MATHEMATICS

Matrices

Part – 1

Elementary Transfomations

Matrices

Elementary Transformation :

(Recall that R and C symbolically represent the rows and columns of a matrix.)

(a) Interchange of any two rows or any two columns. If we interchange the i^{th} row and the j^{th} row of a matrix then after this interchange the original matrix is transformed to a new matrix.
This transformation is symbolically denoted as $R_i \leftrightarrow R_j$ or R_{ij}

Ex. (1) : If $A = \begin{bmatrix} 3 & -5 \\ 2 & 6 \end{bmatrix}$ then $R_1 \leftrightarrow R_2$ gives $\begin{bmatrix} 2 & 6 \\ 3 & -5 \end{bmatrix}$

Ex. (2) : If $B = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 3 \\ 1 & 7 & 8 \end{bmatrix}$ then $C_2 \leftrightarrow C_3$ gives $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 3 & 5 \\ 1 & 8 & 7 \end{bmatrix}$

Note that $A \neq \begin{bmatrix} 2 & 6 \\ 3 & -5 \end{bmatrix}$ and $B \neq \begin{bmatrix} 1 & 4 & 3 \\ 2 & 3 & 5 \\ 1 & 8 & 7 \end{bmatrix}$

But however we write $A \sim \begin{bmatrix} 2 & 6 \\ 3 & -5 \end{bmatrix}$ and $B \sim \begin{bmatrix} 1 & 4 & 3 \\ 2 & 3 & 5 \\ 1 & 8 & 7 \end{bmatrix}$

Note : The symbol $\sim$ is read as equivalent to.

(b) Multiplication of the elements of any row or column by a non-zero scalar : If k is a non-zero scalar and the row R_i is to be multiplied by constant k then we multiply every element of R_i by the constant k and symbolically the transformation is denoted by $R_i \to kR_i$ or kR_i

Ex. (1) : If $A = \begin{bmatrix} 4 & 2 & 3 \\ 8 & -6 & 4 \\ 2 & 4 & 9 \end{bmatrix}$ then $R_2 \to \frac{1}{2}R_2$ gives $A \sim \begin{bmatrix} 4 & 2 & 3 \\ 4 & -3 & 2 \\ 2 & 4 & 9 \end{bmatrix}$

Similarly, if any column of a matrix is to be multiplied by a constant then we multiply every element of the column by the constant. It is denoted as $C_i \to kC_i$ or kC_i .

Ex. (2) : If $P = \begin{bmatrix} -1 & 2 & 3 \\ 5 & -6 & 6 \\ 3 & 4 & 9 \end{bmatrix}$ then $C_3 \to \frac{1}{3}R_3$ gives $P \sim \begin{bmatrix} -1 & 2 & 1 \\ 5 & -6 & 2 \\ 3 & 4 & 3 \end{bmatrix}$

(c) Adding the scalar multiples of all the elements of any row (or column) to corresponding elements of any other row (column).

If k is a non-zero scalar and the k -multiples of the elements of $R_i($ or $C_i)$ can be added to the elements of $R_j($ or $C_j)$ then the transformation is symbolically denoted as $R_i \to R_i + kR_j($ or $C_i \to C_i + kC_j$)

Ex. (1) : If $P = \begin{bmatrix} 3 & 2 & 1 \\ -5 & 4 & 2 \end{bmatrix}$ then $C_2 \to C_2 - 2C_3$ gives $P \sim \begin{bmatrix} 3 & 0 & 1 \\ -5 & 0 & 2 \end{bmatrix}$

Examples

(I) Apply given elementary transformations on examples (1) and (2)

(1) $A = \begin{bmatrix} 3 & -5 \\ 2 & 6 \end{bmatrix}$, $R_2 \to R_2 - \frac{2}{3}R_1$ (2) $B = \begin{bmatrix} -2 & 4 \\ 1 & 7 \\ -1 & 3 \end{bmatrix}$, $R_3 \to R_3 + 2R_2$

(3) $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, Transform C into an lower triangular matrix by using a suitable row transformations.

(4) $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$, Transform A into an upper triangular matrix by using a suitable row transformations.

(5) $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, Transform A into an identity matrix by using a suitable column transformations.

Solutions :

(1) Given : $A = \begin{bmatrix} 3 & -5 \\ 2 & 6 \end{bmatrix}$, $R_2 \to R_2 - \dfrac{2}{3}R_1 \quad \therefore\ A \sim \begin{bmatrix} 3 & -5 \\ 2 & 6 \end{bmatrix}$,

(2) Given : $B = \begin{bmatrix} -2 & 4 \\ 1 & 2 \\ -1 & 3 \end{bmatrix}$, $R_3 \to R_3 + 2R_2 \quad \therefore\ B \sim \begin{bmatrix} -2 & 4 \\ 1 & 7 \\ 1 & 7 \end{bmatrix}$,

(3) Given : $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $R_1 \to R_1 - \dfrac{1}{2}R_2 \quad \therefore\ C \sim \begin{bmatrix} -\frac{1}{2} & 0 \\ 3 & 4 \end{bmatrix}$,

(4) Given : $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$

Note : We can apply two transformations simultaneously on two different rows (or columns) at a time but without using the changed row (or column) in the succeeding transformation

$$R_2 \to R_2 - 2R_1 \ , \ R_3 \to R_3 - 3R_1$$

$$A \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 5 & -2 \end{bmatrix}$$

$$R_3 \to R_3 - \frac{5}{3} R_2$$

$$A \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} \quad \text{which is an upper triangular Matrix}$$

(5) Given : $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$C_2 \to C_2 - 2C_1 \ , \ \ C_3 \to C_3 - 2C_1$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & -2 \\ 2 & -2 & -3 \end{bmatrix}$$

$$C_2 \to C_2 - 2C_3$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 2 & 4 & -3 \end{bmatrix}$$

$$C_1 \to C_1 - 2C_2 \ , \ \ C_3 \to C_3 + 2C_2$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -6 & 4 & 5 \end{bmatrix}$$

$$C_3 \to \frac{1}{5} C_3$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -6 & 4 & 1 \end{bmatrix}$$

$$C_1 \to C_1 + 6C_3 \ , \ C_2 \to C_2 - 4C_3$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{which is an Identity Matrix}$$

Elementary transformations on Matrix equations :

Consider the equation $AB = X$ where A, B and X are matrices of same order. In order to apply a sequence of row transformations on the equation $AB = X$ we apply, these operations simultaneously on on the first matrix A of the product AB on LHS and X on RHS Simillarly, In order to apply a sequence of column transformations on the equation $AB = X$ we apply, these operations simultaneously on the second matrix B of the product AB on LHS and X on RHS Here in the product AB the first matrix A is called Pre Factor and the second matrix B is called Post Factor. Only Row operations are valid on Pre Factor.Only column operations are valid on Post Factor.

For example, if $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ then $AB = \begin{bmatrix} 4 & 5 \\ 10 & 9 \end{bmatrix} = C$

Thus we have $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 10 & 9 \end{bmatrix}$

If we apply a row transformation on pre factor i.e. A and on RHS then the equality of the product of reduced matrices must hold

Now, $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 10 & 9 \end{bmatrix}$

Consider $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 2 & -1 \end{bmatrix}$$

$$\begin{aligned} \text{LHS} &= \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 \times 2 + 2 \times 1 & 1 \times -1 + 2 \times 3 \\ 1 \times 2 + 0 \times 1 & 1 \times -1 + 0 \times 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 + 2 & -1 + 6 \\ 2 + 0 & -1 + 0 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 5 \\ 2 & -1 \end{bmatrix} = \text{RHS.} \end{aligned}$$

Inverse of a Matrix

Definition :

If A is a square matrix of order m and if there exists another square matrix B of the same order such that $AB = BA = I,$ where I is the identity matrix of order m, then B is called as the inverse of A and is denoted by A^{-1}, i.e. $B = A^{-1}$. Hence $AA^{-1} = A^{-1}A = I$.

By using the same definition we can say that A is the inverse of B and is denoted by B^{-1}, i.e. $A = B^{-1}$. Hence $B^{-1}B = BB^{-1} = I$.

Note :

(1) Every square matrix A has its corresponding determinant ; $|A|$

(2) A square matrix A has inverse if and only if $|A| \neq 0$

(3) A matrix is said to be invertible if its inverse exists.

The inverse of a matrix (if it exists) can be obtained by using two methods. (i) Elementary row or column transformation

(ii) Adjoint method

Inverse of a nonsingular matrix by elementary transformation :

Consider the equation $AA^{-1} = I$. Here A is the given matrix of of order m and I is the identity matrix of order m. Hence $A^{-1}$is the only unknown matrix. So to find A^{-1}we have to first convert A to I. This can be done by using a sequence of elementary row transformations on both the sides of the equation. And reduce it to the form $IA^{-1} = B$ so that $A^{-1} = B$.

If we start with the equation $A^{-1}A = I$. then apply a sequence column transformations on the post factor and RHS and reduce the equation to the form $A^{-1}I = B$ so that $A^{-1} = B$.

If A is a non-singular matrix of order 3, say $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

For reducing the above matrix to $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

the suitable transformations are as follows :

Row Transformations	Column Transformations
(1). Reduce a_{11} to $'1'$	(1). Reduce a_{11} to $'1'$
(2). Reduce a_{21} and a_{31} to $'0'$	(2). Reduce a_{12} and a_{13} to $'0'$
(3). Reduce a_{22} to $'1'$ using R_2 and R_3	(3). Reduce a_{22} to $'1'$ using C_2 and C_3
(4). Reduce a_{12} and a_{32} to $'0'$	(4). Reduce a_{21} and a_{23} to $'0'$
(5). Reduce a_{33} to $'1'$	(5). Reduce a_{33} to $'1'$
(6). Reduce a_{13} and a_{23} to $'0'$	(6). Reduce a_{31} and a_{32} to $'0'$

Examples

(II) Find the inverse of the following matrices by using row transformations (if they exist)

(1) $\begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$, (2) $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, (3) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

(III) Find the inverse of the following matrices by using column transformations (if they exist)

(1) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, (2) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$

Solutions

(II). (1) Let, A $= \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$, consider $|A| = 18 - 10 = 8 \neq 0 \therefore A^{-1}$ exists

We have $AA^{-1} = I$

i.e. $\begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & -1 \\ 2 & 6 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 \\ 0 & 8 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$R_2 \to \frac{1}{8} R_2$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -\frac{2}{8} & \frac{3}{8} \end{bmatrix}$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 6/8 & -5/8 \\ -2/8 & 3/8 \end{bmatrix}$$

$$\mathrm{I}A^{-1} = \begin{bmatrix} 6/8 & -5/8 \\ -2/8 & 3/8 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{8} \begin{bmatrix} 6 & -5 \\ -2 & 3 \end{bmatrix}$$

(2) Let, $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

consider $|A| = 2(4-1) + 1(-2+1) + 1(1-2) = 6 - 1 - 1 = 4 \neq 0$

$\therefore\ A^{-1}$ exists

We have $AA^{-1} = I$

i.e. $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_1 \to R_1 + R_2$

i.e. $\begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \to R_2 + R_1,\ R_3 \to R_3 - R_1$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & -1 \\ 0 & -2 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

$R_2 \to R_2 + R_3$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

$R_1 \to R_1 - R_2,\ R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$R_3 \to \frac{1}{4} R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1/4 & 1/4 & 3/4 \end{bmatrix}$$

$R_1 \to R_1 + R_3,\ R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{bmatrix}$$

$$\mathrm{I}A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

(2) Let, A $= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

consider $|A| = \cos\theta(\cos\theta - 0) + \sin\theta(\sin\theta - 0) + 0 = \cos^2\theta + \sin^2\theta = 1 \neq 0$

$\therefore\ A^{-1}$ exists

We have $AA^{-1} = I$

i.e. $$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to \cos\theta R_1 + \sin\theta R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - \sin\theta R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta\cos\theta & 1 - \sin^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0\\0 & \cos\theta & 0\\0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\-\sin\theta\cos\theta & \cos^2\theta & 0\\0 & 0 & 1\end{bmatrix}$$

$$R_2 \to \frac{1}{\cos\theta} R_2$$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\-\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix}$$

$$\mathrm{I}A^{-1} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\-\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\-\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix}$$

(III). (1) Let, A $= \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$,

consider $|A| = 3 + 2 = 5 \neq 0 \ \therefore \ A^{-1}$ exists

We have $AA^{-1} = I$

i.e. $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$C_2 \to C_2 + C_1$

$$\begin{bmatrix} 1 & 0 \\ 2 & 5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$C_2 \to \frac{1}{5} C_2$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1/5 \\ 0 & 1/5 \end{bmatrix}$$

$C_1 \to C_1 - 2C_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{bmatrix}$$

$$\mathrm{I}A^{-1} = \begin{bmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

(2) Let, A $= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ $A^{-1} = \dfrac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$

Now, $|A| = 1(4-4) - 2(-4-2) + 3(-2-1) = 0 + 12 - 9 = 3 \neq 0$

$\therefore$ A^{-1} exists

We have $AA^{-1} = I$

i.e. $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$C_2 \to C_2 - 2C_1,\ C_3 \to C_3 - 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to 2C_2 - C_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 5 \\ 1 & -1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 & -3 \\ 0 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$C_1 \to C_1 + C_2,\ C_3 \to C_3 - 5C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 6 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 2 \\ 2 & 2 & -10 \\ -1 & -1 & 6 \end{bmatrix}$$

$$C_3 \to \frac{1}{6} C_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 1/3 \\ 2 & 2 & -5/3 \\ -1 & -1 & 1 \end{bmatrix}$$

$$C_2 \to C_2 + C_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -2/3 & 1/3 \\ 2 & 1/3 & -5/3 \\ -1 & 0 & 1 \end{bmatrix}$$

$$IA^{-1} = \begin{bmatrix} 0 & -2/3 & 1/3 \\ 6/3 & 1/3 & -5/3 \\ -3/3 & 0 & 3/3 \end{bmatrix} \quad \therefore \quad A^{-1} = \frac{1}{3} \begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$$

####### DONE #######

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