

Lecture 3: 12 April, 2021

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Data Mining and Machine Learning
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Market-Basket Analysis

- Items $I = \{i_1, i_2, \dots, i_N\}$, transactions $T = \{t_1, t_2, \dots, t_M\}$
- Identify all **association rules** $X \rightarrow Y$ meeting two thresholds
 - **Confidence**: $\frac{(X \cup Y).count}{X.count} \geq \chi$
 - **Support**: $\frac{(X \cup Y).count}{M} \geq \sigma$
- First identify **frequent itemsets** Z , such that $Z.count \geq \sigma M$
- Apriori algorithm
 - If X is not frequent, no $Y \supseteq X$ can be frequent
 - Find frequent sets levelwise: F_1, F_2, \dots are frequent itemsets of size $1, 2, \dots$
- How do we generate association rules from frequent itemsets?

Association rules

Naïve strategy

- For every frequent itemset Z
 - Enumerate all pairs $X, Y \subseteq Z, X \cap Y = \emptyset$
 - Check $\frac{(X \cup Y).count}{X.count} \geq \chi$
- Can we do better?
- Sufficient to check all partitions of Z
 - If $X, Y \subseteq Z, X \cup Y$ is also a frequent itemset

Association rules

- Sufficient to check all partitions of Z
- Suppose $Z = X \uplus Y$, $X \rightarrow Y$ is a valid rule and $y \in Y$
- What about $(X \cup \{y\}) \rightarrow Y \setminus \{y\}$?
 - Know $\frac{(X \cup Y).count}{X.count} \geq \chi$
 - Check $\frac{(X \cup Y).count}{(X \cup \{y\}).count} \geq \chi$
 - $X.count \geq (X \cup \{y\}).count$, always
 - Second fraction has smaller denominator, so $(X \cup \{y\}) \rightarrow Y \setminus \{y\}$ is also a valid rule

Observation: Can use apriori principle again!

Apriori for association rules

- If $X \rightarrow Y$ is a valid rule, and $y \in Y$,
 $(X \cup \{y\}) \rightarrow Y \setminus \{y\}$ must also be a valid rule
- If $X \rightarrow Y$ is **not** a valid rule, and $x \in X$,
 $(X \setminus \{x\}) \rightarrow Y \cup \{x\}$ **cannot** be a valid rule
- Start by checking rules with single element on the right
 - $Z \setminus z \rightarrow \{z\}$
- For $X \rightarrow \{x, y\}$ to be a valid rule, both
 $(X \cup \{x\}) \rightarrow \{y\}$ and $(X \cup \{y\}) \rightarrow \{x\}$ must be valid
- Explore partitions of each frequent itemset “level by level”

Association rules for classification

- Classify documents by topic
- Consider the table on the right
- Items are regular words and topics
- Documents are transactions — set of words and one topic
- Look for association rules of a special form
 - $\{\text{student, school}\} \rightarrow \{\text{Education}\}$
 - $\{\text{game, team}\} \rightarrow \{\text{Sports}\}$
- Right hand side always a single topic
- **Class Association Rules**

| Words in document | Topic |
|-----------------------------|-----------|
| student, teach, school | Education |
| student, school | Education |
| teach, school, city, game | Education |
| cricket, football | Sports |
| football, player, spectator | Sports |
| cricket, coach, game, team | Sports |
| football, team, city, game | Sports |

Supervised learning

- A set of items
 - Each item is characterized by attributes (a_1, a_2, \dots, a_k)
 - Each item is assigned a class or category c
- Given a set of examples, predict c for a new item with attributes $(a'_1, a'_2, \dots, a'_k)$
- Examples provided are called **training data**
- Aim is to **learn** a mathematical model that **generalizes** the training data
 - Model built from training data should extend to previously unseen inputs
- **Classification** problem
 - Usually assumed to binary — two classes

Example: Loan application data set

| ID | Age | Has_job | Own_house | Credit_rating | Class |
|----|--------|---------|-----------|---------------|------------|
| 1 | young | false | false | fair | No |
| 2 | young | false | false | good | No |
| 3 | young | true | false | good | Yes |
| 4 | young | true | true | fair | Yes |
| 5 | young | false | false | fair | No |
| 6 | middle | false | false | fair | No |
| 7 | middle | false | false | good | No |
| 8 | middle | true | true | good | Yes |
| 9 | middle | false | true | excellent | Yes |
| 10 | middle | false | true | excellent | Yes |
| 11 | old | false | true | excellent | Yes |
| 12 | old | false | true | good | Yes |
| 13 | old | true | false | good | Yes |
| 14 | old | true | false | excellent | Yes |
| 15 | old | false | false | fair | No |

Basic assumptions

Fundamental assumption of machine learning

- Distribution of training examples is identical to distribution of unseen data

What does it mean to learn from the data?

- Build a model that does better than random guessing
 - In the loan data set, always saying **Yes** would be correct about 9/15 of the time
- Performance should ideally improve with more training data

How do we evaluate the performance of a model?

- Model is optimized for the training data. How well does it work for unseen data?
- Don't know the correct answers in advance to compare — different from normal software verification

The road ahead

Many different models

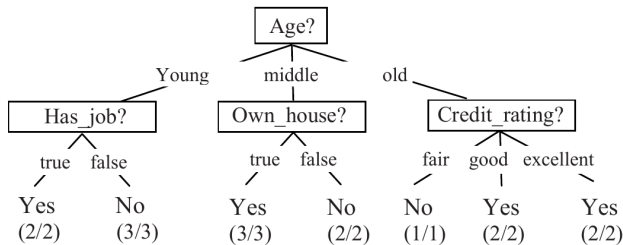
- Decision trees
- Probabilistic models — naïve Bayes classifiers
- Models based on geometric separators
 - Support vector machines (SVM)
 - Neural networks

Important issues related to supervised learning

- Evaluating models
- Ensuring that models generalize well to unseen data
 - A theoretical framework to provide some guarantees
- Strategies to deal with the training data bottleneck

Decision trees

- Play “20 Questions” with the training data
- Query an attribute
 - Partition the training data based on the answer
- Repeat until you reach a partition with a uniform category
- Queries are **adaptive**
 - Different along each path, depends on history



| ID | Age | Has_job | Own_house | Credit_rating | Class |
|----|--------|---------|-----------|---------------|-------|
| 1 | young | false | false | fair | No |
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Decision tree algorithm

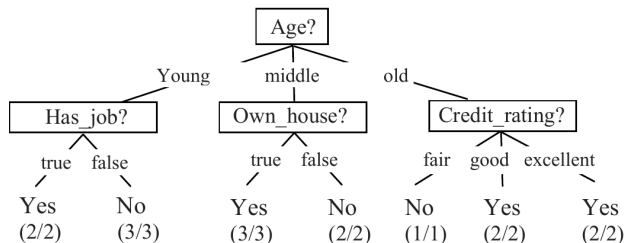
A : current set of attributes

Pick $a \in A$, create children corresponding to resulting partition with attributes $A \setminus \{a\}$

Stopping criterion:

- Current node has uniform class label
- A is empty — no more attributes to query

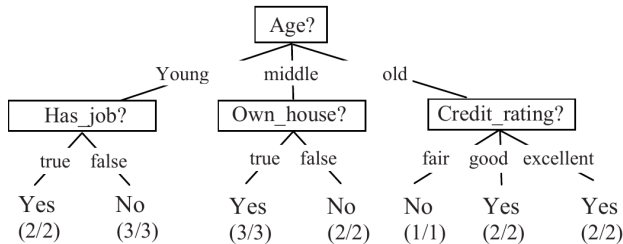
If a leaf node is not uniform, use majority class as prediction



- Non-uniform leaf node — identical combination of attributes, but different classes
- Attributes do not capture all criteria used for classification

Decision trees

- Tree is not unique
- Which tree is better?
- Prefer small trees
 - Explainability
 - Generalize better (see later)



Unfortunately

- Finding smallest tree is NP-complete — for any definition of “smallest”
- Instead, greedy heuristic

