1 SET THEORY

: SYLLABUS :

Set Theory : Introduction, Representation, Operation and its properties, Ven– Diagram, Cartesian product and graph.

1. Introduction : Concept of a Set :

The students of your class is a set of students and you are a member of that set. The college cricket team is a set of players and the captain of that team is a member of that set. Thus, a set is well defined collection of distinct objects and there should exist a rule with the help of which we should be in a position to tell whether a particular object belongs to that collection or not. The objects forming a set are known as elements or members of the set. The followings are some examples of sets :

(i) The set of ministers in Gujarat state.

(ii) The set of alphabates of English language.

(iii) The set of professors of your college.

The sets are generally denoted by capital letters A, B, C, D, X, Y, Z etc. and the elements of the set are denoted by small letters a, b, c, x, y, z. If a is a member of a set A, then we write $a \in A$ and read it as `a belongs to A or a is a member of A. On the other hand if y does not belong to set A, we write $y \notin A$.

2. Methods of Representing Sets :

(i) Tabular form : In this method we write the members of the set in parantheses and separate them by putting commas between them. e.g., The set of even numbers between 1 and 13 can be represented as

$\{2, 4, 6, 8, 10, 12\}$.

(ii) Set–builder form : In this method we write the properties which all the elements of the set must satisfy and write an element x to represent all the elements of the set.

e.g., $A = \{x : x \text{ is a vowel in English alphabates}\}$

$B = \{x : x^2 - 3x - 4 = o\}$

$C = \{x : x \text{ is an integer and } 5 < x < 15\}$

3. Types of Sets :

(i) Finite Set : A set having finite number of elements is called a finite set.

e.g., $A = \{1, 2, 3, 4, 5\}$

$B = \{x : x \text{ is minister of Gujarat state}\}$

(ii) Infinite Set : A set in which the number of elements is not finite is called an infinite set.

e.g., $A = \{1, 2, 3, 4, 5, 6, 7.......\}$

$B = \{x : x = n^2 \text{ and } n \text{ is a natural number}\}$

(iii) Singleton Set : A set having only one element is known as a singleton set.

e.g., $A = \{2\}$

(iv) Empty Set or Null Set : A set having no element is called an empty set or a null set and it is denoted by ϕ.

(v) Equal Sets :* Two sets A and B are said to be equal if all the elements of A are the elements of B and also all the elements of B are elements of A. Symbolically we write A = B.

i.e., if for each $x \in A \Rightarrow x \in B$ and

$x \in B \Rightarrow x \in A$, then $A = B$

* Define : Equal sets (Oct./Nov. 09 - [illegible], Dec., 13)

e.g. {1, 2, 3}; B = {3, 2, 1} are equal sets.

Similarly A = $\{x : x^2 - 5x + 6 = 0\}$, and B = {2, 3} are equal sets.

(vi) Equivalent Sets :* If the elements of one set can be put into one – to – one correspondence with the elements of another set then the two sets are called equivalent sets. Symbolically we can write $A \equiv B$.

e. g. A = {1, 2, 3, 4}

B = {1, 4, 9, 16}

Here $A \equiv B$

(vii) Subset of a Set :@ If all the elements of a set A are the elements of a set B then A is said to be a subset of B. Symbolically we write,

$A \subseteq B$. i.e, if $x \in A \Rightarrow x \in B$ then $A \subset B$.

e.g., If A = {1, 2, 3}; B = {1, 2, 3, 4, 5}, then $A \subseteq B$.

It should be noted that if $A \subseteq B$ and $B \subseteq A$, then A and B are equal sets i.e. A = B.

(1) The empty set ϕ is a subset of every set.

(2) Every set is a subset of itself.

(viii) Proper Sub-sets :#**

If all elements of set A are the elements of set - B and at least one element of superset B is not an element of set A, then set A is called proper subset of superset B.

Symbolically we write $A \subset B$.

e.g., A = {2, 4, 6} B = {2, 4, 6, 8, 10}

Here $A \subset B$.

(ix) Power Set :*** The family of all the subsets of a given set is called its power set.

e.g., If A = {1, 2, 3}, then its power set denoted by P(A) can be given by,

P(A) = {ϕ. {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}

The set having n elements will have 2^n subsets in its power set.

e.g., for A = {3, 5} the number of sets in its power set $= 2^2 = 4$

and for A = $\{a, b, c, d\}$ the number of sets in its power set $= 2^4 = 16$.

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* Define : Equivalent Sets (Nov./Dec. 2008, Dec., 15)

** How do you separate proper and improper subset of a set ? Give illustration (Nov./Dec., 2008, April, 2010; Mar, 15)

Explain proper subset of non empty sets with illustration. (Dec., 14)

*** Define power set with illustration. (Mar., 15)

@ Define subset of a set with illustration. (Dec., 11, 16)

(x) Universal Set : A parent set from which all different subsets are considered is known as an Universal set for that particular situation. Generally universal set is denoted by U.

e.g., U = {1, 2, 3, 4, 5, 6, 7........}

A = {1, 3, 5, 7..........}

B = {2, 4, 6, 8.........}

C = {1, 4, 9, 16..........}.

Here A, B and C are the subsets of universal set U or U is a universal set of the sets A, B and C.

The set of all the members of a family can be considered as an universal set and set of brothers, set of sisters etc. are its subsets.

4. Some Important Number Sets :

We are familiar with the following number sets

1. Set of natural numbers N = {1, 2, 3, 4........}
2. Set of integers Z = {......., –3, –2, –1, 0, 1, 2, 3.......}
3. Set of rational numbers

   $$Q = \{\frac{a}{b} \mid a, b, \in Z \text{ and } b \neq 0\}$$

   Q is a set of rational numbers. Rational numbers include all positive and negative integers, positive and negative fractions and zero.
4. Set of real numbers

   R = Set of real numbers, which includes all rational numbers and all irrational numbers like $\sqrt{2}$, $\sqrt{3}$, ...

6. Operation on Sets :

(i) Intersection of sets :*

The intersection of two sets A and B is the set of all elements which belong to both A and B and it is denoted by $A \cap B$.

i.e. $A \cap B = \{x/x \in A, \text{ and } x \in B\}$

❑ Properties of intersection :

(i) $(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$

(ii) $A \cap \phi = \phi$

(iii) $A \cap A = A$

(iv) $A \cap B = B \cap A$ (Commutative property)

(v) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative property)

Illustration 1 : If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 9, 11\}$, $C = \{2, 11, 18, 22\}$ find : $A \cap B$, $B \cap C$, $C \cap A$, $A \cap B \cap C$.

Also verify that : $(A \cap B) \cap C = A \cap (B \cap C)$,

Ans :

Here $A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 9, 11\}$

$\therefore$ $A \cap B = \{3, 4\}$

$B \cap C = \{11\}$

$C \cap A = \{2\}$

$A \cap B \cap C = \phi$

Now we want to verify that $(A \cap B) \cap C = A \cap (B \cap C)$

$A \cap B = \{3, 4\}$, $C = \{2, 11, 18, 22\}$

$\therefore$ $(A \cap B) \cap C = \phi$

Also $A = \{1, 2, 3, 4\}$, $B \cap C = \{11\}$

$\therefore$ $A \cap (B \cap C) = \phi$

$\therefore$ $(A \cap B) \cap C = A \cap (B \cap C)$

Illustration 2 : If $A = \{x/x \in N, x \leq 5\}$ $B = \{x/x \in N, 2 \leq x \leq 8\}$ $C = \{x/x \in N, x \leq 3\}$, find $A \cap B$, $B \cap C$ and $C \cap A$

Ans : Here

$A = \{1, 2, 3, 4, 5\}$

$B = \{2, 3, 4, 5, 6, 7, 8\}$

$C = \{1, 2, 3\}$

$\therefore$ $A \cap B = \{2, 3, 4, 5\}$

$B \cap C = \{2, 3\}$

$C \cap A = \{1, 2, 3\}$

Define : Intersection of two sets (Sep./Oct. 2009 - Old Course

$\in$ = Epsilon greek letter } Belong to

R1 Prove that $(A \cap B) \cap C = A \cap (B \cap C)$*

In order to prove $(A \cap B) \cap C = A \cap (B \cap C)$

We shall have to prove that

(i) $(A \cap B) \cap C \subseteq A \cap (B \cap C)$ and

(ii) $A \cap (B \cap C) \subseteq (A \cap B) \cap C$

For (i) Let x be any element of $(A \cap B) \cap C$

$\Rightarrow x \in (A \cap B) \cap C$

$\Rightarrow x \in (A \cap B)$ and $x \in C$

$\Rightarrow (x \in A$ and $x \in B)$ and $x \in C$

$\Rightarrow x \in A$ and $(x \in B$ and $x \in C)$

$\Rightarrow x \in A$ and $x \in (B \cap C)$

$\Rightarrow x \in A \cap (B \cap C)$

Thus every element x of $(A \cap B) \cap C$ is also an element of $A \cap (B \cap C)$

$\therefore (A \cap B) \cap C) \subseteq A \cap (B \cap C)$ .... (i)

For (ii) Let y be any element of $A \cap (B \cap C)$

$\Rightarrow y \in A \cap (B \cap C)$

$\Rightarrow y \in A$ and $y \in (B \cap C)$

$\Rightarrow y \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow (y \in A$ and $y \in B)$ and $y \in C$

$\Rightarrow y \in (A \cap B)$ and $y \in C$

$\Rightarrow y \in (A \cap B) \cap C.$

Thus every element y of $A \cap (B \cap C)$ is also an element of $(A \cap B) \cap C$

$\therefore A \cap (B \cap C) \subseteq (A \cap B) \cap C$ .... (ii)

From (i) and (ii) We have

$(A \cap B) \cap C = A \cap (B \cap C)$

Illustration 3 : Show that $(A \cap B) \cap C = A \cap (B \cap C)$ by Venn diagrams.

Ans.

We are required to verify $(A \cap B) \cap C = A \cap (B \cap C)$

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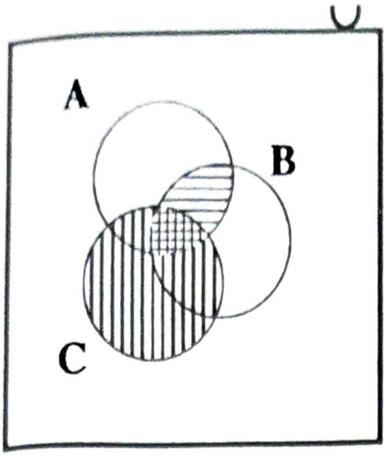
* Prove that $(A \cap B) \cap C = A \cap (B \cap C)$ (March/April, 2007, 2009)

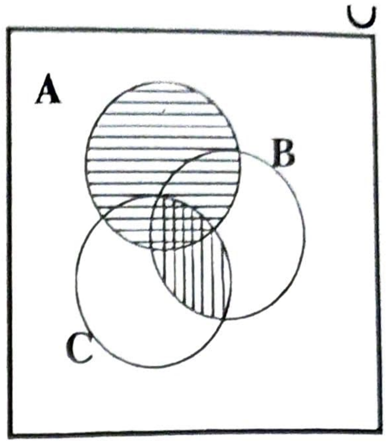


$(A \cap B) \cap C$

$(A \cap B) =$

$C =$

$(A \cap B) \cap C =$



$A \cap (B \cap C)$

$A =$

$B \cap C =$

$A \cap (B \cap C) =$

(ii) Union of Sets*

The union of two sets A and B is the set of all elements which belong to either A or B or both and it is denoted by $A \cup B$.

i.e. $A \cup B = \{x/x \in A \text{ or } x \in B \text{ or } x \in \text{ both } A \text{ and } B\}$

Properties of union of Sets

(i) $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$

(ii) $A \cup \phi = A$

(iii) $A \cup A = A$

(iv) $A \cup B = B \cup A$ (Commutative property)

(v) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative property)

(vi) $A \cup B = \phi \Rightarrow A = \phi$ and $B = \phi$

(vii) $A \cap B \subset A \subset A \cup B$

Illustration 4 : $A = \{x/x \in N, x^2 < 10\}$,

$B = \{x/x \in N, x \leq 1\}$,

$C = \{x/x \in N, 1 \leq x < 5\}$

Find $A \cup B$, $B \cup C$, $C \cup A$ and verify that

$(A \cup B) \cup C = A \cup (B \cup C)$

Ans. :

$A = \{1, 2, 3\}$, $B = \{1\}$, $C = \{1, 2, 3, 4\}$,

$A \cup B = \{1, 2, 3\}$

* Define Union of two sets with illustration.
(March/April 2007, Sept./Oct. 2009 - Old Course).

$B \cup C = \{1, 2, 3, 4\}$

$C \cup A = \{1, 2, 3, 4\}$

Now, $(A \cup B) \cup C = \{1, 2, 3\} \cup \{1, 2, 3, 4\} = \{1, 2, 3, 4\}$

And, $A \cup (B \cup C) = \{1, 2, 3\} \cup \{1, 2, 3, 4\} = \{1, 2, 3, 4\}$

$\therefore (A \cup B) \cup C = A \cup (B \cup C)$

R_2 Prove that : $(A \cup B) \cup C = A \cup (B \cup C)$

In order to prove $(A \cup B) \cup C = A \cup (B \cup C)$ we shall prove that

(i) $(A \cup B) \cup C \subseteq A \cup (B \cup C)$

(ii) $A \cup (B \cup C) \subseteq (A \cup B) \cup C$

(i) Let x be any element of $(A \cup B) \cup C$

$\Rightarrow x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ or $(x \in B \cup C)$

$\Rightarrow x \in A \cup (B \cup C)$

Thus, every element of $(A \cup B) \cup C$ is also an element of $A \cup (B \cup C)$

$\therefore (A \cup B) \cup C \subseteq A \cup (B \cup C)$ .... (i)

(ii) Let. y be any element of $A \cup (B \cup C)$.

$\Rightarrow y \in A \cup (B \cup C)$

$\Rightarrow y \in A$ or $y \in (B \cup C)$

$\Rightarrow y \in$ or $y \in B$ or $y \in C$

$\Rightarrow (y \in A$ or $y \in B)$ or $y \in C$

$\Rightarrow y \in (A \cup B)$ or $y \in C$

$\Rightarrow y \in (A \cup B) \cup C$

Thus, every element of $A \cup (B \cup C)$ is also an element of $(A \cup B) \cup C$

$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C$ .... (ii)

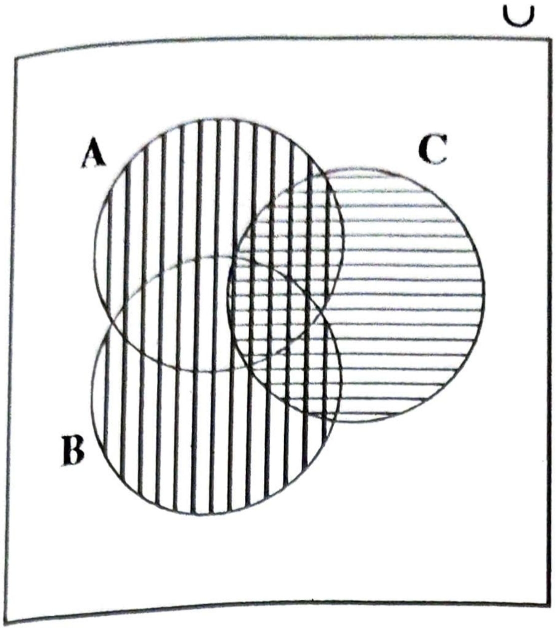
From (i) & (ii) by the definition of equality of sets, we have

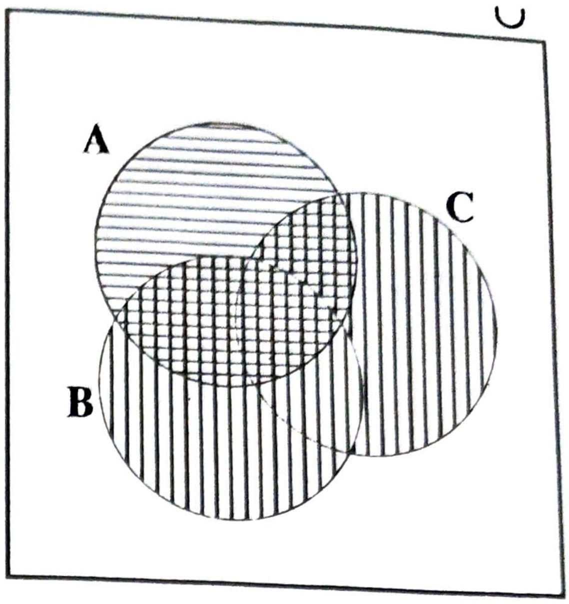
$(A \cup B) \cup C = A \cup (B \cup C)$

Illustration 5 : Verify that $(A \cup B) \cup C = A \cup (B \cup C)$ by Venn diagrams.

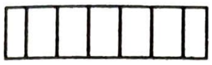
Ans.

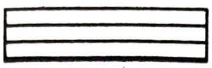
We are required to verify $(A \cup B) \cup C = A \cup (B \cup C)$





$(A \cup B) \cup C$

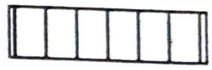
$A \cup B =$ 

$C =$ 

$(A \cup B) \cup C =$ Total shaded region.

$A \cup (B \cup C)$

$A =$ 

$B \cup C =$ 

$A \cup (B \cup C) =$ Total shaded region.

7. Distributive Law of Union over Intersection

Union is distributive over intersection

i.e. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Illustration 6 : If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$, $C = \{1, 3, 5\}$ verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Ans.

Here, $A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5\}$

$C = \{1, 3, 5\}$

L. H. S $= A \cup (B \cap C)$

$= \{1, 2, 3, 4\} \cup \{3, 5\}$

$= \{1, 2, 3, 4, 5\}$

R.H.S. $= (A \cup B) \cap (A \cup C)$

$= \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5\}$

$\therefore$ L. H. S = R. H. S.

(R3) If A. B and C are any three sets prove that*

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

In order to prove the result we shall prove that

(i) $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

(ii) $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

(i) Let x be any element of $A \cup (B \cap C)$

$\Rightarrow x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A$ or ($x \in B$ and $x \in C$)

$\Rightarrow$ ($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

$\Rightarrow (x \in A \cup B)$ and $(x \in A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

Thus, every element of $A \cup (B \cap C)$ is also an element of $(A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ .... (i)

(ii) Let. y be any element of $(A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B)$ and $y \in (A \cup C)$

$\Rightarrow$ ($y \in A$ or $y \in B$) and ($y \in A$ or $y \in C$)

$\Rightarrow y \in A$ or $y \in B$ and $y \in C$

$\Rightarrow y \in A$ or $y \in (B \cap C)$

$\Rightarrow y \in A \cup (B \cap C)$

Thus, each element of $(A \cup B) \cap (A \cup C)$ is also an element of $A \cup (B \cap C)$

$\therefore (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ ...... (ii)

From (i) & (ii) by the definition of equality of sets we have

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Illustration 7 : Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, using Venn diagram.

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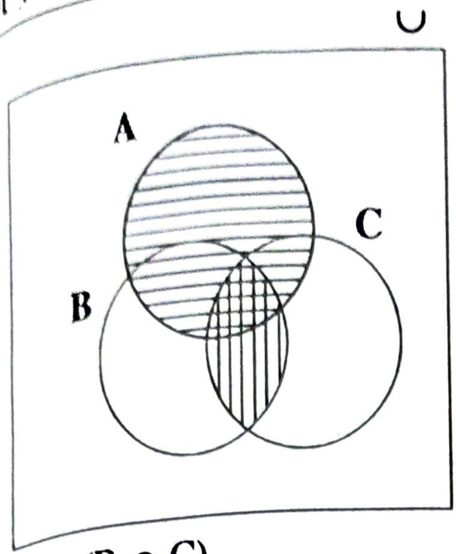
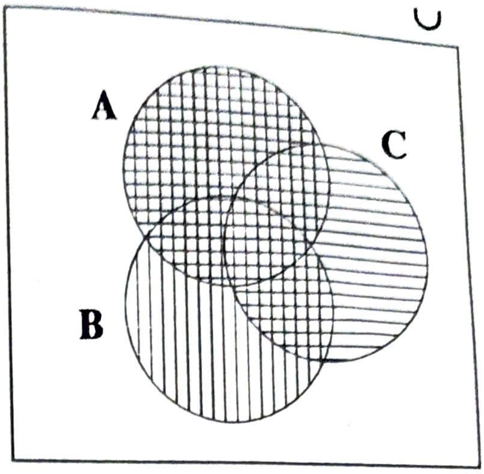
* State and prove distributive law of union over intersection.

(Dec, 2007, 16)

* In usual notations, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(April 2010, Oct./Nov. 2009)





$A \cup (B \cap C)$

$A =$

$B \cap C =$

$A \cup (B \cap C) =$ Total shaded area

$(A \cup B) \cap (A \cup C)$

$A \cup B =$

$A \cup C =$

$(A \cup B) \cap (A \cup C)$

8.	**Distributive Law of Intersection over Union** **Intersection is distributive over union**

i. e. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

R4 **If A, B, & C are any three sets, prove that***

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

In order to prove the given result we shall prove that

(i) $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

(ii) $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

(i) Let x be any element of $A \cap (B \cup C)$

$\Rightarrow x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ and ($x \in B$ or $x \in C$)

$\Rightarrow$ ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$)

$\Rightarrow x \in (A \cap B)$ or $x \in (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

Thus, every element of $A \cap (B \cup C)$ is also an element of $(A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) \subseteq (A \cup B) \cup (A \cap C)$ ...(i)

* Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(Sep./Oct. 2009 – Old Course)

* State & prove distributive law for intersection over union.

(Dec., 14, March/April, 14)

(ii) Let y be any element of $(A \cap B) \cup (A \cap C)$

$\Rightarrow y \in (A \cap B) \cup (A \cap C)$

$\Rightarrow y \in (A \cap B)$ or $y \in (A \cap C)$

$\Rightarrow (y \in A$ and $y \in B)$ or $(y \in A$ and $y \in C)$

$\Rightarrow y \in A$ and $(y \in B$ or $(y \in C)$

$\Rightarrow y \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow y \in A \cap (B \cup C)$

Thus, each element of $(A \cap B) \cup (A \cap C)$ is also an element of $A \cap (B \cup C)$

$\therefore (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ ...... (ii)

Form (i) and (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

9. Complement of a Set *

Complement of a set is always with respect to universal set. The complement of a set A is a set of all elements which do not belong to set A but belong to the universal set. The complement of a set A is denoted by A'

i.e., $A' = U - A = \{x/x \in U \text{ but } x \notin A\}$

e.g. $U = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4\}$

$A' = U - A = \{1, 3, 5, 6\}$

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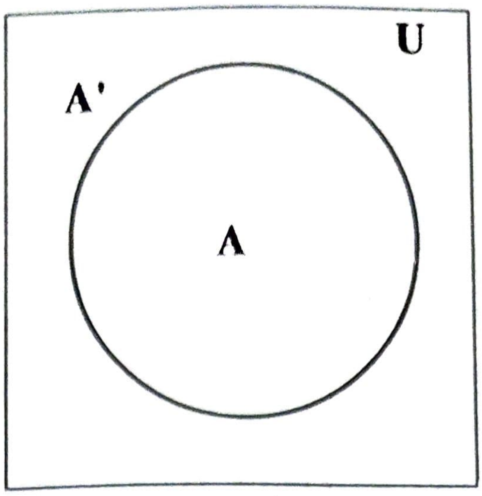
* Define complement of a set. (Nov. Dec., 2008, Dec., 2007, 12)



❑ Properties of Complement

(i) $A \cap A' = \phi$

(ii) $A \cup A' = U$

(iii) $U' = \phi$ and $\phi' = U$

(iv) $(A')' = A$

(v) If $A \subset B$ then $B' \subset A'$

(vi) $(A \cap B) \cup (A \cap B') = A$

(vii) $(A \cup B) \cap (A \cup B') = A$

10. De Morgan's Law for Union

Complement of Union of two sets is the intersection of their complements

i.e., $(A \cup B)' = A' \cap B'$

Illustration 9 : If U = {1, 2, 3,4, 5,6}, A = {2, 3, 6}, B = {3, 5, 6} then verify that (i) $(A \cup B)' = A' \cap B'$

Ans. (i) $A \cup B = \{2, 3, 5, 6\}$

$(A \cup B)' = \{1, 4\}$

$A' = U - A = \{1, 4, 5\}$

$B' = U - B = \{1, 2, 4\}$

$A' \cap B' = \{1, 4\}$

H

$\therefore (A \cup B)' = A' \cap B'$

R5 **If A and B be any two sets prove that $(A \cup B)' = A' \cap B'$***

To prove $(A \cup B)' = A' \cap B'$. We shall prove that

(i) $(A \cup B)' \subseteq A' \cap B'$

(ii) $A' \cap B' \subseteq (A \cup B)'$

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* State and prove De Morgans Law for union. (Dec. – 2007, Mar. 14)
* Prove that $(A \cup B)' = A' \cap B'$ (Nov./Dec. - 2008)

(i) Let x be any element of $(A \cup B)'$

$\Rightarrow x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in (A' \cap B')$

i.e., every element of $(A \cup B)'$ is also an element of $A' \cap B'$

$\therefore (A \cup B)' \subseteq A' \cap B'$ .... (i)

(ii) Let y be any element of $A' \cap B'$

$\Rightarrow y \in (A' \cap B')$

$\Rightarrow y \in A'$ and $y \in B'$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)'$

i.e, each element of $(A' \cap B')$ is also an element of $(A \cup B)'$

$\therefore A' \cap B' \subseteq (A \cup B)'$ .... (ii)

Form (i) & (ii), we conclude that $(A \cup B)' = (A' \cap B')$

11. De Morgan Law for Intersection*

Complement of intersection of two sets is the union of their complements.

i.e $(A \cap B)' = A' \cup B'$

R6 Prove that $(A \cap B)' = A' \cup B'$

In order to prove the result we shall prove that

(i) $(A \cap B)' \subseteq A' \cup B'$ (ii) $A' \cup B' \subseteq (A \cap B)'$

(i) Let x be any element of $(A \cap B)'$

$\Rightarrow x \in (A \cap B)'$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A' \cup B'$

So, each element of $(A \cap B)'$ is also an elements of $A' \cup B'$

$\therefore (A \cap B)' \subseteq (A' \cup B')$ ....(i)

(ii) Let y be any element of $A' \cup B'$

$\therefore y \in (A' \cup B')$

$\Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin A \cap B$

$\Rightarrow y \in (A \cap B)'$

So each element of $A' \cup B'$ is also an element of $(A \cap B)'$

$\therefore A' \cup B' \subseteq (A \cap B)'$ .... (ii)

From (i) and (ii) we have $(A \cap B)' = A' \cup B'$

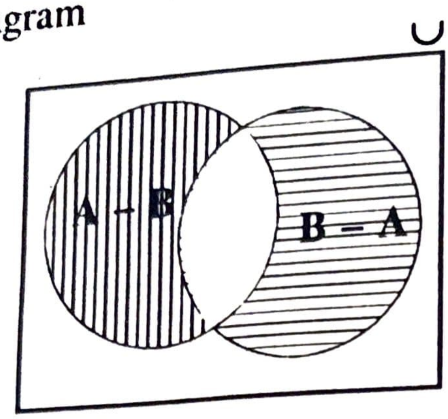
12. Difference of Two Sets*

The difference of two sets A and B is the set of all elements which belong to A but not to B. It is denoted by A – B

i.e. $A - B = \{x/x \in A \text{ but } x \notin B\}$

$B - A = \{x/x \in B \text{ but } x \notin A\}$

By Venn. diagram



A – B =

B – A =

❑ Properties of Difference of two sets

(i) $A - B$, $A \cap B$, $B - A$ are mutually disjoint sets,

(ii) $A - (A - B) = A \cap B$ and $B - (B - A) = A \cap B$

(iii) $A - B \subseteq A$ and $B - A \subseteq B$

De Morgan's law on difference of Sets :

If A, B, C, are any there sets, then $A - (B \cup C) = (A - B) \cap (A - C)$

Illustration 12 : If $A = \{x/x \leq 9, x \in N\}$,

$B = \{y / 3 \leq y \leq 7$, and y is odd number$\}$

$C = \{z / 1 \angle z \angle 7$, and z is even number$\}$ then prove that

$A - (B \cup C) = (A - B) \cap (A - C)$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B = \{3, 5, 7\}$

$C = \{2, 4, 6\}$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B \cup C = \{2, 3, 4, 5, 6, 7\}$

L.H.S $= A - (B \cup C) = \{1, 8, 9\}$ ....(i)

Now, $A - B = \{1, 2, 4, 6, 8, 9\}$

$A - C = \{1, 3, 5, 7, 8, 9\}$

R.H.S $= (A - B) \cap (A - C) = \{1, 8, 9\}$ .... (ii)

From (i) and (ii) We have $A - (B \cup C) = (A - B) \cap (A - C)$

* Define : difference of two sets (March/April 2009, March, 15)

R$_7$ prove that $A - (B \cup C) = (A - B) \cap (A - C)$

To prove the result we shall prove that

(i) $A - (B \cup C) \subseteq (A - B) \cap (A - C)$

(ii) $(A - B) \cap (A - C) \subseteq A - (B \cup C)$

(i) Let x be any element of $A - (B \cup C)$

$\Rightarrow x \in A - (B \cup C)$

$\Rightarrow x \in A$ but $x \notin (B \cup C)$

$\Rightarrow x \in A$ but ($x \notin B$ and $x \notin C$)

$\Rightarrow x \in$ (A but $x \notin B$) and ($x \in A$ but $x \notin C$)

$\Rightarrow x \in (A - B)$ and $x \in (A - C)$

$\Rightarrow x \in (A - B) \cap (A - C)$

Thus, each element x of $A - (B \cup C)$ is also an element of $(A - B) \cap (A - C)$

$\therefore A - (B \cup C) \subseteq (A - B) \cap (A - C)$ ..... (i)

(ii) Let y be any element of $(A - B) \cap (A - C)$

$\Rightarrow y \in (A - B) \cap (A - C)$

$\Rightarrow y \in (A - B)$ and $y \in (A - C)$

$\Rightarrow$ ($y \in A$ but $y \in B$) and ($y \in A$ but $y \notin C$)

$\Rightarrow y \in A$ but ($y \notin B$ and $y \notin C$)

$\Rightarrow y \in A$ but $y \notin (B \cup C)$

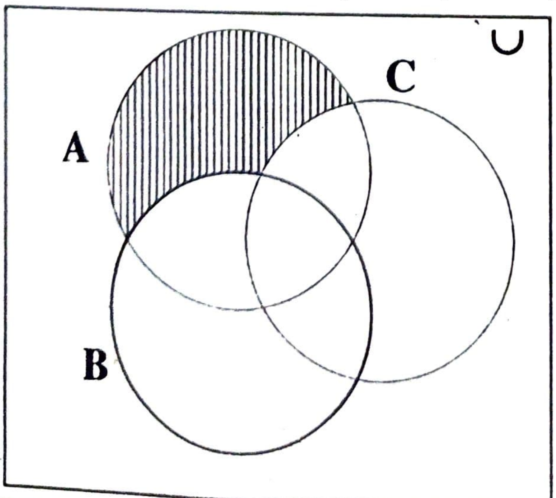
$\Rightarrow y \in A - (B \cup C)$

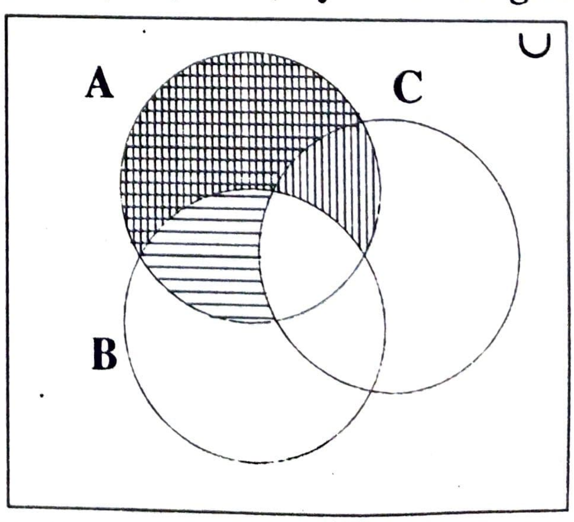
Thus, each element y of $(A - B) \cap (A - C)$ is also an element of $A - (B \cup C)$

$\therefore (A - B) \cap (A - C) \subseteq A - (B \cup C)$ .... (ii)

From (i) & (ii) we can say $A - (B \cup C) = (A - B) \cap (A - C)$

Illustration 13 : Verify $A - (B \cup C) = (A - B) \cap (A - C)$ by Venn–diagram

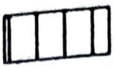




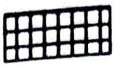
* Prove that $A - (B \cup C) = (A - B) \cap (A - C)$

(March/April 2007, March, 2016)

$A - (B \cup C) =$ 

$A - B =$ 

$A - C =$ 

$(A - B) \cap (A - C) =$ 

R₇ If A, B, & C be any three sets then prove that*

$A - (B \cap C) = (A - B) \cup (A - C)$

In order to prove the result we shall prove that

(i) $A - (B \cap C) \subseteq (A - B) \cup (A - C)$ (ii) $(A - B) \cup (A - C) \subseteq A - (B \cap C)$

(i) Let x be any element of $A - (B \cap C)$

$\Rightarrow x \in A - (B \cap C)$

$\Rightarrow x \in A$ but $x \notin (B \cap C)$

$\Rightarrow x \in A$ but $x \in (B$ or $C)$

$\Rightarrow (x \in A$ but $x \notin B)$ or $(x \in A$ but $x \notin C)$

$\Rightarrow x \in (A - B)$ or $x \in (A - C)$

$\Rightarrow x \in (A - B) \cup (A - C)$

$\therefore A - (B \cap C) \subseteq (A - B) \cup (A - C)$ .... (i)

(ii) Let y be any element of $(A - B) \cup (A - C)$

$\Rightarrow y \in (A - B) \cup (A - C)$

$\Rightarrow y \in (A - B)$ or $y \in (A - C)$

$\Rightarrow (y \in A$ but $y \notin B)$ or $(y \in A$ but $y \notin C)$

$\Rightarrow y \in A$ but $(y \notin B$ and $y \notin C)$

$\Rightarrow y \in A$ but $y \notin (B \cap C)$

$\Rightarrow y \in A - (B \cap C)$

$\therefore (A - B) \cup (A - C) \subseteq A - (B \cap C)$ ....(ii)

From (i) & (ii) we can say that $A - (B \cap C) = (A - B) \cup (A - C)$

Illustration 14 : Verify that $A - (B \cap C) = (A - B) \cup (A - C)$ by Venn- diagram.

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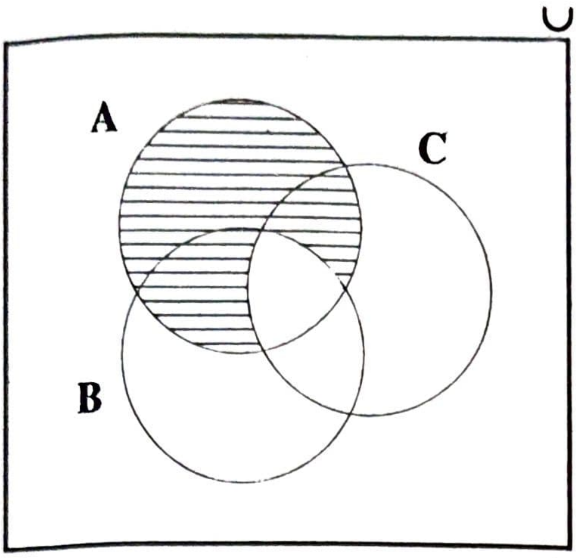
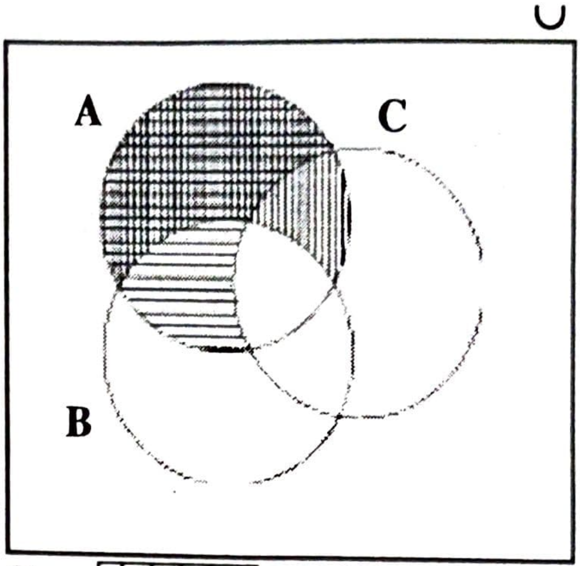
* Prove that $A - (B \cap C) = (A - B) \cup (A - C)$

(Nov./Dec. 2008, Dec. 2007)





$A - B =$

$A - C =$

$A - (B \cap C) =$ $(A - B) \cup (A - C)$ All shaded area

13. Cartesian Poduct of Two Sets*

If A and B be are two sets then the set of all ordered pairs whose first element belongs to set A and second element belongs to set B is called the cartesian product of A and B in that order and denoted by $A \times B$. read as `A cross B.

In other words if A, B are two sets then the set of all ordered pairs like (x,y) where $x \in A$ and $y \in B$ is called the cartesian product of the sets A and B.

Symbolically,

$A \times B = \{(x, y) / x \in A \text{ and } y \in B\}$

e.g. $A = \{1, 2, 3\}, B = \{2, 3\}$

$A \times B = \{(1, 2). (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$ and

$B \times A = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

$\therefore A \times B \neq B \times A$

Some important results

(i) $A \times B$ and $B \times A$ have the same number of elements.

(ii) Generally $A \times B \neq B \times A$.

(iii) If $A \cap B = \phi$ then $(A \times B) \cap (B \times A) = \phi$

(iv) If either A or B is null then the set $A \times B$ is also a null set.

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* Explain cartesian product of two non empty sets with illustration.
(March, 2015, Dec., 2016)

R8 Prove that : $A \times (B \cap C) = (A \times B) \cap (A \times C)$*

In order to prove the result we shall prove that

(i) $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$

(ii) $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$

(i) Let (x, y) be any pair of $A \times (B \cap C)$

$\Rightarrow (x, y) \in A \times (B \cap C)$

$\Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow x \in A$ and ($y \in B$ and $y \in C$)

$\Rightarrow$ ($x \in A$ and $y \in B$) and ($x \in A$ and $y \in C$)

$\Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ ... (i)

(ii) Let. (u, v) be any pair of $(A \times B) \cap (A \times C)$

$\Rightarrow (u, v) \in (A \times B) \cap (A \times C)$

$\Rightarrow (u, v) \in (A \times B)$ and $(u, v) \in (A \times C)$

$\Rightarrow$ ($u \in A$ and $v \in B$) and ($u \in A$ and $v \in C$)

$\Rightarrow u \in A$ and ($v \in B$ and $v \in C$)

$\Rightarrow u \in A$ and $v \in (B \cap C)$

$\Rightarrow (u, v) \in A \times (B \cap C)$

$\therefore (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ ....(ii)

From (i) & (ii) by equality of sets we can say

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

R9 Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$#

In order to prove the result we shall prove that

(i) $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$

(ii) $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$

(i) Let (x, y) be any pair of $A \times (B \cup C)$

$\Rightarrow (x, y) \in A \times (B \cup C)$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow x \in A$ and ($y \in B$ or $y \in C$)

$\Rightarrow$ ($x \in A$ and $y \in B$) or ($x \in A$ and $y \in C$)

$\Rightarrow (x, y) \in (A \times B)$ or $(x, y) \in (A \times C)$

---

* Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (March 12, 13, 15, Dec., 13)

In usual notations, prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (Dec., 15, 14)

$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$

$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ .....(i)

(ii) Again, Let (u, v) be any pair of $(A \times B) \cup (A \times C)$

$\Rightarrow (u,v) \in (A \times B) \cup (A \times C)$

$\Rightarrow (u,v) \in A \times B$ or $(u,v) \in (A \times C)$

$\Rightarrow (u \in A$ and $v \in B)$ or $(u \in A$ and $v \in C)$

$\Rightarrow u \in A$ and $(v \in B$ or $v \in C)$

$\Rightarrow u \in A$ and $v \in (B \cup C)$

$\Rightarrow (u, v) \in A \times (B \cup C)$

$\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ ...... (ii)

From (i) & (ii) we can say that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

14. Number of Elements in Finite Sets :

Let A and B be two finite sets. Let n(A), n(B) denote the number of elements in finite sets A and B respectively.

(1) If A and B are disjoint sets then the number of elements in $A \cup B$ is $n(A \cup B) = n(A) + n(B)$

(2) If A and B are not disjoint, sets then the number of elements in $A \cup B$ is $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

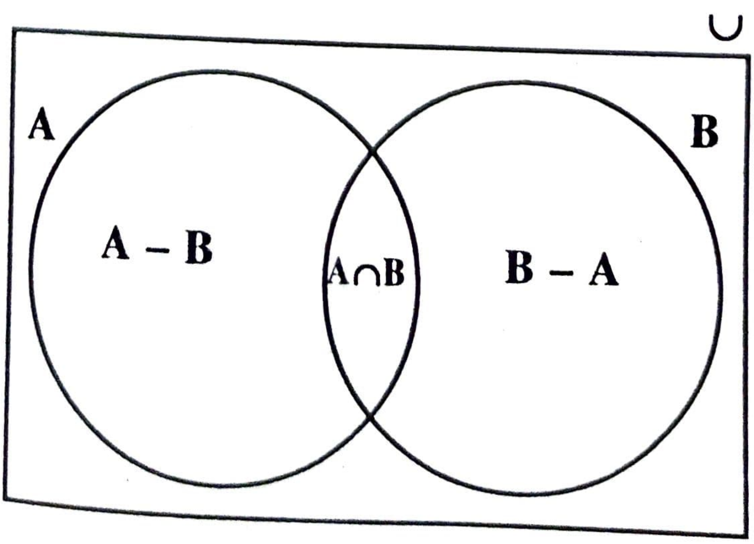
(3) If A, B and C are disjoint sets, then the number of elements in $A \cup B \cup C$ is $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

(4) If A, B and C are not disjoint sets, then the number of elements in $A \cup B \cup C$ is $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Prove that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Let A & B be any two not disjoint sets

from Venn-diagram it is clear that



$(A - B)$, $(A \cap B)$ and $(B - A)$ are disjoint sets.

$A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$

$\therefore n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

From the venn-diagram it is seen that

$n(A - B) = n(A) - n(A \cap B)$

$n(B - A) = n(B) - n(A \cap B)$

$\therefore n(A \cup B) = n(A) - n(A \cap B) + n(A \cap B) + n(B) - n(A \cap B)$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

R_{10} Prove that $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$ $- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Let A, B and C are the three finite sets and let $D = (A \cup B)$

$\therefore n(A \cup B \cup C) = n(D \cup C)$

$= n(D) + n(C) - n(D \cap C)$

Also, $n(D) = n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) - n(A \cap B) + n(C) - n[(A \cup B) \cap C]$

$= n(A) + n(B) - n(A \cap B) + n(C) - [n(A \cap C) \cup n(B \cap C)]$

$= n(A) + n(B) - n(A \cap B) + n(C) - \{n(A \cap C) + n(B \cap C)$

$- n\{(A \cap C) \cap (B \cap C)]\}$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C)$

$+ n(A \cap B \cap C)$

$\therefore$ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Illustration 15 : If $A = \{x / x \in N, |x^3 - 2| \leq 25\}$,

$B = \{y / y \in N, 1 \angle y \angle 5\}$

$C = \{z / z \in N, Z^4 = 81\}$

Verify that = $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Ans :

$A = \{x / x \in N, |x^3 - 2| \leq 25\}$

i.e. **$A = \{1, 2, 3\}$**

$B = \{y / y \in N, 1 < y < 5\}$

i.e. **$B = \{2, 3, 4\}$**

$C = \{z / z \in N, z^4 = 81\}$

i.e. **$C = \{3\}$**

---

* Prove that $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(Dec., 2012)

Now $A = \{1, 2, 3\}$

$A \cup B = \{1, 2, 3, 4\}$

$B \cap C = \{3\}$

$A \cup C = \{1, 2, 3\}$

L.H.S $= A \cup (B \cap C) = \{1, 2, 3\}$

R.H.S. $= (A \cup B) \cap (A \cup C)$

$= \{1, 2, 3\}$

$\therefore$ L.H.S $=$ R.H.S

Illustration 16 : If $A = \{a / a^2 - 1 \angle 10, a \in Z\}$,
$B = \{b \mid b - 1 \mid \angle 2, b \in N\}$
$C = \{c / \mid c \mid \leq 1, c \in Z\}$
Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Ans :

Here $A = \{-3, -2, -1, 0, 1, 2, 3\}$,

$B = \{1, 2\}$

$C = \{-1, 0, 1\}$

Now $B \cap C = \{1\}$

L.H.S $= A \times (B \cap C) = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1)\}$

$A \times B = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1), (-3, 2), (-2, 2), (-1, 2), (0, 2), (1, 2), (2, 2), (3, 2)\}$

$A \times C = \{(-3, -1), (-2, -1), (-1, -1), (0, -1), (1, -1), (2, -1), (3, -1), (-3, 0), (-2, 0), (-1, 0), (0, 0), (1, 0), (2, 0), (3, 0), (-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1)\}$

R.H.S $= (A \times B) \cap (A \times C) = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1)\}$

$\therefore$ L.H.S $=$ R.H.S

Illustration 17 : In a class of 42 students, each play atleast one of the three games Cricket, Hockey and Football. It is found that 14 play Cricket, 20 play Hockey and 24 play Football, 3 play both Cricket and Football, 2 play both Hockey and Football. None play all the three games. Find the number of students who play Cricket but not Hockey.

Ans.

Let C denote the set of students who play cricket $\Rightarrow n(C) = 14$.

Let H denote the set of students who play Hockey $\Rightarrow n(H) = 20$

Let F denote the set of students who play Football $\Rightarrow n(F) = 24$

Also, $n(C \cup H \cup F) = 42$

$C \cap F$ = {students who play both Cricket & Football} $\Rightarrow n(C \cap F) = 3$

$H \cap F$ = {students who play Hockey & Football} $\Rightarrow n(H \cap F) = 2$

$C \cap H \cap F$ = {students who play Cricket, Hockey & Football}

$\Rightarrow n(C \cap H \cap F \cap) = 0$

Here, we are required to find the number of students who play cricket but not Hockey i.e. $n(C \cap H')$

Now, $n(C \cup H \cup F) = n(C) + n(H) + n(F) - n(C \cap H) - n(H \cap F) - n(F \cap C) + n(C \cap H \cap F)$

$\therefore\ 42 = 14 + 20 + 24 - n(C \cap H) - 2 - 3 + 0$

$\therefore\ 42 = 53 - n(C \cap H)$

$\therefore\ n(C \cap H) = 11$

Now $n(C) = n(C \cap H') + n(C \cap H)$

$\therefore\ 14 = n(C \cap H') + 11$

$\therefore\ n(C \cap H') = 3$

$\therefore$ 3 students play Cricket but not Hockey.

Illustration 18 : If $U = \{x / x \in N, x \leq 10\}$,

$A = \{x / x \in N, x^2 \angle 10\}$

$B = \{2, 4, 6\}$

$C = \{x / x^3 - 3x^2 - 4x = 0\}$

Verify that, (i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A' - B' = B - A$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$, $C = \{0, -1, 4\}$

$[\therefore x^3 - 3x^2 - 4x = 0,$

$x(x^2 - 3x - 4) = 0,$

$x(x - 4)(x + 1) = 0,$

$x = 0$ or $x = 4$ or $x = -1]$

(i) $B - C = \{2, 6\}$

$\therefore\ A \cap (B - C) = \{2\}$ ....(i)

Now $A \cap B = \{2\}$, $A \cap C = \{\ \}$,

$(A \cap B) - (A \cap C) = \{2\}$ ....(ii)

From (i) and (ii)

$\therefore\ A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A' = \{4, 5, 6, 7, 8, 9, 10\}$

$B' = \{1, 3, 5, 7, 8, 9, 10\}$

L.H.S = A' – B' = {4, 6}

R.H.S = B – A = {4, 6}

∴ LHS = RHS

Illustration 19 : Examine the validity of the following statements and justify your answer :

(i) If A = $\{x / x^3 - 5x^2 + 6x = 0\}$

B = $\{x / x \in Z, |x| < 1\}$ then $A \cap B = \phi$

(ii) If $A - B = A$, Then $A \cap B = A$

(iii) If $A = \{\phi, a\}$ Then $\{\phi\} \in P(A)$

(iv) If $x \notin (A \cup B)$ Then $x \notin A$ or $x \notin B$

(v) If $A = \{a, b, c\}$, $B = \{2, 5, 7\}$

then $B \times A = \{2a, 5b, 7c\}$

Ans. :

(i) **False :** Here A = {0, 2, 3}, B = {0}

∴ $A \cap B = \{0\}$, and not ϕ

∴ Statement is false

(ii) **False :** A – B = A

i.e.No element of B is in A Hence $A \cap B = \phi$. The statement is therefore false.

(iii) **True :** because $P(A) = \{\{\}, \{\phi, a\}, \{\phi\}, \{a\}\}$

Thus $\{\phi\} \in P(A)$ is true

(iv) **False :** because If $x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$.

(v) **False :** because

A = {a, b, c}, B = {2, 5, 7}

Then B × A = {(2, a), (2, b), (2, c), (5, a), (5, b), (5, c), (7, a), (7, b), (7, c)}

Illustration 20 : If A = [1, 3, a, {1}, {1, a}], state whether the following statements are true or false.

(i) $1 \in A$,

(ii) $\{1\} \in A$,

(iii) $\phi \in A$,

(iv) $\{1, a\} \subset A$

(v) $\phi \subset A$

(vi) $\{1, a\} \in A$

Ans. :

(i) $1 \in A$, True

(ii) $\{1\} \in A$, True

(iii) $\phi \in A$, False

(iv) $\{1, a\} \subset A$, True,

(v) $\phi \subset A$, True

(vi) $\{1, a\} \in A$, True

Illustration 21 : Prove that (A')' = A

We shall prove that

(i) $(A')' \subseteq A$ (ii) $A \subseteq (A')'$

(i) Let x be any element of $(A')'$ $\Rightarrow x \in (A')'$

$\Rightarrow x \notin A'$

$\Rightarrow x \in A$

$\Rightarrow (A')' \subseteq A$ .....(i)

(ii) Let y be any element of A $\Rightarrow y \in A$

$\Rightarrow y \notin A'$

$\Rightarrow y \in (A')'$

$\Rightarrow A \subseteq (A')'$ .....(ii)

From (i) & (ii), $(A')' = A$

Illustration 22 : Prove that : A – (A – B) = A ∩ B#

We shall prove that (i) $A - (A - B) \subseteq (A \cap B)$

(ii) $A \cap B \subseteq A - (A - B)$

(i) Let x be any element of $A - (A - B)$

$\Rightarrow x \in A - (A - B)$

$\Rightarrow x \in A$ but $x \notin (A - B)$

$\Rightarrow x \in A$ but ($x \notin A$ and $x \in B$)

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in A \cap B$

$\Rightarrow A - (A - B) \subseteq A \cap B$ ....(i)

(i) Let y be any element of $A \cap B$

$\Rightarrow y \in (A \cap B)$

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in A$ but ($y \notin A$ and $y \in B$)

$\Rightarrow y \in A$ but $y \notin (A - B)$

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* Prove that $(A')' = A$ (March, 2011, Dec., 2015)

\# Prove that $A - (A - B) = A \cap B$ (Dec., 2015)

$\Rightarrow y \in A - (A - B)$

$\Rightarrow A \cap B \subseteq (A - (A - B)$ ......(ii)

From (i) & (ii)

$A - (A - B) = (A \cap B)$