CS101 Part C (Graph Theory)
Tutorial 1

Note. All graphs are undirected.

1. Let T(V,E) be a tree. Prove or disprove:

   If T has a vertex with degree d, then T has at least d leaves.

2. Let D=(d_1, d_2, …, d_n), where d_1 >= d_2 >= … >= d_n, be the degree sequence of a graph on n vertices.

   Prove or disprove:

   There exists a tree T with degree sequence D if and only if d_1+d_2+...+d_n = 2n-2

3. Let T(V,E) be a tree. Let v be a vertex of degree d in T. Let F be the graph obtained by removing vertex v, and all edges incident on v, from tree T. Prove or disprove:

   F has exactly d connected components.

4. Let G(V,E) be a connected graph on n vertices. Prove or disprove:

   G has exactly one cycle if and only if G has exactly n edges.

   What happens when G is not a connected graph?

5. Let G(V,E) be a graph. Suppose there exist three distinct paths P_1, P_2, P_3 between two vertices u and v of G. Prove or disprove:

   There exist at least three cycles in G.

6. Let G(V,E) be a graph on n vertices, such that no edge of G is a bridge. What is the smallest number of edges possible for G?

7. Let G(V,E) be a graph. Prove or disprove:

   G always has an even number of vertices of odd degree.

8. Let G(V,E) be a graph on n vertices with exactly 5 connected components.

   What is the maximum and minimum number of edges possible for G?

9. Let G(V,E) be a graph, and let W be a closed walk in G. Let e be an edge which occurs

exactly 5 times in walk W. Prove or disprove:

There is a cycle containing edge e in graph G.

10. Let G(V,E) be a connected graph. Let D be the maximum length of any path in G. Let P_1 and P_2 be any two paths of maximum length D in G. Prove or disprove:

    P_1 and P_2 share at least one vertex.