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A New Approach to the Design of a DELTA Robot with a Desired Workspace

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Abstract. In this paper, a new design method considering a desired workspace and swing range of spherical joints of a DELTA robot is presented. The design is based on a new concept, which is *the maximum inscribed workspace* proposed in this paper. Firstly, the geometric description of the workspace for a DELTA robot is discussed, especially, the concept of *the maximum inscribed workspace* for the robot is proposed. The inscribed radius of the workspace on a workspace section is illustrated. As an applying example, a design result of the DELTA robot with a given workspace is presented and the reasonability is checked with the conditioning index. The results of the paper are very useful for the design and application of the parallel robot.

Key words: workspace, parallel robots, kinematics design, conditioning index.

1. Introduction

In the past two decades, parallel robots have attracted more and more researchers' attention in terms of industrial applications, especially in the field of machine tools, for their relative advantages, e.g., high stiffness, high accuracy, low moving inertia, and so on. For such reason, more and more parallel mechanisms with specified number and type of degrees of freedom (DoFs) have been proposed. In these designs, the 6-DoF parallel robots are the most popular robots, which are studied by more researchers. But they suffer the problems of relatively small useful workspace and design difficulties, for which parallel robots with less than 6 DoFs are becom-

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ing increasingly popular in the machine tool industry. There is no doubt that the DELTA robot and its topologies are the most successful parallel robot designs.

The DELTA robot is such a parallel robot that is built using parallelogram mechanisms and the moving platform has three translational and one rotational DoFs with respect to the base (Clavel, 1988). The robot came up firstly at 1986 by means of a WIPO patent (Clavel, 1986). After that many studies have been contributed to the DELTA robot and its topology architectures. Pierrot et al. (1990) gave the equations corresponding to different models such as forward and inverse kinematics as well as inverse dynamics. Codourey (1996) studied the dynamic modeling and mass matrix evaluation of the DELTA robots based on direct application of the virtual work principle. Fischer (1996) investigated methods for improving the accuracy of the DELTA robots. Sternheim (1988) presented a three-dimensional CAD simulation tool for the DELTA4 parallel robot. Recently, the topologies have been designed as several versions of parallel kinematics machines (Demaurex, 1999; Holy and Steiner, 2000; Company et al., 2000). A three translational DoFs parallel cube-manipulator, which is both stiffness and velocity isotropy in its original position, was proposed based on the concept of DELTA robot (Liu et al., 2003). The DELTA related architectures are also applied to other fields. The detailed information can be found in (Bonev, 2001), in which the story of the robot has been summarized.

A parallel robot is such a system that the moving platform is connected to the base by at least two legs, which leads to complex kinematics and interference between legs. For such reasons, parallel robots have the disadvantages in terms of relatively small useful workspace. Workspace is one of the most important issues because it determines the region where the robot can reach. The workspace is, therefore, one of the most important indices to design a robot (Kosinska et al., 2002; Ottaviano and Ceccarelli, 2002; Stamper et al., 1997). In the design process based on a workspace, most common methods are firstly to develop an objective function and then to reach the result using the numerical method with an algorithm (Kosinska et al., 2002; Ottaviano and Ceccarelli, 2002; Ryu and Cha, 2003; Stamper et al., 1997). These methodologies have the disadvantages in common, i.e., the objective function is highly non-linear and the process is iterative and time consuming. In this paper, the issue of the workspace-based design of a DELTA robot with linear actuators is discussed geometrically, and a new design method is presented. Basically, the workspace of the robot is analyzed, and a concept of *the maximum inscribed workspace* is proposed, which is the foundation of the new design method used in this paper. The method is proved to be very simple and effective by a given example and to be reasonable as well because each configuration is far from singularity illustrated by the distribution of conditioning index on a workspace section. The results of the paper are very useful for the design and application of the parallel robot. The method proposed in this paper can be used in other robots with linear actuators whose reachable workspace can be obtained geometrically.

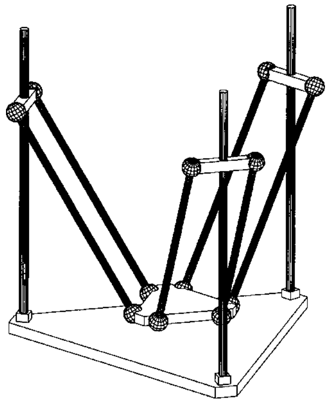


Figure 1. A DELTA robot with linear actuators.

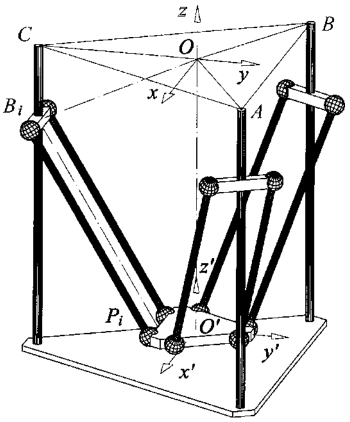


Figure 2. A kinematics model of the DELTA robot.

2. Inverse Kinematics Problem of the DELTA Robot

The DELTA robot with linear actuators is shown in Figure 1, and the geometric parameters are shown in Figure 2, where the moving platform is connected to the base by three identical serial chains OB_iP_iO' $(i = 1, 2, 3)$. Each of the three chains contains one spatial parallelogram, the vertices of which are four spherical joints. Each parallelogram is connected to the base by a prismatic joint. The moving platform of the robot has three translational DoFs with respect to the base. And the output can be obtained through the combination of the actuation to the three prismatic joints.

Closed-form solutions for both the inverse and forward kinematics for the DELTA robot have been developed (Company et al., 2000; Liu, 2001). Here, for convenience, we recall the inverse kinematics problem briefly. A kinematics model of the robot is developed as shown in Figure 2. The center of the link for each of the three chains connected to the base by prismatic joints is denoted as B_i $(i = 1, 2, 3)$, and the center of the interval between the two spherical joints connecting the

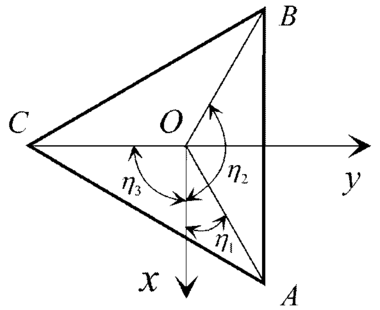


Figure 3. Top view of the base frame.

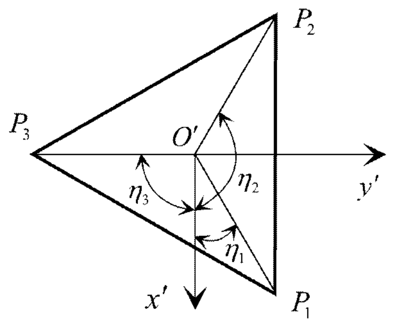


Figure 4. Top view of the moving platform.

moving platform in each chain is denoted as P_i ($i = 1, 2, 3$). A fixed global reference system $\Re$: O-xyz is located at the center of the regular triangle ABC with the z-axis normal to the base and the y-axis directed along CO, as shown in Figure 3. Another reference system $\Re'$: O'-$x'y'z'$ is located at the center of the regular triangle $P_1P_2P_3$. The z'-axis is perpendicular to the output platform and y'-axis directed along P_3O', as shown in Figure 4. Related geometric parameters are $OA = OB = OC = R$, $O'P_i = r$ and $B_iP_i = L$, where $i = 1, 2, 3$. The objective of the inverse kinematics is to find the inputs of the robot with the given position of reference point O'.

The position vector $[\mathbf{c}]_{\Re}$ of point O' in frame $\Re$ can be written as

$$[\mathbf{c}]_{\Re} = (x \quad y \quad z)^{\mathrm{T}}. \tag{1}$$

As shown in Figure 4, the coordinate of the point P_i in the frame $\Re'$ can be described by the vector $[\mathbf{p}_i]_{\Re'}$ ($i = 1, 2, 3$), which can be expressed as

$$[\mathbf{p}_i]_{\Re'} = (r\cos\eta_i \quad r\sin\eta_i \quad 0)^{\mathrm{T}}, \quad i = 1, 2, 3, \tag{2}$$

where

$$\eta_i = \frac{4i-3}{6}\pi, \quad i = 1, 2, 3, \tag{3}$$

which is the angle between the line $O'P_i$ and x'-axis, as shown in Figure 4. Then vectors $[\mathbf{p}_i]_{\Re}$ ($i = 1, 2, 3$) in frame O-xyz can be written as

$$[\mathbf{p}_i]_{\Re} = [\mathbf{p}_i]_{\Re'} + [\mathbf{c}]_{\Re} = (r\cos\eta_i + x \quad r\sin\eta_i + y \quad z)^{\mathrm{T}}, \quad i = 1, 2, 3. \tag{4}$$

As shown in Figure 3, vectors $[\mathbf{b}_i]_{\Re}$ ($i = 1, 2, 3$) will be defined as the position vectors of points B_i in frame $\Re$, and

$$[\mathbf{b}_i]_{\Re} = (R\cos\eta_i \quad R\sin\eta_i \quad z_i)^{\mathrm{T}}, \quad i = 1, 2, 3. \tag{5}$$

Then the inverse kinematics of the DELTA robot can be solved by writing following constraint equations:

$$\|[\mathbf{p}_i - \mathbf{b}_i]_{\Re}\| = L, \quad i = 1, 2, 3, \tag{6}$$

that is,

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = L^2, \quad i = 1, 2, 3, \tag{7}$$

in which, $x_i = (R - r)\cos\eta_i$, $y_i = (R - r)\sin\eta_i$, and z_i are the inputs of the robot and can be rearranged from Equation (7) as

$$z_i = \pm\sqrt{L^2 - (x - x_i)^2 - (y - y_i)^2} + z, \quad i = 1, 2, 3, \tag{8}$$

from which one can see that there are eight inverse kinematics solutions for a given position of the parallel robot. In this paper, we just consider the configuration when the sign "$\pm$" in Equation (8) is "+", i.e., the configuration as shown in Figure 2.

3. Jacobian Matrix and Conditioning Index (CI)

The Jacobian matrix is defined as the matrix that maps the relationship between the velocity of the moving platform and the vector of actuated joint rates. Then we should firstly consider the velocity equation of the robot to get the Jacobian matrix. Equations (7) can be differentiated with respect to time to obtain the velocity equations, which leads to

$$(z - z_i)\dot{z}_i = (x - x_i)\dot{x} + (y - y_i)\dot{y} + (z - z_i)\dot{z}, \quad i = 1, 2, 3. \tag{9}$$

Rearranging Equations (9) leads to an equation of the form

$$\dot{\boldsymbol{\rho}} = \mathbf{J}\dot{\mathbf{p}}, \tag{10}$$

where $\dot{\mathbf{p}}$ is the vector of output velocities defined as

$$\dot{\mathbf{p}} = (\dot{x} \quad \dot{y} \quad \dot{z})^{\mathrm{T}} \tag{11}$$

and $\dot{\boldsymbol{\rho}}$ is the vector of input velocities defined as

$$\dot{\boldsymbol{\rho}} = (\dot{z}_1 \quad \dot{z}_2 \quad \dot{z}_3)^{\mathrm{T}}. \tag{12}$$

Then, the Jacobian matrix of the robot can be written as

$$\mathbf{J} = \begin{bmatrix} \dfrac{x-x_1}{z-z_1} & \dfrac{y-y_1}{z-z_1} & 1 \\ \dfrac{x-x_2}{z-z_2} & \dfrac{y-y_2}{z-z_2} & 1 \\ \dfrac{x-x_3}{z-z_3} & \dfrac{y-y_3}{z-z_3} & 1 \end{bmatrix}. \tag{13}$$

The conditioning index (CI) is defined as the reciprocal of the condition number of the Jacobian matrix (Gosselin and Angeles, 1991; Liu et al., 2000). That is,

$$\mu = \frac{1}{\kappa}, \tag{14}$$

where κ is the condition number of the Jacobian matrix, and $\kappa = \|\mathbf{J}^{-1}\|\|\mathbf{J}\|$, in which $\|\cdot\|$ denotes the any norm of a matrix. In fact, the condition number of a matrix is used in numerical analysis to estimate the error generated in the solution of a linear system of equations by the error on the data (Strang, 1976). When applied to the Jacobian matrix, the condition number will give a measure of the accuracy of the Cartesian velocity of the moving platform and the static load acting on the moving platform. μ is a value between 0 and 1. Actually, the CI μ is such an all-around index that can evaluate the dexterity, isotropy, as well as the static stiffness of a robot (Angeles and López-Cajún, 1992; Salisbury and Graig, 1982; Liu et al., 2000). It can also be used to evaluate the distance to the singularity. $\mu = 1$ means the robot is in its isotropy configuration. When $\mu = 0$, the robot is in its singularity, where the robot will be out of control. Therefore, the larger the CI, the farther the distance to the singularity.

4. Workspace Analysis

One of the disadvantages of parallel robots is their relatively small workspace. Moreover, there are some irregular protuberances (Liu et al., 2003), where a robot is always in or near the singular pose. In order to facilitate the design and the application of the DELTA robot considered in this paper, in this section, the concept of *the maximum inscribed workspace* is proposed and the inscribed radius of the reachable workspace section is presented. And the geometric shape of the maximum inscribed workspace of a DELTA robot is illustrated.

4.1. THE WORKSPACE OF THE DELTA ROBOT

From Equation (7), one can see that the workspace of each of the three legs for the DELTA robot is a sphere if z_i is specified. The sphere is centered at point $O_{iz_i}(x_i, y_i, z_{ij})$, and its radius is L, and z_{ij} is given by

$$z_{ij} \in [z_{i\,\min}, z_{i\,\max}] \tag{15}$$

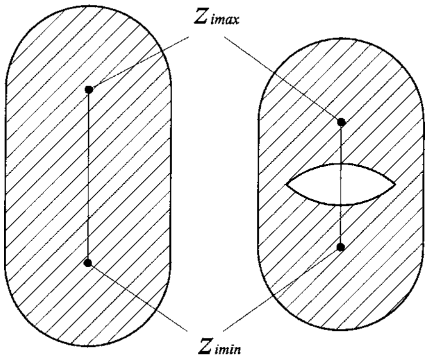


Figure 5. Sections through the centerline of the workspace for a single leg.

in which $[z_{i\,\min}, z_{i\,\max}]$ is the actuation range for each of the three chains. Then the workspace of each of the three legs of the DELTA robot is the enveloping solid of a sphere whose center is moving along a line between two points $(x_i, y_i, z_{i\,\min})$ and $(x_i, y_i, z_{i\,\max})$. The enveloping solid can be described as:

- If $|z_{i\,\min} - z_{i\,\max}| \geqslant 2L$, the enveloping solid consists of two solid spheres whose radii are L and one solid cylinder with the radius L and height $|z_{i\,\min} - z_{i\,\max}|$. And there is no void within the workspace. The workspace section through the centerline of the workspace for this case is shown in Figure 5(a).
- When $|z_{i\,\min} - z_{i\,\max}| < 2L$, there is void in the enveloping solid, which is shown in Figure 5(b). The void is the intersection of two solid spheres $(x - x_i)^2 + (y - y_i)^2 + (z - z_{i\,\max})^2 = L^2$ and $(x - x_i)^2 + (y - y_i)^2 + (z - z_{i\,\min})^2 = L^2$.

The workspace of a DELTA robot is, then, the intersection of three such enveloping solids.

4.2. CHARACTERISTICS OF THE WORKSPACE OF A SINGLE LEG

The Equation (7) can be rearranged as

$$(x - x_i)^2 + (y - y_i)^2 = L^2 - (z - z_i)^2 \tag{16}$$

which means that, for a single leg of the robot, the z section of the workspace is the set of innumerable circles centered at point (x_i, y_i), and their radii can be expressed as

$$r_{ij} = \sqrt{L^2 - (z - z_{ij})^2}, \quad z_{ij} \in [z_{i\,\min}, z_{i\,\max}]. \tag{17}$$

From which one can see that the condition to no null set for the robot is that, for a given value z , there is at least one value z_{ij} within $[z_{i\,\min}, z_{i\,\max}]$ satisfying

$$|z - z_{ij}| < L. \tag{18}$$

And the condition to no void in such set region is that, for a given value z, there is one value z_{ij} within $[z_{i\,\min}, z_{i\,\max}]$ satisfying

$$|z - z_{ij}| = L. \tag{19}$$

In such condition, the z section of the workspace is a circle face, otherwise, it is an annulus face. The maximum and minimum radii of the z section can be expressed, respectively, as

$$r_{i\,\max} = \max_j(r_{ij}), \qquad r_{i\,\min} = \min_j(r_{ij}). \tag{20}$$

Let $r_{\max} = r_{i\,\max}$ ($i = 1, 2, 3$) and $r_{\min} = r_{i\,\min}$ ($i = 1, 2, 3$). Along the z axis, the upper and lower bounds of the set region, i.e., the workspace of a single leg are, as following,

$$z_{\max} = L + z_{i\,\max}, \qquad z_{\min} = z_{i\,\min} - L. \tag{21}$$

Then one can reach following results from above analysis:

$$r_{\max} = \begin{cases} L, & \text{if } z \in [z_{i\,\max}, z_{i\,\min}], \\ \sqrt{L^2 - (z - z_{i\,\max})^2}, & \text{if } z > z_{i\,\max}, \\ \sqrt{L^2 - (z - z_{i\,\min})^2}, & \text{if } z < z_{i\,\min}. \end{cases} \tag{22}$$

If $|z_{i\,\min} - z_{i\,\max}| \geqslant 2L$, the minimum radius is zero, that is,

$$r_{\min} = 0, \tag{23}$$

otherwise, i.e., if $|z_{i\,\min} - z_{i\,\max}| < 2L$,

$$r_{\min} = \begin{cases} 0, & \text{if } z \geqslant L + z_{i\,\min} \text{ or } z \leqslant z_{i\,\max} - L, \\ \sqrt{L^2 - (z - z_{i\,\min})^2}, & \text{if } z \in \left[\dfrac{z_{i\,\max} + z_{i\,\min}}{2}, z_{i\,\min} + L\right), \\ \sqrt{L^2 - (z - z_{i\,\max})^2}, & \text{if } z \in \left(z_{i\,\max} - L, \dfrac{z_{i\,\max} + z_{i\,\min}}{2}\right). \end{cases} \tag{24}$$

4.3. THE MAXIMUM INSCRIBED WORKSPACE

When $r_{\max} \geqslant R - r$, there are two intersection points between two circles $(x - x_1)^2 + (y - y_1)^2 = r_{1\,\max}^2$ and $(x - x_2)^2 + (y - y_2)^2 = r_{2\,\max}^2$. The minimal distance between the two intersection points and point (x_3, y_3, z) is denoted as r_{int}, as shown in Figure 6(a), and

$$r_{\text{int}} = y_1 - \sqrt{r_{\max}^2 - x_1^2} - y_3. \tag{25}$$

From the analysis of Section 4.2, a z workspace section of the robot is the intersection of three same circle or annulus faces, and it can be classified as:

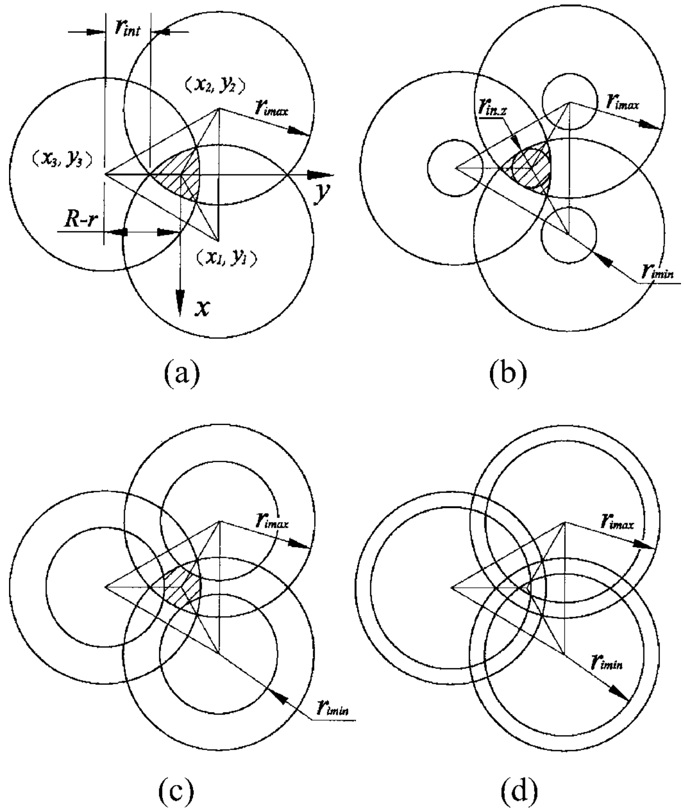


Figure 6. The classification of the workspace section for the DELTA robot.

- On the section, if $r_{\min} = 0$ and $r_{\max} \geqslant R - r$, the workspace section is the intersection of three same circle faces, the boundary consists of three same arcs, as shown in Figure 6(a). From Equation (22), if the section is a point, there are $z = \pm\sqrt{L^2 - (R-r)^2} + z_{i\,\max}$ and $z = \pm\sqrt{L^2 - (R-r)^2} + z_{i\,\min}$, which are also the upper and lower limits of the workspace for the robot along z direction.
- When $r_{\min} < r_{\text{int}}$ and $r_{\max} \geqslant R - r$, the section is the intersection of three same annulus faces, and its boundary is made up of three same arcs, one example of such a case is shown in Figure 6(b).
- If $R - r \geqslant r_{\min} \geqslant r_{\text{int}}$ and $r_{\max} \geqslant R - r$, the section is also the intersection of three same annulus faces, but the boundary consists of six arcs, as shown in Figure 6(c).
- In the case that both $r_{\max}$ and $r_{\min}$ are greater than $R - r$, the section is also the intersection of three same annulus faces, but the set is null, as shown in Figure 6(d).
- When $0 < r_{\max} < R - r$, the section is empty.

According to the above-mentioned classification, if the workspace section is not null, it is the planar region enclosed by three or six arcs. There must exist a maximum inscribed circle within the section, as shown in Figure 6(b). The radius of the inscribed circle is denoted as $r_{\text{in}.z}$, and there are

- $r_{\text{in}.z} = r_{\max} - (R - r)$, if the boundary of the workspace section is composed of three arcs;
- $r_{\text{in}.z} = \min[(R - r) - r_{\min}, r_{\max} - (R - r)]$, if the boundary of the section consists of six arcs, i.e., $R - r \geqslant r_{\min} \geqslant r_{\text{int}}$ and $r_{\max} \geqslant R - r$.

The equation of the circle in frame $\Re$ can be expressed as

$$x^2 + y^2 = r_{\text{in}.z}^2 \tag{26}$$

which is in the plane paralleling to O-xy plane, and the height to O-xy plane is z. The $r_{\text{in}.z}$ will reach its maximum value $r_{\text{in}.z} = L - (R - r)$, if $z = z_{i\max} - L$ or $z = z_{i\min} + L$. The set of all maximum inscribed circles in the workspace section of a DELTA robot is defined as *the maximum inscribed workspace* of the robot.

According to the analysis, in the process of design of the DELTA robot, if the value $r_{\text{it}.z}$ satisfies the condition of desired workspace in the corresponding workspace section, the robot will be the ideal one. This is very useful to design such a robot in terms of a given workspace using the concept, *the maximum inscribed workspace*, which will be proved by an example in Section 5.

4.4. EXAMPLE FOR WORKSPACE ANALYSIS

Based on the above workspace analysis, the workspace and the maximum inscribed workspace can be figured out easily using the commercial CAD software such as AutoCAD. For example, a DELTA robot with linear actuators and the geometric parameters are shown in Figure 7, that is $R = 811$, $r = 260$, $z_{i\max} = -150.2$,

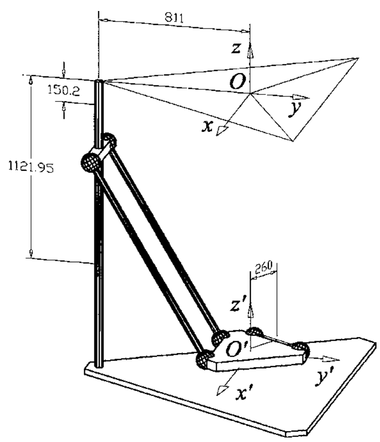


Figure 7. The geometric parameters of an example.

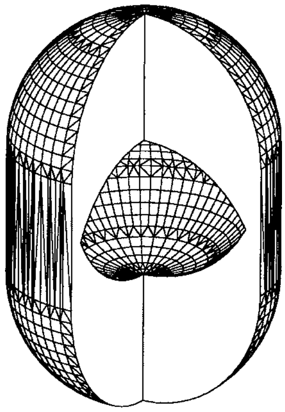


Figure 8. The two thirds part of the workspace for one leg.

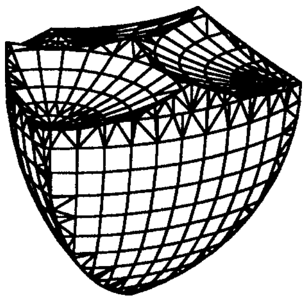


Figure 9. The workspace of a DELTA robot.

$z_{i\,\min} = -1121.95$ and $L = 1000$, which are non-dimensional. Then there are

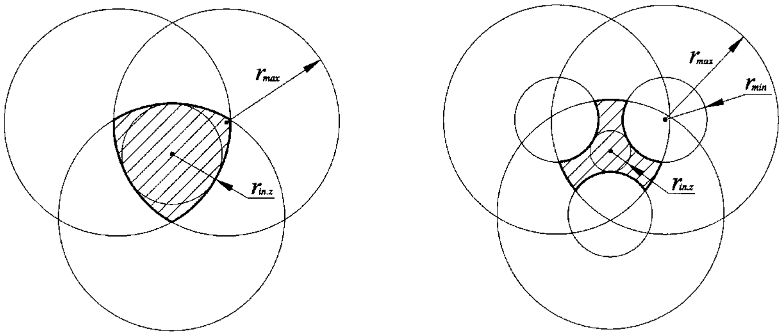
$$R - r = 551, \tag{27}$$

$$\mathbf{x}_o = (x_1, x_2, x_3)^{\mathrm{T}} = (551 \cos 30°, 551 \cos 150°, 0)^{\mathrm{T}}, \tag{28}$$

$$\mathbf{y}_o = (y_1, y_2, y_3)^{\mathrm{T}} = (551 \sin 30°, 551 \sin 150°, -551)^{\mathrm{T}}. \tag{29}$$

The workspace of the robot has following characteristics:

- For the reason of $|z_{i\,\max} - z_{i\,\min}| = 971.75 < 2L$, there is void in the workspace of a single leg, which is the intersection of two solid spheres $(x - x_i)^2 + (y - y_i)^2 + (z + 150.2)^2 = 1000^2$ and $(x - x_i)^2 + (y - y_i)^2 + (z + 1121.95)^2 = 1000^2$.
- The workspace of each one of the three legs is bounded by $z_{\max} = 849.8$ and $z_{\min} = -2121.95$ along z axis and the two thirds of the theoretic workspace is shown in Figure 8.
- The workspace of the robot is the intersection of workspaces of its three legs. The lower part of the intersection is shown in Figure 9, which is the workspace of the robot with the assembly mode as shown in Figure 1. The volume of the workspace can be obtained as 408419044.1447 using AutoCAD.



(a) (b)

Figure 10. Workspace sections of the the robot.

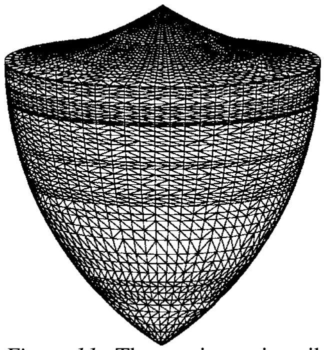


Figure 11. The maximum inscribed workspace.

- When $z = -1079.0$, there are $r_{\max} = 1000.0$ and $r_{\min} = 370.58$. In this condition, the workspace section is the intersection of three same annulus faces, whose maximum and minimum radii are $r_{\max}$ and $r_{\min}$, respectively. The radius of the maximum inscribed circle of this section is $r_{\text{in}.z} = 180.42$, as shown in Figure 10(a).
- For the case of $z = -1250.0$, there are $r_{\max} = 991.77$ and $r_{\min} = 0$, which means that the workspace section is the intersection of three same circle faces, and the radius of each of which is $r_{\max}$. And the radius of the corresponding maximum inscribed circle is $r_{\text{in}.z} = 440.77$, which is shown in Figure 10(b).
- In the case of $z = -1150.2$, the $r_{\text{in}.z}$ reaches to its maximum value $r_{in.z} = 449$. The maximum inscribed workspace is shown in Figure 11.

5. Design of the DELTA Robot with a Desired Workspace

5.1. DESCRIPTION OF THE DESIGN APPROACH

The design of a robot is to determine link lengths with respect to several indices. The process is a complex program and should consider many items, such as workspace, conditioning index, singularity, application, and so on. This paper just considers the design in terms of desired workspace, which is the foundation to the future work. It is based on a new concept, *the maximum inscribed workspace*, which is proposed in last section. Then the objective of the process for the DELTA robot is to determine parameters $R-r$, L, $z_{i\,\max}$ and $z_{i\,\min}$. In the case of the DELTA robot, the performance is related to $R-r$ but neither only R nor only r , because of its pure translational DoFs (Codourey, 1996; Company et al., 2000; Liu, 2001).

According to the analysis results of the workspace for the DELTA robot, one can see that there always exists a maximum inscribed workspace for a robot, and exists a maximum inscribed circle for a z workspace section. The robot is with linear actuators as shown in Figure 1, and it has the workspace advantage along z axis, which can give us the information that if the actuation range $[z_{i\,\min}, z_{i\,\max}]$ is long enough, we can take the maximum workspace of the robot approximately as a cylinder. Based on the assumption, we can design such a robot with desired workspace using the concept of the maximum inscribed workspace. Actually, in many industrial applications, the working space of the device is always given by a cylinder or a cuboid. In the case of a cuboid, we can use an inscribed cylinder to replace it. Therefore, in this paper, the desired workspace is described as a cylinder, whose radius is denoted as r_o. On a z section, let the radius of the maximum inscribed circle to be r_o, that is $r_{\text{in}.z} = r_o$, and $z_o = z$ (z_o can be random for the reason of the above assumption. In this paper, we let $z_o = 0$). In this case, one can obtain the design results $R - r$, L, $z_{i\,\max}$ and $z_{i\,\min}$. The detailed process will be described as following.

As shown in Figure 12, the moving platform is moving on the plane $z = 0$. Point $'O'$ denotes the nearest position of the reference point O' on the moving platform and $''O'$ the farthest position. $'O'P_i'B_i'$ and $''O'P_i''B_i''$ are the corresponding poses of the leg. At these poses, there are

$$\cos\alpha = \frac{R + r_o - r}{L}, \tag{30}$$

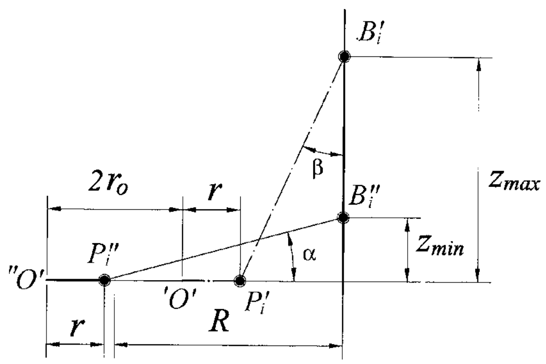


Figure 12. The illustration of designing a DELTA robot.

$$\sin \beta = \frac{R - r_o - r}{L}, \tag{31}$$

where the angles α and β are very relative to the swing angle ψ of spherical joints in this robot, which is

$$\psi = 90^\circ - \alpha - \beta. \tag{32}$$

If α and β are specified, geometric parameters L and $R - r$ can be obtained from Equations (30) and (31) as

$$L = \frac{2r_o}{\cos \alpha - \sin \beta}, \tag{33}$$

$$R - r = L \sin \beta + r_o. \tag{34}$$

Then the actuation range of the robot will be

$$z_{i\,\min} = L \sin \alpha, \tag{35}$$

$$z_{i\,\max} = L \cos \beta + h \tag{36}$$

in which parameter h is the height of the cylinder, i.e., the desired workspace.

From Equations (32)–(36), one can see that the design of the DELTA robot with respect to the desired workspace is very simple using the concept of the maximum inscribed workspace. The process also considers the swing angle of the spherical joints used in the robot.

5.2. DESIGN EXAMPLE

As an application example to the design of the robot using this method, a desired workspace is given as a cylinder with radius $r_o = 250$ and height $h = 500$. The objective is to determine the geometric parameters $R - r$, L, $z_{i\,\max}$ and $z_{i\,\min}$.

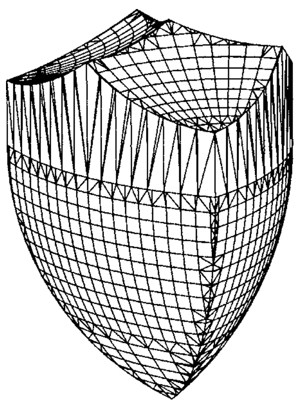


Figure 13. The workspace of the design example.

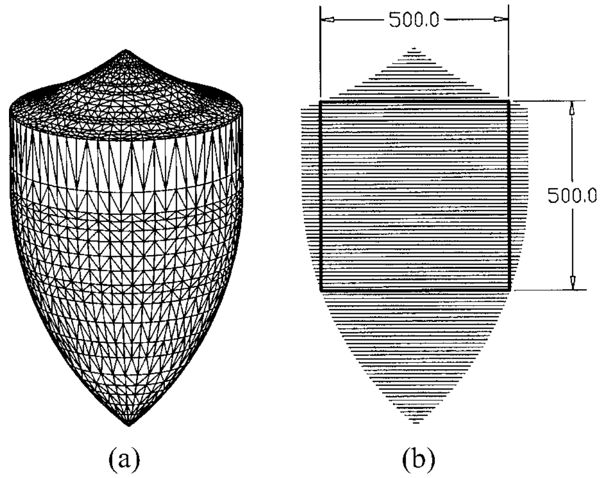


Figure 14. The maximum inscribed workspace of the design example.

Supposing the swing angle ψ of the spherical joint to be 50°, i.e., $\psi = 50°$, there is $\alpha + \beta = 40°$ from Equation (32). Letting $\alpha = \beta$, which leads to $\alpha = \beta = 20°$, the geometric parameters of the robot will be reached from Equations (33)–(36), that is $R - r = 536.13$, $L = 836.58$, $z_{i\,\max} = 1286.13$ and $z_{i\,\min} = 286.13$. The workspace of the designed robot is shown in Figure 13. Figure 14 shows the maximum inscribed workspace, in which the desired workspace, i.e., the cylinder, is embodied, completely.

5.3. DISTRIBUTION OF CI IN THE WORKSPACE

In this section, the distribution of CI is mainly used to illustrate how far each configuration is from the singularity. As mentioned in Section 5.1, the robot has the workspace advantage along z axis, for which we can just present the distribution of CI on a z section of the workspace. For example, when $z = 0.0$, the distribution of CI on the workspace section is shown in Figure 15. Here, the geometric parameters of the robot of the design example in Section 5.2 is used, that is $R - r = 536.13$ (for example, $R = 786.13$ and $r = 250$), $L = 836.58$, $z_{i\,\max} = 1286.13$ and $z_{i\,\min} = 286.13$.

From Figure 15, one can see that (a) the CI value is increased from the boundary to the center of the section and (b) each configuration (each point in the workspace corresponds one configuration) is far from the singularity. This implies that this design method is reasonable and can be used in the design of such a DELTA robot.

6. Conclusions

This paper concerns a new approach to the design of a DELTA robot with a desired workspace. The method is based on a new concept, *the maximum inscribed*

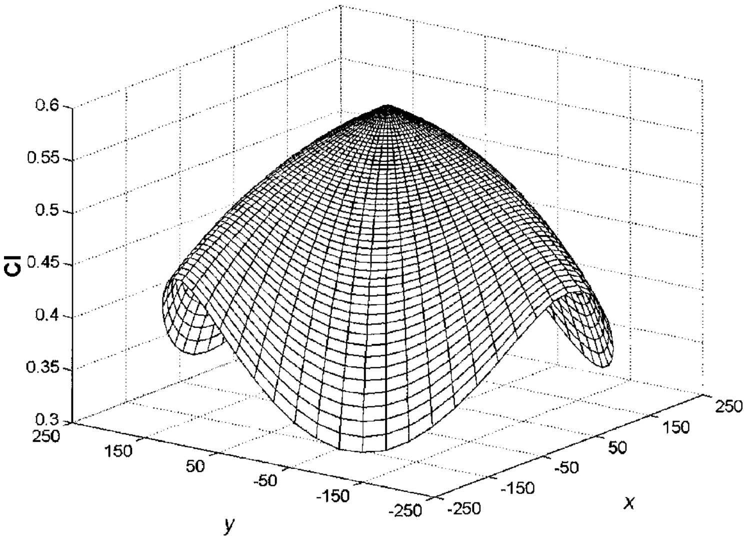


Figure 15. Distribution of CI on the workspace section.

workspace, proposed in this paper. The maximum inscribed workspace can really picture the workspace performance of such a DELTA robot. And the inscribed radius on a workspace section is presented and classified. The design approach also considers the swing range of a spherical joint, and is proved to be effective and simple by an example. Finally, the distribution of the conditioning index for the design example is illustrated on the workspace section, which shows that the proposed design can avoid the singularity. The results of the paper are very useful for the design and application of the parallel robot. And the method proposed in this paper can be applied to other robots with linear actuators whose reachable workspace can be obtained geometrically.

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