EWMA & CUSUM Charts

Prof. Sayak Roychowdhury

IIT Kharagpur

Department of Industrial & Systems Engineering

Limitations of Shewhart Charts

- In Shewhart chart, the plotted point represents information corresponding to observation only.
- It does not use information from previous observations.
- This makes Shewhart charts insensitive to small shifts.
- They are less useful in phase II.
- Warning limits and patterns can be useful, but they reduce the simplicity of the control charts

CUSUM Chart and EWMA Chart

- To detect small process shifts, CUSUM (Cumulative sum) and EWMA (Exponentially weighted moving average) charts are used as alternatives to Shewhart Control Chart
- These are good alternatives for phase II process monitoring
- Sometimes process needs to be monitored when sample size n =1, both CUSUM and EWMA charts work well in this situation.
- EWMA charts are particularly robust against non-normality (read section Montgomery 9.2.3).

CUSUM Chart

- First proposed by Page (1954)
- The CUSUM chart plots the quantity

$$C_i = \sum_{j=1}^{i} (\bar{x}_j - \mu_0)$$

Where $\bar{x}_j$ is the average of the j^{th} sample

μ_0 is the target for process mean

Also applicable for $n = 1$

- So CUSUM charts are particularly useful in chemical and process industries and discrete part manufacturing, where frequently subgroup size is 1.

CUSUM Chart (Ch 9.1 Montgomery)

- There are 2 ways to represent CUSUM charts, tabular method and V-mask method. We will discuss tabular method.
- $C_i^+ = \max(0, x_i - (\mu_0 + K) + C_{i-1}^+)$ (Upper CUSUM)
- $C_i^- = \max(0, (\mu_0 - K) - x_i + C_{i-1}^-)$ (Lower CUSUM)
- $C_0^+ = C_0^- = 0$
- $C_i^+ > H$ or $C_i^- > H$ indicate the process mean has shifted
- K is called reference value, allowance or slack value, H is called decision interval

CUSUM Chart

- $K = k\sigma$, typically halfway between μ_0 and out of control mean that we want to detect
- $K = \frac{|\mu_1 - \mu_0|}{2} = k\sigma$
- $H = h\sigma$
- K and H are chosen to provide good ARL performance
- Generally $k = 0.5$, and $h = 4\ or\ 5$ are chosen. Read (9.1.3)

CUSUM Chart Parameter Values and ARL

■ **TABLE 9.3**

ARL Performance of the Tabular Cusum with $k = \frac{1}{2}$ and $h = 4$ or $h = 5$

Shift in Mean (multiple of σ)	$h = 4$	$h = 5$
0	168	465
0.25	74.2	139
0.50	26.6	38.0
0.75	13.3	17.0
1.00	8.38	10.4
1.50	4.75	5.75
2.00	3.34	4.01
2.50	2.62	3.11
3.00	2.19	2.57
4.00	1.71	2.01

■ **TABLE 9.4**

Values of k and the Corresponding Values of h That Give $ARL_0 = 370$ for the Two-Sided Tabular Cusum [from Hawkins (1993a)]

k	0.25	0.5	0.75	1.0	1.25	1.5
h	8.01	4.77	3.34	2.52	1.99	1.61

EWMA chart (ch 9.2 Montgomery)

- Control chart to detect small shift in the process, ideally used with individual observations.
- The exponentially weighted moving average is defined as

$$z_i = \lambda x_i + (1-\lambda) z_{i-1} \ \ (z_0 = \mu_0\ (target)\ \ or\ \bar{\bar{X}}\ )$$

- λ should be between 0.05 , 0.25 (use smaller λ for smaller shifts)
- Limits
  - $UCL = \bar{\bar{X}} + L\sigma\sqrt{\lambda/(2-\lambda)[1-(1-\lambda)^{2i}]}$
  - $CL = \bar{\bar{X}}$
  - $LCL = \bar{\bar{X}} - L\sigma\sqrt{\lambda/(2-\lambda)[1-(1-\lambda)^{2i}]}$
  - Use μ_0 (target mean) in place of $\bar{\bar{X}}$ if given

- Usually L is taken to be 3. For a steady state process $\sqrt{\lambda/(2-\lambda)[1-(1-\lambda)^{2i}]}$ becomes $\sqrt{\lambda/(2-\lambda)}$
- σ can be estimated by process standard deviation, or $\bar{R}/d_2$ if $\bar{R}$ can be obtained. Sometimes process history is used for estimation.
- EWMA is often used with Shewhart Chart, so that the combined chart can detect small shifts and large shifts quickly enough.
- Minitab > Stat> Control charts > Time weighted charts> EWMA

EWMA Parameters and ARL

■ TABLE 9.11

Average Run Lengths for Several EWMA Control Schemes [Adapted from Lucas and Saccucci (1990)]

Shift in Mean (multiple of σ)	$L = 3.054$ $\lambda = 0.40$	2.998 0.25	2.962 0.20	2.814 0.10	2.615 0.05
0	500	500	500	500	500
0.25	224	170	150	106	84.1
0.50	71.2	48.2	41.8	31.3	28.8
0.75	28.4	20.1	18.2	15.9	16.4
1.00	14.3	11.1	10.5	10.3	11.4
1.50	5.9	5.5	5.5	6.1	7.1
2.00	3.5	3.6	3.7	4.4	5.2
2.50	2.5	2.7	2.9	3.4	4.2
3.00	2.0	2.3	2.4	2.9	3.5
4.00	1.4	1.7	1.9	2.2	2.7

- EWMA chart is fairly robust against non-normal distribution, compared to Shewhart charts for individual measurement (sec 9.2.3 Montgomery)

IIT Kharagpur | Department of Industrial & Systems Engineering