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Data Mining and Machine Learning
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Bayesian classifiers

- As before
  - Attributes $\{A_1, A_2, \ldots, A_k\}$ and
  - Classes $C = \{c_1, c_2, \ldots c_\ell\}$
- Each class c_i defines a probabilistic model for attributes
  - $Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i)$
- Given a data item $d = (a_1, a_2, \ldots, a_k)$, identify the best class c for d
- Maximize $Pr(C = c_i \mid A_1 = a_1, \ldots, A_k = a_k)$

Bayesian classification

- Maximize $Pr(C = c_i \mid A_1 = a_1, \ldots, A_k = a_k)$
- By Bayes' rule,

$$Pr(C = c_i \mid A_1 = a_1, \ldots, A_k = a_k)$$

$$= \frac{Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i) \cdot Pr(C = c_i)}{Pr(A_1 = a_1, \ldots, A_k = a_k)}$$

$$= \frac{Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i) \cdot Pr(C = c_i)}{\sum_{j=1}^{\ell} Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_j) \cdot Pr(C = c_j)}$$

- Denominator is the same for all c_i, so sufficient to maximize

$$Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i) \cdot Pr(C = c_i)$$

Example

- To classify $A = g, B = q$

$P(C=t \mid A=g, B=q)$

- $Pr(C = t) = 5/10 = 1/2$
- $Pr(A = g, B = q \mid C = t) = 2/5$
- $Pr(A = g, B = q \mid C = t) \cdot Pr(C = t) = 1/5$

$2/5$ $1/2$

- $Pr(C = f) = 5/10 = 1/2$
- $Pr(A = g, B = q \mid C = f) = \underline{1/5}$
- $Pr(A = g, B = q \mid C = f) \cdot Pr(C = f) = 1/10$

$1/5$ $1/2$

- Hence, predict $C = t$

A	B	C
m	b	t
m	s	t
g	q	t
h	s	t
g	q	t
g	q	f
g	s	f
h	b	f
h	q	f
m	b	f

Example . . .

- What if we want to classify $A = m, B = q$?
- $Pr(A = m, B = q \mid C = t) = 0$
- Also $Pr(A = m, B = q \mid C = f) = 0$!

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Example . . .

- What if we want to classify $A = m, B = q$?
- $Pr(A = m, B = q \mid C = t) = 0$
- Also $Pr(\underline{A = m, B = q} \mid C = f) = 0$!
- To estimate joint probabilities across all combinations of attributes, we need a much larger set of training data

$\underline{A \times B}$

A	B	C
m	b	t
m	s	t
g	q	t
h	s	t
g	q	t
g	q	f
g	s	f
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Naïve Bayes classifier

- Strong simplifying assumption: attributes are pairwise independent

$$Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i) = \prod_{j=1}^{k} Pr(A_j = a_j \mid C = c_i)$$

  - $Pr(C = c_i)$ is fraction of training data with class c_i
  - $Pr(A_j = a_j \mid C = c_i)$ is fraction of training data labelled c_i for which $A_j = a_j$

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- Final classification is

$$\arg\max_{c_i} \; Pr(C = c_i) \underline{\prod_{j=1}^{k} Pr(A_j = a_j \mid C = c_i)}$$

$P(C=c_i) \cdot \underline{P(A_1 = a_1 \ldots A_k = a_k \mid C = c_i)}$

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- For instance, text classification
  - Items are documents, attributes are words (absent or present)
  - Classes are topics
  - Conditional independence says that a document is a set of words: ignores sequence of words
  - Meaning of words is clearly affected by relative position, ordering

Naïve Bayes classifier . . .

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  - Items are documents, attributes are words (absent or present)
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  - Conditional independence says that a document is a set of words: ignores sequence of words
  - Meaning of words is clearly affected by relative position, ordering
- However, naive Bayes classifiers work well in practice, even for text classification!
  - Many spam filters are built using this model

Example revisited

- Want to classify $A = m, B = q$
- $Pr(A = m, B = q \mid C = t) = Pr(A = m, B = q \mid C = f) = 0$

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- $Pr(A = m \mid C = t) = 2/5$
- $Pr(B = q \mid C = t) = 2/5$

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- $Pr(A = m \mid C = f) = 1/5$
- $Pr(B = q \mid C = f) = 2/5$
- $\underbrace{Pr(A = m \mid C = t) \cdot Pr(B = q \mid C = t)}_{P(A=m,\, B=q \mid C=t)} \cdot Pr(C = t) = 2/25$

$\frac{2}{5} \cdot \frac{\cancel{2}}{5} \cdot \frac{1}{\cancel{2}} \rightarrow \frac{2}{25}$

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- $Pr(B = q \mid C = f) = 2/5$
- $Pr(A = m \mid C = t) \cdot Pr(B = q \mid C = t) \cdot Pr(C = t) = 2/25$
- $Pr(A = m \mid C = f) \cdot Pr(B = q \mid C = f) \cdot Pr(C = f) = 1/25$

$\frac{1}{5} \times \frac{2}{5} \times \frac{1}{2}$

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- $Pr(A = m \mid C = t) \cdot Pr(B = q \mid C = t) \cdot Pr(C = t) = 2/25$
- $Pr(A = m \mid C = f) \cdot Pr(B = q \mid C = f) \cdot Pr(C = f) = 1/25$
- Hence predict $C = t$

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Zero counts

- Suppose $A = a$ never occurs in the test set with $C = c$

$P(A=a|C=c)$

Zero counts

- Suppose $A = a$ never occurs in the test set with $C = c$
- Setting $Pr(A = a \mid C = c) = 0$ wipes out any product $\prod_{i=1}^{k} Pr(A_i = a_i \mid C = c)$ in which this term appears

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- Setting $Pr(A = a \mid C = c) = 0$ wipes out any product $\prod_{i=1}^{k} Pr(A_i = a_i \mid C = c)$ in which this term appears
- Assume A_i takes m_i values $\{a_{i1}, \ldots, a_{im_i}\}$

$A_i = a_{ik}$ does not occur for $C = c$

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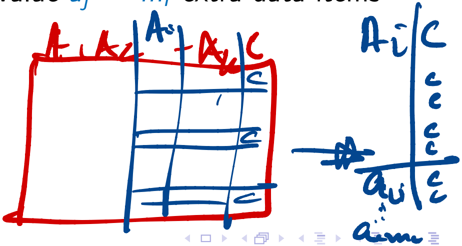
- Assume A_i takes m_i values $\{a_{i1}, \ldots, a_{im_i}\}$
- "Pad" training data with one sample for each value a_j — m_i extra data items



Zero counts

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- Setting $Pr(A = \underset{i}{a} \mid C = \underset{j}{c}) = 0$ wipes out any product $\prod_{i=1}^{k} Pr(A_i = a_i \mid C = c)$ in which this term appears

$$P(A=a_i \mid C=c_j) = \frac{n_{ij}+1}{n_j+m_i}$$

- Assume A_i takes m_i values $\{a_{i1}, \ldots, a_{im_i}\}$
- "Pad" training data with one sample for each value a_j — m_i extra data items
- Adjust $Pr(A_i = a_i \mid C = c_j)$ to $\dfrac{n_{ij}+1}{n_j + m_i}$

  where
  - n_{ij} is number of samples with $A_i = a_i$, $C = c_j$
  - n_j is number of samples with $C = c_j$

A_i	C
a_i	c_j
	c_j
	c_j

$n_j + m_i$

Smoothing

- Laplace's law of succession

$$Pr(A_i = a_i \mid C = c_j) = \frac{n_{ij} + 1}{n_j + m_i}$$

Smoothing

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$$Pr(A_i = a_i \mid C = c_j) = \frac{n_{ij} + 1}{n_j + m_i}$$

+1 perturbs count

- More generally, Lidstone's law of succession, or smoothing

$$Pr(A_i = a_i \mid C = c_j) = \frac{n_{ij} + \lambda}{n_j + \lambda m_i}$$

$\lambda \leq 1$

Smoothing

$A = m,\ B = q$

- Laplace's law of succession

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m, q, t

- More generally, Lidstone's law of succession, or smoothing

$$Pr(A_i = a_i \mid C = c_j) = \frac{n_{ij} + \lambda}{n_j + \lambda m_i}$$

- $\lambda = 1$ is Laplace's law of succession

Text classification

- Classify text documents using topics

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- Want to use a naïve Bayes classifier
- Need to define a generative model
- How do we represent documents?

Set of words model

- Each document is a set of words over a vocabulary $V = \{w_1, w_2, \ldots, w_m\}$

ignore multiplicity

ignore order

A hit B

B hit A

Set of words model

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- Topics come from a set $C = \{c_1, c_2, \ldots, c_k\}$

Focus on words of interest

× the, a, in ...

✓ HP Pavilion,

IBM Thinkpad

CZ22X7

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- $Pr(d \mid c) = \underline{\prod_{w_i \in D} Pr(w_i \mid c)} \; \underline{\prod_{w_i \notin D} (1 - Pr(w_i \mid c))}$

$p^k (1-p)^{n-k}$

$d \subseteq V$

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- $Pr(d \mid c) = \prod_{w_i \in \cancel{D}\, d} Pr(w_i \mid c) \prod_{w_i \notin \cancel{D}\, d} (1 - Pr(w_i \mid c))$
- $Pr(d) = \sum_{c \in C} Pr(d \mid c) \cdot Pr(c)$

$P(c)$

$P(w|c)$

$\downarrow$

$P(d|c)$

$P(d)$

- Training set $D = \{d_1, d_2, \ldots, d_n\}$
  - Each $d_i \subseteq V$ is assigned a unique label from C

$$
\begin{array}{c|cccccccc|}
 & w_1 & w_2 & \cdots & & & & w_m & c \\
\hline
d_i & 0 & 1 & \cdots & 0 & 0 & \cdots & 1 & 7
\end{array}
$$

$$\text{Set } X \subseteq U \equiv f_X : U \to \{0,1\}$$

$$f_X(x) = 1 \text{ iff } x \in X$$

Naïve Bayes classifier

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$w_i = 1$

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- Given a new document $d \subseteq V$, we want to compute $\arg\max_c Pr(c \mid d)$
- By Bayes' rule, $Pr(c \mid d) = \dfrac{Pr(d \mid c)Pr(c)}{Pr(d)}$ $= \sum_{c} P(d|c) \cdot P(c)$
  - As usual, discard the common denominator and compute $\arg\max_c Pr(d \mid c)Pr(c)$

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- Recall $Pr(d \mid c) = \prod_{w_i \in d} Pr(w_i \mid c) \prod_{w_i \notin d} (1 - Pr(w_i \mid c))$

Bag of words model

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- As before
  - Each topic c has probability $Pr(c)$
  - Each word $w_i \in V$ has conditional probability $Pr(w_i \mid c_j)$ with respect to each $c_j \in C$
  - Note that $\sum_{i=1}^{m} Pr(w_i \mid c_j) = 1$
  - Assume document length is independent of the class

Bag of words model

- Generating a random document d
  - Choose a document length ℓ with $Pr(\ell)$
  - Choose a topic c with probability $Pr(c)$
  - Recall $|V| = m$.
    - To generate a single word, throw an m-sided die that displays w with probability $Pr(w \mid c)$
    - Repeat ℓ times

$V = \{w_1, \ldots, w_i, \ldots, w_m\}$

$P(w_i \mid c_j)$

ℓ-words

$u_1 \quad u_2 \quad \ldots \quad u_\ell$

w_i?

Bag of words model

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7h 3t

hhhhh th

tht

p^k

- Let n_j be the number of occurrences of w_j in d

- $Pr(d \mid c) = Pr(\ell)\ \ell! \prod_{j=1}^{m} \frac{Pr(w_j \mid c)^{n_j}}{n_j!}$

$P(w_j \mid c)^{n_j}$

Parameter estimation

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w_1	w_2	w_m	C
n_1	n_2	n_m	c_j

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- As before, $Pr(c_j)$ is fraction of D labelled c_j
- $Pr(w_i \mid c_j)$ — fraction of occurrences of w_i over documents $D_j \subseteq D$ labelled c_j
  - n_{id} — occurrences of w_i in d
  - $Pr(w_i \mid c_j) = \dfrac{\sum_{d \in D_j} n_{id}}{\sum_{t=1}^{m} \sum_{d \in D_j} n_{td}}$

D_j = document labelled c_j

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  - n_{id} — occurrences of w_i in d
  - $$Pr(w_i \mid c_j) = \frac{\sum_{d \in D_j} n_{id}}{\sum_{t=1}^{m} \sum_{d \in D_j} n_{td}} = \frac{\sum_{d \in D} n_{id} \, Pr(c_j \mid d)}{\sum_{t=1}^{m} \sum_{d \in D} n_{td} \, \underline{Pr(c_j \mid d)}},$$

$Pr(c_j|d)$

$d \in D_j \Rightarrow c_j = 1$

$d \notin D_j \Rightarrow c_j = 0$

$d \in D \setminus D_j$

$Pr(c|d) = 0$

$d \in D_j$

$d \in D$

since $Pr(c_j \mid d) = \begin{cases} 1 & \text{if } d \in D_j, \\ 0 & \text{otherwise} \end{cases}$

Classification

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d_1 dog^1 dog^2 cat^3 $c=0$

d_2 dog^4 cat^5 wolf^6 $c=0$

$P(\text{dog} \mid c=0) = \frac{2}{2} = 1$

set of words

$P(\text{dog} \mid c=0) = \frac{3}{6}$ — dog — total