$A_I = \{1, 2, 3, \ldots\}$

$A_{II} = \{S \subseteq N : v(S) > 0\}$

$$u(i,S) = \begin{cases} 1/v(S) & \text{if } i \in S \\ 0 & \text{otherwise.} \end{cases}$$

MSNE:

value of $G = \delta$.

step 2: $\delta > 0$

step 3: $\delta < 1$

$x = \{x_1, x_2, \ldots, x_n\}$ ms of player I

↳ probability distribution of set of actions.

not negati as in pict. [illegible] dis

core empty $\Rightarrow x \notin \mathcal{C}(N, v) \rightarrow$ There exists $S \subseteq N$ s.t $v(S) > x(S) \geq 0$

$\Rightarrow v(S) > 0$.

$$u(x,S) = \sum_{i \in S} x_i \, u(i,S) = \sum_{i \in S} \frac{x_i}{v(S)} = \frac{x(S)}{v(S)} < 1$$

$\therefore \delta < 1$

step 3: $\langle N, v \rangle$ is not balanced

$\exists$ BCWS $(\lambda_S)_{S \in \mathcal{C}}$ s.t

$$\sum_{S \in \mathcal{C}} \lambda_S v(S) > v(N)$$

$y^* (y_S)_{\{S : v(S) > 0\}}$ optimal strategy of player II.

Date
Page

$C = \{S \subseteq N : v(S) > 0\} \cup \{\{1\}, \{2\}, \ldots, \{n\}\}$.

winning coalition with 0-1 game.

$\lambda_S = \dfrac{y_S}{\cancel{\Sigma(v(S))}\ \delta\, v(S)}$ ← probability assigned to set y_S . $v(S) > 0$

$$\lambda_{\{i\}} = 1 - \sum_{\{S \subseteq N : i \in S,\ v(S) > 0\}} \lambda_S \qquad \forall i \in N.$$

To show :- $(\lambda_S)_{S \in C}$ is BCWs

(i) $\sum_{\{S \in C : S \ni i\}} \lambda_S = 1 \quad \forall i \in N.$

(ii) $\lambda_S \in [0, 1]$, $\forall S \subseteq N$.

$\forall i \in N \quad \sum_{\{S \subseteq N : i \in S,\ v(S) > 0\}} \lambda_S + \lambda_{\{i\}} = 1.$

$$\sum_{\{S \subseteq N : i \in S,\ v(S) > 0\}} \lambda_S = \frac{1}{\delta} \sum y_S / v(S) = \frac{1}{\delta}\, w(i, y) \le \frac{\delta}{\delta} = 1$$

$0 \le \lambda_{\{i\}} \le 1$.

To show $\sum_{S \in C} \lambda_S\, v(S) > v(N)$

$$\sum_{S \in C} \lambda_S v(S) = \sum_{i \in N} \lambda_{\{i\}}\, v(i) + \sum_{\{S \subseteq N : v(S) > 0\}} \lambda_S\, v(S)$$

$= 0 + \cancel{\Sigma_1 y_S} \sum_{\{S \subseteq N : v(S) > 0\}} \frac{y_S}{\delta\, v(S)} \cdot v(S)$

because of 0-1 game

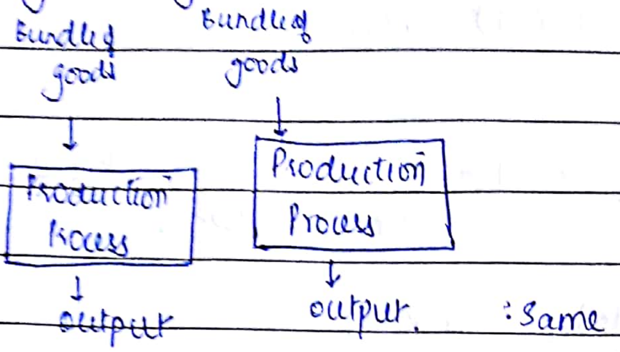
$= \frac{1}{\delta} \sum_{\{S \subseteq N : v(S) > 0\}} y_S$

$= \frac{1}{\delta} \cdot 1 > 1 = v(N)$

Classical economy

~~Model~~ market : consisting player called as agents.

$agent_1$, $agent_2$ . . . . — .



A market with transferrable payoff consists of

- a finite set N (the set of agents)
- a +ve integers of ℓ (the number of input goods)
- for each agent $i \in N$ a vector $w_i \in \mathbb{R}^{\ell}_{+}$ (the endowment of agent i)
- for each agent $i \in N$ a continuous, non-decreasing, and concave function

$$f_i : \mathbb{R}^{\ell}_{+} \rightarrow \mathbb{R}_{+} \text{ (the production function of agent } i\text{)}$$

Ex $N = \{1,2,3\}$

$\ell = 2$: 2 commodities.

endowments of agents

$w_1 = (1,0)$, $w_2 = (0,1)$, $w_3 (2,2)$.

Production functions

$f_1(z) = z_{1,1} + z_{1,2}$

$f_2(z) = z_{2,1} + 2 \cdot z_{2,2}$.

$f_3(z) = \sqrt{z_{3,1}} + \sqrt{z_{3,2}}$

input vector is a member of $\mathbb{R}^{\ell}_{+}$. profile $(z_i)_{i \in N}$ of input vector for which $\sum_{i \in N} z_i = \sum_{i \in N} w_i$ is an allocation.

model a market $\langle N, \ell, (w_i), (f_i) \rangle$ as CG-TP

$\langle N, v \rangle$

- N is set of agents
- for each coalition S we have

$$v(S) = \max_{(z_i)_{i \in N}} \left\{ \sum_{i \in S} f_i(z_i) : z_i \in \mathbb{R}_+^{\ell} \text{ and } \sum_{i \in S} z_i = \sum_{i \in S} w_i \right\}$$

Note : 1. output of production process are same.

2. players outside coalition (S/N) don't affect production

$N = \{1, 2, 3\}$

find v for each coalition.