

Chapter 5 – Support Vector Machines

This notebook contains all the sample code and solutions to the exercises in chapter 5.



Run in Google Colab (https://colab.research.google.com/github/ageron/handson-ml2/blob/master/05_support_vector_machines.ipynb)

Setup

First, let's import a few common modules, ensure Matplotlib plots figures inline and prepare a function to save the figures. We also check that Python 3.5 or later is installed (although Python 2.x may work, it is deprecated so we strongly recommend you use Python 3 instead), as well as Scikit-Learn ≥ 0.20 .

```
In [1]: # Python  $\geq 3.5$  is required
import sys
assert sys.version_info >= (3, 5)

# Scikit-Learn  $\geq 0.20$  is required
import sklearn
assert sklearn.__version__ >= "0.20"

# Common imports
import numpy as np
import os

# to make this notebook's output stable across runs
np.random.seed(42)

# To plot pretty figures
%matplotlib inline
import matplotlib as mpl
import matplotlib.pyplot as plt
mpl.rc('axes', labelsize=14)
mpl.rc('xtick', labelsize=12)
mpl.rc('ytick', labelsize=12)

# Where to save the figures
PROJECT_ROOT_DIR = "."
CHAPTER_ID = "svm"
IMAGES_PATH = os.path.join(PROJECT_ROOT_DIR, "images", CHAPTER_ID)
os.makedirs(IMAGES_PATH, exist_ok=True)

def save_fig(fig_id, tight_layout=True, fig_extension="png", resolution=300):
    path = os.path.join(IMAGES_PATH, fig_id + "." + fig_extension)
    print("Saving figure", fig_id)
    if tight_layout:
        plt.tight_layout()
    plt.savefig(path, format=fig_extension, dpi=resolution)
```

Large margin classification

The next few code cells generate the first figures in chapter 5. The first actual code sample comes after:

```
In [2]: from sklearn.svm import SVC
from sklearn import datasets
```

```
iris = datasets.load_iris()
X = iris["data"][:, (2, 3)] # petal length, petal width
y = iris["target"]

setosa_or_versicolor = (y == 0) | (y == 1)
X = X[setosa_or_versicolor]
y = y[setosa_or_versicolor]

# SVM Classifier model
svm_clf = SVC(kernel="linear", C=float("inf"))
svm_clf.fit(X, y)
```

```
Out[2]: SVC(C=inf, cache_size=200, class_weight=None, coef0=0.0,
decision_function_shape='ovr', degree=3, gamma='auto_deprecated',
kernel='linear', max_iter=-1, probability=False, random_state=None,
shrinking=True, tol=0.001, verbose=False)
```

```

In [3]: # Bad models
x0 = np.linspace(0, 5.5, 200)
pred_1 = 5*x0 - 20
pred_2 = x0 - 1.8
pred_3 = 0.1 * x0 + 0.5

def plot_svc_decision_boundary(svm_clf, xmin, xmax):
    w = svm_clf.coef_[0]
    b = svm_clf.intercept_[0]

    # At the decision boundary, w0*x0 + w1*x1 + b = 0
    # => x1 = -w0/w1 * x0 - b/w1
    x0 = np.linspace(xmin, xmax, 200)
    decision_boundary = -w[0]/w[1] * x0 - b/w[1]

    margin = 1/w[1]
    gutter_up = decision_boundary + margin
    gutter_down = decision_boundary - margin

    svs = svm_clf.support_vectors_
    plt.scatter(svs[:, 0], svs[:, 1], s=180, facecolors='#FFAAAA')
    plt.plot(x0, decision_boundary, "k-", linewidth=2)
    plt.plot(x0, gutter_up, "k--", linewidth=2)
    plt.plot(x0, gutter_down, "k--", linewidth=2)

fig, axes = plt.subplots(ncols=2, figsize=(10,2.7), sharey=True)

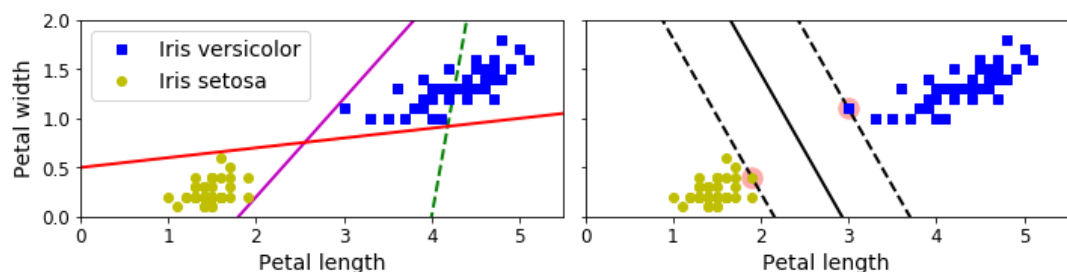
plt.sca(axes[0])
plt.plot(x0, pred_1, "g--", linewidth=2)
plt.plot(x0, pred_2, "m-", linewidth=2)
plt.plot(x0, pred_3, "r-", linewidth=2)
plt.plot(X[:, 0][y==1], X[:, 1][y==1], "bs", label="Iris versicolor")
plt.plot(X[:, 0][y==0], X[:, 1][y==0], "yo", label="Iris setosa")
plt.xlabel("Petal length", fontsize=14)
plt.ylabel("Petal width", fontsize=14)
plt.legend(loc="upper left", fontsize=14)
plt.axis([0, 5.5, 0, 2])

plt.sca(axes[1])
plot_svc_decision_boundary(svm_clf, 0, 5.5)
plt.plot(X[:, 0][y==1], X[:, 1][y==1], "bs")
plt.plot(X[:, 0][y==0], X[:, 1][y==0], "yo")
plt.xlabel("Petal length", fontsize=14)
plt.axis([0, 5.5, 0, 2])

save_fig("large_margin_classification_plot")
plt.show()

```

Saving figure large_margin_classification_plot



Sensitivity to feature scales

```

In [4]: Xs = np.array([[1, 50], [5, 20], [3, 80], [5, 60]]).astype(np.float64)
ys = np.array([0, 0, 1, 1])
svm_clf = SVC(kernel="linear", C=100)

```

```

svm_clf.fit(Xs, ys)

plt.figure(figsize=(9,2.7))
plt.subplot(121)
plt.plot(Xs[:, 0][ys==1], Xs[:, 1][ys==1], "bo")
plt.plot(Xs[:, 0][ys==0], Xs[:, 1][ys==0], "ms")
plot_svc_decision_boundary(svm_clf, 0, 6)
plt.xlabel("$x_0$", fontsize=20)
plt.ylabel("$x_1$", fontsize=20, rotation=0)
plt.title("Unscaled", fontsize=16)
plt.axis([0, 6, 0, 90])

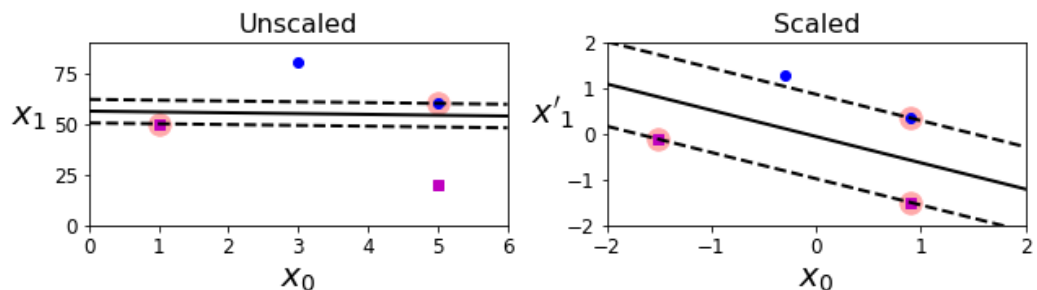
from sklearn.preprocessing import StandardScaler
scaler = StandardScaler()
X_scaled = scaler.fit_transform(Xs)
svm_clf.fit(X_scaled, ys)

plt.subplot(122)
plt.plot(X_scaled[:, 0][ys==1], X_scaled[:, 1][ys==1], "bo")
plt.plot(X_scaled[:, 0][ys==0], X_scaled[:, 1][ys==0], "ms")
plot_svc_decision_boundary(svm_clf, -2, 2)
plt.xlabel("$x_0$", fontsize=20)
plt.ylabel("$x'_1$", fontsize=20, rotation=0)
plt.title("Scaled", fontsize=16)
plt.axis([-2, 2, -2, 2])

save_fig("sensitivity_to_feature_scales_plot")

```

Saving figure sensitivity_to_feature_scales_plot



Sensitivity to outliers

```

In [5]: X_outliers = np.array([[3.4, 1.3], [3.2, 0.8]])
y_outliers = np.array([0, 0])
Xo1 = np.concatenate([X, X_outliers[:,1]], axis=0)
yo1 = np.concatenate([y, y_outliers[:,1]], axis=0)
Xo2 = np.concatenate([X, X_outliers[1:]], axis=0)
yo2 = np.concatenate([y, y_outliers[1:]], axis=0)

svm_clf2 = SVC(kernel="linear", C=10**9)
svm_clf2.fit(Xo2, yo2)

fig, axes = plt.subplots(ncols=2, figsize=(10,2.7), sharey=True)

plt.sca(axes[0])
plt.plot(Xo1[:, 0][yo1==1], Xo1[:, 1][yo1==1], "bs")
plt.plot(Xo1[:, 0][yo1==0], Xo1[:, 1][yo1==0], "yo")
plt.text(0.3, 1.0, "Impossible!", fontsize=24, color="red")
plt.xlabel("Petal length", fontsize=14)
plt.ylabel("Petal width", fontsize=14)
plt.annotate("Outlier",
             xy=(X_outliers[0][0], X_outliers[0][1]),
             xytext=(2.5, 1.7),
             ha="center",

```

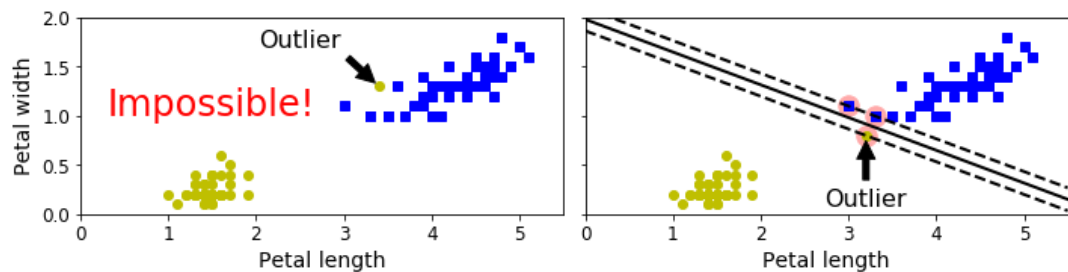
```

        arrowprops=dict(facecolor='black', shrink=0.1),
        fontsize=16,
    )
plt.axis([0, 5.5, 0, 2])

plt.sca(axes[1])
plt.plot(Xo2[:, 0][yo2==1], Xo2[:, 1][yo2==1], "bs")
plt.plot(Xo2[:, 0][yo2==0], Xo2[:, 1][yo2==0], "yo")
plot_svc_decision_boundary(svm_clf2, 0, 5.5)
plt.xlabel("Petal length", fontsize=14)
plt.annotate("Outlier",
             xy=(X_outliers[1][0], X_outliers[1][1]),
             xytext=(3.2, 0.08),
             ha="center",
             arrowprops=dict(facecolor='black', shrink=0.1),
             fontsize=16,
            )
plt.axis([0, 5.5, 0, 2])

save_fig("sensitivity_to_outliers_plot")
plt.show()
Saving figure sensitivity_to_outliers_plot

```



Large margin vs margin violations

This is the first code example in chapter 5:

```

In [6]: import numpy as np
        from sklearn import datasets
        from sklearn.pipeline import Pipeline
        from sklearn.preprocessing import StandardScaler
        from sklearn.svm import LinearSVC

        iris = datasets.load_iris()
        X = iris["data"][:, (2, 3)] # petal length, petal width
        y = (iris["target"] == 2).astype(np.float64) # Iris virginica

        svm_clf = Pipeline([
            ("scaler", StandardScaler()),
            ("linear_svc", LinearSVC(C=1, loss="hinge", random_state=42)),
        ])

        svm_clf.fit(X, y)

Out[6]: Pipeline(memory=None,
                 steps=[('scaler', StandardScaler(copy=True, with_mean=True, with_std=True)),
                        ('linear_svc', LinearSVC(C=1, class_weight=None, dual=True, fit_intercept=True,
                        intercept_scaling=1, loss='hinge', max_iter=1000, multi_class='ovr',
                        penalty='l2', random_state=42, tol=0.0001, verbose=0))])

```

```
In [7]: svm_clf.predict([[5.5, 1.7]])
```

```
Out[7]: array([1.])
```

Now let's generate the graph comparing different regularization settings:

```
In [8]: scaler = StandardScaler()
svm_clf1 = LinearSVC(C=1, loss="hinge", random_state=42)
svm_clf2 = LinearSVC(C=100, loss="hinge", random_state=42)

scaled_svm_clf1 = Pipeline([
    ("scaler", scaler),
    ("linear_svc", svm_clf1),
])
scaled_svm_clf2 = Pipeline([
    ("scaler", scaler),
    ("linear_svc", svm_clf2),
])

scaled_svm_clf1.fit(X, y)
scaled_svm_clf2.fit(X, y)
```

/Users/ageron/miniconda3/envs/tf2b/lib/python3.7/site-packages/sklearn/svm/base.py:931: ConvergenceWarning: Liblinear failed to converge, increase the number of iterations.

"the number of iterations.", ConvergenceWarning)

```
Out[8]: Pipeline(memory=None,
               steps=[('scaler', StandardScaler(copy=True, with_mean=True, with_std=True)),
                      ('linear_svc', LinearSVC(C=100, class_weight=None, dual=True, fit_intercept=True,
        intercept_scaling=1, loss='hinge', max_iter=1000, multi_class='ovr',
        penalty='l2', random_state=42, tol=0.0001, verbose=0))])
```

```
In [9]: # Convert to unscaled parameters
b1 = svm_clf1.decision_function([-scaler.mean_ / scaler.scale_])
b2 = svm_clf2.decision_function([-scaler.mean_ / scaler.scale_])
w1 = svm_clf1.coef_[0] / scaler.scale_
w2 = svm_clf2.coef_[0] / scaler.scale_
svm_clf1.intercept_ = np.array([b1])
svm_clf2.intercept_ = np.array([b2])
svm_clf1.coef_ = np.array([w1])
svm_clf2.coef_ = np.array([w2])

# Find support vectors (LinearSVC does not do this automatically)
t = y * 2 - 1
support_vectors_idx1 = (t * (X.dot(w1) + b1) < 1).ravel()
support_vectors_idx2 = (t * (X.dot(w2) + b2) < 1).ravel()
svm_clf1.support_vectors_ = X[support_vectors_idx1]
svm_clf2.support_vectors_ = X[support_vectors_idx2]
```

```
In [10]: fig, axes = plt.subplots(ncols=2, figsize=(10,2.7), sharey=True)

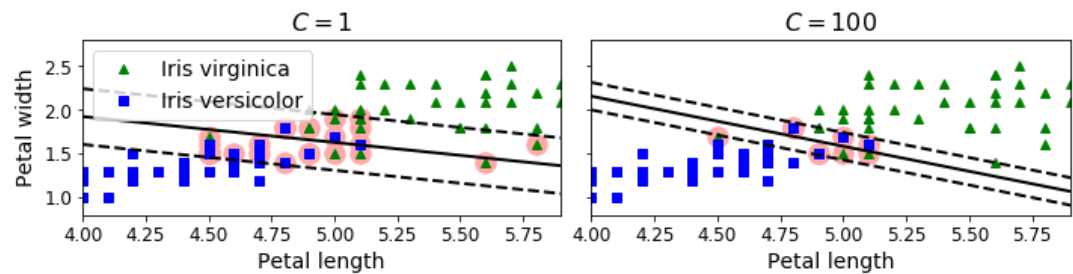
plt.sca(axes[0])
plt.plot(X[:, 0][y==1], X[:, 1][y==1], "g^", label="Iris virginica")
plt.plot(X[:, 0][y==0], X[:, 1][y==0], "bs", label="Iris versicolor")
plot_svc_decision_boundary(svm_clf1, 4, 5.9)
plt.xlabel("Petal length", fontsize=14)
plt.ylabel("Petal width", fontsize=14)
plt.legend(loc="upper left", fontsize=14)
plt.title("$C = {}$".format(svm_clf1.C), fontsize=16)
plt.axis([4, 5.9, 0.8, 2.8])

plt.sca(axes[1])
plt.plot(X[:, 0][y==1], X[:, 1][y==1], "g^")
plt.plot(X[:, 0][y==0], X[:, 1][y==0], "bs")
plot_svc_decision_boundary(svm_clf2, 4, 5.99)
```

```
plt.xlabel("Petal length", fontsize=14)
plt.title("$C = {}".format(svm_clf2.C), fontsize=16)
plt.axis([4, 5.9, 0.8, 2.8])
```

```
save_fig("regularization_plot")
```

Saving figure regularization_plot



Non-linear classification

```

In [11]: X1D = np.linspace(-4, 4, 9).reshape(-1, 1)
X2D = np.c_[X1D, X1D**2]
y = np.array([0, 0, 1, 1, 1, 1, 1, 0, 0])

plt.figure(figsize=(10, 3))

plt.subplot(121)
plt.grid(True, which='both')
plt.axhline(y=0, color='k')
plt.plot(X1D[:, 0][y==0], np.zeros(4), "bs")
plt.plot(X1D[:, 0][y==1], np.zeros(5), "g^")
plt.gca().get_yaxis().set_ticks([])
plt.xlabel(r"$x_1$", fontsize=20)
plt.axis([-4.5, 4.5, -0.2, 0.2])

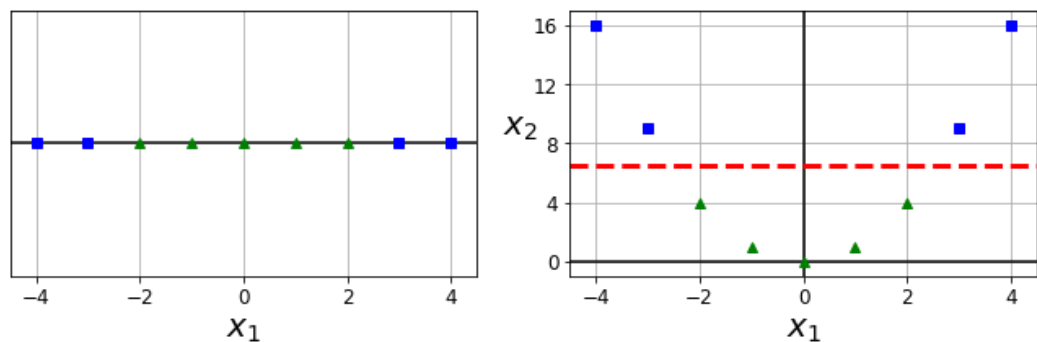
plt.subplot(122)
plt.grid(True, which='both')
plt.axhline(y=0, color='k')
plt.axvline(x=0, color='k')
plt.plot(X2D[:, 0][y==0], X2D[:, 1][y==0], "bs")
plt.plot(X2D[:, 0][y==1], X2D[:, 1][y==1], "g^")
plt.xlabel(r"$x_1$", fontsize=20)
plt.ylabel(r"$x_2$", fontsize=20, rotation=0)
plt.gca().get_yaxis().set_ticks([0, 4, 8, 12, 16])
plt.plot([-4.5, 4.5], [6.5, 6.5], "r--", linewidth=3)
plt.axis([-4.5, 4.5, -1, 17])

plt.subplots_adjust(right=1)

save_fig("higher_dimensions_plot", tight_layout=False)
plt.show()

```

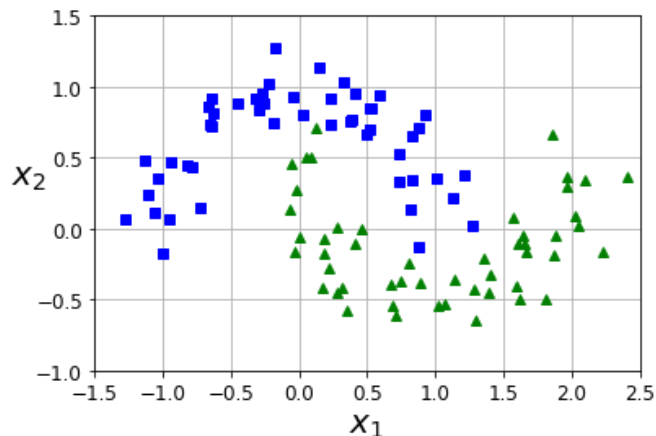
Saving figure higher_dimensions_plot




```
In [12]: from sklearn.datasets import make_moons
X, y = make_moons(n_samples=100, noise=0.15, random_state=42)

def plot_dataset(X, y, axes):
    plt.plot(X[:, 0][y==0], X[:, 1][y==0], "bs")
    plt.plot(X[:, 0][y==1], X[:, 1][y==1], "g^")
    plt.axis(axes)
    plt.grid(True, which='both')
    plt.xlabel(r"$x_1$", fontsize=20)
    plt.ylabel(r"$x_2$", fontsize=20, rotation=0)

plot_dataset(X, y, [-1.5, 2.5, -1, 1.5])
plt.show()
```



```
In [13]: from sklearn.datasets import make_moons
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import PolynomialFeatures

polynomial_svm_clf = Pipeline([
    ("poly_features", PolynomialFeatures(degree=3)),
    ("scaler", StandardScaler()),
    ("svm_clf", LinearSVC(C=10, loss="hinge", random_state=42))
])

polynomial_svm_clf.fit(X, y)
```

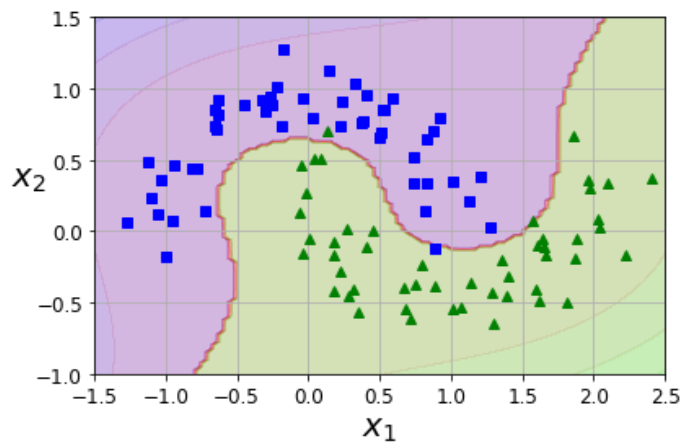
```
Out[13]: Pipeline(memory=None,
               steps=[('poly_features', PolynomialFeatures(degree=3, include_bias=True,
interaction_only=False)), ('scaler', StandardScaler(copy=True, with_mean=True,
with_std=True)), ('svm_clf', LinearSVC(C=10, class_weight=None, dual=True, fit
_intercept=True,
            intercept_scaling=1, loss='hinge', max_iter=1000, multi_class='ovr',
penalty='l2', random_state=42, tol=0.0001, verbose=0))])
```

```
In [14]: def plot_predictions(clf, axes):
    x0s = np.linspace(axes[0], axes[1], 100)
    x1s = np.linspace(axes[2], axes[3], 100)
    x0, x1 = np.meshgrid(x0s, x1s)
    X = np.c_[x0.ravel(), x1.ravel()]
    y_pred = clf.predict(X).reshape(x0.shape)
    y_decision = clf.decision_function(X).reshape(x0.shape)
    plt.contourf(x0, x1, y_pred, cmap=plt.cm.brg, alpha=0.2)
    plt.contourf(x0, x1, y_decision, cmap=plt.cm.brg, alpha=0.1)

plot_predictions(polynomial_svm_clf, [-1.5, 2.5, -1, 1.5])
plot_dataset(X, y, [-1.5, 2.5, -1, 1.5])

save_fig("moons_polynomial_svc_plot")
plt.show()
```

Saving figure moons_polynomial_svc_plot



In [15]: `from sklearn.svm import SVC`

```
poly_kernel_svm_clf = Pipeline([
    ("scaler", StandardScaler()),
    ("svm_clf", SVC(kernel="poly", degree=3, coef0=1, C=5))
])
poly_kernel_svm_clf.fit(X, y)
```

Out[15]: Pipeline(memory=None, steps=[('scaler', StandardScaler(copy=True, with_mean=True, with_std=True)), ('svm_clf', SVC(C=5, cache_size=200, class_weight=None, coef0=1, decision_function_shape='ovr', degree=3, gamma='auto_deprecated', kernel='poly', max_iter=-1, probability=False, random_state=None, shrinking=True, tol=0.001, verbose=False))])

```
poly100_kernel_svm_clf = Pipeline([
    ("scaler", StandardScaler()),
    ("svm_clf", SVC(kernel="poly", degree=10, coef0=100, C=5))
])
poly100_kernel_svm_clf.fit(X, y)
```

Out[16]: Pipeline(memory=None, steps=[('scaler', StandardScaler(copy=True, with_mean=True, with_std=True)), ('svm_clf', SVC(C=5, cache_size=200, class_weight=None, coef0=100, decision_function_shape='ovr', degree=10, gamma='auto_deprecated', kernel='poly', max_iter=-1, probability=False, random_state=None, shrinking=True, tol=0.001, verbose=False))])

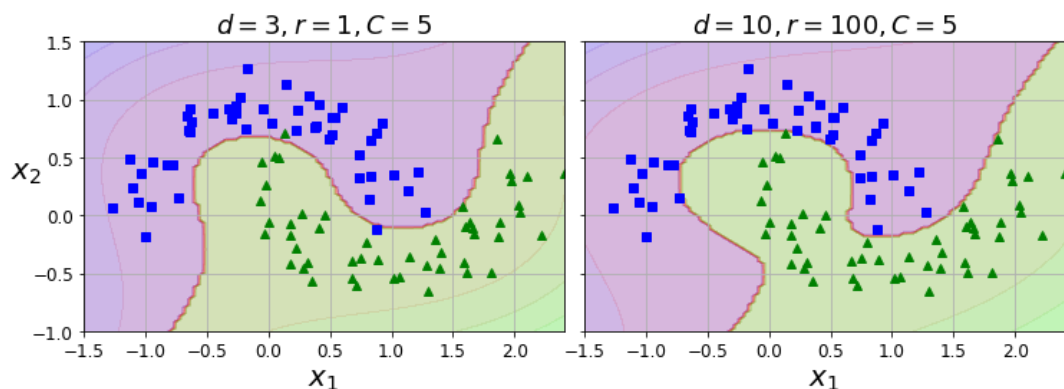
```
In [17]: fig, axes = plt.subplots(ncols=2, figsize=(10.5, 4), sharey=True)

plt.sca(axes[0])
plot_predictions(poly_kernel_svm_clf, [-1.5, 2.45, -1, 1.5])
plot_dataset(X, y, [-1.5, 2.4, -1, 1.5])
plt.title(r"$d=3, r=1, C=5$", fontsize=18)

plt.sca(axes[1])
plot_predictions(poly100_kernel_svm_clf, [-1.5, 2.45, -1, 1.5])
plot_dataset(X, y, [-1.5, 2.4, -1, 1.5])
plt.title(r"$d=10, r=100, C=5$", fontsize=18)
plt.ylabel("")

save_fig("moons_kernelized_polynomial_svc_plot")
plt.show()
```

Saving figure moons_kernelized_polynomial_svc_plot



```
In [18]: def gaussian_rbf(x, landmark, gamma):
    return np.exp(-gamma * np.linalg.norm(x - landmark, axis=1)**2)

gamma = 0.3

x1s = np.linspace(-4.5, 4.5, 200).reshape(-1, 1)
x2s = gaussian_rbf(x1s, -2, gamma)
x3s = gaussian_rbf(x1s, 1, gamma)

XK = np.c_[gaussian_rbf(X1D, -2, gamma), gaussian_rbf(X1D, 1, gamma)]
yk = np.array([0, 0, 1, 1, 1, 1, 0, 0])

plt.figure(figsize=(10.5, 4))

plt.subplot(121)
plt.grid(True, which='both')
plt.axhline(y=0, color='k')
plt.scatter(x=[-2, 1], y=[0, 0], s=150, alpha=0.5, c="red")
plt.plot(X1D[:, 0][yk==0], np.zeros(4), "bs")
plt.plot(X1D[:, 0][yk==1], np.zeros(5), "g^")
plt.plot(x1s, x2s, "g--")
plt.plot(x1s, x3s, "b:")
plt.gca().get_yaxis().set_ticks([0, 0.25, 0.5, 0.75, 1])
plt.xlabel(r"$x_1$", fontsize=20)
plt.ylabel(r"$Similarity$", fontsize=14)
plt.annotate(r'$\mathbf{x}$',
             xy=(X1D[3, 0], 0),
             xytext=(-0.5, 0.20),
             ha="center",
             arrowprops=dict(facecolor='black', shrink=0.1),
             fontsize=18,
             )
plt.text(-2, 0.9, "$x_2$", ha="center", fontsize=20)
plt.text(1, 0.9, "$x_3$", ha="center", fontsize=20)
```

```

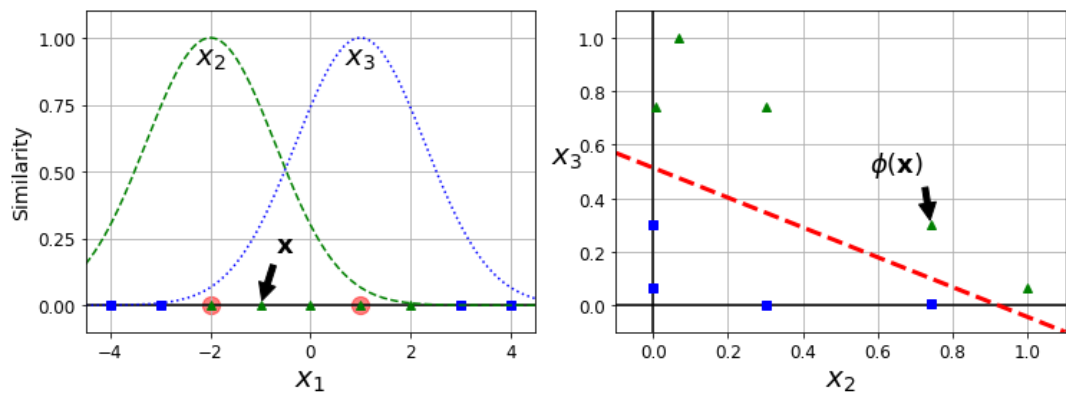
plt.axis([-4.5, 4.5, -0.1, 1.1])

plt.subplot(122)
plt.grid(True, which='both')
plt.axhline(y=0, color='k')
plt.axvline(x=0, color='k')
plt.plot(XK[:, 0][yk==0], XK[:, 1][yk==0], "bs")
plt.plot(XK[:, 0][yk==1], XK[:, 1][yk==1], "g^")
plt.xlabel(r"$x_2$", fontsize=20)
plt.ylabel(r"$x_3$", fontsize=20, rotation=0)
plt.annotate(r'$\phi(\mathbf{x})$',
             xy=(XK[3, 0], XK[3, 1]),
             xytext=(0.65, 0.50),
             ha="center",
             arrowprops=dict(facecolor='black', shrink=0.1,
                             fontsize=18,
                             )
            )
plt.plot([-0.1, 1.1], [0.57, -0.1], "r--", linewidth=3)
plt.axis([-0.1, 1.1, -0.1, 1.1])

plt.subplots_adjust(right=1)

save_fig("kernel_method_plot")
plt.show()
Saving figure kernel_method_plot

```



```

In [19]: x1_example = X1D[3, 0]
for landmark in (-2, 1):
    k = gaussian_rbf(np.array([[x1_example]]), np.array([[landmark]]), gamma)
    print("Phi({}, {}) = {}".format(x1_example, landmark, k))

```

```

Phi(-1.0, -2) = [0.74081822]
Phi(-1.0, 1) = [0.30119421]

```

```

In [20]: rbf_kernel_svm_clf = Pipeline([
        ("scaler", StandardScaler()),
        ("svm_clf", SVC(kernel="rbf", gamma=5, C=0.001))
    ])
rbf_kernel_svm_clf.fit(X, y)

```

```

Out[20]: Pipeline(memory=None,
                 steps=[('scaler', StandardScaler(copy=True, with_mean=True, with_std=True)),
                        ('svm_clf', SVC(C=0.001, cache_size=200, class_weight=None, coef0=0.0,
                                decision_function_shape='ovr', degree=3, gamma=5, kernel='rbf',
                                max_iter=-1, probability=False, random_state=None, shrinking=True,
                                tol=0.001, verbose=False))])

```

```

In [21]: from sklearn.svm import SVC

gamma1, gamma2 = 0.1, 5
C1, C2 = 0.001, 1000
hyperparams = (gamma1, C1), (gamma1, C2), (gamma2, C1), (gamma2, C2)

```

```

svm_clfs = []
for gamma, C in hyperparams:
    rbf_kernel_svm_clf = Pipeline([
        ("scaler", StandardScaler()),
        ("svm_clf", SVC(kernel="rbf", gamma=gamma, C=C))
    ])
    rbf_kernel_svm_clf.fit(X, y)
    svm_clfs.append(rbf_kernel_svm_clf)

fig, axes = plt.subplots(nrows=2, ncols=2, figsize=(10.5, 7), sharex=True, sharey=True)

for i, svm_clf in enumerate(svm_clfs):
    plt.sca(axes[i // 2, i % 2])
    plot_predictions(svm_clf, [-1.5, 2.45, -1, 1.5])
    plot_dataset(X, y, [-1.5, 2.45, -1, 1.5])
    gamma, C = hyperparams[i]
    plt.title(r"$\gamma = {}, C = {}".format(gamma, C), fontsize=16)
    if i in (0, 1):
        plt.xlabel("")
    if i in (1, 3):
        plt.ylabel("")

save_fig("moons_rbf_svc_plot")
plt.show()
Saving figure moons_rbf_svc_plot

```

