PROJECT REPORT

BITS Pilani K K Birla Goa Campus

A sustainable, equitable and economically feasible water distribution model for a region using principles of Game theory

In partial fulfillment of the course BITS F314 - Game Theory and its Application
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November 2018

ACKNOWLEDGEMENTS

We would like to thank Dr. Himadri Mukherjee for his patient guidance as well as the encouragement throughout the course of this project. We would also like to thank sir for his valuable inputs at every juncture of the project, right from the conception stage to the final stages.

ABSTRACT

A 2014 survey of the world's 500 largest cities estimates that one in four are in a situation of "water stress". According to UN-endorsed projections, global demand for fresh water will exceed supply by 40 % in 2030, thanks to a combination of climate change, human action and population growth. In recent times, Sao Paulo and Cape Town were two of the major cities that came very close to running out of water, a situation named as "Day 0", due to a mixture of environmental and man made events. On the other hand, several cities face water crunch, such as Bengaluru, which is predicted to run out of groundwater by 2020 by Niti Aayog.
In this project we develop a sustainable, equitable and economically feasible water distribution model for a region using principles of Game theory. We model water allocation as co-operative and non co-operative games and attempt to find a Nash equilubruim. Finally, we apply the model to water management in the affected city - Bengaluru.

Contents

1. INTRODUCTION

In the following world map, you can notice that many countries are currently classified as under moderate to severe stress with respect to availability of clean water. Moreover, the same countries have huge population and face severe pollution problems. Eventually, as populations increase and clean available water decreases, conflicts might arise.
Moreover, since water resources are not equally distributed among different countries, these conflicts might escalate into war. Hence, resolution of conflicts related to water between countries, as well as states is important.
Also, there is another dimension to water resource management, that is how the governments distribute the available water between different players such as citizens, industry, etc. This when modeled as a game, opens up interesting possibilities.

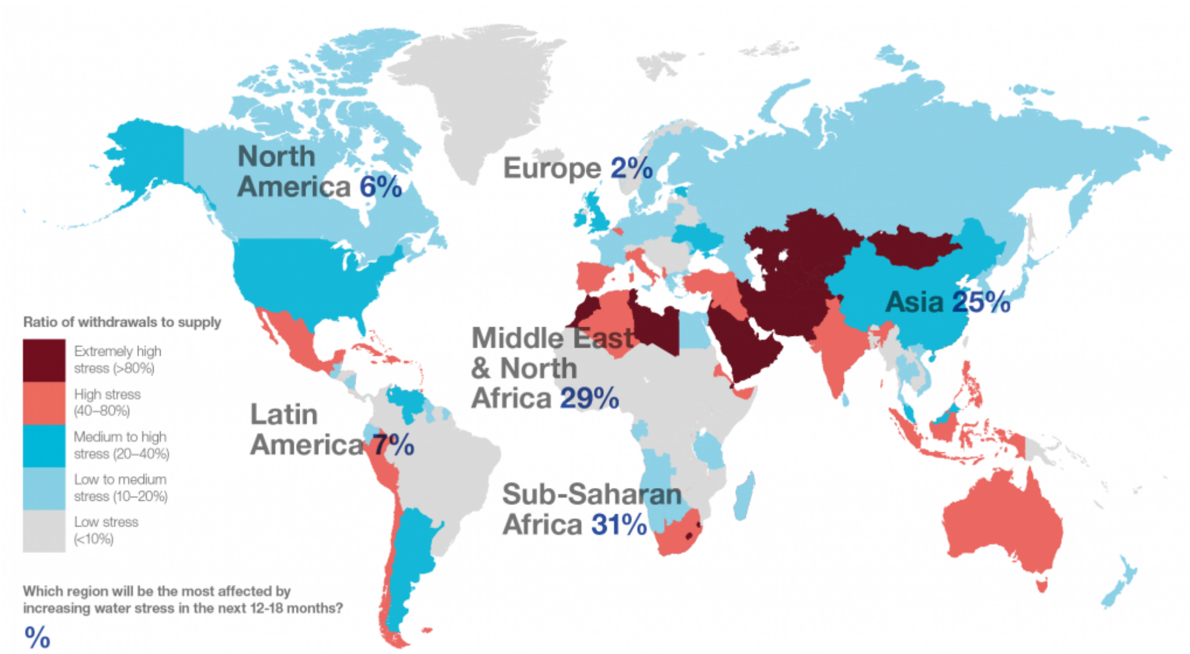
stress.png 

Figure 1.1: Water Stress

1.1. Terminology

A - The amount of water in dispute.

1. With subscript "I" - refer to Industry.
a. γ_I - Revenue - per unit water.
b. p_I - Cost to treat pollutants: The cost incurred by industries due to water treatment standards imposed by environmental bodies.
c.U_{I_i} - Water supplied to industry by state i.
c. u_I - Monetary payoff made by industry.
d. v_I Satisfaction payoff of the industry.
e. V_I - Demand by Industry: Demand for water by industries, considering both freshwater and recycled water.
f. κ_I - GDP Growth ratio: Defines how change in water allocation to industry affects GDP.
g. β_{I_i} - Scaling factor for GDP growth. h. t_{I_i} - Tariff on industry in state i.
i. p_{IC} - cost incurred by industries in state i to treat water 100%.

2. With subscript "C" - refer to Citizens.
a. α_{C_i} - Coefficient of rainwater harvesting: Willingness of citizens to practice rainwater harvesting.
b. η_{C_i} - Coefficient of water tax dissatisfaction.
c. V_{C_i} = Demand of citizens.
d. t_{C_i} = Water tax on citizens.
e. l_1 = Monetary scaling of rainwater harvesting.
f. u_{C_i} = Monetary payoff of citizens. g. v_{C_i} = Satisfaction payoff of citizens.

3. With subscript "S" - refer to State.
a. V_{S_i} - Demand of state i: The entire water demand in a state, including the demand by industry and citizens.
b. t_{C_i} - Water tax on citizens of state i: Tariff imposed on citizens of a state for water, including domestic and agriculture needs.
c. t_{I_i} - Water tax on Industry of state i.
d. c_{S_i} - Cost of water infrastructure for state i: The cost incurred by the state government of state i in building the necessary infrastructure - dams, water distribution pipelines, etc.
e. U_{I_i} - Supply to industry: The amount of water the state i supplies to the industry.
f. U_{C_i} - Supply to citizens: The amount of water the state i supplies to the citizens.
g. λ_i - Population of state i.
h. U_{S_i} - The amount of water allocated to state i.
i. u_{S_i} = Monetary payoff of state i. j. v_{S_i} = Satisfaction payoff of state i.

4. With Subscript "E" - refer to environmental bodies.
a. κ_E = Water treatment ratio.

b. v_E = Satisfaction payoff of Environmental bodies.

5. General
a. l_1 - Monetary scaling of rainwater harvesting.
b. l_2 - Monetary scaling of reservoir and dams.
c. k_1 - Satisfaction scaling of water supply.
c. k_2 - Satisfaction scaling of treatment cost ratio.
d. k_3 - Amount of water saved via rainwater harvesting per year.
e. k_4 - Satisfaction scaling of GDP of industry.
f. k_5 - Capacity of reservoir or dams.
g. k_6 - Scaling of citizen dissatisfaction.

1.2. Strategies and Payoffs

1.Industry (I):

Strategies:

a. $V_I = \begin{cases} 0 & \gamma_I - t_I - p_I \leq 0 \\ > 0 & otherwise \end{cases}$
b. κ_I

Other factors:

$$\beta_{I_i} = \frac{U_{I_i} \kappa_I (t_{I_{i\,max}} - t_{I_{i\,min}})}{A(t_{I_i} - t_{I_{i\,min}})}$$
$$t_I = \sum_{i=1}^{n} t_{I_i}$$

Payoffs:
a. $u_I = (\gamma_I$ - t_I - $p_I)$
b. $v_I = k_{I_i}$ $(U_{I_i}$ - $v_I)$

2. State governments (S_i):

Strategies:
a. U_{C_i}
b. U_{I_i}

c. t_{C_i}
d. t_{I_i}
e. c_{S_i}

Other factors:
$U_{S_i} = U_{I_i} + \lambda_i \, U_{C_i}$
$U_I = \sum_{i=1}^{n} U_{I_i}$

Payoff:
a. $u_{S_i} = t_{C_i} \, l_3 \, \lambda_i + t_{I_i}(1\text{-}l_3) - l_2 \, c_{S_i}$
b. $v_{S_i} = k_1 \, (U_{S_i} - V_{S_i}) + (k_4 \, \beta_i) + k_5 \, c_{S_i} \, k_1 - k_6 \, \eta_{C_i}$

3. Citizens(C_i):

Strategies:
a. α_{C_i}

Other factors:

$$\eta_{C_i} = \frac{t_{C_i} - t_{C_{i\,min}}}{t_{C_{i\,max}} - t_{C_{i\,min}}} \qquad 0 <= \eta_{C_i} <= 1$$

Payoff:
a. $u_{C_i} = -\,(l_1 \, \alpha_{C_i} + t_{C_i})$
b. $v_{C_i} = k_1 \, (U_{C_i} - V_{C_i} + k_3 \, \alpha_{C_i})$

4.Environmental Body (E):

Strategies:

k_E varies as $0 <= k_E <= 1$.

Other factors:
$p_I = k'_E p_{IC}$

Payoff:
$v_E = k_2 \kappa_E - \frac{k_4}{n} \sum_{i=1}^{n} \beta_{I_i}$

2. MODEL OF THE GAME

The game is played at two levels:

1. Allocation game - The central governing body allocates water to all the states from the total amount A.
2. Reaction game - Once the allocation has been done, each state distributes water among its citizens and Industry and also decides the tariffs. Once these values have been decided, the other agents (Industry, Environment Body, Citizens) decide the values of their strategies as a reaction to this allocation.

2.1. ALLOCATION GAME

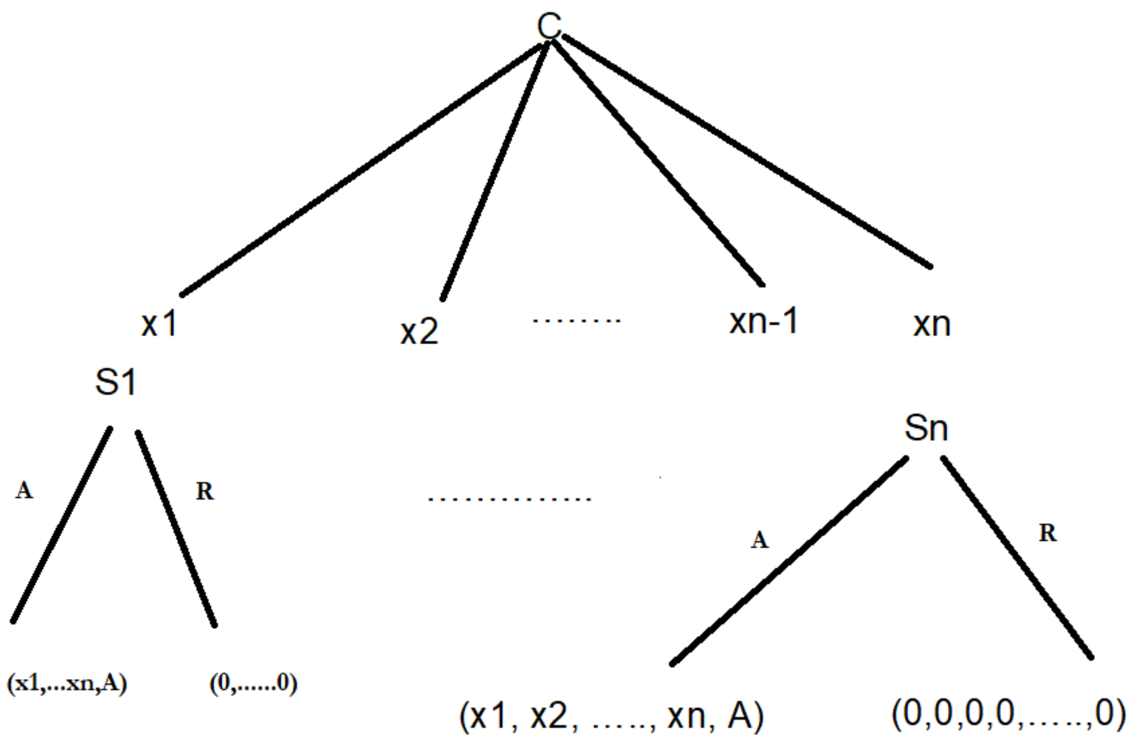


Figure 2.1: Model of allocation game.

The Allocation Game is played as follows :
The central governing body allocates water such that state S_{I_i} gets amount x_i. Each state can accept or reject the allocation made to it. If all the states accept their allocation, then

payoff of each state is U_{S_i}, the value of which is obtained in the reaction game, and the payoff of the central governing body is $\sum_{i=1}^{n} U_{S_i}$. If any of the states reject their allocation, all the states as well as the central governing body get a payoff of 0.
Whether a state accepts or rejects its allocation is decided by a threshold value ϵ_i. Using this threshold value, we can define the best response function of each state as

$$BR_i = \begin{cases} Accept & x_i >= \epsilon_i \\ Reject & otherwise \end{cases} \tag{2.1}$$

For the non co-operative case, ϵ_i will be A/n for all the states since no state wishes to compromise and this is the maximum possible value of ϵ_i that will prevent any state from rejecting their allocation.
For the co-operative case, ϵ_i < A/n for all the states since the states are willing to compromise. Because of this, the central governing body is able to allocate more water to states in dire need and overall the net payoff of the central governing body will increase.

2.2. Reaction game

The reaction game is four-player perfect information extensive game. The players are State, Industry, Environmental bodies and Citizens and they make their choices in the same order (Fig 2.2).
We solve this game using backward induction starting with maximizing the payoff for the Citizens and moving upwards. This results in a subgame perfect Nash equilibrium.
This game can be solved by using backward induction algorithm. We begin by maximizing the citizen's payoff, by differentiating it with respect to their strategies, which is coefficient of rainwater harvesting (α_{C_i}), equating it to 0, and solving.
Next, we maximize the payoff of the Environmental Body, by differentiating it with respect to their strategy κ_E, equating to 0 and solving.
Similarly, we next move to the Industry. We maximize their payoff by differentiating it with respect to both κ_I and V_I.
Finally, we maximize the payoff of the State i by differentiating their payoff function with respect to t_{I_i},t_{C_i}, U_{I_i}, U_{C_i} and V_i.
All these will result in a system of partial differential equations, which on solving would give the equilibrium points for all the strategies of agents involved.

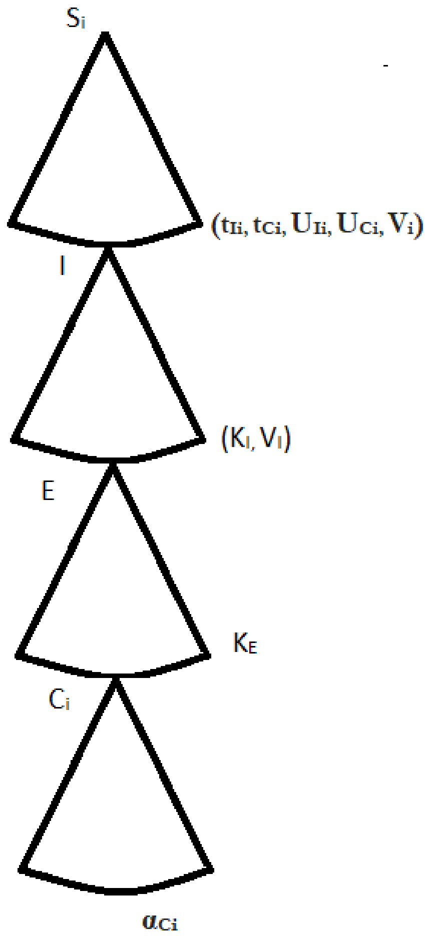


Figure 2.2: Reaction game model.

2.3. Solving the Non co-operative model

The reaction game for the non-cooperative case is solved as explained above.
These values of the different strategies obtained form an equilibrium. Since the allocation is already fixed for the non-cooperative model (each state gets exactly A/n), we have solved the complete game for non-cooperative case.

2.4. Solving the Co-operative model

The Reaction game of the Co-operative model is solved in a similar way as that of the Non Co-operative model (Fig 2.2).
The Allocation game is solved differently in this case as the allocation is not fixed (since threshold of each state is less than A/n).

Once the equilibrium values for different strategies of the Reaction game have been obtained, we calculate individual payoffs of the different states. Using these payoffs, we solve for the Shapley value, which gives us an optimal allocation among the states that would maximize the overall payoffs of all states.

We demonstrate the calculation of Shapley value for the case of 3 states :
Let the states be s_1, s_2, s_3. Let their respective payoffs be v_1, v_2 and v_3.
The following are the payoffs of possible coalitions:
1. $p_1 = \{s_1,s_2\}$
2. $p_2 = \{s_1,s_3\}$
3. $p_3 = \{s_2,s_3\}$
4. $p_4 = \{s_1,s_2,s_3\}$

We find the shapley values using the following formula:

$$\phi_i(v) = \frac{1}{|N|!}\sum_R [v(P_i^R \bigcup \{i\}) - v(P_i^R)] \tag{2.2}$$

Shapley value for each state :
ϕ_1(v) = (1/6) ($p_1 + 2v_1 + 2p_4$ - $2p_3 + p_2$ - v_3 - v_2)
ϕ_2(v) = (1/6) ($p_3 + p_1$ - $v_1 + 2p_4$ - $2p_2 + 2v_2$ -v_3)
ϕ_3(v) = (1/6) ($p_3 + 2p_4$ - $2p_1 + p_2 + p_3$ - v_1 - $v_2 + 2v_3$)

3. Case study - Cauvery River basin

3.1. Non Co-Operative model

3.1.1. Assumptions

- Demand of citizens is constant and is equal to their minimum requirement.
- Demand of the industry is assumed to be constant.
- Demand of the state is constant and equal to the sum of demand of all its citizens and industry.
- When calculating the total payoff of any agent, both the monetary and satisfaction payoffs have been assumed to have equal weightage (except for Environmental Body which has only satisfaction payoff).
- The scaling factors (k_i's and l_i's) have been chosen so that all the terms of payoff have approximately the same order of magnitude.

3.1.2. Citizens

Values of coefficients and other data:
l_1 = Rs 2.64 per m^3 of water per year.
$k_1 = 0.1$
$k_3 = 1.8$ m^3 of water per year.
$t_{C_{i\,max}}$ = Rs 12 per m^3 of water per year.
$t_{C_{i\,min}}$ = Rs 8 per m^3 of water per year.
$w_1 = w_2 = 50$.

Strategies:
α_{C_i}

Other factors:
$\eta_{C_i} = (t_{C_i} - 8)/4$ [Refer A.3 for values.]

Payoff:
$P_{C_i} = -123\ \alpha_{C_i} - 50\ t_{C_i} + 5\ U_{C_i}$

Formation of PDE:

$$\frac{\partial P_{C_i}}{\partial \alpha_{C_i}} = 0 \tag{3.1}$$

$$\implies \alpha_{C_i} = 0$$

3.1.3. ENVIRONMENTAL BODY

Values of coefficients and other data:
p_{IC} = Rs 12 per m^3 of water.
$k_2 = 10$
$k_4 = 30$
A = $321.5 \times 10^8 m^3$ of water per year.
$w_1 = 0, w_2 = 1$

Strategies:
κ_E

Other factors:
$p_I = 12\kappa_E$

Payoff:
$P_E = 10\ \kappa_E$ - $(46.656 \times 10^{-10})\kappa_I \sum_{i=1}^{3} \frac{U_{I_i}}{t_{I_i}-15}$

Formation of PDE:

$$\frac{\partial P_E}{\partial \kappa_E} = 10 - (46.656 \times 10^{-10})(\kappa_I \sum_{i=1}^{3} \frac{\partial(\frac{U_{I_i}}{t_{I_i}-15})}{\partial \kappa_E} - \frac{\partial \kappa_I}{\partial \kappa_E} \sum_{i=1}^{3} \frac{U_{I_i}}{t_{I_i} - 15}) \tag{3.2}$$

3.1.4. INDUSTRY

Values of coefficients and other data:
γ_I = Rs 109.5 per m^3 of water per year. [Refer A.4 for values.]
$k_1 = 10^{-8}$
$t_{I_{i\,max}}$ = Rs 30 per m^3 of water per year.
$t_{I_{i\,min}}$ = Rs 15 per m^3 of water per year. [Refer A.1 for values.]
$w_1 = w_2 = 50$.

Strategies:
κ_I

Other factors:
$\beta_{I_i} = (46.656 \times 10^{-11}) \frac{U_{I_i}}{t_{I_i}-15} \kappa_I$

Payoff:
P_I = 5475 - 50$\sum_{i=1}^{3} t_{I_i}$ - 600κ_E + $(5 \times 10^{-7})\sum_{i=1}^{3} U_{I_i}$

Formation of PDE:

$$\frac{\partial P_I}{\partial \kappa_I} = -50\sum_{i=1}^{3}\frac{\partial t_{I_i}}{\partial \kappa_I} - 600\frac{\partial \kappa_E}{\partial \kappa_I} + (5 \times 10^{-7})\sum_{i=1}^{3}\frac{\partial U_{I_i}}{\partial \kappa_I} \tag{3.3}$$

3.1.5. State

Values of coefficients and other data:
$k_1 = 10^{-8}$
$k_4 = 50$
$k_5 = 9.5 \times 10^9$ per m^3 of water per year.
l_2 = Rs 0.016 to save one m^3 of water per year.
$l_3 = 4.91 \times 10^{-8}$
$w_1 = w_2 = 50$.

Strategies:
$t_{I_i}, t_{C_i}, U_{I_i}, U_{C_i}, c_{S_i}$

Other factors:
-

Payoff:
$P_{S_i} = (245.5 \times 10^{-8})\lambda_i t_{S_i} + 50(1 - 4.91 \times 10^{-8})t_{I_i} - 0.08c_{S_i} + (5 \times 10^{-7})(\lambda_i U_{C_i} + U_{I_i})\kappa_I +$
$(116.64 \times 10^{-8})\frac{U_{I_i}}{t_{I_i} - 15} + 4750c_{S_i} - 625(t_{C_i} - 4)$

Formation of PDE:
A total of five PDEs will be obtained by taking partial derivatives of P_{S_i} with each of $t_{I_i}, t_{C_i}, U_{I_i}, U_{C_i}, c_{S_i}$.

3.2. Co-Operative model

The Supreme court has allocated water to the states Karnataka (s_1), Tamil Nadu (s_2), and Kerala (s_3) in the following way.
Karnataka (w_1): 284.75 TMC
Tamil Nadu(w_2): 404.25 TMC
Kerala(w_3): 30 TMC

The following are the payoffs of possible coalitions:
1. v1 = [s_1]
2. v2 = [s_2]
3. v3 = [s_3]
4. p_1 = [s_1,s_2]
5. p_2 = [s_1,s_3]
6. p_3 = [s_2,s_3]
7. p_4 = [s_1,s_2,s_3]

$p_1 = (w_1/w_2)\text{v1} + (w_3/w_1)\text{v2}$
$p_2 = (w_1/w_3)\text{v1} + (w_3/w_1)\text{v3}$
$p_3 = (w_2/w_3)\text{v2} + (w_3/w_2)\text{v3}$
$p_4 = (w_1/(w_2 + w_3))\text{v1} + (w_2/(w_1 + w_3))\text{v2} + (w_3/(w_1 + w_2))\text{v3}$

$p_1 = 0.7044\ \text{v1} + 1.4197\ \text{v2}$
$p_2 = 9.4917\ \text{v1} + 0.1056\ \text{v3}$
$p_3 = 13.4150\ \text{v2} + 0.0742\ \text{v3}$
$p_4 = 0.6557\ \text{v1} + 1.2843\ \text{v2} + 0.0435\ \text{v3}$

Shapley values:
$\phi_1(\text{v}) = (1/6)\ (p_1 + 2v_1 + 2p_4 - 2p_3 + p_2 - v_3 - v_2)$
$\phi_2(\text{v}) = (1/6)\ (p_3 + p_1 - v_1 + 2p_4 - 2p_2 + 2v_2 - v_3)$
$\phi_3(\text{v}) = (1/6)\ (p_3 + 2p_4 - 2p_1 + p_2 + p_3 - v_1 - v_2 + 2v_3)$

Substituting values:
$\phi_1(\text{v}) = 2.2512\ \text{v1} - 3.9736\ \text{v2} - 0.1594\ \text{v3}$
$\phi_2(\text{v}) = -2.9946\ \text{v1} + 3.2339\ \text{v2} - 0.175\ \text{v3}$
$\phi_3(\text{v}) = 1.3990\ \text{v1} + 2.0240\ \text{v2} + 0.3778\ \text{v3}$

Once the payoffs of different states are obtained from the reaction game above, we can plug them in here to obtain the Shapley values, which would help us move towards the direction of the optimal allocation of water among these states.

4. Conclusion

The game is of two levels. The first level is about how the water is allocated between the states by the concerned central authorities (like the Supreme Court, the Central Government). This game results in a Nash Equilibrium. The value of the Nash Equilibrium depends on the way we view the game. It can be viewed either in the Non-cooperative sense, or the cooperative sense. In this report, we have considered both.

The second level is a reaction to the allocation game played by the states. The Environmental body (E), the Industry (I), the Citizens (C_i), and the states(S_i) are the players in the reaction game. Each player then changes their strategies according to the allocations made: to maximize their individual payoffs in the case of the non-cooperative model, and to cooperate and maximize their overall gain, not necessarily maximizing their individual gain.

List of Figures

5. References

Sources of data:
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A. APPENDIX A

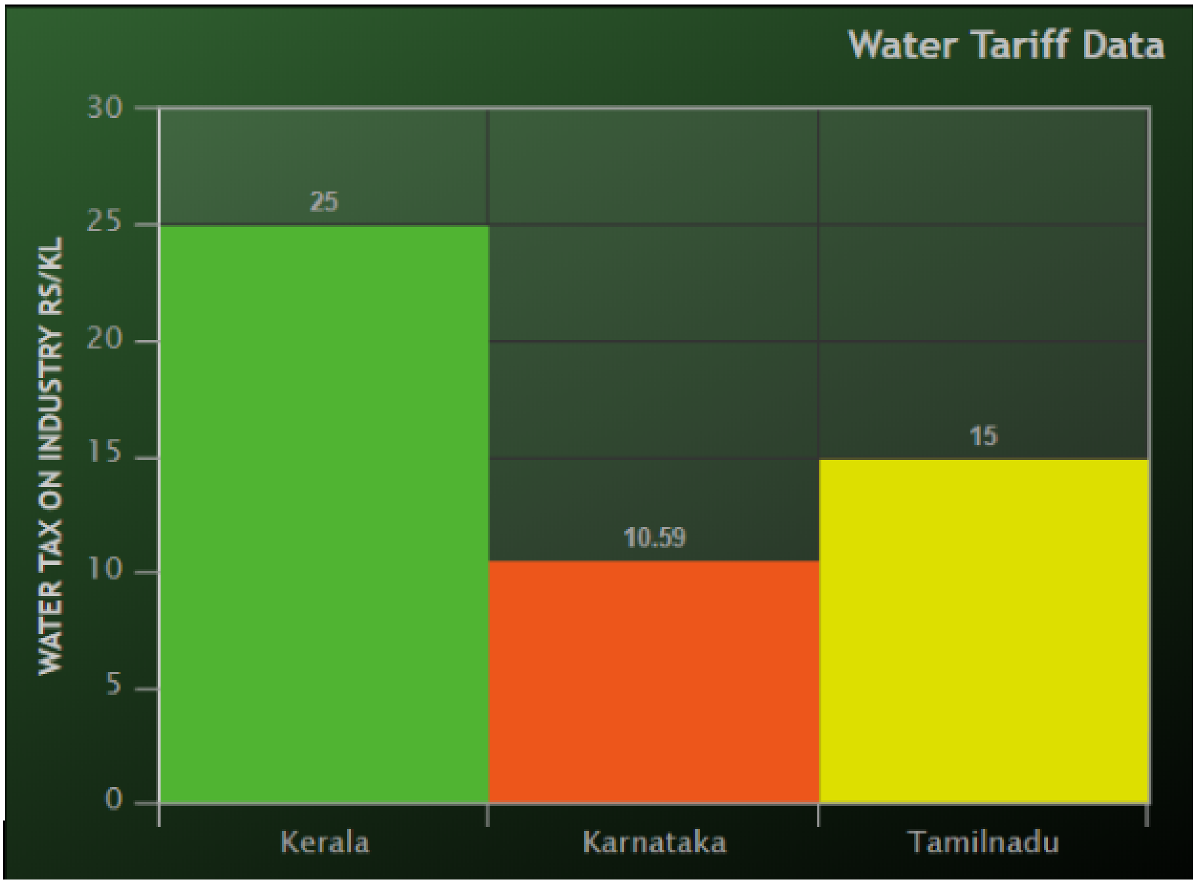

Figure A.1: Water Tax on industry.

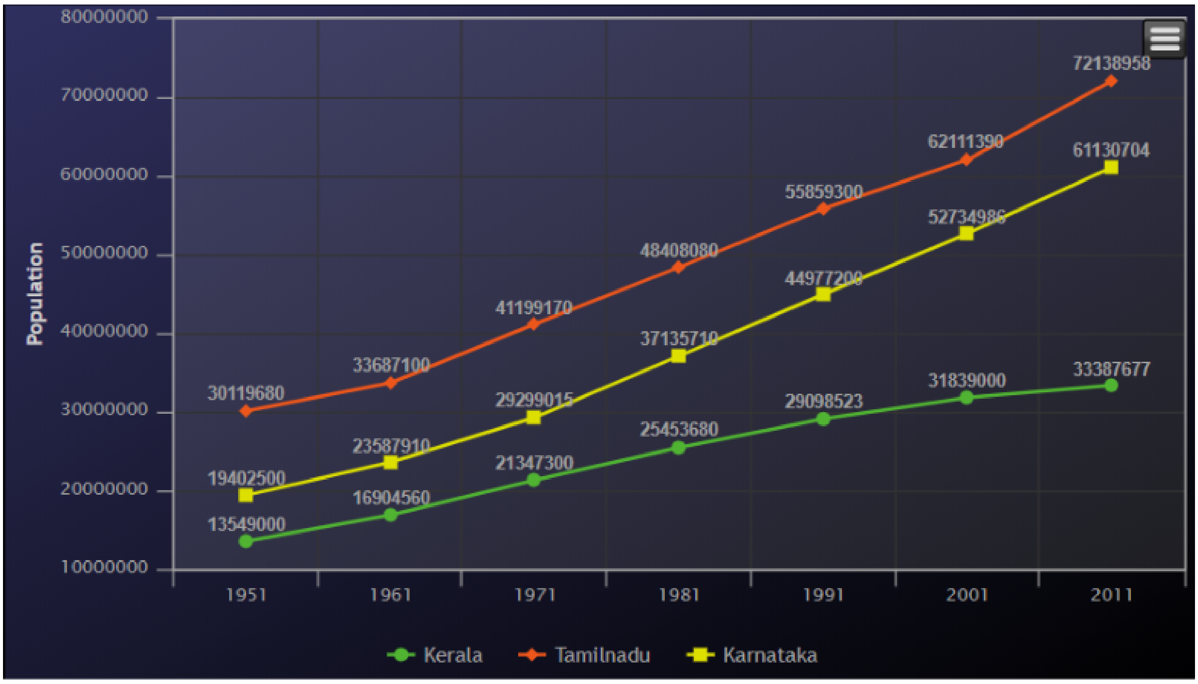

Figure A.2: Population time series data.

<table>
<tr><th colspan="5">Tariff for Domestic connections</th></tr>
<tr><th>Slab</th><th>Water Tariff, Rs</th><th>Sanitary</th><th>Sanitary for Borewell, Rs</th><th>Meter Cost (15mm), Rs</th></tr>
<tr><td>0-8000 litres</td><td>7</td><td>Rs.14 /-</td><td rowspan="4">Rs.100</td><td>30</td></tr>
<tr><td>8001-25000</td><td>11</td><td rowspan="3">25%</td><td>50</td></tr>
<tr><td>25001-50000</td><td>26</td><td>75</td></tr>
<tr><td>Above 50000</td><td>45</td><td>150</td></tr>
</table>

Figure A.3: Water tax on citizens.

Sector	No. of Unit	Investment(Crore)	Water Requirement (Million liter/day)	Profit(Crore)
Micro	150000	25000	15	3750
Small	15000	50000	10	7500
Medium	3000	50000	15	7500
Large	1500	75000	30	11250
Mega	300	100000	20	15000
Ultra Mega	150	100000	50	15000
Super Mega	75	100000	100	15000
Total	170025	500000	240	75000

Figure A.4: Estimation of profit to industry.