Trajectory Planning

Assume the angle to be a cubic function of time

$\theta(t) = A + Bt + Ct^2 + Dt^3$ ------- Equation 1

Boundary Conditions:

- $\theta(0) = \theta_i$
- $\theta(t_f) = \theta_f$

Initial and Final Conditions:

- $\frac{d^2\theta}{dt^2}(0) = 0$
- $\frac{d^2\theta}{dt^2}(t_f) = 0$

Solving the Equation 1 with the above conditions

$$\theta(t) = \theta_i + 3\left(\frac{(\theta_f - \theta_i)}{{t_f}^2}\right)t^2 - 2\left(\frac{(\theta_f - \theta_i)}{{t_f}^3}\right)t^3$$

$$\frac{d\theta(t)}{dt} = 6\left(\frac{(\theta_f - \theta_i)}{{t_f}^2}\right)t - 6\left(\frac{(\theta_f - \theta_i)}{{t_f}^3}\right)t^2$$

$$\frac{d^2\theta(t)}{dt^2} = 6\left(\frac{(\theta_f - \theta_i)}{{t_f}^2}\right) - 12\left(\frac{(\theta_f - \theta_i)}{{t_f}^3}\right)t$$

$$t_f \text{ is time taken for a single PTP translation}$$

As per the cubic relation of angle with time, the angular acceleretion at t_0 and t_f is assumed to be 0.

$$\text{Value of } t_f \text{ depends on the maximum acceleration required as the usecase}$$

$$\text{at } t = 0$$

$$t_f = \sqrt{\frac{6(\theta_f - \theta_i)}{\alpha}}$$

EXAMPLE:

The end effector moves from A (0,0, -327.161) to B (50,0, -327.161)

- Calculate the angles using Inverse Kinematics

```
>> invKineDelta(0,0,-327.161)

ans =

   -0.0288   -0.0288   -0.0288

>> invKineDelta(50,0,-327.161)

ans =

   10.9473   -3.9745   -3.9745
```

$$angular\ acceleration\ is\ set\ to\ 0.5\frac{rad^2}{s}\ \ then\ t_f\ is\ 11.46\ s$$

Angular velocity – Parabolic

Angular acceleration – linear