

Network AnalysisTopics:

Basic definitions

Types of elements

Nodal Analysis

Mesh Analysis

Superposition Theorem

1) Definitions: P, V, I, Emf2) Basic elements: L, C, R

3) KCL, KVL

8) Mesh Analysis

4) I div, V division

9) Types of elements

5) Network terminology

10) Types of sources

6) Nodal Analysis

7) Series & parallel Connection

Voltage:-

Atoms have two charges +ve & -ve there is a need of some amount of energy to separate both the charges, and make the particle smooth to certain distance. So, there will be certain amount of potential energy between both the charges which makes them separated.

Potential Difference:

The difference in PE in charge is called potential difference. And in electrical terminology it is

Known as Voltage

→ The units of voltage or PD is volts (V)

$$V = \frac{W}{Q}$$

W - Joules

Q - Coulombs

Current:

The free electrons in conductive & semi-conductive materials move in free in different directions. And these free electrons are applied with some amount of voltage. Then they try to move in one direction depending upon the polarity of applied voltage. This movement to the other end constitute an electric current.

a flow of electricity which results from the ordered directional movement of electrically charged particles:

- It is denoted by I .
- Units are Ampere

$$I = Q/t$$

- Current always flows from higher to lower potential.

Power

13/07/2023

It is rate of change of energy or capacity of doing work

$$P = \frac{dW}{dt} \text{ or } P = V \times I$$

- Units of power is ~~Gauss~~ "Watts."

Ohm's Law

Resistance: The property of the material, to restrict the flow of electrons. It is denoted by R . Units are Ω (ohms).

Resistor: $I \propto V/R$

$$V = I \times R$$

- The electrons move through the material and collide with atoms. Because of this some energy is lost in the form of heat.
- The power absorbed by the resistor is given by $P = VI$

$$P = I^2 R$$

- The energy loss in resistor $\Rightarrow W = \frac{V^2}{R}$

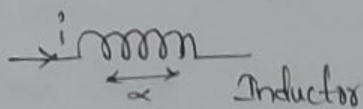
Inductance (L)

When a wire is twisted into a coil it becomes an inductor.

- When I is passed through it then emf is generated
- L stores energy in the form of electromagnetic field.
- If I increases then emf also increases.

→ The current and voltage equations for an inductor.

henry



$$V = L \frac{di}{dt}$$

$$I(t) = \int_0^t \frac{1}{L} V dt \quad [V = P = V \times I]$$

$$V dt = L di$$

$$\int_0^t \frac{1}{L} V dt = \int_{i(0)}^{i(t)} di$$

→ power absorb by the inductor. $P = V \times I$

$$P = \left(L \frac{di}{dt} \right) \times i$$

$$P = L i \frac{di}{dt}$$

→ Energy accepted by inductor

$$W = \int P dt = \int_0^t \frac{di}{dt} \times i = \frac{1}{2} Li^2 = W$$

$$W = \frac{1}{2} Li^2$$

→ It will not allow sudden changes in current (I).

Capacitance:

→ when two conducting surfaces are separated by dielectric or insulating medium then it gives the property of capacitor.

→ The Capacitor will store energy in the \rightarrow (1) \leftarrow (1) \leftarrow dielectric material

→ It is denoted by C.

Units: Farad

$$i = C \frac{dv}{dt}$$

$$i dt = C dv$$

$$\int_0^t i dt = \int_{v(0)}^{v(t)} C dv$$

$$dv = \int_0^t \frac{1}{C} dt$$

$$P = V \times i$$

$$= V \times C \frac{dv}{dt}$$

$$= VC \frac{dv}{dt}$$

$$W = \int P dt = \frac{1}{2} CV^2$$

→ Energy accepted by Capacitor

Ohm's Law:

The current flowing in a circuit is directly proportional to P which occurs in the circuit and inversely proportional to the resistance of the circuit provided a temperature remains constant.

$$I \propto \frac{1}{R}$$

$$I = \frac{V}{R} \text{ (Amperes)}$$

$$V = I \times R \text{ (Volts)}$$

→ These Law gives relation b/w P and current (I) and resistance in a AC Circuit.

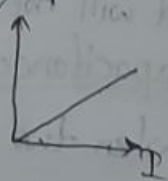
Limitations:

- * It is not applicable to non-linear devices.
- * It does not hold good for non-metallic devices such as Silicon, Carbide.

Charge:

Some electrons are very loosely bonded to their nucleus. If some electrons are removed from an atom then it becomes positively charge. If electrons are added then it becomes negative charge.

→ Total efficiency or addition of excess electrons into an atom



is called charge.

Units: coulomb

$$1 \text{ electron charge} = 1.6 \times 10^{-19} \text{ Coulombs}$$
$$= 1.602 \times 10^{-19} \text{ Coulombs}$$

$$1 \text{ coulomb} = \frac{1}{1.602 \times 10^{-19}} = 6.24 \times 10^{18} \text{ electrons.}$$

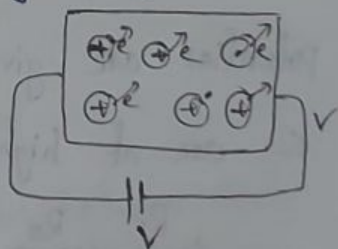
Electromotive force:

As it is known that the flow of electrons will be in a free form in a conductor there is a need of external force which should be applied to move electrons in uniform direction.

→ When some external electric force is applied to a conductor then electrons are free to move in a particular direction.
→ The direction of electrons is dependent on applied electrical force.

Def: The electrical effort required to drift free electrons in a particular direction in a conductor is called electromotive force.

→ In metals, charged particles are present when external electric force is applied.
→ A negatively charged particles gets attracted towards positive of cell in this way the electrons get aligned in one particular direction.



23/07/2023

* Resistance (R)

Inductance (L)

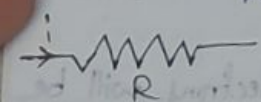
Capacitance (C)

$$\text{Voltage (V)} = IR$$

$$\text{Current (I)} = V/R$$

$$\text{Power (P)} = I^2 R$$

$$\text{Work (W)} = \frac{V^2}{R}$$



* Units: ohms (Ω)

* Energy released as heat.

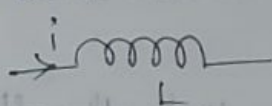
Inductance (L)

$$\text{Voltage} = L \frac{di}{dt}$$

$$\text{Current} = \frac{Vt}{L} \int \frac{1}{L} \int V dt$$

$$\text{Power} = Li \frac{di}{dt}$$

$$\text{Work} = \frac{1}{2} Li^2$$



* Units: Henry

* Energy stored as Electromotive field (E.M.F)

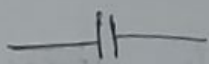
Capacitance (C)

$$V = \frac{q}{C} \text{ } \cancel{C} = \frac{q}{V}$$

$$i = C \frac{dv}{dt}$$

$$P = CV \frac{dv}{dt}$$

$$W = \frac{1}{2} CV^2$$



* Units: Farad

* Energy stored as electrostatic field (E.S.F)

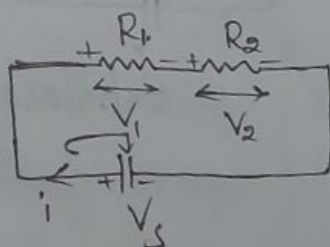
Kirchoff's Law: There are two types of Kirchoff's Law they are

1) KCL - Kirchoff's Current Law

2) KVL -

1) KVL: It states that algebraic sum of voltages around a closed path in a circuit is always zero.

* Current leaves positive terminal and enter into negative terminal. As it current passess their will be voltage law in the circuit, so sum of voltage drops around the loop should be equal to total voltage in that loop. Are polarities are given in order to mention voltages at a c are at higher potential than at b & d.



$$V_1 = I * R_1$$

$$V_2 = I * R_2$$

$$V_s = V_1 + V_2$$

$$V_s = IR_1 + IR_2 = I(R_1 + R_2)$$

$$I = \frac{V_s}{R_1 + R_2}$$

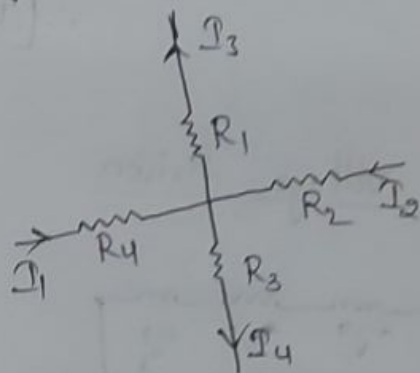
2) KCL:- It states that algebraic sum of current entering into a point is equal to sum of current leaving that point. (or)
 * The algebraic sum of current meeting at a point is equal to zero.

$$I_1 + I_2 - I_3 - I_4 = 0$$

(or)

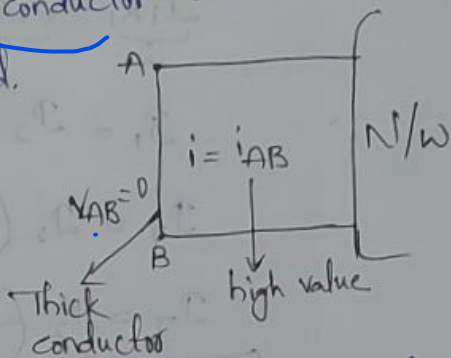
$$\textcircled{1} = I_1 + I_2$$

$$\textcircled{2} = I_3 + I_4$$



Short Circuit:-

If two points in a network are joined directly are with a thick conductor then the two points are said to be short circuited.



$$R = 0$$

$$V_{AB} = I_{AB} \times R_{AB}$$

$$V_{AB} = I_{AB} \times 0$$

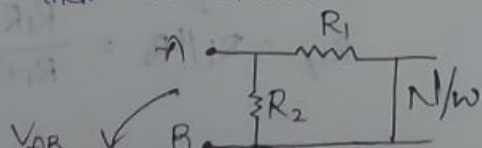
$$\therefore \boxed{V_{AB} = 0}$$

→ From the above, derivation according to ohm's law the equations have been solved for calculating voltage in a short-circuit network.

→ In a short-circuit network the resistance is considered to be equal to zero theoretically and current (I) flowing in the circuit is taken as I_{AB} & voltage flowing in the circuit is taken as V_{AB} .

Open Circuit:-

If two points have no connection between them then it has only some amount of voltage present across two points then the circuit is called open circuit.



$$I_{AB} = 0$$

$$R_{AB} = \infty$$

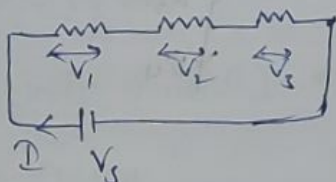
$$I_{AB} = \frac{V_{AB}}{R_{op}}$$

$$I_{AB} = \frac{V_{AB}}{\infty}$$

$$\therefore \boxed{I_{AB} = 0}$$

25/07/2023

Voltage division



Apply KVL

$$V = IR$$

$$V_1 = I_1 R_1$$

$$V_2 = I_2 R_2$$

$$V_3 = I_3 R_3$$

$$V_s = V_1 + V_2 + V_3$$

$$V_s = I(R_1 + R_2 + R_3)$$

$$I = \frac{V_s}{R_1 + R_2 + R_3}$$

$$V_{R1} = IR_1$$

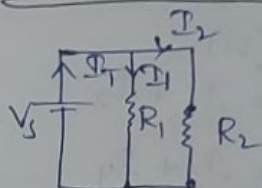
$$V_{R1} = \left(\frac{V_s}{R_1 + R_2 + R_3} \right) R_1$$

$$= V_s \left(\frac{R_1}{R_1 + R_2 + R_3} \right)$$

$$V_{R2} = V_s \left(\frac{R_2}{R_1 + R_2 + R_3} \right)$$

$$V_{R3} = V_s \left(\frac{R_3}{R_1 + R_2 + R_3} \right)$$

Current division



$$I_1 \rightarrow R_1$$

$$I_2 \rightarrow R_2$$

$$\boxed{I_T = I_1 + I_2} \quad \text{--- (1)}$$

$$\frac{I_1 R_1}{\downarrow} = \frac{I_2 R_2}{\downarrow}$$

$$I_1 = I_2 \left(\frac{R_2}{R_1} \right)$$

$$I_T = I_2 \left(\frac{R_2}{R_1} \right) + I_2$$

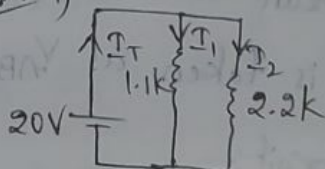
$$I_T = I_2 \left(\frac{R_2 + R_1}{R_1} \right)$$

$$\boxed{I_2 = I_T \left(\frac{R_1}{R_1 + R_2} \right)} \quad \text{--- (2)}$$

$$I_T = I_1 + I_2$$

$$\boxed{I_1 = I_T \left(\frac{R_2}{R_1 + R_2} \right)} \quad \text{--- (3)}$$

Ex: 1)



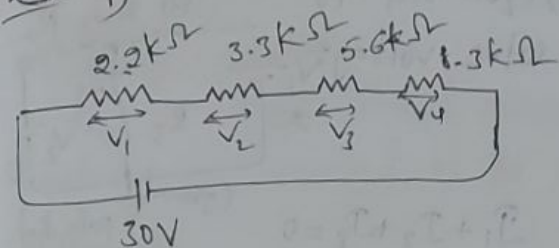
$$I_1 = I_T \left(\frac{2.2}{2.2 + 1.1} \right)$$

$$I_T = V/R_{eq}$$

$$R_{eq} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

→ so voltage drop across any resistance is equal to the ratio of same resistance to the sum of total resistance into supply voltage.

Ex: 1)



$$V_1 = V_T \left(\frac{2.2}{2.2 + 3.3 + 5.6 + 8.3} \right)$$

$$= 30 \left(\frac{2.2}{19.4} \right)$$

$$V_1 = 3.402V$$

$$V_2 = \left(\frac{3.3}{19.4} \right) 30$$

$$= 5.103V$$

$$V_3 = \left(\frac{5.6}{19.4} \right) 30 = 8.69V$$

$$V_4 = \left(\frac{8.3}{19.4} \right) 30 = 12.8V$$

$$V_s = V_1 + V_2 + V_3 + V_4$$

$$= 29.964 \approx 30V$$

$$I_T = \frac{20}{0.73} = 27.39A$$

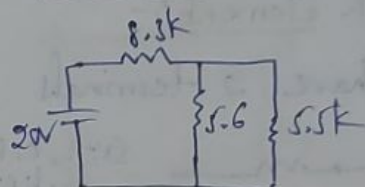
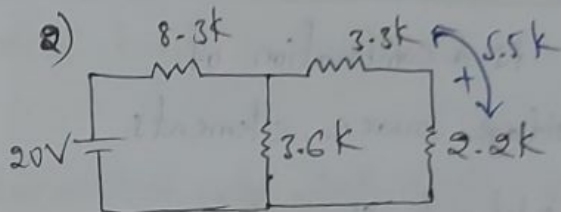
$$I_1 = 27.39 \left(\frac{2.2}{3.3} \right)$$

$$= 18.26A$$

$$I_2 = 27.39 \left(\frac{1.1}{3.3} \right)$$

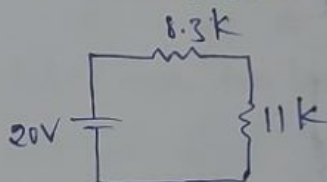
$$= 9.1A$$

$$I_T = I_1 + I_2$$



$$R_{eq} = (5.6 // 5.5)$$

$$= \frac{5.6 \times 5.5}{5.6 + 5.5} = 11\Omega$$



$$I_T = \frac{V}{R_{eq}} = \frac{20}{11} = 1.81mA$$

$$I_1 = I_T \left(\frac{5.5}{5.6 + 5.5} \right)$$

$$= 1.81 \left(\frac{5.5}{11.1} \right)$$

$$I_1 = 0.896mA$$

$$I_2 = 1.81 \left(\frac{5.6}{11.1} \right) = 0.913mA$$

$$I_T = I_1 + I_2$$

$$= 0.896 + 0.913$$

$$I_T = 1.809mA$$

Network Terminology

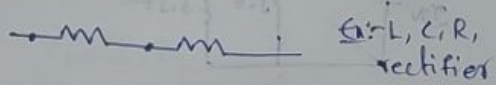
- 1) Electrical Network
- 2) Network Element
- 3) Branch
- 4) Node
- 5) Circuit (ckt)

1) Electrical Network:-

is a combination of voltage source, elements.

2) Network Element:-

should have 2 terminals



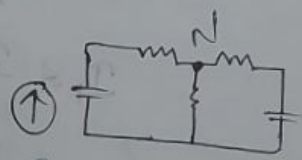
3) Branch:-

should have atleast one (1) element in it.



4) Node:-

two or more than branches are connected at a point is called a single Node.



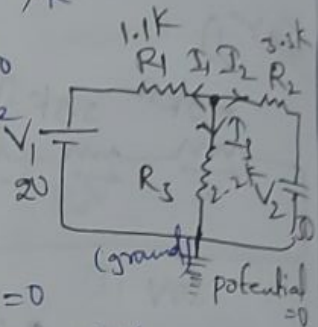
5) Circuit:-

Combination of above all terminologies.

Nodal Analysis

- 1) Identify Node
- 2) Denote current (I) directions
- 3) Apply KCL
 $I = V/R$

4) Equations to solve voltage V_1



$$I_1 + I_2 + I_3 = 0$$

$$I_1 = \frac{V}{R} = \frac{V - V_1}{R_1}$$

$$I_2 = \frac{V - V_2}{R_2}$$

$$I_3 = \frac{V - 0}{R_3}$$

$$\frac{V - V_1}{R_1} + \frac{V - V_2}{R_2} + \frac{V - 0}{R_3} = 0$$

Ex:-

$$\frac{V - 20}{1.1} + \frac{V - 30}{3.3} + \frac{V}{2.2} = 0$$

$$\frac{V - 20}{1.1} + \frac{V}{3.3} - \frac{30}{3.3} + \frac{V}{2.2} = 0$$

$$V \left(\frac{1}{1.1} + \frac{1}{3.3} + \frac{1}{2.2} \right) = \frac{30}{3.3} + \frac{20}{1.1}$$

$$V = 16.36$$

$$I_1 = \frac{16.36 - 20}{1.1} = -3.30 \text{ mA}$$

$$I_2 = \frac{16.36 - 30}{3.3} = -4.1 \text{ mA}$$

$$I_3 = \frac{16.36}{2.2} = 7.42 \text{ mA}$$

Superposition Theorem:-

Network Terminology:-

1) Electrical Network:- Arrangement of various electrical energy source and elements is called electrical network.

(or)

interconnection of elements and devices.

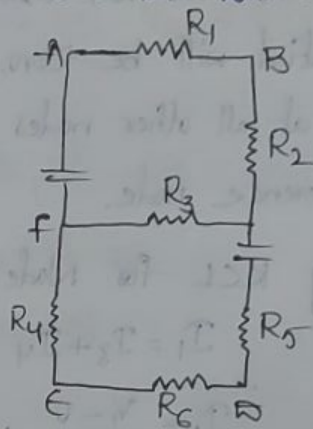
2) Network Elements:-

Any individual circuit element with two terminals which can be connected to the other element in the circuit.

3) Branch:-

A portion of circuit between two nodes which consists of atleast 1 element in it.

AB, BC, CD, EF, AF,
FC, DE are branches



4) Node:-

The point at which two or more elements joined together.

Note:- While solving a problem we take more two branches connected.

A, B, C, D, E, F are nodes.

5) Mesh/loop:-

A loop is a closed path which originates from a particular node and terminates at the same node without travelling through the same point twice.

A B C F A

6) Circuit:-

Network with one or more closed path is a circuit.

Nodal Analysis:- (** 10 marks) (steps)

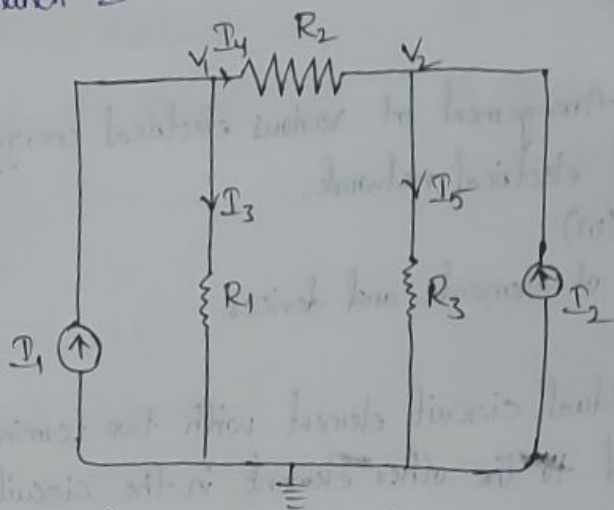
1) Identify Node

2) 'I' direction

3) KCL

4) Solve Vnode

5) Branch I



This method is based on KCL

→ In this method we will consider a node as a reference node whose potential will be zero.

→ Equations at all other nodes should be written with respect to this reference node.

Apply KCL for Node 1

$$I_1 = I_3 + I_4$$

$$I_1 = \frac{V_1 - 0}{R_1} + \frac{V_1 - V_2}{R_2} \quad \text{--- (1)}$$

Apply KCL for Node 2

$$I_4 + I_2 = I_5$$

$$\frac{V_1 - V_2}{R_2} + I_2 = \frac{V_2 - 0}{R_3} \quad \text{--- (2)}$$

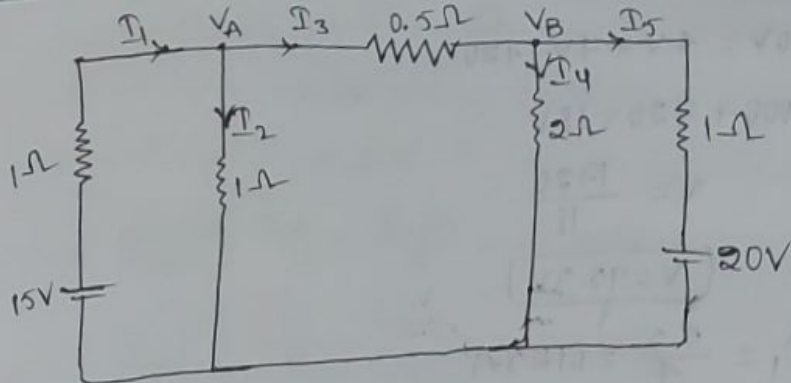
Steps for Nodal Analysis:

1) choose the node and set the node voltages.

2) choose the current at each branch preferably as current leaving the node

3) Apply KCL at each node with proper sign convention. obtain each equation in terms of node voltages and solve for node voltages.

4) obtain branch currents



for Node 1:

$$I_1 = I_2 + I_3$$

$$\frac{15 - V_A}{1} = \frac{V_A - 0}{1} + \frac{V_A - V_B}{0.5} \quad \text{--- (1)}$$

$$(0.5)(15 - V_A) = (0.5)(V_A) + (V_A - V_B) \quad \text{--- (2)}$$

for Node 2:

$$7.5 - 0.2V_A = -V_B \Rightarrow 2V_A - V_B = 7.5$$

$$I_3 = I_4 + I_5$$

$$\frac{V_A - V_B}{0.5} = \frac{V_B - 0}{2} + \frac{V_B - 20}{1} \quad \text{--- (3)}$$

$$2(V_A - V_B) = (0.5)V_B + V_B - 20$$

$$2V_A - 3.5V_B = -20 \quad \text{--- (4)}$$

from (3) & (4)

$$2.5V_B = 27.5$$

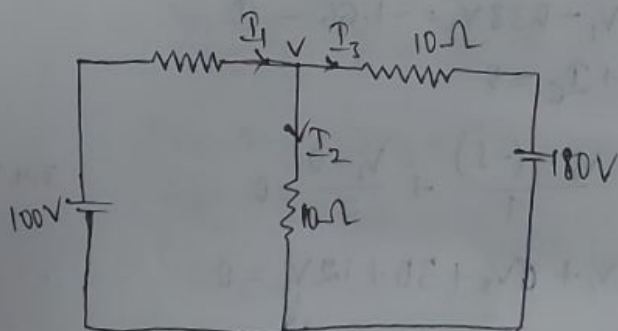
$$\boxed{V_B = 11V}$$

$$\boxed{V_A = 9.25V}$$

$$I_{2\Omega} = \frac{V_B - 0}{2}, \quad I_{0.5\Omega} = \frac{V_A - V_B}{0.5}$$

$$\boxed{I_{2\Omega} = \frac{11}{2} A}$$

$$= \frac{9.25 - 11}{0.5} = -3.5A \text{ (opposite direction)}$$



$$I_1 = I_2 + I_3$$

$$\frac{100 - V}{4} = \frac{V - 0}{10} + \frac{V - 180}{10}$$

$$1000 - 10V = 4V + 4V - 720$$

$$1000 + 720 = 18V$$

$$V = \frac{1720}{18}$$

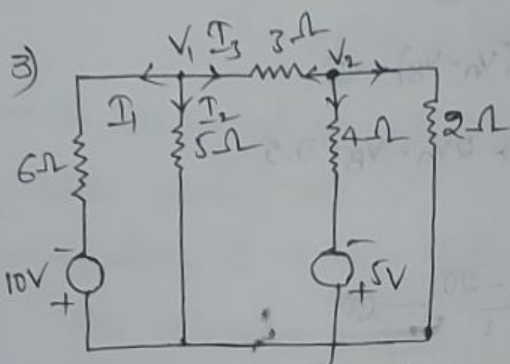
$$V = 95.5V$$

$$I_1 = \frac{5.5}{4} = 1.125A$$

$$I_2 = \frac{95.5}{10} = 9.55A$$

$$I_3 = \frac{95.5 - 180}{10} = -8.45A$$

04/10/2023



Apply KCL for node ① & ②

Node 1: $I_1 + I_2 + I_3 = 0$

$$\frac{V_1 - (-10)}{6} + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{3} = 0$$

$$\frac{V_1 + 10}{6} + \frac{V_1}{5} + \frac{V_1 - V_2}{3} = 0$$

$$15V_1 + 150 + 18V_1 + 30V_1 - 30V_2 = 0$$

$$63V_1 - 30V_2 = -150$$

$$21V_1 - 10V_2 = -50$$

$$0.69V_1 - 0.33V_2 = -1.66 \quad \text{--- ①}$$

Node 2:

$$I_4 + I_5 + I_6 = 0$$

$$\frac{V_2 - V_1}{3} + \frac{V_2 - (-5)}{4} + \frac{V_2 - 0}{2} = 0$$

$$8V_2 - 8V_1 + 6V_2 + 30 + 12V_2 = 0$$

$$26V_2 - 8V_1 = -30$$

$$8V_1 - 26V_2 = 30$$

$$4V_1 - 13V_2 = 15$$

$$\begin{array}{r} 3 \times 6, 1, 5 \\ 3 \times 2, 5, 3 \\ \hline 2, 5, 1 \\ 15 \\ \hline 90 \\ \hline 15 \\ \hline 105 \end{array}$$

$$3, 4, 2 = 24$$

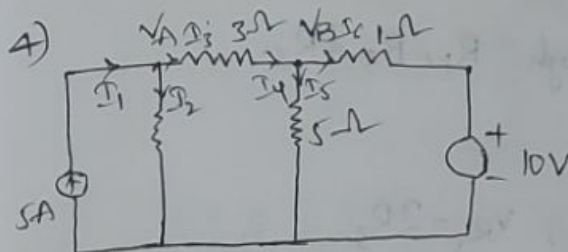
$$1.08V_2 - 0.33V_1 = -1.25 \quad \text{--- (2)}$$

$$V_1 = -3.4V$$

$$V_2 = -2.21V$$

$$I_{2\Omega} = V_2/2 = \frac{-2.21}{2} = -1.10A$$

$$I_{5\Omega} = \frac{V_1}{5} = \frac{-3.4}{5} = -0.68A,$$



for Node 1: $I_1 = I_2 + I_3$

$$5 = \frac{V_A - 0}{10} + \frac{V_A - V_B}{3} \quad \text{--- (1)}$$

At Node 2:

$$I_4 = I_5 + I_6$$

$$\frac{V_A - V_B}{3} = \frac{V_B}{5} + \frac{V_B - 10}{1} \quad \text{--- (2)}$$

from eq-1

$$5 = \frac{V_A}{10} + \frac{V_A - V_B}{3}$$

$$V_A + V_A - V_B = 5 \quad 5 = 3V_A + 10V_A - 10V_B$$

$$5 = 3V_A + 10V_A - 10V_B$$

$$13V_A - 10V_B = 150$$

from eq-2

$$\frac{V_A - V_B}{3} = \frac{V_B}{5} + \frac{V_B - 10}{1}$$

$$5V_A - 5V_B = 3V_B + 15V_B - 150$$

$$5V_A - 23V_B = -150$$

$$13V_A - 10V_B = 150$$

$$5V_A - 23V_B = -150$$

$$V_A = 19.89V$$

$$V_B = 10.84V$$

$$I_{3\Omega} = 3A$$

$$I_{1\Omega} = 0.84A$$

Series Connection:

When an element is connected one after the other then it forms a series circuit. It is also called as end to end connection or cascaded connection.

Resistor in Series:

R_1, R_2 & R_3 are said to be in series. So, there will be same amount of 'I' flow through R_1, R_2 & R_3 .

$$V = IR$$

$$V_{R_1} = IR_1, V_{R_2} = IR_2, V_{R_3} = IR_3$$

$$V_S = V_1 + V_2 + V_3$$

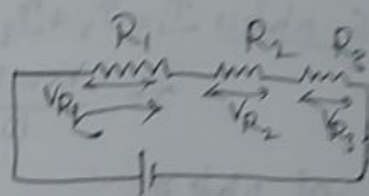
$$V_S = I(R_1 + R_2 + R_3)$$

$$V_S = I(R_{eq})$$

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_n$$

For 2 R's

$$R_{eq} = R_1 + R_2$$



→ where V_1, V_2 & V_3 are voltages across the terminals of resistance.

→ Total (or) Equivalent resistance of series circuit is arithmetic sum of the resistances connected in series.

Characteristics of Series Circuit:

Same current flows through all resistances. the supply voltage is equal to sum of individual voltage drops across the resistance.

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_n$$

= The largest of all resistance.

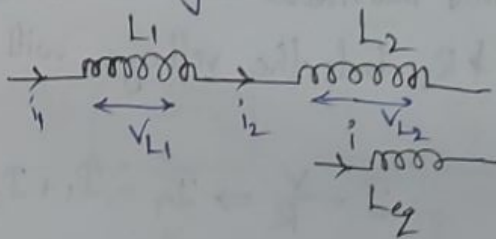
Inductors in Series:-

The two inductors are connected in series which are L_1 & L_2 . The current through L_1 & L_2 are I_1 & I_2 respectively. Voltage developed are V_{L_1} & V_{L_2} respectively.

$$V = L \frac{di}{dt}$$

$$V_{L_1} = L_1 \frac{di}{dt}, \quad V_{L_2} = L_2 \frac{di}{dt}$$

$$V_{L_1} + V_{L_2} = [L_1 + L_2] \frac{di}{dt} \\ = L_{eq} \frac{di}{dt}$$



→ The equivalent inductance is equal to sum of individual inductance connected in series.

$$\therefore L_{eq} = L_1 + L_2 + \dots + L_n$$

Capacitance in Series:-

C_1 & C_2 are connected in series the currents & the voltages develops across C_1 & C_2 are I_1 & I_2 & V_{C_1} , V_{C_2} respectively. The reciprocal of equivalent capacitor of series combination is the sum of reciprocal of individual capacitance.

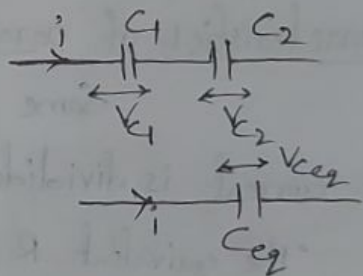
$$V = \frac{1}{C} \int i dt$$

$$V_{C_1} = \frac{1}{C_1} \int i dt + \frac{1}{C_2} \int i dt$$

$$V_{C_1} = \frac{1}{C_{eq}} \int i dt$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \text{ only for 2 capacitors}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n} \text{ for } n \text{ capacitors.}$$

Parallel Connection:-

06/10/2023

Resistors in Parallel:-

In parallel circuit, the total current is divided into individual currents I_1, I_2, I_3 across three resistors R_1, R_2 & R_3 but the voltage will remain same across each resistor.

$$I = \frac{V}{R} \Rightarrow I_T = I_1 + I_2 + I_3$$

$$I_T = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$= V \left(\frac{1}{R_{eq}} \right)$$

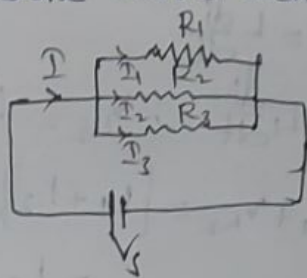
2 R's connected in ||

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

for n R's connected in ||

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$



Characteristics of parallel circuit:-

Same voltage across all parallel resistance

the current is divided.

The equivalent R is small is compared to all R 's.

Inductors in parallel:-

The currents through L_1 & L_2 are I_1 & I_2 and voltages across them.

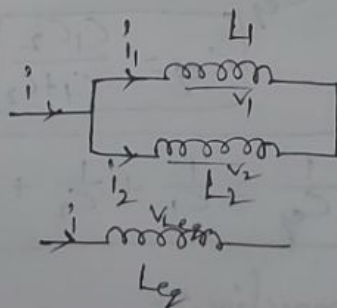
$$i = \frac{1}{L} \int v dt$$

$$= \frac{1}{L_1} + \frac{1}{L_2} \int v dt$$

$$= \frac{1}{L_{eq}} \int v dt$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$



for n L's connected in ll

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}$$

Capacitance in parallel:-

$$i = C \frac{dv}{dt}$$

$$i_1 = C_1 \frac{dv}{dt}, i_2 = C_2 \frac{dv}{dt}$$

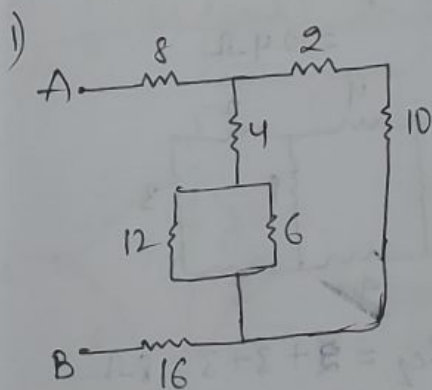
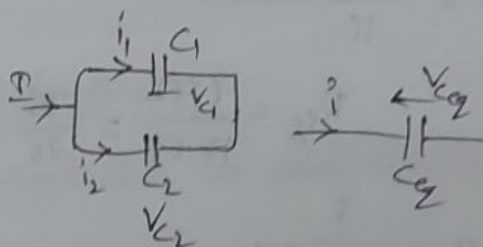
$$I_T = i_1 + i_2 = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$$

$$= (C_1 + C_2) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

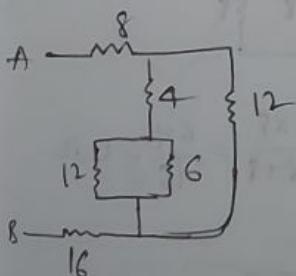
$$\therefore \boxed{C_{eq} = C_1 + C_2}$$

for n C's

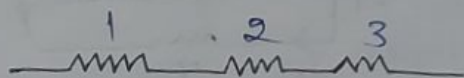
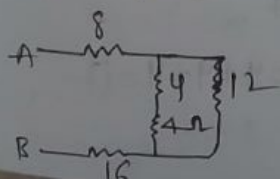
$$C_{eq} = C_1 + C_2 + \dots + C_n$$



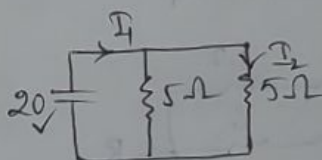
$$R_{eq} = 2 + 10 = 12 \Omega$$



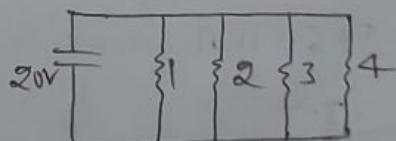
$$R_{eq} = \frac{12 \times 6}{12 + 6} = \frac{72}{18} = 4 \Omega$$



$$R_{eq} = R_1 + R_2 + R_3 = 1 + 2 + 3 = 6 \Omega$$



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 5}{5 + 5} = \frac{25}{10} = 2.5 \Omega$$

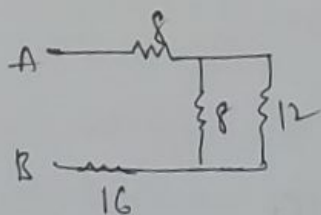


$$R_{eq} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

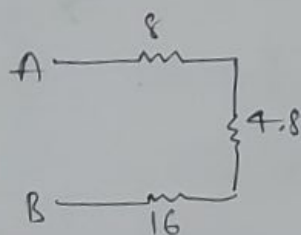
$$= \frac{24 + 12 + 8 + 6}{24}$$

$$= \frac{50}{24} = 2.08 \Omega$$

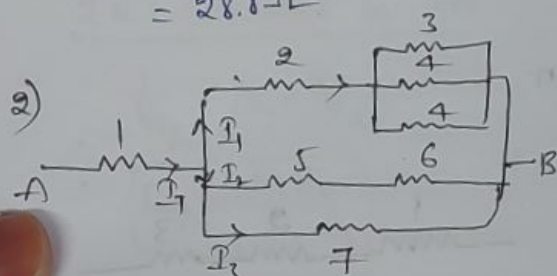
$$R_{eq} = 4 + 4 = 8 \Omega$$



$$R_{eq} = \frac{8 \times 12}{12 + 8} = \frac{96}{20} = 4.8 \Omega$$

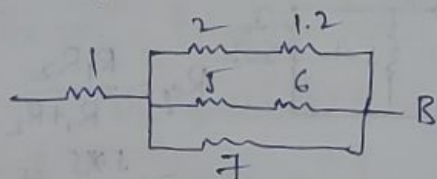


$$R_{eq} = 8 + 16 + 4.8 = 28.8 \Omega$$



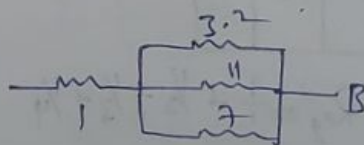
find R_{eq} b/w A & B.

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{4} = 1.2 \Omega$$



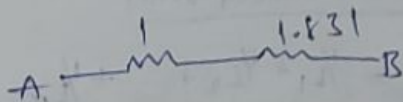
$$R_{eq} = 2 + 1.2 = 3.2 \Omega$$

$$R_{eq} = 5 + 6 = 11 \Omega$$



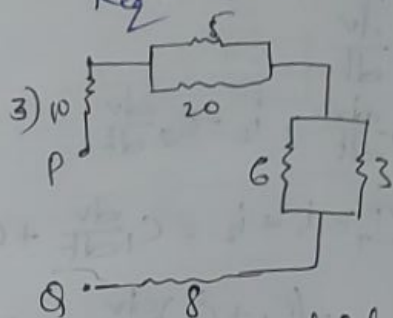
$$\frac{1}{R_{eq}} = \frac{1}{3.2} + \frac{1}{11} + \frac{1}{7}$$

$$= 0.546 \Omega \quad \frac{1}{R_{eq}} = 1.831 \Omega$$

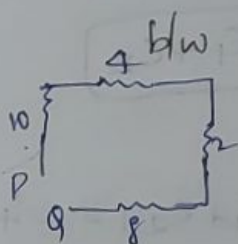


$$R_{eq} = 1 + 1.831$$

$$R_{eq} = 2.831 \Omega$$



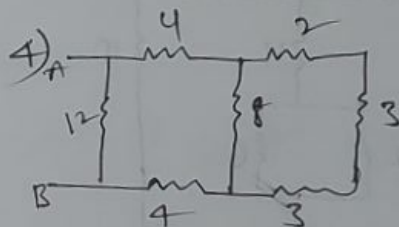
Calculate R_{eq}



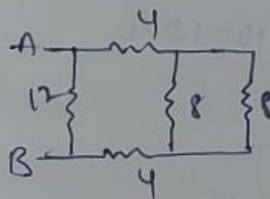
$$R_{eq} = \frac{5 \times 20}{5 + 20} = 4$$

$$R_{eq} = \frac{3 \times 6}{3 + 6} = 2$$

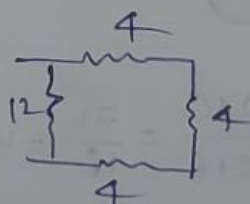
$$R_{eq} = 4 + 2 + 8 + 10 = 24 \Omega$$



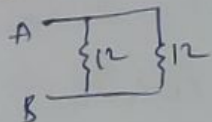
$$R_{eq} = 4 + 3 + 3 = 8 \Omega$$



$$R_{eq} = \frac{8 \times 8}{8 + 8} = \frac{64}{16} = 4 \Omega$$



$$R_{eq} = 4 + 4 + 4 = 12$$

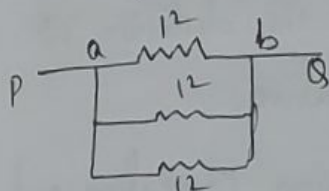
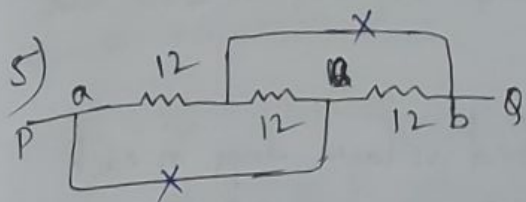


$$R_{eq} = \frac{12 \times 12}{12 + 12} = \frac{144}{24} = 6 \Omega$$

$$R_{eq} = \frac{10 \times 10}{10 + 10} = \frac{100}{20} = 5 \Omega$$

$$\therefore (R_{eq} = 5 \Omega)$$

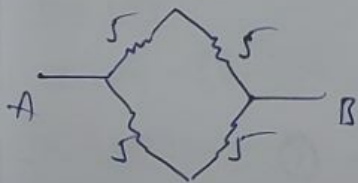
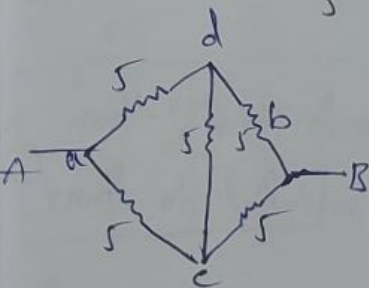
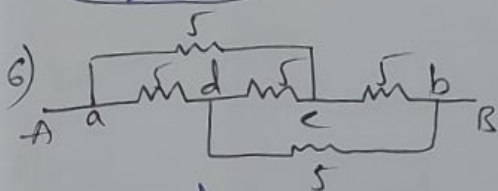
09/10/2023



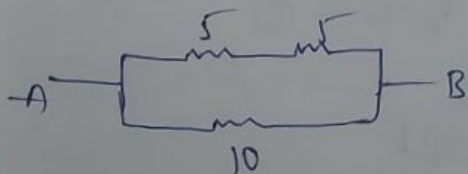
$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$

$$= \frac{3}{12} = \frac{1}{4} \Omega$$

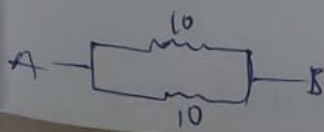
$$R_{eq} = 4 \Omega$$



$$R_{eq} = 5 + 5 = 10 \Omega$$



$$R_{eq} = 5 + 5 = 10 \Omega$$



Mesh Analysis:-

→ Applicable for planar circuits

Steps:-

- 1) Define currents, meshes (clock wise direction)
- 2) Identify voltage drops, polarities
- 3) Calculate KVL $\Rightarrow \Sigma = \frac{V}{R}$
- 4) Write equations for Σ 's
- 5) Solve

Mesh Analysis / Loop Analysis:-

→ This method of analysis is useful for the circuits

that have voltage sources, many nodes and many loops.

In whiststone bridge we cannot count middle

→ The advantage of this method is it reduces the complexity of solving the network with less no. of unknowns.

Points to Note:-

→ While assuming loop currents make sure atleast one loop current links with every element.

→ Note two loops should be identical.

→ Choose minimum no. of loops current.

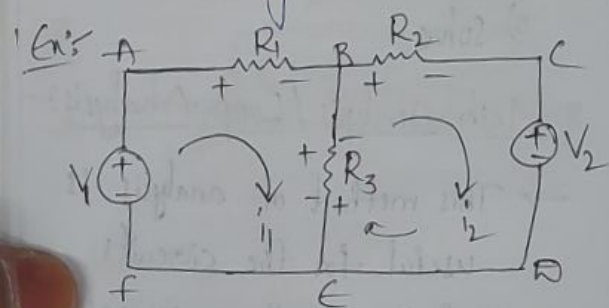
→ Mesh analysis is applicable only for planar circuits.

Steps for Loop Analysis:-

1. Choose various loops and draw loops currents in clock wise direction preferably.

2. Indicate polarities of associated voltage drops in each loop. The drop of resistor should be in the direction of loop current as positive and then negative.

3. Apply KVL to the loops following sign convention and write loop equations. Assume main loop currents as positive and remaining ^{loop} currents are -ve for remaining common branches.



* For the branch BE two loop currents i_1 & i_2 flow. As currents always flow from higher potential to lower potential.

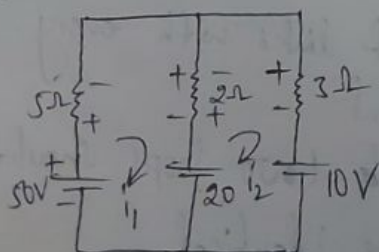
For Loop 1:

$$+V_1 - R_1 i_1 - R_3 (i_1 - i_2) = 0 \quad \text{--- (1)}$$

For Loop 2:

$$-R_3 (i_2 - i_1) - i_2 R_2 - V_2 = 0 \quad \text{--- (2)}$$

Ex-
Prblm



Sol:- For Loop 1

$$+50 - 5(i_1) - 2(i_1 - i_2) - 20 = 0$$

For Loop 2

$$+20 - 2(i_2 - i_1) - 3(i_2) - 10 = 0$$

from ①
 $50 - 5i_1 - 2i_1 + 2i_2 - 20 = 0$
 $-7i_1 + 2i_2 = -30 \text{ --- ①}$

from ②
 $20 - 2i_2 + 2i_1 - 3i_2 - 10 = 0$

$$2i_1 + 5i_2 = -10 \text{ --- ②}$$

Solving eq ① & ② we get,

$$i_1 = 5.47 \text{ A}$$

$$i_2 = 4.19 \text{ A}$$

$$i_{2\Omega} = i_1 - i_2 = 5.47 - 4.19$$

$$= 1.285 \text{ A}$$

$$i_{5\Omega} = 5.47 \text{ A}$$

$$i_{3\Omega} = 4.19 \text{ A}$$

$$-6i_1 + 4i_2 = -10$$

from ②

$$4i_2 - 4i_1 - i_2 - 6i_2 + 6i_3 = 0$$

$$4i_1 - 11i_2 + 6i_3 = 0$$

$$-4i_1 + 6i_2 - 3i_3 = 0$$

$$-4i_1 + 3i_2 + 6i_3 = 0$$

from ③

$$-6i_3 + 6i_2 - 4i_3 - 20 = 0$$

$$-10i_3 + 6i_2 = 20$$

$$i_1 = 0.915 \text{ A}$$

$$i_2 = -1.126 \text{ A}$$

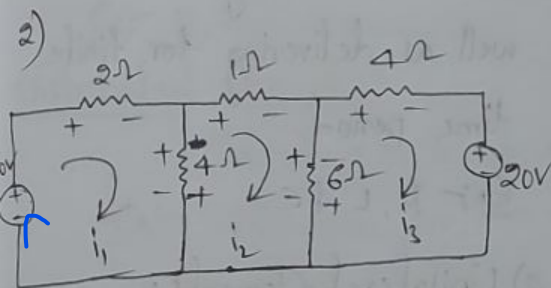
$$i_3 = -2.67 \text{ A}$$

$$i_{2\Omega} = 0.915 \text{ A}$$

$$i_{4\Omega} = i_1 - i_2 = 0.915 + 1.126$$

$$= 2.041$$

11/10/2023



for Loop 1:-

$$10 - 2i_1 - (i_1 - i_2)4 = 0 \text{ --- ①}$$

for Loop 2:-

$$4(i_2 - i_1) - i_2 - 6(i_2 - i_3) = 0 \text{ --- ②}$$

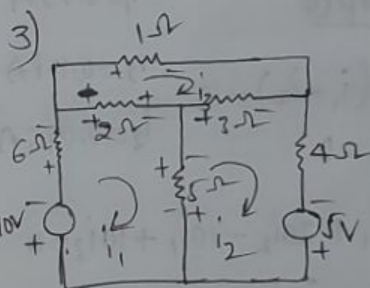
for Loop 3:-

$$-6(i_3 - i_2) - 4i_3 - 20 = 0 \text{ --- ③}$$

from ①

$$10 - 2i_1 - (i_1 + i_2)4 = 0$$

$$10 - 2i_1 - 4i_1 + 4i_2 = 0$$



Loop 1:

$$-10 - 6i_1 - 2(i_1 - i_3) - 5(i_1 - i_2) = 0$$

Loop 2:

$$-5(i_2 - i_1) - 3(i_2 - i_3) - 4i_2 + 5 = 0$$

Loop 3:

$$-2(i_3 - i_1) - 3(i_3 - i_2) - i_3 = 0$$

from ①

$$-10 - 6i_1 - 2i_1 + 2i_3 - 5i_1 + 5i_2 = 0$$

$$-13i_1 + 5i_2 + 2i_3 = 10$$

from ②

$$-5i_2 + 5i_1 - 3i_2 + 3i_3 - 4i_2 + 5 = 0$$

$$5i_1 - 12i_2 + 3i_3 = -5$$

from ③

$$-i_3 - 3i_3 + 3i_2 - 2i_3 + 2i_1 = 0$$

$$2i_1 + 3i_2 - 6i_3 = 0$$

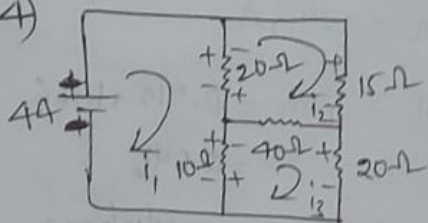
Solving eq-①, ② & ③ we get

$$i_1 = \frac{-150}{157} = -0.802 \text{ A}$$

$$i_2 = \frac{10}{561} = 0.017 \text{ A}$$

$$i_3 = \frac{-145}{561} = -0.258 \text{ A}$$

4)



from Loop ①

$$i_1 = 0.22 \text{ A}$$

$$44 - 20(i_1 - i_2) - 10$$

$$i_2 = 0.128 \text{ A}$$

$$(i_1 - i_3) = 0$$

$$i_3 = 0.096 \text{ A}$$

$$44 - 20i_1 + 20i_2 - 10i_1 + 10i_3 = 0$$

$$-30i_1 + 20i_2 + 10i_3 = -44 \text{ --- ①}$$

from Loop ②

$$-20(i_2 - i_1) - 15i_2 - 40(i_2 - i_3) = 0$$

$$-20i_2 + 20i_1 - 15i_2 - 40i_2 + 40i_3 = 0$$

$$20i_1 - 75i_2 + 40i_3 = 0 \text{ --- ②}$$

from Loop ③

$$10(i_3 - i_1) - 40(i_3 - i_2) - 20i_3 = 0$$

$$10i_3 - 10i_1 - 40i_3 + 40i_2 - 20i_3 = 0$$

$$-10i_1 + 40i_2 - 50i_3 = 0 \text{ --- ③}$$

Types of Elements

- 1) Active and passive elements
- 2) Unilateral & Bilateral
- 3) Linear & Non linear
- 4) Lumped & distributed

1) Active Elements:-

Delivers energy/power to some external device over infinite time interval.

Ex:- voltage & current source

Passive Elements:-

are capable of receiving power but some elements like L & C are storing power as well as delivering for finite time period.

Ex:- R, L & C

2) Unilateral Elements:-

Elements which does not allow equal current to flow from both directions in there V & I relationship is not same in both directions.

Ex:- vacuum & silicon diode

→ But In practical case,

there will be some reverse current which close but these current is very

minimum so it will be neglected.

Bilateral Elements:-

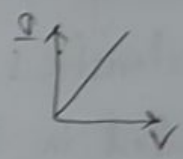
Elements which allow equal currents to flow in both directions i.e., $V-I$ relationship is same in both directions.

Ex:- R, L, C

3) Linear Elements:-

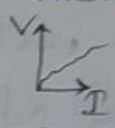
The element is said to be linear if the output is exactly linearly proportional to input.

→ DILE they/their own be any linear device so whatever is linear to linear will be considered.



Non-Linear Elements:-

In this, output is not at all linear with input system.



4) Lumped Elements:-

The elements which are separated physically.

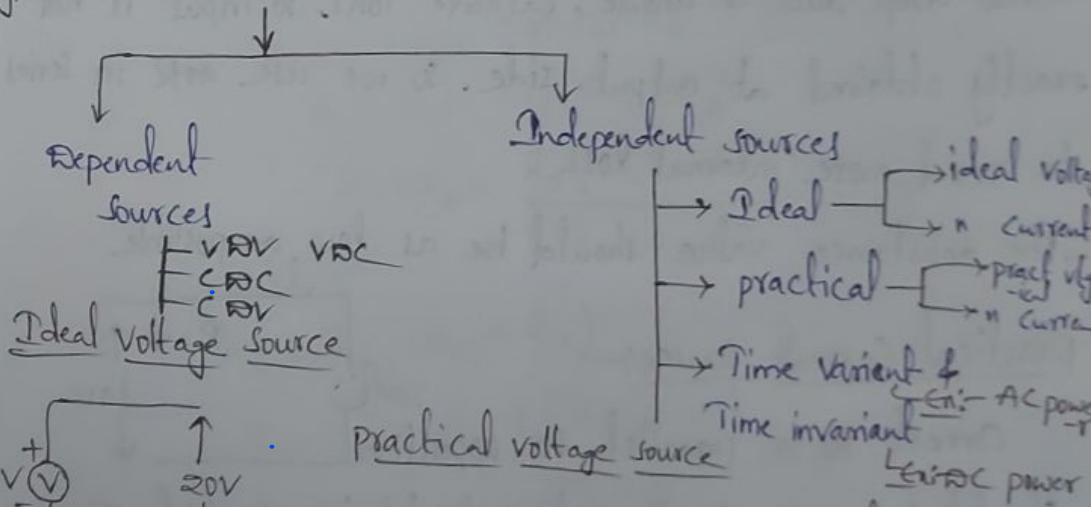
Ex:- R, L, C

Distributed Elements:-

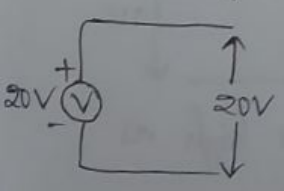
The elements which are not separable for analysis.

Ex:- Transmission line

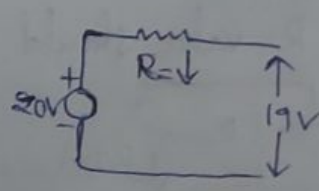
Types of Sources:-



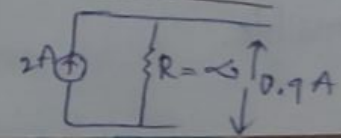
Ideal Voltage Source



practical voltage source



Current source



Dependent Sources

Types

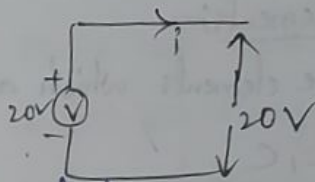
- 1) Voltage dependent V source
- 2) Current dependent I source
- 3) V dependent I source
- 4) I dependent V source

Types of Sources:-

→ Depending upon the terminal voltage and terminal current characteristics, energy sources are classified into as represented in the above figure.

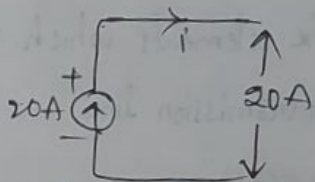
Ideal voltage source:-

There won't be any internal drop, so, input voltage is obtained that output also.



Ideal current source:-

There won't be any internal drop.



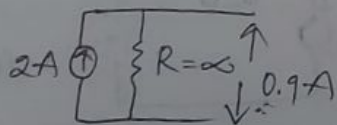
Practical voltage source:-

In this, there is some internal drop due to anode, cathode ions, so input is not exactly obtained at output side. So we use R in series to avoid more internal loss.

→ The resistance value should be as low as possible.

Practical current source:-

Connect R in parallel so opposition should be more. So R value should be as high as possible.



Time invariant:-
 → The sources in which voltage do not vary with time.
 Ex:- DC source.

Time variant:-
 The sources in which voltage varies with time.

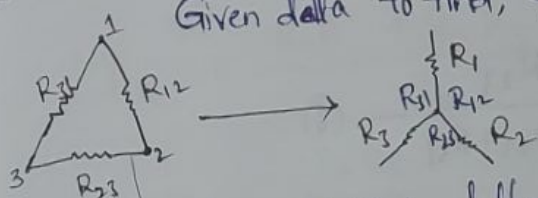
Ex:- AC source.

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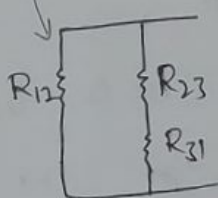
Star to Delta:- ($\Delta \rightarrow Y$)

Delta to star transformation:-

Given delta to find, equivalent star transformation.



Between Nodes ① & ② in delta the equivalent circuit will become



for ~~delta~~ delta:

$$\Rightarrow \frac{R_{12}}{R_1} \parallel (R_{23} + R_{31})$$

$$= \frac{R_{12} * (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{--- ①}$$

$$* R_{12} = R_{21}$$

$$* R_{23} = R_{32}$$

$$* R_{13} = R_{31}$$

for star

$$R_1 + R_2 = R_{eq} \quad \text{--- ②}$$

Now equate ① & ②

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{--- ③}$$

for delta & star

$$R_2 + R_3 = R_{23} \parallel (R_{12} + R_{31})$$

$$R_2 + R_3 = \frac{R_{23} * (R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{--- ④}$$

$$R_{31} = R_{13} \parallel (R_{23} + R_{12})$$

$$R_1 + R_3 = \frac{R_{13} * (R_{23} + R_{12})}{R_{12} + R_{23} + R_{31}} \quad \text{--- ⑤}$$

Subtract ③ & ④ eq^{ns}

$$R_1 + R_2 - R_2 + R_3 = \frac{R_{12}R_{23} + R_{31}R_{12} - R_{23}R_{12} - R_{31}R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_1 - R_3 = \frac{R_{31} R_{12} - R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} \quad \text{--- (5)}$$

Adding (5) & (6) eq^{ns}

$$R_1 + R_3 + R_1 - R_3 = \frac{R_{13} R_{23} + R_{13} R_{12} + R_{13} R_{12} - R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$

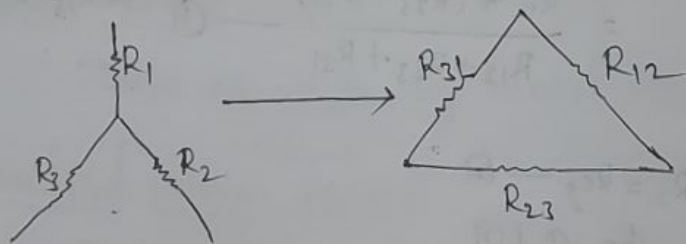
$$2R_1 = \frac{2(R_{13} R_{12})}{R_{12} + R_{23} + R_{31}}$$

$$R_1 = \frac{R_{13} R_{12}}{R_{12} + R_{23} + R_{31}} \quad \text{--- (7)}$$

$$R_2 = \frac{R_{21} R_{23}}{R_{12} + R_{23} + R_{31}} \quad \text{--- (8)}$$

$$R_3 = \frac{R_{31} R_{32}}{R_{12} + R_{23} + R_{31}} \quad \text{--- (9)}$$

Star to Delta Transformation:-



$$\left. \begin{matrix} R_1 R_2 \\ R_2 R_3 \\ R_3 R_1 \end{matrix} \right\} \begin{matrix} R_1 R_2 + \\ R_2 R_3 + \\ R_3 R_1 \end{matrix}$$

$$R_1 R_2 = \frac{R_{13} R_{12} * R_{23} R_{21}}{R_{12} + R_{13} + R_{31}}$$

$$R_1 R_2 = \frac{R_{12}^2 R_{13} R_{23}}{(R_{12} + R_{13} + R_{31})^2}$$

$$R_2 R_3 = \frac{R_{21} R_{23} * R_{31} R_{32}}{R_{12} + R_{13} + R_{31}} = \frac{R_{23}^2 R_{21} R_{31}}{(R_{12} + R_{13} + R_{31})^2}$$

$$R_3 R_1 = \frac{R_{31} R_{32} * R_{12} R_{13}}{R_{12} + R_{13} + R_{31}} = \frac{R_{31}^2 R_{32} R_{12}}{(R_{12} + R_{13} + R_{31})^2}$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{(R_{12} + R_{23} R_{31})(R_{12} R_{23} R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\therefore R_1 = \frac{R_{13} R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = R_1 * R_{23}$$

Similarly for R_2 & R_3

$$R_{13} = \frac{R_1 R_2 + R_3 R_2 + R_1 R_3}{R_2}$$

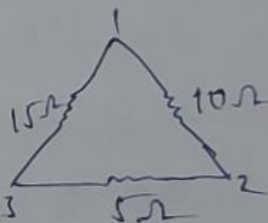
$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

Ex:-
1) Convert Δ to Y $R_{12} = 10\Omega$, $R_{13} = 5\Omega$, $R_{23} = 15\Omega$

$$R_1 = \frac{R_{13} R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$= \frac{5 \times 10}{10 + 5 + 15} = \frac{50}{30} = 1.67\Omega$$



$$R_2 = \frac{R_{21} R_{32}}{R_{12} + R_{23} + R_{31}} = \frac{10 \times 15}{30} = \frac{150}{30} = 5\Omega$$

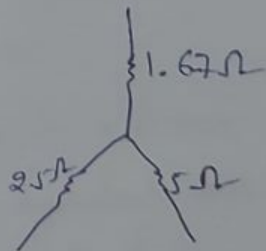
$$R_3 = \frac{R_{31} R_{32}}{R_{12} + R_{23} + R_{31}} = \frac{5 \times 15}{30} = \frac{75}{30} = 2.5\Omega$$

Y to Δ :

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

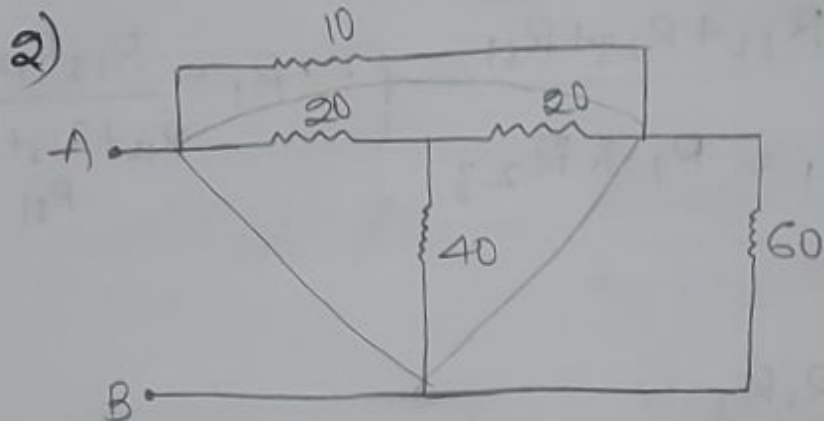
$$= \frac{(1.67)(5) + (5)(2.5) + (1.67)(2.5)}{2.5}$$

$$= \frac{8.35 + 12.5 + 4.175}{2.5} = \frac{25.025}{2.5} = 10.01\Omega$$



$$R_{13} = \frac{25.025}{5} = 5.005 \Omega$$

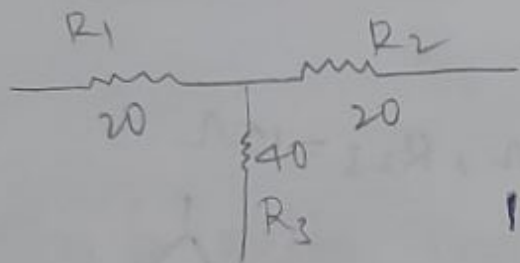
$$R_{23} = \frac{25.025}{2.5} = 10.01 \Omega$$



delta to star: (wrong because 40 is connected b/w both)

$$R_1 = \frac{R_{12} R_{13}}{R_{12} + R_{13} + R_{23}} = \frac{10 \times 20}{10 + 20 + 20} = \frac{200}{50} = 4 \Omega$$

Star to delta:




Dependent Sources:-

The value of source voltage or current in the circuit depends on the other voltage or current which are present somewhere in the circuit.

1) Voltage dependent voltage source:-

produces voltage as a function of voltage else where in the circuit.

$$V = KV_1$$



2) Current dependent current source:-

produces current as a function of current else where in the circuit.

$$I = KI_1$$



Current dependent voltage source:-

produces voltage as a function of current else where in the circuit.

$$V = KI_1$$


Voltage dependent current source:-

produces current as a function of voltage else where in the circuit

$$I = KV_1$$


$K = \text{constant}$

V_1 & I_1 are voltages & ~~current~~

present else where in the circuit

and these are also called as controlled sources.

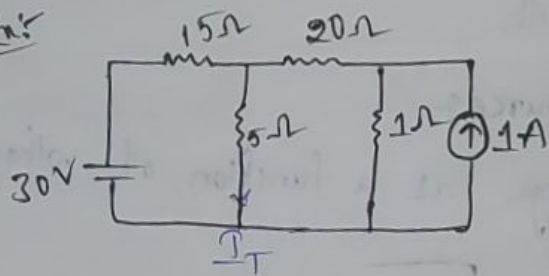
Superposition Theorem:-

This theorem states that in a linear network comprising of 'n' number of independent sources. The total response in any branch of a network is equal to the algebraic sum of individual responses acting alone i.e., considering one source at a time and making all other sources to zero. But dependent

Sources must be retained in the network.

Note:- If voltage source is there it must be short-circuited.
And if current source is there it must be open-circuited.

Ex:-

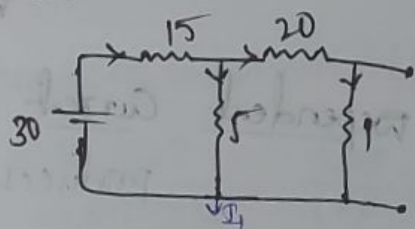


$$I_1 = 0.84A$$

$$I_2 = 0.95A$$

Case-i:-

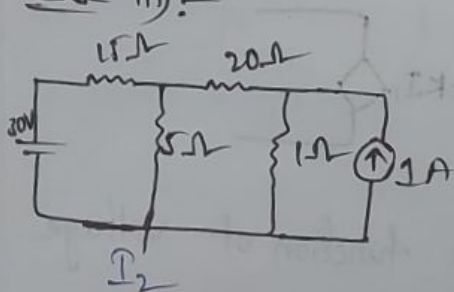
Case-ii:-



$$I = V/R_{eq}$$

$$R_{eq} = ((20+1) // 5 + 15)$$

Case-iii:-



Advantages of AC:-

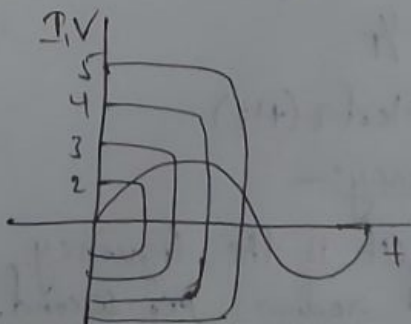
- * Variation of voltages is possible by device is called transformer in dc circuit it is not possible.
- * When AC voltages can be exsased less current can be flow through transmission lines. so conducting materials required will be very less so cost decreases.
- * High voltages will help in building high ~~repe~~ speed ac generators of large capacity. The construction and ^{also} cost of generators are very low. This is not possible in dc.
- * AC electrical motors are simple, cheap and required less maintaranace.
- * AC supply can be converted into dc easily. The practical advantage of ac is dominating dc.

Types of Wave forms:-

- The waveform of ac, current and voltage is shown in pure sine but practically will be getting different waveforms with variations in instantaneous values, both in magnitude & direction by considering 0 reference.

Instantaneous value:-

At any given time it has some instantaneous value at different points along wave form.



1 - 2sec
2 - 3sec of
voltages

Wave form:

The Graph of instantaneous values of ac quantity against time is called wave form.

Advantages of Considering sine wave as a theoretically wave form:-

- Mathematically, equations can be written very easily for sine wave.
- In AC, only sine & cosine values can pass through linear circuit ~~are~~ ~~the~~ ~~to~~ R, L, C without distortion.
- Integration & derivation is with sine wave against sine function. So Analysis will become very easy.

Basic definitions

Cycle:- Each repetition of +ve & -ve instantaneous values of ac quantity.

Periodic wave form:-

repetition of 1 cycle in regular intervals of time.

Time period:-

The time taken to complete 1 cycle by ac quantity.

Units: seconds

Frequency:-

The no. of cycles completed by ac quantity in 1 seconds.

$$f = 1/T$$

Units: Hertz (Hz)

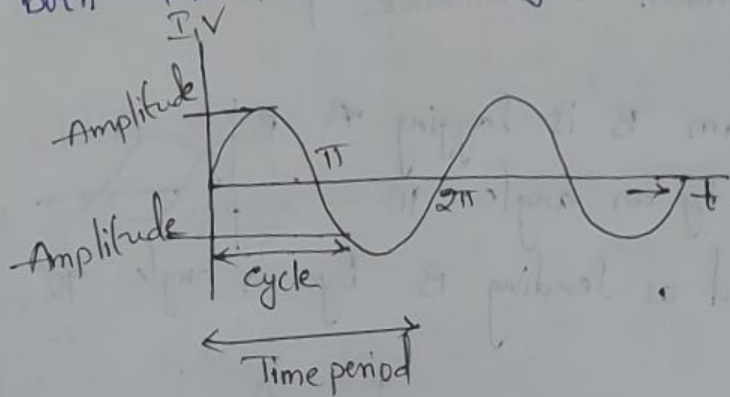
Angular frequency:-

It is the frequency which is expressed in electrical radian per second.

$$1 \text{ cycle} = 2\pi \text{ radians}$$

$$\omega = 2\pi \times \text{cycles per second} \Rightarrow \omega = 2\pi \times \text{radians/second}$$

Amplitude: The maximum value attained by alternative quantity in both +ve & -ve half cycles.



Prblms

1) The period of sine wave is given as 20ms. what is its frequency?

$$f = \frac{1}{T} = \frac{1}{20\text{ms}} \\ = 0.05 \text{ Hz}$$

2) Calculate the timeperiod for each value of frequency

- a) 50Hz b) 100kHz c) 1Hz d) 2mHz

a) 0.02 sec

b) $10 \mu\text{sec}$

c) $1\text{Hz} = 1\text{sec}$

d) $2\text{mHz} = 0.5\text{msec}$

Phase of Sine Wave:-

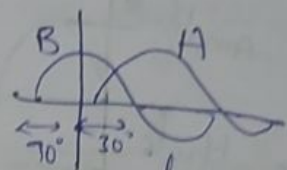
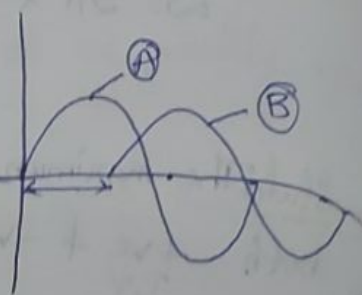
A phase of sine wave is angular measurement which specifies the position of sine wave related to reference.

In this, A is taken as reference wave form.

So, the wave form B is lagging

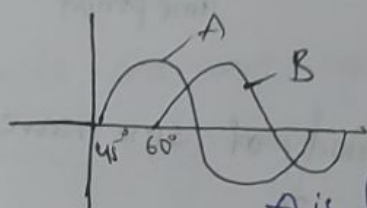
behind A by an angle 90°

or A is called as leading B by an angle 90° .



Sine wave:-

Sine wave equation



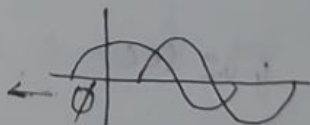
A is leading B by an angle 15°

The sine wave is graphical representation by having amplitude of sine wave represented on vertical axis and angular measurement is represented on horizontal axis.

$$V(t) = V_m \sin \omega t$$

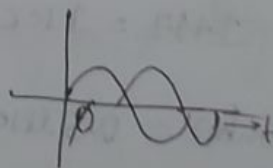
→ If sin wave is shifted to left by an angle ' ϕ '

$$V(t) = V_m \sin \omega t + \phi$$



→ If sin wave is shifted towards the right by an angle ' ϕ '

$$V(t) = V_m \sin \omega t - \phi$$



V, I, sine wave

- 1) Instantaneous values
- 2) Peak values
- 3) Peak to peak values

4) Rms values

5) Average values

Analytical
practical
Theoretical

voltages and currents of a sine wave:-
The magnitude of wave is not constant, so the wave is measured in different values/waves.

V, I sine waves

1) Average values:-

It is defined as the value which is obtained by adding all the instantaneous values over a period of half cycle or symmetrical ac wave form. The average value over a complete cycle will be equal to zero. so average value is defined for half cycle only.

Graphical method / Analytical method:-

$$V_{\text{avg}} = \frac{V_1 + V_2 + V_3 + \dots + V_n}{n}$$

$$I_{\text{avg}} = \frac{i_1 + i_2 + \dots + i_n}{N}$$

2) practical / theoretical method:-

$$V_{\text{avg}} = v(t) = V_m \sin \omega t$$

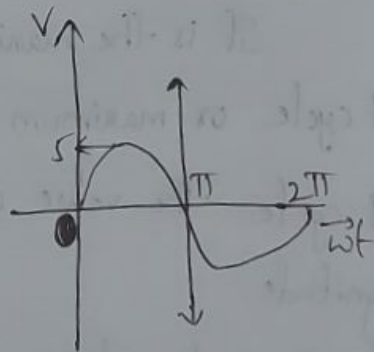
$$V_{\text{avg}} = \frac{1}{T} \int_0^t v(t) dt$$

$$V_{\text{avg}} = \frac{1}{T} \int_0^t V_m \sin \omega t d\omega t$$

$$= \frac{1}{T} \int_0^{\pi} V_m \sin \omega t d\omega t$$

$$= \frac{1}{T} V_m (-\cos \omega t)_0^{\pi}$$

$$= \frac{1}{\pi} V_m \times 2$$



$$V_{\text{avg}} = \frac{2V_m}{\pi}$$

$$V_{rms}:- \text{Heat} \rightarrow$$

$$= \sqrt{\frac{1}{T} \int_0^T (V_m \sin^2 \omega t) d\omega t}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (V_m \sin \omega t)^2 d\omega t}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_m^2 \left(1 - \frac{\cos 2\omega t}{2}\right) d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi} \left(\omega t + \frac{\sin 2\omega t}{2}\right)_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi} (2\pi)}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

$$\therefore \boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

01/11/2023

Peak Value:-

It is the maximum value of a wave, during +ve half cycle or maximum value of a wave during -ve half cycle the value of these two waves have equal magnitude.

Peak to Peak Value:-

It is the value from +ve to -ve Peak (P).

Peak factor:-

It is the ratio of peak value of wave by rms value,

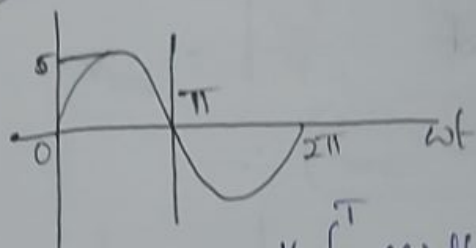
$$\text{Peak factor} = \frac{V_m}{\text{Rms value}} = \frac{V_m}{\frac{V_m}{\sqrt{2}}} = 1.414$$

Form factor:-

It is the ratio of rms value to the average value of wave.

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Avg value}} = \frac{V_m / \sqrt{2}}{\frac{2V_m}{\pi}} = 1.11$$

Prblm



$$V_{\text{avg}} = \frac{1}{T} \int_0^T v(t) dt$$

$$= \frac{1}{\pi} \int_0^{\pi} V_m \sin \omega t d\omega t$$

$$= \frac{V_m}{\pi} \left[-\cos \omega t \right]_0^{\pi}$$

$$V_{\text{avg}} = \frac{2V_m}{\pi} = \frac{2 \times 5}{\pi} = 10/\pi \text{ Volts}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T (v(t))^2 dt}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (V_m \sin \omega t)^2 d\omega t}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (5)^2 \sin^2 \omega t d\omega t}$$

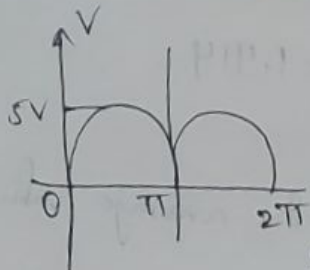
$$= \sqrt{\frac{25}{2\pi} \int_0^{2\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) d\omega t}$$

$$= \sqrt{\frac{25}{4\pi} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_0^{2\pi}}$$

$$= \sqrt{\frac{25}{4\pi} \times (2\pi)} = \sqrt{\frac{25}{2}}$$

$$= \frac{5}{\sqrt{2}} \therefore V_{\text{rms}} = \frac{\sqrt{25}}{\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ V}$$

2)



$$V_{avg} = \frac{1}{T} \int_0^T V_m \sin \omega t \, d\omega t$$

$$= \frac{1}{\pi} \int_0^{\pi} 5 \sin \omega t \, d\omega t$$

$$= \frac{1}{\pi} (5) \int_0^{\pi} \sin \omega t \, d\omega t$$

$$= \frac{2(5)}{\pi} = \frac{10}{\pi}$$

$$\boxed{V_{avg} = \frac{10}{\pi}} \text{ volts}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T (V_m \sin \omega t)^2 \, d\omega t}$$

$$= \sqrt{\frac{1}{\pi} \int_0^{\pi} (V_m \sin \omega t)^2 \, d\omega t}$$

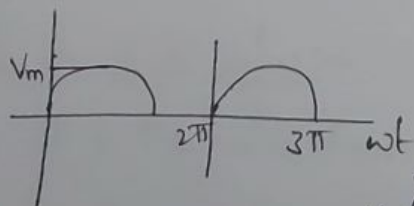
$$= \sqrt{\frac{1}{\pi} \int_0^{\pi} (5)^2 (\sin^2 \omega t) \, d\omega t}$$

$$= \sqrt{\frac{25}{\pi} \left[\frac{1 - \cos \omega t}{2} \right]_0^{\pi} \, d\omega t}$$

$$= \sqrt{\frac{25}{2\pi} \times \pi}$$

$$\boxed{V_{rms} = \sqrt{\frac{25}{2}}} \text{ V}$$

3)



$$V_{avg} = \frac{1}{T} \int_0^T V_m \sin \omega t \, d\omega t$$

$$= \frac{1}{\pi} \int_0^{\pi} V_m \sin \omega t \, d\omega t$$

$$V_{avg} = \frac{2V_m}{\pi} \text{ Volts}$$

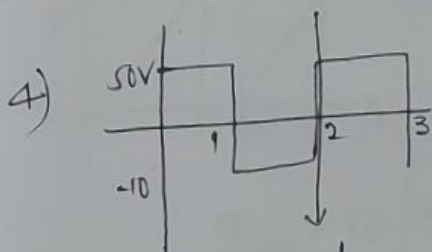
$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T (V_m \sin \omega t)^2 dt}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (V_m \sin \omega t)^2 dt}$$

$$= \sqrt{\frac{V_m^2}{4\pi} [\pi]}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{4}} = \frac{V_m}{2}$$

$$\therefore V_{rms} = \frac{V_m}{2} \checkmark$$



$$0 < t < 1 \rightarrow 50$$

$$1 < t < 2 \rightarrow -10$$

$$V_{avg} = \frac{1}{T} \int_0^T v(t) dt + \frac{1}{T} \int_1^2 v(t) dt$$

$$= \frac{1}{2} \int_0^1 50 dt + \frac{1}{2} \int_1^2 -10 dt$$

$$= \frac{1}{2} (50t)_0^1 + \frac{1}{2} (-10t)_1^2$$

$$= \frac{1}{2} (50 - 10) = \frac{1}{2} \times 40$$

$$V_{avg} = 20 \text{ Volts}$$

$$V_{rms} = \sqrt{\frac{1}{T} \left(\int_0^T (v(t))^2 dt + \int_1^2 (v(t))^2 dt \right)}$$

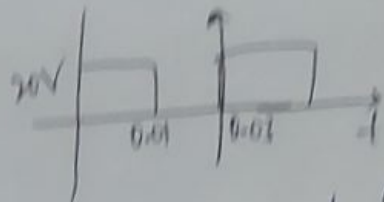
$$= \sqrt{\frac{1}{2} \left(\int_0^1 (50)^2 dt + \int_1^2 (-10)^2 dt \right)}$$

$$= \sqrt{\frac{1}{2} \left((2500t)_0^1 + (100t)_1^2 \right)}$$

$$= \sqrt{\frac{1}{2} (2600)} = \sqrt{1300} = 36.0 \text{ V}$$

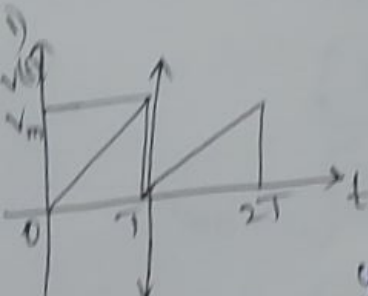
$$\therefore V_{rms} = 36.0 \text{ V}$$

5)



$$\begin{aligned}
 V_{avg} &= \frac{1}{T} \left(\int_0^{0.01} v(t) dt + \int_{0.01}^{0.03} v(t) dt \right) \\
 &= \frac{1}{0.03} \left(\int_0^{0.01} 20 dt + \int_{0.01}^{0.03} 0 dt \right) \\
 &= \frac{1}{0.03} [0.20] = 6.66 \text{ V}
 \end{aligned}$$

$$V_{avg} = 6.66 \text{ V}$$



$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

there, $(x_1, y_1) = (0, 0)$

$$(x_2, y_2) = (T, V_m)$$

$$(x, y) = (t, v(t))$$

$$(v(t) - 0) = \frac{V_m - 0}{T - 0} (t - 0)$$

$$v(t) = \frac{V_m}{T} (t)$$

$$V_{avg} = \frac{1}{T} \int_0^T v(t) dt$$

$$V_{avg} = \frac{1}{T} \int_0^T \frac{V_m}{T} (t) dt$$

$$= \frac{1}{T} \frac{V_m}{T} \left(\frac{t^2}{2} \right)_0^T$$

$$V_{avg} = \frac{V_m}{T^2} \left(\frac{T^2}{2} \right)$$

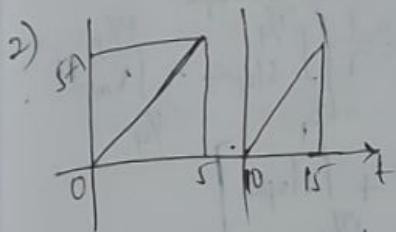
$$\boxed{V_{avg} = \frac{V_m}{2}}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T (v(t))^2 dt}$$

$$= \sqrt{\frac{1}{T} \int_0^T \left(\frac{V_m}{T} t\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \left(\frac{V_m^2}{T^2} \frac{t^3}{3}\right)_0^T}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{3}}}$$



$$I_{avg} = \frac{1}{10} \int_0^5 i(t) dt + \int_5^{10} 0 dt$$

$$= \frac{1}{10} \int_0^5 5 dt$$

$$= \frac{1}{10} (5t)_0^5 = \frac{1}{10} (25) = \frac{25}{10} = 2.5 \text{ Volts. (wrong)}$$

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$(x, y) = (t, I(t))$$

$$(x_1, y_1) = (0, 0)$$

$$(x_2, y_2) = (5, 5)$$

$$I(t) - 0 = \frac{5 - 0}{5 - 0} (t - 0)$$

$$I(t) = \frac{5}{5} (t)$$

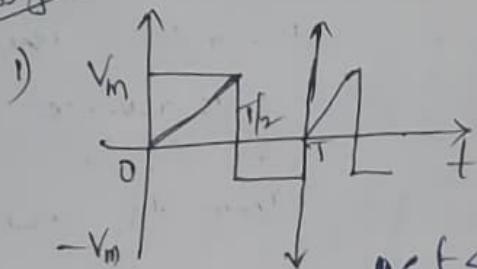
$$\boxed{I(t) = t}$$

$$I_{avg} = \frac{1}{10} \int_0^5 t dt$$

$$= \frac{1}{10} \left[\frac{t^2}{2} \right]_0^5$$

$$I_{avg} = \frac{1}{10} \left[\frac{25}{2} \right] = \frac{25}{4} = 5/4 = 1.25 \text{ Amp}$$

Assignment



$$0 < t < T/2$$

$$T/2 < t < T$$

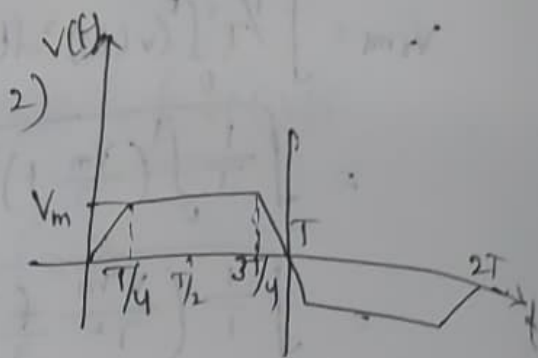
$$v(t) = \text{slope}$$

$$v(t) = (y - y_1) = \frac{(y_2 - y_1)}{x_2 - x_1} (x - x_1)$$

$$v(t) = \frac{V_m}{T/2} (t - 0)$$

$$= \frac{2V_m t}{T}$$

$$V_{avg} = \frac{1}{T} \left[\int_0^{T/2} \text{slope} \, T + \int_{T/2}^T -V_m \, dt \right]$$



Case 1: slope

Case 2: V_m

Case 3: slope

$$V_{avg} = \frac{1}{T} \left[\int_0^{T/4} \text{slope} \, dt + \int_{T/4}^{T/2} V_m \, dt + \int_{T/2}^{3T/4} \text{slope} \, dt \right]$$