Chapter 10 : Continuity

(2 - Marks)

1. Prove that the function $\dfrac{\sin x}{|x|}$ is discontinuous at $x = 0$.

2. Let f (x) be a continuous function and g (x) be a discontiuous function, prove that f (x) + g (x) is discontinuous function.

3. Discuss the continuity of $f(x) = \dfrac{x^5\sqrt{x} - 32\sqrt{2}}{x^3\sqrt{x} - 8\sqrt{2}}$, $x \neq 2$

$$= \frac{44}{7}, \quad \text{at } x = 2$$

4. If $f(x) = \dfrac{\sqrt[n]{x^n} - 1}{\sqrt[m]{x^n} - 1}$ for $x \neq 1$ is continuous at $x = 1$, find f (1)

5. Discuss the continuity of the function

$$\begin{aligned} f(x) &= x^2/a - a, && x < a \\ &= 0, && x = a \\ &= a - x^2/a && x > a \end{aligned}$$

, at $x = a$

6. Find the point (s) in the interval [–1, 2] where the function $f(x) = x$ for $x \neq 0$ and $f(0) = 1$ is discontinuous

7. If $f(x) = \dfrac{\sin[4(x-3)]}{x^2 - 9}$, $x \neq 3$ is continuous at $x = 3$ then find f (3).

8. If the function $f(x) = \dfrac{\cos kx - \cos 4x}{x^2}$, $x \neq 0$

$= 6$, $x = 0$

is continuous at $x = 0$, find k.

9. If the function $f(x) = \dfrac{\log x - 1}{x - e}$, for $x \neq e$

is continuous at $x = e$ find f (e).

10. Show that the function $f(x) = 2x - |x|$ is continuous at $x = 0$.

3 - Marks

1. If $f(x) = \left(1 + |\sin x|\right)^{\frac{a}{|\sin x|}}$, $-\pi/6 < x < 0$

$= b$, $x = 0$

$= e^{\tan 2x / \tan 3x}$, $0 < x < \pi/6$

Find a and b so that the function is continuous at $x = 0$

2. Find the value of f (0) so that the function

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$ is continuous for all x.

3. If the function $f(x) = \dfrac{x + x^2 + x^3 + x^4 + x^5 - 62}{x - 2}$, $x \neq 2$

$= 3k$, $x = 2$

is continuous at $x = 2$, find k.

4. If $f(x) = \dfrac{\sin x - \sin a}{\cos x - \cos a}$, $x \neq a$

$= 1$, $x = a$ is continuous, at $x = a$ find a

5. If the function $f(x) = \dfrac{(a + x)^2 \sin (a + x) - a^2 \sin x}{x}$, $x \neq 0$

is continuous at $x = 0$ find f (0)

6. If $f(x) = \dfrac{x \quad a^x - x}{\sqrt{1 + x^2} - \sqrt{1 - x^2}}$, $x \neq 0$

$= k$, $x = 0$ is continuous at $x = 0$, find k.

7. If the function $f(x) = \dfrac{(5^x - 1)^4}{x \tan (x/5) \log (1 + x^2/5)}$, $x \neq 0$

is continuous at $x = 0$, find f (0).

8. Discuss the continuity of $f(x) = \dfrac{2^{2x-2} - 2^x + 1}{\tan^2(x-1)}$, $x \neq 1$

$= 2 \log 2$, $x = 1$ at $x = 1$

9. If the function $f(x) = \dfrac{x + 1 - \sqrt{x + 13}}{x - 3}$, $x \neq 3$

$= k$ $x = 3$

is continuous at $x = 3$, find k.

10. If $f(x) = \dfrac{(a^x - 1)^3}{\sin(x \log a) \, log\,(1 + x^2\, log\, a^2)}$, $x \neq 0$

is continuous at $x = 0$, find f (0).

4 - Marks

1. Find the value of A so that the function

$f(x) = \dfrac{2^{x+2} - 16}{4^x - 2^4}$, $x \neq 2$

$= A$, $x = 2$ is continuous at $x = 2$

2. If $f(x) = \dfrac{2^{x-} + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$, $x \neq 2$ is continuous

at x = 2, find f (2)

3. If $f(x) = \dfrac{1 - \cos(x^2 - x - 6)}{(x - 3)^2}$, $x \neq 3$ is continuous at $x = 3$ find f (3)

4. If the function

$f(x) = \dfrac{1}{x^8} [1 - \cos(x^2/2) - \cos(x^2/4) + \cos(x^2/2)\cos(x^2/4)]$, $x \neq 0$

$= \dfrac{k^2}{4}$, $x = 0$ is continuous at $x = 0$ find k.

5. Discuss the continuity of the function

$$f(x) = \frac{(1 - \tan(x/2))(1 - \sin x)}{(1 + \tan(x/2))(\pi - 2x)^3}, \quad x \neq \pi/2$$

$$= \frac{1}{16}, \quad x = \pi/2 \qquad \text{at } x = \pi/2$$

6. Let $f(x) = \dfrac{\log(1 + x - x^2) + \log(1 - x + x^2)}{\sec x - \cos x}, \quad x \neq 0$

then find the value of f (0) so that f (x) is continuous at $x = 0$

7. If the function

$$f(x) = \frac{x \sin a - a \sin x}{x - a}, \quad x \neq a$$

is continuous at $x = a$, find f (a)

8. If the function

$$f(x) = x + a^2 \sqrt{2} \sin x, \quad 0 \leq x < \pi/4$$

$$= x \cot x + b, \quad \pi/4 \leq x < \pi/2$$

$$= b \sin 2x - a \cos 2x \quad \pi/2 \leq x \leq \pi$$

is continuous on $[0, \pi]$, find a & b

9. Discuss the continuity of the function.

$$f(x) = \frac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x}, \quad x \neq \pi/4$$

$$= 3/\sqrt{2}, \quad x = \pi/4 \qquad \text{at } x = \pi/4$$

10. Find K it the fuction

$$f(x) = \frac{\log(1 + 2x) - 2\log(1 + x)}{x^2}, \quad x \neq 0$$

$$= kx^2 + 5x + 3k, \quad x = 0 \text{ is continuous at } x = 0$$

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Chapter 11 : Differentation

(2 - Marks)

1. If $y = x \left[\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right) + \sin x\right] + \dfrac{1}{2\sqrt{x}}$ find $\dfrac{dy}{dx}$

2. If $y = \tan^{-1} \dfrac{(ax - b)}{(bx + a)}$ then find ${}^{dy}/_{dx}$.

3. If $y = \sec^{-1}\left(\dfrac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\dfrac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ then find ${}^{dy}/_{dx}$.

4. If $y = \cos^{-1}\left(\sqrt{\dfrac{1+x}{2}}\right)$ find ${}^{dy}/_{dx}$.

5. If $f(x) = |x - 1| + |x - 3|$ then find $f^{1}(2)$

6. Find the deivative of $f(x) = \left[\cos^{-1}\left(\sin\sqrt{\dfrac{1+x}{2}}\right) + x^{x}\right]$ *w.r.t.x* at x=1

7. If $x^{y}y^{x} = 16$ then find $\dfrac{dy}{dx}$ at (2, 2)

8. If $f(x) = \dfrac{x}{1 + |x|}$, for $x \in R$ then find $f'(0)$

9. If $f'(x) = \sqrt{3x^2-1}$ and $y = f(x^2)$ then find $\dfrac{dy}{dx}$

10. If $y = \tan^{-1}\left[\dfrac{\log(e/x^3)}{\log(ex^3)}\right] + \tan^{-1}\left[\dfrac{\log(e^4x^3)}{\log(e/x^{12})}\right]$ show that $\dfrac{d^2y}{dx^2} = 0$

3 - Marks

1. Let f (x) = e^x, g (x) = $\sin^{-1}x$ and h (x) = f (g (x)) then find $\dfrac{h'(x)}{h(x)}$

2. Find the deivative of f (tan x) w.r.t. to g (sec x) at $x = \pi/4$ where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$

3. If $y = \tan^{-1}\left(\dfrac{1}{1+x+x^2}\right) + \tan^{-1}\left(\dfrac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\dfrac{1}{x^2+5x+7}\right)$

   + ... + n terms then find $y'(0)$

4. If f (x) = $\cos x \ \cos 2x \ \cos 4x \ \cos (8x) \,.\, \cos 16x$ then find $f'(\pi/4)$

5. If f be twice differentiable function such that $f^{11}(x) = -f(x)$ and $f'(x) = g(x)$, $h(x) = [f(x)]^2 + [g(x)]^2$ if h (5) = 10 then find h (10).

6. If $y = \sin^2\left[\cot^{-1}\left(\dfrac{1}{\sqrt{\dfrac{1+x}{1-x}}}\right)\right]$ then find $\dfrac{dy}{dx}$

7. If y = x cos y, show that $\dfrac{dy}{dx} = \dfrac{\cos^2 y}{\cos y + y \sin x}$

8. If $y = \dfrac{x^{\sin x}}{1+x+x^2}$ find $\dfrac{dy}{dx}$

9. If $y = e^{f(x)}$ where $f(x) = \sqrt{\dfrac{x-1}{x+1}}$ then show that $\dfrac{dy}{dx} = \dfrac{y \log y}{x^2-1}$

10. If $y = \tan x + {}^2/_3 \tan^3 x + {}^1/_5 \tan^5 x$ then find ${}^{dy}/_{dx}$ in terms of sec x.

4 - Marks

1. If $\log y = \log(\sin x) - x^2$ then show that $\frac{d^2y}{d^2x} + 4x\frac{dy}{dx} + (4x^2 + 3)\,y = 0$

2. If $2y = \sqrt{x + 1} + \sqrt{x - 1}$ show that $4(x^2 - 1)\, {}^{d^2y}/_{dx^2} + 4x\, {}^{dy}/_{dx} - y = 0$

3. If $y = |\cos x| + |\sin x|$ then find $\frac{dy}{dx}$ at $x = 2\pi/3$

4. Find $\frac{dy}{dx}$ if $y = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$ at $x = 1$

5. If $\sqrt{x + y} - \sqrt{y - x} = 8$ then prove that $\frac{d^2y}{d^2x} = {}^{2}/_{c^2}$.

6. If $(a - b\tan y)(a + b\tan x) = a^2 + b^2$ show that ${}^{dy}/_{dx}$ is constant and state its value.

7. If $y = \sin^{-1}[15x - 500x^3] + \cos^{-1}[1372x^3 - 21x]$ find ${}^{dy}/_{dx}$.

8. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots\ldots\infty}}}}$

   show that $\frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$

9. If $x^2 + y^2 = t + t^{-1}$ and $x^4 + y^4 = t^2 + t^{-2}$ then show that $\frac{dy}{dx} = {}^{-y}/_{x}$.

10. If $y = \frac{a + b\tan x}{b - a\tan x}$ show that $\frac{dy}{dx} = 1 + y^2$.

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Chapter 12 : Application of derivative

(2 - Marks)

1. An inverted conical container, R = 4 cm, H = 20 cm is full of water. Due to a small leak at the vertex, the volume of the water in the container reduce at $12 cm^3/sec$. Find the rate at which water level falls when water level is 5 cm.

2. A wire length 2 units is cuts in to two parts which are bent to form a square of side x units and a circle of radius r units. If the sum of the area of the square and the circle so formed is minimum then find the relation between x and r.

3. Find c if $f(x) = \dfrac{x^2 - 4x}{x + 2}$, $0 \le x \le 4$ and Rolle's theorem is applicable.

4. If $f(x) = x^n$, $0 < a < b < c$ and $n > 1$ is an odd number. Then show that $(b - c)a^n + (c - a)b^n + (a - b)c^n$ is negative.

5. Find the approximate value of $\sqrt{37}$.

6. Find the approximate value of $\log_e 99$ if $\log_e 10 = 2.3023$.

7. If $1° = 0.0175$ radians . $\sin 60° = 0.8660$ then find the approximate value of $\cos(60° 40')$.

8. If e = 2.7183. Find the approximate value of $e^{1.005}$.

9. Find the approximate value of $f(x) = x^3 + 5x^2 - 7x + 10$ at $x = 1.1$

10. If $f(x) = x(2 - x)$, $x \in [0, 1]$ verifies LMVT. Then find c.

11. On the interval [0, 1] the function $x^{25}(1 - x)^{75}$ what point takes its maximum value.

12. Find the anlge between the curves $y = a^x$ and $y = b^x$.

13. The distance covered by a particle in t second is give be $x = 3 + 8t - 4t^2$. Find the velocity after 1 second.

3 - Marks

1. Verifty Rolle's theorem for function $f(x) = x^2 - 8x + 12$ on $[2, 6]$
2. Verifty Rolle's theorem for function $f(x) = x(x-4)^2$ on $[0, 4]$
3. Verifty Rolle's theorem for function $f(x) = \dfrac{\sin x}{e^x}$ on $[0, \pi]$
4. Verifty Rolle's theorem for function $f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$
5. Discuss the applicability of Roll's theorem for function $f(x) = |x|$ in $[-1, 1]$
6. Discuss the applicability of Roll's theorem for the function

$$f(x) = \begin{cases} x^2 + 1 & 0 \le x < 1 \\ 3 - x & 1 \le x \le 2 \end{cases}$$

7. Verify LMVT for the function $f(x) = \sin x - \sin 2x - x$ on $[0, \pi]$
8. Verify LMVT for the function $f(x) = (x-3)(x-6)(x-9)$ on $[3, 5]$
9. Verify LMVT for the function $f(x) = \log x$ on $[1, 2]$
10. Using LMVT, find the point on the curve $y = \sqrt{x-2}$ defined on the interval $[2, 3]$ where the tangent is parallel to the chord joining the end points of the curve.
11. Using LMVT, find a point on the curve $y = x^2 + x$, where the tangent is parallel to the chord joining (0,0) and (1, 2).
12. A stone is thrown vertically upward from the top of a tower 64 m high accordingly the law of motion given by $s = 48t - 16t^2$ what is the greatest height attained by the stone above the ground.
13. If the surface area of a sphere of radius r in increasing uniformaly at the rate 8 cm^2/s then show that the rate of change of volume is propertional to radius r.

4 - Marks

1. Each side of an eqilateral triangle is increasing at the rate of $\sqrt{3}$ cm/sec. Find the rate of which its area is increasing when its side 2 meters.

2. A circular blot of ink increases in area in such a way that the radius 'r' cm at a time 't' sec. is given by $r = 2t^2 - t^3/4$ what is the rate of increase of the area when t = 4.

3. Water is being poured at the rate $36m^2$/min in to cylindrical vessel whose base is a circle of radius 3 meters. Find the rate at which the level of water is rising.

4. The height of cone is 30 cm and it is constant, the radius of the base is increasing at the rate of 2.5 cm/sec. Find the rate of increase of volume of the cone when the radius is 10cm.

5. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the point on the curve where the y - coordinate is changing twice as fast as the x - co-ordinate.

6. An edge of a veriable cube is increasing at the rate of 10 cm/sec. How fast is the volume of the cube increasing when the edge is 5 cm long ?

7. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.

8. Find the slope of the tangent and the normal when $x = a(\theta - \sin\theta)$; $y = a(1 - \cos\theta)$ at $= \pi/2$

9. Find a point on the curve $xy = -4$ where the tangents are inclined at an angle 45^0 with x - axis.

10. Show that the tangents to the curve $y = 2x^2 - 3$ at a point $x = 2$ and $x = -2$ are parallel.

11. FInd the point on the curve $y = 6x - x^2$ where the tangents has slope –4. Also find the equation of the tanent at that point.

12. Find the second degree polynomial $f(x)$ satisfying $f(0) = 0$, $f(1) = 1$, $f^1(x) > 0$ for all $x \in (0, 1)$

13. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ attains its maximum and minimum at p and q respectively such that $p^2 = q$ then find the value of a.

14. If $x + y = 8$ then show that maximum value of x^2y is $\frac{2048}{27}$

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Chapter 13 : Indefinite Integeals

SECTION B
(2 - Marks)

1. Evaluate $\displaystyle\int \sqrt{\frac{e^x - 1}{e^x + 1}}\, dx$

2. Evaluate $\displaystyle\int \frac{\sec^2 x - 7}{\sin^7 x}\, dx$

3. Evaluate $\displaystyle\int \sin(101\, x) \sin^{99} x\, dx$

4. Evaluate $\displaystyle\int \frac{\sqrt{\cot x}}{\sin x \,.\, \cos x}\, dx$

5. Evaluate $\displaystyle\int \frac{\log(e^x + 1)}{e^x}\, dx$

6. Evaluate $\displaystyle\int \frac{x^4 + 1}{x^6 + 1}\, dx$

7. Evaluate $\displaystyle\int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}}\, dx$

8. Evaluate $\displaystyle\int \frac{1 + x}{1 + \sqrt[3]{x}}\, dx$

9. Evaluate $\displaystyle\int x \,.\, (x^x)^x (2 \log x + 1)\, dx$

10. Show that $\displaystyle\int \frac{dx}{\sin^4 x}$ is a polynomial of degree three in cot x.

SECTION C

3 - Marks

1. Evaluate $\int \tan^{-1}\left(\sqrt{\dfrac{1-x}{1+x}}\right) dx$

2. Evaluate $\int \dfrac{dx}{\cos^3 x \sqrt{\sin 2x}}$

3. Evaluate $\int \dfrac{dx}{x^2 (x^4 + 1)^{3/4}}$

4. Evaluate $\int \dfrac{1}{1 + x + x^2 + x^3} dx$

5. Evaluate $\int \dfrac{1}{x} \sqrt{\dfrac{1-x}{1+x}} dx$

6. Evaluate $\int \dfrac{(3 \sin\theta - 2) \cos\theta}{5 - \cos^2\theta - 4 \sin\theta} d\theta$

7. Evaluate $\int x \sin x . \sec^3 x \, dx$

8. Evaluate $\int \dfrac{\sin x}{\sqrt{1 + \sin x}} dx$

9. Evaluate $\int \dfrac{\cos 4x - 1}{\cot x - \tan x} dx$

10. Show that $\int (\sqrt{\tan x} + \sqrt{\cot x}) \, dx = \sqrt{2} \tan^{-1}\left(\dfrac{\tan x - 1}{\sqrt{2 \tan x}}\right) + c$

SECTION D

4 - Marks

1. Evaluate $\int \tan^{-1}(1 + \sqrt{x})\, dx$

2. Evaluate $\int \dfrac{1}{\sin^3 x + \cos^3 x}\, dx$

3. Evaluate $\int \log \left| \sqrt{1 - x} + \sqrt{1 + x} \right| dx$

4. Evaluate $\int \dfrac{1}{\cos^2 x + \cot^2 x}\, dx$

5. Evaluate $\int \dfrac{1}{\sec x + \operatorname{cosec} x}\, dx$

6. Evaluate $\int \dfrac{\sin x}{\sin 4x}\, dx$

7. If $\int [\log(\log x) + (\log x)^{-2}]\, dx = f(x) + c$ then find $f(x)$.

   Also find f (x) when graph of y = f (x) passes through the point (e, e)

8. Evaluate $\int \sin^{-1}\left(\sqrt{\dfrac{x}{a + x}}\right) dx$

9. Evaluate $\int \dfrac{(x + 1)\sqrt{x + 2}}{\sqrt{x - 2}}\, dx$

10. Evaluate $\int \dfrac{dx}{\sqrt{2x + 3} + \sqrt{x + 2}}\, dx$

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Chapter 14 : Definite Integrals and It's Applications

SECTION B

2 - Marks

1. Evaluate $\int_{0}^{\pi/2} \frac{3\sin\theta + 4\cos\theta}{\sin\theta + \cos\theta}$

2. Evaluate $\int_{1/3}^{1} \frac{(x - x^3)^{\frac{1}{3}}}{x^4}\, dx$

3. Show that $\int_{0}^{\pi/2} \cos^{99}x\ dx = 0$

4. Find the area of the region enclosed by the lines y = x, x = e and the curve y = $\frac{1}{x}$ and posiitve x - axis.

5. Evaluate $\int_{1}^{e^{37}} \frac{\pi\sin(\pi\log x)}{x}\ dx$

6. Evaluate $\int_{-3}^{3} f(x - [x])\, dx$ where f is signum function and [.] denotes greates integer function.

7. Evaluate $\int_{\alpha}^{\beta} x\,|\,x\,|\, dx$ where $\alpha < 0 < \beta$.

8. Show that $\int_{0}^{\pi/2} (2\log\sin x - \log\sin 2x)\, dx = \frac{-\pi}{2}\log 2$

9. Evaluate $\int_{\frac{1}{e}}^{e} |\log x|\ dx$

10. Show that $\int_{0}^{\pi} x\, f(\sin x)\, dx = \pi\int_{0}^{\pi/2} f(\cos x)\, dx.$

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SECTION C

3 - Marks

1. Evaluate $\displaystyle\int_0^{\pi/2} \frac{1}{1 + e^{\sin x}}\, dx$

2. Evaluate $\displaystyle\int_0^{1} [\, 5x \,]\, dx$. Where [.] denotes greatest integer function.

3. Find the area bounded between the parablas $x^2 = \dfrac{y}{4}$ and $x^2 = 9y$ and the strainght line $y = 2$.

4. Show that $\displaystyle\int_0^{\pi/4} \log (1 + \tan^2\theta + 2 \tan \theta)\, d\theta = \frac{\pi}{4}\log 2$

5. Evaluate $\displaystyle\int_{-1}^{3} | x - 2 | + [x])\, dx$, where [.] denotes greatest integer function.

6. If $f(x) = \displaystyle\int_1^{x} \sqrt{2 - t^2}$ at then find the roots of the equation $x^2 - f'(x) = 0$

7. Find the area of the region bounded by the curves $y = \begin{cases} x^2, & x < 0 \\ x, & x > 0 \end{cases}$ and the line $y = 4$

8. Evaluate $\displaystyle\int_0^{\pi/2} (\sin 2x \,.\, \tan^{-1} (\sin x))\, dx$.

9. If $f(x)$ is a continuous function such taht $f(2 - x) + f(x) = 0$ for all x, then find

$$\int_0^{2} \frac{1}{1 + 2^{f(x)}}\, dx$$

10. Evaluate $\displaystyle\int_0^{1} \sin \left(2 \tan^{-1} \left(\sqrt{\frac{1 + x}{1 - x}}\right)\right) dx$

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SECTION D
4 - Marks

1. Evaluate $\int_0^1 \tan^{-1}(1 - x + x^2)\, dx.$

2. Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} | x \cos \pi x |\, dx.$

3. Evaluate $\int_0^\infty \frac{x \log x}{(1 + x^2)^2}\, dx.$

4. Evaluate $\int_0^1 \frac{2 - x^2}{(1 + x)\sqrt{1 - x^2}}\, dx.$

5. $\int_{\frac{1}{e}}^{e} \frac{| \log x |}{x^2}\, dx.$

6. Find the area bounded by the curves $x = y^2$ and $x = 3 - 2y^2$.

7. Evaluate $\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(| x | + \frac{\pi}{3}\right)}\, dx.$

8. Find the area enclosed within the curves $| x | + | y | = 1$

9. If $I = \int_2^4 \frac{\sqrt{x^2 - 4}}{x^4}\, dx$ then show that $I = \sqrt{3}$

10. Let $f(x)$ = maximum of $\{ x + | x |, x - [x] \}$ where $[.]$ denotes greatest integer function then find $\int_{-2}^{2} f(x)\, dx.$

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Chapter 15 : Differential Equation

(2 - Marks)

1. Find the differential equation of all circles passing through the origin and having their centres on the X - axis.

2. Verify $y = \frac{1}{4}e^{-2x} + cx + d$, is the solution of the differential equation $\frac{d^2y}{dx^2} = e^{-2x}$

3. Find the equation of the curve whose slope $\frac{dy}{dx} = \frac{2y}{x}$, x, $y > 0$ which passes through the point (1, 1)

4. Find the equation of curve passing through (1, $\pi/4$) and having slope $\frac{\sin 2y}{x + \tan y}$ at (x, y)

5. Find the integrating factor (I.F.) of differential equation $\frac{dy}{dx} = e^{x-y}(1 - e^{y})$

6. If $\sin x$ is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ then find the value of P.

7. Find the integrating factor of the differential equation $(xy - 1)\frac{dy}{dx} + y^2 = 0$

8. State order and degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} \left(\frac{dy}{dx}\right)^{\frac{1}{3}} = 5$

9. By eliminating arbitary constant of equation $y = c^2 + \frac{c}{x}$ find differential equation.

10. If $y = \sin^{-1} x$ then show that $(1 - x^2)\frac{d^2y}{dx^2} = x\frac{dy}{dx}$

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3 - Marks

1. Find the differential equation associated to the primitive $y = ae^{4x} - be^{-3x} + c$
2. Find A if $x = 4t^3$, $y = 4t^2 - t^4$ constitute a solution of the differential equation

$$36 \frac{d^2y}{dx^2} \left[y - (2x)^{2/3} \right] = A + \left(\frac{x}{4}\right)^{2/3}$$

3. Find the particular solution of the differenital equation given as

$e^{\frac{dy}{dx}} = x + 1$ at $y(0) = 3$

4. Find $y(\pi/2)$ if $(2 + \sin x)\frac{dy}{dx} + (y + 1) \cos x = 0$ and $y(0) = 1$
5. Find the equation of the curve passing through $(2, -2)$ and having slope $\frac{y+1}{x^2 - x}$
6. If curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differenital

quation $y(1 + xy)\,dx = x\,dy$, then find $f\left(-\frac{1}{2}\right)$

7. Find the general solution of the differenital equation

$$(e^{x^2} + e^{y^2})\, y \frac{dy}{dx} + e^{x^2}(xy^2 - x) = 0$$

8. The differential equation $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ whose general solution is given by

$f(x, y)\, e^{e^x} =$ constant, then find $f(0, 0)$

9. Find the general solution of the differential equation

$\cos x\, dy = y(\sin x - y)\, dx, \quad 0 < x < \pi/2$

10. If $f'(x) = f(x)$ and $f(-1) = 1$ then find $f(5)$.
11. Form and differential equation satisfying $\sqrt{1 - x^4} + \sqrt{1 - y^4} = a(x^2 - y^2)$ and find the degree.
12. If the population of a country doubles in 50 years. Then find the number of years in which population will be triple under the assumption that the rate of increase of population proportional to the number of inhabitants.
13. A ray of light coming from origin after reflecting at the point $P \equiv (x, y)$ of any curve become parallel to x - axis find the equation of curve.

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4 - Marks

1. The rate at which radioactive substance decay is known to be proportional to the number of such nuclei that are present at that time in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergoan disintegration in a period of 100 years. Find what percentage of the original radioactive nuclei will remain after 1000 years.

2. In a college hostel accomodating 1000 students, one of them came in carrying a flue virus, then the hostel was isoleted. If the rate of which the virus spreads is assumed to be proportional to the product of the number N of infected students and the number of non infected students and if the number of infected students is 50 after 4 days. Show that more than 95% of the students will be infected after 10 days.

3. Water flows from the base of rectangular tank of depth 16 meters. The rate of flowing the water is proportional to the square root of depth at any time 't'. After 2 hours depth of water is 4 meter, find the depth of water after 4 hours.

4. Water at 100^0C cools in 10 minutes to 88^0C, in the room temperature of 25^0C. Find the temperature of water after 20 minutes.

5. A tank of 100 m^3 capacity is full with pure water. Begining at t = 0 brine containing 1 kg/m^3 of salt runs in at the rate 1 m3/min. The mixture is kepts uniform by stirring. It runs out at the same rate when will there by 50 kg. of dissolved salt in the tank ?

6. Show that the singular solution of the differential equation $y = mx + m - m^3$ where

   $m = \dfrac{dy}{dx}$ passes through the point (–1, 0)

7. Let the population of rabiits surviving at a time 't' be governed by the differential equation

   $$\frac{dp\,(t)}{dt} = -\tfrac{1}{2} P\,(t) - 200.$$ If p (0) = 100 then find p (t).

8. A hemispherical tank (radius = 1m) is initially full of water and has an out let of 12 cm^2 at the bottom. When the out let is opened the flow of water is according to the law

   $v\,(t) = \lambda \quad h(t)$ where v (t) is the velocity of flow in cm/sec. h (t) is the water level in cms. and λ is constant. Find the time taken to empty of tank is sec.

9. A mothball evaporate at a rate proportional to the instantaneus surface area. Its radius to half in value in one day. Find the requaired for the ball to disapper compleletly.

10. An inverted conical tank of 2 m radius and 4n height is initially full of water has an out let at bottom. The outlet is opened at some instant. The rate of flow through the outlet at any time t is $6\,h^{3/2}$, where h is height of water level above the out let at time t. Then fine the time it takes to empty the tank.

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Chapter 16 : Probability Distribution- VSA Qns

(2 - Marks)

1. Two cards are drawn from a pack of 52 cards. Prepare the probability distribution of the random variable defined as, number of black cards.

2. State with reasons whether the following represent the p.m.f of a random variable.

x :	0	1	2	3
p(x) :	0.1	0.2	-0.1	0.7

y	0	1	2
P(y)	0.1	0.2	0.5

3. Verify whether the following function is p.m.f. of continuous r.v.X.

$$f(x) = x/2 \qquad -2 < x < 2$$
$$= 0 \qquad \text{, otherwise}$$

4. Verify whether the following function can be regarded as the p . m. f for the given values of X

$$P(X = x) = \frac{x - 2}{5} \qquad X = 1,2,3,4,5$$
$$= 0 \quad , \quad \text{otherwise}$$

5. Verify whether the following function can be regarded as the p.m.f. for the given values of X

$$P(X = x) = 1/5 \qquad \text{for } x = 0,1,2,3,4$$
$$= 0 \qquad \text{otherwise}$$

6. Determine k such that the following function is a p.m.f.

$$P(X = x) = kx \ , \qquad x = 1,2,3,4,5$$
$$= 0 \ , \qquad \text{otherwise}$$

7. Determine k such that the following function is a p.m.f.

$$P(X = x) = k(.2^x/x!) \ , \ x = \ 0,1,2,3$$
$$= 0 \qquad \text{otherwise}$$

8. In a Pizza Hut , the following distribution is found for a daily demand of Pizzas. Find the expected daily demand.

No. of Pizzas :	5	6	7	8	9	10
Probability :	0.07	0.2	0.3	0.3	0.07	0.06

3 - Marks

1. Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces.

2. Find the probability distribution of number of doublets in three throws of a pair of dice.

3. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some constant.

$$P(X = x) = \begin{cases} 0.1 & \text{if } x = 0, \\ kx & \text{if } x = 1 \text{ or } 2, \\ k(5 - x) & \text{if } x = 3 \text{ or } 4, \\ 0, & \text{otherwise.} \end{cases}$$

Then a) Find the value of k ,

b) What is the probability that you study atleast two hours ?

4. Find the probability distribution of

i) Number of heads in two tosses of a coin

ii) Number of tails in the simultaneous tosses of three coins

iii) Number of heads in four tosses of a coin.

5. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

6. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice. Find the probability distribution of number of tails.

7. A random variable X has the following probability distribution:

X :	0	1	2	3	4	5	6	7
$P(X = x)$:	0	k	2k	2k	3k	k^2	$2k^2$,	$7k^2 + k$

Determine ; i) k, ii) $P(X < 3)$, iii) $P(0 < X < 3)$.

8. The r.v X has the following probability distribution function as

$$\begin{aligned} P(x) &= k && \text{if } x = 0 \\ &= 2k, && \text{if } x = 1, \\ &= 3k, && \text{if } x = 2, \\ &= 0, && \text{otherwise.} \end{aligned}$$

Then find i) k ii) $P(X < 2)$ iii) $P(X \leq 2)$.

9. Given below is the probability distribution of a discrete r.v. X:

X :	1	2	3	4	5	6
P (X = x) :	k	0	2k	5k	k	3k

Find k and hence find $P(2 \leq X \leq 3)$

10. Two fair dice are rolled. X denotes the sum of numbers appearing on the uppermost faces of the dice. Find i) $P(X < 4)$ ii) $P(3 < X < 7)$.

11. It is known that a box of 8 batteries contains 3 defective pieces and a person randomly selects 2 batteries from this box. Find the probability distribution of the number of defective batteries.

12. The probability distribution of a r.v X is as follows:

X :	-1.5	-0.5	0.5	1.5	2.5
P (X = x) :	0.05	0.2	0.15	0.25	0.35

Construct c.d.f. F(x) of X.

13. In the following table c.d.f of r.v X is given

X :	–2	–1	0	1	2
F (X_i) :	0.2	0.5	0.65	0.9	1

Find p.m.f. of X. Also find $P[X \leq 0]$.

14. An urn contains 4 white and 6 red balls . 4 balls are drawn at random from the urn . Find the probability distribution of the number of white balls.

15. 4 bad oranges are mixed accidently with 16 good oranges. Find the probability distribution of the number of bad oranges in a draw of 2 oranges.

16. In a roadside joint, veg and non-veg samosas are served along with other things. Profit per samosa is 3 rupees for a veg samosa and 5 rupees for a non-veg samosa .The probability distribution for the demand of veg and non-veg samosa are as follows:

i)

Demand (veg) :	10	15	20	25	30
Probability :	0.3	0.2	0.3	0.15	0.05

ii)

Demand (non-veg) :	5	7	9	11
Probability :	0.4	0.3	0.15	0.15.

Which type of samosa brings in more expected profit?

17. Show that the function f(x) defined by , $f(x) = 1/7$ for $1 \leq x < 8$,

$= 0$, otherwise,

is a probability density function for a random variable. Hence find $P(3 < X < 10)$.

18. Find the c.d.f F (x) associated with the following pdf. f (x)

f (x) = $12x^2 (1 - x), 0 < x < 1,$

= 0 , otherwise. Also, find P $\left(\frac{1}{3} < x < \frac{1}{2}\right)$ by using c.d.f. and sketch the graph of F(x).

19. A die is tossed twice. Getting a number greater than 4 is considered a "success". Find the mean and variance of the probability distribution of the number of successes.

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Chapter 17 : Binomail Distribution -VSA

(2 - Marks)

1. Write the Binomial distribution if mean for the distribution is 3 and the standard deviation is 3/2.
2. If X ~ B (6, p) and 2 P (X = 3) = P (X = 2) then find the value of p.
3. If a die is thrown twice, then find the probability of occurence of 4 atleast once.
4. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn one-by-one with replacement then find the variance of the number of yellow balls.
5. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting point 0, 1, 2 are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent. Find the probability of India getting at least 7 points .
6. 100 identical coins, each with probability p, of showing up heads are tossed once. If $0 < p < l$ and the possibility of heads showing on 50 coins is equal to that of heads showing on 51 coins, then find the value of p .

3 Marks

1. If the mean and the variance of the binomial varite X are 2 and 1 respectively, then find the probability that X takes a value greater than one.
2. Numbers are selected at random, one at a time, from the two-digit numbers 00, 01, 02, 03,....99 with replacement. An event E occurs if and only if the product of the two digits of a selected number is 18. If four numbers are selected, find probability that the event E occurs at least 3 times.
3. A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability at the end of eleven steps he is one step away from the starting point.
4. Find the minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96.
5. A lot of 100 pens contains 10 defective pens. 5 pens are selected at random from the lot and sent to the retail store. What is the probability that the store will receive at least one defective pen?
6. In a production process, producing bulbs, the probability of getting a defective bulb remains constant and it is 0.3. If we select a sample of 10 bulbs, what is the probability of getting 3 defective bulbs?

7. In a bag containing 100 eggs, 10 eggs are rotten. Find the probability that out of a sample of 5 eggs none are rotten, if the sampling is with replacement.
8. A fair coin is tossed six times. What is the probability of obtaining four or more heads?
9. Each of two persons A and B toss 3 fair coins. Find the probability that both get the same number of heads.
10. The probability of India winning a test match against England is 2/3. Assuming independence from match to match, find the probability that in a 7 match series India's third win occurs at the 5^{th} match..
11. Let x denotes the number of times heads occur in n tosses of a fair coin, if P(X = 4), P (X = 5) and P(X = 6) are in A.P., find n.

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