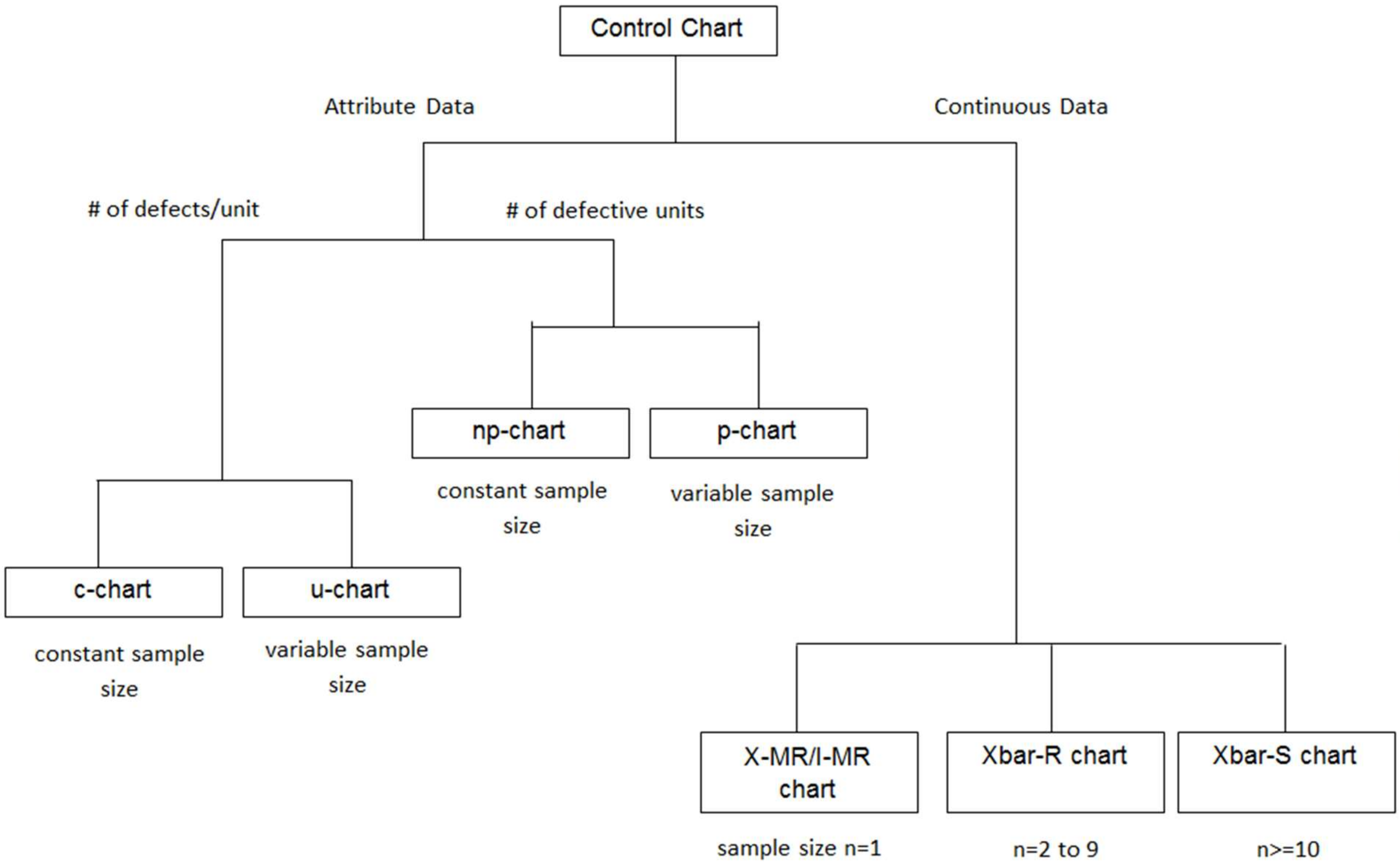
Attribute Control Charts, I-MR Chart

Prof. Sayak Roychowdhury

IIT Kharagpur | Department of Industrial & Systems Engineering

Control Charts



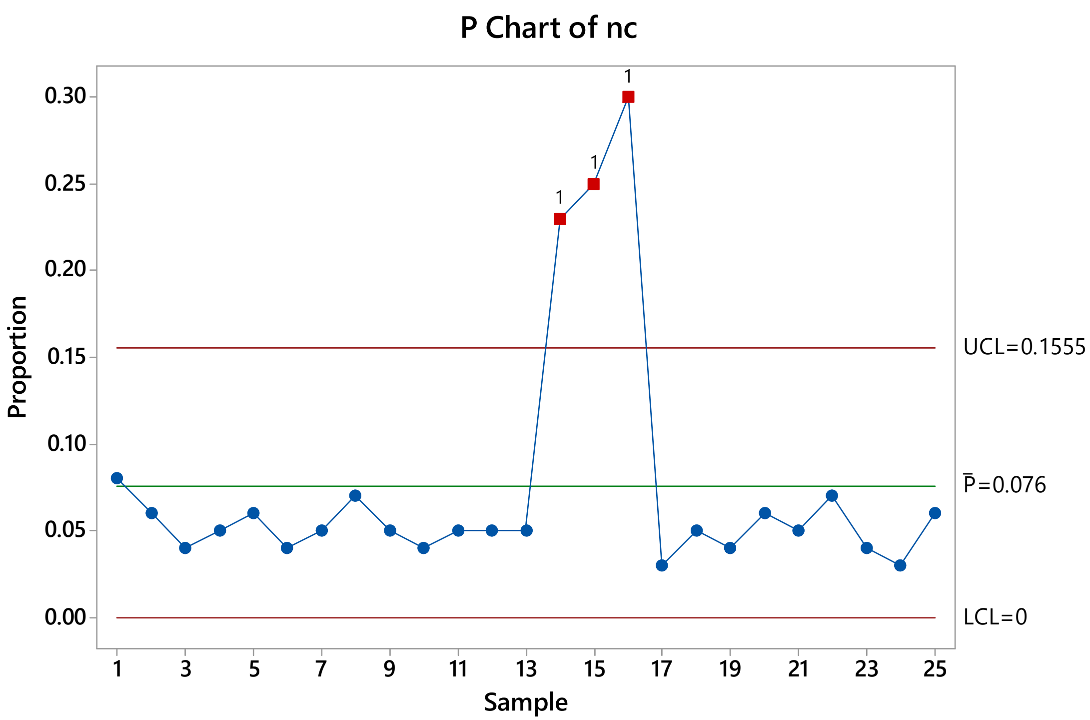
P-chart (fraction of non-conforming)

- *Step 1*. (Startup) Obtain the total fraction of nonconforming units or systems using 25 rational subgroups each of size *n*.
- *Step 2*. (Startup) Calculate "trial" limits:
  - $UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ (Minitab>Stat>Control charts>Attribute Charts>P)
  - $LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$
- *Step 3*. (Startup) Identify all the periods for which p = fraction nonconforming in that period and $p < LCL_{trial}$ or $p > UCL_{trial}$. Investigate, remove if unfair.
- *Step 4*. (Startup) Calculate the total fraction nc. using remaining. **New $\bar{p}$ = "process capability".** Calculate revised limits.
- *Step 5*. (Steady State) Plot the fraction nonconforming, p_j, for each period j and alert designated local authority if out-of-control signals occur.

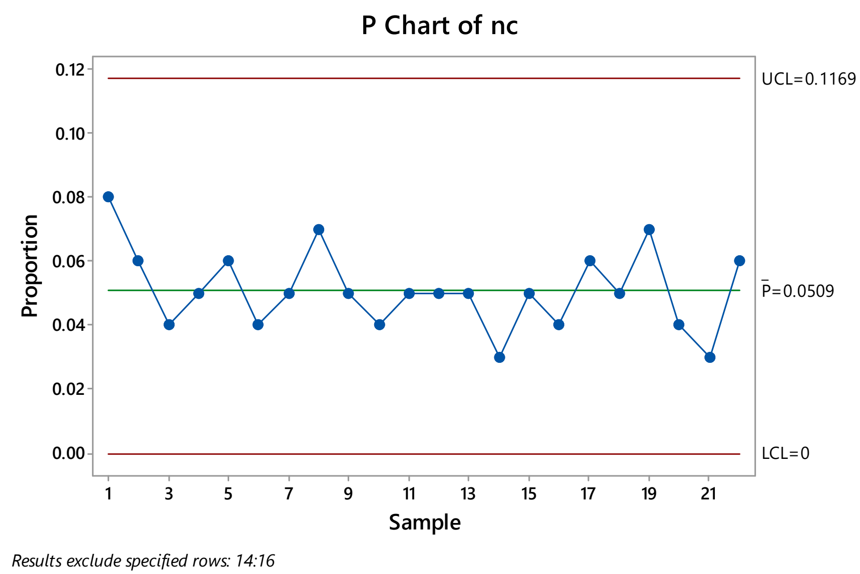
https://www.spcforexcel.com/knowledge/attribute-control-charts/p-control-charts#example

P-chart

New Trainee



Training implemented



Capability = 0.0909

np-chart (# of non-conforming)

- Subgroup size needs to be constant
- Calculate “trial” limits:
  - $UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$
  - $CL = n\bar{p}$
  - $LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

Variable Sample Size for Non conforming Attribute Control Charts

1. Variable Control Limits:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}, \quad LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

2. Control limits based on average sample size:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}, LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$ where

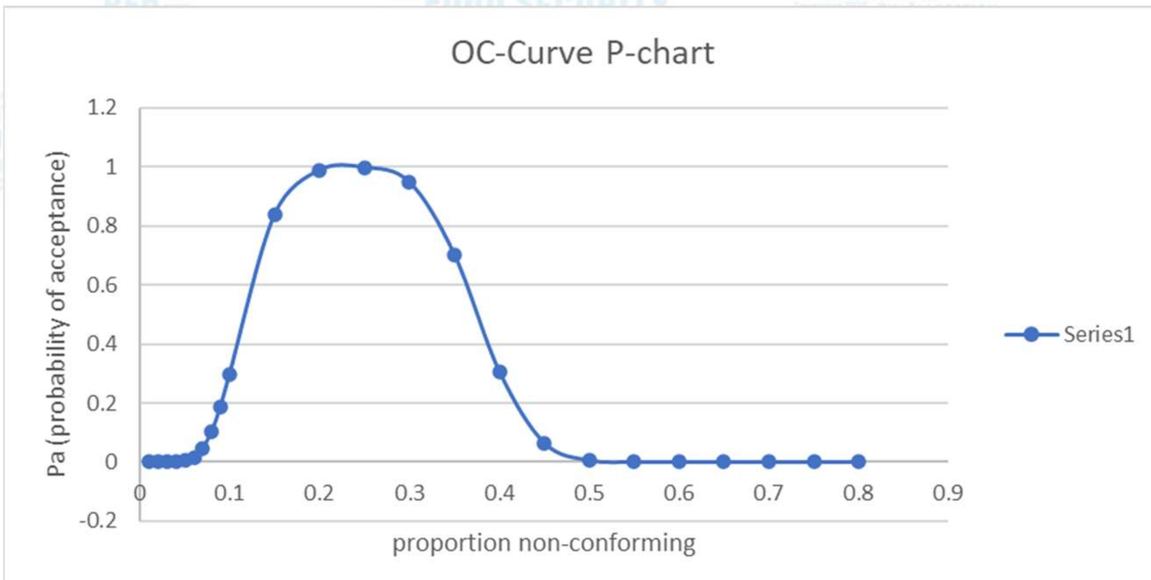
$\bar{n} = \frac{\sum_{i=1}^{m} n_i}{m}$ is the average sample size.

3. Standardized control chart:

$Z_i = \frac{p_i - \bar{p}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}}$ with control limits +3, and -3.

OC-Curve for p-chart (7.2.4)

- Probability of Type II Error
- $\beta = P(\hat{p} \leq UCL|p) - P(\hat{p} \leq LCL|p) = P(D \leq nUCL|p) - P(D \leq nLCL|p)$
- $P(x \leq nUCL|p) = \sum_{x=0}^{nUCL} \binom{n}{x} p^x (1-p)^{1-x}$
- $P(x \leq nLCL|p) = \sum_{x=0}^{nLCL} \binom{n}{x} p^x (1-p)^{1-x}$



C-chart (count of non-conformities)

- *Step 1*. (Startup) Collect data
- *Step 2*. (Startup) Subgroup size is one inspected unit (e.g. 1 airplane, 1 case of pencils)
- *Step3.*Calculate trial limits
  - $UCL_{trial} = \bar{c} + 3\sqrt{\bar{c}}$, $CL_{trial} = \bar{c}$ (average count of non-conformities)
  - $LCL_{trial} = Max\,\{\bar{c} - 3\sqrt{\bar{c}}, 0\}$
- *Step 3*. (Startup) Find out of control signals. Remove if unfair.
- *Step 4*. (Startup) Revise limits **Revised $\bar{c}$ is process capability**
- *Step 5*. (Steady State) Plot and local authority investigates if out-of-control signals occur (can act).

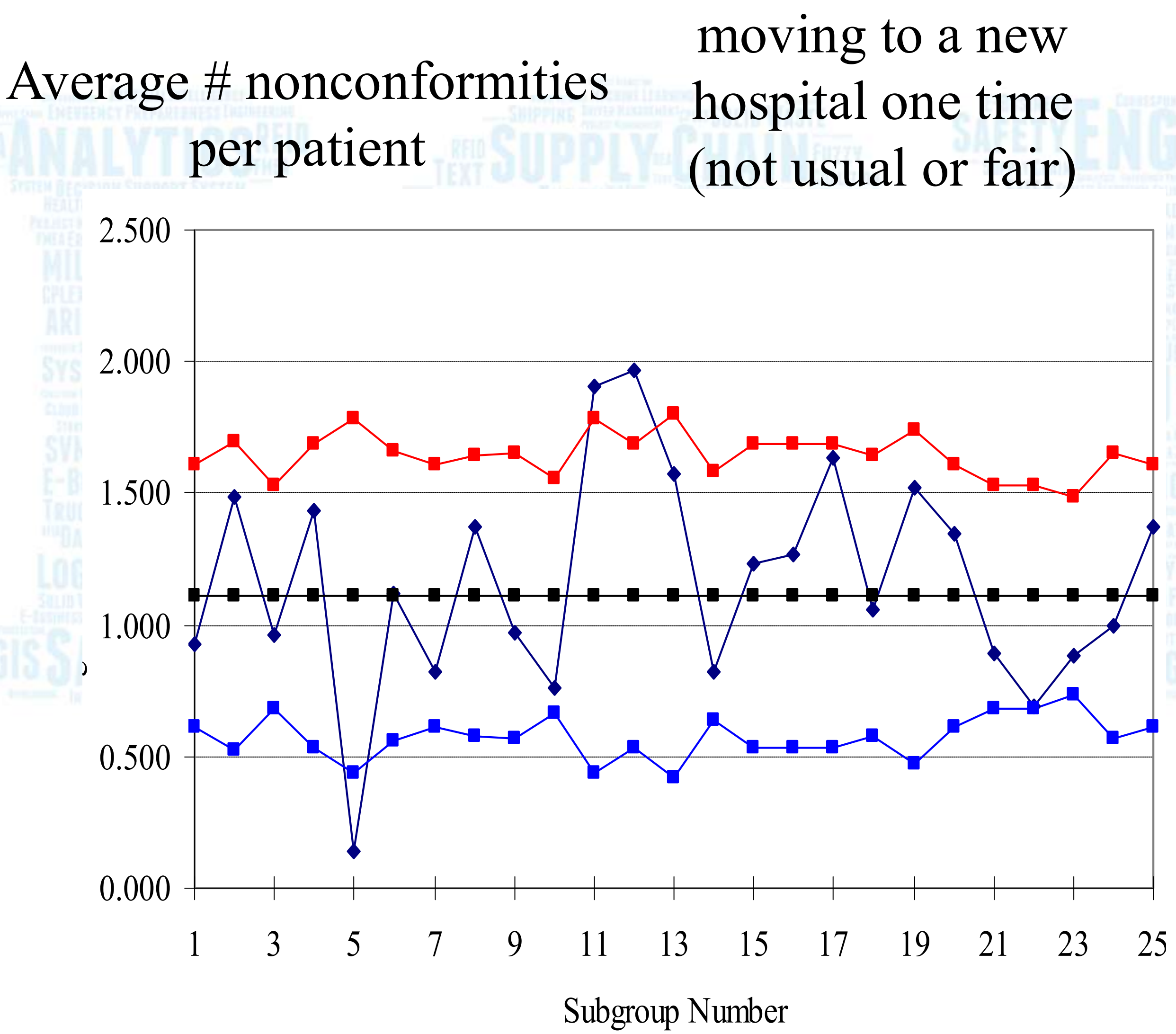
(Minitab>Stat>Control charts>Attribute Charts>C)

U-chart (count of non-conformities/unit)

- *Step 1*. (Startup) Collect data for m periods. $\bar{u} = \sum_{i=1}^{m} c_i \,/\, \sum_{i=1}^{m} n_i$ where c_i is the count of defects (non conformities) in each subgroup
- *Step 2*. (Startup) Calculate "trial" limits for i^{th} sample:
  - $UCL_{trial} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$, $CL_{trial} = \bar{u}$
  - $LCL_{trial} = Max\,\{\bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}, 0\}$
- *Step 3*. (Startup) Find out of control signals. Remove if unfair.
- *Step 4*. (Startup) Revise limits **Revised $\bar{u}$ is process capability**
- *Step 5*. (Steady State) Plot and local authority investigates if out-of-control signals occur (can act).

(Minitab>Stat>Control charts>Attribute Charts>U)

U-chart



Average # nonconformities per patient
moving to a new hospital one time (not usual or fair)
2.500
2.000
1.500
1.000
0.500
0.000
1
3
5
7
9
11
13
15
17
19
21
23
25
Subgroup Number

Demerit Chart

- Determine 4 classes of non-conformities, very serious (A), serious (B) moderately serious (C), and minor (D). Assign weights w_A, w_B, w_C and w_D.
- For sample size n, let c_A, c_B, c_C, c_D denote the total number of defects of each class.
- Determine standard non-conformities per unit $\bar{u}_A, \bar{u}_B, \bar{u}_C, \bar{u}_D$
- $D = w_A c_A + w_B c_B + w_C c_C + w_D c_D$
- Demerits per unit is given by $\mathrm{U} = \frac{D}{n}$
- CL = $\bar{U} = w_A \bar{u}_A + w_B \bar{u}_B + w_C \bar{u}_C + w_D \bar{u}_D$
- $\sigma_{0u} = \sqrt{\frac{w_A^2 \bar{u}_A + w_B^2 \bar{u}_B + w_C^2 \bar{u}_C + w_D^2 \bar{u}_D}{n}}$
- $UCL = \bar{U} + 3\sigma_{0u}$ $LCL = \bar{U} - 3\sigma_{0u}$

Chart Comparison

Method	Advantages	Disadvantages
Xbar & R charting	Uses fewer inspections, gives greater sensitivity	**Requires 2 or more charts for single type of unit**
p-charting	Requires only go-no-go data, intuitive	**Requires many more inspections, less sensitive**
Demerit charting	Addresses differences between nonconformities	**Requires more inspections (but less than p), less sensitive**
u-charting	Relatively simple version of demerit charts	**Requires more inspections (but less than p), less sensitive**
c-charting	Simple (u with n = 1)	**Requires more inspections (but less than p), less sensitive**
Np-charting	**Simpler (equivalent to p just no ÷ n)**	**Same as p plus cannot have variable n**

I-MR (Individual, Moving Range) Chart

- For some cases, rate of production is low, it is not feasible for a sample size to be greater than 1.
- When testing process is destructive, and the cost of item is high, then sample size might be chosen to be 1.
- If every unit is inspected, sample size is 1.
- Data comes slowly, so samples with elements produced at long intervals creates problem for rational subgrouping.

I-MR chart (ch 6.4)

- Control chart for individual measurements
- Useful for detection of system stability
- $R_i = |x_i - x_{i-1}|$ (moving ranges)
- Trial Limits
  - $UCL = \bar{x} + 3 * \frac{\overline{MR}}{d_2}$ (for n=2, d_2=1.128) $UCL_R = D_4\overline{MR},\ \ D_4 = 3.267\ for\ n = 2$
  - $CL = \bar{x}$ $CL_R = \overline{MR}$
  - $LCL = \bar{x} - 3 * \frac{\overline{MR}}{d_2}$ $LCL_R = 0$
- Revised Limits
  - $\sigma_0 = 0.8865MR_0$, $x_0 = \bar{x}_{new}$
  - $UCL = x_0 + 3\sigma_0$ $UCL_R = 3.686\sigma_0$
  - $LCL = x_0 - 3\sigma_0$ $LCL_R = 0$
- Minitab > Stat> Control charts > Variable charts for individuals > I-MR
- Researchers have indicated that MR chart cannot really provide additional information on variability, MR values are not independent.

EWMA & CUSUM Charts

Prof. Sayak Roychowdhury

IIT Kharagpur | Department of Industrial & Systems Engineering

Limitations of Shewhart Charts

- In Shewhart chart, the plotted point represents information corresponding to observation only.
- It does not use information from previous observations.
- This makes Shewhart charts insensitive to small shifts.
- They are less useful in phase II.
- Warning limits and patterns can be useful, but they reduce the simplicity of the control charts

CUSUM Chart and EWMA Chart

- To detect small process shifts, CUSUM (Cumulative sum) and EWMA (Exponentially weighted moving average) charts are used as alternatives to Shewhart Control Chart
- These are good alternatives for phase II process monitoring
- Sometimes process needs to be monitored when sample size n =1, both CUSUM and EWMA charts work well in this situation.
- EWMA charts are particularly robust against non-normality (read section Montgomery 9.2.3).

CUSUM Chart

- First proposed by Page (1954)
- The CUSUM chart plots the quantity

$$C_i = \sum_{j=1}^{i} (\bar{x}_j - \mu_0)$$

Where $\bar{x}_j$ is the average of the j^{th} sample

μ_0 is the target for process mean

Also applicable for $n = 1$

- So CUSUM charts are particularly useful in chemical and process industries and discrete part manufacturing, where frequently subgroup size is 1.

CUSUM Chart (Ch 9.1 Montgomery)

- There are 2 ways to represent CUSUM charts, tabular method and V-mask method. We will discuss tabular method.
- $C_i^+ = \max(0, x_i - (\mu_0 + K) + C_{i-1}^+)$ (Upper CUSUM)
- $C_i^- = \max(0, (\mu_0 - K) - x_i + C_{i-1}^-)$ (Lower CUSUM)
- $C_0^+ = C_0^- = 0$
- $C_i^+ > H$ or $C_i^- > H$ indicate the process mean has shifted
- K is called reference value, allowance or slack value, H is called decision interval

CUSUM Chart

- $K = k\sigma$, typically halfway between μ_0 and out of control mean that we want to detect
- $K = \frac{|\mu_1 - \mu_0|}{2} = k\sigma$
- $H = h\sigma$
- K and H are chosen to provide good ARL performance
- Generally $k = 0.5$, and $h = 4\ or\ 5$ are chosen. Read (9.1.3)

CUSUM Chart Parameter Values and ARL

TABLE 9.3

ARL Performance of the Tabular Cusum with $k = \frac{1}{2}$ and $h = 4$ or $h = 5$

Shift in Mean (multiple of σ)	$h = 4$	$h = 5$
0	168	465
0.25	74.2	139
0.50	26.6	38.0
0.75	13.3	17.0
1.00	8.38	10.4
1.50	4.75	5.75
2.00	3.34	4.01
2.50	2.62	3.11
3.00	2.19	2.57
4.00	1.71	2.01

TABLE 9.4

Values of k and the Corresponding Values of h That Give $ARL_0 = 370$ for the Two-Sided Tabular Cusum [from Hawkins (1993a)]

k	0.25	0.5	0.75	1.0	1.25	1.5
h	8.01	4.77	3.34	2.52	1.99	1.61

EWMA chart (ch 9.2 Montgomery)

- Control chart to detect small shift in the process, ideally used with individual observations.
- The exponentially weighted moving average is defined as

$$z_i = \lambda x_i + (1-\lambda) z_{i-1} \;\; (z_0 = \mu_0\,(target)\;\; or\; \bar{\bar{X}}\,)$$

- λ should be between 0.05 , 0.25 (use smaller λ for smaller shifts)
- Limits
  - $UCL = \bar{\bar{X}} + L\sigma\sqrt{\lambda/(2-\lambda)[1-(1-\lambda)^{2i}]}$
  - $CL = \bar{\bar{X}}$
  - $LCL = \bar{\bar{X}} - L\sigma\sqrt{\lambda/(2-\lambda)[1-(1-\lambda)^{2i}]}$
  - Use μ_0 (target mean) in place of $\bar{\bar{X}}$ if given

- Usually L is taken to be 3. For a steady state process

$\sqrt{\lambda/(2-\lambda)[1-(1-\lambda)^{2i}]}$ becomes $\sqrt{\lambda/(2-\lambda)}$

- σ can be estimated by process standard deviation, or $\bar{R}/d_2$ if $\bar{R}$ can be obtained. Sometimes process history is used for estimation.
- EWMA is often used with Shewhart Chart, so that the combined chart can detect small shifts and large shifts quickly enough.
- Minitab > Stat> Control charts > Time weighted charts> EWMA

EWMA Parameters and ARL

■ **TABLE 9.11**

Average Run Lengths for Several EWMA Control Schemes [Adapted from Lucas and Saccucci (1990)]

Shift in Mean (multiple of σ)	$L = 3.054$ $\lambda = 0.40$	2.998 0.25	2.962 0.20	2.814 0.10	2.615 0.05
0	500	500	500	500	500
0.25	224	170	150	106	84.1
0.50	71.2	48.2	41.8	31.3	28.8
0.75	28.4	20.1	18.2	15.9	16.4
1.00	14.3	11.1	10.5	10.3	11.4
1.50	5.9	5.5	5.5	6.1	7.1
2.00	3.5	3.6	3.7	4.4	5.2
2.50	2.5	2.7	2.9	3.4	4.2
3.00	2.0	2.3	2.4	2.9	3.5
4.00	1.4	1.7	1.9	2.2	2.7

- EWMA chart is fairly robust against non-normal distribution, compared to Shewhart charts for individual measurement (sec 9.2.3 Montgomery)