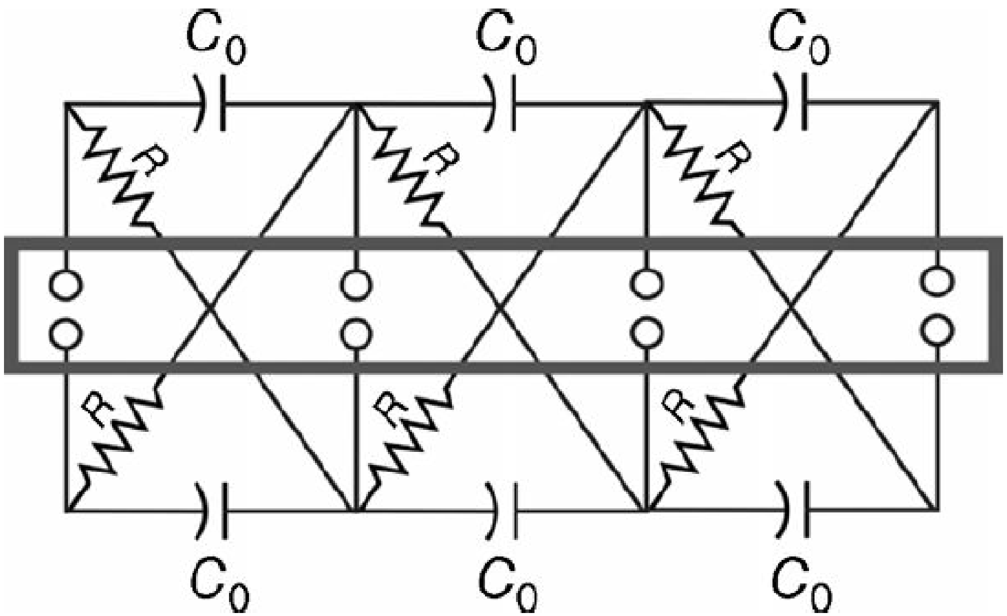
Switches + capacitors + connections



- Marx equivalent erected capacitance $C_M = \frac{C_0}{N}$
- Elected Marx open-circuit voltage $V_{OC} = -NV_0$
- Marx erected impedance $Z_M = \sqrt{\frac{L_M}{C_M}}$
- Marx intrinsic discharge time $T_M = \sqrt{L_M C_M}$

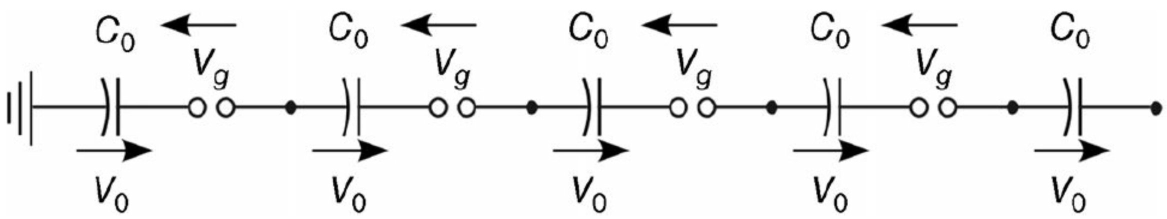
Marx erection

- The maximum output voltage is ensured when the first spark gap initiates the discharge and fires each successive stage.
- A convenient method of ensuring low jitter in simple Marx generators is to use the ultraviolet light generated by the firing of the first spark gap to trigger the next stage.
- The switches are arranged in line-of-sight.



Switch over-voltages in an ideal Marx

- The unfired Marx draws no current and $V_{OC} = 0$. Applying Kirchoff's voltage law (KVL) to each stage, the voltage across each spark gap is equal in magnitude and opposite in sense to the stage capacitor voltage.



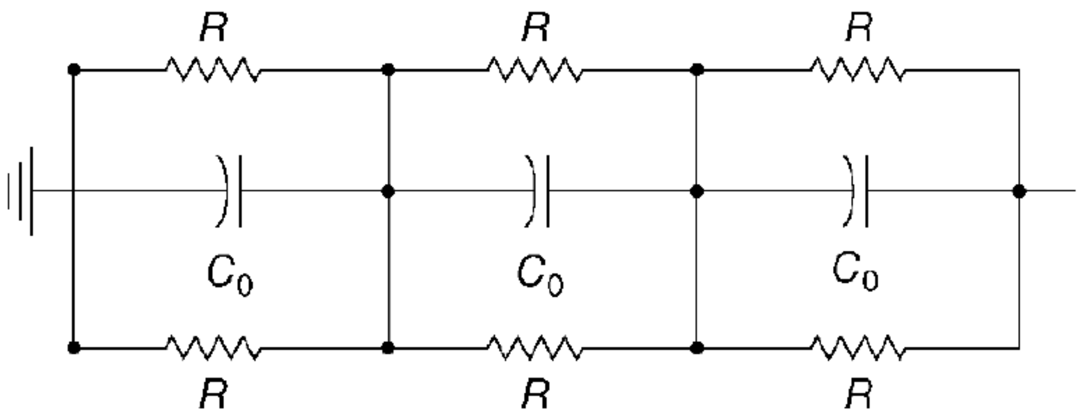
- When the first spark gap fires, its voltage goes to zero and the voltage at its upper node becomes the stage voltage V0. The Marx remains unfired and the voltage is redistributed across the remaining unfired spark gap.

$$V_{OC} = 0 = \sum_{n=1}^{N} V_0 - \sum_{n=1}^{N-1} V_g = NV_0 - (N-1)V_g \qquad V_g = \frac{N}{N-1} V_0$$

Before firing, voltage on first gap V_{g1}	$NV_0 = NV_g$	$V_g = V_0$
First gap fires:	$NV_0 = (N-1)V_g$	$V_g = \frac{N}{N-1} V_0$
Second gap fires:	$NV_0 = (N-2)V_g$	$V_g = \frac{N}{N-2} V_0$
nth gaps fires:	$NV_0 = (N-2n)V_g$	$V_g = \frac{N}{N-n} V_0$
$(N-1)$th gap fires:	$NV_0 = V_g$	$V_g = NV_0 = V_{OC}$

No fire: self-discharge

- When the final output gap of the Marx does not fire, the circuit is represented by the equivalent circuit as

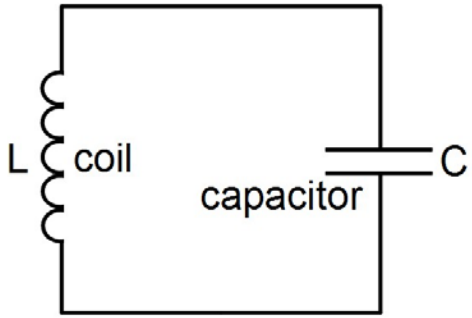


- Self-discharge time

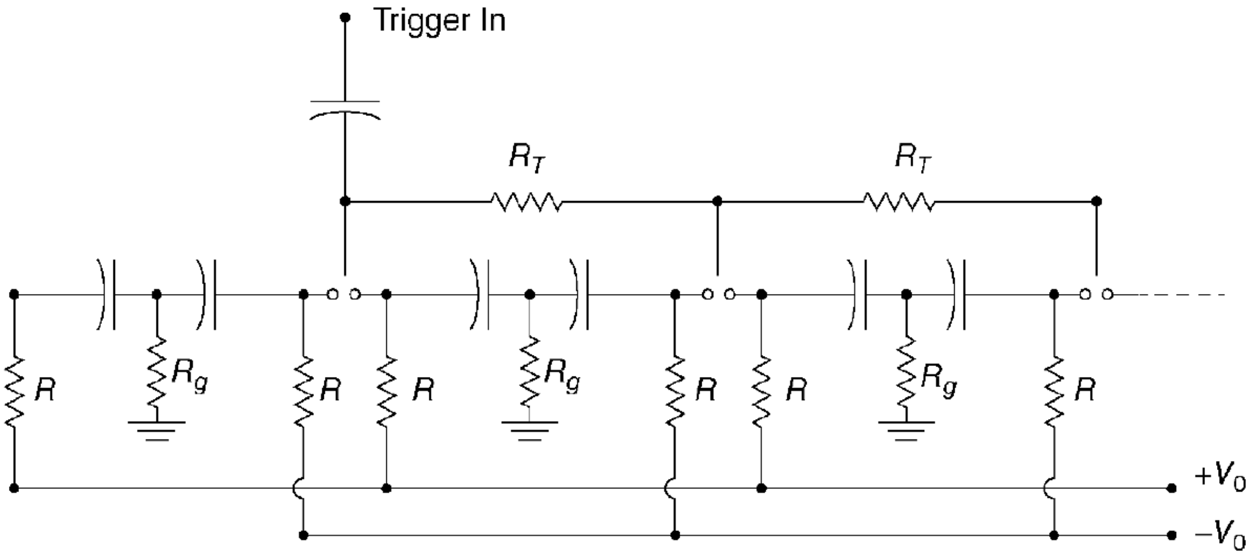
$$\tau_{discharge} = \frac{1}{2} R C_0$$

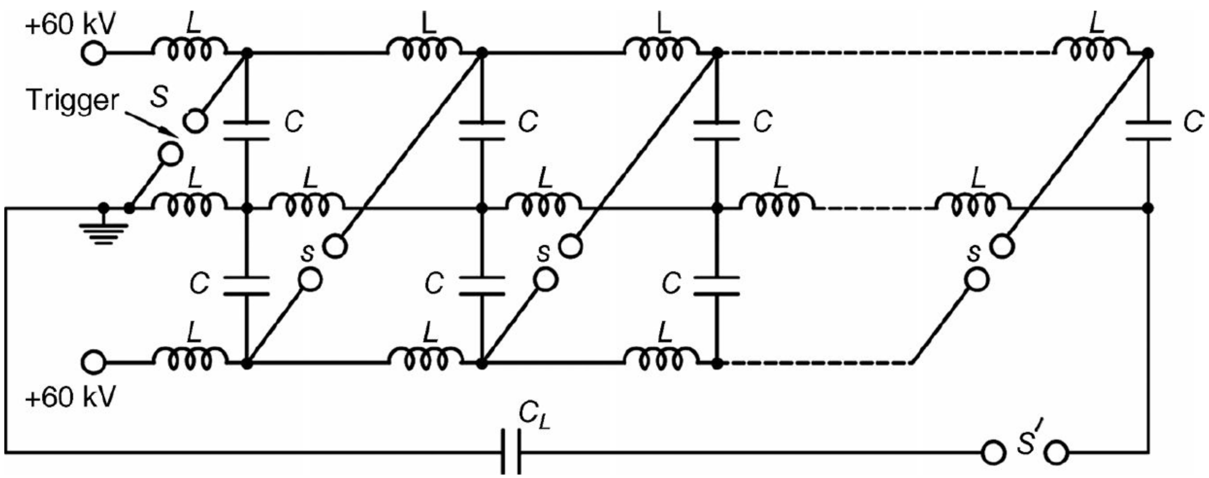
- When L is used instead of R

$$\tau_{discharge} = 2\pi \sqrt{\frac{1}{2} L C_0}$$



Bipolar Marx





$$C_M = \frac{C_0}{2N}$$

$$V_{OC} = 2NV_0$$

$$L_M = NL_S + 2NL_C$$

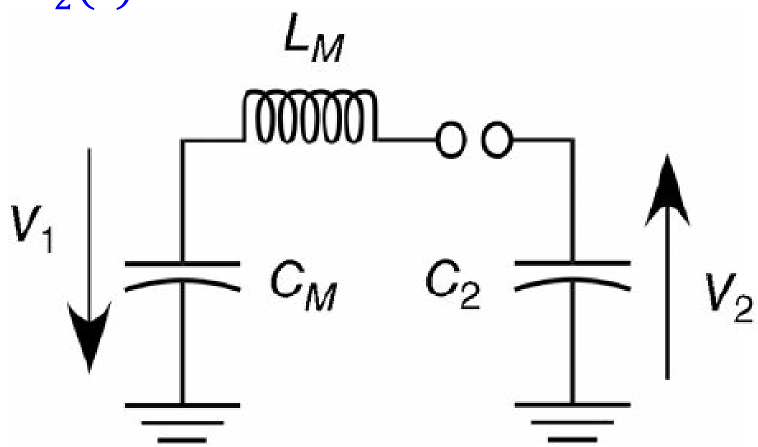
$$E_s = \frac{1}{2}(2NC_0)V_0^2$$

Capacitive loads

- A charged capacitor can transfer almost all of its energy to an uncharged capacitor if connected through an inductor. It is the basis for the intermediate storage capacitor architecture used in multi-gigawatt pulsed power machines.

- KVL $$V_1(t) - L_M \frac{di}{dt} = V_2(t)$$

$$V_1(t) - V_M = \frac{1}{C_M} \int i(t)dt$$



$$V_2(t) = \frac{1}{C_2} \int i(t)dt$$

- Find solution (H/W)

$$i(t) = \frac{V_M}{\omega L} \sin \omega t \qquad \omega = \sqrt{\frac{C_M + C_2}{L C_M C_2}}$$

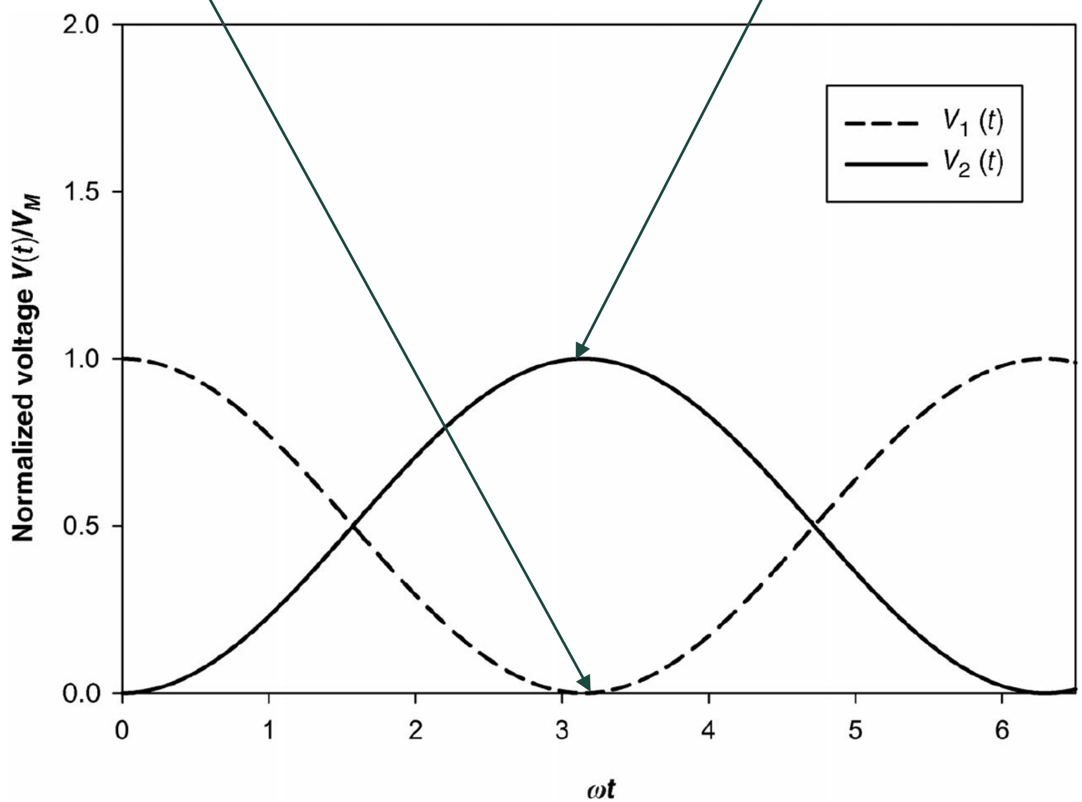
$$V_1(t) = V_M \left[1 - \frac{C_2}{C_M + C_2} [1 - \cos \omega t] \right] \qquad V_2(t) = \frac{C_M V_M}{C_M + C_2} [1 - \cos \omega t]$$

Capacitive loads: energy transfer for pulse compression

- In the case of $C_M \approx C_2$, the energy from the charged Marx generator can be transferred efficiently to the load.

$$V_1\left(t=\frac{\pi}{\omega}\right) \approx 0$$

$$V_2\left(t=\frac{\pi}{\omega}\right) = \frac{2C_M V_M}{C_M + C_2} \approx V_M$$

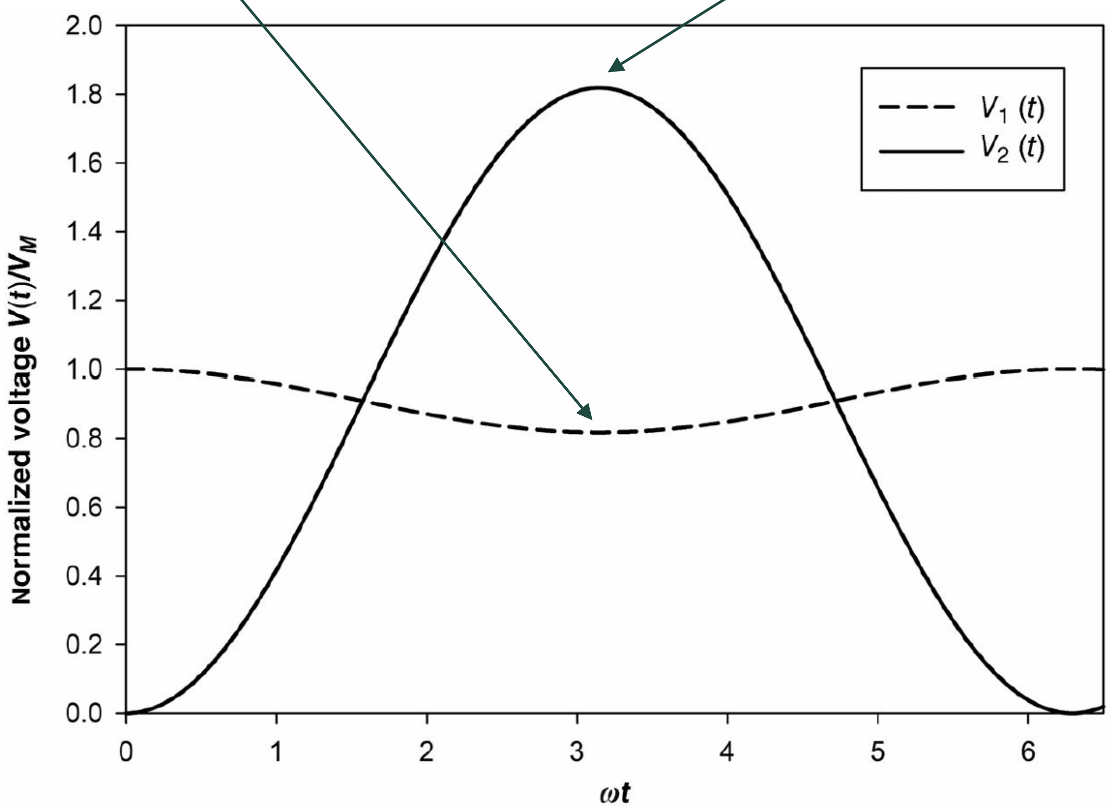


Capacitive loads: peaking circuit

- In the case of $C_M \gg C_2$, The voltage $V_2(t)$ across the capacitor C_2 is driven to nearly twice the Marx voltage, while the voltage $V_1(t)$ remains nearly the same.

$$V_1\left(t = \frac{\pi}{\omega}\right) \approx V_M$$

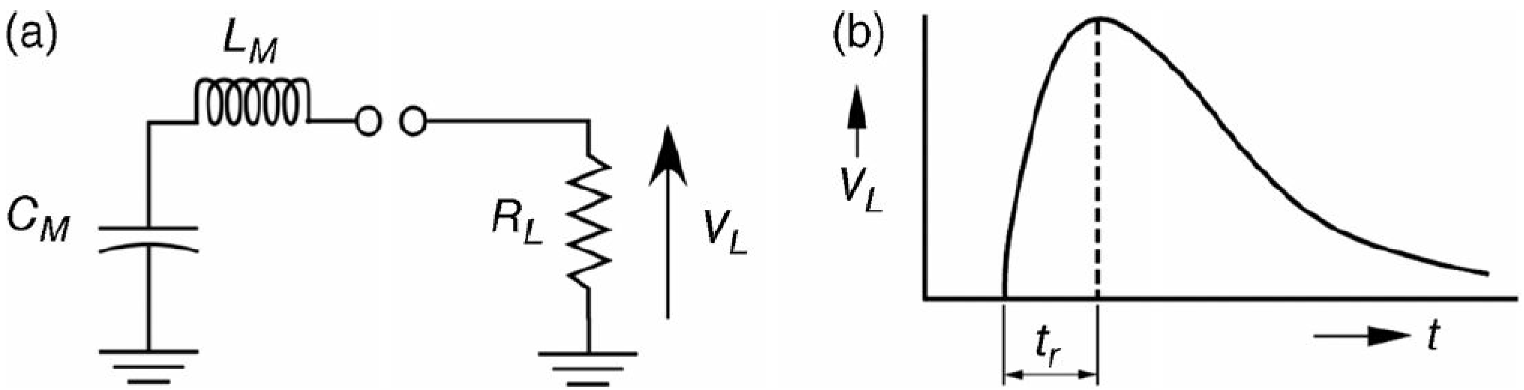
$$V_2\left(t = \frac{\pi}{\omega}\right) = \frac{2C_M V_M}{C_M + C_2} \approx 2V_M$$



Resistive loads

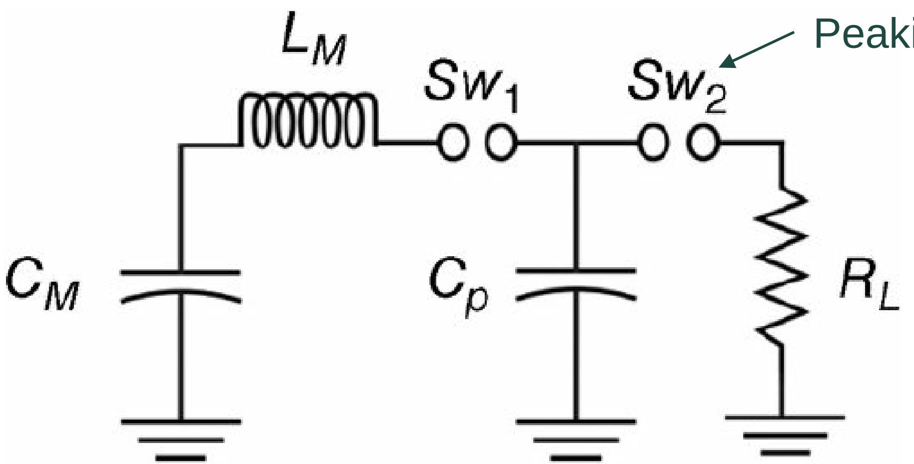
- The equivalent circuit for a Marx generator driving a resistive load is the same as typical LCR circuit.
- For a relatively large resistive load, for example electron or ion beam applications, the circuit has a characteristics of over-damped.
- The solution is

$$i(t) = \frac{V_M}{\gamma L_M}\left(1 - e^{-\gamma t}\right)\exp\left(-\frac{1}{2}\left|\frac{R}{L_M} - \gamma\right|\right)t \qquad \gamma = \sqrt{\left(\frac{R}{L_M}\right)^2 - \frac{4}{L_M C_M}}$$



Peaking circuit driving a resistive load

- A peaking capacitor ($C_2 = C_p$) can be used in conjunction with a Marx generator to sharpen the rise time of a Marx.



Peaking switch: close when the voltage is maximum

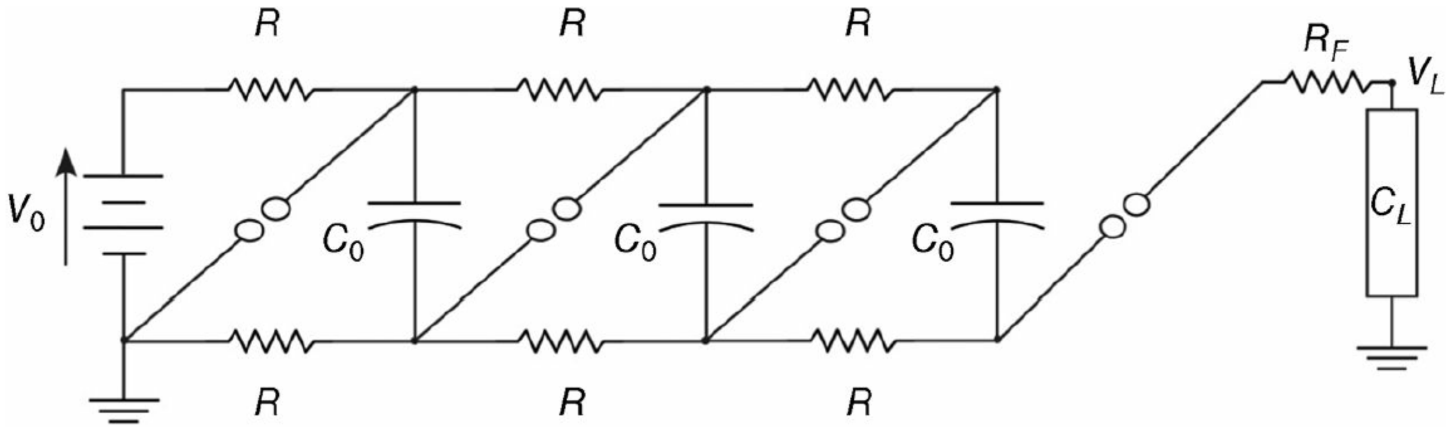
- Capacitance of the peaking capacitor $$C_p = \frac{L_M C_M}{R_L^2 C_M + L_M}$$

- The time of switch closing for the peaking switch

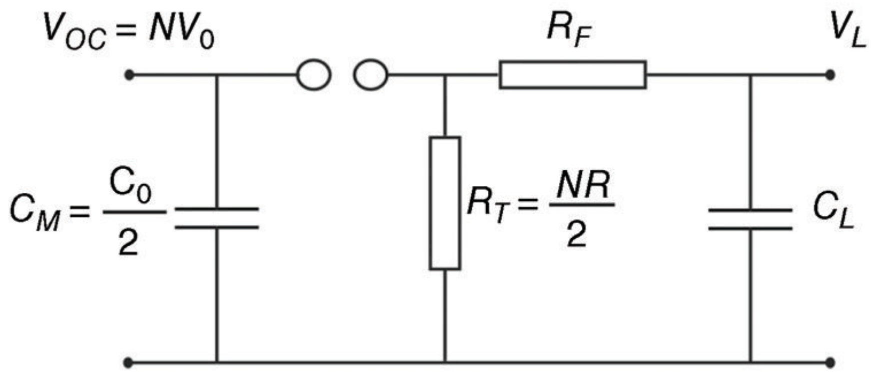
$$t_p = \frac{1}{\omega}\cos^{-1}\left(-\frac{C_p}{C_M}\right) = \sqrt{\frac{L_M C_M C_p}{C_M + C_p}}\cos^{-1}\left(-\frac{C_p}{C_M}\right)$$

Impulse generator

- Impulse generators are an important and long-used application of Marx generators. The pulse duration can be adjusted by choosing proper values for the front resistor R_F and tail resistor R_T.



- Equivalent circuit

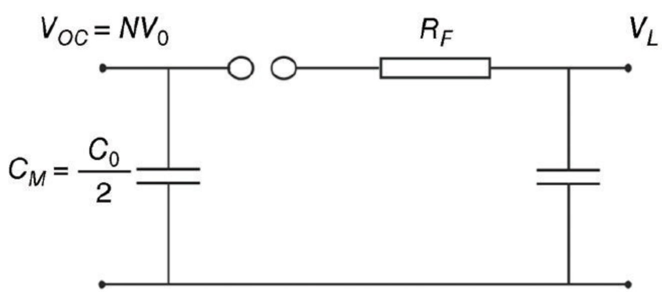


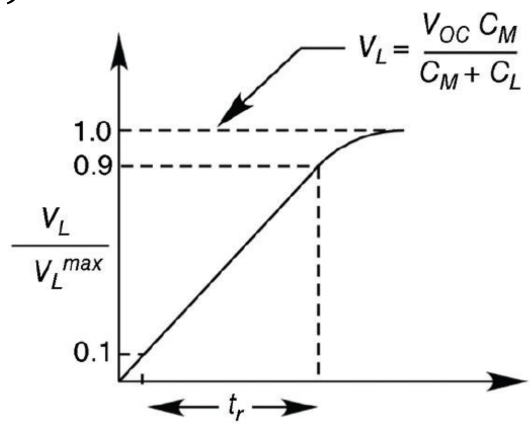
Impulse generator

- Approximate equivalent circuit for the front time behavior

$$V_L(t) \approx \frac{V_{OC}C_M}{C_M + C_L}\left(1 - e^{-t/(R_F C_T)}\right)$$

$$C_T = \frac{C_M C_L}{C_M + C_L}$$

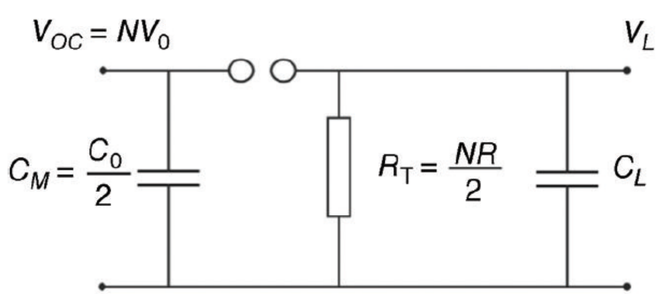


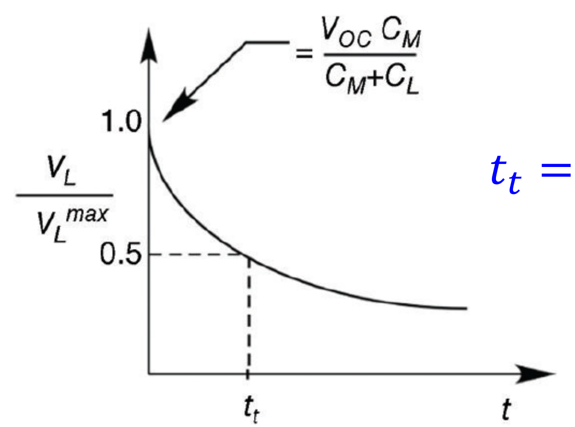


$$t_r = 2.2 C_T R_F$$

$$t_f = 1.25\, t_r$$

- Approximate equivalent circuit for the tail time behavior

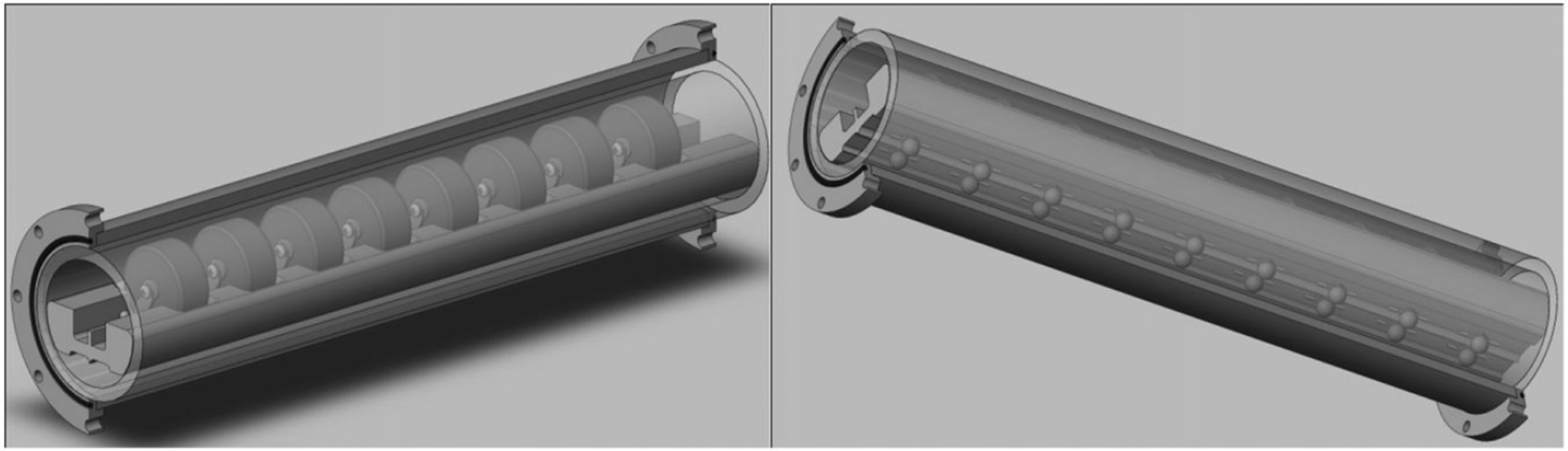




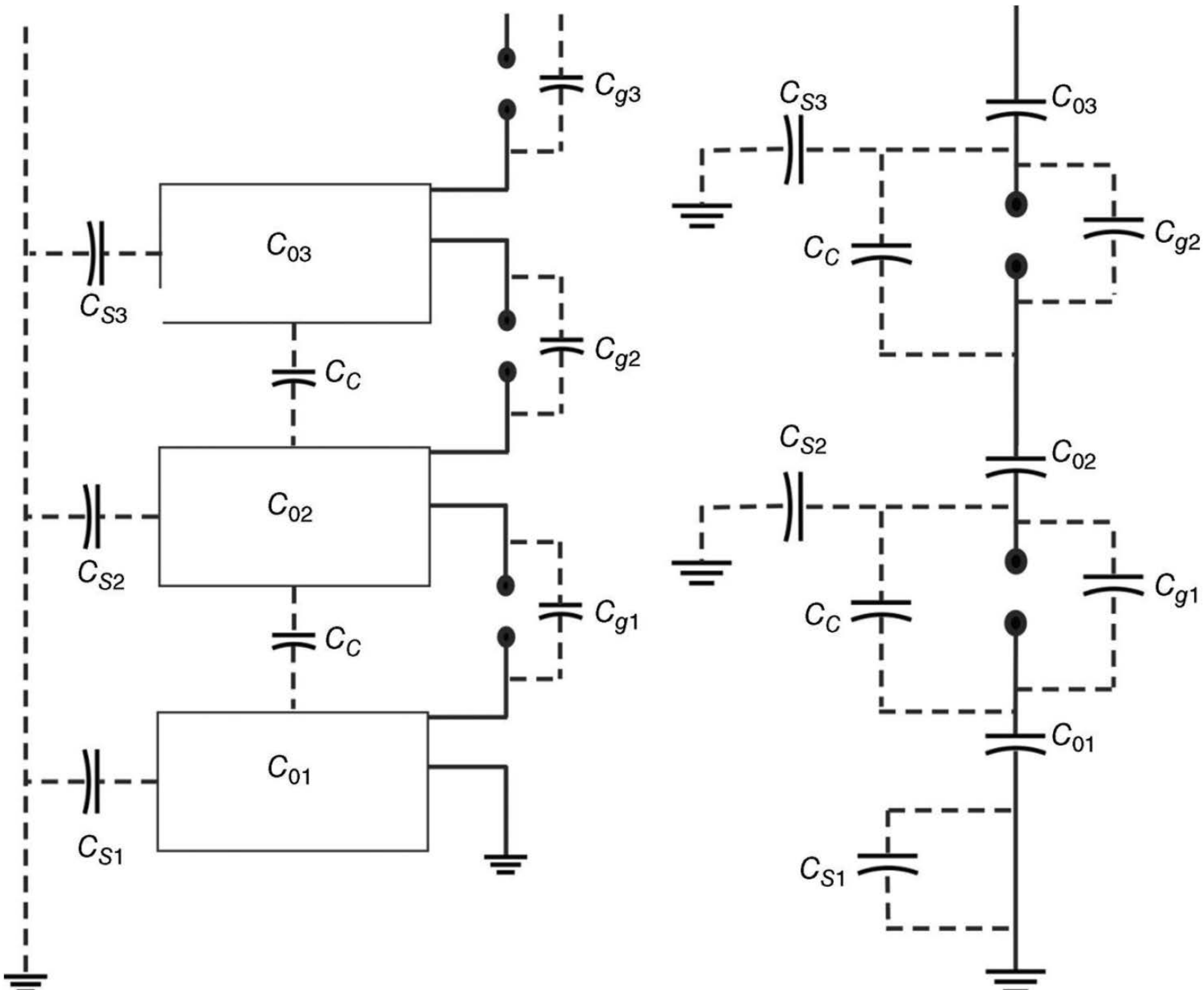
$$t_t = 0.7\, R_T(C_L + C_M)$$

Sources of stray capacitance

- The first is the stray capacitance across the spark gaps (C_g). Prior to closing, a spark gap consists of two electrodes separated by an insulator, which also describes a capacitor.
- The second source is the conducting connections of each stage of the Marx that are isolated from the system ground and are represented by the stray capacitance C_s.
- The third source is stray capacitance between stages. In many cases, the electrodes of the energy storage capacitors C_0 have only small separations between adjacent stages, leading to the stages being coupled by a capacitance C_c.



Sources of stray capacitance



Effects of stray capacitance

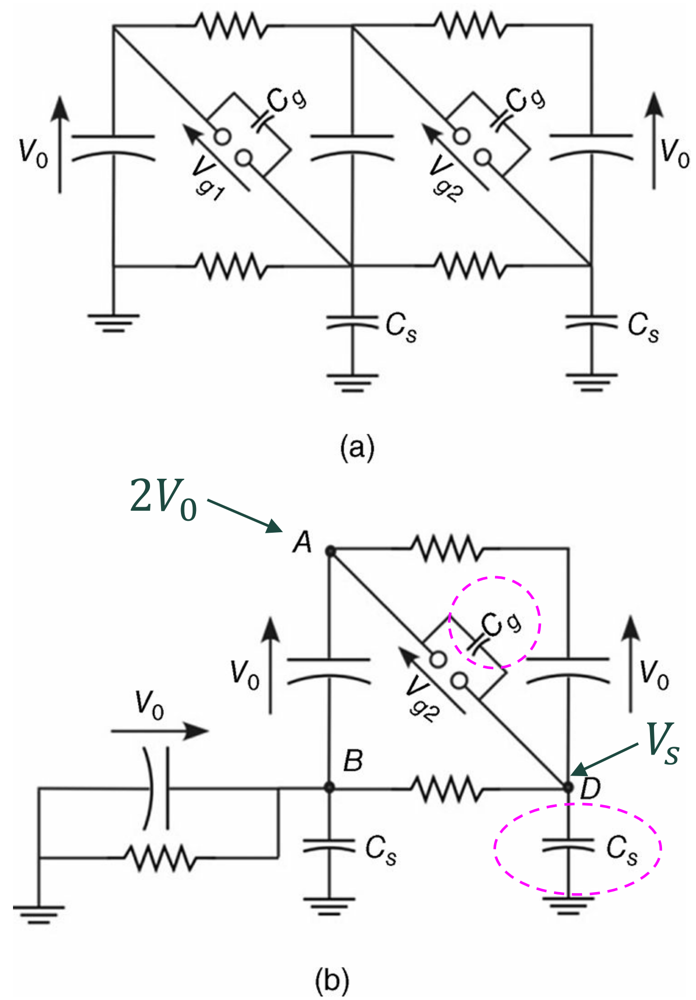
- Figure shows a Marx generator with some stray capacitance (a) during the charge cycle and (b) after the first stage has fired.
- After the first stage erects, the voltage at point B, V_B, equals V_0 because one terminal of the first-stage capacitance is connected directly to ground. The voltage at point A is $V_A = 2V_0$.
- The circuit A-D-G becomes a capacitive voltage divider circuit, so that

$$V_s = V_D = 2V_0 \frac{C_g}{C_s + C_g}$$

- The voltage across gap 2 is given by

$$V_{g2} = V_A - V_D = \frac{2V_0}{1 + C_g/C_s} < 2V_0$$

  - The overvoltage on gap 2 is maximized from when $C_g \ll C_s$.

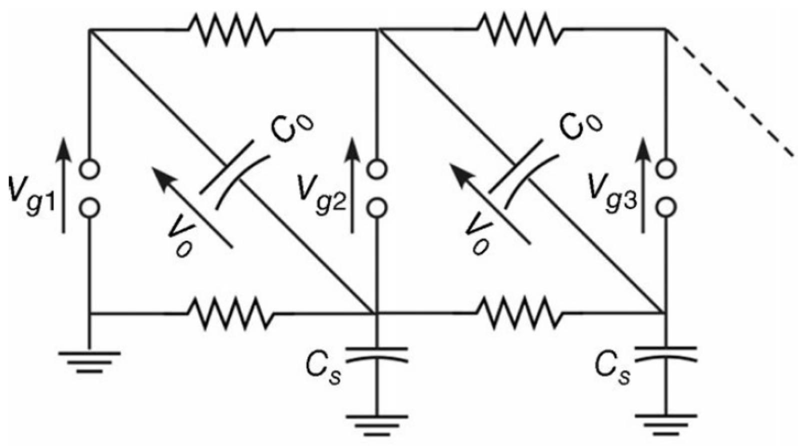


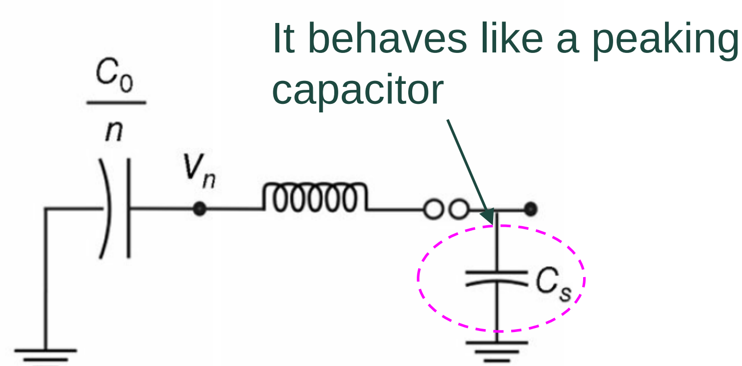
Exploiting stray capacitance

- The stray capacitance to ground (C_s) maybe utilized to produce a Marx generator with a very fast rise time by designing the Marx to act like a cascading peak circuit.
- The value of C_s may be controlled by a suitably designed, grounded metal enclosure. The stage capacitance C_0 and the total number of stages N are selected to satisfy

$$\frac{C_0}{n} = C_n \gg C_s \ \ (n < N)$$

- Each Marx stage becomes a peaking circuit charging the next stage in an increasingly fast erection rate.





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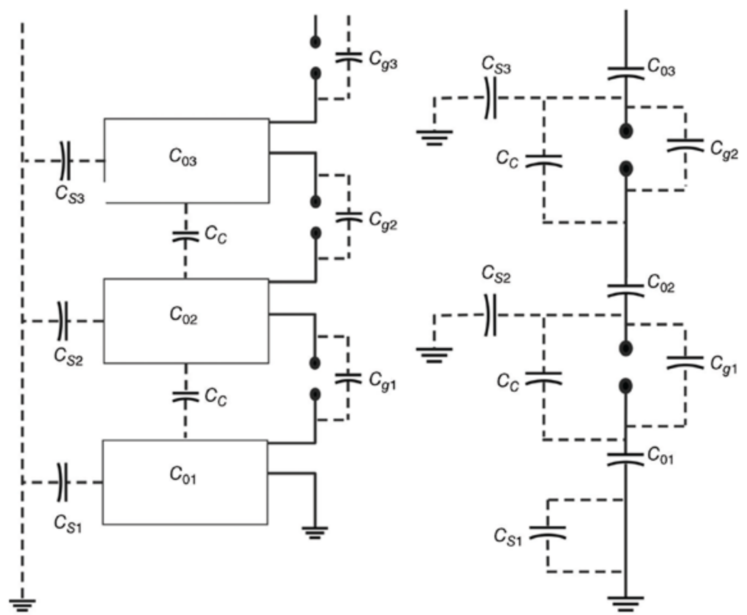
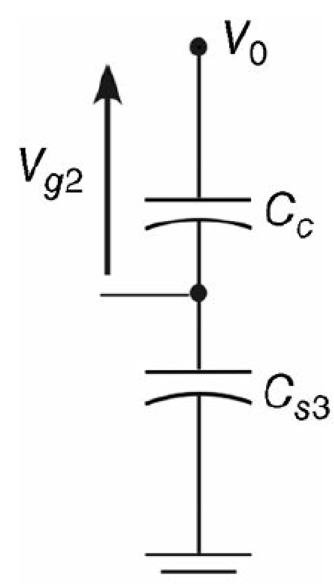
Effects of inter-stage coupling capacitance

- The coupling capacitance slows the erection process, which can only proceed as fast as the inter-stage capacitances can be charged and discharged.
- Only the voltage contribution from the erection of the first stage of the Marx sees the voltage divider. The charge voltage on stage 2 must be added to the divided contribution from stage 1, yielding the voltage on spark gap 2 to be

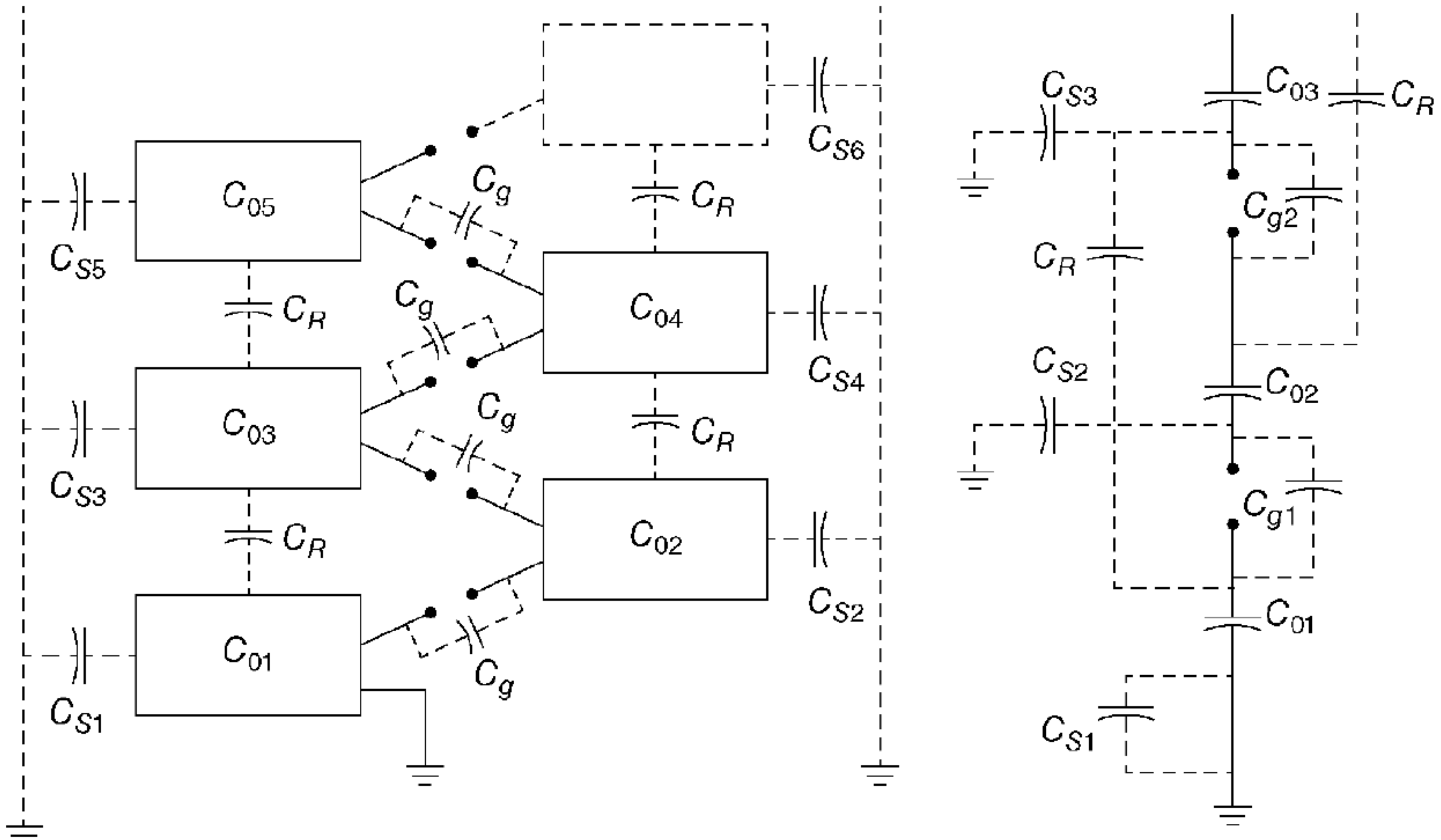
$$V_{g2} = V_0 + \frac{V_0 C_{s3}}{C_{s3} + C_c}$$





Capacitive back-coupling

- The inter-stage coupling capacitance can be reduced by arranging the Marx into columns, with each column containing alternating stages of the Marx.



PFN-Marx generator

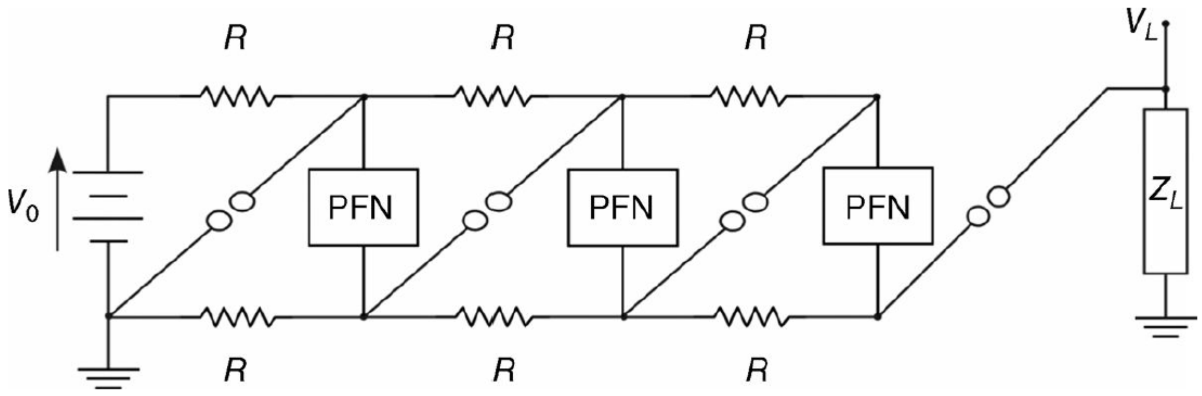
- Conventional Marx designs do not produce rectangular pulse. The principles of Marx operation may be modified, however, to produce a flat-top pulse by making the Marx "capacitors" energy storage elements that produce a pulse with a flat-top pulse. Erection of the Marx adds the voltages of each stage while preserving the pulse shape.
- PFN-Marx parameters

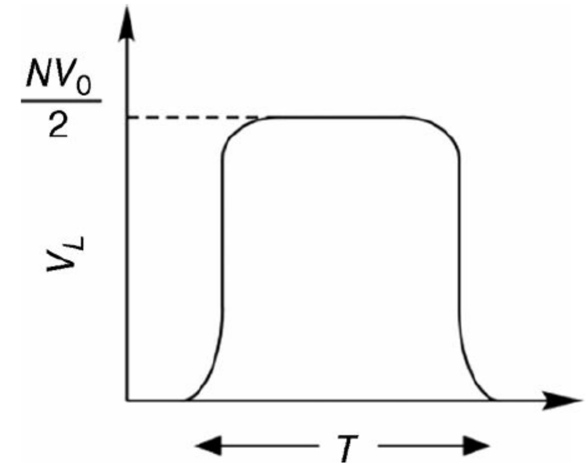
No. of capacitors in PFN

$$Z_{PFN} = \sqrt{L/C} \qquad T = 2n\sqrt{LC}$$

$$V_L = NV_0/2 \qquad R_L = NZ_{PFN}$$

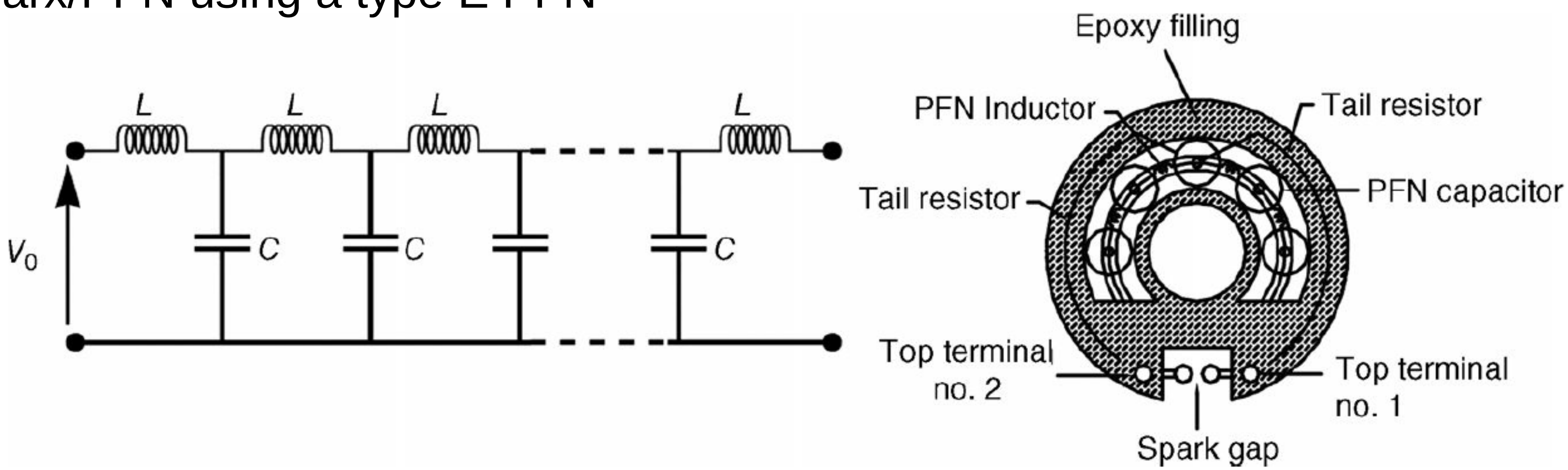
No. of Marx stages



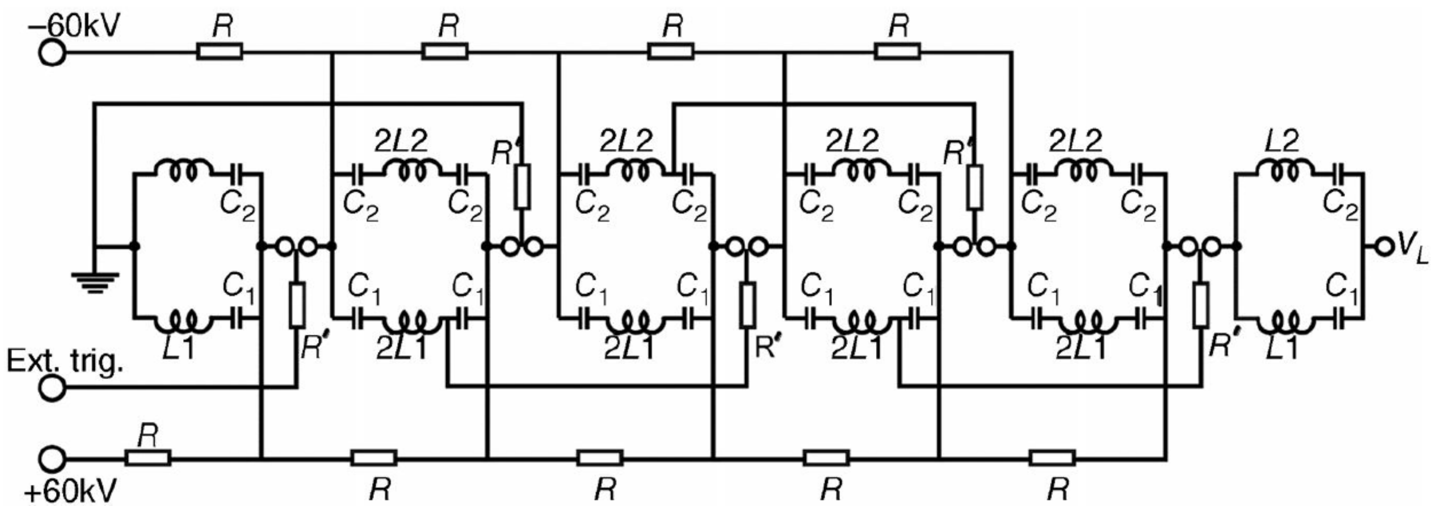


PFN-Marx generator

- Marx/PFN using a type E PFN

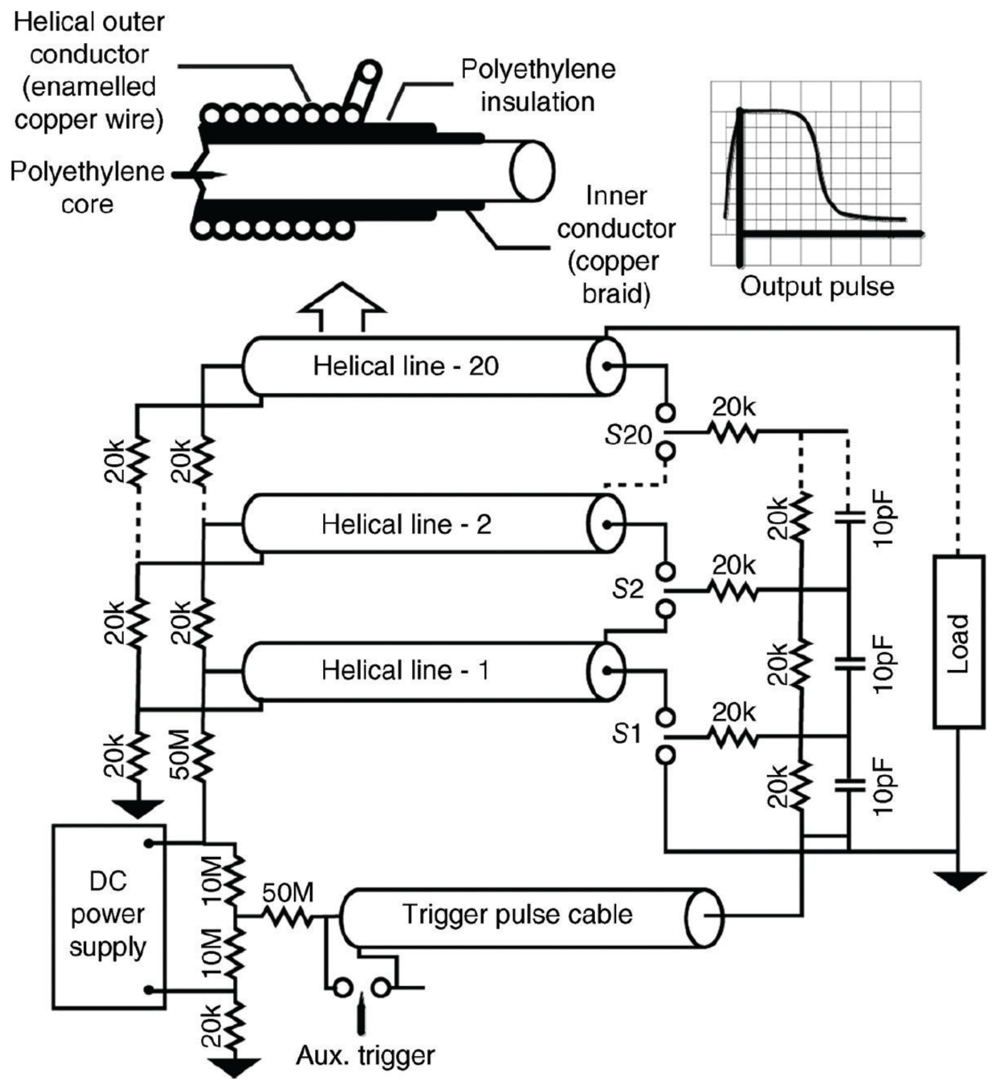


- Marx/PFN using a type C Guilleman network



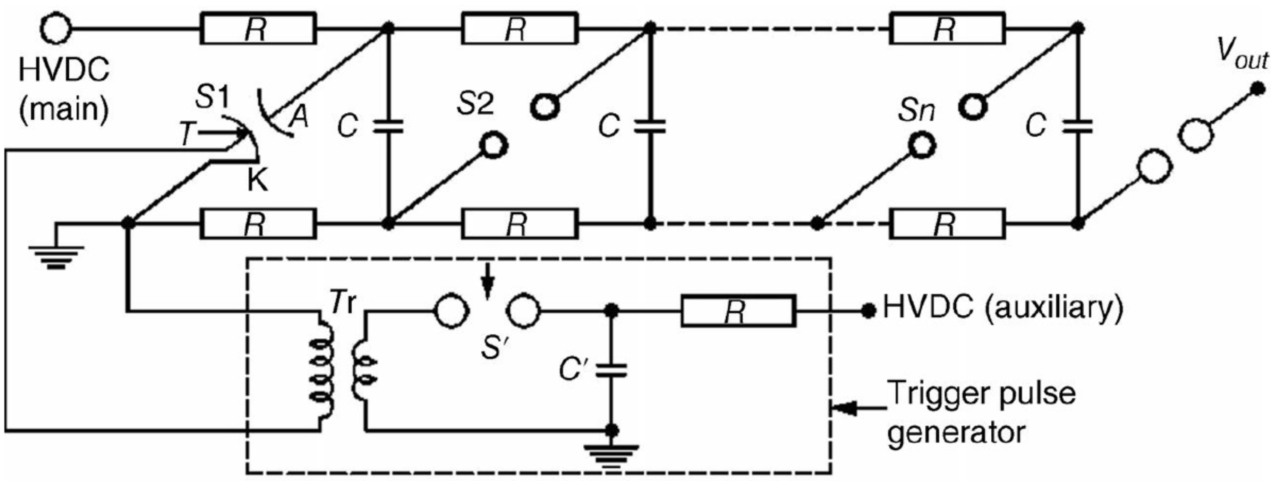
Helical line Marx generator

- The helical line Marx-PFN is useful for the generation of flat-top pulses of long duration(a few microseconds).
- The Marx capacitors are replaced by helical lines made by replacing the braided outer conductor of a typical low-inductance coaxial cable with a helical winding.

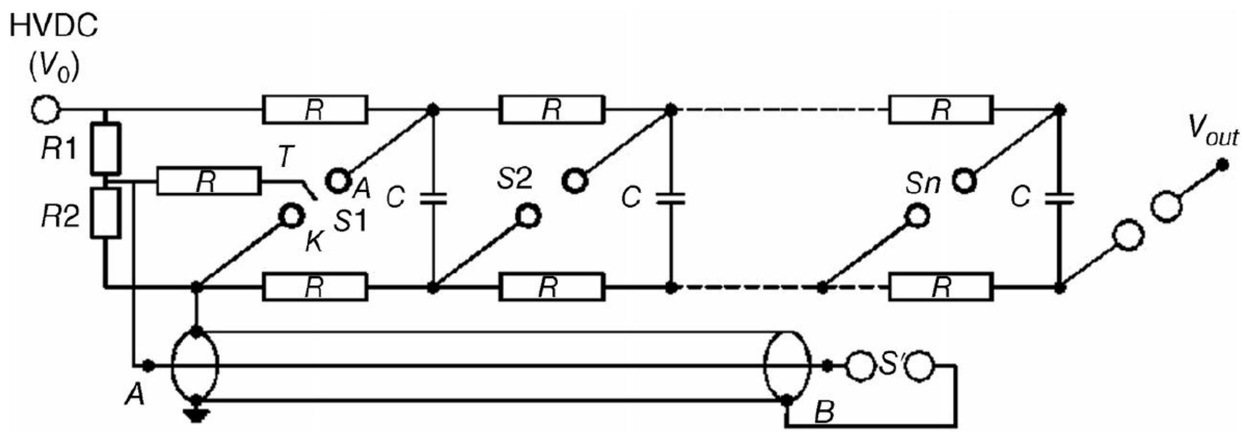


Marx initiation

- Using a trigatron

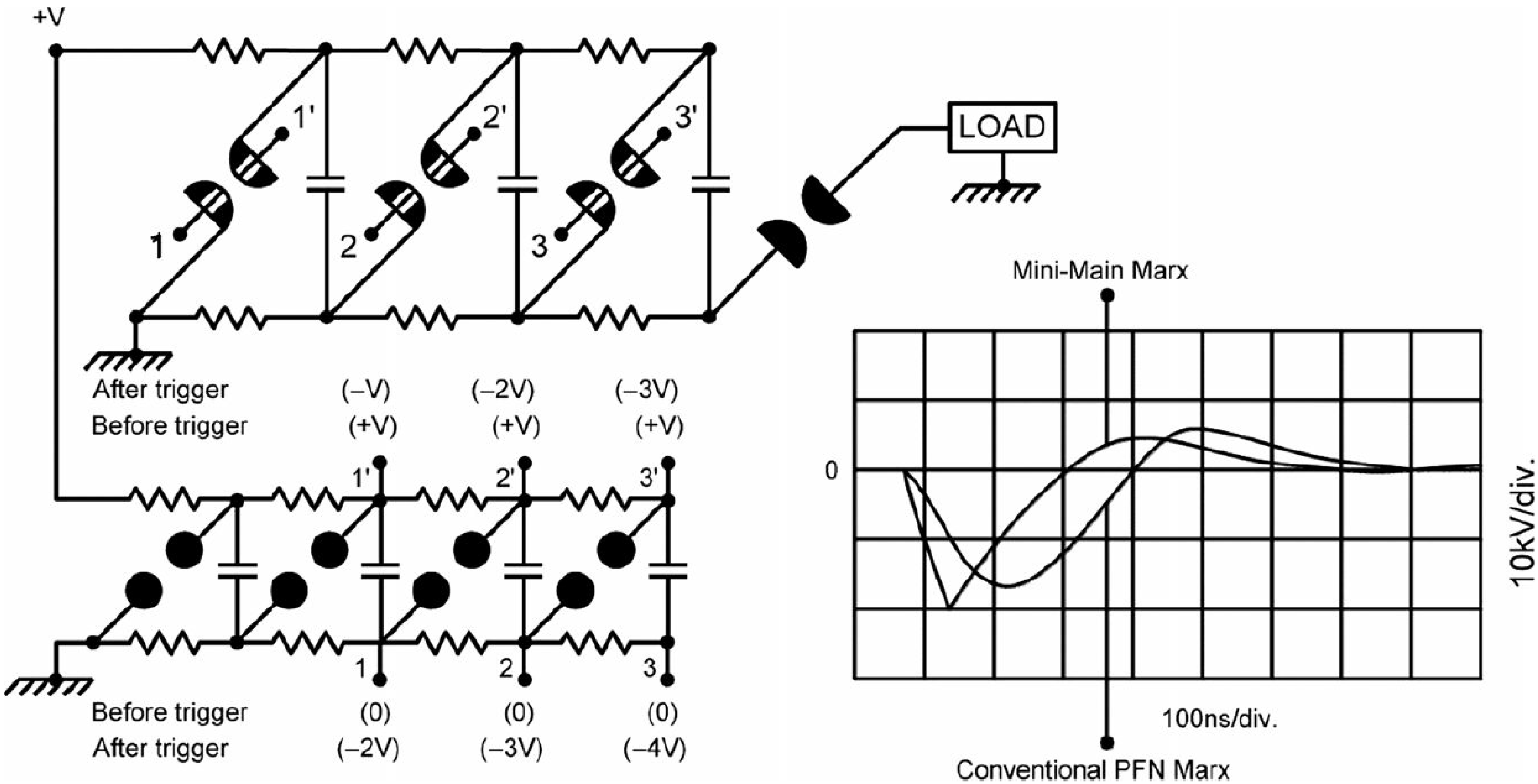


- Using a transmission line

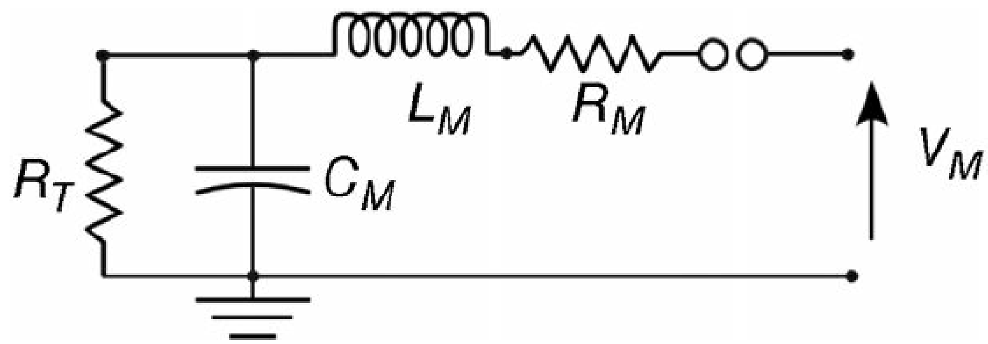


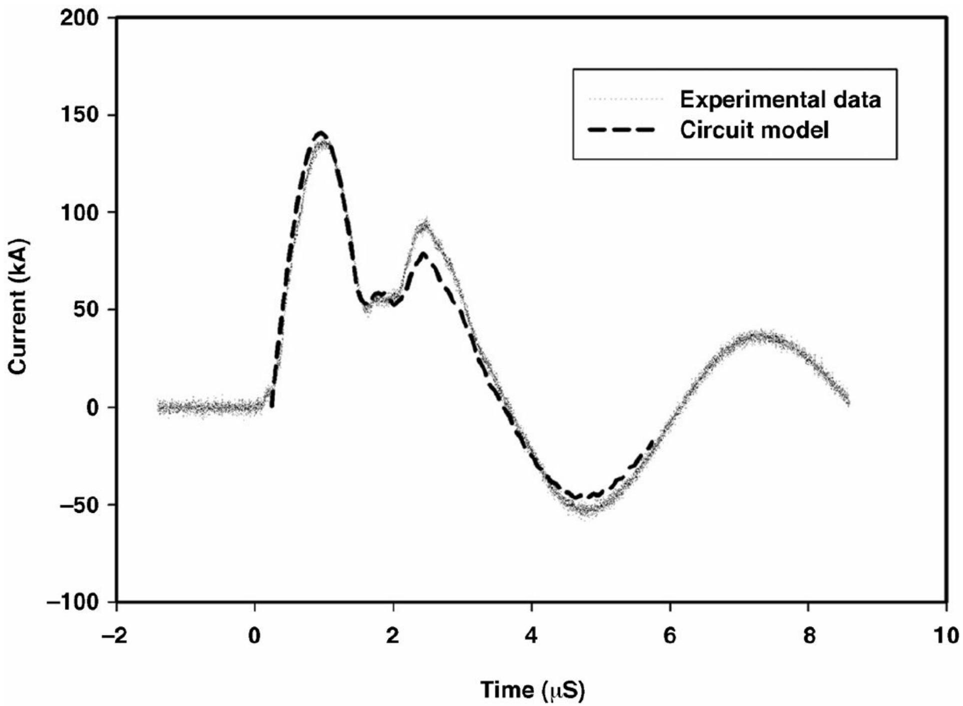
Marx initiation

- Using a smaller Marx



Circuit modeling





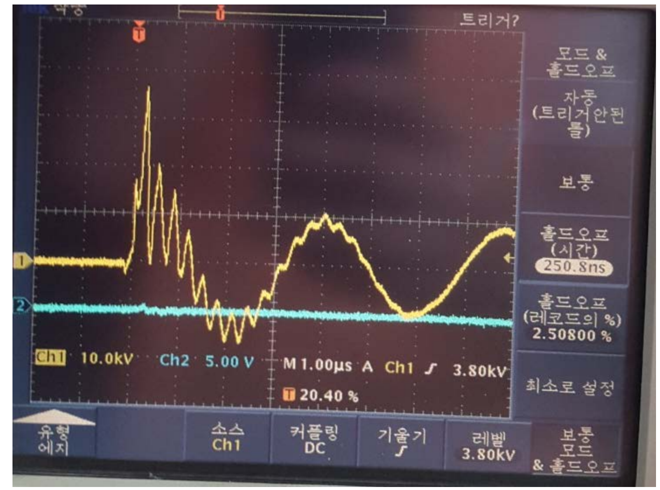
Example: conventional Marx generator

갭 스위치

캐패시터

인덕터

- 1단의 전압 파형



[스펙]

- 최대 전압 : 20 kV x 5 stage = 100 kV
- 저장에너지 : 100 J @100 kV
- 단락 전류 : 약 10 kA
- 펄스상승시간 : 200 ns
- 펄스 반복률 : 0.5 Hz

Example: solid-state Marx generator

- Spark gap switches and resistors can be replaced with solid-state switches and diodes, respectively.