Due on 28 January 2023

Computer vision - Assignment 2

In the first few questions we will check our understanding of some basic elements of projective geometry.

1. Consider the point $p(4, 18, 2)$ in projective space. Which point(s) in the real 2D plane does it correspond to?
2. What are the possible equivalent representations in $\mathbb{P}^2$ of the point $(3, 8)$ in $\mathbb{R}^2$?
3. What is the intersection point of the lines in $\mathbb{P}^2$ described by $(5, 2, 1)$ and $(5, 2, 9)$?
4. Consider a projective transformation $H = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and let C be the conic given by $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$. Find the transformation of C under H.
5. Consider the homography matrix H between two images I_1 and I_2 of the same scene. $H = \begin{pmatrix} 1 & 3 & 5 \\ 4 & 3 & 7 \\ 5 & -4 & 2 \end{pmatrix}$. Suppose $(5, 6) \in \mathbb{R}^2$ is a point in I_1. What is the corresponding point in I_2?
6. Recall that the line at infinity containing all ideal points is given by $L_\infty = (0, 0, 1)$. Consider a transformation given by $H = \begin{pmatrix} 2 & 3 & -4 \\ -4 & 3 & 2 \\ 3 & 1 & 1 \end{pmatrix}$. Compute the vanishing line in the transformed space.

In the next three questions we will learn how to find a homography and explore its uses.

7. Download the 3 images in the folder named CalvinandHobbes from the computer vision class folder `https://drive.google.com/drive/folders/1igbFa7RhAbqQf61HUQ3WChbHW63Nq-ED?usp=sharing`. We want to find a homography that maps image1.jpg to image2.jpg.
   (a) For this, select sets of four point correspondences on the two images, store them in two arrays called pts1 and pts2 respectively. Display these arrays clearly.
   (b) Compute a homography that maps image1 to image2 using DLT (you may use the inbuilt SVD function, but not the inbuilt function for computing DLT or homography). The system of equations being used to solve for elements of the homography must be clearly stated. What is the rank of the matrix of this system?
   (c) Compare your result with results of using the function findHomography in OpenCV.
   (d) Observe how the homography affects the other parts of the image. Can you explain why?
8. Use the code in the above exercise to find a homography that 'straightens' image1 as follows: you are given that the aspect ratio (width/height) of the book is 30/23. Mark and store the 4 corner points of the book in image1; choose 4 points in the destination image that you will map the chosen points to (keeping the aspect ratio in mind). Then compute the homography and apply it to image1. The resulting image should be akin to what is seen in image3.
9. Find at least two different use cases of a homography and demonstrate these cases using either the given image or your own images.