

• Dimension's of Quality :-

Quality can be described in several ways. Garvin provides an excellent discussion of eight component's / dimension's of quality. They are as follows:-

1. **Performance** :- Customers usually evaluates a product based on how it will perform certain specific functions, and how well it performs.
2. **Reliability** :- (How often do a product fail?). Complex products require some repair in their service time/life. How often does it require a repair is the reliability of that product.
3. **Durability** :- (How long does it lasts?) The effective service life of a product. Customers obviously want's their product to perform satisfactorily over a long period of time.
4. **Serviceability** :- (How easy is it to repair?). Many industries, customers quality view is dependent on how quickly and economically a product's repair or maintenance is accomplished.
5. **Aesthetics** :- (What does product look like). It is the visual appeal of product. Some of the Aesthetic factors are colors, style, shape, packaging, etc.
6. **Features** :- Usually, customer need's high quality products that have added features- others than the basic one's. of the competitions.
7. **Perceived Quality** :- (What is reputat' of Company /product) In many cases, customers rely on past reputation of product / company. This reputat' is influenced by failures of product in past & how the customer is treated when quality-related problems are reported. Perceived Quality, Customer loyalty, and repeated business are inter-connected.
8. **Conformance to Standards** :- (Is product made exactly as designer intended?) We usually think highly-quality product as that exactly meets the requirements placed on it. An Automobile consists of several thousand parts. If each one is slightly too big/small, many components will not fit. A vehicle may not perform as designer intended.

In Service and Transactional business Organized, we add the following:-

1. **Responsiveness** :- How long a service provider takes to reply to your request for service and how ^{willing to} helpful was he?
2. **Professionalism** :- The knowledge and skill of service provider the competency to provide the required services.
3. **Attentiveness** :- How caring and personalized attention you get from service providers.

- **Traditional Definition of Quality** :- Based on the view-point that product and services must meet the requirements of those who use them. Quality means fitness for use.

These are two general aspects of fitness for use.

① **Quality of Design** :- All goods and services produced have various grades/level of quality. These variations are intentional and consequently appropriate technical term is Quality of Design.

② **Quality of Conformance** :- How well product conforms to specific required by design. Influenced by factors → Choice of manufacturing process; type of process control, test etc.

• **Modern Definition** :- Quality is inversely proportional to Variability. This implies as the variability increases the Quality of Product Decreases. It results to defining the quality Improvement.

• **Quality Improvement** :- It is the reduction of Variability of process and products.

② **Reduction of waste** :- particularly effective in Service Industries where there may not be as many things than can be directly measured. In service industry, by improving service process, the wasted effort & expense (for an error/mistake) can be avoided.

Non-Conforming:- those that fail to meet one or more of their specificⁿ

non-conformity:- Not necessarily unfit for use. Ex - A detergent with less conc. (use more amt.)

Prime Merit

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Date

• Management Aspects of Quality

Improvement
Assurance :-

① Quality Planning:- A strategic activity as vital as organization's long term business success plan, financial plan, marketing plan. Without this plan, enormous amount of time, money and effort will be wasted dealing with faulty designs, manufacturing defects, etc. It involves identifying customers, both external & those operate internal to business. & identifying their needs.

② Quality Assurance:- It is a set of activities that ensures quality levels of product and services are properly maintained & suppliers and customer's quality issues are properly resolved. Quality System documentation involves four components : ① policy ② procedure ③ Work instructions ④ Specifications and records

① Policy → deals with what to be done and why?

② Procedure → focus on methods & personnel that implement policy.

③ Work instructions → are usually product, department, tool / machine-oriented specifications.

④ Records → way of documenting policies, procedures & work ins. & spec. & also can be used to track specific unit, so that to determine how it was produced.

③ Quality Control and Improvement:- involves set of activities used to ensure that products & services meet requirements and are improved on a continuous basis. Since Variability is major issue, (Statistical techniques, design experiment are major tools) including SPC & tools of quality control and improvements.

- W. Edwards Deming's 14 points:-

He believed most of the opportunities for quality improvement require management action and very few opportunities lies at workforce or operator level. This philosophy is summarized in this 14 points:

1. Create a constancy of purpose focused on improvement of products & services.

Management should constantly try to improve product design and performance. This must include investment in research, development & innovation, which have long-term payback to organization.

2. Adopt new philosophy that recognizes we are in different economic era.

Reject defective products / bad service. It cost as much to produce defective units as it does to produce good ones (sometimes more). Cost to deal with scrap, rework created by defective is enormous drain of company resources.

3. Do not rely on mass inspection to "control" quality.

"Inspect" just costs out defectives, but does not save cost to produce them, and "Inspect" itself is expensive & occurs too late in process & is inefficient.

4. Do not award business to supplier's on the basis of price alone, but also consider quality.

Price is meaningful measure of supplier's product only if it is considered in relation with ^{measure of} quality. Preference should be given to suppliers who use modern methods of quality improv. in their business and demonstrate process control and capability. It is imp. to built effective, long term relationship with supplier.

5. Focus on Continuous improvement.

Constantly try to improve the product and services using statistical method's and tool's.

6. Practice Modern training method's and invest it on the job training for all employees.

Everyone should be trained in the technical aspects of their jobs and in modern quality and productivity method as well as encourage

7. Improve leadership and practice modern supervision methods.

Supervision should not only focus on surveillance of workers, but also to help the employee's improve the system in which they work. It should be to improve work system & product.

8. Drive out fear:-

Management should eliminate the fear of workers to ask question's, report problems & point out that barrier's to quality & effective product.

9. Break down the barrier's b/w functional area of business.

Teamwork among diff. organization units are essential for quality & productivity improvement to take place.

10. Eliminate target's, slogan, and numerical goals for Workforce.

A target such as "Zero defect" is useless without a plan for achievement.

Rather, work to improve the system and provide information on that.

11. Eliminate numerical quotas and work standard's.

These std's have historically been set without regard to quality. Work std. are often symptoms of management's inability to understand work process. and to provide an effective management system focused on improving process.

12. Remove the barrier's that discourage employee from doing their jobs.

Management must often listen to employee's suggestion, since he knows about job he's doing and may have valuable data to make process work more efficient.

13. Institute an ongoing program of education for all employees.

Education is simple, powerful statistical technique. Should be mandatory for all employees. As employees understand their uses, they will be more likely to look for ~~poor~~ cause of poor quality and to identify process improvement, SPC Problem solving tools and

14. Create a structure in top management that will vigorously advocate the first 13 points.

• Juran Trilogy:-

The Juran's quality management philosophy focuses on 3 components.

① Planning ② Control ③ Improvement. These are known as Juran's Trilogy.

• Armand V. Feigenbaum's Total Quality Control (TQC) :-

Inferences About Process Quality

→ Statistics and Sampling Distribution's:-

The objective of statistical inference is to draw conclusion or make decision about population based on a sample selected from population.

We define x_1, x_2, \dots, x_n as a random sample of size n if it is ~~oft~~ selected so that $\{x_i\}$ are independent and identically distributed.

$$\textcircled{1} \text{ The Sample Mean } = \frac{1}{n} \sum_{i=1}^n x_i = \bar{x}$$

$$\textcircled{2} \text{ Sample Variance } = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} = s^2$$

$$\textcircled{3} \text{ Sample Std. Dev } = \sqrt{s^2} = s$$

The probability Distribution of Sample Statistics / Estimators is known as Sampling Distribution

① Sampling from Normal Distribution :-

If mean is μ and variance is σ^2 of normally distributed random variables

x . If $x_1, x_2, x_3, \dots, x_n$ is a random sample of size n from this process, then.

\bar{x} be the sample mean, Now distribution of Sample mean \bar{x} is ~~N($\mu, \sigma^2/n$)~~

* Note:- From Central Limit theorem regardless of distribution of population, the distribution of random variable samples i.e. Sampling distribution of Sample Mean is approximately $\bar{x} \sim N(\mu, \sigma^2/n)$

• Chi-Square Distribution :-

IF $x_1, x_2, x_3, \dots, x_n$ are normally and independently distributed random variables

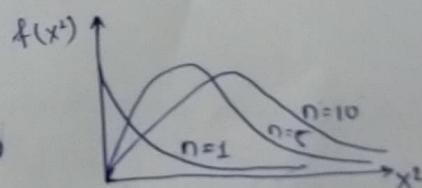
with mean zero and variance 1, then the random variable,

$$X_n^2 = x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2 \text{ is distributed } \chi^2 \text{ (Chi-Square)}$$

With. Degree of freedom n

$$f(x^2) = \frac{1}{2^{n/2} \Gamma(n/2)} (x^2)^{(n/2)-1} e^{-x^2/2}, x^2 > 0$$

Here, $\mu=n$ and $\sigma^2=2n$. The distribution is skewed.



If $x_1, x_2, x_3, \dots, x_n$ is a random sample from an $N(\mu, \sigma^2)$. Then random variable

$$\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{\sigma^2} \sim \chi_{n-1}^2 \text{ i.e. } \frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2 \quad x_{n-1} = \sqrt{n-1} \frac{S}{\sigma}$$

The sampling distribution of $(n-1)S^2/\sigma^2$ is χ_{n-1}^2 , if population is Normal distribution.

● t-Distribution:-

If \bar{x} and S^2 are independent (Std. Normal and Chi-square), then

$t_k = \frac{\bar{x}}{\sqrt{S^2/k}}$ is distributed t with k degree of freedom.

$$f(t) = \frac{\Gamma((k+1)/2)}{\sqrt{k\pi} \Gamma(k/2)} \times \left(\frac{t^2}{k}\right)^{-(k+1)/2} \quad -\infty < t < \infty$$

$$\mu=0; \sigma^2 = k/(k-2) \text{ for } k>2$$

If x_1, x_2, \dots, x_n is from $N(\mu, \sigma^2)$ distribution. \bar{x} and S^2 are its statistic.

$$\frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{(\bar{x} - \mu)/\sigma/\sqrt{n}}{S/\sigma} = \frac{N(0,1)}{\sqrt{X_{n-1}^2/(n-1)}} \sim t_{n-1} \quad [(n-1)S^2/\sigma^2 \sim \chi_{n-1}^2]$$

→ F-Distribution:-

If X_u^2 and X_v^2 are two independent Chi-square distribution with u and v. as d.f. then the ratio.

$$F_{u,v} = \frac{X_u^2/u}{X_v^2/v} \text{ is F-distributed.}$$

$$g(f) = \frac{\Gamma(u+v)}{\Gamma(u/2)\Gamma(v/2)} \left(\frac{u}{v}\right)^{u/2} \times \frac{f^{(u/2)-1}}{\left[\left(\frac{u}{v}\right)f + 1\right]^{(u+v)/2}} \quad 0 < f < \infty$$

For our purposes:-

$$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1, n_2-1}$$

$$F_{1-\alpha/2, n_1, n_2} = \frac{1}{F_{\alpha/2, n_2, n_1}}$$

Note:- Imp:-

① If σ is known, the sample distribution follows $N(\bar{x}, \sigma^2/n)$ or Z-test.

② If σ is unknown, the sample distribution follows $t_{n-1}(\bar{x}, S)$

② Sampling from Bernoulli Distribution:-

x_1, x_2, \dots, x_n is taken from Bernoulli process. $\sim (MP)$
 $x = x_1 + \dots + x_n \rightarrow$ has parameters n and p

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \in \{0, 1/n, 2/n, \dots, 1\}$$

$$\therefore P(\bar{x} \leq a) = P(x \leq an) = \sum_{k=0}^{[an]} \binom{n}{k} p^k (1-p)^{n-k}$$

$$\text{Here:- } \mu_{\bar{x}} = p ; \sigma_{\bar{x}}^2 = \frac{p(1-p)}{n}$$

③ Sampling from a Poisson Distribution:-

if x_1, \dots, x_n are taken from Poisson distribution with parameter λ ,
then, $x = x_1 + x_2 + \dots + x_n$ is also Poisson with parameter $n\lambda$.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \in \{0, 1/n, 2/n, \dots\}$$

$$P(\bar{x} \leq a) = P(x \leq an) = \sum_{k=0}^{[an]} \frac{e^{-n\lambda} (n\lambda)^k}{k!}$$

$$\text{Here:- } \bar{x} = \lambda, \sigma_{\bar{x}}^2 = \frac{\lambda}{n}$$

• Estimation of Process Parameters:-

① Point Estimators:-

Two most important properties required for a good point estimator are:-

- ① Point estimator should be unbiased. Expected value of point parameter.
Should be parameter being estimated.

- ② Point estimator should have minimum Variance.

\bar{x} and s^2 are unbiased estimators of population mean and variance.

but, s is not an unbiased estimator for population std. Dev.

Ex:-

- (A) Poisson's Process:- $\mu = \lambda$ and $\sigma^2 = \lambda$

$$\hat{\lambda} = \frac{1}{n} \sum_{i=1}^n x_i = \bar{x} \rightarrow \text{point estimator of } \lambda.$$

- (B) Binomial Distribution:- $\mu = np$ and $\sigma^2 = np(1-p)$

$$\hat{P} = \frac{1}{n} \sum_{i=1}^n x_i = \bar{x} \rightarrow \text{point estimator of } P$$

② Interval Estimators:-

$$P(L \leq \mu \leq U) = 1 - \alpha \quad \text{confidence coefficient}$$

The resulting interval $L \leq \mu \leq U$ is called $100(1-\alpha)\%$ confidence interval for unknown mean μ .

$$P(L \leq \mu) = 1 - \alpha \quad \text{and} \quad P(U \geq \mu) = 1 - \alpha$$

One sided lower confi. Limit One sided Upper Confidence Limit.

→ Confidence Intervals For Various Cases:-

① ON Mean for known Variance:- (Z-distribn)

- a) $\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ (for $100(1-\alpha)\%$ two-sided interval on μ)
- b) $\mu \leq \bar{x} + \frac{Z_{\alpha} \sigma}{\sqrt{n}}$ → For upper confidence - One sided.
- c) $\mu \geq \bar{x} - \frac{Z_{\alpha} \sigma}{\sqrt{n}}$ → For Lower confidence - One sided.

② ON Mean of Normal Distbⁿ with Unknown Variance : (t-distribution)

- a) $\bar{x} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$ → Two Sided Test
- b) $\mu \leq \bar{x} + t_{\alpha, n-1} \frac{s}{\sqrt{n}}$ → upper confidence.
- c) $\mu \geq \bar{x} - t_{\alpha, n-1} \frac{s}{\sqrt{n}}$ → Lower Confidence

③ ON Variance of Normal Distribution: (χ^2 -distribution)

- a) $\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}$
- b) $\sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha, n-1}}$
- c) $\sigma^2 \geq \frac{(n-1)s^2}{\chi^2_{\alpha, n-1}}$

④ ON Difference in Two Means with Known Variance: (Z-distribn)

- a) $(\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
- b) $(\mu_1 - \mu_2) \leq (\bar{x}_1 - \bar{x}_2) + Z_{\alpha} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
- c) $(\mu_1 - \mu_2) \geq (\bar{x}_1 - \bar{x}_2) - Z_{\alpha} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

(5)

⑤ ON Difference b/w means with Unknown Variances (t-distⁿ)

① Case I :- Assume $\sigma_1^2 = \sigma_2^2 = \sigma^2$

Combined estimate of common variance

$$(\text{Pooled Estimate}) S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$$\textcircled{a} (\bar{x}_1 - \bar{x}_2) - t_{\alpha/2, n_1+n_2-2} \times S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, n_1+n_2-2} \times S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\textcircled{b} \quad \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + t_{\alpha, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\textcircled{c} \quad \mu_1 - \mu_2 \geq (\bar{x}_1 - \bar{x}_2) - t_{\alpha, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Case-II :- $\sigma_1^2 \neq \sigma_2^2$ (Behren's-fisher Problem)

$$\textcircled{a} \quad (\bar{x}_1 - \bar{x}_2) - t_{\alpha/2, \nu} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, \nu} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$\textcircled{b} \quad \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + t_{\alpha, \nu} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$\textcircled{c} \quad (\mu_1 - \mu_2) \geq (\bar{x}_1 - \bar{x}_2) - t_{\alpha, \nu} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

where,

$$U = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{(n_1-1)} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{(n_2-1)}}$$

⑥ ON Ratio of Variances of Two Normal Distributions (F-distributⁿ)

$$\textcircled{a} \quad \frac{S_1^2}{S_2^2} F_{1-\alpha/2, n_2-1, n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2}{S_2^2} F_{\alpha/2, n_2-1, n_1-1}$$

$$\textcircled{b} \quad \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2}{S_2^2} F_{\alpha/2, n_2-1, n_1-1}$$

$$\textcircled{c} \quad \frac{\sigma_1^2}{\sigma_2^2} \geq \frac{S_1^2}{S_2^2} F_{1-\alpha/2, n_2-1, n_1-1}$$

$$\frac{S_1^2}{S_2^2} \times \frac{1}{F_{\alpha/2, n_2-1, n_1-1}} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2}{S_2^2} \times \frac{f_{\alpha/2, n_2-1, n_1-1}}{f_{\alpha/2, n_2-1, n_1-1}}$$

$$F_{1-\alpha/2, n_2-1, n_1-1} = \frac{1}{F_{\alpha/2, n_1-1, n_2-1}}$$

⑦ ON Binomial Parameter's:- (Normal Distributⁿ)

① $\hat{P} - Z_{\alpha/2} \sqrt{\frac{\hat{P}(1-\hat{P})}{n}} \leq p \leq \hat{P} + Z_{\alpha/2} \sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$ @ (if n is large and $P \geq 0.1$, Normal distⁿ approximated)

- ② (a) if n is large and p is small, then Poisson Approximatⁿ is useful to construct interval for Binomial.
- (b) For small n Binomial table should be used.

⑧ ON Difference b/w two Binomial Proportion's:-

⑨ $(\hat{P}_1 - \hat{P}_2) - Z_{\alpha/2} \sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}} \leq (P_1 - P_2) \leq (\hat{P}_1 - \hat{P}_2) + Z_{\alpha/2} \sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}$

Same rule's as above for using distributⁿ.

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Hypothesis Testing :-

P-value :- Smallest level of significance that would lead to rejection of the null hypothesis. i.e. if P-value is less than (α) it^(H₀) would be rejected.

For Normal Distributⁿ =

$(\alpha > P \rightarrow \text{for reject of } H_0)$

$$P = \begin{cases} 2(1 - \Phi(|z|)) & \rightarrow \text{for two-tailed} \\ 1 - \Phi(z_0) & \rightarrow \text{for an upper-tailed test} \\ \Phi(z) & \rightarrow \text{for a lower-tailed test} \end{cases}$$

$H_0: \mu = \mu_0$	$H_1: \mu \neq \mu_0$
$H_0: \mu = \mu_0$	$H_1: \mu > \mu_0$
$H_0: \mu = \mu_0$	$H_1: \mu < \mu_0$