Practical No 01 - Locus

1. Let $A \equiv (-15, 0)$, $B \equiv (15, 0)$. Let $P(x, y)$ be a point on the locus. Then $l(PA) - l(PB) = 24$

$\sqrt{(x + 15)^2 + (y - 0)^2} - \sqrt{(x - 15)^2 + (y - 0)^2} = 24$

$\therefore \sqrt{(x + 15)^2 + (y - 0)^2} = 24 + \sqrt{(x - 15)^2 + (y - 0)^2}$

$\therefore \sqrt{(x + 15)^2 + y^2} = 24 + \sqrt{(x - 15)^2 + y^2}$

$\therefore (x + 15)^2 + y^2 = 576 + 48\sqrt{(x - 15)^2 + y^2} + (x - 15)^2 + y^2$

$\therefore x^2 + 30x + 225 + y^2 = 576 + 48\sqrt{(x - 15)^2 + y^2} + x^2 - 30x + 225 + y^2$

$\therefore 60x = 576 + 48\sqrt{(x - 15)^2 + y^2}$

$\therefore 5x = 48 + 4\sqrt{(x - 15)^2 + y^2}$

$\therefore (5x - 48)^2 = 16(x - 15)^2 + 16y^2$

$\therefore 25x^2 - 480x + 2304 = 16x^2 - 480x + 3600 + 16y^2$

$\therefore 25x^2 - 480x + 2304 = 16x^2 - 480x + 3600 + 16y^2$

$\therefore 9x^2 - 16y^2 = 1296$

$\therefore \frac{x^2}{144} - \frac{y^2}{81} = 1$

2. If origin is shifted to $(h, k) \equiv (-3, 1)$, new co-ordinates are (X, Y) and old co-ordinates are (x, y) then

$x = X + h, \ y = Y + k$

i.e $x = X - 3, \ y = Y + 1$

Putting this in the given locus eqn,

$x^2 + 3y^2 + 6x + 10 = 0$

$\therefore (X - 3)^2 + 3(Y + 1)^2 + 6(X - 3) + 10$

$X^2 + 3Y^2 + 6Y + 4 = 0$

i.e $x^2 + 3y^2 + 6y + 4 = 0$

Practical No. 02 - Logarithums

1. Let $x = \frac{12.49 \times 0.6872}{(4.232)^2}$

$\therefore \log x = \log(12.49) + \log(0.6872) - 2\log(4.232)$

$= 1.0966 + \bar{1}.8371 - 2(0.6265)$

$= \bar{1}.6807$

$\therefore x = \text{antilog } \bar{1}.6807$

$= 0.4794$

2. Let $x = \frac{23.8 \times (7.3)^2}{\sqrt{0.82}}$

$\therefore \log x = \log(23.8) + 2\log(7.3) - \frac{1}{2}\log(0.82)$

$= 1.3766 + 2(0.8633) - \frac{1}{2}(\bar{1}.9138)$

$= 3.1463$

$\therefore x = \text{antilog } 3.1463$

$= 1401$

Practical No. 03 - Applications of Determinants

1. $\frac{1}{\cancel{2}}\begin{vmatrix} 3 & -5 & 1 \\ -2 & k & 1 \\ 1 & 4 & 1 \end{vmatrix} = \pm \frac{33}{\cancel{2}}$

$\therefore 3(k - 4) + 5(-2 - 1) + 1(-8 - k) = \pm 33$

$\therefore 3k - 12 - 15 - 8 - k = \pm 33$

$\therefore 2k - 35 = \pm 33$ i.e. $2k = \pm 33 + 35$

$\therefore 2k = 68$ or 2 $\therefore k = 34$ or 1

2. We rewrite the eqns as ...

$a + b + c = 3$

$2a - b + 3c = 4$

$3a + 4b - 2c = 5$

$\boxed{\begin{array}{l} a = \sin x \\ b = \cos y \\ c = \tan z \end{array}}$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 4 & -2 \end{vmatrix} = 1(2-12)-1(-4-9)+1(8+3) = 14$

$D_a = \begin{vmatrix} 3 & 1 & 1 \\ 4 & -1 & 3 \\ 5 & 4 & -2 \end{vmatrix} = 3(2-12)-1(-8-15)+1(16+5) = 14$

$D_b = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 3 \\ 3 & 5 & -2 \end{vmatrix} = 1(-8-15)-3(-4-9)+1(10-12) = 14$

$D_c = \begin{vmatrix} 1 & 1 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1(-5-16)-1(10-12)+3(8+3) = 14$

$a = \frac{D_a}{D} = \frac{14}{14} = 1$

$b = \frac{D_b}{D} = \frac{14}{14} = 1$

$c = \frac{D_c}{D} = \frac{14}{14} = 1$

$x = \sin^{-1}a = \sin^{-1}1 = 90°$

$y = \cos^{-1}b = \cos^{-1}1 = 0$

$z = \tan^{-1}c = \tan^{-1}1 = 45°$

Practical No. 04 - Complex Numbers

1. Let $x + yi = \sqrt{-2i}$

$\therefore \quad (x + yi)^2 = -2i$

$\therefore \quad x^2 + 2xyi + y^2i^2 = -2i$

$\therefore \quad x^2 + 2xyi + y^2(-1) = -2i$

$\therefore \quad (x^2 - y^2) + (2xy)i = 0 + (-2)i$

Equating real and imaginary parts,

$x^2 - y^2 = 0 \quad$ --- (1)

$2xy = -2$

$xy = -1 \quad$ --- (2)

From (2), $\quad y = \dfrac{-1}{x}$

Substituting in (1),

$$x^2 - \left(\frac{-1}{x}\right)^2 = 0$$

$\therefore \quad x^2 - \dfrac{1}{x^2} = 0$

$\therefore \quad (x^2)^2 - 1 = 0$

$\therefore \quad (x^2 - 1)(x^2 + 1) = 0$

$\therefore \quad x^2 = 1$ or $x^2 = -1$

Since x is real, only $x^2 = 1$ is valid

$x = \pm 1$

For $x = 1$, (2) gives $y = -1$

For $x = -1$, (2) gives $y = 1$

Hence the square root is

$1 - i$ or $-1 + i$

i.e $\pm(1 - i)$

2. $$\begin{aligned}(1 + \omega - \omega^2)^6 &= (-\omega^2 - \omega^2)^6 \\ &= (-2\omega^2)^6 \\ &= (-2)^6 (\omega^2)^6 \\ &= 2^6 (\omega^3)^4 \\ &= 64\,(1)^4 \\ &= 64.\end{aligned}$$

$\boxed{\text{Since } 1 + \omega + \omega^2 = 0 \quad 1 + \omega = -\omega^2}$

$\boxed{\omega^3 = 1}$

Practical No. 05 - Algebra of Matrices

1. A(B + C)

$$= \begin{bmatrix} 1 & -4 \\ -2 & 3 \end{bmatrix} \left(\begin{bmatrix} -1 & 6 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & -5 \\ 2 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -17 & 5 \\ 9 & -5 \end{bmatrix}$$

AB + AC

$$= \begin{bmatrix} 1 & -4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 6 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 4 & -5 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -13 & 14 \\ 11 & -18 \end{bmatrix} + \begin{bmatrix} -4 & -9 \\ -2 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} -17 & 5 \\ 9 & -5 \end{bmatrix} \quad \therefore \ A(B + C) = AB + AC$$

2. $$AB = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} -2 + 5 & 8 + 7 \\ -5 + 15 & 16 + 21 \end{bmatrix} = \begin{bmatrix} 3 & 15 \\ 11 & 37 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 4 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} -2 + 16 & -1 + 12 \\ 10 + 28 & 5 + 21 \end{bmatrix} = \begin{bmatrix} 14 & 11 \\ 38 & 26 \end{bmatrix}$$

Hence $AB \neq BA$

From this we conlude that matrix multiplication is not commutative.

Practical No. 06 - Special Series

1. We have

$$\sum_{r=1}^{n} (6r^2 - 2r + 6)$$

$$= 6 \sum_{r=1}^{n} r^2 - 2 \sum_{r=1}^{n} r + \sum_{r=1}^{n} 6$$

$$= \cancel{6} \frac{n(n + 1)(2n + 1)}{\cancel{6}} - \cancel{2} \frac{n(n + 1)}{\cancel{2}} + 6n$$

$$= n(n + 1) [(2n + 1) - 1] + 6n$$

$\boxed{\text{Taking } n(n + 1) \text{ common}}$

$$= n(n + 1)(2n) - 6n$$

$$= 2n(n^2 + n + 3)$$

$\boxed{\text{Taking } 2n \text{ common}}$

2. Reqd Sum is

$$\sum_{r=1}^{n} r(r+1)(r+2)$$

$$= \sum_{r=1}^{n} (r^3 + 3r^2 + 2r)$$

$$= \sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2 + 2\sum_{r=1}^{n} r$$

$$= \frac{n^2(n+1)^2}{4} + \frac{3n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$$

$$= n(n+1)\left[\frac{n(n+1)}{4} + \frac{2n+1}{2} + 1\right]$$

$$= n(n+1)\left[\frac{n(n+1) + 4n + 2 + 4}{4}\right]$$

$$= \frac{n(n+1)}{4}\,[\,n^2 + 5n + 6\,]$$

$$= \frac{n(n+1)(n+2)(n+3)}{4}$$

Practical No. 07 - Applications of Vectors

1. Given $\bar{a} = 4\bar{i} + \bar{j} + 3\bar{k}$, $\bar{b} = \bar{i} + 3\bar{j} + 2\bar{k}$, $\bar{c} = 2\bar{i} + 7\bar{k}$

$\overline{AB} = \bar{b} - \bar{a}$
$= (1 - 4)\bar{i} + (3 - 1)\bar{j} + (2 - 3)\bar{k}$
$= -3\bar{i} + 2\bar{j} - \bar{k}$

$\overline{AC} = \bar{c} - \bar{a}$
$= (2 - 4)\bar{i} + (0 - 1)\bar{j} + (7 - 3)\bar{k}$
$= -2\bar{i} - \bar{j} + 4\bar{k}$

$\overline{AB} \cdot \overline{AC} = (-3)(-2) + (2)(-1) + (-1)(4) = 0$

$\angle A = 90°$ $\therefore$ Δ ABC is a right angled Δ.

2. Given $\bar{a} = 2\bar{i} - \bar{j} + 3\bar{k}$ and $\bar{b} = 3\bar{i} + \bar{j} - 4\bar{k}$

$$\therefore\ \bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -1 & 3 \\ 3 & 1 & -4 \end{vmatrix} \qquad \text{(Oct '96)}$$

$= \bar{i}(4 - 3) - \bar{j}(-8 - 9) + \bar{k}(2 + 3)$
$= \bar{i} + 17\bar{j} + 5\bar{k}$

$\therefore\ |\bar{a} \times \bar{b}| = \sqrt{1^2 + 17^2 + 5^2} = \sqrt{315}$

Hence the unit vector $\perp^r$ to $\bar{a}$ & $\bar{b}$ is

$$\pm \frac{\bar{a} \times \bar{b}}{|\bar{a} \times \bar{b}|} = \pm \frac{\bar{i} + 17\bar{j} + 5\bar{k}}{\sqrt{315}}$$

Practical No. 08 - Limits

1. $$\lim_{x \to a} \frac{\sin x - \sin a}{x - a}$$

$$= \lim_{x \to a} \frac{2\cos\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)}{x - a}$$

$$= \lim_{x \to a} \cos\left(\frac{x+a}{2}\right) \times \lim_{x \to a} \frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}$$

$$\boxed{\begin{array}{l} \text{Put } (x - a)/2 = t \\ \text{As } x \to a,\ t \to 0 \end{array}}$$

$$= \cos\left(\frac{a+a}{2}\right) \times \lim_{t \to 0} \frac{\sin t}{t}$$

$= \cos a \times 1$

$= \cos a$

2. $$\lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{x \sin 2x}$$

$$= \lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{x^2} \times \frac{x^2}{x \sin 2x}$$

$$= \lim_{x \to 0} \frac{2^x 5^x - 2^x - 5^x + 1}{x^2} \times \lim_{x \to 0} \frac{x}{\sin 2x}$$

$$= \lim_{x \to 0} \frac{2^x(5^x - 1) - (5^x - 1)}{x^2} \times 2$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{(2^x - 1)(5^x - 1)}{x^2}$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{2^x - 1}{x} \times \lim_{x \to 0} \frac{5^x - 1}{x}$$

$$= \frac{1}{2} (\log 2)(\log 5)$$

$$\lim_{x \to 0} \frac{x}{\sin 2x} = \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^{-1}$$

$$= \left(2 \lim_{x \to 0} \frac{\sin 2x}{2x}\right)^{-1}$$

$$= 2^{-1}\left(\lim_{x \to 0} \frac{\sin 2x}{2x}\right)^{-1}$$

$$= \frac{1}{2}$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \log a$$

Practical No. 09 - Family of Lines

1. Let the required point be P(h, k).
Since it is on the line $x + y + 3 = 0$,

$\therefore\ h + k + 3 = 0$ --- (1)

Also, distance of P from $x + 2y + 2 = 0$ is given as $\sqrt{5}$.

$$\therefore\ \left|\frac{h + 2k + 2}{\sqrt{(1)^2 + (2)^2}}\right| = \sqrt{5}$$

$h + 2k + 2 = \pm 5$ (removing | |)

$\therefore\ h + 2k + 2 \pm 5 = 0$

$\therefore\ h + 2k + 7 = 0$ --- (2)

OR $h + 2k - 3 = 0$ --- (3)

Solving (1) & (2), h= 1 and k = -4
and solving (1) & (3), h= -9 and k = 6

$\therefore$ Required point is (1, -4) or (-9, 6)

2. The required line is of the form

$(x - y - 5) + k(2x - y - 8) = 0$

$\therefore\ (1 + 2k)x - (1 + k)y + (-5 - 8k) = 0$

$\therefore$ Slope of this line is $\dfrac{1 + 2k}{1 + k}$

Slope of the parallel line $x + 3y = a$ is $-1/3$

$\therefore\ \dfrac{1 + 2k}{1 + k} = -\dfrac{1}{3}\quad \therefore\quad k = -\dfrac{4}{7}$

Hence the required eqn is

$\therefore\ (x - y - 5) - \dfrac{4}{7}(2x - y - 8) = 0$

$\therefore\ 7x - 7y - 35 - 8x + 4y + 32 = 0$

$\therefore\ -x - 3y - 3 = 0$

$\therefore\ x + 3y + 3 = 0$

Practical No. 10 - Permutations and Combination

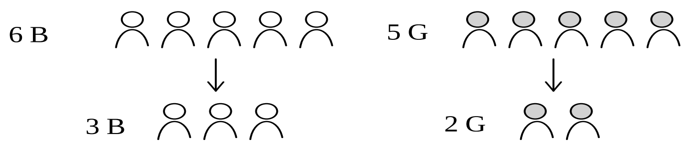
1. (a) There are 2 girls. Considering these 2 girls as as one girl, we have 7 persons in all. They can be arragend amongst themseleves in 7! ways Further two girls can themselves be seated in 2! different ways, the req. is $\underline{7! \times 2! = 10080}$

B B B G G B B B

(b) The two girls can occupy the two end seats in 2! different ways. Having occupied the end seats by two girls, the remaining 6 seats can be occupied the remaining 6 persons in 6! different ways Hence the required answer is $\underline{\underline{2! \times 6! = 1440}}$

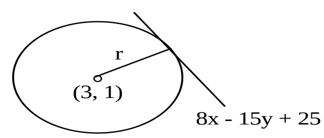
G B B B B B B G

2. 3 boys can be selected from 6 in ${}^6C_3$ different ways. Also 2 girls can be selected from 5 in ${}^5C_2$ ways. Hence by the fundmental principle the ans. is ${}^6C_3 \times {}^5C_2 = 20 \times 10 = \underline{200}$



Practical No. 11 - Circle

1. The centre is (3, 1) and $8x - 15y + 25 = 0$ is tangent to the circle. Therefore



Dist from the centre = Radius

$\therefore\ \left| \dfrac{8(3) - 15(1) + 25}{\sqrt{(8)^2 + (-15)^2}} \right| = r$ (Radius)

$\therefore\ \left| \dfrac{34}{17} \right| = r$

$\therefore\ r = 2$

Hence the equation of the circle is

$(x - h)^2 + (y - k)^2 = r^2$

i.e $(x - 3)^2 + (y - 1)^2 = 2^2$

i.e $x^2 + y^2 - 6x - 2y + 6 = 0$ --- Ans

2. Let C(h,k) be the centre $\therefore CP^2 = CQ^2$

$\therefore\ (h + 2)^2 + (k - 6)^2 = (h - 5)^2 + (k + 1)^2$

$\therefore\ h - 1k + 1 = 0$ ---(1)

Similarly, $CQ^2 = CR^2$, gives

$-2h + 1k + 1 = 0$ ---(2)

Solving (1) & (2) we get,

$h = 2$ and $k = 3$

Now $r = \sqrt{(-2 - 2)^2 + (6 - 3)^2}$

$= \sqrt{25}$

By the Centre-Radius form,

$(x - 2)^2 + (y - 3)^2 = 25$

$\therefore\ x^2 + y^2 - 4x - 6y - 12 = 0$

Practical No. 12 - Binomial Theorem

1. $(\sqrt{3} + 1)^4 - (\sqrt{3} - 1)^4 = {}^4C_0(\sqrt{3})^{4-0}(1)^0 + {}^4C_1(\sqrt{3})^{4-1}(1)^1 + \cdots\cdots + {}^4C_4(\sqrt{3})^{4-4}(1)^4$

$- \ {}^4C_0(\sqrt{3})^{4-0}(-1)^0 + {}^4C_1(\sqrt{3})^{4-1}(-1)^1 + \cdots\cdots + {}^4C_4(\sqrt{3})^{4-4}(-1)^4$

$= 2\left[{}^4C_1(\sqrt{3})^{4-1}(1)^1 + {}^4C_3(\sqrt{3})^{4-3}(1)^3 \right]$

$= 2\left[4(3\sqrt{3}) + 4\sqrt{3} \right]$

$= 32\sqrt{3}$ --- Ans

$(2 + \sqrt{5})^5 + (2 - \sqrt{5})^5 = {}^5C_0(2)^{5-0}(\sqrt{5})^0 + {}^5C_1(2)^{5-1}(\sqrt{5})^1 + \cdots\cdots + {}^5C_5(2)^{5-5}(\sqrt{5})^5$

$+ \ {}^5C_0(2)^{5-0}(-\sqrt{5})^0 + {}^5C_1(2)^{5-1}(-\sqrt{5})^1 + \cdots\cdots + {}^5C_5(2)^{5-5}(-\sqrt{5})^5$

$= 2\left[{}^5C_0(2)^{5-0}(\sqrt{5})^1 + {}^5C_2(2)^{5-2}(\sqrt{5})^2 + {}^5C_4(2)^{5-4}(\sqrt{5})^4 \right]$

$= 2\left[(1)(32) + 10(40) + 5(2)(25) \right]$

$= 1364$ --- Ans

2. The general term is

$$^{12}C_r (2x)^{12-r} \left(\frac{-y}{2}\right)^r$$

For the 3^{rd} term, $r = 2$

Hence the required term is

$$= {}^{12}C_2 (2x)^{12-2} \left(\frac{-y}{2}\right)^2$$

$$= {}^{12}C_2 \times 2^{10} \times x^{10} \times \frac{y^2}{2^2}$$

$$= 16896x^{10}y^2$$

Practical No. 13 - Applications of Conics - 1

1. Given Ellipse is

$\therefore \frac{x^2}{25} + \frac{y^2}{9} = 1$

$\therefore$ $a = 5$ and $b = 3$ $(a > b)$

$\therefore$ Length of major axis = $2a = 2(5) = \underline{\underline{10}}$

Length of minor axis = $2b = 2(3) = \underline{\underline{6}}$

$$e^2 = \frac{a^2 - b^2}{a^2} = \frac{25 - 9}{25} = \frac{16}{25}$$

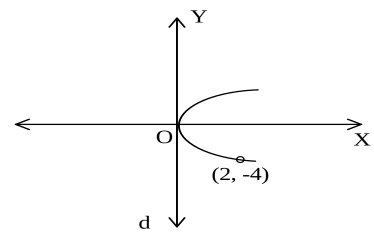
$\therefore$ $e = \frac{4}{5}$

2. From the given data it follows that the required parabola is a stdard parabola with vertex at the origin. So let $y^2 = 4ax$ be the required equation. Since the parabola passes through (2, -4),

$$(-4)^2 = 4a(2)$$

$$a = 2$$

Consequently, the required eqn of the parabola is $y^2 = 4(2)x$ i.e $y^2 = 8x$



Practical No. 14 - Mathematical Induction

1. Let P(n) : $3^{2n} + 7$ is divisible by 8

For n = 1, $3^2 + 7 = 16$, is divisible by 8

$\therefore$ P(1) is true.

Let P(k) be true.

$\therefore$ $3^{2k} + 7$ is divisible by 8 ---- (1)

Now $3^{2(k+1)} + 7 = 3^{2k+2} + 7$

$= 3^{2k}3^2 + 7$

$= 9 \cdot 3^{2k} + 7$

$= 9(3^{2k} + 7) - 56$

$\therefore$ But $9(3^{2k} + 7)$ is divisible by 8, by (1)

$\therefore$ Also 56 is divisible by 8.

$\therefore$ Hence $9(3^{2k} + 7) - 56$ is divisible by 8

$\therefore$ $3^{2(k+1)} + 7$ is divisible by 8

$\therefore$ P(k+1) is true.

$\therefore$ Hence the result by induction.

2. Let P(n) : $1 + 3 + 5 + \cdots + 2n - 1 = n^2$

For n = 1, $1 = 1^2$ $\therefore$ P(1) is true.

Let P(k) be true.

$\therefore$ $1 + 3 + 5 + \cdots + 2k - 1 = k^2$

For n = k + 1, we have

$1 + 3 + 5 + + \cdots + (2k - 1) + (2k + 1)$ --- (1)

$= k^2 + (2k + 1)$, by (1)

$= (k + 1)^2$

This shows that P(k + 1) is true.

P(k) is true $\Rightarrow$ P(k + 1) is true

P(n) is true $\forall$ n $\in$ N.

Practical No. 15 - Applications of Conics - 2

1. Given Ellipse is

$\therefore \frac{x^2}{25} + \frac{y^2}{9} = 1$

$\therefore$ $a = 5$ and $b = 3$ $(a > b)$

$\therefore$ $e^2 = \frac{a^2 - b^2}{a^2} = \frac{25 - 9}{25} = \frac{16}{25}$

$\therefore$ $e = \frac{4}{5}$

Now, foci are $(\pm ae, 0) = (\pm 4, 0)$

directrices are $x = \pm \frac{a}{e}$

i.e $x = \pm \frac{25}{4}$

2. Given hyperbola is $\frac{x^2}{4} - \frac{y^2}{36} = 1$

$\therefore$ $a = \sqrt{4} = 2$ and $b = \sqrt{36} = 6$

$\therefore$ $e = \sqrt{\frac{a^2 + b^2}{a^2}} = \sqrt{\frac{4 + 36}{4}} = \sqrt{10}$

Latus rectum is $\frac{2b^2}{a} = \frac{2(36)}{2} = 36$ units

and the foci are

$(\pm ae, 0) \equiv (\pm 2\sqrt{10}, 0)$

Practical No. 16 - Differentiation

1. By definition $f'(x)$ is

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\log[(x+h)+7] - \log(x+7)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \log\left(\frac{x+7+h}{x+7}\right)$$

$$= \lim_{h\to 0} \frac{1}{h} \log\left(1 + \frac{h}{x+7}\right)$$

$$= \lim_{h\to 0} \log\left(1 + \frac{h}{x+7}\right)^{\frac{1}{h}}$$

$$= \lim_{h\to 0} \log\left[\left(1 + \frac{h}{x+7}\right)^{\frac{x+7}{h}}\right]^{\frac{1}{x+7}}$$

$$= \log\left\{\left[\lim_{h\to 0}\left(1 + \frac{h}{x+7}\right)^{\frac{x+7}{h}}\right]^{\frac{1}{x+7}}\right\}$$

$$= \log e^{\frac{1}{x+7}} = \frac{1}{x+7} \quad \text{--- Ans}$$

2. Let $y = \dfrac{x^4e^x}{\sin x + 1}$ $\therefore \dfrac{dy}{dx}$

$$= \frac{(\sin x+1)\dfrac{d}{dx}(x^4e^x) - (x^4e^x)\dfrac{d}{dx}(\sin x+1)}{(\sin x+1)^2}$$

$$= \frac{(\sin x+1)(x^4e^x + 4x^3e^x) - (x^4e^x)(\cos x)}{(\sin x+1)^2}$$

$$= \frac{x^3e^x(\sin x+1)(x+4) - x^4e^x\cos x}{(\sin x+1)^2}$$

Practical No. 17 - Linear Equations

1. Consider $4x + 3y = 12$

x	0	3
y	4	0
(x, y)	(0, 4)	(3, 0)

A = (0, 4) and B = (3, 0)

For x = y = 0 , 4(0) + 3(0) = 0 $\leqslant$ 12

$\therefore$ Origin side is the solution set

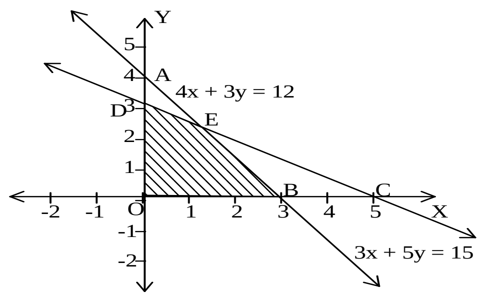
Consider $3x + 5y = 15$

x	0	5
y	3	0
(x, y)	(0, 3)	(5, 0)

D = (0, 3) and C = (5, 0)

For x = y = 0 , 3(0) + 5(0) = 0 $\leqslant$ 15

$\therefore$ Origin side is the solution set



$\therefore$ Bounded shaded portion OBED is the common solution set

2. Consider $3x + 2y = 6$

x	0	2
y	3	0
(x, y)	(0, 3)	(2, 0)

A = (0, 3) and B = (2, 0)

For x = y = 0 , 3(0) + 2(0) = 0 $\geqslant$ 6 (absurd)

$\therefore$ Non-origin side is the solution set

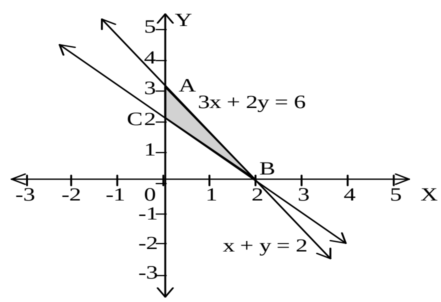
Consider $x + y = 2$

x	0	2
y	2	0
(x, y)	(0, 2)	(2, 0)

C = (0, 2) and B = (2, 0)

For x = y = 0 , 0 - 0 = 0 $\leqslant$ 2

$\therefore$ Origin side is the solution set



$\therefore$ Bounded shaded portion CAB is the common solution set

Practical No. 18 - Integration

1. $$\int \frac{15}{\sqrt{2x+11} + \sqrt{2x-4}}\,dx$$

$$= \int \frac{15}{\sqrt{2x+11} + \sqrt{2x-4}} \times \frac{\sqrt{2x+11} - \sqrt{2x-4}}{\sqrt{2x+11} - \sqrt{2x-4}}\,dx$$

$$= 15 \int \frac{\sqrt{2x+11} - \sqrt{2x-4}}{2x+11-2x+4}\,dx$$

$$= \cancel{15} \int \frac{\sqrt{2x+11} - \sqrt{2x+4}}{\cancel{15}}\,dx$$

$$= \int \left[(2x+11)^{1/2} - (2x-4)^{1/2}\right] dx$$

$$= \frac{(2x+1)^{3/2}}{2\times(3/2)} - \frac{(2x-4)^{3/2}}{2\times(3/2)} + c$$

$$= \frac{1}{3}\left[(2x+11)^{3/2} - (2x-4)^{3/2}\right] + c$$

2. $I = \int \sqrt{1 - \sin x}\ dx$

$= \int \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}\ dx$

$= \int \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) dx$

$= \int \cos\frac{x}{2}\ dx - \int \sin\frac{x}{2}\ dx$

$= \frac{\sin(x/2)}{1/2} - \frac{[-\cos(x/2)]}{1/2} + c$

$= 2\sin\frac{x}{2} + 2\cos\frac{x}{2} + c$

Practical No. 19 - Probability

1. Group of 4 can be selected from 7 (3 B, 4G) in ${}^7C_4$ different ways.

$\therefore\ n(S) = {}^7C_4$

Let A be the event of selecting 3 boys and 1 girl.

$\therefore\ n(A) = {}^3C_3 \times {}^4C_1$

$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{{}^3C_3 \times {}^4C_1}{{}^7C_4} = \frac{4}{35}$

Let B be the event of selecting 1 boy and 3 girls.

$\therefore\ n(B) = {}^3C_1 \times {}^4C_3 \qquad \therefore\ P(B) = \frac{n(B)}{n(S)} = \frac{{}^3C_1 \times {}^4C_3}{{}^7C_4} = \frac{12}{35}$

Since A and B are mutually exclusuve, $P(A \cup B) = P(A) + P(B) = \frac{4}{35} + \frac{12}{35} = \frac{16}{35}$

2. Sample space S = { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}. $\therefore\ n(S) = 8$

Let A : atleast two heads

A = {HHT, HTH, THH, HHH} $\quad n(A) = 4$

$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

Let B : all 3 coins show head.

$\therefore$ B = {HHH} $\quad \therefore\ n(B) = 4$

$\therefore\ P(B) = \frac{n(B)}{n(S)} = \frac{1}{8}$

$A \cap B = \{HHH\}$

$\therefore\ n(A \cap B) = 1$

$\therefore\ P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{8}$

By conditional probability

$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/8}{1/2} = \frac{1}{4}$

Practical No. 20 - Measures of Dispersion

1. We prepare cumulative frequency table (less than)

Class Intervals	Frequency f	Cum. Freq < type
0 - 10	22	22
10 - 20	38	60
20 - 30	46	106
30 - 40	35	141
40 - 50	20	161

$\frac{N}{4} = \frac{161}{4} = 40.25 \quad \therefore\ Q_1$ lies in 10 - 20

L = 10 , h = 10, c.f. = 22 (prev)

$Q_1 = L + \frac{h\left(\frac{N}{4} - c.f.\right)}{f}$

$= 10 + \frac{10(40.25 - 22)}{38}$

$= 14.802$

$\frac{3N}{4} = \frac{3 \times 161}{4} = 120.75 \quad \therefore\ Q_3$ lies in 30 - 40

L = 30 , h = 10, c.f. = 106 (prev)

$Q_1 = L + \frac{h\left(\frac{3N}{4} - c.f.\right)}{f}$

$= 30 + \frac{10(120.75 - 106)}{38}$

$= 34.214$

$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{34.214 - 14.802}{2} = \underline{\underline{9.706}}$

2. Arranging in ascendinbg order

23, 29, 38, 41, 53

Here n = 5, $\therefore\ \frac{n+1}{2} = 3$

3rd Observation = 38 is the medan

Table to Calculate Median :

x_i	$\lvert x_i - M \rvert$
23	15
29	09
38	00
41	03
53	15
	42

Mean deviation about median $= \frac{\Sigma \lvert x_i - M \rvert}{n} = \frac{42}{5} = 8.4$

--- End ---