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A Closed Form Inverse Dynamics Model of the Delta Parallel Robot

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Abstract: A complete kinematics and inverse dynamics model in closed form of the Delta Parallel Robot is developped. The modelling approach uses only inverse kinematics and Newton laws to obtain an inverse dynamics model called "in the two spaces" since it is parametrized by the robot's state in both task space and joint space.The proposed modelling process can be applied to all fully parallel robot structures and it is shown that the number of computations required to evaluate the model is not larger than the standard case of a serial robot.

Keywords: parallel robot, kinematics and dynamics modelling.

1 INTRODUCTION

Parallel robots are based on mechanical structures where more than one kinematic chain joins the robot's *base* to its end-effector. Therefore, these *sub-chains* form closed *kinematic chains*, so there are *non-actuated joints* along these loops to avoid internal constraints. The advantages of such structures over serial arms of comparable size are a higher stiffness and a lower mobile inertia and therefore a higher mechanical bandwidth. The main drawback of parallel robots is their limited workspace volume compared to a serial arm.

Here, we consider only the class of *fully parallel robots* which consist in n identical sub-chains each having only one actuated joint. We call *platform* the rigid body supporting the end-effector and where all sub-chains are connected. The whole structure is then a n degrees-of-freedom (dof) manipulator. The most well known fully parallel robot is a 6 dof machine often referred to as the *Stewart platform* [12]based on a structure from Mc Cough. Many parallel manipulators derived from this original idea were developed and studied are now used in many robotic applications. Merlet [10] gives an extensive list of the existing machines and related research and a complete review of the most important results obtained in the field.

This paper concentrates on a 3 dof fully parallel robot known as Delta [1] which is the fastest manipulator currently available. We propose here the first complete inverse dynamics model of the Delta in a closed form formulation which allows both analysis and implementation of a *computed torque* based control scheme. We show that the complete model can be put in the same handy form as the simplified model we proposed in [7]. We call this formulation a dynamic model "*in the two spaces*" since it can be used either to design a control loop in joint space or in task space using kinematics relations. We showed in [7] that a task space control was more suitable for implementation efficiency. Here, we quantify the complexity of the described algorithm.

2 THE DELTA ROBOT

The Delta 3 dof manipulator (Fig.1) was designed by Clavel [1] for very fast pick & place applications. Its structure belongs to Merlet's second class [10]since each sub-chain consists of two bodies, the first one (the arm) being linked to the base by an actuated revolute joint. The main idea behind the design is the "space parallelogram" formed by the parallel bars which build the fore-arms. This guarantees that the platform and the basis remain coplanar. Therefore, the platform moves in Cartesian space with no added rotations.

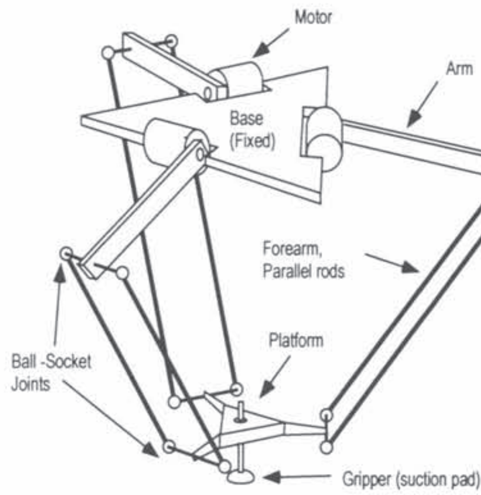

Fig.1: Delta manipulator.

The Delta is used upside-down, the base "hanged on the ceiling"; it was specially designed for very fast pick-and-place tasks: its typical task is to pick a 10 g mass and place it 30 cm further along a half-elliptic trajectory in 0.15 ms, allowing a 3 cycles/s cadency. The platform can reach accelerations of 300 m/s^2, travelling near 10 m/s [3]. To the best of our knowledge, it is the fastest robot in the world. When considering dynamics, it is the first robot where the Coriolis and centripetal effects can become dominant, i.e. most of the torque applied to the motors is used to compensate that effects.

3 POSITION KINEMATICS

Let us define q as the position vector in joint space of dimension 3 and then 3-dimensional vector p as the position vector in (Cartesian) task space. Definitions of the other geometrical constants and variables used further can be found in Fig.2

It is show in [10] that the inverse kinematics model $q = F^{-1}(p)$ of parallel robots always has an explicit form and that the forward model $p = F(q)$ doesn't have a closed form in general. The Delta is an exception since a closed form forward kinematics relation exists ([2], [3]). Since an inverse kinematics algorithm was presented in [7] in the same context, we introduce here below only the notations that are used further for the inverse dynamics model derivation.

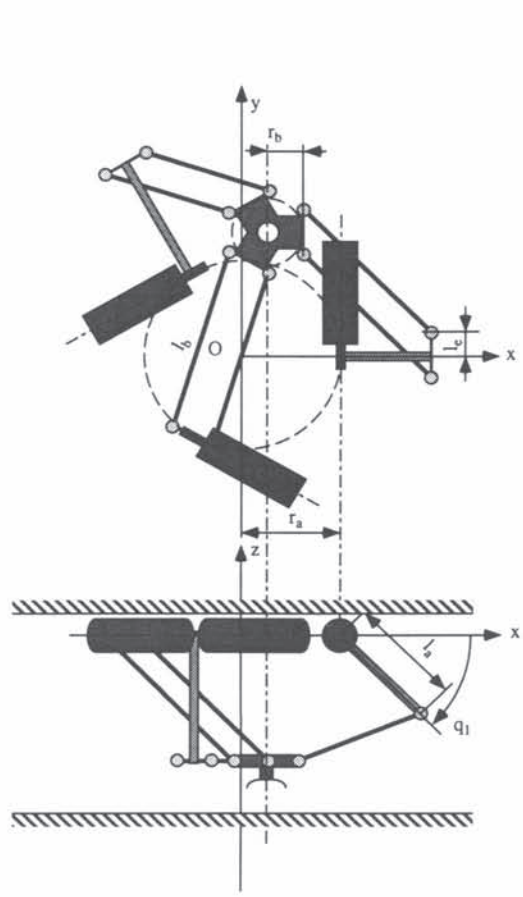

Fig.2: Delta robot geometry.

Three Cartesian frames $\{O, x_i, y_i, z\}$ are introduced to describe each sub chain. They are obtained by rotation of angle φ_i around axis z where $\varphi = [0, 120°, 240°]$.

$$p_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \phi_i \cdot p \text{ where } \phi_i = \begin{bmatrix} \cos\varphi_i & \sin\varphi_i & 0 \\ -\sin\varphi_i & \cos\varphi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

is the Cartesian position of the platform in the corresponding frame. Note that $\phi_i^{-1} = \phi_i^T$.

$$e_i = \begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix} + L_a \alpha_i \quad (2)$$

is the position of the i^{th} elbow in the (O, x_i, z) frame with $R = R_a - R_b$ and

$$\alpha_i = \begin{bmatrix} \cos q_i \\ 0 \\ \sin q_i \end{bmatrix} \quad (3)$$

is the unit direction vector of the i^{th} arm. The unit direction vector of the corresponding forearm is

$$\beta_i = \frac{1}{L_b}(p_i - e_i) \tag{4}$$

We have therefore $p_i = e_i + L_b\beta_i$. (5)

4 SPEED KINEMATICS

Differentiating (5) yields $\dot{p}_i = L_a \alpha_i \dot{q}_i + L_b \dot{\beta}_i$ where

$$\alpha_i = \frac{\partial \alpha_i}{\partial q_i} = \begin{bmatrix} -\sin q_i \\ 0 \\ \cos q_i \end{bmatrix}. \tag{6}$$

After some transformations (see [7]), the equations system obtained for i=1,2,3 can be written in the form:

$$M \cdot \dot{p} = L_a V \cdot \dot{q} \tag{7}$$

where

$$M = \begin{bmatrix} \beta_1^T \cdot \phi_1 \\ \beta_2^T \cdot \phi_2 \\ \beta_3^T \cdot \phi_3 \end{bmatrix} = [\phi_1^{-1} \cdot \beta_1, \phi_2^{-1} \cdot \beta_2, \phi_3^{-1} \cdot \beta_3]^T \tag{8}$$

and V is the diagonal matrix

$$V = \begin{bmatrix} \beta_1^T \cdot \alpha_1 & 0 & 0 \\ 0 & \beta_2^T \cdot \alpha_2 & 0 \\ 0 & 0 & \beta_3^T \cdot \alpha_3 \end{bmatrix}. \tag{9}$$

The inverse Jacobian is then $J^{-1} = \frac{1}{L_a} V^{-1} \cdot M$. It is known [10] that it is easier to obtain the inverse Jacobian matrix $J^{-1}(p) = \frac{\partial}{\partial p} F^{-1}(p)$ of a parallel robot than its forward Jacobian J, which can usually be obtained only through inversion of J^{-1}. One can here easily see that the analytic form of the forward Jacobian $J = L_a M^{-1} \cdot V$ would be too complex to be of any practical use.

5 ACCELERATION KINEMATICS

Differentiating (7) gives the relation between task space and joint space accelerations:

$$M \cdot \ddot{p} = L_a V \cdot \ddot{q} - \dot{M} \cdot \dot{p} + L_a \dot{V} \cdot \dot{q}. \tag{10}$$

$$\dot{M}\dot{p} = \begin{bmatrix} \dot{\beta}_1^T \cdot \phi_1 \\ \dot{\beta}_2^T \cdot \phi_2 \\ \dot{\beta}_3^T \cdot \phi_3 \end{bmatrix} \dot{p} \tag{11}$$

Since $\dot{\beta}_i = \frac{1}{L_b}(\dot{p}_i - \dot{e}_i) = \frac{1}{L_b}(\dot{p}_i - L_a \alpha_i \dot{q}_i)$, (12)

$$\dot{M}\dot{p} = \frac{1}{L_b}(\dot{p}^T \cdot \dot{p}) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{L_a}{L_b} \dot{q} \bar{P} \dot{p} \tag{13}$$

$$\text{where } \bar{P} = \begin{bmatrix} \alpha_1^T \cdot \phi_1 \\ \alpha_2^T \cdot \phi_2 \\ \alpha_3^T \cdot \phi_3 \end{bmatrix}. \tag{14}$$

To remain coherent, let us also define

$$P = \begin{bmatrix} \alpha_1^T \cdot \phi_1 \\ \alpha_2^T \cdot \phi_2 \\ \alpha_3^T \cdot \phi_3 \end{bmatrix} \tag{15}$$

that will appear in the dynamics. The last term of (10) involves

$$\dot{V}\dot{q} = \nabla \cdot \dot{q} - \dot{q}\left[\bar{\nabla} \cdot \dot{q}\right] \text{ where} \tag{16}$$

$$\nabla = \begin{bmatrix} \dot{\beta}_1^T \cdot \alpha_1 & 0 & 0 \\ 0 & \dot{\beta}_2^T \cdot \alpha_2 & 0 \\ 0 & 0 & \dot{\beta}_3^T \cdot \alpha_3 \end{bmatrix} \text{ and} \tag{17}$$

$$\bar{\nabla} = -\begin{bmatrix} \dot{\beta}_1^T \cdot \alpha_1 & 0 & 0 \\ 0 & \dot{\beta}_2^T \cdot \alpha_2 & 0 \\ 0 & 0 & \dot{\beta}_3^T \cdot \alpha_3 \end{bmatrix}. \tag{18}$$

Replacing (13) and (16) in (10) gives the inverse kinematics model for accelerations:

$$\begin{aligned} \ddot{q} = V^{-1}\Big[& \frac{1}{L_a} M \cdot \ddot{p} + \frac{1}{L_a L_b} \dot{p}^2 - \bar{\nabla} \cdot \dot{q} - \\ & - \frac{1}{L_b} \dot{q} \, [\bar{P} \cdot \dot{p}] + \dot{q}\left[\bar{\nabla} \cdot \dot{q}\right] \Big] \end{aligned} \tag{19}$$

Note that a forward model (i.e. $\ddot{p}$ as a function of $\ddot{q}$) would require M^{-1} since the forward Jacobian is implicitely used.

6 DYNAMICS

Since dynamics is a key for the Delta control because of its speed and accelerations, much effort has been put in developing models by different approaches. Dayer [4] used Newton-Euler relations and obtained a system of dimension 21 (!!!) to be solved at every sampling period. This model used the robot state in joint space, therefore forward kinematics relations were implicitly used. Codourey [3] proposed the first model which could be used for real-time control. The parallel rods were modeled by two punctual masses at the shoulder and on the platform; therefore, their inertia momentum were neglected. Miller [11] removed this assumption and solved the problem using Lagrange multipliers. These two models required numerous numeric differentiation since the kinematics relations for speeds and accelerations were obtained later by Guglielmetti [6]. In [7] we used these kinematic relations to write the simplified model from Codourey [3] under a closed form and proposed a task space control scheme based on this model. We here below remove the assumption made about the forearms inertia and write the complete model so that it would be usable for analysis purposes.

We first consider the i^{th} forearm as separated from the rest of the structure. Since the forearm consists of two identical bars which always remain parallel, we model each forearm as a single bar of mass m_b and inertia momentum j_b relatively to any axis intersecting the elbow point e_i and orthogonal to the bar. The position of the forearm in the $\{O, x_i, y_i, z\}$ frame is specified by the platform point p_i and the elbow point e_i. The acceleration of the latter is obtained by differentiating Eq. (2) twice as $\ddot{e}_i = L_a(\bar{\alpha}_i\ddot{q}_i - \alpha_i\dot{q}_i^2)$. Since we consider a perfectly rigid bar, the acceleration of the platform point p_i must be compatible with the acceleration of the other edge and can therefore be written under the form $\ddot{p}_i = \ddot{e}_i + \ddot{s}_i + \ddot{r}_i$ with $\ddot{s}_i$ being the centripetal acceleration (colinear to β_i) due to the rotation of the bar around the elbow e_i and $\ddot{r}_i$ the tangent acceleration related to angular accelerations of the bar around the elbow. $\ddot{e}_i$ can be viewed as the translation acceleration of the bar. The force that should be applied on the platform to obtain the desired bar acceleration is then given by

$$d_i = \frac{m_b}{2}[\ddot{e}_i + \phi_i\ddot{g}] + \frac{j_b}{L_b}\ddot{r}_i + m_b \cdot \ddot{s}_i . \tag{20}$$

We have $\ddot{s}_i = \beta_i \cdot \beta_i^T[\ddot{p}_i - \ddot{e}_i]$ and (21)

$$\ddot{r}_i = \ddot{p}_i - \ddot{e}_i - \ddot{s}_i = [I - \beta_i \cdot \beta_i^T] \cdot [\ddot{p}_i - \ddot{e}_i] , \tag{22}$$

thus

$$\begin{aligned} d_i = {} & \frac{m_b}{2}\phi_i\ddot{g} + \left[\frac{j_b}{L_b}I - \left(\frac{j_b}{L_b} - m_b\right)\beta_i \cdot \beta_i^T\right] \cdot \ddot{p}_i + \\ & + \left[\left(\frac{j_b}{L_b} - m_b\right)\beta_i \cdot \beta_i^T + \left(\frac{m_b}{2} - \frac{j_b}{L_b}\right)I\right] \cdot \ddot{e}_i \end{aligned} \tag{23}$$

Moreover, the force $m_n \cdot (\ddot{p} + \ddot{g})$ should be applied on the platform to accelerate it. Therefore, the total force that must be applied on the platform is

$$:f = m_n[\ddot{p} + \ddot{g}] + \sum_{i=1}^{3} \phi_i^{-1} \cdot d_i . \tag{24}$$

f is transmitted from the motors to the platform through 3 forces of module h_i applied along each forearm. Vector h is obtained by solving the linear equation system $M^T h = f$. Since M^T is regular except at singular positions of the robot which are excluded from the workspace, we can write $h = M^{-T} \cdot f$.

Finally, we consider each arm separately. j_a being the inertia of the arm and m_a the mass submitted to gravity effects. Considering the center of gravity of the arm at the middle of its length for simplicity but without loss of generality, the torques at the actuated joint are given by

$$\begin{bmatrix}\chi_i\\ \tau_i\\ \psi_i\end{bmatrix} = \begin{bmatrix}0\\ j_a\\ 0\end{bmatrix}\ddot{q}_i + \left[\frac{m_b + m_a}{2}\phi_i\ddot{g} + h_i\beta_i + \frac{m_b}{2}\ddot{e}_i\right] \times L_a\alpha_i \tag{25}$$

where only τ_i is the motor torque, χ_i and ψ_i being applied on the bearings. Considering the second line only, the outer product with $L_a\alpha_i$ is equivalent to the inner product of the line vector $L_a\bar{\alpha}^T_i$ with the force applied on the platform. We can now write the inverse dynamics model in the matrix form

$$\tau = \left(j_a + \frac{m_b}{2}L_a^2\right)I\ddot{q} + J^T f + + \frac{L_a(m_b + m_a)}{2}\alpha^T{}_i \cdot \phi_i \cdot \ddot{g} \quad (26)$$

Note that the term $J^T f$ corresponds to the well known result that forces applied on the end-effector are mapped onto motor torques using the transposed Jacobian. This yields both for parallel and serial robots.

Expanding all terms from (26) and regrouping them as factors of velocities and accelerations leads to

$$\tau = A_q\ddot{q} + J^T A_p\ddot{p} + (C_q + J^T C_p)\dot{q}^2 + (H + m_n J^T)\ddot{g} \quad (27)$$

where:

$$A_q = \left(j_a + \frac{m_b}{2}L_a^2\right)I + L_a^2\left(\frac{j_b}{L_b} - m_b\right)[V^T \cdot V] + + L_a\left(\frac{m_b}{2} - \frac{j_b}{L_b}\right)\bar{P} \quad (28)$$

$$A_p = \left(m_n + 3\frac{j_b}{L_b}\right)I + 3\left(\frac{j_b}{L_b} - m_b\right)[M^T \cdot M], \quad (29)$$

$$C_q = L_a\left(\frac{j_b}{L_b} - m_b\right)I, \; C_p = L_a\left(\frac{j_b}{L_b} - \frac{m_b}{2}\right)P \quad (30)$$

$$\text{and } H = \frac{L_a(m_b + m_a)}{2}\bar{P} + \frac{3m_b}{2}I. \quad (31)$$

We call (27) an *"inverse dynamics model in the two spaces"* since it uses the robot state both in joint space and in task space. The square matrix A_q is called the *"joint space inertia matrix"* and A_p the *"task space inertia matrix"* by analogy to the standard form of the inverse dynamics model of a serial robot. It is important to remark here that a formulation in joint space of the usual form $\tau = A(q)\ddot{q} + H(q, \dot{q})$ would be dramatically more complex in the Delta case and does not even exist for a general parallel robot since forward kinematics of such structures has no closed form. However, introducing the inverse kinematics relations for positions (see [7]), speeds (7) and accelerations (10) into (26) would lead to a purely task space formulation of the inverse dynamics of the form $\tau = A(p)\ddot{p} + H(p, \dot{p})$, which would be very interesting for analysis purposes. This approach still has to be studied.

From a more practical point of view, Eq. (27) is very useful for simulation since every single force acting on the robot structure can be isolated. However, the inversion of the inverse Jacobian to obtain the J^T matrix and the matrix multiplications with the Jacobian can be avoided in a real-time implementation. Eq. (27) is rewritten as

$$\tau = A_q\ddot{q} + H\ddot{g} + C_q\dot{q}^2 + \sigma \text{ with} \quad (32)$$

$$J^{-T}\sigma = A_p\ddot{p} + C_p\dot{q}^2 + m_n\ddot{g}. \quad (33)$$

σ is obtained by solving the linear equations system (33) which is much more efficient.

7 COMPLEXITY ANALYSIS

In [7] we presented the implementation of a task space control based on a simplified model of the form (27) on a multiprocessor command and described the inherent parallelism of the inverse dynamics model computation. Here, we concentrate on the number of computations required. In order to compare our proposed inverse dynamics model with previous work from Codourey [3] and Miller [11], we first consider that $\{p, \dot{p}, \ddot{p}, \sin q, \cos q, \dot{q}, \ddot{q}\}$ are available as inputs. The following table gives the number of additions/substractions and multiplications/divisions required for each stage of the evaluation of the inverse dynamics model:

	Eq.#	* /	+ -
p_i for i=1,2,3	(1)	8	4
β for i=1,2,3	(4)	9	9
matrix M	(8)	12	6
vector V	(9)	9	3
P and $\bar{P}$	(14)(15)	6	6
A_q inertia matrix	(28)	12	12
$A_q \cdot \ddot{q}$		9	6
$A_p \cdot \ddot{p}$		45	27
$C_q \cdot \dot{q}^2$	(30)		6
$C_p \cdot \dot{q}^2$	(30)		12
$H\ddot{g}$	(31)	6	1
$m_n\ddot{g}$			1
J^{-1} and LU decomp.		8	8

	Eq.#	*/	+-
σ	(33)	8	6
final evaluation of τ	(32)	6	
Total		140	107

The total required number of computations is about 20% more than Miller's results (127 mult. and 79 add.)[11] with the Lagrange multipliers approach and twice more than Codourey's simplified model [3] (78 mult. and 55 add.). All these approaches also need a forward kinematics algorithm to obtain the p from the measured q and where sin/cos of the q_i's are computed.

However, the advantage of our approach appears when considering the cost of the other required kinematics algorithms which were not investigated by previous research [2], [3] and [11]. Obtaining $\dot{p}$ from the measured $\dot{q}$ is easy since the inverse Jacobian is available in a LU decomposed form; it requires only 8 multiplications and 6 additions. The transformation of joint speed accelerations $\ddot{q}$ into task space accelerations $\ddot{p}$ which is required in a "task space computed torque scheme" given in Eq. (19) requires 48 multiplications and 30 additions. As shown in [7], since the forward kinematics would require the forward Jacobian, the standard computed torque scheme in joint space would be less efficient than task space control.

The complexity of the whole proposed model is about the same order of magnitude as for a serial robot : reference [5] gives 149 multiplications and 124 additions for a general 3 dof serial arm.

8 CONCLUSION

The first closed form inverse dynamics model of the Delta robot is proposed. Its formulation called "in the two spaces" allow simulation, analysis and model based control to be done either in joint space or in task space. The inverse Jacobian matrix is obtained analytically and its inversion to obtain the forward Jacobian is avoided and replaced by a linear system solution. The complexity of the whole model is shown to be of the same order of magnitude as in the case of a 3dof general serial arm. The approach used in this work is very general and can be applied to other fully parallel robots with little modifications.

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