

MM19B012_Question-2

Consider a dust particle that gets flushed down the sink. The particle takes a helical path down as the liquid swirls around while flowing down as it drains. This can be modeled using the following assumptions: The radial distance of the particle from the center of the sink decreases with increasing time. The vertical distance of the particle decreases with increasing time. The angular velocity of the particle is constant. Model the trajectory of the particle and show the same in a 3D line plot

Answer:-

For the spiral we need to implement two things

- downward motion (and)
- decreasing radius

A helix is a radial path which is extended along the axis perpendicular to its plane.

Solution methodology:-

- The spiral in 2-D shall be treated as a separate problem .
 - We need r to decrease constantly with θ so we use a logarithmic spiral¹
 - As particle is under constant angular velocity θ is a linear function of time
 - Transforming the function to x, y we apply components to function at any points. Then we change function of x, y in terms of time parameter

¹ From : www.britannica.com

The equiangular, or [logarithmic](#), spiral was discovered by the French scientist [René Descartes](#) in 1638. In 1692 the Swiss mathematician [Jakob Bernoulli](#) named it *spira mirabilis* (“miracle spiral”) for its mathematical properties; it is carved on his tomb. The general equation of the logarithmic spiral is $r = ae^{\theta \cot b}$, in which r is the radius of each turn of the spiral, a and b are constants that depend on the particular spiral, θ is the angle of rotation as the curve spirals, and e is the base of the natural logarithm

- For z co-ordinate we write z as a quadratic function of t assuming constant acceleration, then a 3-D parametric plot is plotted
- for a linear z case spiral has also been plotted

Choice of program used:-

Sagemath 8.9: Sagemath has the following features

- great ease of operation in mathematical manipulations,
- plotting functions and 3-D plotting
- parametric and polar supporting
- version 8.9 plots 3-D plots with relative ease
- more visual than octave