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QUALITATIVE APTITUDE TRICKS & SHORTCUTS FOR CAT

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Tip 1 – Logarithms, Surds and Indices

- “Logarithms, Surds and Indices” is one of the easiest topics in the quantitative section of the CAT exam.

- Although the number of formulae is high, the basic concepts are very simple to understand and apply.

- There are no shortcuts to remember and the scope of the questions that can be asked is very limited.

- The accuracy of answering questions from this section is very high and good students tend to score very well here.

Tip 2 – Logarithms, Surds and Indices

If X,Y > 0 and m,n are rational numbers then

- $X^m \times X^n = X^{m+n}$
- $X^0 = 1$
- $\frac{X^m}{X^n} = X^{m-n}$
- $(X^m)^n = X^{mn}$
- $X^m \times Y^m = (X \times Y)^m$
- $\frac{X^m}{Y^m} = (X/Y)^m$
- $X^{-m} = \frac{1}{X^m}$

Tip 3 – Logarithms, Surds and Indices

If X and Y are positive real numbers and a,b are rational numbers.

- $(X/Y)^{-a} = (Y/X)^{a}$
- $X^{1/a} = \sqrt[a]{X}$
- $X^{a/b} = \sqrt[b]{X^{a}}$
- $\sqrt[a]{X} \times \sqrt{\quad} = \sqrt[a]{XY}$
- $\sqrt[a]{X} / \sqrt[a]{Y} = \sqrt[a]{X/Y}$
- $\frac{1}{\sqrt{N+1}-\sqrt{N}} = \sqrt{N+1}+\sqrt{N}$

Tip 4 – Logarithms, Surds and Indices

- Surd is an irrational number involving a root ex : $\sqrt{5}$, $\sqrt[3]{7}$, $\sqrt[5]{2}$
- Like surds are two surds having same number under radical sign.
- Like surds can be added or subtracted. $6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$

Tip 5 – Logarithms, Surds and Indices

- If $a+\sqrt{b} = c+\sqrt{d}$, then a = c and b = d
- The conjugate of $a+\sqrt{b}$ is $a-\sqrt{b}$
- $\sqrt{a\sqrt{a\sqrt{a \ldots\ldots\infty}}} = a$
- $\sqrt{a\sqrt{a\sqrt{a \ldots\ldots x \text{ times}}}} = a^{1-[1/(2^x)]}$
- To find $\sqrt{\sqrt{x}+\sqrt{y}}$, $\sqrt{x}+\sqrt{y}$ should be written in the form of $m+n+2\sqrt{mn}$ where $x = m+n$ and $4mn = y$ and $\sqrt{\sqrt{x}+\sqrt{y}} = \pm(\sqrt{m}+\sqrt{n})$

Tip 6 – Logarithms, Surds and Indices

If $N = a^x$ then, x is defined as the logarithm of N to base a

or $x = \log_a N$

Logarithm of a negative number or zero is not defined.

- $\log_a 1 = 0$
- $\log_a xy = \log_a x + \log_a y$
- $\log_a b^c = c \log_a b$
- $\log_a a = 1$
- $x^{\log_b y} = y^{\log_b x}$

Tip 7 – Logarithms, Surds and Indices

- $\log_a \sqrt[n]{b} = \frac{\log_a b}{n}$
- $\log_a x = \frac{1}{\log_x a}$
- $b^{\log_b x} = x$
- $\log_a b = \frac{\log_c b}{\log_c a}$
- $\log_a b * \log\ a = 1$
- $\log_a(X/Y) = \log_a X - \log_a Y$

Tip 8 – Logarithms, Surds and Indices

- If $0 < a < 1$, then $\log_a x < \log_a$ (if $x>y$)
- If $a > 1$ then $\log_a x > \log_a y$ (if $x>y$)

Tip 1 – Permutations & Combinations

- Permutations & Combinations is an extremely important topic in CAT.
- This topic can be the most rewarding topic in quant section.
- Unlike number systems questions, most of these questions generally take lesser time to solve.
- Also, they are generally fairly basic in nature.

Tip 2 – Permutations & Combinations

- The more questions you solve, the better you will get at this topic.
- So look through the formula list a few times and understand the formulae.
- But the best way to tackle this subject is by solving questions.
- Solve as many questions as you can, from this topic that you will start to see that all of them are generally variations of the same few themes that are listed in the formula list.

Tip 3 – Permutations & Combinations

- $N! = N(N-1)(N-2)(N-3)\ldots\ldots 1$
- $0! = 1! = 1$
- ${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$
- ${}^{n}P_{r} = \frac{n!}{(n-r)!}$

Tip 4 – Permutations & Combinations

- **Arrangement** :

n items can be arranged in n! ways

- **Permutation** :

A way of selecting and arranging r objects out of a set of n objects, ${}^{n}P_{r} = \frac{n!}{(n-r)!}$

- **Combination** :

A way of selecting r objects out of n (arrangement does not matter)

$${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

Tip 5 – Permutations & Combinations

- Selecting r objects out of n is same as selecting (n-r) objects out of n, ${}^{n}C_{r} = {}^{n}C_{n-r}$

- Total selections that can be made from 'n' distinct items is given $\sum_{k=0}^{n} {}^{n}C_{k} = 2^{n}$

Tip 6 – Permutations & Combinations

Partitioning :

- Number of ways to partition n identical things in r distinct slots is given by $^{n+r-1}C_{r-1}$
- Number of ways to partition n identical things in r distinct slots so that each slot gets at least 1 is given by $^{n-1}C_{r-1}$
- Number of ways to partition n distinct things in r distinct slots is given by r^n
- Number of ways to partition n distinct things in r distinct slots where arrangement matters = $\frac{(n+r-1)!}{(r-1)!}$

Tip 7 – Permutations & Combinations

Arrangement with repetitions :

If x items out of n items are repeated, then the number of ways of arranging these n items is $\frac{n!}{x!}$ ways. If a items, b items and c items are repeated within n items, they can be arranged in $\frac{n!}{a!\,b!\,c!}$ ways.

Tip 8 – Permutations & Combinations

Rank of a word :

- To get the rank of a word in the alphabetical list of all permutations of the word, start with alphabetically arranging the n letters. If there are x letters higher than the first letter of the word, then there are at least x*(n-1)! Words above our word.

- After removing the first affixed letter from the set if there are y letters above the second letter then there are y*(n-2)! words more before your word and so on. So rank of word = x*(n-1)! + y*(n-2)! + .. +1

Tip 9 – Permutations & Combinations

Integral Solutions :

- Number of positive integral solutions to $x_1+x_2+x_3+.....+x_n= s$ where $s \geq 0$ is ${}^{s-1}C_{n-1}$

- Number of non-negative integral solutions to $x_1+x_2+x_3+.....+x_n = s$ where $s \geq 0$ is ${}^{n+s-1}C_{n-1}$

Tip 10 – Permutations & Combinations

Circular arrangement :

Number of ways of arranging n items around a circle are 1 for n = 1,2 and (n-)! for n≥3. If its a necklace or bracelet that can be flipped over, the possibilities are $\frac{(n-1)!}{2}$

Derangements :

If n distinct items are arranged, the number of ways they can be arranged so that they do not occupy their intended spot is

$$D = n!\left(\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots \frac{(-1)^n}{n!}\right)$$

Tip 1 – S.I and C.I

- Simple Interest (S.I) and Compound Interest (C.I) is one of the easiest topics in quantitative section.
- Every year, a significant number of questions appear from each of these sections and students should aim to get all the questions right from these topics.
- The number of concepts in these topics is limited and most of the problems can be solved by applying the formulae directly.
- Many students commit silly mistakes in this topic due to complacency and this should be avoided.

Tip 2 – S.I & C.I

- In Simple Interest the principal and the Interest (occurred every period) remains constant

- In Compound Interest the Interest earned over the period is added over to the existing principal after every compounding period. So the principal and the Interest over a period changes after every compounding period.

- For the same principal, positive rate of interest and time period (>1 year), the compound interest on the loan is always greater than the simple interest.

Tip 3 – S.I

- The sum of principal and the interest is called Amount.

Amount (A) = Principal (P) + Interest (I)

- The Simple Interest (I) occurred over a time period (T) for R% (rate of Interest per annum),

$$I = \frac{PTR}{100}$$

Tip 4 – C.I

- The amount to be paid, if money is borrowed at Compound Interest for N number of years,

$$A = P\left(1 + \frac{R}{100}\right)^{N}$$

- The Interest occurred, I = A – P

$$I = P\left(1 + \frac{R}{100}\right)^{N} - P$$

Tip 5 – C.I

If R is rate of interest per year, N is number of years, P is the principal

- If interest is compounded half yearly, then Amount,

$$A = P\left(1 + \frac{R/2}{100}\right)^{2N}$$

- If interest is compounded quarterly, then Amount,

$$A = \left(1 + \frac{R/4}{100}\right)^{4N}$$

Tip 6 – S.I & C.I

- If interest Rate is R_1% for first year, R_2% for second year and R_3% for 3rd year,

then the Amount, A = P $\left(1+\frac{R_1}{100}\right)\ 1+\frac{R_2}{100}\Big)\left(1+\frac{R_3}{100}\right)$

- If a difference between C.I and S.I for certain sum at same rate of interest is given, then Principal = Difference $(100/R)^2$

- When interest is compounded annually but time is in fraction, let $a\frac{b}{c}$ then

the Amount, A = P $\left(1+\frac{R}{100}\right)^a\left(1+\frac{R\frac{b}{c}}{100}\right)$

Tip 7 – S.I & C.I

If R is the rate per annum, then present worth of Rs. K due to N years hence is given by

$$\text{Present worth} = \frac{K}{\left(1+\frac{R}{100}\right)^{N}}$$

Tip 7 – Quadratic Equations

If $A_n X^n + A_{n-1} X^{n-1} + \ldots + A_1 X + A\ \ = 0$, then

- Sum of the roots = $-A_{n-1}/A_n$
- Sum of roots taken two at a time = A_{n-2}/A_n
- Sum of roots taken three at a time = $-A_{n-3}/A_n$ and so on
- Product of the roots = $[(-1)^n A_0\ ]\ /A_n$

Tip 1 – Profit, Loss and Discount

- Profit, Loss and Discount is very important topic for CAT and significant number of questions are asked from this topic every year.
- The number of concepts in these topics is limited and most of the problems can be solved by applying the formulae directly
- This document covers various formulas, tips and shortcuts of Profit, Loss and Discount topic.

Tip 2 – Profit, Loss and Discount

- **Cost Price**

The amount paid to purchase an article or the cost of manufacturing an article is called Cost Price (C.P)

- **Selling Price**

The price at which a product is sold is called Selling price (S.P)

- **Marked Price**

The price at which an article is marked is called Marked price (M.P)

Tip 3 – Profit, Loss and Discount

- If S.P>C.P, then Profit or Gain, P = S.P – S.P
- If C.P>S.P, then Loss, L = C.P – S.P
- % Profit or Gain percentage or Profit Percentage = $\frac{\text{Profit}}{\text{C.P}} \times 100$
- %Loss = $\frac{\text{Loss}}{\text{C.P}} \times 100$
- Discount = M.P – S.P (If no discount is given, then M.P = S.P)
- %Discount = $\frac{}{} \times 100$

Tip 4 – Profit, Loss and Discount

- Total increase in price due to two subsequent increases of X% and Y% is (X+ Y+ $\frac{X\,Y}{100}$)%

- If two items are sold at same price, each at Rs. x, one at a profit of P% and other at a loss of P% then there will be overall loss of $\frac{P^2}{100}$ %

  The absolute value of loss = $\frac{2P^2x}{100^2 - P^2}$

Tip 5 – Profit, Loss and Discount

- If C.P of two items is same, and by selling of each item he earned p% profit on one article and p% loss on another, then the there will be no loss or gain.

- If a trader professes to sell at C.P but uses false weight, then

$$\text{Gain\%} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

Tip 6 – Profit, Loss and Discount

- S.P = $\left(\frac{100 + \text{Profit}\%}{100}\right)$ C.P (If S.P > C.P)
- S.P = $\left(\frac{100 - \text{Loss}\%}{100}\right)$ C.P (If S.P < C.P)
- C.P = $\frac{100 \times \text{S.P}}{100 + \text{Profit}\%}$ (If S.P > C.P)
- C.P = $\frac{100 \times \text{S.P}}{100 - \text{Loss}\%}$ (If S.P < C.P)

Tip 7 – Profit, Loss and Discount

- Buy x get y free, then the %discount = $\frac{y}{x+y} \times 100$.

  (here x+y articles are sold at C.P of x articles.)

- When there are two successive discounts of a% and b% are given then the,

$$\text{Resultant discount} = \left(a + b - \frac{a*b}{100}\right)$$

- If C.P of x article is equal to the selling price of y articles then the,

$$\text{Resultant profit \% or loss \%} = \frac{y-x}{y} \times 100$$

Tip 6 – Quadratic Equations

- Minimum and maximum values of $ax^2+bx+c = 0$
- If $a > 0$: minimum value = $\frac{4ac-b^2}{4a}$ and occurs at $x = -b/2a$
- If $a < 0$: maximum value = $\frac{4ac-b^2}{4a}$ and occurs at $x = -$

Tip 1 – Sets and Venn diagrams

- Its one of the easiest topics of CAT.
- Most of the formulae in this section can be deduced logically with little effort.
- The difficult part of the problem is translating the sentences into areas of the Venn diagram.
- While solving, pay careful attention to phrases like and, or, not, only, in as these generally signify the relationship.

Tip 2 – Sets and Venn diagrams

- Set is defined as a collection of well-defined objects.
  Ex. Set of whole numbers
- Every object is called Element of the set.
- The number of elements in the set is called cardinal number

Tip 3 – Sets and Venn diagrams

Types of Sets

1. <u>Null set:</u>

A set with zero or no elements is called Null set. It is denoted by { } or Ø. Null set cardinal number is 0

2. <u>Singleton set:</u>

Sets with only one element in them are called singleton sets.
Ex. {2}, {a}, {0}

3. <u>Finite and Infinite set:</u>

A set having finite number of elements is called finite set. A set having infinite or

Tip 4 – Sets and Venn diagrams

Types of Sets

4. Universal set:

A set which contains all the elements of all the sets and all the other sets in it, is called universal set.

5. Subset:

A set is said to be subset of another set if all the elements contained in it are also part of another set. Ex. If A = {1,2}, B = {1,2,3,4} then, Set “A” is said to be subset of set B.

Tip 5 – Sets and Venn diagrams

Types of Sets

6. Equal sets:

Two sets are said to be equal sets when they contain same elements
Ex. A = {a,b,c} and B = {a,b,c} then A and B are called equal sets.

7. Disjoint sets:

When two sets have no elements in common then the two sets are called disjoint sets
Ex. A = {1,2,3} and B = {6,8,9} then A and B are disjoint sets.

Tip 6 – Sets and Venn diagrams

Types of Sets

8. <u>Power set:</u>

- A power set is defined as the collection of all the subsets of a set and is denoted by P(A)

- If A = {a,b} then P(A) = { { }, {a}, {b}, {a,b} }

- For a set having n elements, the number of subsets are 2^n

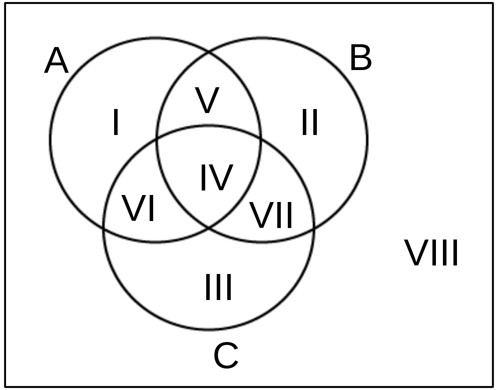
Tip 7 – Sets and Venn diagrams

Properties of Sets:

- The null set is a subset of all sets
- Every set is subset of itself
- $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (B \cap C) = (A \cap B) \cap C$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup \emptyset = A$

Tip 8 – Sets and Venn diagrams

Venn diagrams: A Venn diagram is a figure to represent various sets and their relationship.



I,II,III are the elements in only A, only B and only C respectively
IV – Elements which are in all of A, B and C.
V - Elements which are in A and B but not in C.
VI – Elements which are in A and C but not in B.
VII – Elements which are in B and C but not in A.
VIII – Elements which are not in either A or B or C.

Tip 9 – Sets and Venn diagrams

Union of sets is defined as the collection of elements either in A or B or both. It is represented by symbol “U”. Intersection of set is the collection of elements which are in both A and B.

- Let there are two sets A and B then,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

- If there are 3 sets A, B and C then,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - \quad \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

Tip 10 – Sets and Venn diagrams

To maximize overlap,

- Union should be as small as possible
- Calculate the surplus = n(A) + n(B) + n(C) - n(AUBUC)
- This can be attributed to $n(A \cap B \cap C')$, $n(A \cap B' \cap C)$, $n(A' \cap B \cap C)$, $n(A \cap B \cap C)$.
- To maximize the overlap, set the other three terms to zero.

Tip 11 – Sets and Venn diagrams

To minimize overlap,

- Union should be as large as possible
- Calculate the surplus = n(A) +n(B) +n(C) - n(AUBUC)
- This can be attributed to $n(A \cap B \cap C')$, $n(A \cap B' \cap C)$, $n(A' \cap B \cap C)$, $n(A \cap B \cap C)$.
- To minimize the overlap, set the other three terms to maximum possible.

Tip 12 – Sets and Venn diagrams

Some other important properties

- A’ is called complement of set A, or A’ = U-A
- $n(A-B) = n(A) - n(A \cap B)$
- $A-B = A \cap B'$
- $B-A = A' \cap B$
- $(A-B) \cup B = A \cup B$

Tip 1 – Mixtures and Alligations

- The topic mixtures and alligations is basically an application of averages concept in CAT.
- The theory involved in this topic is very limited and students should be comfortable with the some basic formulas and concepts.
- This pdf covers all the important formulas and concepts related to mixtures and alligations.

Tip 2 – Mixtures and Alligations

A mixture is created when two or more substances are mixed in a certain ratio.

Types of mixtures

1. Simple mixture

A simple mixture is formed by the mixture of two or more different substances.

Ex. Water and Wine mixture

2. Compound mixture

Compound mixture is formed by the mixture of two or more simple mixtures.

Tip 3 – Mixtures and Alligations

Item 1		Item 2
M_1		M_2
	MA	
Q_1		Q_2

If M_1 and M_2 are the values, Q_1 and Q_2 are the quantities of item 1 and item 2 respectively and M_A is the weighted average of the two items, then

$$\frac{Q_1}{Q_2} = \frac{M_2 - MA}{M_A - M_1}$$

Weighted average M_A can be calculated by, $M_A = \frac{Q_1 M_1 + Q_2 M_2}{Q_1 + Q_2}$

Tip 4 – Mixtures and Alligations

The alligation rule can be applied when cheaper substance is mixed with expensive substance

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{Price of dearer} - \text{Mean price}}{\text{Mean price} - \text{Price of cheaper}}$$

Tip 5 – Mixtures and Alligations

If two mixtures M_1 and M_2,having substances S_1 and S_2 in the ratio a:b and p:q respectively are mixed, then in the final mixture,

$$\frac{\text{Quantity of } S_1}{\text{Quantity of } S_2} = \frac{M_1\left[\frac{a}{a+b}\right]+M_2\left[\frac{p}{p+q}\right]}{M_1\left[\frac{b}{a+b}\right]+M_2\left[\frac{q}{p+q}\right]}$$

Tip 6 – Mixtures and Alligations

If there is a container with 'a' liters of liquid A and if 'b' liters are withdrawn and equal amount is replaced by another liquid B and if the operation is repeated for 'n' times

After nth operation,

- Liquid A in the container = $\left[\frac{a-b}{a}\right]^n \times$ Initial quantity of A in the container

- $\frac{\text{Liquid A after nth operation}}{\text{Liquid B after nth operation}} = \frac{\left[\frac{a-b}{a}\right]^n}{1-\left[\frac{a-b}{a}\right]^n}$

Tip 1 – Progressions and Series

- Progressions and Series is one of the important topics for CAT and significant number of questions appear in the examination from this section every year.

- of the questions from this section can be very tough and time consuming while the others can be very easy.

- The trick to ace this section is to quickly figure out whether a question is solvable or not and not waste time on very difficult questions.

- Some of the questions in this section can be answered by ruling out wrong choices among the options available. This method will both save time and improve accuracy.

- There are many shortcuts which will be of vital importance in answering this section.

- This formula sheet contains an exhaustive list of various formulas and shortcuts.

Tip 3 – Progressions and Series

There are 3 standard types of progressions

- Arithmetic Progression
- Geometric Progression
- Harmonic Progression

Tip 4 – Progressions and Series

Arithmetic progression (A.P)

- If the sum or difference between any two consecutive terms is constant then the terms are said to be in A.P
- Ex. 2,5,8,11 or a, a+d, a+2d, a+3d…
- If ‘a’ is the first term and ‘d’ is the difference then the general ‘n’ term is $T_n = a + (n-1)d$
- Sum of first ‘n’ terms in A.P = $\frac{n}{2}[2a+(n-1)d]$
- Number of terms in A.P = $\frac{\text{Last term} - \text{First term}}{\text{Common difference}} + 1$

Tip 5 – Progressions and Series

Properties of A.P

If a, b , c, d,…. are in A.P and ‘k’ is a constant then

- a-k, b-k, c-k,… will also be in A.P
- ak, bk, ck,…will also be in A.P
- a/k, b/k, c/k also be in A.P

Tip 6 – Progressions and Series

Geometric Progression

- If in a succession of numbers the ratio of any term and the previous term is constant then that numbers are said to be in Geometric Progression.
- Ex :1, 3, 9, 27 or a, ar, ar^2, ar^3
- The general expression of an G.P, $T_n = ar^{n-1}$(where a is the first terms and ‘r’ is the common ratio)
- Sum of ‘n’ terms in G.P, Sn = $\frac{a(1-rn)}{1-r}$ (If r<1) or $\frac{a(rn-1)}{r-1}$ (If r>1)

Tip 7 – Progressions and Series

Properties of G.P

If a, b , c, d,…. are in G.P and ‘k’ is a constant then

1. ak, bk, ck,…will also be in G.P
2. a/k, b/k, c/k will also be in G.P

Sum of term of infinite series in G.P, $S_\infty = \frac{a}{1-r}$ (-

Tip 8 – Progressions and Series

Harmonic Progression

- If a, b, c, d,……are unequal numbers then they are said to be in H.P if 1/a, 1/b, 1/c,……are in A.P
- The ‘n’ term in H.P is 1/(nth term in A.P)

Properties of H.P :

If a, b, c, d,…are in H.P, then

$$a+d > b+c$$

$$ad > bc$$

Tip 9 – Progressions and Series

Arithmetic Geometric Series

- A series will be in arithmetic geometric series if each of its term is formed by product of the corresponding terms of an A.P and G.P.
- The general form of A.G.P series is a, (a+d)r, (a+2d)r^2,……
- Sum of ‘n’ terms of A.G.P series

$$S_n = \frac{a}{1-r} + rd\frac{(1-r^{n-1})}{1-r} + rn\frac{[a+(n-1)d]}{1-r} \quad \neq 1)$$

$$S_n = \frac{n}{2}\,[2a + (n-d)]$$

Tip 10 – Progressions and Series

Arithmetic Geometric Series

- Sum of infinite terms of A.G.P series

$$S_\infty = \frac{a}{1-r} + rd\frac{1}{(1-r)^2} \quad (r<1)$$

Tip 11 – Progressions and Series

Standard Series

- The sum of first ‘n’ natural number = $\frac{n(n+1)}{2}$
- The sum of squares of first ‘n’ natural numbers = $\frac{n(n+1)(2n+1)}{6}$
- The sum of cubes of first ‘n’ natural numbers = $(\frac{n(n+1)}{2})^2$
- 2
- The sum of first ‘n’ even natural numbers = n(n+1)
- In any series $T_n = S_n - S_{n-1}$

Tip 12 – Progressions and Series

Arithmetic mean

- The arithmetic mean = $\frac{\text{Sum of all the terms}}{\text{Number of Terms}}$

- If two number A and B are in A.P then arithmetic mean = $\frac{\text{a+b}}{2}$

Arithmetic mean

- Inserting ‘n’ means between two numbers a and b
- The total terms will become n+2, a is the first term and b is the last term
- Then the common difference d = $\frac{b-a}{n+1}$
- The last term b = a+(n+1)d
- The final series is a, a+d, a+2d,….

Tip 14 – Progressions and Series

Geometric Mean

- If a, b, c,… n terms are in G.P then G.M = $\sqrt[n]{a \times b \times c \times \cdots n \text{ terms}}$
- If two numbers a, b are in G.P then their G.M = $\sqrt{a \times b}$

Tip 15 – Progressions and Series

Geometric Mean

- Inserting ‘n’ means between two quantities a and b with common ration ‘r’
- Then the number of terms are n+2 and a, b are the first and last terms
- $r^{n+1} = \frac{b}{a}$ or $r = \frac{\sqrt[n+1]{b}}{a}$
- The final series is a, ar, ar^2,…

Tip 16 – Progressions and Series

Harmonic Mean

- If a, b, c, d,.. are the given numbers in H.P then the Harmonic mean of 'n' terms = $\dfrac{\text{Number of terms}}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\cdots..}$

- If two numbers a and b are in H.P then the Harmonic mean = $\dfrac{2ab}{a+b}$

Tip 17 – Progressions and Series

Relationship between AM, GM and HM for two numbers a and b,

- AM = $\frac{a+b}{2}$
- G.M = $\sqrt{a \times b}$
- H.M = $\frac{2ab}{a+b}$
- G.M = $\sqrt{AM \times HM}$
- A.M ≥ G.M ≥ H.M

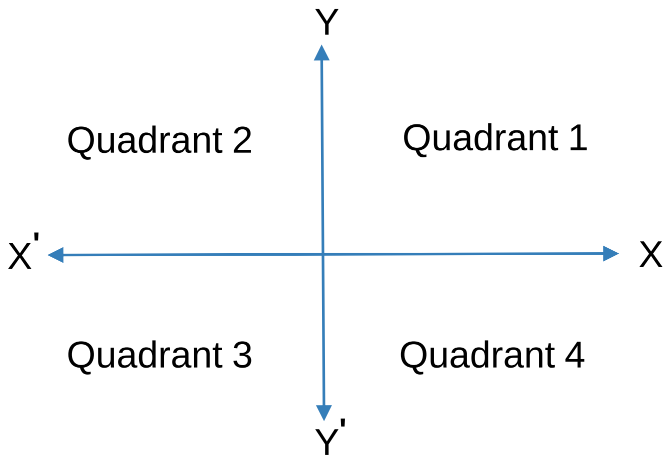
Tip 1 - Geometry

- Geometry is one of the hardest sections to crack without preparation and one of the easiest with preparation.
- With so many formulas to learn and remember, this section is going to take a lot of time to master.
- Remember, read a formula, try to visualize the formula and solve as many questions related to the formula as you can.

Tip 2 - Geometry

- Knowing a formula and knowing when to apply it are two different abilities.
- The first will come through reading the formulae list and theory but the latter can come only through solving many different problems.
- So in this document we are going to provide an exhaustive list of formulas and tips for making geometry section a lot easier.
- Try to remember all of them and don’t forget to share.

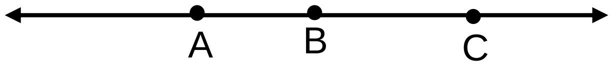
Tip 3 - Geometry



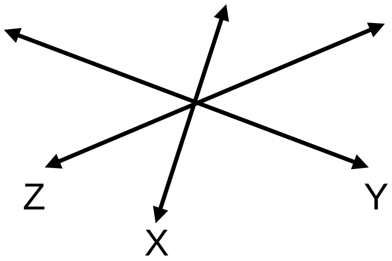
Quadrant I	X is Positive	Y is Positive
Quadrant II	X is Negative	Y is Positive
Quadrant III	X is Negative	Y is Negative
Quadrant IV	X is Positive	Y is Negative

Tip 4 - Geometry

- Collinear points : Three or more points lying on the single straight line. In this diagram the three points A,B and C are collinear



- Concurrent lines : If three or more lines lying in a same plane intersect at a single point then that lines are called concurrent lines. The three lines X, Y and Z are here concurrent lines.



Tip 5 - Geometry

- The distance between two points with coordinates (X_1,Y_1), (X_2,Y_2) is given by $D = \sqrt{(X_2-X_1)^2 + (Y_2-Y_1)^2}$
- Slope, $m = \frac{y_2-y_1}{x_2-x_1}$ (If $x_2 = x_1$ then the lines are perpendicular to each other)
- Mid point between two points $A(x_1,y_1)$ and $B(x_2,y_2)$ is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$
- When two lines are parallel, their slopes are equal i.e. $m_1 = m_2$
- -1 i.e. $m_1 * m_2 = -1$

Tip 6 - Geometry

- If two intersecting lines have slopes m_1 and m_2 then the angle between two lines will be $\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$ (where θ is the angle between the lines)
- The length of perpendicular from a point (X_1, Y_1) on the line $AX+BY+C = 0$ is

$$P = \frac{AX_1 + BY_1 + C}{\sqrt{A^2 + B^2}}$$

- The distance between two parallel lines $Ax+By+C_1 = 0$ and $Ax+By+C_2 = 0$ is

$$\left| \frac{C_1 - C_2}{\sqrt{A^2 + B^2}} \right|$$

Tip 7 - Geometry

Equations of a lines :

General equation of a line	Ax + By = C
Slope intercept form	y = mx + c (c is y intercept)
Point-slope form	$y - y_1 = m (x - x_1)$
Intercept form	$\frac{x}{a} + \frac{y}{b} = 1$
Two point form	$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

Tip 8 - Geometry

General equation of a circle :

The general equation of a circle is $x^2+y^2+2gx+2fy+c=0$

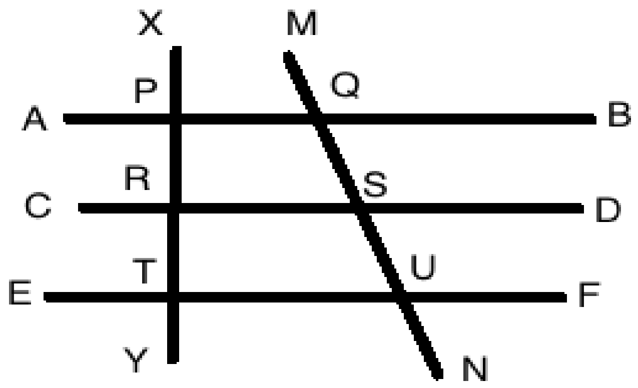
- Centre of the circle is (-g,-f)
- Radius of the circle = $\sqrt{g^2+f^2-c}$
- If the origin is the centre of the circle then equation of the circle is $^2+y^2=r^2$

Tip 9 - Geometry

- When two angles A and B are complementary, sum of A and B is 90°
- When two angles A and B are supplementary, sum of A and B is 180°
- When two lines intersect, opposite angles are equal. Adjacent angles are supplementary
- When any number of lines intersect at a point, the sum of all the angles formed = 360°

Tip 10 - Geometry

- Consider parallel lines AB, CD and EF as shown in the figure.



- XY and MN are known as transversals
- ∠XPQ = ∠PRS = ∠RTU as corresponding angles are equal

Tip 11 - Geometry

- Interior angles on the side of the transversal are supplementary. i.e. ∠PQS + ∠QSR = 180°
- Exterior angles on the same side of the transversal are supplementary. i.e. ∠MQB + ∠DSU = 180°
- Two transversals are cut by three parallel lines in the same ratio i.e. $\frac{PR}{RT} = \frac{QS}{SU}$

- Interior angles on the side of the transversal are supplementary. i.e. ∠PQS + ∠QSR = 180°
- Exterior angles on the same side of the transversal are supplementary. i.e. ∠MQB + ∠DSU = 180°
- Two transversals are cut by three parallel lines in the same ratio i.e. $\frac{PR}{RT} = \frac{QS}{SU}$

Tip 12 - Geometry

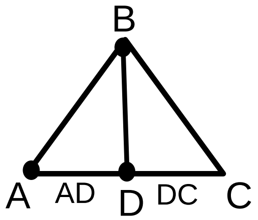
- Sum of all angles in a triangle is 180°
- An angle less than 90° is called an acute angle. An angle greater than 90° is called an obtuse angle.
- A triangle with all sides unequal is called scalene triangle
- A triangle with two sides equal is called isosceles triangle. The two angles of an isosceles triangle that are not contained between the equal sides are equal
- A triangle with all sides equal is called equilateral triangle. All angles of an equilateral triangle equal 60°.

Tip 13 - Geometry

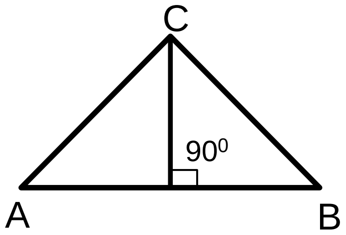
- If in a triangle all of its angles are less than 90^o than that triangle is called as acute angled triangle
- A triangle with one of its angle equal to 90^o than that triangle is called as Right angled triangle
- A triangle with one of its angle greater than 90^o than that triangle is called as Obtuse angled triangle
- If one side of a triangle is produced then that exterior angle formed is equal to the sum of opposite remote interior angles

Tip 14 - Geometry

- A line joining the mid point of a side with the opposite vertex is called a median. (Here D is the midpoint of AC side or AD = DC). BD is the median of this triangle ABC.



- A perpendicular drawn from a vertex to the opposite side is called the altitude



Tip 15 - Geometry

- A line that bisects and also makes right angle with the same side of the triangle is called perpendicular bisector
- A line that divides the angle at one of the vertices into two parts is called angular bisector
- All points on an angular bisector are equidistant from both arms of the angle.
- All points on a perpendicular bisector of a line are equidistant from both ends of the line.
- In an equilateral triangle, the perpendicular bisector, median, angle bisector and altitude (drawn from a vertex to a side) coincide.

Tip 16 - Geometry

- The point of intersection of the three altitudes is the Orthocentre
- The point of intersection of the three medians is the centroid.
- The three perpendicular bisectors of a triangle meet at a point called the Circumcentre. A circle drawn from this point with the circumradius would pass through all the vertices of the triangle.
- The three angle bisectors of a triangle meet at a point called the incentre of a triangle. The incentre is equidistant from the three sides and a circle drawn from this point with the inradius would touch all the sides of the triangle.

Tip 17 - Geometry

- Sum of any two sides of a triangle is always greater than it's third side
- Difference of any two sides of a triangle is always lesser than it's third side

Pythagoras theorem :

In a right angled triangle ABC where $\angle B=90^0$, $AC^2 = AB^2 + BC^2$

Apollonius theorem

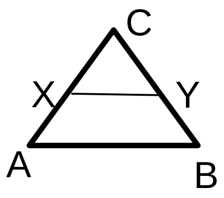
In a triangle ABC, if AD is the median to side BC then by Apollonius theorem,

$$2*(AD^2+BD^2) = AC^2 + AB^2$$

Tip 18 - Geometry

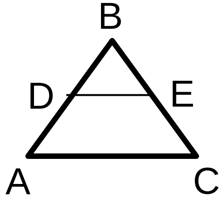
Mid Point Theorem :

The line joining the midpoint of any two sides in a triangle is parallel to the third side and is half the length of the third side.
If X is the midpoint of CA and Y is the midpoint of CB
Then XY will be parallel to AB and XY = ½ * AB



Basic proportionality theorem :

If a line is drawn parallel to one side of a triangle and it intersects the other two sides at two distinct points then it divides the two sides in the ratio of respective sides
If in a triangle ABC, D and E are the points lying on AB and BC respectively and DE is parallel to AC then
AD/DB = EC/BE

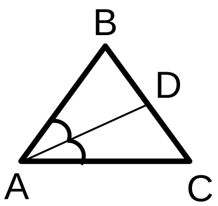


Tip 19 - Geometry

Interior Angular Bisector theorem :

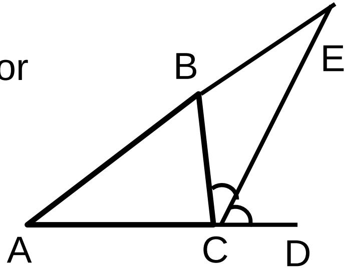
In a triangle the angular bisector of an angle divides the side opposite to the angle, in the ratio of the remaining two sides. In a triangle ABC if AD is the angle bisector of angle A then AD divides the side BC in the same ratio as the other two sides of the triangle.

i.e. BD/ CD= AB/AC.



Exterior Angular Bisector theorem :

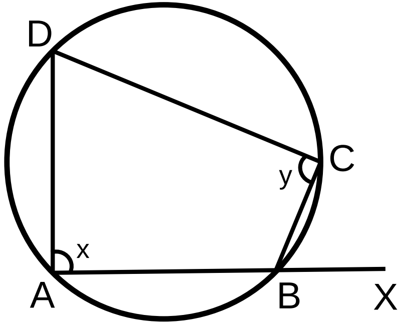
The angular bisector of exterior angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle. In a triangle ABC, if CE is the angular bisector of exterior angle BCD of a triangle, then AE/BE = AC/BC



Tip 20 - Geometry

Cyclic Quadrilateral :

If a quadrilateral has all its vertices on the circle and its opposite angles are supplementary (here $x+y = 180^o$) then that quadrilateral is called cyclic quadrilateral.



- In a cyclic quadrilateral the opposite angles are supplementary.
- Area of a cyclic quadrilateral is $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ where $s = (a+b+c+d)/2$
- $\angle CBX = \angle ADC$)

Tip 21 - Geometry

- If x is the side of an equilateral triangle then the

  Altitude (h) = $\frac{\sqrt{3}}{2}$ x

  Area = $\frac{\sqrt{3}}{4}$ x^2

  Inradius = $\frac{1}{3}$ * h

  $\frac{2}{3}$ * h

- Area of an isosceles triangle = $\frac{a}{4}\sqrt{4c^2 - a^2}$

(where a, b and c are the length of the sides of BC, AC and AB respectively and b = c)

Tip 22 - Geometry

Similar triangles :

If two triangles are similar then their corresponding angles are equal and the corresponding sides will be in proportion.

For any two similar triangles :

- Ratio of sides = Ratio of medians = Ratio of heights = Ratio of circumradii = Ratio of Angular bisectors
- Ratio of areas = Ratio of the square of the sides.

Tests of similarity : (AA / SSS / SAS)

Tip 23 - Geometry

Congruent triangles

If two triangles are congruent then their corresponding angles and their corresponding sides are equal.

Tests of congruence : (SSS / SAS / AAS / ASA)

Tip 24 - Geometry

Area of a triangle, A :

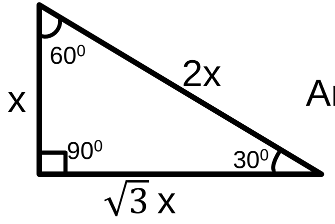
- A = $\sqrt{s(s-a)(s-b)(s-c)}$ where s = (a+b+c)/2.
- A = 1/2 * base * altitude
- A = 1/2 * ab * sinC (C is the angle formed between sides a and b)
- A = $\frac{abc}{4R}$ where R is the circumradius
- A = r * s where r is the inradius and s is the semi perimeter.

(where a, b and c are the lengths of the sides BC, AC and AB)

Tip 25 - Geometry

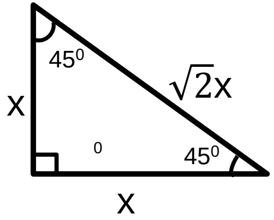
Special triangles :

30^0, 60^0 and 90^0



Area = $\frac{\sqrt{3}}{2}$ x^2

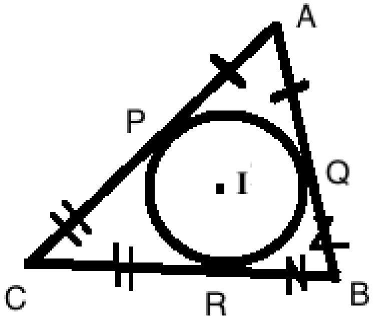
45^0, 45^0, 90^0



Area = $\frac{x^2}{2}$

Tip 26 - Geometry

- Consider the triangle ABC with incentre I, and the incircle touching the triangle at P,Q,R as shown in the diagram. As tangents drawn from a point are equal, AP=AQ, CP=CR and BQ=BR.

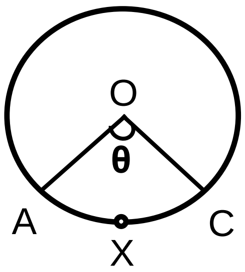


- If a is the side of an equilateral triangle, circumradius = $a/\sqrt{3}$ and inradius = $a/(2\sqrt{3})$

Tip 27 - Geometry

- The angle subtended by a diameter of circle on the circle = 90^0
- Angles subtended by a equal chord are equal. Also, angles subtended in the major segment are half the angle formed by the chord at the centre
- Equal chords of a circle or equidistant from the centre
- The radius from the centre to the point where a tangent touches a circle is perpendicular to the tangent
- Tangents drawn from the same point to a circle are equal in length
- A perpendicular drawn from the centre to any chord, bisects the chord

Tip 28 - Geometry

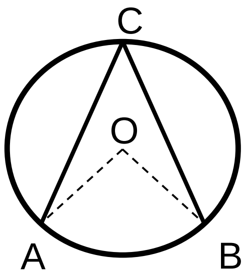


Area of sector OAXC = $\frac{\theta}{360} * \pi r^2$

Area of minor segment AXC = $\frac{\theta}{360}\pi r^2 - \frac{1}{2} r^2 \sin\theta$

Tip 29 - Geometry

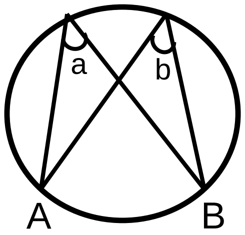
Inscribed angle Theorem :



$$2\angle ACB = \angle AOB$$

The angle inscribed by the two points lying on the circle, at the centre of the circle is twice the angle inscribed at any point on the circle by the same points.

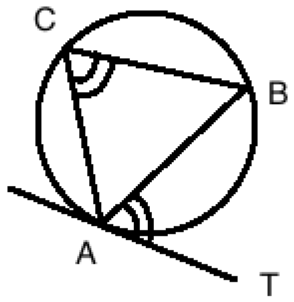
Tip 30 - Geometry



Angles subtended by the same segment on the circle will be equal. So here angles a and b will be equal

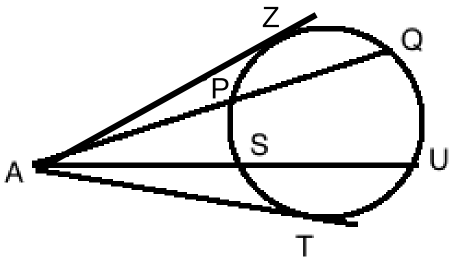
Tip 31 - Geometry

The angle made by a chord with a tangent to one of the ends of the chord is equal to the angle subtended by the chord in the other segment. As shown in the figure, ∠ACB = ∠BAT.



Tip 32 - Geometry

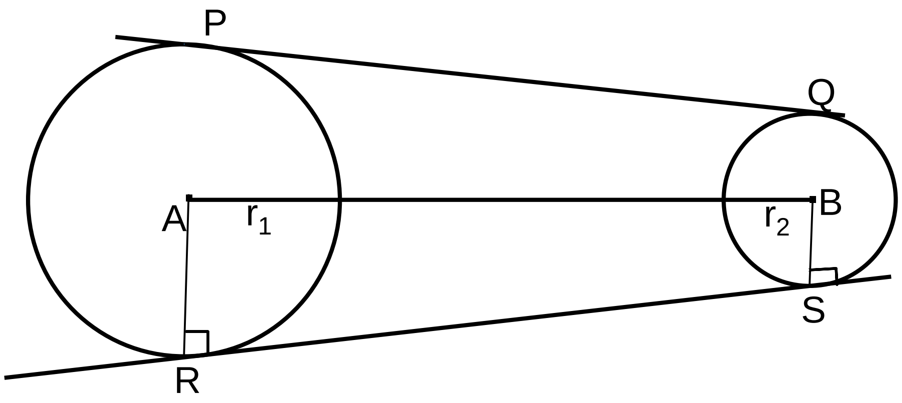
Consider a circle as shown in the image. Here, AP * AQ = AS * AU = AT^2



Two tangents drawn to a circle from a external common point will be equal in length. So here AZ = AT

Tip 33 - Geometry

Direct common tangent :

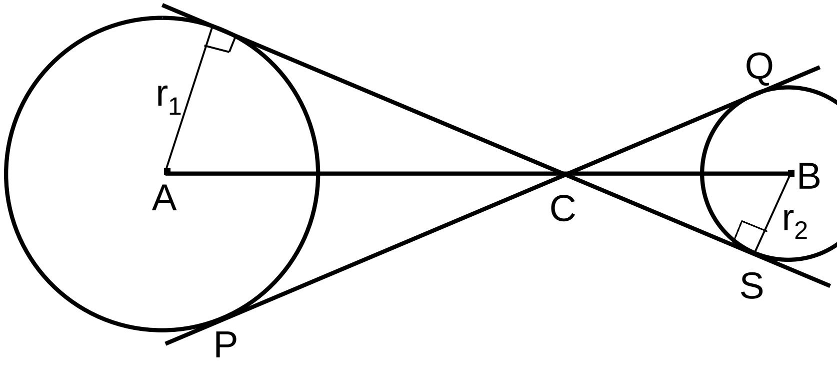


In this figure PQ and RS are the direct common tangents and let AB (Distance between the two centres) = D

$$PQ^2 = RS^2 = D^2 - (r_1 - r_2)^2$$

Tip 34 - Geometry

Transverse common tangent :



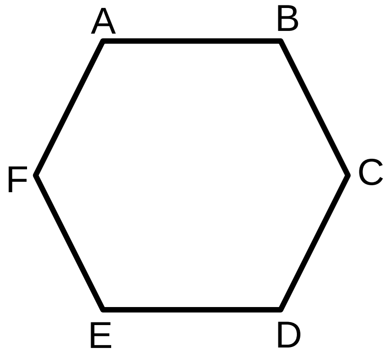
In this figure PQ and RS are the transverse common tangents and let AB (Distance between the two centres) = D

$$PQ^2 = RS^2 = D^2-(r_1+r_2)^2$$

Tip 35 - Geometry

- If all sides and all angles are equal, then the polygon is a regular polygon
- A regular polygon of n sides has n(n-3)/2 diagonals
- In a regular polygon of n sides, each exterior angle is 360/n degrees.
- Sum of measure of all the interior angles of a regular polygon is 180 (n-2) degrees (where n is the number of sides of the polygon)
- Sum of measure of all the exterior angles of regular polygon is 360 degrees

Tip 36 - Geometry



ABCDEF is a regular hexagon with each side equal to ‘x’ then

- Each interior angle = 120^o
- Each exterior angle = 60^o
- Sum of all the exterior angles = 360^o
- Sum of all the interior angles = 720^o
- Area = $\frac{3\sqrt{3}}{2}a^2$

Tip 37 - Geometry

Areas of different geometrical figures

Triangle	½ * base * height
Rectangle	length * width
Trapezoid	½ * sum of bases * height
Parallelogram	base * height
Circle	$\pi *$ radius^2
Rhombus	½ * product of diagonals
square	side^2 or ½ diagonal^2
Kite	½ * product of the diagonals

Tip 38 - Geometry

Volume of different solids

Cube	length^3
Cuboid	Length $*$ base $*$ height
Prism	Area of base $*$ height
Cylinder	$\pi r^2 h$
Pyramid	1/3 $*$ Area of base $*$ Height
Cone	$1/3 * \pi r^2 * h$
Cone Frustum (If R is the base radius, r is the upper surface radius and h is the height of the frustum)	$1/3\ \pi h(R^2+Rr+r^2)$
	$4/3 * \pi * r^3$
Hemi-sphere	$2/3\ \pi r^3$

Tip 39 - Geometry

Total Surface area of different solids

Prism	2* base area + base perimeter*height
Cube	6 * $length^2$
Cuboid	2(lh+bh+lb)
Cylinder	$2\pi rh + 2\pi r^2$
Pyramid	1/2 *Perimeter of base *slant height +Area of base
Cone (l is the slant height)	$\pi r(l+r)$
Cone Frustum (where R and r are the radii of the base faces and l is the slant height)	$\pi(R^2+r\ +Rl+rl)$
Sphere	$4\pi r^2$
Hemi-sphere	$3\pi r^2$

Tip 40 - Geometry

Lateral/Curved surface area

Prism	base perimeter*height
Cube	4 * length^2
Cuboid	2length*height + 2*breadth*height
Cylinder	$2\pi rh$
Pyramid	1/2 *Perimeter of base *slant height
Cone (l is the slant height)	πrl
Cone Frustum (where R and r are the radii of the base faces and l is the slant height)	π

Tip 41 - Geometry

- The angle subtended by a diameter of circle on the circle = 90 degrees
- Angles subtended by a equal chords are equal. Also, angles subtended in the major segment are half the angle formed by the chord at the centre
- The radius from the centre to the point where a tangent touches a circle is perpendicular to the tangent
- Tangents drawn from the same point to a circle are equal in length
- A perpendicular drawn from the centre to any chord, bisects the chord
- In a regular polygon of n sides, each exterior angle is 360/n degrees.

Time, Speed, Distance and Work Tips

- Time, Distance and Work is the most important topic for all Competitive examinations.

- The questions from this topic varies from easy to difficult.

- This formula sheet covers the most importance tips that helps you to answer the questions in a easy, fast and accurate way.

Tip 1 – Time, Speed, Distance & Work

Distance = Speed × Time

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Tip 2 – Time, Speed, Distance & Work

If the ratio of the speeds of A and B is a : b, then

- The ratio of the times taken to cover the same distance is 1/a : 1/b or b : a.
- The ratio of distance travelled in equal time intervals is a : b

Tip 3 – Time, Speed, Distance & Work

- Average speed = $\dfrac{\text{Total Distance travelled}}{\text{Total Time taken}}$

- If a part of a journey is travelled at speed S_1 km/hr in T_1 hours and remaining part at speed S_2 km/hr in T_2 hours then,

  Total distance travelled = $S_1T_1 + S_2T_2$ km

  Average speed = $\dfrac{S_1T_1 + S_2T_2}{T_1 + T_2}$ km/hr

Tip 4 – Time, Speed, Distance & Work

- In a journey travelled with different speeds, if the distance covered in each stage is constant, the average speed is the harmonic mean of the different speeds.

- Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr

Then the average speed during the whole journey is $\frac{2xy}{x+y}$ km/hr

Tip 5 – Time, Speed, Distance & Work

- In a journey travelled with different speeds, if the time travelled in each stage is constant, the average speed is the arithmetic mean of the different speeds.

- If a man travelled for certain time at the speed of x km/hr and travelled for equal amount of time at the speed of y km/hr then

The average speed during the whole journey is $\frac{x+y}{2}$ km/hr

Tip 6 – Time, Speed, Distance & Work

Constant distance :

Let the distances travelled in each part of the journey be d_1, d_2, d_3 and so on till d_n and the speeds in each part be s_1, s_2, s_3 and so on till s_n.

$_1 = d_2 = d_3 = ... = d_n = d$, then the average speed is the harmonic mean of the speeds s_1, s_2, s_3 and so on till s_n.

Constant time :

Let the distances travelled in each part of the journey be d_1, d_2, d_3 and so on till d_n and the time taken for each part be t_1, t_2, t_3 and so on till t_n.

If $t_1 = t_2 = t_3 = ... = t_n = t$, then the average speed is the arithmetic mean of the speeds s_1, s_2, s_3 and so on till s_n.

Tip 7 – Time, Speed, Distance & Work

Circular Tracks :

If two people are running on a circular track with speeds in ratio a:b where a and b are co-prime, then

- They will meet at a+b distinct points if they are running in opposite direction.
- They will meet at |a-b| distinct points if they are running in same direction

Tip 8 – Time, Speed, Distance & Work

Circular Tracks

If two people are running on a circular track having perimeter l, with speeds m and n,

- The time for their first meeting = l / (m +n)
  (when they are running in opposite directions)

- The time for their first meeting = l / (|m-n|)
  (when they are running in the same direction)

Tip 9 – Time, Speed, Distance & Work

If a person P starts from A and heads towards B and another person Q starts from B and heads towards A and they meet after a time 't' then,

$$t = \sqrt{(x*y)}$$

where x = time taken (after meeting) by P to reach B and
y = time taken (after meeting) by Q to reach A.

Tip 10 – Time, Speed, Distance & Work

A and B started at a time towards each other. After crossing each other, they took T_1 hrs, T_2 hrs respectively to reach their destinations. If they travel at constant speeds S_1 and S_2 respectively all over the journey, Then

$$\frac{S_1}{S_2} = \sqrt{\frac{T_2}{T_1}}$$

Tip 11 – Time, Speed, Distance & Work

***Trains* :**

- Two trains of length L_1 and L_2 travelling at speeds of S_1 and S_2 cross each other in a time

$$= \frac{L_1+L_2}{S_1+S_2}$$ (if they are going in opposite directions)

$$= \frac{L_1+L_2}{|S_1-S_2|}$$ (if they are going in the same direction)

Tip 12 – Time, Speed, Distance & Work

Work **:**

- If X can do a work in 'n' days, the fraction of work X does in a day is 1/n

- If X can do a work in 'x' days, and Y can do a work in 'y' days,

  The number of days taken by both of them together is $\frac{x*y}{x+y}$

- If M_1 men work for H_1 hours per day and worked for D_1 days and completed W_1 work, and if M_2 men work for H hours per day and worked for D_2 days and completed W_2 work, then

$$\frac{M_1H_1D_1}{W1} = \frac{M_2H_2D_2}{W2}$$

Tip 13 – Time, Speed, Distance & Work

Boats and Streams **:**

If the speed of water is 'W' and speed of a boat in still water is 'B‘

- Speed of the boat (downstream) is B+W
- Speed of the boat (upstream) is B-W

The direction along the stream is called **downstream.**
And, the direction against the stream is called **upstream.**

Tip 14 – Time, Speed, Distance & Work

Boats and Streams :

- If the speed of the boat downstream is x km/hr and the speed of the boat upstream is y km/hr, then

$$\text{Speed of boat in still water} = \frac{x+y}{2}\ \text{km/hr}$$

$$\text{Rate of stream} = \frac{x-y}{2}\ \text{km/hr}$$

- While converting the speed in m/s to km/hr, multiply it by 3.6 (18/5).

  1 m/s = 3.6 km/h

- While converting km/hr into m/sec, we multiply by 5/18

Tip 15 – Time, Speed, Distance & Work

***Pipes and Cisterns* :**

Inlet Pipe : A pipe which is used to fill the tank is known as Inlet Pipe.

Outlet Pipe : A pipe which can empty the tank is known as Outlet Pipe.

- If a pipe can fill a tank in ‘x’ hours then the part filled per hour = 1/x
- If a pipe can empty a tank in ‘y’ hours, then the part emptied per hour = 1/y
- If a pipe A can fill a tank in ‘ ’ hours and pipe can empty a tank in ‘y’ hours, If they are both active at the same time, then

The part filled per hour $= \frac{1}{x} - \frac{1}{y}$ (If y > x)

The part emptied per hour $= \frac{1}{y} - \frac{1}{x}$ (If x > y)

Tip 16 – Time, Speed, Distance & Work

- Some of the questions may consume a lot of time. While solving, write down the equations as far as possible to avoid mistakes. The few extra seconds can help you avoid silly mistakes.

- Check if the units of distance, speed and time match up. So if you see yourself adding a unit of distance like m to a unit of speed m/s, you would realize you have missed a term.

- Choose to apply the concept of relative speed wherever possible as it can greatly reduce the complexity of the problem.

- Like speed and distance, in time and work while working with terms ensure that you convert all terms to consistent units like man-hours.

Number Systems Tips

- Number Systems is the most important topic in the quantitative section.

- It is a very vast topic and a significant number of questions appear in CAT every year from this section.

- Learning simple tricks like divisibility rules, HCF and LCM, prime number and remainder theorems can help improve the score drastically.

- This document presents best short cuts which makes this topic easy and helps you perform better.

Tip 1 - Number systems

HCF and LCM

- HCF * LCM of two numbers = Product of two numbers
- The greatest number dividing a, b and c leaving remainders of x_1, x_2 and x_3 is the HCF of $(a-x_1)$, $(b-x_2)$ and $(c\ x_3)$.
- The greatest number dividing a, b and c $(a<b<c)$ leaving the same remainder each time is the HCF of (c-b), (c-a), (b-a).
- If a number, N, is divisible by X and Y and HCF(X,Y) = 1. Then, N is divisible by X*Y

Tip 2 - Number systems

Prime and Composite Numbers

- Prime numbers are numbers with only two factors, 1 and the number itself.

- Composite numbers are numbers with more than 2 factors. Examples are 4, 6, 8, 9.

- 0 and 1 are neither composite nor prime.

- There are 25 prime numbers less than 100.

Tip 3 - Number systems

Properties of Prime numbers

- To check if n is a prime number, list all prime factors less than or equal to $\sqrt{n}$. If none of the prime factors can divide n then n is a prime number.

- For any integer a and prime number p, $a^p - a$ is always divisible by p

- All prime numbers greater than 2 and 3 can be written in the form of 6k+1 or 6k-1

- If a and b are co-prime then $a^{(b-1)} \bmod b = 1$.

Tip 4 - Number systems

Theorems on Prime numbers

Fermat's Theorem:

Remainder of a^(p-1) when divided by p is 1, where p is a prime

Wilson's Theorem:

Remainder when (p-1)! is divided by p is (p-1) where p is a prime

Tip 5 - Number systems

Theorems on Prime numbers

Remainder Theorem

- If a, b, c are the prime factors of N such that $N= a^p * b^q * c^r$

Then the number of numbers less than N and co-prime to N is

$\phi(N)= N (1-1/a) (1 \quad 1/b) (1 - 1/c).$

This function is known as the Euler's totient function.

Euler's theorem

- If M and N are co-prime to each other then remainder when $M^{\phi(N)}$ is divided by N is 1.

Tip 6 - Number systems

- Highest power of n in m! is $[m/n]+[m/n^2]+[m/n^3]+\ldots..$

Ex: Highest power of 7 in 100! = [100/7] +[100/49] = 16

- To find the number of zeroes in n! find the highest power of 5 in n!
- If all possible permutations of n distinct digits are added together the sum = (n-1)! * (sum of n digits) * (11111... n times)

Tip 7 - Number systems

- If the number can be represented as $N = a^p * b^q * c^r \ldots$ then number of factors the is (p+1) * (q+1) * (r+1)

- Sum of the factors = $\dfrac{a^{p+1}-1}{a-1} * \dfrac{b^{q+1}-1}{b-1} * \dfrac{c^{r+1}-1}{c-1}$

-

- If there are n factors, then the number of pairs of factors would be n/2. If N is a perfect square then number of pairs (including the square root) is (n+1)/2

Tip 8 - Number systems

If the number can be expressed as $N = 2^p * a^q * b^r$ . . . where the power of 2 is p and a, b are prime numbers

- Then the number of even factors of N = p (1+q) (1+r) . . .
- The number of odd factors of N = (1+q) (1+r)…

Tip 9 - Number systems

Number of positive integral solutions of the equation x^2 - y^2 = k is given by

- $\frac{\text{Total number of factors of k}}{2}$ (If k is odd but not a perfect square)
- $\frac{(\text{Total number of factors of k}) - 1}{2}$ (If k is odd and a perfect square)
- $\frac{\text{Total number of factors of } \frac{k}{4}}{2}$ (If k is even and not a perfect square)
- $\frac{(\text{Total number of factors of } \frac{k}{4}) - 1}{2}$ (If it is even and a perfect square)

Tip 10 - Number systems

- Number of digits in a^b = [b $\log_m(a)$] + 1 ; where m is the base of the number and [.] denotes greatest integer function

- Even number which is not a multiple of 4, can never be expressed as a difference of 2 perfect squares.

Tip 11 - Number systems

- Sum of first n odd numbers is n^2
- Sum of first n even numbers is $n(n+1)$
- The product of the factors of N is given by $N^{a/2}$, where a is the number of factors

Tip 12 - Number systems

- The last two digits of a^2, $(50 - a)^2$, $(50+a)^2$, $(100 - a)^2$ . . .. are same.

- 2^{10n}

When n is odd, the last 2 digits are 24.

When n is even, the last 2 digits are 76.

Tip 13 - Number systems

Divisibility

- Divisibility by 2: Last digit divisible by 2
- Divisibility by 4: Last two digits divisible by 4
- Divisibility by 8: Last three digits divisible by 8
- Divisibility by 16: Last four digit divisible by 16

Tip 14 - Number systems

Divisibility

- Divisibility by 3: Sum of digits divisible by 3
- Divisibility by 9: Sum of digits divisible by 9
- Divisibility by 27: Sum of blocks of 3 (taken right to left) divisible by 27
- Divisibility by 7: Remove the last digit, double it and subtract it from the truncated original number. Check if number is divisible by 7
- Divisibility by 11: (sum of odd digits) - (sum of even digits) should be 0 or divisible by 11

Tip 15 - Number systems

Divisibility properties

- For composite divisors, check if the number is divisible by the factors individually. Hence to check if a number is divisible by 6 it must be divisible by 2 and 3.

- The equation $a^n - b^n$ is always divisible by a-b. If n is even it is divisible by a+b. If n is odd it is not divisible by a+b.

- The equation $a^n + b^n$, is divisible by a+b if n is odd. If n is even it is not divisible by a+b.

Tip 16 - Number systems

- Converting from decimal to base b. Let R_1, R_2 . . . be the remainders left after repeatedly dividing the number with b. Hence, the number in base b is given by ... R_2R_1.

- Converting from base b to decimal - multiply each digit of the number with a power of b starting with the rightmost digit and b^0.

- A decimal number is divisible by b-1 only if the sum of the digits of the number when written in base b are divisible by b-1.

Tip 17 - Number systems

Cyclicity

- To find the last digit of a^n find the cyclicity of a. For Ex. if a=2, we see that
- $2^1=2$
- $2^2=4$
- $2^3=8$
- $2^4=16$
- $2^5=32$

Hence, the last digit of 2 repeats after every 4^{th} power. Hence cyclicity of 2 = 4. Hence if we have to find the last digit of a^n, The steps are:

1. Find the cyclicity of a, say it is x
2. Find the remainder when n is divided by x, say remainder r
3. Find a^r if r>0 and a^x when r=0

Tip 18 - Number systems

- $(a + b)(a - b) = (a^2 - b^2)$
- $(a + b)^2 = (a^2 + b^2 + 2ab)$
- $(a - b)^2 = (a^2 + b^2 - 2ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Tip 19 - Number systems

- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a \quad b)(a^2 + ab + b^2)$
- $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
- When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Tip 1 – Ratio and Proportion

- Ratio and Proportions is one of the easiest concepts in CAT. It is just an extension of high school mathematics.

- Questions from this concept are mostly asked in conjunction with other concepts like similar triangles, mixtures and alligations.

- Hence fundamentals of this concept are important not just from a stand-alone perspective, but also to answer questions from other concepts

Tip 2 – Ratio and Proportion

- A ratio can be represented as fraction a/b or using the notation a:b. In each of these representation ′a′ is called the antecedent and ′b′ is called the consequent.

- For a ratio to be defined, the quantities of the items should be of same nature. We can not compare the length of the rod to the area of a square.

- However if these quantities are represented in numbers, i.e., length of a rod is a cm and area of a square is b sq.km, we can still define the ratio of these numbers as a:b

Tip 3 – Ratio and Proportion

Properties of Ratios :

- A ratio need not be positive. However, if we are dealing with quantities of items, their ratios will be positive. In this concept we will consider only positive ratios.

- A ratio remains the same if both antecedent and consequent are multiplied or divided by the same non-zero number, i.e.,

$$\frac{a}{b} = \frac{pa}{pb} = \frac{qa}{qb}, \; p, q \neq 0$$

$$\frac{a}{b} = \frac{a/p}{b/p} = \frac{a/q}{b/q}, \; p, q \neq 0$$

Tip 4 – Ratio and Proportion

- Two ratios in their fraction notation can be compared just as we compare real numbers.

$$\frac{a}{b} = \frac{p}{q} \Leftrightarrow aq = bp$$

$$\frac{a}{b} > \frac{p}{q} \Leftrightarrow aq > bp$$

$$\frac{a}{b} < \frac{p}{q} \Leftrightarrow aq < bp$$

- If antecedent > consequent, the ratio is said to be ratio of greater inequality.

- If antecedent < consequent, the ratio is said to be ratio of lesser inequality.

- If the antecedent = consequent, the ratio is said to be ratio of equality

Tip 5 – Ratio and Proportion

If a, b, x are positive, then

- If $a > b$, then $\frac{a+x}{b+x} < \frac{a}{b}$
- If $a < b$, then $\frac{a+x}{b+x} > \frac{a}{b}$
- If $a > b$, then $\frac{a-x}{b-x} > \frac{a}{b}$
- If $a < b$, then $\frac{a-x}{b-x} < \frac{a}{b}$
- If $\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{d}{s} = \ldots$, then $a:b:c:d:\ldots = p:q:r:s:\ldots$

Tip 6 – Ratio and Proportion

If two ratios a/b and c/d are equal

- $\frac{a}{b} = \frac{c}{d} \Longrightarrow \frac{b}{a} = \frac{d}{c}$ (Invertendo)
- $\frac{a}{b} = \frac{c}{d} \Longrightarrow \frac{a}{c} = \frac{b}{d}$ (Alternendo)
- $\frac{a}{b} = \frac{c}{d} \Longrightarrow \frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)
- $\frac{a}{b} = \frac{c}{d} \Longrightarrow \frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)
- $\frac{a}{b} = \frac{c}{d} \Longrightarrow \frac{}{} = \frac{c+d}{c-d}$ (Componendo-Dividendo)
- $\frac{a}{b} = \frac{c}{d} \Longrightarrow \frac{pa+qb}{ra+sb} = \frac{pc+qd}{rc+sd}$, for all real p, q, r, s such that $pa+qb \neq 0$ and $rc+sd \neq 0$

Tip 7 – Ratio and Proportion

If a, b, c, d, e, f, p, q, r are constants and are not equal to zero

- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ then each of these ratios is equal to $\frac{a+c+e..}{b+d+f..}$
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ then each of these ratios is equal to ____________
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ then each of these ratios is equal to $\frac{(pna+qnc+rne+..)1/^{n}}{(p^{n}b+q^{n}d+rnf+..)1/^{n}}$.
- Duplicate Ratio of a : b is $a^2 : b^2$
- Sub-duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$
- Triplicate Ratio of a : b is $a^3 : b^3$
- Sub-triplicate ratio of a : b is $a^{1/3} : b^{1/3}$

Tip 8 – Ratio and Proportion

Proportions :

A proportion is an equality of ratios. Hence a:b = c:d is a proportion. The first and last terms are called extremes and the other two terms are called means.

If four terms a, b, c, d are said to be proportional, then a:b = c:d. If three terms a, b, c are said to be proportional, then a:b = b:c

Tip 9 – Ratio and Proportion

Properties of proportions :

If a:b = c:d is a proportion, then

- Product of extremes = product of means i.e., ad = bc
- Denominator addition/subtraction: a:a+b = c:c+d and a:a-b = c:c-d
- a, b, c, d,.... are in continued proportion means, a:b = b:c = c:d = ...
- a:b = b:c then b is called mean proportional and b^2 = ac
- The third proportional of two numbers, a and b, is c, such that, a:b = b:c
- d is fourth proportional to numbers a, b, c if a:b = c:d

Tip 10 – Ratio and Proportion

Variations :

- If x varies directly to y, then x is said to be in directly proportional with y and is written as $x \propto y$

  $x = ky$ (where k is direct proportionality constant)
  $x = ky + C$ (If x depends upon some other fixed constant C)

- If x varies inversely to y, then x is said to be in inversely proportional with y and is written as $x \propto \frac{1}{y}$

  $x = k\frac{1}{y}$ (where k is indirect proportionality constant)
  $x = k\frac{1}{y} + C$ (If x depends upon some other fixed constant C)

Tip 11 – Ratio and Proportion

Variations :

- If $x \propto y$ and $y \propto z$ then $x \propto z$
- If $x \propto y$ and $x \propto z$ then $x \propto (y \pm z)$
- If $a \propto$ $\propto y$ then $ax \propto by$

Tip 1 – Inequalities

- The topic Inequalities is one of the few sections in the quantitative part which can throw up tricky questions. The questions are often asked in conjunction with other sections like ratio and proportion, progressions etc.

- The theory involved in Inequalities is very limited and students should be comfortable with the basics involving addition, multiplication and changing of signs of the inequalities.

- The scope for making an error is high in this section as a minor mistake in calculation (like forgetting the sign) can lead to a completely different answer.

Tip 2 – Inequalities

- The modulus of x, |x| equals the maximum of x and –x

$$-|x| \leq x \leq |x|$$

- For any two real numbers 'a' and 'b',

$$|a| + |b| \geq |a + b|$$

$$|a| - |b| \leq |a - b|$$

$$|a \cdot b| = |a|\,|b|$$

Tip 3 – Inequalities

- For any three real numbers X, Y and Z; if X > Y then X+Z > Y+Z

- If X > Y and

  1. Z is positive, then XZ > YZ
  2. Z is negative, then XZ < YZ
  3. If X and Y are of the same sign, 1/X < 1/Y
  4. If X and Y are of different signs, 1/X > 1/Y

Tip 4 – Inequalities

- For any positive real number, $x + \frac{1}{x} \geq 2$
- For any real number $x > 1$,

$$2 < \left[1 + \frac{1}{x}\right]^x < 2.8.$$

As x increases, the function tends to an irrational number called 'e' which is approximately equal to 2.718

Tip 5 – Inequalities

- If $|x| \le k$ then the value of x lies between –k and k, or $-k \le x \le k$
- If $|x| \ge k$ then $x \ge k$ or $x \le -k$

Tip 6 – Inequalities

- If $ax^2+bx+c < 0$ then $(x-m)(x-n) < 0$, and if $n > m$, then $m < x < n$
- If $ax^2+bx+c \quad 0$ then $(x-m)(x-n) > 0$ and if $m < n$, then $x < m$ and $x > n$
- If $ax^2+bx+c > 0$ but $m = n$, then the value of x exists for all values, except x is equal to m,

  i.e., $x < m$ and $x > m$ but $x \neq m$

Tip 1 – Quadratic Equations

- Quadratic Equations is one of the important topics for CAT
- The theory involved in this topic is very simple and students should be comfortable with the some basic formulas and concepts.
- The techniques like option elimination, value assumption can help to solve questions from this topic quickly.
- This pdf covers all the important formulas and concepts related to Quadratic Equations.

Tip 2 – Quadratic Equations

General Quadratic equation will be in the form of $ax^2+bx+c = 0$

The values of ‘x’ satisfying the equation are called roots of the equation.

- The value of roots, p and q = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Sum of the roots = p+q = $\frac{-b}{a}$
- Product of the roots = p*q = $\frac{c}{a}$
- If c and a are equal then the roots are reciprocal to each other
- If b = 0, then the roots are equal and are opposite in sign.

Tip 3 – Quadratic Equations

Let D denote the discriminant, $D = b^2 - 4ac$. Depending on the sign and value of D, nature of the roots would be as follows:

- $D < 0$ and $|D|$ is not a perfect square:

Roots will be in the form of p+iq and p-iq where p and q are the real and imaginary parts of the complex roots. p is rational and q is irrational.

- $D < 0$ and $|D|$ is a perfect square:

Roots will be in the form of p+iq and p-iq where p and q are both rational.

- $D = 0$

Roots are real and equal. $X = -b/2a$

Tip 4 – Quadratic Equations

- $D > 0$ and D is not a perfect square:
Roots are conjugate surds

- $D > 0$ and D is a perfect square:
Roots are real, rational and unequal

Tip 5 – Quadratic Equations

Signs of the roots: Let P be product of roots and S be their sum

- $P > 0, S > 0$: Both roots are positive
- $P > 0, S < 0$: Both roots are negative
- $P < 0, S > 0$: Numerical smaller root is negative and the other root is positive
- $P < 0, S < 0$: Numerical larger root is negative and the other root is positive

Tip 6 – Quadratic Equations

- Minimum and maximum values of $ax^2+bx+c = 0$
- If $a > 0$: minimum value = $\frac{4ac-b^2}{4a}$ and occurs at $x = -b/2a$
- If $a < 0$: maximum value = $\frac{4ac-b^2}{4a}$ and occurs at $x = -b/2a$

Tip 7 – Quadratic Equations

If $A_nX^n + A_{n-1}X^{n-1} + \ldots + A_1X + A_0 = 0$, then

- Sum of the roots = $-A_{n-1}/A_n$
- Sum of roots taken two at a time = A_{n-2}/A_n
- Sum of roots taken three at a time = $-A_{n-3}/A_n$ and so on
- Product of the roots = $[(-1)^n A_0]/A_n$

Tip 8 – Quadratic Equations

Finding a quadratic equation:

- If roots are given: $(x-a)(x-b)=0 \Rightarrow x^2-(a+b)x+ab = 0$

  If sum s and product p of roots are given: $x^2-sx+p = 0$

- If roots are reciprocals of roots of equation $ax^2+bx+c = 0$, then equation is $cx^2+bx+a = 0$

- If roots are k more than roots of $ax^2+bx+c = 0$ then equation is $a(x-k)^2+b(x-k)+c = 0$

- If roots are k times roots of $ax^2+bx+c = 0$ then equation is $a(x/k)^2+b(x/k)+c = 0$

Tip 9 – Quadratic Equations

- Descartes Rules: A polynomial equation with n sign changes can have a maximum of n positive roots. To find the maximum possible number of negative roots, find the number of positive roots of f(-x).

- An equation where highest power is odd must have at least one real root

Tip 1 – Linear equations

- Linear equations is one of the foundation topics in the Quant section of CAT.
- Hence, fundamentals of this concept are useful in solving the questions of the other topics by assuming the unknown values as variables.
- Be careful of silly mistakes in this topic as that is how students generally lose marks here.
- Generally, the number of equations needed to solve the given problem is equal to the number of variables

Tip 2 – Linear equations

- A linear equation is an equation which gives straight line when plotted on a graph.
- Linear equations can be of one variable or two variable or three variable.
- Let a, b, c and d are constants and x, y and z are variables. A general form of single variable linear equation is $ax+b = 0$.
- A general form of two variable linear equation is $ax+by = c$.
- A general form of three variable linear equation is $ax+by+cz = d$.

Tip 3 – Linear equations

Equations with two variables:

- Consider two equations ax+by = c and mx+ny = p. Each of these equations represent two lines on the x-y co-ordinate plane. The solution of these equations is the point of intersection.

- If $\frac{\mathrm{a}}{\mathrm{m}} = \frac{\mathrm{b}}{\mathrm{n}} \neq \frac{\mathrm{c}}{\mathrm{p}}$ then the slope of the two equations is equal and so they are parallel to each other. Hence, no point of intersection occurs. Therefore no solution.

- If $\frac{\mathrm{a}}{\mathrm{m}} \neq \frac{\mathrm{b}}{\mathrm{n}}$ then the slope is different and so they intersect each other at a single point. Hence, it has a single solution.

- If $\frac{\mathrm{a}}{\mathrm{m}} \quad \frac{\mathrm{b}}{\mathrm{n}} = \frac{\mathrm{c}}{\mathrm{p}}$ then the two lines are same and they have infinite points common to each other. So, infinite solutions occurs

Tip 4 – Linear equations

General Procedure to solve linear equations:

- Aggregate the constant terms and variable terms
- For equations with more than one variable, eliminate variables by substituting equations in their place.
- Hence, for two equations with two variables x and y, express y in terms of x and substitute this in the other equation.
- For Example, let $x+y = 14$ and $x+4y = 26$ then $x = 14-y$ (from equation 1) substituting this in equation 2, we get $14-y+4y = 26$. Hence, $y = 4$ and $x = 10$.

Tip 5 – Linear equations

General Procedure to solve linear equations:

For equations of the form $ax+by = c$ and $mx+ny = p$, find the LCM of b and n. Multiply each equation with a constant to make the y term coefficient equal to the LCM. Then subtract equation 2 from equation 1.

Example:

Let $2x+3y = 13$ and $3x+4y = 18$ are the given equations (1) and (2).

- LCM of 3 and 4 is 12.
- Multiplying (1) by 4 and (2) by 3, we get $8x+12y = 52$ and $9x+12y = 54$.
- (2)-(1) gives $x=2$, $y=3$

Tip 6 – Linear equations

- If the system of equations has *n* variables with *n-1* equations then the solution is indeterminate
- If system of equations has *n* variables with *n-1* equations with some additional conditions like the variables are integers then the solution may be determinate
- If system of equations has *n* variables with *n-1* equations then some combination of variables may be determinable.
- For example, if ax+by+cz = d and mx+ny+pz = q, if a, b, c are in Arithmetic progression and m, n and p are in AP then the sum x+y+z is determinable.

Tip 7 – Linear equations

Equations with three variables:
Let the equations be $a_1x+b_1y+c_1z = d_1$, $a_2x+b_2y+c_2z = d_2$ and $a_3x+b_3y+c_3z = d_3$. Here we define the following matrices.

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad D_x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix} \quad D_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix} \quad D_z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

- If Determinant of $D \neq 0$, then the equations have a unique solution.
- If Determinant of $D = 0$, and at least one but not all of the determinants D_x, D_y or D_z is zero, then no solution exists.
- If Determinant of $D = 0$, and all the three of the determinants D_x, D_y and D_z are zero, then there are infinitely many solution exists.
- Determinant can be calculated by $D = a_1(b_2c_3-c_2b_3)-b_1(a_2c_3-c_2a_3)+c_1(a_2b_3-b_2a_3)$