Portfolio Optimization Using Various Correlation Matrices

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Introduction

In this project, we aim to estimate the correlation matrix between correlated assets and to investigate how Modern Portfolio Theory (MPT) can be used to optimize a portfolio. While also building an asset portfolio that maximizes expected returns for a specific risk tolerance, We use various optimization techniques by considering a set of assets, their historical returns, and different ways of calculating correlation matrices.

Literature review

Markowitz's Portfolio theory Markowitz, 1952 is the most fundamental theory which started the foundation of the contemporary theory of asset allocation. The literature considers rational investors and minimizes portfolio variance with a fixed expected return for the whole portfolio. Though, the model is impractical and most investment companies don't use Markowitz's mean-variance optimization methods, almost all the good strategies are based on this bedrock.

Correlation Matrix

The correlation matrix is a squared, symmetric, and positive semi-definite matrix with all the principal diagonal elements equal to 1 and which shows the correlation co-efficients between different assets. In portfolio optimization techniques, correlation plays a significant role and that's why calculating a suitable correlation matrix is very crucial.
We have used 4 different methods of estimating Correlation Matrices viz, Pearson's Corr., Kendall Corr., and Spearman Rank Corr. & Distance Corr.

Problem Description

MAXIMIZE: $w^T.E$

SUBJECT TO: $w^T.Cov.w <= \gamma^2$

$\Sigma(w) = 1$ & $w_i >= 0$ for all $i = 1,2,3 \ldots$

Decision Variable: (Asset allocation (Weights)) → the distribution of investments across different asset classes (e.g. stocks, bonds, etc.). Weights will be assigned to assets (say w_1, w_2,...) where

$$w_i = \frac{\textit{Capital invested in 'ith' asset}}{\textit{Total Capital}}$$

E is the expected value of daily return.

Parameters:

- Daily return (i^{th} day): $[W^T.P_{i+1}/(W^T.P)_i - 1]*100$
- Target Risk: Investor's risk tolerance = γ

Expected Values, E is estimated by the time-averaged prices
P is the Price matrix where j^{th} column is the price of j^{th} asset.
*Data for 120 days is taken for Analysis from Yahoo Finance

Results and discussion

Model estimates

Below given is a plot of the efficient frontier for our data. We can also observe the desired output of minimum risk at the rightmost part of the graph labeled as 'Minimum Risk Portfolio'.

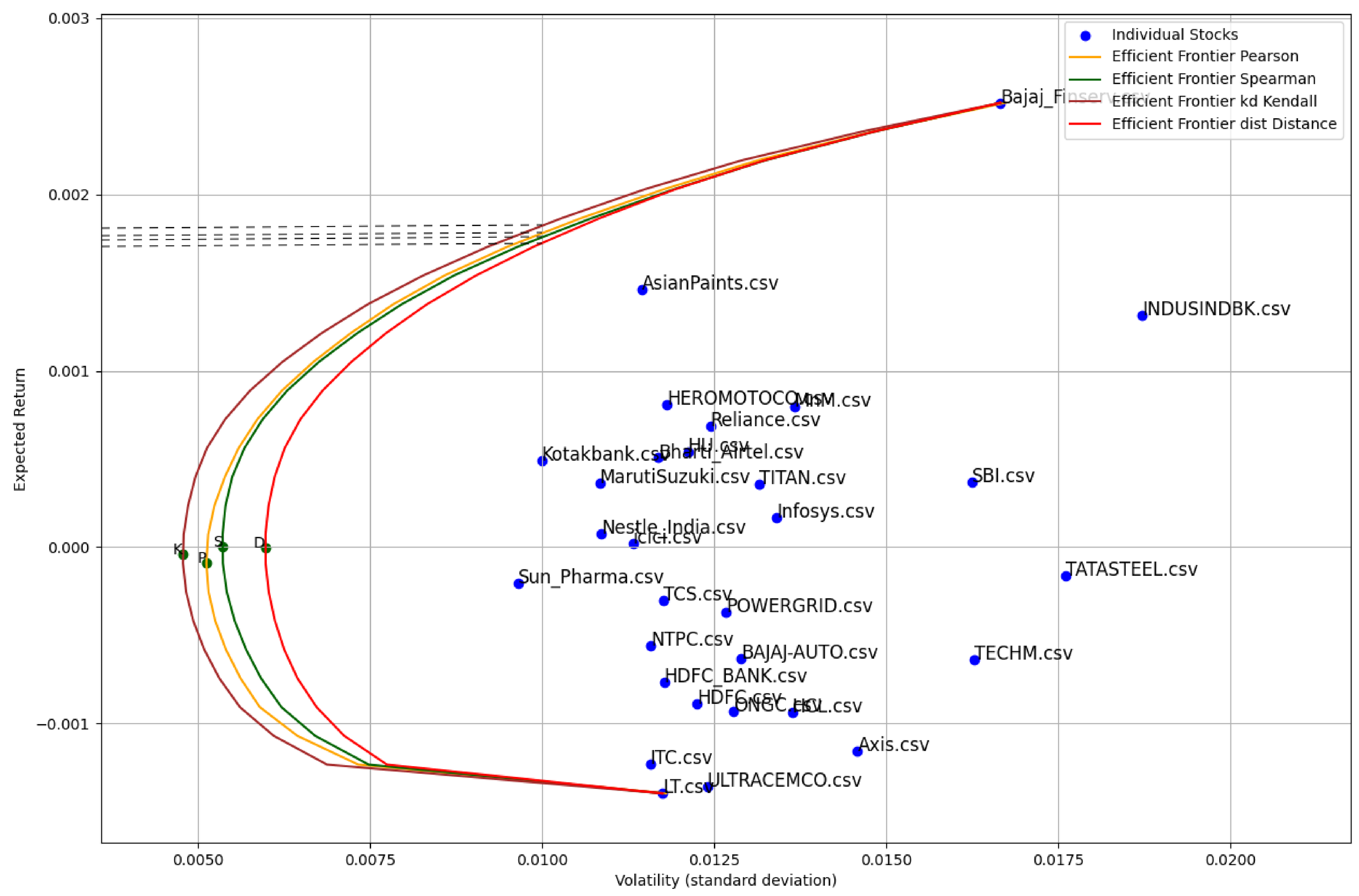

Figure 1. Efficient Frontier

The optimized weights with Kendall covariance which we found after the optimization was done considering the minimizing risk constraint can be found in the table below:

Company	Weight	Company	Weight	Company	Weight
HDFC	0	AXISBANK	0	ULTRACEMCO	0
HDFCBANK	0	LT	0	TITAN	0
RELIANCE	0	MARUTI	0	TECHM	0
INFY	0	BAJAJFINSV	45.08	BAJAJ-AUTO	4.592
ICICIBANK	0	ASIANPAINT	39.65	POWERGRID	0
TCS	0	HCLTECH	0	HEROMOTOCO	0
HINDUNILVR	0	SBIN	0	INDUAINDBK	0
ITC	10.66	NESTLEIND	0	NTPC	0
KOTAKBNK	0	SUNPHARMA	0	ONGC	0
BHARTIARTL	0	M&M	0	TATASTEEL	0

Table 1. Weight distribution(%)

Conclusions

- After comparing for different correlation matrics, we get the order of returns as Pearson Corr < Spearman Rank Corr < Distance Corr < Kendall Corr

References

https://www.wallstreetmojo.com/markowitz-model/
https://www.gurobi.com
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https://finance.yahoo.com/