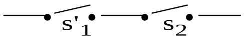
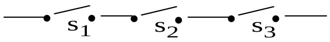
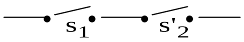
Practical No. 01 - Applications of Logic

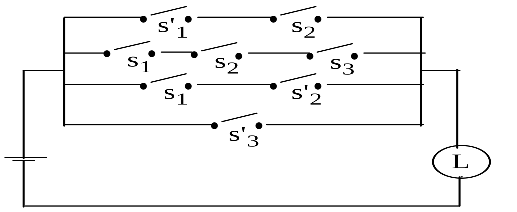
1. Let p : Switch S_1 be closed
q : Switch S_2 be closed
r : Switch S_3 be closed
$\therefore$ The switches $S_1 = \sim p$, $S_2 = \sim q$
and $S_3 = \sim r$ are open

Now, $\sim p \wedge q$ = 

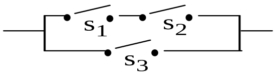
$p \wedge q \wedge r$ = 

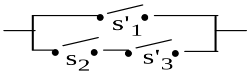
$\sim p \wedge q$ = 

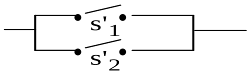
$\therefore$ The switching circuit for the given statement is



2. Let p : Switch S_1 be closed, q : Switch S_2 be closed, r : Switch S_3 be closed
$\therefore$ The switches $S'_1 = \sim p$, $S'_2 = \sim q$ and $S'_3 = \sim r$ are open

Now,  $= (p \wedge q) \vee r$

 $= \sim p \vee (q \wedge \sim r)$

 $= \sim p \wedge \sim q$

$\therefore$ The given circuit can be expressed as a logical statement as follows

$$[(p \wedge q) \vee r] \wedge [\sim p \vee (q \wedge \sim r)] \wedge [\sim p \wedge \sim q]$$

The switching table for the above logical statement is

1	2	3	4	5	6	7	8	9	10	11	12	13
p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \wedge q$	$D \vee r$	$q \wedge C$	$A \vee F$	$A \wedge B$	$E \vee G$	$H \vee I$
			A	B	C	D	E	F	G	H	I	I
1	1	1	0	0	0	1	1	0	0	0	1	1
1	1	0	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	0	0	1	0	0	0	1	1
1	0	0	0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	1	0	1	0	1	1
0	1	0	1	0	1	0	0	1	1	0	1	1
0	0	1	1	1	0	0	1	0	1	1	1	1
0	0	0	1	1	1	0	0	0	1	1	1	1

Practical No. 02 - Inverse of a Matrix by Adjoint Method

1, Let the given matrix be $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$
We have

$M_{11} = \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -2$, $A_{11} = (-1)^2(-2) = -2$

$M_{12} = \begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} = 3$, $A_{12} = (-1)^3(3) = -3$

$M_{13} = \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = 6$, $A_{13} = (-1)^4(6) = 6$

$M_{21} = \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 0$, $A_{21} = (-1)^2(-2) = -2$

$M_{22} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$, $A_{22} = (-1)^4(1) = -1$

$M_{23} = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2$, $A_{23} = (-1)^5(2) = -2$

$M_{31} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$, $A_{31} = (-1)^4(2) = 2$

$M_{32} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -2$, $A_{32} = (-1)^5(-2) = 2$

$M_{33} = \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} = -2$, $A_{33} = (-1)^6(-6) = -6$

Now det A = -2

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{Adj } A) = -\frac{1}{2}\begin{bmatrix} -2 & 0 & 2 \\ -3 & 1 & 2 \\ 6 & -2 & -6 \end{bmatrix}$$

Solving the system :
Writing given system in matrix form...

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 9 \end{bmatrix}$$

i.e $AX = B$
Pre-multiplying by A^{-1},
$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -2 & 0 & 2 \\ -3 & 1 & 2 \\ 6 & -2 & -6 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}$$

$\therefore$ $x = -2$, $y = 4$, $z = 1$

2. Let the given matrix be $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix}$

We have

$A_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = -11$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 3 \\ 3 & -4 \end{vmatrix} = 13$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$

$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 1 & -4 \end{vmatrix} = -3$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 1 \\ 3 & -4 \end{vmatrix} = -11$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = -5$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} = -11$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -5$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 5$

Now det A = - 40

$\therefore \quad A^{-1} = \frac{1}{|A|} (\text{Adj } A) = -\frac{1}{40} \begin{bmatrix} -11 & -3 & -5 \\ 13 & -11 & -5 \\ -5 & -5 & 5 \end{bmatrix}$

Solving the system :

Writing given system in matrix form...

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

i.e $\quad AX = B$

Pre-multiplying by A^{-1},

$X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{40} \begin{bmatrix} -11 & -3 & -5 \\ 13 & -11 & -5 \\ -5 & -5 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore \quad x = 1, \quad y = 2, \quad z = 1$

Practical No. 03 - Inverse of a Matrix by Elementary Transformation

1. We have $AA^{-1} = I$

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 + R_1, \quad R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & -3 & -5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$\frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3/2 \\ 0 & -3 & -5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1/2 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$R_3 + 3R_2, \quad R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3/2 \\ 0 & 0 & -1/2 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 1/2 & 1/2 & 0 \\ -1/2 & 3/2 & 1 \end{bmatrix}$$

$-2 R_3$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 1/2 & 1/2 & 0 \\ 1 & -3 & -2 \end{bmatrix}$$

$R_1 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & -7 & -4 \\ 1/2 & 1/2 & 0 \\ 1 & -3 & -2 \end{bmatrix}$$

$R_2 - \frac{3}{2} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & -7 & -4 \\ -1 & 5 & 3 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 2 & -7 & -4 \\ -1 & 5 & 3 \\ 1 & -3 & -2 \end{bmatrix}$$

Solving the system :

Writing given system in matrix form...

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$$

i.e $\quad AX = B$

Pre-multiplying by A^{-1},

$X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -7 & -4 \\ -1 & 5 & 3 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x = 1, \quad y = 1, \quad z = 1$

2. We have $AA^{-1} = I$

$$\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\frac{1}{7} R_1$

$$\begin{bmatrix} 1 & -3/7 & -3/7 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/7 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 + R_1, \quad R_3 + R_1$

$$\begin{bmatrix} 1 & -3/7 & -3/7 \\ 0 & 4/7 & -3/7 \\ 0 & -3/7 & 4/7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/7 & 0 & 0 \\ 1/7 & 0 & 0 \\ 1/7 & 0 & 0 \end{bmatrix}$$

$\frac{7}{4} R_2$

$$\begin{bmatrix} 1 & -3/7 & -3/7 \\ 0 & 1 & -3/4 \\ 0 & -3/7 & 4/7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/7 & 0 & 0 \\ 1/4 & 7/4 & 0 \\ 1/7 & 0 & 0 \end{bmatrix}$$

$R_3 + \frac{3}{7} R_2, \quad R_1 + \frac{3}{7} R_2$

$$\begin{bmatrix} 1 & 0 & -3/4 \\ 0 & 1 & -3/4 \\ 0 & 0 & 1/4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/4 & 3/4 & 0 \\ 1/4 & 7/4 & 0 \\ 1/4 & 3/4 & 1 \end{bmatrix}$$

$4 R_3$

$$\begin{bmatrix} 1 & 0 & -3/4 \\ 0 & 1 & -3/4 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/4 & 3/4 & 0 \\ 1/4 & 7/4 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

$R_1 + \frac{3}{4} R_2, \quad R_2 + \frac{3}{4} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

The given system is

$7x - 3y - 3z = 7, \quad -x + y = -2, \quad -x + z = 0.$

Writing given system in matrix form...

$$\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 0 \end{bmatrix}$$

i.e $\quad AX = B$

Pre-multiplying by A^{-1},

$X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 7 \\ -2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$x = 1, \quad y = -1, \quad z = 1$

Practical No. 04 - Solutions of Triangle.

1. Given that b = 10, and c = 12 and $\sin\left(\frac{A}{2}\right) = \frac{1}{2\sqrt{10}}$

We know that, $\sin\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$

$$\therefore \quad \frac{1}{2\sqrt{10}} = \sqrt{\frac{(s-10)(s-12)}{(10)(12)}}$$

squaring both sides we get,

$$\frac{1}{\cancel{40}} = \frac{(s-10)(s-12)}{\cancelto{3}{120}}$$

$$3 = s^2 - 22s + 120$$

$$s^2 - 22s + 117 = 0$$

$$s^2 - 13s - 9s + 117 = 0$$

$$s(s-13) - 9(s-13) = 0$$

$$(s-13)(s-9) = 0$$

$s - 13 = 0$ OR $s - 9 = 0$

$s = 13$ OR $s = 9$

$s = 9$ is rejected as $b + c = 22$ $\therefore$ $s = 13$.

But $s = \frac{a+b+c}{2}$ $\quad\therefore$ $2s = a + b + c$

$\therefore$ $2(13) = a + 10 + 12$

$\therefore$ $a = 26 - 22 = 4.$ $\quad\therefore$ $a = 4$

2. Given that, $b : c = 2 : \sqrt{3}$ and $\angle A = 30^\circ$

$$\therefore \quad \frac{b}{c} = \frac{2}{\sqrt{3}}$$

Let $b = 2k$ and $c = \sqrt{3}k$ where $k \in R$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos 30^\circ = \frac{4k^2 + 3k^2 - a^2}{2(2k)\sqrt{3}k}$$

$$\frac{\sqrt{3}}{\cancel{2}} = \frac{7k^2 - a^2}{\cancelto{2}{4}\sqrt{3}k^2}$$

$$6k^2 = 7k^2 - a^2$$

$$k^2 = a^2 \Rightarrow k = a$$

$\therefore$ $b = 2k = 2a$ and $c = \sqrt{3}k = \sqrt{3}a$

By sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\therefore \quad \frac{a}{\sin 30^\circ} = \frac{2a}{\sin B}$$

$$\therefore \quad \sin B = 2.\sin 30^\circ$$

$$\therefore \quad \sin B = 2\left(\frac{1}{2}\right) = 1$$

$$\therefore \quad B = 90^\circ = \frac{\pi}{2}$$

Practical No. 05 - Inverse Trigonometric Functions.

1. Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$ and $\beta = \cos^{-1}\left(\frac{12}{13}\right)$.........(1)

$\therefore$ $\cos\alpha = \frac{4}{5}$ and $\cos\beta = \frac{12}{13}$

where , $0 < \alpha < \pi/2$, $0 < \beta < \pi/2$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\sin\beta = \sqrt{1 - \cos^2\beta} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

$$= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}$$

$$\cos(\alpha + \beta) = \frac{48}{65} - \frac{15}{65}$$

$$= \frac{48 - 15}{65}$$

$$\cos(\alpha + \beta) = \frac{33}{65}$$

$$\alpha + \beta = \cos^{-1}\left(\frac{33}{65}\right)$$

$\therefore$ From (1) we get

$$\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$$

2. We know that , $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$\text{L.H.S.} = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left[\frac{\frac{1}{3} + \frac{1}{8}}{1 - \left(\frac{1}{3}\right)\left(\frac{1}{8}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\frac{8+3}{3.8}}{\frac{3.8-1}{3.8}}\right]$$

$$= \tan^{-1}\left(\frac{11}{24-1}\right)$$

$$= \tan^{-1}\left(\frac{11}{23}\right)$$

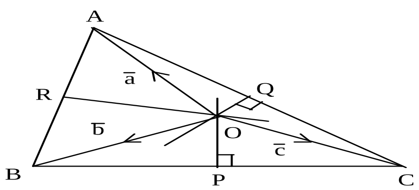
$$= \tan^{-1}\left(\frac{11}{23}\right)$$

$$= \text{R.H.S.}$$

$$\therefore \quad \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \tan^{-1}\left(\frac{11}{23}\right)$$

Practical No. 06 - Geometrical Applications Of Vectors.

1. Let ABC be a triangle. Let P, Q, R be mid-points of the sides BC, CA and AB.
Let $\perp$ bisectors of the sides AB and AC intersect at O. Take O as the origin.
It is enough to show that $OP \perp BC$ or $\overline{OP} \cdot \overline{BC} = 0$
Let $\bar{a}, \bar{b}, \bar{c}, \bar{p}, \bar{q}, \bar{r}$ be the p.v.'s of A, B, C, P, Q, R resp.



Now $\overline{OQ} \cdot \overline{AC} = 0$ (since $OQ \perp BC$)

$\therefore$ $\bar{q} \cdot (\bar{c} - \bar{a}) = 0$

$\therefore$ $2\bar{q} \cdot (\bar{c} - \bar{a}) = 0$

$\therefore$ $(\bar{c} + \bar{a}) \cdot (\bar{c} - \bar{a}) = 0$

$\therefore$ $c^2 - a^2 = 0$ --- (1)

Q is mid-pt of CA

$$\boxed{\bar{q} = \frac{\bar{c} + \bar{a}}{2}}$$

Also $\overline{OR} \cdot \overline{BA} = 0$ (since $OR \perp BA$)

$\therefore$ $\bar{r} \cdot (\bar{a} - \bar{b}) = 0$

$\therefore$ $2\bar{r} \cdot (\bar{a} - \bar{b}) = 0$

$\therefore$ $(\bar{a} + \bar{b}) \cdot (\bar{a} - \bar{b}) = 0$

$\therefore$ $a^2 - b^2 = 0$ --- (2)

R is mid-pt of AB

$$\boxed{\bar{r} = \frac{\bar{a} + \bar{b}}{2}}$$

Adding (1) and (2) ,

$\therefore$ $c^2 - b^2 = 0$

$\therefore$ $b^2 - c^2 = 0$

$\therefore$ $(\bar{b} + \bar{c}) \cdot (\bar{b} - \bar{c}) = 0$

$\therefore$ $2\bar{p} \cdot (\bar{b} - \bar{c}) = 0$

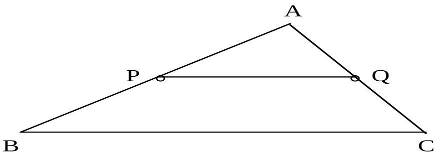
$\therefore$ $\bar{p} \cdot (\bar{b} - \bar{c}) = 0$

$\therefore$ $\overline{OP} \cdot \overline{BC} = 0$

P is mid-pt of BC

$$\boxed{\bar{p} = \frac{\bar{b} + \bar{c}}{2}}$$

2. Let ABC be a triangle and P and Q are the mid-points of the sides AB and AC respectively.



Let $\overline{a}, \overline{b}, \overline{c}, \overline{p}$ & $\overline{q}$ are position vectors of the points A, B, C, D, P, & Q respectively.

$\therefore \quad \overline{p} = \dfrac{\overline{a} + \overline{b}}{2}$ and $\overline{q} = \dfrac{\overline{a} + \overline{c}}{2}$

$$\therefore \quad \overline{PQ} = \overline{q} - \overline{p}$$

$$= \frac{\overline{a} + \overline{c}}{2} - \frac{\overline{a} + \overline{b}}{2}$$

$$= \frac{1}{2}(\overline{c} - \overline{b})$$

$$= \frac{1}{2}\overline{BC}$$

//repeated in PQR for searching

Hence $\overline{BC}$ is a scalar multiple of $\overline{AB}$

$\overline{BC}$ is parallel to $\overline{AB}$

seg PQ is parallel to seg BC

Also $|\overline{PQ}| = \dfrac{1}{2}|\overline{BC}|$

$l(PQ) = \dfrac{1}{2}\, l(BC)$

Practical No. 07 - Three Dimensional Geometry (dr's ans dc's)

1. $A \equiv (5, 6, 4)$, $B \equiv (4, 4, 1)$, $C \equiv (8, 2, k)$

$\therefore$ d.r.'s of lines AB and BC are

4 - 5, 4 - 6, 1 - 4 and 4 -2, 2 - 4, k -1

i.e -1, -2, -3 and 2, -2, k -1

i.e 1, 2, 3 and 2, -2, k -1

Let a_1, b_1, c_1 and a_2, b_2, c_2 be the d.rs of AB and BC respectively

$\therefore \quad a_1 = 1, b_1 = 2, c_1 = 3$ and $a_2 = 2, b_2 = -2, c_2 = k - 1$.

Given : ΔABC is right angled at B $\therefore$ $AB \perp BC$

$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore \quad (1)(2) + (2)(-2) + (3)(k -1) = 0$

$\therefore \quad 2 - 2 + 3(k -1) = 0 \Rightarrow 3(k - 1) = 0$

$\therefore \quad k - 1 = 0 \Rightarrow k = 1$

2. Let a_1, b_1, c_1 be the d.rs of the first line and a_2, b_2, c_2 be the d.rs of the second line.

Given that ,

$a_1 = 2, b_1 = 3, c_1 = 6$ & $a_2 = 1, b_2 = -2, c_2 = 2$

Let θ be the acute angle betweeen the lines

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

$$= \left| \frac{(2)(1) + (3)(-2) + (6)(2)}{\sqrt{(2)^2 + (3)^2 + (6)^2}\sqrt{(1)^2 + (-2)^2 + (2)^2}} \right|$$

$$= \left| \frac{2 - 6 + 12}{\sqrt{4 + 9 + 36}\sqrt{1 + 4 + 4}} \right|$$

$$= \left| \frac{8}{(7)(3)} \right|$$

$$= \left| \frac{8}{21} \right|$$

$$\cos\theta = \frac{8}{21}$$

$$\therefore \quad \theta = \cos^{1}\left(\frac{8}{21}\right)$$

Practical No. 08 - Applications of scalar triple product of vectors.

1. Let $A \equiv (3, 2, -1)$, $B \equiv (5, 4, 2)$

$C \equiv (6, 3. 5)$, $D(1, 0, x)$

$\overline{AB} = (5 - 3)\overline{i} + (4 - 2)\overline{j} + (2 + 1)\overline{k}$

$= 2\overline{i} + 2\overline{j} + 3\overline{k}$

$\overline{AC} = (6 - 3)\overline{i} + (3 - 2)\overline{j} + (5 + 1)\overline{k}$

$= 3\overline{i} + 6\overline{j} + \overline{k}$

$\overline{AD} = (1 - 3)\overline{i} + (0 - 2)\overline{j} + (x + 1)\overline{k}$

$= -2\overline{j} - 2\overline{j} + (x + 1)\overline{k}$

Since A, B, C, D are coplanar ,

$\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar.

$\therefore \quad [\,\overline{AB}, \overline{AC}, \overline{AD}\,] = 0$

$$\therefore \quad \begin{vmatrix} 2 & 2 & 3 \\ 3 & 6 & 1 \\ -2 & -2 & x+1 \end{vmatrix} = 0$$

$\therefore \quad 2(6x+6+2) - 2(3x+3+2) + 3(-6+12) = 0$

$\therefore \quad 12x + 16 - 6x -10 + 18 = 0$

$\therefore \quad 6x + 24 = 0$

$\therefore \quad x = -4$

2, Given : $\overline{a} = \overline{i} - 2\overline{j} - \overline{k}$, $\overline{b} = 3\overline{i} + 2\overline{j} + \overline{k}$,

and $\overline{c} = \overline{i} + \overline{j} + 5\overline{k}$

$$\therefore \quad [\,\overline{a}\,\overline{b}\,\overline{c}\,] = \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ 1 & 1 & 5 \end{vmatrix}$$

$= (1)[9] - (-2)[14] + (-1)[1]$

$= 9 + 28 - 1$

$= 36$

$\therefore$ The required volume is 36 cu. units

Practical No. 09 - Three Dimentional Geometry - Line.

1. Since the required line is perpendicular to the line with d.r.'s 1, -2, - 4 & 3, 2, 5.

Hence the req. line is $\perp^{r}$ to the vectors

$\overline{a} = \overline{i} - 2\overline{j} - 4\overline{k}$ & $\overline{b} = 3\overline{i} + 2\overline{j} + \overline{5}$

$$\overline{a} \times \overline{b} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 1 & -2 & -4 \\ 3 & 2 & 5 \end{vmatrix}$$

$= \overline{i}(-10 + 8) - \overline{j}(5 + 12) + \overline{k}(2 + 6)$

$= -2\overline{i} - 17\overline{j} + 8\overline{k}$

$\therefore$ The d.r.'s of the line are -2, -17, 8.

i.e. -2, -17, 8

Given that the line passes through (0, 1, 2).

Hence using symmetrical form the req. eqn is

$$\frac{x - 0}{-2} = \frac{y - 1}{-17} = \frac{z - 2}{8}$$

$$\frac{x}{-2} = \frac{y - 1}{-17} = \frac{z - 2}{8}$$

2. The line passes through (2, 2, 3)

Given that the line is parallel to $2\overline{i} - \overline{j} + 5\overline{k}$.

$\therefore$ Direction ratios of the line are 2, -1, 5.

Using symmetrical form, the reqd. eqn is

$$\frac{x - 2}{2} = \frac{y - 2}{-1} = \frac{z - 3}{5}$$

Practical No, 10 - Three Dimentional Geometry - Plane.

1. Eqn of the plane passing through the points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Eqn of the plane passing through the points (0, -1, 0), (2, 1, -1), (1, 1, 1) is

$$\begin{vmatrix} x - 0 & y + 1 & z - 0 \\ 2 & 2 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$(x - 0)(4) - (y + 1)(3) + (z - 0)(2) = 0$

$\therefore \quad 4x - 3y + 2z - 3 = 0$ --- Ans

2. Let $\bar{a} = 4\bar{i} + 5\bar{j} + 3\bar{k}$ and $\bar{b} = 2\bar{i} + 3\bar{j} + 2\bar{k}$

Since $\bar{a}$ and $\bar{b}$ are both parallel to the the plane, $\bar{a} \times \bar{b}$ is $\perp^r$ to the plane.

Now

$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 4 & 5 & 3 \\ 2 & 3 & 2 \end{vmatrix}$$

$$= \bar{i}(10 - 9) - \bar{j}(8 - 6) + \bar{k}(12 - 10)$$

$$= \bar{i} - 2\bar{j} + 2\bar{k}$$

$\therefore$ $\bar{i} - 2\bar{j} + 2\bar{k}$ is normal to the plane with the direction ratios 1, -2, 2

Also, the plane passes through (3, 4, 2),

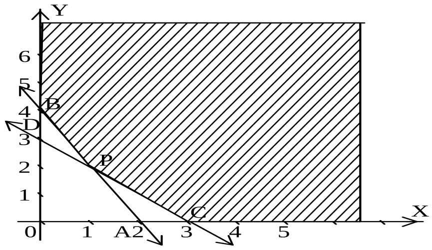
$\therefore$ its equation is

$1(x - 3) + (-2)(y - 4) + 2(z - 2) = 0$

$x - 3 - 2y + 8 + 2z - 4 = 0$

$x - 2y + 2z + 1 = 0$

Practical No. 11 - Formations and Solutions of LPP

1. 

Let x packets of A and y packets of B and be purchased.Cost of 1 packet of food A is Rs.15 and that of B is Rs.10. So the cost of x packets of food A is Rs.15x cost of x packets of food B is Rs.10y

$\therefore$ Total cost $Z = 15x + 10y$ ....(1M)

Vitamin	Food A	Food B	Required
Vitamin A1	2 x	y	4
Vitamin B1	4 x	4 y	12

$\therefore$ The given problem can be formulated as an LPP as follows

Minimize $Z = 15x + 10y$

Subjected to $2x + y \geq 4$

$4x + 4y \geq 12$

$x \geq 0,\ y \geq 0$

Draw, graphs of lines $2x + y = 4$ ....(1)

$4x + 4y = 12$ ....(2)

P is the point of intersection of (1) qnd (2)

Solving (1) and (2), we get P = (1, 2)

Feasible region is shaded unbounded area with corner points B, P and C

Vertex	Obj.Function	Value of Z
B = (0, 4)	Z=15(0)+10(4)	40
P = (1, 2)	15(1) + 10(2)	35
C = (3, 0)	15(3) + 10(0)	45

$\therefore$ Minimum value of Z is 35 at P (1, 2)

$\therefore$ 1 packet of food type A and 2 packets of food type Bto be purchased

2. Let x be the no. of chairs and y be the no. of tables. Then we have the following

Formation : Maximize $z = 20x + 30y$, s.t.

$3x + 3y \leqslant 36$, $5x + 2y \leqslant 50$,

$2x + 6y \leqslant 60$, $x \geqslant 0$, $y \geqslant 0$

Solution : Draw the lines - $3x + 3y = 36$

$5x + 2y = 50$ and $2x + 6y = 60$

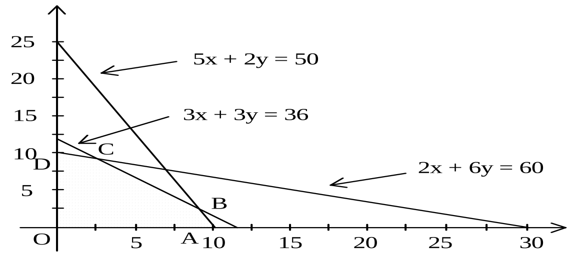
The vertices of the convex region are O(0, 0), A(10, 0), B(26/3, 10/3), C(3, 9) and D(0,10). Profits at these points are

$Z_O = 20(0) + 30(0) = 0$

$Z_A = 20(10) + 30(0) = 200$

$Z_B = 20(26/3) + 30(10/3) = 820/3$

$Z_C = 20(3) + 30(9) = 330$



$Z_D = 20(0) + 30(10) = 300$

$\therefore$ $Z_{max} = 330$ at C(3, 9). Hence 3 chairs and 9 tables should be produced. Maximum profit is Rs 330.

Practical No. 12 - Applications Of Derivatives (Geometric applications)(DS)

1. Equation of the curve is $\sqrt{x} - \sqrt{y} = 1$ ....(1)

Diff.(1) w.r.t.x. we get,

$$\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \quad \therefore \quad -\frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \sqrt{\frac{y}{x}}$$

Given that P (9, 4). $\therefore$ Slope of the tangent at P is given by $m_1 = \left(\frac{dy}{dx}\right)_{(9,4)} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

Equation of the tangent is given by,

$$y - 4 = \frac{2}{3}(x - 9)$$

$3y - 12 = 2x - 18 \Rightarrow 2x - 3y - 6 = 0$

Normal is perpendicular to tangent

$\therefore$ Slope of the normal is $m_2 = -\frac{1}{dy/dx} = -\frac{3}{2}$

Equation of the tangent is given by,

$y - 4 = -\frac{3}{2}(x - 9)$ i.e. $2y - 8 = -3x + 27$

$3x + 2y - 35 = 0$

2. Let P (x_1, y_1) be the required point on the curve

$$y = x - \frac{4}{x} \quad \text{..........(1)}$$

Diff. w.r.t.x,

$$\frac{dy}{dx} = 1 + \frac{4}{x^2}$$

$\therefore$ Slope of tangent at P on (1) is

$$\left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} = 1 + \frac{4}{x^2_1}$$

Given that the tangent is parallel to the line $y = 2x$ whose slope is 2

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} = 2$$

$$1 + \frac{4}{x^2_1} = 2$$

$\therefore$ $x^2_1 + 4 = 2.x_1^2$

$\therefore$ $x^2_1 = 4$

$\therefore$ $x_1 = \pm 2$

If $x_1 = 2$, then from (1) we get $y = 0$

The point is (2, 0)

If $x_1 = -2$, then from (1) we get $y = 0$

The point is (-2, 0)

The required points are (2, 0) and (–2, 0)

Practical No. 13 - Applications of Derivatives - Rate, Measure(DS)

1. Let v be the velocity and f be the acc^n of the particle at any time t.

Since $s = 160t - 16t^2$

$\therefore \quad v = \frac{ds}{dt} = 160 - 32t$

Velocity at $t = 1$ is $v_{(t=1)}$
$= 160 - 32(1)$
$= 128$ m/s

Velocity at $t = 9$ is $v_{(t=9)}$
$= 160 - 32(9)$
$= -128$ m/s

Since two velocities have opp. signs & equal magnitude, the required result

Also when $v = 0$, $160 - 32t = 0$

$t = 5$

$\therefore \quad s = 160(5) - 3(5)^2 = 400$ m

2, Let r be the radius and V be the volume.

$$\text{Now } V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{24}\pi(2r)^3$$
$$= \frac{1}{6}\pi D^3, \text{ where D is diameter}$$
$$\frac{dV}{dt} = \frac{1}{2}\pi D^2 \frac{dD}{dt}$$

When r = 5 cms i.e when D = 10 cm,

$$3 = \frac{1}{2}\pi(10)^2 \frac{dD}{dt} \qquad \boxed{\frac{dV}{dt} = 3 \quad \text{(Given)}}$$

$$\therefore \quad \frac{dD}{dt} = \frac{3}{50\pi} \quad \text{cm/sec}$$

Practical No. 14 - Applications of Derivatives - Maxima and Minima(DS)

1. Let the parts be x and 64 - x

Let $f(x) = x(64 - x)$
$= 64x - x^2$

$\therefore \quad f'(x) = 64 - 2x$

$\therefore \quad f'(x) = 0$ gives $x = 32$

Further, $f''(x) = -2$
< 0

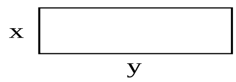
Hence by the second derivative test f is maximum at $x = 32$ $\therefore$ Parts are 32 & 32

2. Let x, y be the sides of the rectangle.

Then $2x + 2y = 108$ i.e $x + y = 54$ -- [1]

Rectangle Area $A = xy$
$= x(54 - x)$ by [1]
$= 54x - x^2$

$\therefore \quad \frac{dA}{dx} = 54 - 2x$



Now $\frac{dA}{dx} = 0 \Rightarrow 54 - 2x = 0 \therefore x = 27$

Finally $\left(\frac{d^2A}{dx^2}\right)_{x=27} = -2 < 0$

Hence by the second derivative test A is Maximum at $x = 27$

Consequently [1] gives $y = 54 - 27 = 27$ $\therefore$ Dimensions are 27×27 cm .

Practical No. 15 - Applications of Derivatives - Rolle's Theorem, LMVT

1. Polynomial function is known to be continuous and differentiable for all x.

Also $f(-2) = (-2)^2 - (-2) - 12 = -6$ and $f(3) = (3)^2 - (3) - 12 = -6$

Thus (i) f is continuous on [-2, 3] (ii) f is differentiable on (-2, 3) and (iii) f(-2) = f(3)

Hence all the condition of Rolle's Theorem are satisfied. Therefore there exists atleast one $c \in (-2, 3)$ such that $f'(c) = 0$.

Now $f'(x) = 0$ gives $2x - 1 = 0 \Rightarrow x = 1/2$. Thus $c = 1/2 \in (-2, 3)$ s.t $f'(c) = 0$

Hence Rolle's Theorem is verified.

2. Polynomial function is known to be continuous and differentiable for all x.

Thus (i) f is continuous on [1, 4] (ii) f is differentiable on (1, 4)

$\therefore$ All the conditions of Lagrange's Mean Value Theorem are satisfied. Therefore there exists

atleast one $c \in (1, 4)$ such that $\frac{f(4) - f(1)}{4 - 1} = f'(c)$ --- (1) $\therefore \frac{7 - (-4)}{4 - 1} = 4c - 7 \quad \therefore \frac{11}{3} = 4c - 7$

$c = 8/3 \in (1, 4)$ which satisfies (1). Hence Lagranges Mean Value Theorem is verified.

Practical No. 16 - Applications of Definite Integrals - Limit of Sum(DS)

1. $$\int_1^3 (4x + 5)\,dx \qquad \boxed{h = \frac{b - a}{n} = \frac{2}{n}}$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left[4(1 + rh) + 5\right] h$$
$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left[4rh^2 + 9h\right]$$
$$= \lim_{n\to\infty} \left[4h^2 \sum_{r=1}^{n} r + \sum_{r=1}^{n} 9h\right]$$
$$= \lim_{n\to\infty} \left[4h^2 \frac{n(n+1)}{2} + 9hn\right]$$
$$= \lim_{n\to\infty} \left[4\,\frac{4}{n^2}\,\frac{n(n+1)}{2} + 9(2)\right], \quad h = \frac{2}{n}$$
$$= \lim_{n\to\infty} \left[8 + \frac{8}{n} + 18\right]$$
$$= 8 + 0 + 18$$
$$= 26$$

2. $\displaystyle\int_1^2 e^x\,dx$ $\qquad \boxed{h = \dfrac{b-a}{n} = \dfrac{1}{n}}$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left[e^{1+rh}\right] h$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left[e\, e^{rh}\, h\right]$$

$$= \lim_{n\to\infty} \left[eh \sum_{r=1}^{n} e^{rh}\right]$$

$\boxed{\begin{array}{l}\text{as } n\to\infty \\ h\to 0\end{array}}$

$$= \lim_{h\to 0} \left[eh\ \frac{e^h(e-1)}{e^h-1}\right]$$

$$= e(1)e^0(e-1)$$

$$= e(e-1) \quad \text{--- Ans}$$

$\boxed{\lim_{h\to 0} \dfrac{h}{e^h - 1} = 1}$

$$\boxed{\begin{aligned}\sum_{r=1}^{n} e^{rh} &= e^h + e^{2h} + e^{3h} + \text{ - - - } + e^{nh}\\ &= \frac{e^h\left[(e^h)^n - 1\right]}{e^h - 1}\\ &= \frac{e^h(e^{nh}-1)}{e^h - 1}\\ &= \frac{e^h(e-1)}{e^h-1} \qquad , \ nh = 1\end{aligned}}$$

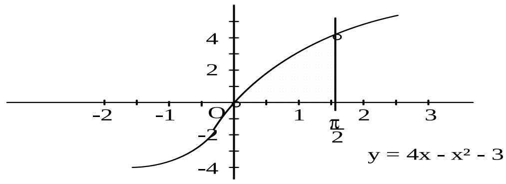
Practical No. 17 - Applications Of Definite Integral - Area (DS)

1. The required area (shaded part) is

$$= \int_0^{\pi/2} (2x + \sin x)\,dx$$

$$= \left[\,x^2 - \cos x\,\right]_0^{\pi/2}$$

$$= \left[\frac{\pi^2}{4} - 0\right] - [\,0 - 1\,] \quad = \frac{\pi^2}{4} + 1$$

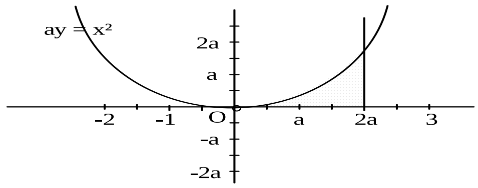


2. The required area (shaded part) is

$$= \int_0^{2a} \frac{1}{a}\, x^2\,dx$$

$$= \frac{1}{a}\left[\frac{x^3}{3}\right]_0^{2a}$$

$$= \frac{1}{a}\left[\frac{8a^3}{3} - 0\right] = \frac{8a^2}{3} \qquad \text{--- Ans}$$



Practical No. 18 - Applications of Differential Equations(DS)

1. Let B : No. of bacteria present at time t hrs

Given $B = N$ when $t = 0$ --- (1)

$B = 2N$ when $t = 3$ --- (2)

Also $\dfrac{dB}{dt} \propto B$

$\dfrac{dB}{dt} = kB$, where k is +ve constant

$\dfrac{dB}{B} = k\,dt$

Integrating,

$\log B = kt + c$ --- (3)

$\log N = k(0) + c$, using (1)

$\therefore\ c = \log N$

Thus (3) becomes

$\log B = kt + \log N$

Now, (2) gives

$\log 2N = k(3) + \log N$

$\therefore\ \log\left(\dfrac{2N}{N}\right) = 3k$

$\therefore\ k = \dfrac{\log 2}{3}$ $\qquad\therefore$ (3) becomes

$\therefore\ \log B = \left(\dfrac{\log 2}{3}\right) t + \log N$

Finally, after 6 hrs, i.e when $t = 6$,

$\log B = \left(\dfrac{\log 2}{3}\right)(6) + \log N$

$\log B = 2\log 2 + \log N$

$\log B = \log 4N$

$B = 4N$

2. Let the amt of ice be Q at t minutes.

Let Q_0 be the initial amount present.

Then $Q = Q_0$ when $t = 0$ --- (1)

$Q = Q_0/2$ when $t = 20$ --- (2)

and $\dfrac{dQ}{dt} \propto Q$

$\therefore\ \dfrac{dQ}{dt} = -kQ$, (where $k > 0$)

$\therefore\ \dfrac{dQ}{Q} = -k\,dt$

Integrating,

$\log Q = -kt + c$ --- (3)

$\therefore\ \log Q_0 = -k(0) + c$ Using (1)

$\therefore\ c = \log Q_0$ --- (20)

Also $\log Q_0/2 = -k(20) + c$ By (2), (3)

$\therefore\ \log Q_0/2 = -20k + \log Q_0$ By (20)

$\therefore\ k = \dfrac{\log 2}{20}$

Hence (3) becomes

$\log Q = \log Q_0 - \left(\dfrac{\log 2}{20}\right) t$

Finally, after 1 hour, ($t = 60$ min),

$\therefore\ \log Q = \log Q_0 - \left(\dfrac{\log 2}{20}\right)(60)$

$\therefore\ \log Q = \log Q_0 - 3\log 2$

$\therefore\ \log Q = \log Q_0 - \log 8$

$\therefore\ \log Q = \log (Q_0/8)$

$Q = Q_0/8$ at $t = 60$

$\therefore$ The (1/8)th amount of ice is left after 1 hour.

Practical No. 19 - Expected value, Variance and SD of Random Variable (DS)

1. Mean $\overline{X} = E(X)$

$$= \sum_{x \in S} xP(x)$$

$$= 1\left(\frac{1}{8}\right) + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{1}{4}\right)$$

$$= \frac{5}{2}$$

$$V(X) = E(X^2) - [E(X)]^2$$

$$= 1^2\left(\frac{1}{8}\right) + 2^2\left(\frac{1}{2}\right) + 3^2\left(\frac{1}{8}\right) + 4^2\left(\frac{1}{4}\right) - \left[\frac{5}{2}\right]^2$$

$$= \frac{1}{8} + 2 + \frac{9}{8} + 4 - \frac{25}{4} = 1$$

$$SD = \sqrt{\text{Variance}} = \sqrt{1} = 1$$

2. $P(x<1) = \int_{-3}^{1} f(x)\,dx$

$$= \int_{-3}^{1} \frac{x^2}{5}\,dx$$
$$= \left[\frac{x^3}{15}\right]_{-3}^{1}$$
$$= \left[\frac{1}{15} + \frac{27}{15}\right]$$
$$= \frac{28}{15}$$
$$= 1.8666$$

$P(|x<1|) = \int_{-1}^{1} f(x)\,dx$

$$= \int_{-1}^{1} \frac{x^2}{5}\,dx$$
$$= \left[\frac{x^3}{15}\right]_{-1}^{1}$$
$$= \left[\frac{1}{15} + \frac{1}{15}\right]$$
$$= \frac{2}{15}$$
$$= 0.13333$$

Practical No. 20 - Binomial Distribution

1, Let p : Probability of getting head $= \frac{1}{2}$

q : Probability of not getting head $= \frac{1}{2}$

X : Number of heads

n : Number of trials. = 7

The Probability Mass Function of X is given by

$P(X = x) = P(x) = {}^nC_x\, p^x q^{n-x}$

X = Exactly 5 heads i.e. x = 5.

$$P(5) = {}^7C_5 \left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{7-5} = {}^7C_2 \left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2$$
$$= \left(\frac{7\times \cancel{6}^{3}}{\cancel{2}\times 1}\right)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2$$
$$= \frac{21}{2^7} = \frac{21}{128}$$

$P(\text{Exactly 5 heads}) = \frac{21}{128}$

X = heads atleast once i.e. $x \geqslant 1$

$$P(\text{getting heads atleast once}) = P(X \geqslant 1)$$
$$= 1 - P(X = 0)$$
$$= 1 - P(0)$$
$$= 1 - {}^7C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{7-0}$$
$$= 1 - \left(\frac{1}{2}\right)^7$$
$$= 1 - \frac{1}{128}$$

$P(\text{getting heads atleast once}) = \frac{127}{128}$

2, Let p : Probability of hitting the target $= \frac{2}{3}$

q : Probability of not hitting the target $= \frac{1}{3}$

X : Number of bullets hitting the target.

n : Number of bullets fired = 7

The Probability Mass Function of X is given by

$P(X = x) = P(x) = {}^nC_x\, p^x q^{n-x}$

X = Exactly 3 bullets hit the target, x = 3

$$P(3) = {}^7C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^{7-3}$$
$$= {}^7C_2\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^4$$
$$= \frac{7.\cancel{6}^{3}}{\cancel{2}.1} \times \frac{2^3}{3^3} \times \frac{1}{3^4}$$
$$= \frac{7\times 3\times 8}{3^7} = \frac{56}{3^6} = \frac{56}{729}$$

--- End ---