

**Std. XII**

**MATHEMATICS (40)**

**Specimen  
Question Bank**

# Chapter 1 : Mathematical Logic

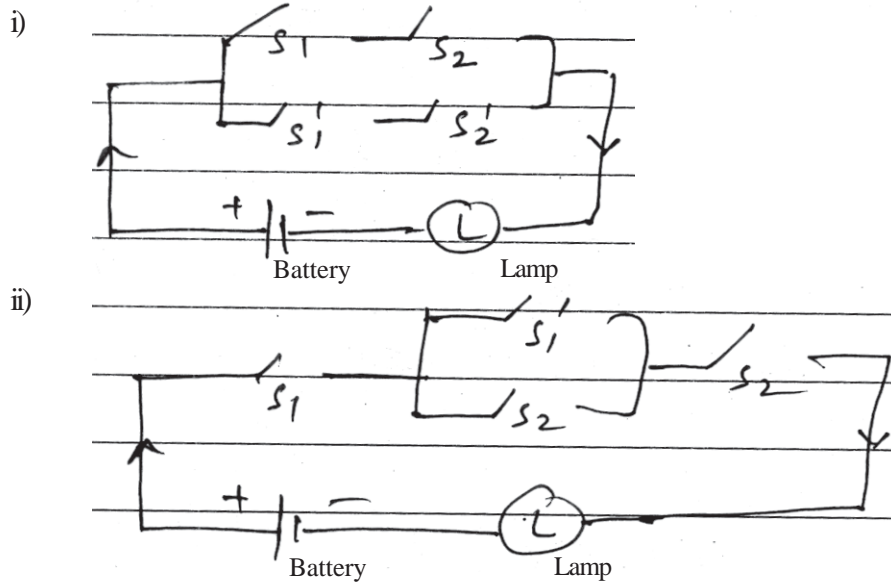
(2 - Marks)

1. Write the truth values of the following statements.
  - i) 2 is a rational number and it is the only even prime number.
  - ii)  $\exists x \in \mathbb{N}$  such that  $x + 3 > 5$
  - iii)  $3 + 2i$  is a real number or it is a complex number.
  - iv) It is false that New Delhi is not a capital of India.
  - v) The cube roots of unity are in G.P.
2. If p, q, r are statements with truth value T, F T respectively, determine the truth values the of following :
  - i)  $(p \vee r) \sim q$
  - ii)  $(p \rightarrow q) \leftrightarrow r$
  - iii)  $(p \leftrightarrow q) \wedge (q \leftrightarrow r)$
  - iv)  $\sim (r \wedge \sim q) \vee (p \wedge \sim r)$
  - v)  $(r \wedge q) \leftrightarrow \sim p$
3. Write the negations of the following statements.
  - i) He is rich and happy.
  - ii) If I become a teacher, then I will open a school.
  - iii)  $\forall x \in \mathbb{N}, x + 5 > 8$
  - iv) A person is busy if and only if he is a doctor.
  - v)  $\sim p \rightarrow (q \vee r)$
  - vi) All parents care for their children.
4. Prepare the truth table for each of the following statement pattern.
  - i)  $p \wedge (q \rightarrow p)$
  - ii)  $(\sim p \vee \sim q) \leftrightarrow (p \wedge q)$
  - iii)  $(p \wedge \sim p) \vee q$
  - iv)  $(p \leftrightarrow q) \vee (q \leftrightarrow p)$
  - v)  $\sim q \rightarrow \sim p$
  - vi)  $p \vee \sim p$

5. Write the following statments in symbolic form :

- i) Manisha does not live in Mumbai.
- ii) If a number  $n^2$  is even, then  $n$  is even.
- iii) Rohit is neither healthy nor wealthy.
- iv) If  $\Delta ABC$  is right angled at  $B$ , then  $AB^2 + BC^2 = AC^2$ .
- v) It is raining if and only if the weather is humid.

6. Express the given circuits in symbolic form.



7. If  $p$  : The earth is round.

$q$  : The moon rotates around the earth.

and  $r$  : The sun is hot.

Write the following in verbal form.

- i)  $p \wedge q$  (ii)  $p \leftrightarrow q$  (iii)  $p \rightarrow (q \vee r)$
- (iv)  $(\sim p \wedge q) \vee r$  (v)  $q \rightarrow r$

8. If  $A = \{4, 5, 7, 9\}$ , determine the truth value of each of the following. quantified statements.

- i)  $\exists x \in A$ , such that  $x + 2 = 7$
- ii)  $\forall x \in A$ ,  $x + 3 < 10$
- iii)  $\exists x \in A$ , such  $x + 5 > 9$
- iv)  $\exists x \in A$ , such that  $x$  is even.
- v)  $\forall x \in A$ ,  $2x < 17$

9. Write duals of the following statements.
- $(p \vee q) \vee r$
  - $(p \vee q) \wedge T$
  - $\sim (p \vee q) \wedge [p \vee \sim (q \wedge \sim r)]$
  - Sohan and Kavita can not read french.
  - $(\sim p \wedge \sim q) \equiv \sim (p \vee q)$
  - $(p \wedge T) \vee (F \wedge \sim q)$
10. Prove the following results ; using truth tables.
- $p \rightarrow q \equiv \sim p \vee q$
  - $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
  - $\sim (p \wedge q) \equiv \sim p \vee \sim q$
  - $p \wedge q \equiv \sim (p \rightarrow \sim q)$
  - $\sim (p \rightarrow q) \equiv p \wedge \sim q$

### 3 Marks

- State the converse, inverse and contrapositive of the following conditional statement.
  - If the teacher is absent, then the students are happy.
  - If  $2 + 3 < 7$  then  $7 + 3 > 2$
  - If  $f(x)$  is differentiable function then it is continuous
  - $[p \wedge (p \rightarrow q)] \rightarrow q$
  - A family becomes literate if the woman in it are literate.
- Prepare the truth table of the following statement patterns.
  - $(p \wedge q) \rightarrow (\sim p \vee \sim q)$
  - $[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$
  - $(\sim p \wedge q) \leftrightarrow (p \rightarrow \sim q)$
  - $(p \leftrightarrow r) \wedge (q \leftrightarrow p)$
- Using truth tables. Prove the following logical equivalences.
  - $(p \leftrightarrow q) \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
  - $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$
  - $\sim (\sim p \wedge q) \wedge (p \vee q) \equiv p$

4. Using truth tables, examine whether each of the following statement patterns is a tautology or a contradiction or a contingency.

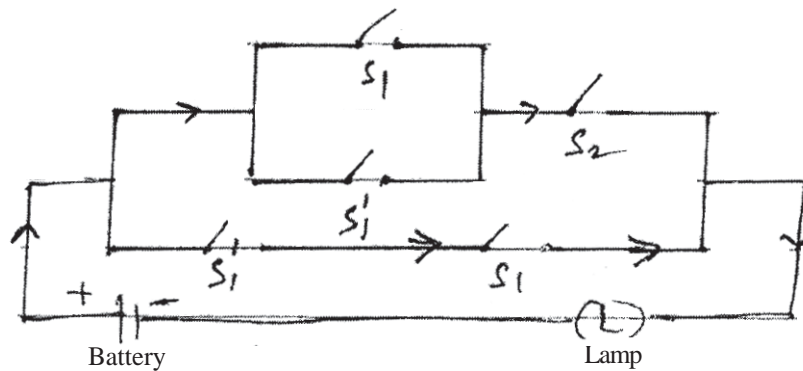
- i)  $[(p \rightarrow q) \wedge p] \rightarrow q$
- ii)  $(p \rightarrow \sim p) \wedge (\sim p \rightarrow p)$
- iii)  $(\sim p \wedge q) \wedge (q \rightarrow p)$
- iv)  $(p \vee q) \vee r \leftrightarrow p \vee (q \vee r)$
- v)  $(p \vee q) \wedge (p \vee r)$

5. Using the rules of negation, write the negation of the following.

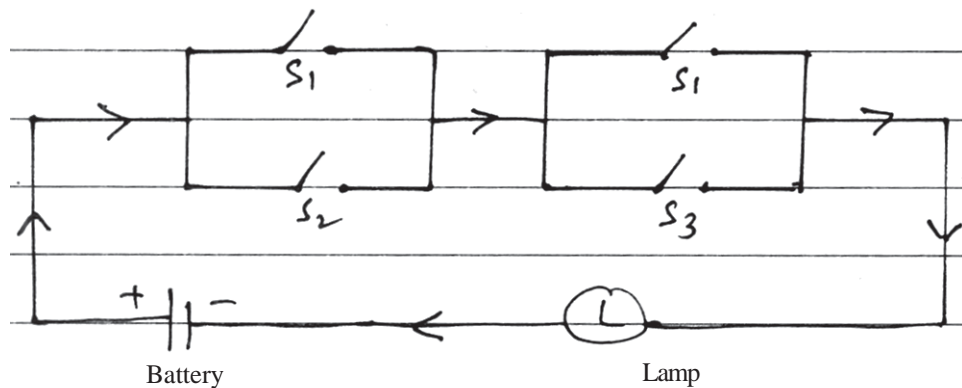
- i)  $p \wedge \sim (q \wedge r)$
- ii)  $(\sim p \wedge q) \vee (p \wedge \sim q)$
- iii)  $(p \rightarrow q) \wedge r$
- iv) If  $10 > 5$  and  $5 < 8$  then  $8 < 7$
- v) It is false that the sky is not blue.

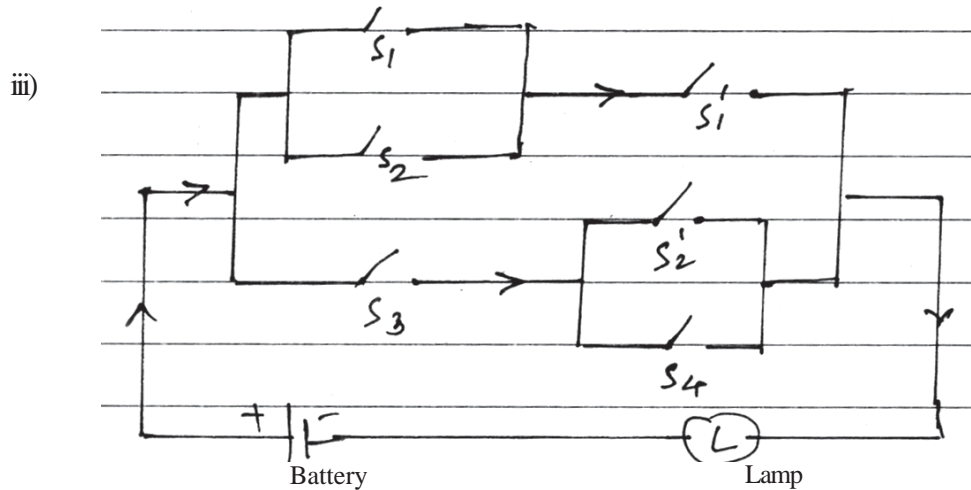
6. Express the following circuits in symbolic form and write input output table.

i)



ii)





7. Construct the switching circuits of the following statement patterns.

- i)  $[p \wedge (q \vee r) \vee (\sim p \vee s)]$
- ii)  $(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$
- iii)  $[(p \vee q) \wedge \sim p] \vee [r \wedge (\sim q \vee s)]$
- iv)  $p \wedge [q \vee (r \wedge \sim p)] \wedge s$

8. Write the following compound statement in symbolic form and write their negations :

- i) Mahesh is fat but not lazy.
- ii) It is neither cold nor raining.
- iii) Some countries are digital and all people are technosavy.
- iv) If I drive fast and do not follow traffic rules, then I will meet with an accident.

9. If p, q are true statements and r, s are false statements, then find the truth values of the following compound statements.

- i)  $(\sim p \vee q) \rightarrow (s \wedge \sim r)$
- ii)  $[(p \rightarrow q) \rightarrow r] \rightarrow s$
- iii)  $(p \leftrightarrow s) \wedge (p \rightarrow q)$
- iv)  $[p \wedge (q \vee r)] \vee [s \wedge \sim q]$
- v)  $p \wedge [q \wedge (\sim p \wedge r) \vee \sim s] \vee \sim r$

10. i) Write the contrapositive of the inverse of the statement “If two numbers are not equal, then their squares are not equal”.

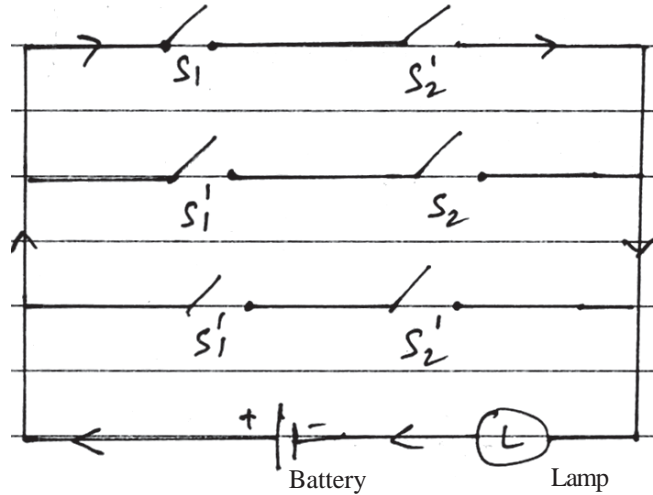
ii) If  $(p \wedge q) \rightarrow r$  is false, then find the truth value of the negation of the statement.

$$(p \vee \sim r) \rightarrow (q \wedge \sim p)$$

iii) Show that the dual of  $(p \rightarrow \sim q) \vee q$  is a contradiction.

**(D) for 4 marks**

1. Simplify the following so that the new circuit has minimum number of switches. Also, draw the simplified circuit.



2. Without using truth table, prove that

i)  $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q) \equiv \sim p \vee \sim q$

ii)  $[p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p] \equiv p$

iii)  $p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

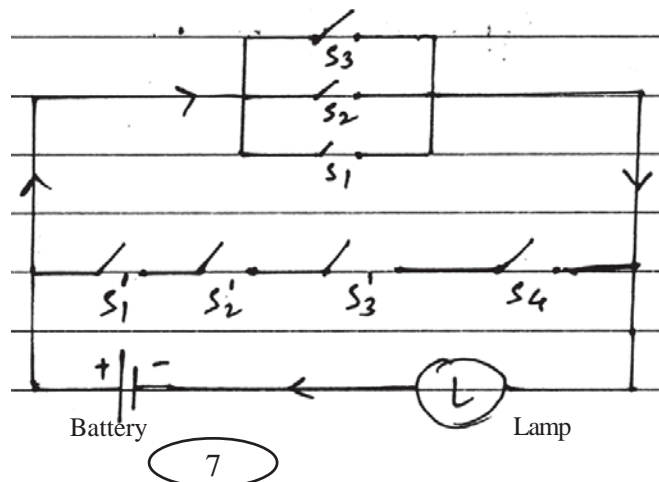
3. Identify the pairs of following statements having same meaning.

- i) If a person is a social, then he is happy.
- ii) If a person is not social, then he is not happy.
- iii) If a person is unhappy, then he is not social.
- iv) If a person is happy, then he is social.

4. Write the following statement in four different ways, conveying the same meaning.

“If you drive over 80 km per hour, then you will get a fine.”

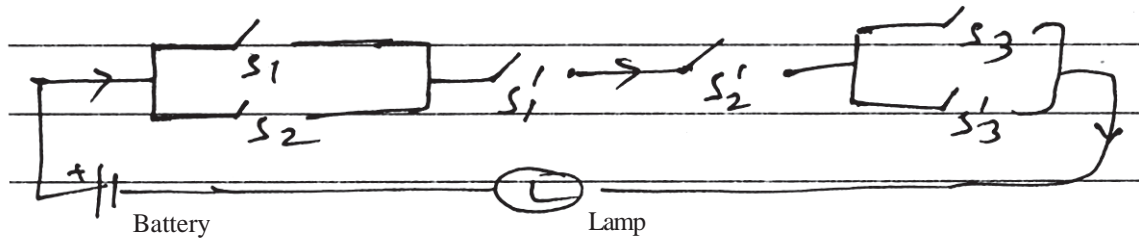
5. Show that, the following, circuit can be simplified and reconstructed so as to reduce its number of switches from 7 to 4.



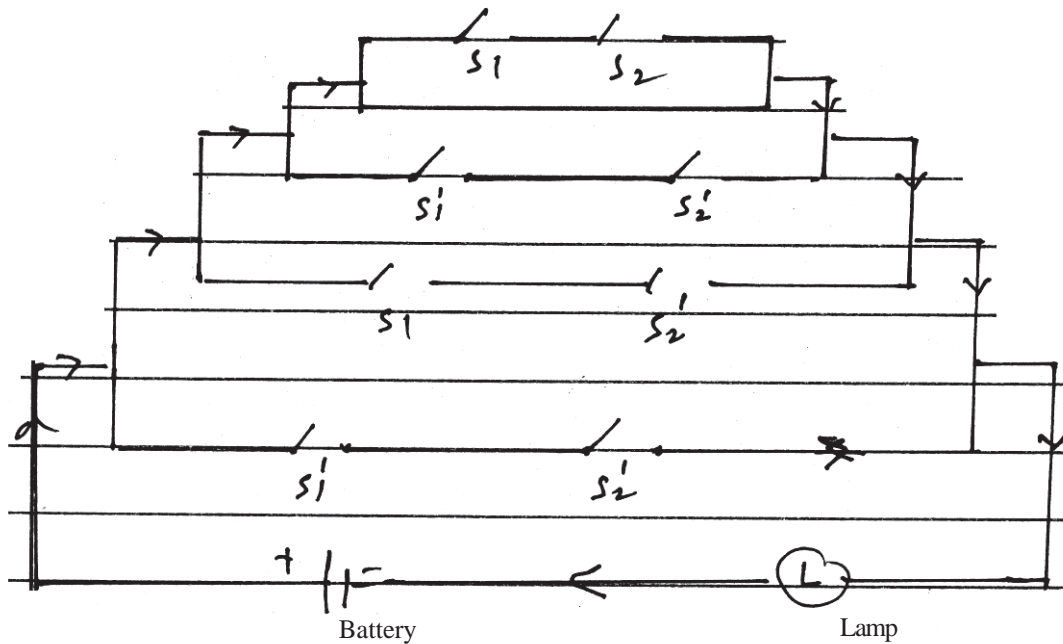
6. State the dual of the following statement by applying the principle of duality. Also, prove that both sides of the dual are equivalent.

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

7. Simplify the following circuit and reconstruct an alternative circuit having minimum switches :



8. Write the following circuit symbolically and construct its switching table. What conclusion would you draw from the table ?



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## Chapter 2 : Matrices

(2 - Marks)

1. Find the adjoint of the following matrices.

i)  $\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$

ii)  $\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}$

iii)  $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$

2. Find the inverse of the following Matrices using elementary row transformations.

i)  $\begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$

ii)  $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

iii)  $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

iv)  $\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

v)  $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

3. Find the inverse of the following Matrices using elementary column transformations.

i)  $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

ii)  $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

iii)  $\begin{bmatrix} \operatorname{cosec} \theta & \cot \theta \\ -\cot \theta & \operatorname{cosec} \theta \end{bmatrix}$

iv)  $\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$

v)  $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

4. Find the inverse of the following Matrices using adjoint method.

i)  $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

ii)  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

iii)  $\begin{bmatrix} \sin \theta & 1 \\ 0 & \cos \theta \end{bmatrix}$

iv)  $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

v)  $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 4 & 5 \end{bmatrix}$

5. Express the following equations in matrix form and solve them by using.

a) reduction method

b) Inversion method.

i)  $x + y = 2, \quad 3x + 2y = 5$

ii)  $2x + y = 5, \quad 3x + 5y = -3$

iii)  $x + 3y = 4, \quad 4x - y = 3$

iv)  $2x - y = -2, \quad 3x + 4y = 3$

v)  $4x + 3y = 1, \quad 2x + y = 1$

6. i) Find the matrix X such that  $AX = B$ , where  $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$  and

$$B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

ii) Find the matrix X such that  $AX = I$  where  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

iii) If  $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$  and

$C = \begin{bmatrix} 24 & 7 \\ 31 & 9 \end{bmatrix}$ , then find the matrix X such that  $AXB = C$

iv) If  $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ , then find the matrix X such

that  $A^{-1}X = B$

7. i) If  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

then find matrix  $(AB)^{-1}$ .

ii) If  $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ , then find  $\text{adj}(\text{adj } A)$

iii) If  $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$  and  $A^{-1} = KA$ , then find the value of K.

8. i) If  $A = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$  and  $B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$  find  $AB'$ .
- ii) If  $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ ,  $B = [3 \quad 1 \quad -2]$  verify that  $(AB)' = B'A'$ .
- iii) If  $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$  show that  $A^2 - 5A - 14I = 0$
- iv) If  $f(x) = x^2 - 2x - 3$ , find  $f(A)$  when  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

9. If  $A$  is invertible matrix of order 3 and  $|A| = 5$ , then find the value of  $|\text{adj } A|$ .

10. If  $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 4 \\ 6 & 19 \end{bmatrix}$

verify that  $\text{adj}(AB) = (\text{adj } B)(\text{adj } A)$

### 3 Marks

1. i) If  $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ , show that  $A^2 - 5A + 7I = 0$  and hence find  $A^{-1}$ .

ii) If  $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ , show that  $A^{-1} = \frac{1}{6}(A - 5I)$

iii) If  $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}$ , then

find  $(AB)^{-1}$

iv) If  $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$  then prove that  $A^2 = I$ . Hence show that  $A^{-1} = A$

2. Find adj A, if  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

3. Nina and Meena want to buy pens and books. Nina wants 2 pens and 5 books while Meena wants 6 pens and 8 books. They both go to a shop and buy them. When the shopkeeper gives them the pens and the books. Nina pays him Rs. 110 and Meena pays Rs. 190. Find the prices of one pen and one book using matrices.

4. If  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

then find the matrix X such that  $XA = B$ .

5. Find the adjoint of the matrix

$A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$  and verify that  $A (\text{adj } A) = (\text{adj } A) A = |A| I$

6. Find the inverse of the matrix using adjoint method.  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$

7. If A and B are two invertible matrices of the same order, then prove that  $(AB)^{-1} = B^{-1}A^{-1}$

8. If  $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ , show that  $\text{adj } A = A$

9. If  $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 4 \\ 0 & -1 & 2 \end{bmatrix}$  with usual notations verify that

i)  $a_{31} C_{31} + a_{32} C_{32} + a_{33} C_{33} = |A|$

ii)  $a_{21} C_{31} + a_{22} C_{32} + a_{23} C_{33} = 0$ , Where  $C_{ij}$  the co factor of  $a_{ij}$

10. Using elementary transformations show that the inverse of the matrix.

$\begin{bmatrix} a & -b & 0 \\ b & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$  is  $\begin{bmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$  if  $a^2 + b^2 = 1$

## 4 Marks

1. Find the inverse of the following matrices using

(a) elementary row transformation. (b) elementary column transformation.

$$\text{i) } \begin{bmatrix} 2 & 1 & 3 \\ 5 & 3 & 1 \\ 3 & 2 & 3 \end{bmatrix} \quad \text{ii) } \begin{bmatrix} 2 & 0 & 1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\text{iii) } \begin{bmatrix} 3 & 1 & 2 \\ 5 & 2 & 4 \\ -2 & 3 & 9 \end{bmatrix} \quad \text{iv) } \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$\text{v) } \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ 2 & -1 & -4 \end{bmatrix} \quad \text{vii) } \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

2 i) If the matrix  $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$  and

$$B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix} \text{ then compute } (AB)^{-1}.$$

ii) For the matrix  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ ,

verify that  $A^3 - 6A^2 + 9A - 4I = 0$ . Hence find  $A^{-1}$ .

iii) Find the matrix A such that

$$A \begin{bmatrix} 2 & -1 & 0 \\ -5 & 4 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 5 & 6 \\ 7 & -5 & 4 \\ -7 & 7 & 4 \end{bmatrix}$$

3. Express the following equations in matrix form and solve them by (a) Reduction method (b) Inversion method

i)  $2x - y + z = 1$ ,  $x + 2y + 3z = 8$ ,  $3x + y - 4z = 1$

ii)  $x + y + z = 3$ ,  $3x - 2y + 3z = 4$ ,  $5x + 5y + z = 11$

iii)  $x + y + z = 3$ ,  $2x - y + z = 2$ ,  $x - 2y + 3z = 2$

iv)  $2x + 3y + 3z = 5$ ,  $x - 2y + z = -4$ ,  $3x - y - 2z = 3$

v)  $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$ ,  $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

4. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs. 60. The cost of 2 kg onion, 4kg wheat and 6 kg rice is Rs. 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs. 70. Find cost of each item per kg by matrix method.

5. If  $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & 2 \\ 3 & -4 & -1 \end{bmatrix}$ , find  $A^{-1}$ .

Using  $A^{-1}$ , solve the following system of linear equations.

$$2x - y + 3z = 13, \quad x + 3y + 2z = 1, \quad 3x - 4y - z = 8$$

6. A salesman has the following record of sales during the past three months for three items A, B and C which have the different rates of commission.

Months	Sales of Units			Total commission (in Rs.)
	A	B	C	
Janaury	90	100	20	800
February	130	50	40	900
March	60	100	30	850

Find out the rates of commission on items A, B and C

7. If  $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$  show that  $(f(x))^{-1} = f(-x)$ .

8. Ramesh buys half a dozen pencils, 2 erasers and 2 sharpeners from a shop and pays Rs. 14 from the same shop, Suresh buys 15 pencils, 5 erasers and 3 sharpeners and pays Rs. 35, whereas their friend Mahesh, who accompanied them to the shop, buys as a token 1 pencil, 1 eraser and 1 sharpener for the payment of Rs. 3. Find the price of each item at the shop, by using matrices.
9. Three cricket fans, nick named as Soni, Moni and Dhoni, went to play for a country match. Their individual scores being  $x$ ,  $y$  and  $z$  respectively. Find  $x$ ,  $y$ ,  $z$  using inversion method from the following data :
- the sum of their scores is a century.
  - if we subtract the sum of Soni and Moni's score from twice of Dhoni's score it is still a half century.
  - four times Moni's score minus Soni's score equal to Dhoni's score.
10. Solve the following equations by using Reduction method.
- $$\log_e x + e^y + z^2 = 3$$
- $$\log_e x + 2 e^y + 3 z^2 = 6$$
- $$2 \log_e x + 3 e^y + 4 z^2 = 1$$

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## Chapter 3 : Trigonometric Functions

(2 - Marks)

1. Find the general solution of  $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$ .
2. Find the number of ordered pairs  $(x, y)$  satisfying  $y = 2 \sin x$  and  $y = 5x^2 + 2x + 3$
3. Find the number of solutions of the equation  $1 + \sin x \cdot \sin^2 \left( \frac{x}{2} \right) = 0$  in  $[-\pi, \pi]$
4. If  $2 \cos^2 x + 3 \sin x - 3 = 0$ ,  $0^\circ \leq x \leq 180^\circ$  then find the value of  $x$ .
5. If  $\tan 2x = \tan \left( \frac{x}{2} \right)$ , then find the value  $x$ .
6. Find the general solution of  $4 \sin^2 x - 3 = 0$
7. Find the principal solution of  $\sqrt{3} \sec x + 2 = 0$
8. Find the general solution of  $\cos x = -\frac{1}{2}$
9. Find the polar co-ordinates of point whose cartesian co-ordinates are  $(1, \sqrt{3})$
10. Find cartesian co-ordinates of the point whose polar co-ordinates are  $(2, \pi/4)$
11. Show that  $a \cos B - b \cos A = a^2 - b^2$ .

3 Marks

1. Two adjacent sides of a cyclic quadrilateral are 2 and 5. The angle between them is  $60^\circ$ . If the third side is 3, then find the fourth side.
2. Find the number of values of  $x$  in  $[0, 2\pi]$  satisfying the equation  $3 \cos x - 10 \cos^2 x + 7 = 0$
3. If in  $\Delta ABC$ ,  $b = \sqrt{3}$ ,  $c = 1$  and  $B - C = 90^\circ$ , then find the angle  $A$ .
4. Find the value of  $x$  if  $\sin^{-1}(1/3) + \sin^{-1}\left(\frac{2}{3}\right) = \sin^{-1}x$ .
5. Find 'x' if  $\sec^2 2x = 1 - \tan 2x$ .
6. Solve the equation  $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$
7. The general solution of  $\tan\left(\frac{2x}{3}\right) = \sqrt{3}$



8. Find the principal value of  $\operatorname{cosec}(x) = 2$
9. In right angled triangle  $\Delta ABC$ , right angled at C. Show that  $\tan A + \tan B = \frac{c^2}{ab}$
- 10) In  $\Delta ABC$   
 $\sin(A/2) \sin(C/2) = \sin(B/2)$  and '2s' is the perimeter of the triangle then find 's'.

#### 4 Marks

1. In  $\Delta ABC$  prove that  $\frac{b-c}{a} = \frac{\tan(B/2) - \tan(C/2)}{\tan(B/2) + \tan(C/2)}$
2. In  $\Delta ABC$ ,  $a \cos^2(C/2) + c \cos^2(A/2) = \frac{3b}{2}$  then prove that a, b and c are in A.P.
3. In  $\Delta ABC$ , if  $a^2, b^2, c^2$  are in A.P. then prove that  $\cot A, \cot B, \cot C$  are in A.P.
4. Find the value of the expression  $\tan \left\{ \frac{1}{2} \left[ \sin^{-1} \left( \frac{2x}{1+x^2} \right) + \cos^{-1} \left( \frac{1-y^2}{1+y^2} \right) \right] \right\}$

Where  $x > 0, y > 0$  Such that  $xy < 1$

5. In  $\Delta ABC$  if  $\angle C = \frac{\pi}{2}$  then prove that  $\sin(A-B) = \frac{a^2 - b^2}{a^2 + b^2}$
6. Show that  $\tan^{-1} \left( \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$ ,  
 for  $\frac{-1}{\sqrt{2}} < x < 1$ .
7. Prove the following  $\sin^{-1} \left( \frac{3}{5} \right) + \cos^{-1} \left( \frac{12}{13} \right) = \sin^{-1} \left( \frac{56}{65} \right)$
8. If  $\tan^{-1} \left( \frac{x-1}{x-2} \right) + \cot^{-1} \left( \frac{x+2}{x+1} \right) = \frac{\pi}{4}$ , then find x.
9. Prove that  $\sin^{-1} \left( \frac{-1}{2} \right) + \cos^{-1} \left( \frac{-\sqrt{3}}{2} \right) = \cos^{-1} \left( \frac{-1}{2} \right)$
10. Show that  $\sin^{-1} \left( \frac{8}{17} \right) + \sin^{-1} \left( \frac{3}{5} \right) = \sin^{-1} \left( \frac{77}{85} \right)$

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## Chapter 4 : Pairs of Straight lines

### (2 - Marks)

1. Find the condition that the lines joining origin to the points of intersection of the line  $y = mx + c$  and the curve  $x^2 + y^2 = a^2$  will mutually perpendicular.
2. Find the distance between pair of parallel lines given by  $x^2 + 2xy + y^2 - 8ax - 9a^2 = 0$
3. The lines represented by  $x^2 + \lambda xy + 2y^2 = 0$  and the lines represented by  $(1 + \lambda) x^2 - 8xy + y^2 = 0$  are equally inclined, then find  $\lambda$ .
4. Show that the equations  $(y - mx)^2 = a^2 (1 + m^2)$  and  $(y - nx)^2 = a^2 (1 + n^2)$  form a rhombus.
5. For what value of 'k' the sum of the slopes of the lines given by  $3x^2 + kxy - y^2 = 0$ , is zero.
6. Show that the equation  $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$  represents a pair of lines.
7. Find the equations of angle bisectors between the lines  $3x + 4y - 7 = 0$  and  $12x + 5y + 17 = 0$
8. If the angle between the pair of straight lines represented by the equation  $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$  is  $\tan^{-1} (1/3)$  where ' $\lambda$ ' is non negative real number, then find ' $\lambda$ '.
9. The orthocentre of the triangle formed by the lines  $x + y = 1$ ,  $2x + 3y = 6$  and  $4x - y + 4 = 0$  lie in which quadrant ?
10. The slopes of the lines represented by  $x^2 + 2hxy + 2y^2 = 0$  are in the ratio 1 : 2 then find 'h'.

### 3 Marks

1. Find the joint equation of pair of lines through the origin which are perpendicular to the lines represented by  $5x^2 + 2xy - 3y^2 = 0$
2. Find the joint equation of the pair of lines which bisect angles between the lines given by  $x^2 + 3xy + 2y^2 = 0$
3.  $\Delta OAB$  is formed by the lines  $x^2 - 4xy + y^2 = 0$  and the line AB. The equation of the line AB is  $2x + 3y - 1 = 0$ . Find the equation of the median of the triangle drawn from the origin.

4. Show that the lines  $x^2 - 4xy + y^2 = 0$  and  $x + y = \sqrt{6}$  form an equilateral. Also find its area.
5. If the lines represented by the equation  $2x^2 - 3xy + y^2 = 0$  makes angle  $\alpha$  and  $\beta$  with X - axis, find the value of  $\cot^2\alpha + \cot^2\beta$ .
6. Two lines are given by  $(x - 2y)^2 + k(x - 2y) = 0$  then find the value of k, so that the distance between them is 3.
7. Find the difference between slopes of the lines represented by equation  $x^2 (\sec^2 \theta - \sin^2 \theta) - 2xy \tan \theta + y^2 \sin^2 \theta = 0$
8. Find the condition of slope of one of the lines represented by  $ax^2 + 2hxy + by^2 = 0$  is the square of the other.
9. Find the number of lines that are parallel to  $2x + 6y + 7 = 0$  and have intercept of length 10 between the co-ordinate axes.
10. If the lines  $px^2 - qxy - y^2 = 0$  makes the angles  $\alpha$  and  $\beta$  with X - axis then find the value of  $\tan (\alpha + \beta)$

#### 4 Marks

1. Find the condition that the pair of lines  $ax^2 + 2(a + b)xy + by^2 = 0$  lie among diameters of a circle and divide the circle into four sectors such that the area of one of the sector is thrice the area of the another sector.
2. Prove that the product the lengths of perpendicular form  $P(x_1, y_1)$  to the line representd by  $ax^2 + 2hxy + by^2 = 0$  is
 
$$\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a - b)^2 + 4h^2}}$$
3. Find the equation of the bisectors of the angles between the lines.
 
$$(\sqrt{a} + \sqrt{c})x^2 + 2\sqrt{d}xy + (\sqrt{b} + \sqrt{c})y^2 = 0$$
4. Find the measure of the acute angle between the lines.
 
$$(a^2 - 3b^2)x^2 + 8abxy + (b^2 - 3a^2)y^2 = 0$$
5. Find the condition that the equation  $ax^2 + by^2 + cx + cy = 0$  may represents a pair of lines.
6. Show that the equation  $(x - 3)^2 + (x - 3)(y - 4) - 2(y - 4)^2 = 0$  represents a pair of lines also find the acute angle between them.

7. Find the joint equation of pair of lines passing through the origin and making an angle of  $30^\circ$  with the lines  $x + y = 5$ .
8. Find the combined equation of the lines, through the origin forming an equilateral triangle with the line  $x + y = \sqrt{3}$
9. Find the condition that the equation  $hxy - gx - fy + c = 0$  represents a pair of lines.
10. Find 'k' if sum of the slopes of the lines represented by  $x^2 + kxy - 3y^2 = 0$  is twice their product.

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## Chapter 5 : Vectors

(2 - Marks)

1. Find vector  $\vec{c}$  if  $|\vec{c}| = 3\sqrt{6}$  and  $\vec{c}$  is directed along the angle bisectors of the vectors  $\vec{OA} = 7\hat{i} - 4\hat{j} - 4\hat{k}$  and  $\vec{OB} = -2\hat{i} - \hat{j} + 2\hat{k}$ .
2. If  $\vec{a}$  and  $\vec{b}$  are non collinear vectors then find the value of  $x$  for which vectors  $\vec{\alpha} = (x - 2)\vec{a} + \vec{b}$  and  $\vec{\beta} = (3 + 2x)\vec{a} - 2\vec{b}$  are collinear.
3. Let  $\vec{a}, \vec{b}, \vec{c}$  are three non-zero vectors such that any two of them are non-collinear. If  $\vec{a} + 2\vec{b}$  is collinear with  $\vec{c}$  and  $\vec{b} + 3\vec{c}$  is collinear with  $\vec{a}$  then prove that  $\vec{a} + 2\vec{b} + 6\vec{c} = \vec{0}$
4.  $\vec{DA} = \vec{a}$ ;  $\vec{AB} = \vec{b}$ ;  $\vec{CB} = k\vec{a}$ ;  $k > 0$  and X, Y are mid points of DB and AC respectively such that  $|\vec{a}| = 17$  and  $|\vec{XY}| = 4$ . Find value of  $k$ .
5. If  $\vec{a} + \lambda\vec{b} + 3\vec{c}$ ;  $-2\vec{a} + 3\vec{b} - 4\vec{c}$ ;  $\vec{a} - 3\vec{b} + 5\vec{c}$  are coplanar then find the value of  $\lambda$ .
6. Find the value of 
$$\frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{\vec{b} \cdot (\vec{c} \times \vec{a})} + \frac{\vec{b} \cdot (\vec{c} \times \vec{a})}{\vec{c} \cdot (\vec{a} \times \vec{b})} + \frac{\vec{c} \cdot (\vec{a} \times \vec{b})}{\vec{a} \cdot (\vec{b} \times \vec{c})}$$
7. In  $\Delta ABC$ , M is mid-point of side BC. If  $\angle BAM = \theta$  then using vector method prove that 
$$\cos \theta = \frac{\sin C + \sin B \cos A}{\sqrt{\sin^2 B + \sin^2 C + 2\sin B \sin C \cos A}}$$
8.  $\vec{a}, \vec{b}, \vec{c}$  represent three concurrent edges of a rectangular parallelepiped whose lengths are 4, 3, 2 units respectively then find value of  $(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$
9. If D, E, F are three points on the sides BC, CA, AB respectively of a  $\Delta ABC$ , such that AD, BE, CF are concurrent, then using vector method prove that 
$$\left| \frac{BD}{CD} \times \frac{CE}{AE} \times \frac{AF}{BF} \right| = 1$$
10.  $\vec{a}, \vec{b}$  are perpendicular vectors, find projection of the vector 
$$l \frac{\vec{a}}{|\vec{a}|} + m \frac{\vec{b}}{|\vec{b}|} + n \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$
 along the angle bisector of the vectors  $\vec{a}$  and  $\vec{b}$

**(3 - Marks)**

1. If  $\vec{a}, \vec{b}, \vec{c}$  are non co-planer non-zero vectors in the plane and  $\vec{r}$  is any vector in the space then show that  $[\vec{b} \vec{c} \vec{r}] \vec{a} + [\vec{c} \vec{a} \vec{r}] \vec{b} + [\vec{a} \vec{b} \vec{r}] \vec{c} = [\vec{a} \vec{b} \vec{c}] \vec{r}$
2. A parallelogram is constructed on the vector  $\vec{a} = \sqrt{3} \vec{\alpha} - \vec{\beta}$ ,  $\vec{b} = \vec{\alpha} + \sqrt{3} \vec{\beta}$  and  $|\vec{\alpha}| = |\vec{\beta}| = 2$  and angle between  $\vec{\alpha}$  and  $\vec{\beta}$  is  $\frac{\pi}{3}$  then find lengths of the diagonals.
3.  $\vec{a}, \vec{b}, \vec{c}$  are three vectors such that  $|\vec{a}| = |\vec{b}| = |\vec{c}| = 4$  and angles between  $\vec{a}$  and  $\vec{b}$ ;  $\vec{b}$  and  $\vec{c}$ ;  $\vec{c}$  and  $\vec{a}$  are equal to  $\frac{\pi}{3}$ . Find volume of parallelopiped whose adjacent sides  $\vec{a}, \vec{b}, \vec{c}$ .
4.  $\vec{a}, \vec{b}, \vec{c}$  are three vectors and vectors  $\vec{a}', \vec{b}', \vec{c}'$  are three vectors such that  $\vec{a} \cdot \vec{a}' = \vec{b} \cdot \vec{b}' = \vec{c} \cdot \vec{c}' = 1$  and  $\vec{a} \cdot \vec{b}' = \vec{a} \cdot \vec{c}' = \vec{b} \cdot \vec{c}' = \vec{c} \cdot \vec{a}' = \vec{c} \cdot \vec{b}' = 0$   
  
Then prove that  $[\vec{a}' \vec{b}' \vec{c}'] = \frac{1}{[\vec{a} \vec{b} \vec{c}]^3} [\vec{b} \times \vec{c} \quad \vec{c} \times \vec{a} \quad \vec{a} \times \vec{b}]$
5. If  $\vec{OA} = \vec{a}$  and  $\vec{OB} = \vec{b}$  then show that the vector along the angle bisector of  $\angle AOB$  is given by  $\vec{d} = \lambda \left( \frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} \right)$
6. A transversal cuts the sides OL, OM and diagonal ON of the parallelogram at A, B, C respectively. Prove that  $\frac{OL}{OA} + \frac{OM}{OB} = \frac{ON}{OC}$  using vector method.
7. Find all values of  $\lambda$  for which  $(\hat{i} + \hat{j} + 3\hat{k})x + (3\hat{i} - 3\hat{j} + \hat{k})y + (-4\hat{i} + 5\hat{j})z = \lambda(x\hat{i} + y\hat{j} + z\hat{k})$  where  $x, y, z$  are not all equal to 0.
8. A straight line intersects sides AB, AC and AD in point B<sub>1</sub>, C<sub>1</sub>, D<sub>1</sub>.  
If  $\vec{AB}_1 = \lambda_1 \vec{AB}$ ;  $\vec{AD}_1 = \lambda_2 \vec{AD}$ ;  $\vec{AC}_1 = \lambda_3 \vec{AC}$  then prove that  $\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$
9. If  $\cos \alpha \neq 1$ ;  $\cos \beta \neq 1$ ;  $\cos \gamma \neq 1$  Prove that vectors  $\vec{a} = \cos \alpha \hat{i} + \hat{j} + \hat{k}$ ;  $\vec{b} = \hat{i} + \cos \beta \hat{j} + \hat{k}$ ;  $\vec{c} = \hat{i} + \hat{j} + \cos \gamma \hat{k}$  can never coplanar.

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**(4 - Marks)**

1. If  $\vec{a}, \vec{b}, \vec{c}, \vec{u}, \vec{v}, \vec{w}$  are vectors prove that  $[\vec{a} \vec{b} \vec{c}] [\vec{u} \vec{v} \vec{w}] = \begin{vmatrix} \vec{a} \cdot \vec{u} & \vec{b} \cdot \vec{u} & \vec{c} \cdot \vec{u} \\ \vec{a} \cdot \vec{v} & \vec{b} \cdot \vec{v} & \vec{c} \cdot \vec{v} \\ \vec{a} \cdot \vec{w} & \vec{b} \cdot \vec{w} & \vec{c} \cdot \vec{w} \end{vmatrix}$
2. If  $\vec{a}, \vec{b}, \vec{c}$  are vectors prove that  $[\vec{a} \vec{b} \vec{c}]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{b} \cdot \vec{a} & \vec{c} \cdot \vec{a} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} & \vec{c} \cdot \vec{b} \\ \vec{a} \cdot \vec{c} & \vec{b} \cdot \vec{c} & \vec{c} \cdot \vec{c} \end{vmatrix}$
3. If  $\vec{a}, \vec{b}, \vec{c}, \vec{l}, \vec{m}$  are vectors prove that  $[\vec{a} \vec{b} \vec{c}] (\vec{l} \times \vec{m}) = \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a} \cdot \vec{l} & \vec{b} \cdot \vec{l} & \vec{c} \cdot \vec{l} \\ \vec{a} \cdot \vec{m} & \vec{b} \cdot \vec{m} & \vec{c} \cdot \vec{m} \end{vmatrix}$
4. Prove that  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$
5. If  $\vec{a}, \vec{b}, \vec{c}$  are non-coplanar unit vectors each including the angle of measure  $30^\circ$  with the other then find the volume of tetrahedron whose co-terminal edges are  $\vec{a}, \vec{b}, \vec{c}$ .
6. In  $\Delta OAB$ , E is the mid-point of OB and D is a point on AB such that  $AD : DB = 2 : 1$ . If OD and AE intersect at P, determine ratio OP : PD using vector method.
7.  $\vec{a}$  and  $\vec{b}$  are two non-collinear vectors, show that the points having positions vectors.  
 $l_1 \vec{a} + l_2 \vec{b}; m_1 \vec{a} + m_2 \vec{b}; n_1 \vec{a} + n_2 \vec{b}$  are co-linear if  
 $(l_1 m_2 - m_1 l_2) + (m_1 n_2 - n_1 m_2) + (n_1 l_2 - l_1 n_2) = 0$
8. Let the perpendicular lines  $B'B$  and  $C'C$  intersect at A and position vector of A w.r.t. O be  $\vec{a}$ .  $\vec{AB}$  and  $\vec{AC}$  are parallel to  $\vec{b}$  and  $\vec{c}$  respectively. If P is any point on the bisector of  $\angle CAB$  then prove that position vector of P is given by  $\vec{a} + \lambda \left( \frac{\vec{b}}{|\vec{b}|} \pm \frac{\vec{c}}{|\vec{c}|} \right)$
9.  $\vec{a}, \vec{b}, \vec{c}$  are position vectors of points A, B, C and P, Q, R are points BC, CA, AB respectively such that  $BP : PC = CQ : QA = AR : RB = 1 : 2$   
Find position vector of vertices of  $\Delta XYZ$  formed by lines AP, BQ and CR.

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## Chapter 6 : Co-ordinate Geometry

(2 - Marks)

1. Find the values of  $\lambda$  for which the triangle with vertices A (6, 10, 10) ; B (1, 0, -5) ; C (6, -10,  $\lambda$ ) is a right angled triangle at B.
2. Find direction ratios of the line which bisects the angle between the lines whose direction cosines are  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$ .
3. The equation of motion of a particle in space is given by  $x = 2t$  ;  $y = -4t$  ;  $z = 4t$  , where  $t$  is measured in second and co-ordinates of particle in kilometre. Then find the distance covered from the starting point by the particle in 15 seconds.
4. The equation of motion of a particle in space is given by  $x = 2t$  ;  $y = -4t$  ;  $z = 4t$  , where  $t$  is measured in second and co-ordinates of particle in kilometre. Then find the speed of particle in km / sec.
5. If distance of the point P (4, 3, 5) from the Y- axis is  $\lambda$ , then find the value of  $7\lambda^2$ .
6. A (3, 2, 0) ; B (5, 3, 2) ; C (-9, 6, -3) are the vertices of  $\triangle ABC$ . If the bisector of  $\angle BAC$  meets BC at D then find the ratio in which C divides BD.
7. Planes are drawn parallel to the co-ordinate planes through the point (1, 2, 3) and (3, -4, -5). Find the lengths of edges of the parallelepiped so formed.
8. Find the ratio in which the plane  $ax + by + cz + d = 0$  divides the join the points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  .
9. Find  $\vec{r}$ , if direction ratio of vector  $\vec{r}$  are 2, -3 6 and  $|\vec{r}| = 21$  and  $r$  makes obtuse angle with the x -axis.
10. Let PM be the perpendicular drawn from the point P ( $x, y, z$ ) on XY plane and OP makes an angle  $\theta$  with the positive direction of Z - axis, OM makes an anlge  $\theta$  with positive direction of X - axis then Prove that  
$$x = r \sin\theta \cos\phi ; y = r \sin\theta \sin\phi ; z = r \cos \theta$$



**(3 - Marks)**

1. The points  $(0, 1, -2)$  ;  $(3, \lambda, -1)$  ;  $(\mu, -3, -4)$  are collinear show that the point  $(12, 9, 2)$  lies on the same line.
2. If  $\theta$  is the angle between the lines having direction cosines  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$ .  
Then prove that  $\sin \theta = \sqrt{(l_1 m_2 - m_1 l_2)^2 + (m_1 n_2 - n_1 m_2)^2 + (n_1 l_2 - l_1 n_2)^2}$
3.  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$  are direction cosines of perpendicular lines. Find the direction ratios of the line perpendicular to both these lines.
4. If the direction cosines of the line in two adjacent positions are  $l, m, n$  and  $l + \delta l, m + \delta m, n + \delta n$  then show that the small angle  $\delta\theta$  between two positions is given by  $\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$
5. Find the angle included between the lines whose direction cosines are given by the equations  $3l + m + 5n = 0$  and  $6mn - 2nl + 5lm = 0$
6. If direction cosines of the line satisfy the relation  $\lambda(l + m) = n$  and  $mn + nl + lm = 0$  then find the value of  $\lambda$  for which the two lines are perpendicular.
7. If  $l_1, m_1, n_1$  ;  $l_2, m_2, n_2$ ; and  $l_3, m_3, n_3$  are direction cosines of mutually perpendicular vectors  $\overline{OA}, \overline{OB}, \overline{OC}$ . respectively then prove that the line having direction cosines proportional to  $l_1 + l_2 + l_3, m_1 + m_2 + m_3$  and  $n_1 + n_2 + n_3$  make equal angles with  $\overline{OA}, \overline{OB}, \overline{OC}$ .
8. Let PM be the perpendicular drawn from the point P  $(1, 2, 3)$  on XY plane. OP makes an angle  $\theta$  with the positive direction of z - axis. OM makes an angle  $\phi$  with positive direction of X - axis find  $\theta$  and  $\phi$ .
9. A  $(2, 3, 5)$  ; B  $(-1, 3, 2)$  ; C  $(\lambda, 5, \mu)$  are vertices of  $\Delta ABC$  and median through vertex A is equally inclined to the axes then find area of  $\Delta ABC$ .

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## Chapter 7 : Line

(2 - Marks)

1. Find the vector equation of line passing through the point A (5, 3, 8) and parallel to vector  $3\hat{i} + 4\hat{j} + 5\hat{k}$ .
2. A line passes through the point with position vector  $3\hat{i} - 4\hat{j} + \hat{k}$  and is in the direction of  $2\hat{i} + \hat{j} - 2\hat{k}$ , find the equation of the line in vector and cartesian form.
3. Find the equation of line in symmetric form passes through the point (8, 3, 7) and (-2, 5, -3)
4. Find the equation of line in cartesian form passing through the point (2, 1, -2) and perpendicular to the vector  $2\hat{i} - 3\hat{j} + 4\hat{k}$  and  $2\hat{i} - 2\hat{j} + 3\hat{k}$ .
5. Find the vector equation of line perpendicular to the line  $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $x = 5$ ,  $\frac{y-3}{2} = \frac{z-2}{3}$ , and passing through (3, -1, 11)
6. Write symmetric form of the equation of the line  $3x - 1 = 4y + 8 = 3z - 3$
7. Find the angle between the pair of line  
 $\vec{r} = (4\hat{i} + 7\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$  and  
 $\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$
8. Find the distane between the parallel lines  
 $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$  and  
 $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$
9. Find the direction cosines of the line  
 $\frac{2x-1}{3} = 3y = \frac{4z+3}{2}$
10. Find the vecter equation of a line passing through the point with position vector  $2\hat{i} - \hat{j} + \hat{k}$  and parallel to the line joining the points  $-\hat{i} + 4\hat{j} + \hat{k}$  and  $\hat{i} + 2\hat{j} + 2\hat{k}$ .

**(3 - Marks)**

1. Show that the lines  $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda (2\hat{i} + \hat{j} + \hat{k})$  and  $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \mu (\hat{i} + 3\hat{j} + 2\hat{k})$  intersect find their point of intersection.
2. Find the value of k, if the points A (1, 2, -1), B (4, -2, 4) and C (0, 0, k) form a triangle right angled, at C.
3. Find the foot of the perpendicular drawn from the point A (1, 0, 3) to the line joining the points B (4, 7, 1) and C (3, 5, 3)
4. Find the vector equation of the line passing through the point (2, 3, -4) and perpendicular to XZ - Plane, Hence find the equation in cartesian form.
5. Find the distance of P (1, 2, -2) from the line  $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z}{-1}$
6. A line makes the same angle  $\theta$  with each of X and Z - axis. If the angle  $\beta$  which it makes with Y - axis is such that  $\sin^2\beta = 3\sin^2\theta$ , then find the value of  $\cos^2\theta$ .
7. If the line  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$  and  $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{-1}$  intersect then find the value of k.
8. Find the shortest distance between the line  $1+x=2y=-12z$  and  $x=y+2=6z-6$
9. Find the distance of P (2, -1, 3) from the line  $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-2}{2}$
10. Find the two points on the line  $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z-5}{2}$  on either side (2, -3, -5) which are at a distance of 3 units from it.

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## Chapter 8 : Plane

(2 - Marks)

1. Find the vector equation of the plane passing through the point A (4, -2, 3) and parallel to the plane  $x + 2y - 5z + 8 = 0$
2. Find the vector equation of plane which passes through the point A (1, -1, 1) and perpendicular to the vector  $4\hat{i} + 2\hat{j} + 2\hat{k}$ .
3. Find the acute angle between the planes  $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = -5$  and  $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 8$
4. Find the value of p, if the planes  $x - y + pz + 7 = 0$  and  $3x + y - z = 4$  are perpendicular to each other.
5. Find the vector equation of the plane passing through the points (5, 2, -1) (2, 2, 3) and origin.
6. Find the equation of the plane through the point (2, -3, 1) and perpendicular to the line whose d.r's are 3, -1, 2.
7. Find the angle between the line  $\frac{x-1}{5} = \frac{y}{2} = \frac{z}{14}$  and plane  $12x + 4y - 3z = 25$
8. Find the equation of a plane whose distance from the origin is 5 units and normal in the direction of  $\hat{n} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$
9. If the lines  $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{k}$  and  $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$  are co-planar then find the value of K.
10. A plane makes intercept 1, 2, 3 on the co-ordinate axes. If the distance from origin is p then find the value of p.

### 3 - Marks

1. Find the equation of a plane which bisects the line joining the point A (2, 3, 4) and B (4, 5, 8) at right angles.
2. Find the equation of the plane through the intersection of the planes  $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$  and  $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$  whose perpendicular distance from origin is unity.
3. Find the equation of plane containing the line  $2x - 5y + z = 3$  ;  $x + y + 4z = 5$  and parallel to the plane  $x + 3y + 6z = 1$
4. A variable plane  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  at a unit distance from the origin cuts the co-ordinate axes at A, B and C. Centroid (x, y, z) of  $\Delta ABC$  satisfies the equation  $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$  then find the value of k
5. If the angle between the line  $\frac{x}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$  and the plane  $x + 2y + 3z = 4$  is  $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$  then find the value of  $\lambda$
6. Find the distance of the plane passes through (1, -2, 1) and perpendicular to planes  $2x - 2y + z = 0$  and  $x - y + 2z = 4$  from the point (1, 2, 2)
7. Find the vector and cartesian equation of the plane passing through the points (2, 3, 1), (4, -5, 3) and parallel to the X - axis.
8. Find the equation of the plane passing through the point (3, -2, -1) and parallel to the lines whose direction ratios are 1, -2, 4 and 3, 2, -5.
9. Find the angle between the planes  $x - 2y + 2z = 7$  and  $x - y - 3z = 5$
10. Find the equation of plane in vector form and cartesian form if the plane is at a distance of 3 units from the origin and has  $\hat{i} + \hat{j} - 3\hat{k}$  as a normal vector.

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## Chapter 9 : Linear Programming

(2 - Marks)

### ‘SECTION B’

- Find the point at which the maximum value  $(3x + 2y)$  takes place when subject to constraints  $x + y \leq 2$ ,  $x \geq 0$ ,  $y \geq 0$ .
- Find the feasible region graphically of the inequalities  $5x + 10y \geq 100$ ,  $x + y \leq 60$ ,  $x \geq 0$ ,  $y \geq 0$ .
- Find the maximum value of  $z = 6x + 10y$  subject to  $x \leq 6$ ,  $y \leq 2$ ,  $x \geq 0$ ,  $y \geq 0$ .
- Find the corner points of the feasible region determined by the linear inequalities  $2x + y \leq 10$ ,  $x + 3y \leq 15$ ,  $x \geq 0$ ,  $y \geq 0$ .
- Draw the graph of the inequalities  $5x + y \geq 10$ ,  $x + y \geq 6$ ,  $x + 4y \geq 12$ ,  $x \geq 0$ ,  $y \geq 0$  on a graph paper.
- A firm is engaged in producing two models  $X_1$ ,  $X_2$  performing only three operations assembling, painting and testing. The relevant data are as follows.

Model	Unit Sale Price	Hours required for each unit		
		Assembling	Painting	Testing
$X_1$	Rs. 50	1.0	0.2	0.0
$X_2$	Rs. 80	1.5	0.2	0.1

Total number of hours available each week for assembling 600 hours, painting 100 hours and testing 30 hours. The firm wishes to determine its weekly product - mix. So as to maximize revenue. Formulate L.P.P. model for maximize the revenue.

- Find the coordinates of the point for minimum value of  $z = 7x - 8y$ , subject to  $x + y \leq 20$ ,  $y \geq 5$ ,  $x \geq 0$ ,  $y \geq 0$ .
- Find the area of the feasible region for the constraints  $x + 3y \leq 3$ ,  $x \geq 0$ ,  $y \geq 0$ .
- Find the solution set of inequation  $x - 2y \geq 0$ ,  $2x - y + 2 \leq 0$ ,  $x \geq 0$ ,  $y \geq 0$ .
- Draw the geometrical shape of the common region represented by the inequalities  $0 \leq x \leq 6$ ,  $0 \leq y \leq 4$ .



### 3 - Marks

#### 'SECTION 'C'

1. Using graphical method solve Minimize  $z = 7x - 8y$  subject to the constraints  $x + y \leq 20$ ,  $y \geq 5$ ,  $x \geq 0$ ,  $y \geq 0$ .
2. Using graphical method solve Maximize  $z = 7x + 10y$  subject to the constraints  $x + y \leq 30$ ,  $y \leq 12$ ,  $x \geq 6$ ,  $x \geq 0$ ,  $y \geq 0$ .
3. Using graphical method find the maximum value of  $z = 10x + 25y$  subject to the  $0 \leq x \leq 3$ ,  $0 \leq y \leq 3$ ,  $x + y \leq 5$ .
4. Using graphical method find the minimum value of  $z = 7x + y$  subject to  $5x + y \geq 5$ ,  $x + y \geq 3$ ,  $x \geq 0$ ,  $y \geq 0$ .
5. A factory produces two products  $P_1$  and  $P_2$ . Each of the product  $P_1$  requires 2 hrs. for moulding, 3 hours for grinding and 4 hrs. for polishing. and each of the product  $P_2$  requires 4 hrs for moulding, 2 hrs for grinding and 2 hrs for polishing. The factory has moulding machine available for 20 hrs, grinding machine for 24 hrs and polishing machine for 13 hrs. The profit is Rs, 5 per unit of  $P_1$  and Rs. 3 per unit of  $P_2$  and the factory can sell all that it produces. Formulate the problem as a linear programming problem to maximize the profit.
6. A firm can produce three types of cloths say  $C_1$ ,  $C_2$ ,  $C_3$ . Three kinds of wool are required for it say red wool, green wool and blue wool one unit of length  $C_1$  needs 2 meters of red wool, 3 meters of blue wool, one unit of cloth  $C_2$  needs 3 meters of red wool, 2 meters of green wool and 2 meters of blue wool and one unit of cloth  $C_3$  needs 5 meters of green wool and 4 meters of blue wool. The firm has only a stock of 16 meters of red wool, 20 meters of green wool and 30 meters of blue wool. It is assumed that the income obtained from one unit of length of  $C_1$  is Rs. 6, of cloth  $C_2$  is Rs. 10 and of cloth  $C_3$  is Rs. 8. Formulate the problem as a linear programming problem to maximize the income.
7. A company makes two types of leather belts A and B. A is high quality belt and B is of lower quality belt. The profits are Rs. 40 and Rs. 30 per belt. Each belt of type A requires twice as much time as a belt of type B. And if all belts were of type B, the company could make 1000 belts per day. The supply leather is sufficient for only 800 belts per day (both A and B combined) Belt A requires a fancy buckle and only 400 buckles per day are available. There are only 700 buckles available for belt B. What should be the daily production for each type of belt ? Formulate the problem as a LPP.

8. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contain atleast 8 units of Vitamin A and 10 units of Vitamin C. Food 'P' Contains 2 units per kg. of Vitamin A and 1 unit per kg of Vitamin C while food 'Q' contains 1 unit per kg of Vitamin A and 2 units per kg of Vitamin C. It costs Rs. 50/- per kg to purchase food 'P' and Rs. 70/- per kg to purchase food 'Q'. Formulate the above linear programming problem to minimize the cost of such a mixture.
9. A rubber company is engaged in producing three types of tyres A, B and C. Each type requires processing in two plants. Plant I and Plant II. The Capacities of the two plants in number of tyres per day are as follows.

Plant	A	B	C
I	50	100	100
II	60	60	200

- The monthly demand for tyre A, B and C is 2500, 3000 and 7000 respectively. If plant I costs Rs. 2500/- per day and plant II costs Rs. 3500/- per day to operate. How many days should each be run per month to minimize cost while meeting the demand ? Formulate the problem as LPP.
10. A firm is engaged in breeding goats. The goats are fed on various products grown on the farm. They need certain nutrients named as X, Y and Z. The goats are fed on two products A and B. One unit of product A contains 36 units of X, 3 units of Y and 20 units of Z. while one unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs Rs. 20/- per unit and product B costs Rs. 40/- per unit. Formulate the LPP to minimize the cost.



## 4 - Marks

### 'SECTION 'D''

1. Using graphical method solve maximize  $Z = 12x + 3y$  subject to the constraints  $x + y \leq 5$ ,  $3x + y \leq 9$ ,  $x \geq 0$ ,  $y \geq 0$ .
2. Using graphical method solve minimize  $Z = 2x + 2y$  subject to the constraints  $3x + 2y \geq 12$ ,  $x + 3y \geq 11$ ,  $x \geq 0$ ,  $y \geq 0$ .
3. Solve the following LPP graphically.  
Maximize  $Z = x + 2y$  subject to  $x + 2y \geq 100$ ,  $2x - y \leq 0$ ,  $2x + y \leq 200$ ,  
 $x \geq 0$ ,  $y \geq 0$
4. Solve the following LPP graphically  
Minimize :  $Z = -3x + 4y$  subject to  $x + 2y \leq 8$ ,  $3x + 2y \leq 12$ ,  $x \geq 0$ ,  $y \geq 0$
5. Solve the following LPP graphically method  
Minimize :  $Z = 20x + 10y$  subject to  $x + 2y \leq 40$ ,  $3x + y \geq 30$ ,  $4x + 3y \geq 60$ ,  
 $x \geq 0$ ,  $y \geq 0$
6. A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hours of work on machine A and 3 hours on machine B to produce package of nuts while it takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He urns a profit of Rs. 2.50 per package of nuts and Rs. 1.00 per package of bolts. How many package of each should he produce each day so as to maiximize his profit, If he operates his machine for at most 12 hours a day. Formulate this LPP and then solve it.
7. An oil Company requires 12,00, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high grade, medium grade and low grade oil respectively while refinely B produces 200, 400 and 100 barrels per day of high grade, medium grade and low grade oil respectively. If refinery A costs Rs. 400 per day and refinery B costs 300 per day to operate, how many days should each be run to minimize costs while satisfying requirements.
8. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760/- to invest and had space for atmost 20 items. A fan costs him Rs. 360/- and a sweing machine Rs. 240/-. His expectation is that he can sale a fan at a profit of Rs. 22 and sweing machine at a profit of Rs. 18. Assuming that he can sale all the items that he can buy. How

- should he invest his money in order to maximize his profit? Translate this problem mathematically and then solve it.
9. A farm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents (call them X, Y and Z), it is necessary to buy two additional products say A and B. One unit of product A contains 36 units of X, 3 units of Y and 20 units of Z. One unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs Rs. 20/- per unit and product B costs Rs. 40 per unit formulate linear programming. Problem to minimize the total cost and solve the problem by using graphical method.
10. If a young man rides his motorcycle at 25 km / hour. He had to spend Rs. 2/- per km on petrol. If he rides at a faster speed of 140 km / hour the petrol cost increases at Rs. 5/- per km. He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as an LPP and solve it graphically.

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