QDC Assignment-1

Name: Ritam Pradhan   Roll: 20QE30002

1)

Saltwater (y_1)	Oil (y_2)	$(y_1-\bar{y}_1)$	$(y_2-\bar{y}_2)$	$(y_1-\bar{y}_1)^2$	$(y_2-\bar{y}_2)^2$
145	152	-2.6	2.6	6.76	6.76
150	150	2.4	0.6	1.44	0.36
153	147	5.4	-2.4	29.16	5.76
148	155	0.4	5.6	0.16	31.36
141	140	-6.6	-9.4	43.56	88.36
152	146	4.4	-3.4	19.36	11.56
146	158	-1.6	8.6	2.56	73.96
154	152	6.4	2.6	40.96	6.76
139	151	-8.6	1.6	73.96	2.56
148	143	0.4	-6.4	0.16	40.96

$\Sigma y_1 = 1476$ $\Sigma y_2 = 1494$ $\Sigma(y_1-\bar{y}_1)^2 = 222.4$ $\Sigma(y_2-\bar{y}_2)^2 = 268.4$

$\bar{y}_1 = 147.6$ $\bar{y}_2 = 149.4$

$n_1 = 10$ $n_2 = 10$

a) $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

Assuming equal variance, our test statistic would be

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}, \quad S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}},$$

$$S_1^2 = \frac{\Sigma(y_1-\bar{y}_1)^2}{n_1-1}, \quad S_2^2 = \frac{\Sigma(y_2-\bar{y}_2)^2}{n_2-1}$$

$$S_1^2 = \frac{222.4}{9} = 24.71, \quad S_2^2 = \frac{268.4}{9} = 29.82$$

$$S_p = \sqrt{\frac{222.4+268.4}{18}} = \sqrt{\frac{490.8}{18}} = 5.22$$

$$t_0 = \frac{147.6 - 149.4}{5.22\sqrt{\frac{1}{10}+\frac{1}{10}}} = \frac{-1.8}{5.22\sqrt{1/5}} = -0.77$$

$dof = n_1+n_2-2 = 10+10-2 = 18$, $\alpha = 0.05$, $t_{\frac{\alpha}{2},dof} = 2.101$

we get $|t_0| < t_{\alpha/2,dof}$ $[0.77 < 2.101]$, we fail to reject H_0. (Ans)

b) $\bar{y}_1 - \bar{y}_2 = -1.8$

$t_{\frac{\alpha}{2}, dof} = t_{0.025, 18} = 2.101$

$S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = \frac{5.22}{\sqrt{5}} = 2.33$

At 95% confidence, the interval for $\mu_1 - \mu_2 = 0$ will be

$$\bar{y}_1 - \bar{y}_2 - t_{\frac{\alpha}{2}, dof} \cdot S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + t_{\frac{\alpha}{2}, dof} \cdot S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$\Rightarrow -1.8 - 2.101 \times 2.33 \le \mu_1 - \mu_2 \le -1.8 + 2.101 \times 2.33$

$\Rightarrow -6.695 \le \mu_1 - \mu_2 \le 3.095$

$\Rightarrow -6.70 \le \mu_1 - \mu_2 \le 3.10$

$\Rightarrow \boxed{\mu_1 - \mu_2 \in [-6.70, 3.10]}$ (Ans)

c) $F_0 = \frac{S_1^2}{S_2^2} = \frac{24.71}{29.82} = 0.8286$

$F_{\frac{\alpha}{2}, n_1 - 1, n_2 - 1} = F_{0.025, 9, 9} = 4.03$

$F_{1-\frac{\alpha}{2}, n_1 - 1, n_2 - 1} = F_{0.975, 9, 9} = 0.25$

At 95% confidence, the interval for $\frac{\sigma_1^2}{\sigma_2^2}$ will be:

$$\frac{S_1^2}{S_2} F_{1-\frac{\alpha}{2}, n_1 - 1, n_2 - 1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2} F_{\frac{\alpha}{2}, n_1 - 1, n_2 - 1}$$

$\Rightarrow 0.8286 \times 0.25 \le \frac{\sigma_1^2}{\sigma_2^2} \le 0.8286 \times 4.03$

$\Rightarrow 0.207 \le \frac{\sigma_1^2}{\sigma_2^2} \le 3.34$

$\Rightarrow \boxed{\frac{\sigma_1^2}{\sigma_2^2} \in [0.207, 3.34]}$ (Ans)

Plotting the normal probability plot for saltwater and oil, we

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a)

x_1	x_2	x_3	x_4	$\bar{x}$	R
459	449	435	450	448.25	24
443	440	442	442	441.75	3
457	444	449	444	448.50	13
469	463	453	438	455.75	31
443	457	445	454	449.75	14
444	456	456	457	453.25	13
445	449	450	445	447.25	5
446	455	449	452	450.50	6
444	452	457	440	448.25	17
432	463	463	443	450.25	31
445	452	453	438	447.00	15
456	457	436	457	451.50	21
459	445	441	447	448.00	18
441	465	438	450	448.50	27
460	453	457	438	452.00	22
453	444	451	435	445.75	18
451	460	450	457	454.50	10
422	431	437	429	429.75	15
444	446	448	467	451.25	23
450	450	454	454	452.00	4

$$\bar{\bar{x}} = \frac{8973.75}{20} = 448.6875$$

$$\bar{R} = \frac{330}{20} = 16.50$$

From table, $n=4$

$A_2 = 0.729$, $D_3 = 0$, $D_4 = 2.282$

$$UCL = \bar{\bar{X}} + A_2\bar{R} = 448.6875 + 0.729 \times 16.50 = 460.716$$

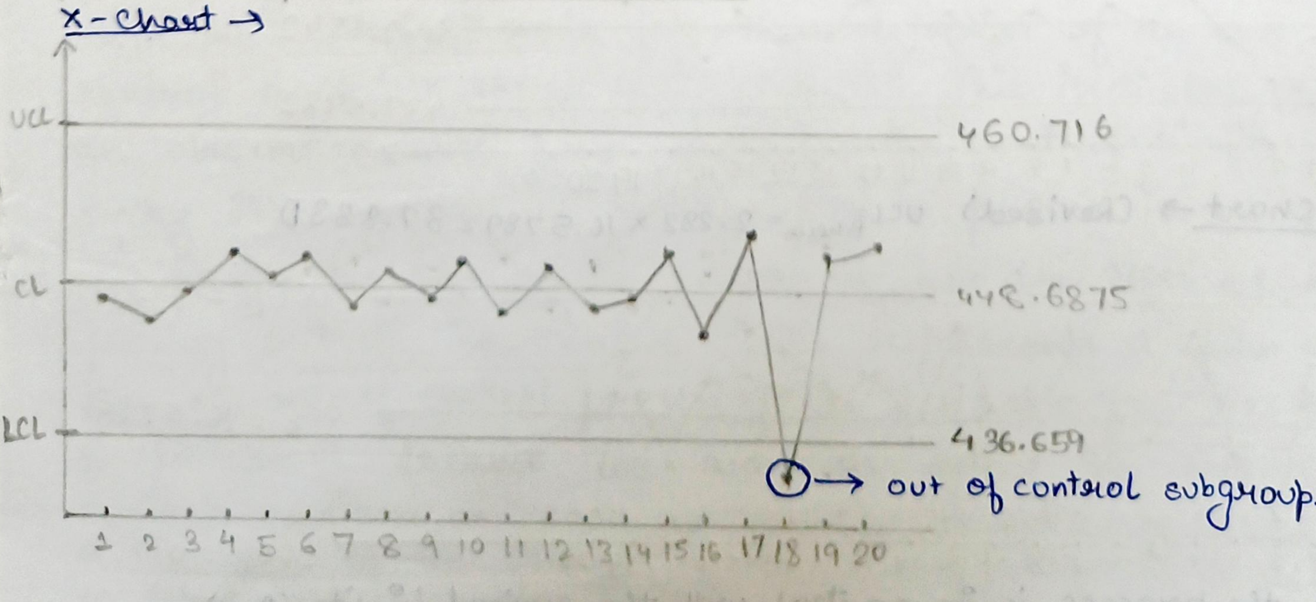
$$LCL = \bar{\bar{X}} - A_2\bar{R} = 448.6875 - 0.729 \times 16.50 = 436.659$$

$$CL = 448.6875 = \bar{\bar{X}}$$

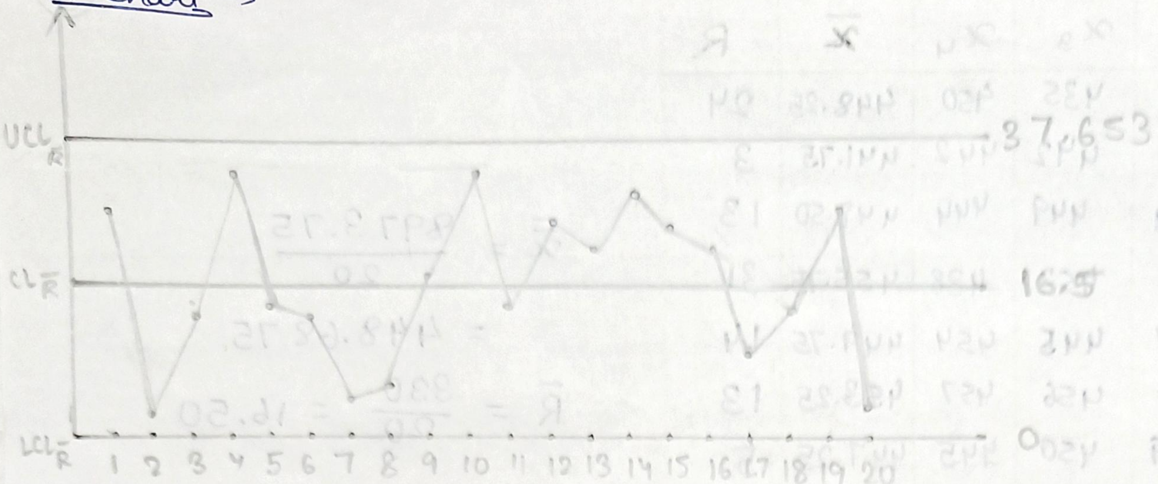
$$UCL_{\bar{R}} = D_4\bar{R} = 2.282 \times 16.50 = 37.653$$

$$LCL_{\bar{R}} = D_3\bar{R} = 0$$

X-Chart →



R-Chart →



Eliminating Subgroup 18 as it is out of control subgroup.
The process is not in control.

$$\bar{\bar{X}}_{new} = \frac{8973.75 - 429.75}{19} = \frac{8544}{19} = 449.6842$$

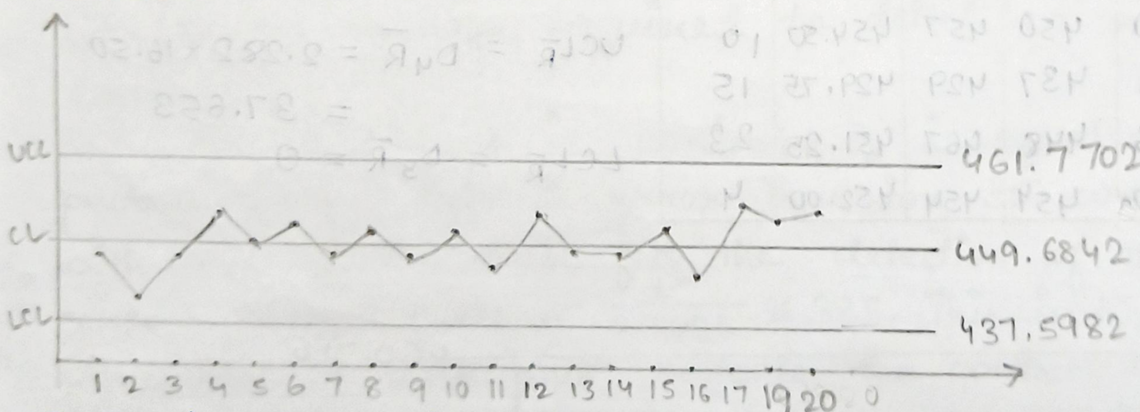
$$\bar{R}_{new} = \frac{330 - 15}{19} = \frac{315}{19} = 16.5789$$

$$UCL_{\bar{X}_{new}} = 449.6842 + 0.729 \times 16.5789 = 461.7702$$

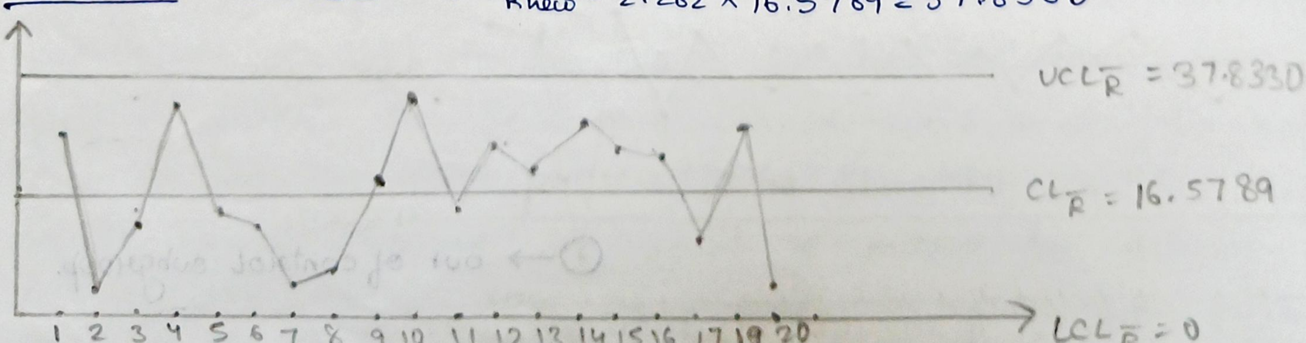
$$LCL_{\bar{X}_{new}} = 449.6842 - 0.729 \times 16.5789 = 437.5982$$

$$CL_{\bar{X}_{new}} = \bar{\bar{X}}_{new} = 449.6842$$

X-Chart → (Revised)



R-Chart → (Revised) $UCL_{\bar{R}_{new}} = 2.282 \times 16.5789 = 37.8330$



Now, the process is in control with the revised limits. (Ans)

b) From table, $n = 4$, $d_2 = 2.059$

$$\delta = \frac{\bar{R}}{d_2} = \frac{16.5}{2.059} = 8.0136$$

we are given, $USL = 480$, $LSL = 420$, $SL = 450$

$$C_p = \frac{USL - LSL}{6\delta} = \frac{60}{6 \times 8.0136} = 1.2479$$

$$C_{PK} = \min\left\{\frac{USL - \bar{\bar{X}}}{3\delta}, \frac{\bar{\bar{X}} - LSL}{3\delta}\right\}$$

$$= \min\left\{\frac{480 - 448.6875}{3 \times 8.0136}, \frac{448.6875 - 420}{3 \times 8.0136}\right\}$$

$$= \min\left\{\frac{31.3125}{3 \times 8.0136}, \frac{28.6875}{3 \times 8.0136}\right\}$$

$$= \frac{28.6875}{3 \times 8.0136} = 1.1933$$

we observe that $C_{PK} < C_p$, hence we conclude that the process is $\boxed{\text{not capable}}$. (Ans)

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(3) ARL is the average run length which is an alternative measure of the performance of a control chart, in addition to the OC curve. This denotes the number of samples, on average, required to detect an out-of-control signal.

For a process in control, P_d is equal to α, the probability of a type I error. Thus, for 3δ control charts with the selected rule for the detection of an out-of-control condition, ARL is $\frac{1}{0.0026} \cong 385$. This indicates that an observation will plot outside the control limits every 385 samples, on average. For a process in control, we prefer the ARL to be large because an observation plotting outside the control limits represents a false alarm. For an out-of-control process, it is desirable for the ARL to be small because we want to detect the out-of-control condition as soon as possible. The values of ARL represent a measure of the strength of the control chart in its ability to detect process changes quickly.

(4) a) H_0: Given data follows a normal distribution

H_1: Given data do not follow a normal distribution

b) The data is as follows:

-0.50	-0.37	2.14	1.61	-1.61
0.93	0.05	+0.58	-1.86	-1.16
0.97	2.25	-2.52	1.05	0.96
1.58	-0.30	1.15	-1.23	-0.07
-0.54	3.55	2.14	0.61	-1.61
0.53	2.25	-0.38	1.86	-4.01
4.97	-3.47	-2.02	3.89	0.96
1.58	-0.47	1.15	-1.50	-0.07

$\mu = \frac{11.91}{40} = 0.30$, $\sigma = \sqrt{\frac{141.0543}{40}} = 1.88$

Highest = 4.97, Lowest = -4.01

Sl. no.	Class Intervals	f_o	f_e	$(f_o - f_e)^2$	$\frac{(f_o-f_e)^2}{f_e}$
1.	-4.01 - -2.214	3	3.164	0.026896	8.5006×10^{-3}
2.	-2.214 - -0.418	11	10.476	0.274576	26.21×10^{-3}
3.	-0.418 - 1.378	15	14.548	0.204304	14.0434×10^{-3}
4.	1.378 - 3.174	8	8.852	0.725904	82.0045×10^{-3}
5.	3.174 - 4.97	3	2.256	0.553536	245.3617×10^{-3}

For f_e,

1. $40\times P(-4.01 \le x \le -2.214) = P\left(\frac{-4.01-0.3}{1.88} \le z \le \frac{-2.214-0.3}{1.88}\right)\times 40$

$= 40\times P(-2.2926 \le z \le -1.3372) = [\Phi(-1.34) - \Phi(-2.29)]\times 40$

$= 40\times(0.0901 - 0.0110) = 40\times 0.0791 = 3.164$

2. $40\times P(-2.214 \le x \le -0.418) = P\left(\frac{-2.214-0.3}{1.88} \le z \le \frac{-0.418-0.3}{1.88}\right)\times 40$

$= 40\times P(-1.3372 \le z \le -0.3819) = [\Phi(-0.3819) - \Phi(-1.3372)]\times 40$

$= 40\times(0.3520 - 0.0901) = 40\times 0.2619 = 10.476$

3. $40\times P(-0.418 \le x \le 1.378) = P\left(\frac{-0.418-0.3}{1.88} \le z \le \frac{1.378-0.3}{1.88}\right)\times 40$

$= 40\times P(-0.3819 \le z \le 0.5734) = [\Phi(0.57) - \Phi(-0.38)]\times 40$

$= 40\times(\overset{-0.3520}{\cancel{0.9370}} + 0.7157) = 40\times 0.3637 = 14.548$

4. $40 \times P(1.378 \le x \le 3.174) = P\left(\frac{1.378-0.3}{1.88} \le z \le \frac{3.174-0.3}{1.88}\right) \times 40$

$= 40 \times P(0.5734 \le z \le 1.5287) = [\Phi(1.53) - \Phi(0.57)] \times 40$

$= 40 \times (0.9370 - 0.7157) = 40 \times 0.2213 = 8.852$

5. $40 \times P(3.174 \le x \le 4.97) = P\left(\frac{3.174-0.3}{1.88} \le z \le \frac{4.97-0.3}{1.88}\right) \times 40$

$= 40 \times P(1.5287 \le z \le 2.4840) = [\Phi(2.48) - \Phi(1.53)] \times 40$

$= 40 \times (0.9934 - 0.9370) = 40 \times 0.0564 = 2.256$

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} = 376.1202 \times 10^{-3} = \boxed{0.3761202} \text{ (Ans)}$$

c) $k = 5$, $c = 2$

$dof = k - c = 5 - 2 = 3$

$\alpha = 0.05$

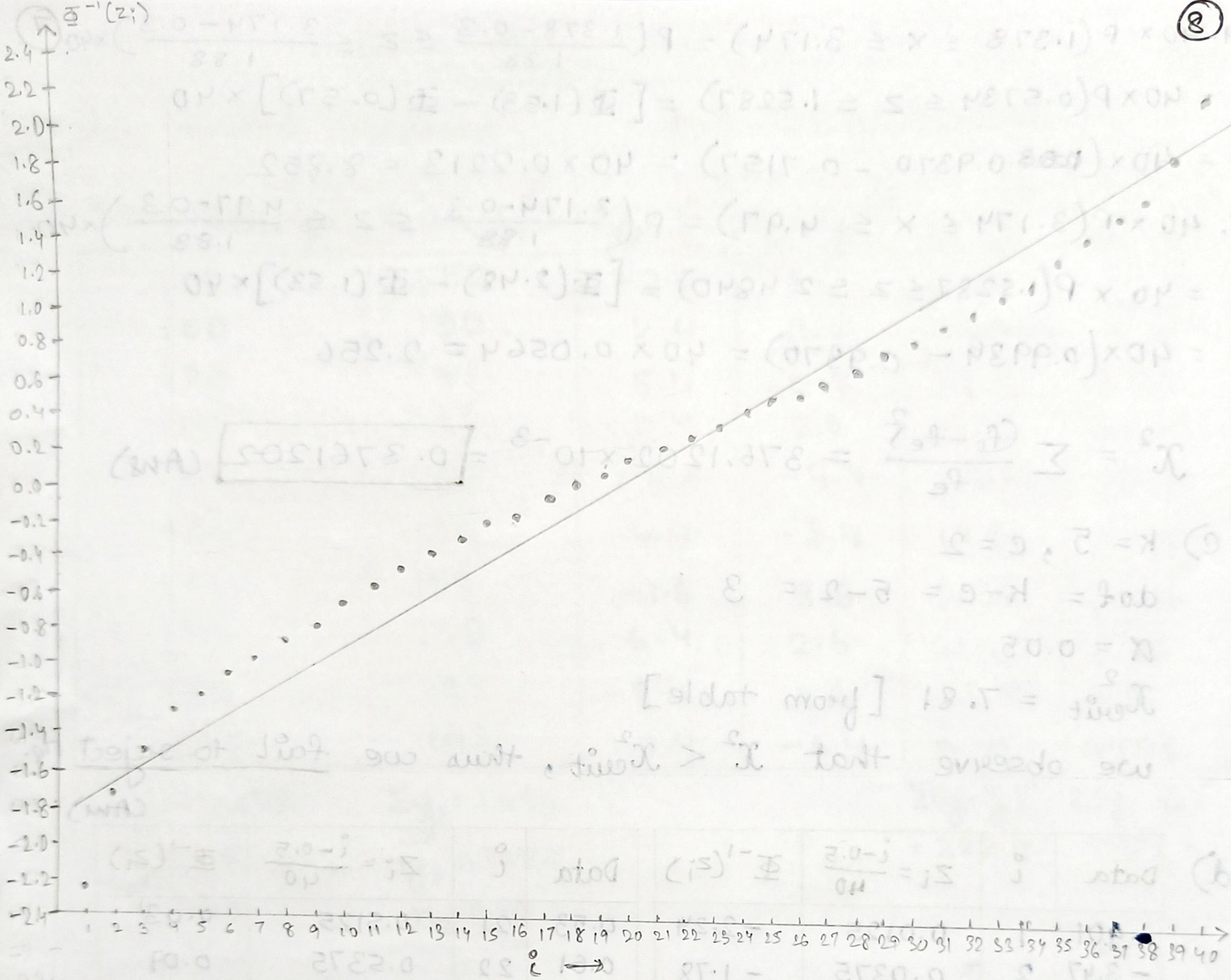
$\chi^2_{crit} = 7.81$ [from table]

we observe that $\chi^2 < \chi^2_{crit}$, thus we fail to reject H_0.

(Ans)

d)

Data	i	$z_i = \frac{i-0.5}{40}$	$\Phi^{-1}(z_i)$	Data	i	$z_i = \frac{i-0.5}{40}$	$\Phi^{-1}(z_i)$
-4.01	1	0.0125	-2.24	0.53	21	0.5125	0.03
-3.47	2	0.0375	-1.78	0.61	22	0.5375	0.09
-2.52	3	0.0625	-1.53	0.93	23	0.5625	0.16
-2.02	4	0.0875	-1.36	0.96	24	0.5875	0.22
-1.86	5	0.1125	-1.21	0.96	25	0.6125	0.29
-1.61	6	0.1375	-1.09	0.97	26	0.6375	0.35
-1.61	7	0.1625	-0.98	1.05	27	0.6625	0.42
-1.50	8	0.1875	-0.89	1.15	28	0.6875	0.49
-1.23	9	0.2125	-0.80	1.15	29	0.7125	0.56
-1.16	10	0.2375	-0.71	1.58	30	0.7375	0.64
-0.58	11	0.2625	-0.64	1.58	31	0.7625	0.71
-0.54	12	0.2875	-0.56	1.61	32	0.7875	0.80
-0.50	13	0.3125	-0.49	1.86	33	0.8125	0.89
-0.47	14	0.3375	-0.42	2.14	34	0.8375	0.98
-0.38	15	0.3625	-0.35	2.14	35	0.8625	1.09
-0.37	16	0.3875	-0.29	2.25	36	0.8875	1.21
-0.30	17	0.4125	-0.22	2.25	37	0.9125	1.36
-0.07	18	0.4375	-0.16	3.55	38	0.9375	1.53
-0.07	19	0.4625	-0.09	3.89	39	0.9625	1.78
0.05	20	0.4875	-0.03	4.97	40	0.9875	2.24



From the above graph, we observe that the points are almost forming a straight line. Hence, we can conclude that the data is normally distributed. (Ans)

Ritam Pradhan
20QE30002