Class Test 1
Quality Control and Design (IM31005)

Full marks: 50 **Duration: 1 hour**

Q. 1) The inside diameters of bearings used in an aircraft landing gear assembly are known to have a standard deviation of σ=0.002cm. A random sample of 15 bearings has an average inside diameter of 8.2535 cm.

(a) Test the hypothesis (the most appropriate test) that the mean inside bearing diameter is 8.25 cm. Use a two-sided alternative. (5)

Hint. Perform 1 sample z test; $z_0 = \frac{8.2535-8.25}{\frac{0.002}{\sqrt{15}}} = 6.78$;

$z_{crit} = Z_{0.975} = 1.96$

(b) Give a range for the *P*-value for this test. (3)

Ans. P-value <0.05, (almost 0 to be precise)

(c) Construct a 95% two-sided confidence interval on the mean bearing diameter. (2)

Q. 2) Two quality control technicians measured the surface finish of a metal part, obtaining the data in Table below. Assume that the measurements are normally distributed.

Technician 1	Technician 2
1.45	1.54
1.37	1.41
1.21	1.56
1.54	1.37
1.48	1.20
1.29	1.31
1.34	1.27
	1.35

(a) Test the hypothesis that the mean surface finish measurements made by the two technicians are equal. Use α=0.05 and assume equal variances. (5)

Hint. Perform 2 sample t test;

(b) What are the practical implications of the test in part (a)? Discuss what practical conclusions you would draw if the null hypothesis were rejected. (2)

Hint. If Null Hyp is rejected, that means average measurement by the two technicians are significantly different. This indicates one of the key contributors to the difference in measurement of the surface finish is the person doing the measurement.

(c) Assuming that the variances are equal, construct a 95% confidence interval on the mean difference in surface-finish measurements. (3)

Q. 3) A high-voltage power supply should have a nominal output voltage of 350 V. A sample of four units is selected each day and tested for process-control purposes. The data shown in below Table give the difference between the observed reading on each unit and the nominal voltage times ten; that is, *xi* = (observed voltage on unit *i* - 350) * 10

(a) Set up $\bar{X}$ and *R* charts on this process. Is the process in statistical control (considering the patterns and rules as well)? (10)

Sample number	x_1	x_2	x_3	x_4	Xbar	R
1	6	9	10	15	10	9
2	10	4	6	11	7.75	7
3	7	8	10	5	7.5	5

4	8	9	6	13	9	7
5	9	10	7	13	9.75	6
6	12	11	10	10	10.75	2
7	16	10	8	9	10.75	8
8	7	5	10	4	6.5	6
9	9	7	8	12	9	5
10	15	16	10	13	13.5	6
11	8	12	14	16	12.5	8
12	6	13	9	11	9.75	7
13	16	9	13	15	13.25	7
14	7	13	10	12	10.5	6
15	11	7	10	16	11	9
16	15	10	11	14	12.5	5
17	9	8	12	10	9.75	4
18	15	7	10	11	10.75	8
19	8	6	9	12	8.75	6
20	13	14	11	15	13.25	4

Hint : $\bar{\bar{X}} = 10.325,\ \bar{R} = 6.25,\ UCL_{\bar{X}}, LCL_{\bar{X}} = \bar{\bar{X}} \pm A_2\bar{R} = 10.325 \pm 0.729 * 6.25;$

$$UCL_{\bar{R}}, LCL_{\bar{R}} = D_4\bar{R}, D_3\bar{R} = 2.282 * 6.25, 0$$

$$\widehat{\sigma_x} = \frac{\bar{R}}{d_2} = \frac{6.25}{2.059} = 3.035$$

(b) If specifications are at 350 V ± 5 V, what can you say about process capability y? Calculate C_p, C_{pk} and C_{pm} indices (assuming T =350V). (Hint: Consider calculation of $mean$ and $s.d.$ for the observed voltage, from the mean and s.d. of x). Comment on the process capability. (5)

Hint: Observed Voltage $Z_i = \frac{X_i}{10} + 350;\ \ \bar{Z} = \frac{\bar{\bar{X}}}{10} + 350 = 351.032,\ \hat{\sigma}_z = \frac{\hat{\sigma}_x}{10} = 0.03$

Calculate $\widehat{C_p}, \hat{C}_{pk}$ and $\hat{C}_{pm}$ using $\bar{Z}$ and $\hat{\sigma}_z$

Q. 4) A process is being controlled with a fraction nonconforming control chart. The process average has been shown to be 0.07. Three-sigma control limits are used, and the procedure calls for taking daily samples of 100 items.

(a) Calculate the upper and lower control limits. (5)

Hint: $UCL_p, LCL_p = \bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.147, 0$

(b) If the process average should suddenly shift to 0.10, what is the probability that the shift would be detected on the first subsequent sample? (4)

Hint: Prob = $1 - P(LCL \le p \le UCL|\ \bar{p} = 0.10) =$

$$1 - \{P(D \le nUCL|\bar{p} = 0.10) - P(D \le nLCL|\bar{p} = 0.10)\}$$

$$= 1 - P(D \le 14.65|\lambda = 10) = 1 - 0.9165 = 0.0835$$

Q. 5) What is the difference between quality control and quality assurance? Write two disadvantages of Shewhart Control Charts. What is rational subgrouping. Give an example. (6)