S. I. W. S. Smt. Thirumalai College of Science – Wadala
MATHEMATICS AND STATISTICS
(ARTS AND SCIENCE)

Time : 3 Hrs Max. Marks : 80

Notes :

1) *All questions are compulsory.*
2) *The question paper consists of 30 questions divided in to four sections A, B, C, D.*
3) *Section A contains 6 questions of 1 mark each.*
   Section B contains 6 questions of 2 mark each.
   Section C contains 6 questions of 3 mark each.
   Section D contains 6 questions of 4 mark each.
4) *Use of Logarithmic table is allowed.*
5) *For each MCQ , correct answer must be written along wit its alphabet.*
   In case of MCQ (Q1 to Q6) evaluation would be done for the first attempt only.
6) *Use of calculator is not allowed.*
7) *In L. P. P. only rough sketch of graph is expected. Graph paper is not necessary.*

SECTION – A

Select and write the most appropriate answer from the given alternatives in each of the following questions.

(1) $p \to (q \wedge r)$ is logically equivalent to
(a) $(p \vee q) \to (p \vee r)$ (b) $(p \to q) \vee (p \to r)$ (c) $(p \wedge q) \to (p \wedge r)$ (d) $(p \to q) \wedge (p \to r)$

(2) If te vectors $\bar{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}, \bar{b} = \hat{i} + 2\hat{j} - 2\hat{k}, \bar{c} = -\hat{i} + 2\hat{j} + m\hat{k}$ are co-planar then
(a) m = - 4 (b) m = - 5 (c) m = - 6 (d) m = 6

(3) The direction ratios of the line joining A (1, 2, 1) and (2, 1, 2) are
(a) 1, 1, -1 (b) -1, -1, -1 (c) -1, 1, -1 (d) -1, 1, 1

(4) If $y = \dfrac{x + \sqrt{x}}{\sqrt{x} + 1}$ then $\dfrac{dy}{dx} =$
(a) $\dfrac{x+1}{(\sqrt{x}+1)^2}$ (b) $\dfrac{1}{2\sqrt{x}}$ c) $\dfrac{x\sqrt{x}-1}{(\sqrt{x}+1)^2}$ (d) None of these

(5) The anti derivative of $cotx$ is
(a) $log\,|sinx| + c$ (b) $log\,|sinx| + c$ (c) $log\,|sinx| + c$ (d) $log\,|sinx| + c$

(6) The order of the differential equation of all circles having radius r is
(a) one (b) two (c) three (d) four

SECTION – B

(7) Write the converse and the inverse of " If $a \in \mathbb{R}$ then $a^2 \geq 0$ "

(8) Prove that the general solution of $cos\theta = cos\alpha$ is $\theta = 2n\pi \pm \alpha, n \in I$

(9) In $\triangle ABC$ if $sin^2A + sin^2B = sin^2C$, then show that the triangle is a right angled triangle.

(10) Differentiate $log(1 + x^2)$ with respect to $tan^{-1}x$

(11) Find the acute angle between the lines whose drs are 1, 2, 2 and -3, 6, 2

OR

Show that there is no line in space which makes angle of 30^0 with each of X and Y - axes

(12) Test if the function $f(x) = 2 - 3x + 3x^2 - x^3$ is increasing or decreasing every $x \in \mathbb{R}$.

(13) Evaluate : $\int tan^2(3x)dx$

(14) Evaluate : $\int_{-\pi/2}^{\pi/2} log\left(\dfrac{5 + sinx}{5 - sinx}\right) dx$

SECTION – C

(15) Without using truth table show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

OR

Construct the switching circuit for the statement $(p \wedge \sim q \wedge r) \vee [p \wedge (\sim q \vee \sim r)]$.

(16) Find the vector equation of the plane passing through (1, 1, -2), (1, 2, 1) and (2, -1, 1).

(17) Find the length of perpendicular from (2, -3, 1) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+2}{-1}$

(18) If y is a differentiable function of u and u is a differentiable function of x then prove that $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

OR

If $\left(x^2 + y^2\right) \sec y = x^2 - y^2$ then prove that $\frac{dy}{dx} = \frac{y}{x}$

(19) Three fair coins are tossed simultaneously. If X denotes the number of heads, then find the probability distribution of X.

(20) There are 10 defective screws in a lot of 100 screws. Five screws are selected at random from the lot. What is the probability that at most one screw is defective.

SECTION – D

(21) Solve the equations $3x - y = 5, 2x - 3y = 8$ by method of inversion.

(22) In $\triangle ABC$, if a, b, c are in A. P. then show that $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A. P.

OR

Find the general solution of $\sin x \cdot \tan x = \tan x - \sin x + 1$.

(23) Show that the equation $9x^2 - 6xy + y^2 + 18x - 6y + 8 = 0$ represents a pair of lines. Find the acute angle between the lines.

(24) Using vector method, show that altitudes of a triangle are concurrent.

OR

If $\overline{a}, \overline{b}$ and $\overline{r}$ are position vectors of the points A, B, and R respectively and R divides the line segment AB internally in the ratio $m : n$ then prove that $\overline{r} = \frac{m\overline{b} + n\overline{a}}{m+n}$. If $2\overline{a} + 3\overline{b} - 5\overline{c} = \overline{0}$, then find the ratio in which C divides AB.

(25) Maximize : $Z = 4x + 6y$ subject to $3x + 2y \leq 12,\ x + y \geq 4,\ x, y \geq 0$

(26) Discuss the continuity of f at x = 0 where,

$$f(x) = \frac{4^x - 2^{x+1} + 1}{1 - cos2x} \quad \text{for} \quad x \neq 0$$

$$= \frac{(log2)^2}{2} \quad \text{for} \quad x = 0.$$

(27) An open box is to be made out of a piece of square card board of side 18 cm by cutting off equal squares from corners and turning up the sides. Find the maximum volume of the box.

(28) Prove that $\int\sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

(29) Evaluate $\int_0^{\pi/4} \log(1 + \tan x)\, dx$

(30) Find the particular solution of the differential equation $(x+1)\frac{dy}{dx} - 1 = 2e^{-y}$ satisfying the condition y = 0 when x = 1.

OR

The rate of decay of certain substance is directly proportional to the amount present at that instant. Initially, there are 27 gm of certain substance and three after it is found that 8 gm are left. Find the amount left after one more hour.

Answers and Hints

Section A

1. (d) $(p \to q) \wedge (p \to r)$ 2. (c) m = - 6 3. (c) 1-, 1, -1

4. (b) $\frac{1}{2\sqrt{x}}$ 5. (a) $log\,|sinx| + c$ 6. (c) three

Section B

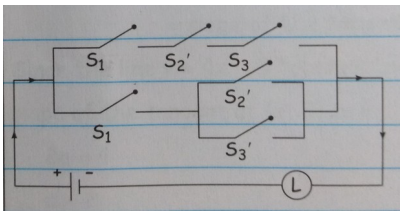
7. (i) If $a^2 \geq 0$ then $a \in \mathbb{R}$, If $a \notin \mathbb{R}$ then $a^2 < 0$

10. 2x 11. $cos^{-1}\left(\frac{13}{21}\right)$ 12. $f'(x) = -3(x-1)^2 \leq 0$, decreasing $\forall x \in \mathbb{R}$

13. $\frac{tan3x}{3} - x + c$ 14. Odd function. So, I = 0.

Section C

15. Circuit Diagram



16. $\vec{r} \cdot \left(9\hat{i} + 3\hat{j} - \hat{k}\right) = 14$

17. $\frac{\sqrt{531}}{14}$ 19. Table 20. $\frac{91854}{10^5} = 0.91854$

Section D

21. x = 1 and y = - 2 22. $x = n\pi + (-1)^n \frac{\pi}{2}, n \in \mathbb{Z}$ or $x = n\pi + \frac{3\pi}{4}, n \in \mathbb{Z}$

23. 24. 3 : 2 25. Max(z) = 36 at x = 0 and y = 6 $\theta = 0$

26. continuous 27. x = 3, Max Vol = 432 cc. 29. $\frac{\pi}{8}\log 2$

30. $2(2 + e^y) = 3(x+1)$ 30. $\frac{16}{3}$ gm.