Specimen Question Paper

MATHEMATICS AND STATISTICS (ARTS AND SCIENCE)(40)-SET-I

Time : 3 hrs **Std.: XII** **Max.Marks : 80**

Note :

1. *All questions are compulsory.*
2. *Figures to the right indicate full marks.*
3. *The question paper consist of 30 questions divided into **FOUR** sections A, B, C, D.*
   - *Section A contains 6 questions of 1 mark each.*
   - *Section B contains 8 questions of 2 marks each. (one of them has internal option)*
   - *Section C contains 6 questions of 3 marks each. (two of them have internal option)*
   - *Section D contains 10 questions of 4 marks each. (three of them have internal option)*
4. *For each MCQ , correct answer must be written along with its alphabet,*
   e.g. (a) .... / (b) .... / c) .... / d) ....
   In case of MCQ (Q1 to Q6) evaluation would be done for the first attempt only.
5. *Start each section on new page only.*
6. *Use of logarithmic tables is allowed. Use of calculator is **not** allowed.*
7. *In L.P.P. only rough sketch of graph is expected. Graph paper is **not** necessary.*

SECTION - A (6 Marks)

Select and write the most appropriate answer from the given alternatives for each question.

Q.1. Which of the following is the principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$?

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{2\pi}{3}$ d) $\frac{3\pi}{2}$ (1)

Q.2. If $|\bar{a}-\bar{b}| = |\bar{a}| = |\bar{b}|$, where $\bar{a}$ and $\bar{b}$ are non zero vectors then the angle between $\bar{a}-\bar{b}$ and $\bar{b}$ is

a) 120^0 b) 45^0 c) 60^0 d) 90^0 (1)

Q.3. If a line makes angles $\theta_1, \theta_2, \theta_3$ with the co-ordinate planes then $\sin^2\theta_1 + \sin^2\theta_2 + \sin^2\theta_3 =$

a) 1 b) 2 c) -1 d) -2 (1)

Q.4. The order of the differential equation of all circles having radius r is

a) 1 b) 2 c) 3 d) 4 (1)

Q.5. Which of the following functions is not continuous on its domain ?

a) $|x-1|$ b) $\sqrt{x}$ c) $\frac{1}{x^2+2x+3}$ d) $f(x) = \begin{cases} \frac{\cos x}{x}, & \text{for } x \neq 0 \\ 1, & \text{for } x = 0 \end{cases}$ (1)

Q.6. For a random variable X if $E(X^2) = 31$, $Var(X) = 6$ then $E(X) =$

a) 2 b) 4 c) 5 d) 25 (1)

SECTION - B (16 Marks)

Q.7. If statement p is true and statements r and s are false then find the truth value of $p \wedge (r \rightarrow s)$. (2)

Q.8. If m_1 and m_2 are slopes of lines represented by equation $3x^2 + 2xy - y^2 = 0$ then find the value of $(m_1)^2 + (m_2)^2$. (2)

Q.9. Find separate equations of the lines represented by $xy - 2x - 3y + 6 = 0$. (2)

Q.10. If the direction ratios of two parallel lines are $4, -3, -1$ and $p + q, 1 + q, 2$ then find the values of p and q. (2)

Q.11. If $y = \sin^{-1}(2x)$ then find $\frac{dy}{dx}$.

OR

If $y = x^{2x}$ then find $\frac{dy}{dx}$. (2)

Q.12. Find the slope of the tangent to the curve $x^2 = 8y$ at point $P(-4, 2)$. (2)

Q.13. Evaluate $\int_0^1 \frac{1}{\sqrt{4-x^2}} dx$ (2)

Q.14. A random variable X has the following probability distribution. Find the value of k. (2)

X	-2	-1	0	1	2	3
$P(X)$	0.1	k	0.2	$2k$	0.3	k

SECTION - C (18 Marks)

Q.15. If $-1 \leq x \leq -\frac{1}{\sqrt{2}}$ then prove that $sin^{-1}(2x\sqrt{1-x^2}) = -2\pi + 2\cos^{-1} x$.

OR

If $|x| \leq 1$ then prove that $\cos^{-1}(-x) = \pi - \cos^{-1} x$ (3)

Q.16. Prove that the equation of the line passing through the point $A(x_1, y_1, z_1)$ and having direction ratios a, b, c is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$. Hence write the equation of the line passing through the point A(1, 2, 4) and is parallel to the line $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z+4}{4}$. (3)

Q.17. Line $\bar{r} = (\hat{\imath} - \hat{\jmath} + \hat{k}) + t(2\hat{\imath} - \hat{\jmath} + \hat{k})$ contained in a plane to which vector $\bar{n} = 3\hat{\imath} - 2\hat{\jmath} + \lambda\hat{k}$ is normal. Find the value of λ. Also find the vector equation of the plane. (3)

Q.18. $f(x) = x^2\left(1 - \cos\left(\frac{2}{x}\right)\right)$ for $x \neq 0$ and $f(0) = k$. If $f(x)$ is continuous at $x = 0$ then find k. (3)

Q.19. Prove that $\int u \cdot v\, dx = u \int v\, dx - \int (\int v\, dx) \cdot \frac{du}{dx}\, dx$.

Or

Prove that $\int e^x \{ f(x) + f'(x) \} dx = e^x f(x) + c$. Hence evaluate $\int e^x\{1 + x\}dx$. (3)

Q.20. A coin is tossed repeatedly until it shows head. Let X be the number of tosses required to get head. Write the probability distribution of X. (3)

SECTION - D (40 Marks)

Q.21. In ΔABC with usual notations if $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ then prove that $a^2 = b^2 + c^2 - 2bc \cos A$ (4)

Q.22. Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ by elementary row transformations.

OR

The sum of three numbers is 9. If we multiply third number by 3 and add to the second number, we get 16. B y adding the first and the third numbers and then subtracting twice the second number from this sum, we get 6. Using this information find the system of linear equations. Find the three numbers using matrices. (4)

Q.23. Write the converse, inverse, contrapositive and negation of the statement : "If a function is differentiable then it is continuous." (4)

Q.24. If $P(\bar{p})$ divides the segment joining $A(\bar{a})$ and $B(\bar{b})$ internally in the ratio $m:n$ then prove that $\bar{p} = \frac{m\bar{b}+n\bar{a}}{m+n}$. Hence, find the position vector of the foot of the perpendicular drawn from the vertex A to the side BC of acute angled triangle ABC. (4)

Q.25. Maximize : $Z = 6x + 4y$ subject to $x \leq 2,\ x + y \leq 3, -2x + y \leq 1,\ x \geq 0, y \geq 0$. (4)

Q.26. If $3x^2 + 4xy - 7y^2 = 0$ then show that (a) $\frac{dy}{dx} = \frac{y}{x}$ and (b) $\frac{d^2y}{dx^2} = 0$

OR

If $x = at^2$, $y = 2at$ then show that $\frac{d^2y}{dx^2} = \frac{-1}{2at^3}$. (4)

Q.27. If $f(a - x) = -f(x)$ then prove that $\int_0^a f(x)\, dx = 0$. Hence show that $\int_0^{\frac{\pi}{2}} \log(\tan x)\, dx = 0$. (4)

Q.28. Find the maximum value of $f(x) = x - e^x$.

OR

Find the values of x, for which the function $f(x) = x^3 + 12x^2 + 36x + 6$ is increasing. (4)

Q.29. Evaluate $\int \frac{1+\cos 4x}{\cot x - \tan x} dx$ (4)

Q.30. Find the general solution of $y \log y \frac{dx}{dy} + x - \log y = 0$. (4)

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