Signal and Data Analysis in Neuroscience
Bar-Ilan, spring 2023

Due date: 21/05/23, 08:00

Project 04

General Instructions:

1. Remember legends, axis labels (for all axes) and units for all graphs/plots.
2. Please add comments to your code and submit only the codes specified in the grading table.
3. Write codes in python.
4. We encourage you to write your solution in English. (5 points bonus)
5. Submit a softcopy (including source code) to biu.sigproc@gmail.com.
6. Note the due date and time.
7. Individual work - No code sharing.

1) ROC (20)

The file leftRate.csv and rightRate.CSV are two vectors each containing 1000 responses of the neuron to left or right moving dots accordingly.

A. Plot the ROC curve differentiating the left and right movement.
   Plot a figure justifying what should be considered as H_0 and what as H_1.
   Evaluate the performance of the discrimination based solely on the ROC plot.
B. What is the rate limit (Z) providing 80% true positives? What is the rate of false positives for this rate?

Grading Table

Grade component	Requirements	Points
Accuracy	Accurate calculations. .	5
Explanations	Clear and concise explanations, describing in full details your figures and the conclusions from them. No point reducing for grammar and spelling mistakes.	10
Figures	Correspondence to instructions, no missing components, clear visibility.	5

2) Population vector (45)

Dr. Kafka studies the population coding in the cockroach positioning system. He would like to create a model of population vector to compare his neural recordings:

A. Generate a function which simulates the firing rate of numerous interneurons in the positioning system of the cockroach. The firing rate of each neuron is given by the function:

$r_i = [r_0 \cdot \cos(\alpha - \alpha_i)]_+$ where $[\quad]_+$ indicates half-wave rectification (only positive values, non-positive are set to 0)

The inputs of the code:
r_0 – baseline firing rate
n – number of interneurons
α_i – vector of the preferred angle α of each neuron
α – real angle
The output of the code:
$\hat{\alpha}$ – the prediction of the neurons using population vector

B. Create the following plots and explain the conclusions from each one of them. For graphs with prediction $\hat{\alpha}$ as function $\alpha \in [0,2\pi]$, address the range of values of α which can be decoded correctly.
   - Dr. Kafke measured three interneurons with $r_0 = 55$, $\alpha_i = [0, \frac{2\pi}{3}, \frac{4\pi}{3}]$:
     i. Plot the r_i as a function of $\alpha \in [0,2\pi]$ of each neuron (all on the same plot)
     ii. Plot their population coding prediction $\hat{\alpha}$ for $\alpha \in [0,2\pi]$.
   - Plot the prediction $\hat{\alpha}$ for $\alpha \in [0,2\pi]$ for different number of interneurons from 3 to 10. Assume that the α_i corresponds to the number of interneurons with equal jumps between each element (like in the case of 3 neurons). Use the same baseline.

C. Create an error analysis by adding an independent Gaussian noise component ($\mu = 0, \sigma = 4$) to the r_i of each neuron ($r_{i_{noisy}} = [r_i + noise]_+$). You don't have to try different σ of the Gaussian.
By repeating the additive noise components for at least 100 trials:
   - plot the $\hat{\alpha}$ as a function of α ($mean \pm STD$ or $mean \pm SEM$ across trials). Add on the plot a graph for perfect decoding for comparison.
   - Plot the averaged RMSE of the $\hat{\alpha}$ as a function of α.

Choose only 3 figures with different numbers of interneurons to display and explain the differences between them. Address the effect of the noise on the decoding performance over the α range.

Grading Table

Grade component	**Requirements**	**Points**
code writing (grading instructions only for section A)	clear code writing (code is working, notes). Accurate calculations. No point reducing for efficiency.	20
Section B: Accuracy, explanations and figures	Accurate calculations, clear and full detailed explanations and figures, clear visibility.	15

Section C: Accuracy, explanations and figures	No point reducing for grammar and spelling mistakes.	10

3) Estimators MAP and MLE (35)

Important note: Solve this question analytically, without using any code. You can use excel or wolfram alpha.

The time difference (in weeks) between occurrences of Shogun addiction symptoms may generally be estimated by the geometric distribution:

$$p(x|k) = \begin{cases} (1-k^2)^{x-1} \cdot e^{0.1k} & x \in \{1,2,3,\dots\} \\ 0 & x \leq 0 \end{cases}$$

The parameter k describing the distribution is different for each patient $(0 \leq k \leq 1)$.
The five observations of Shogun addiction in patient X are spaced by 2, 3, 5, 4, 8 weeks.
Assuming the prior distribution $p(\mathrm{k}) = \mathrm{e}^{-0.5\mathrm{k}}$ known for the general patient population.

A. What is the maximum a-posteriori (MAP) estimator for k? Write a detailed calculation. (**15 points**)
B. Assuming there was a 6th observation of 37 for patient X, is the MAP estimator affected? If so, calculate the new estimator. (**10 points**)
C. Considering the original 5 observations only, How would the estimator be affected for a different prior distribution, defined as:

$$p(k) = \begin{cases} \frac{1}{2} & 0 \leq k \leq 1 \\ 0 & otherwise \end{cases}$$

(**5 points**)
D. What is the maximum likelihood (MLE) estimator for k? Explain the differences. (**10 points**)

Grading Table

Grade component	**Requirements**	**Points**
Accuracy	Accurate calculations and full explanations for the calculations.	35

Good luck!
SDA team.