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Data Mining and Machine Learning
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Information retrieval on the Internet

- Traditional IR
  - Books published after editing, review — trustworthy content

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- Self published documents may omit useful search terms
  - IBM webpage did not mention the word "computer"

Exploiting hypertext

- Hypertext links refer from one document to another
  - `<a href="https://www.cmi.ac.in"> CMI webpage </a>`
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- Use anchor text to index document at target location
  - Reliable indicator of what target document is about
- Hyperlinks also connect internet documents as a directed graph
  - Reason about the World Wide Web (WWW) as a gigantic graph
  - Use techniques from social network analysis

Social network analysis — prestige

- Consider the film industry
    - When is an actor a star? When is a director famous?

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  - Recursive definition
- Network (graph) of actors and directors, matrix M

$a \longrightarrow d$

$$\begin{array}{ccc} & & \textit{Directors} \\ & & j \\ \textit{Actors} & i & \left[\begin{array}{cc} & \vdots \\ \cdots & 1 \\ & \end{array}\right] \end{array}$$

$M[i,j] = 1$ if Actor i works in a film directed by Director j

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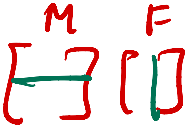

- Each actor i has star value $S[i]$
- Each director j has fame $F[j]$
- Actors derive star value from the famous directors they work with

$$S[i] = \sum_j \underline{M[i,j]} \cdot \underline{F[j]}, \quad \text{or } \underline{S = M \cdot F}$$

Matrix mult

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- Solve for S, F to compute star ratings, fame

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- Structure of the internet, adjacency matrix A

$$\begin{array}{cc} & \textit{Webpages} \\ & j \\ \textit{Webpages} \quad i & \begin{bmatrix} & \vdots & \\ \cdots & 1 & \\ & & \end{bmatrix} \end{array}$$

$A[i,j] = 1$ if webpage i has a link to webpage j

- Suppose $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Prestige for webpages ...

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d2

<a href= "d3"> -- <>

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- Use theory of Markov chains

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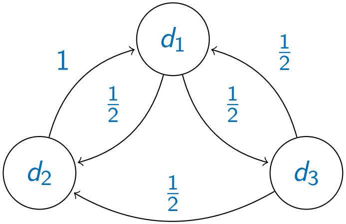
Three state Markov chain



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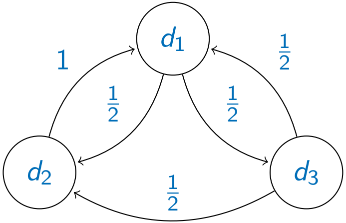

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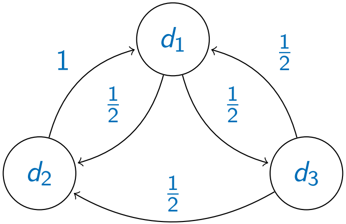

- $P[j]$ is probability of being in document j
- Start in document 1, so initially $P = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Markov chains ...

- After one step: $P^\top A = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

  $1/3 \quad 1/3 \quad 1/3$

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- Continuing our example,

$\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{4} & \frac{3}{8} & \frac{3}{8} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{9}{16} & \frac{5}{16} & \frac{1}{8} \end{bmatrix}$

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- Is it the case that $P[j] > 0$ for all j continuously, after some point?

Ergodicity

- Markov chain A is ergodic if there is some t_0 such that for every P, for all $t > t_0$, for every j, $(P^\top A^t)[j] > 0$.
  - No matter where we start, after $t > t_0$ steps, every state has a nonzero probability of being visited in step t

$$\cancel{P}\left(\left(P^\top A\right) A\right) A$$

$$1 \text{ step} \Rightarrow P_1$$

$$P_1 \cdot A^{\cancel{T}} \to P_2$$

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$P^T A = P^T$

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    - There is a stationary distribution π such that $\pi^\top A = \pi^\top$
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- Properties of ergodic Markov chains
  - There is a stationary distribution π such that $\pi^\top A = \pi^\top$
    - $\pi^\top$ is a left eigenvector of A
  - For *any* starting distribution P, $\lim_{t\to\infty} P^\top A^t = \pi^\top$

Ergodicity ...

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- Sufficient conditions for ergodicity
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- Sufficient conditions for ergodicity
  - Irreducibility: When viewed as a directed graph, A is strongly connected
    - For all states i, j, there is a path from i to j and a path from j to i
  - Aperiodicity: For any pair of vertices i, j, the gcd of the lengths of all paths from i to j is 1
    - In particular, paths (loops) from i to i do not all have lengths that are multiples of some $k \geq 2$
    - Prevents bad cycles

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  - Check that M is stochastic
- By construction,
  - M is strongly connected — direct edge between each pair of documents
  - M is aperiodic — paths of any length exist between i and j
  - M has no dead ends

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- Use recursive doubling to accelerate computation of $\lim_{t \to \infty} P^T M^t$
  - Compute M, M^2, $(M^2)^2 = M^4$, $\ldots$, $(M^{2i})^2 = M^{4i}$, $\ldots$
  - Set a threshold for progress to stop the process

$x \quad (x^2)^2 \quad (x^4)^2 \quad x^8$

$x^y \qquad x \cdot x \cdots x \quad y$ times

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- Some limitations of Page rank
  - Universal property of a webpage, independent of a query
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The Pianist

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  - Define a topic-sensitive page rank
- Page rank was one the keys to the initial success of Google
  - Constant tweaks to ranking algorithm to keep ahead of search engine optimizers (SEO)