HALDIA INSTITUTE OF TECHNOLOGY

(AN AUTONOMOUS INSTITUTION UNDER MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL)

Paper Code: ESC-CS 501

Paper Name: Probability and Statistics

Time Allotted: 3 Hours | *Full Marks*: 70

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Group – A

(Multiple Choice Type Questions)

Choose the correct alternatives from the followings: **15 x 1 = 15**

1. (i) If A and B be events with P(A)=1/3, P(B)=1/4, and $P(A \cup B) = \frac{1}{2}$ then $P\left(\frac{B}{A}\right) =$

a) 3/4 b) 4/3 c) 1/4 d) 1/3

(ii) The probability $P(a \le x \le b)$ is defined by (where F(x) is the distribution function)

a) F(b)-F(a) b) F(b)+F(a) c) F(a)-F(b) d) F(a)F(b)

(iii) If Var(X)=2/3, then Var(3X+5)=?

a) 8 b) 2 c) 6 d) 11

(iv) A random variable has a Poisson distribution such that P(1)=P(2). Then the S.D of X is

a) 0 b) 2 c) $\sqrt{2}$ d) -2

(v) If X has a Poisson distribution with parameter λ, then mean of X is

a) $\lambda(\lambda - 1)$ b) $1/\lambda$ c) λ d) 1

(vi) The distribution for which mean and variance are equal is

a) Poisson b) Normal c) Binomial d) Exponential

(vii) If X and Y are independent then

a) $r_{xy} = 1$ b) Cov(X, Y)=0 c) $Y = aX + b$ d) None

(viii) If X and Y have means 4 and -2 respectively, then mean of $2X + Y - 5$ is

a) 1 b) 0 c) -1 d) -2

(ix) If $x + 5u = 2, 2y + v = 7$ and $r_{xy} = 0.25$, then $r_{uv} =$?

a) 0.25 b) -0.25 c) 0.05 d) 0.50

(x) If regression coefficient of y on x is defined by

a) $r_{xy}\frac{\sigma_y}{\sigma_x}$ b) $r_{xy}\frac{\sigma_x}{\sigma_y}$ c) r_{xy} d) None

(xi) If cov(x,y)=32, var(x)=36, var(y)=64, then correlation coefficient is

a) 3/2 b) 2/3 c) 1/24 d) 1/72

(xii) The regression line x on y is 2x-3y+5=0 then the regression coefficient of x on y is

a) 2/3 b) 3/2 c) 2 d) 3

(xiii) If $H_1(\mu > 60)$ be an alternative hypothesis, then the null hypothesis is

a) $H_0(\mu > 60)$ b) $H_0(\mu < 60)$ c) $H_0(\mu = 60)$ d) $H_0(\mu \ne 60)$

(xiv) In the case of one-way classification with N observation and t treatments, the error degrees of freedom is

a) N-1 b) t-1 c) N-t d) Nt

(xv) In regression line, what does Analysis of variance (ANOVA) calculates?

a) Z-score b) t-score c) χ^2 d) F-ration

Group – B
(Short Answer Type Questions)

Attempt any three from the followings: **3 x 5 = 15**

2. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be hearts. Find the probability of the missing card to be a heart. 5

3. The pmf for poisson distribution is given by $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}; x = 0,1,\ldots,\lambda > 0.$
Show that mean=variance=λ. 5

np npq : q=1-p

4. (i) Write the formula for mean and variance of Binomial distribution.
(ii) Ten coins are thrown simultaneously. Find the probability of getting at least seven heads. 2+3

5. (i) Given $\sum x = 56, \sum y = 40, \sum x^2 = 524, \sum y^2 = 256, \sum xy = 364, n = 8$. Find the correlation coefficient.
(ii) Find the regression equation of x on y. 3+2

6. From a random sample of size 100, mean 105, s.d. 20. Test at 1% level whether the mean of the population can be less than 120. [Given $z_{0.01} = 2.33$]. 5

Group – C
(Long Answer Type Questions)

Attempt any four from the followings: **4 x 10 = 40**

7. (i) Given P(A)=0.12, P(B)=1/8, $P(A \cap B) = 1/17$. Find $P(\bar{A}), P(A \cup B), P(A|B), P(\bar{A} \cap B), P(A \cap \bar{B})$.
(ii) Examine whether the events A and B are mutually exclusive, exhaustive, and independent.
(iii) If A & Bare independent events then show that $\bar{A}$ and $\bar{B}$; $\bar{A}$ and B are independent. 5+3+2

8. (i) Given P(A)=1/2, P(B)=1/3, P(A∩B)=1/4. Find the values of $P(\bar{A}), P(A \cup B), P(A|B), P(\bar{A} \cap B), P(\bar{A} \cap \bar{B}), P(\bar{A} \cup B)$.
(ii) State whether the events A and b are mutually exclusive, exhaustive, equally likely, independent. 6+4

9. (i) A random variable X has the following probability distribution

$X = x_i$	0	1	2	3	4	5	6	7	8
f_i	k	3k	5k	7k	9k	11k	13k	15k	17k

Determinethe value of k
(ii) Determine P(X<3), P(X≥ 3), P(2≤ $X \leq 3$) (iii) The smallest value of c s.t.P(X≤ $c) > \frac{1}{2}$. 2+6+2

10. (i) If X is normally distributed with mean 3 and s.d. 2, find c such that P(X>c)=2P(X≤ c). [Given $\int_{-\infty}^{0.43} \phi(t)dt = 1/3$.]
(ii) The length of bolts produced by a machine is normally distributed with mean 4 and s.d. 0.5. A bolt is defective if its length does not lie in the interval (3.8, 4.3). Find the percentage of defective bolts produced by the machine. [Given $\int_{-\infty}^{0.6} \phi(t)dt = 0.7257, \int_{-\infty}^{0.4} \phi(t)dt = 0.6554$.] 4+6

11. (i) Suppose that the 2D continuous r.v (X,Y) has joint pdf given by
$$f(x) = \begin{cases} 6x^2y, & if\ 0 < x < 1;\ 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$
Find $P(0 < X < 3/4, 1/3 < Y < 2), P(X + Y < 1), P(X > Y)$.
(ii) The joint pdf of the bivariate (X, Y) is
$$f(x,y) = \begin{cases} k(3x + y), 1 \leq x < 3, 0 \leq y \leq 2 \\ \quad 0, \text{elsewhere} \end{cases}$$
Find the value of k, $P(X + Y < 2)$ and the marginal distributions X and Y. 5+5

12. (i) In a random sample of size 400 there are 80 defective items. Test at 5% level whether the proportion of defective items in the population may be regarded as 1/6. [Given $\int_0^{1.96} \phi(t)dt = 0.475$]
(ii) A sample of size 600 persons selected at random from a large city shows that percentage of male in the sample is 53%. It is believed that male to total proportion ratio in the city is ½. Test whether this belief is confirmed by the observation. [Given $\int_{-1.96}^{1.96} \phi(t)dt = 0.025$] 4+6