GATE-2011-AE

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1) The possible set of eigenvalues of a 4×4 skew-symmetric orthogonal real matrix is

a) $\{\pm i\}$ b) $\{\pm i, \pm 1\}$ c) $\{\pm 1\}$ d) $\{0, \pm i\}$

2) The coefficient of $(z - \pi)^2$ in the Taylor series expansion of $f(z) = \begin{cases} \frac{\sin z}{z-\pi} & if z \neq \pi \\ -1 & if z = \pi \end{cases}$ around π is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $\frac{1}{6}$ d) $-\frac{1}{6}$

3) Consider $\mathbb{R}^2$ with the usual topology. Which of the following statements are TRUE for all $A, B \subseteq \mathbb{R}^2$?
P: $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
Q: $\overline{A \cap B} = \overline{A} \cap \overline{B}$.
R: $(A \cup B)^o = A^o \cup B^o$.
S: $(A \cap B)^o = A^o \cap B^o$.

a) P and R only b) P and S only c) Q and R only d) Q and S only

4) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function with $f(1) = 5$ and $f(3) = 11$. If $g(x) = \int_1^3 f(x+t)\, dt$ then $g'(0)$ is equal to ____________.

5) Let P be a 2×2 complex matrix such that $trace(P) = 1$ and $\det(P) = -6$. Then, $trace\left(P^4 - P^3\right)$ is ____________.

6) Suppose that R is a unique factorization domain and that $a, b \in R$ are distinct irreducible elements. Which of the following statements is **TRUE**?
a) The ideal $\langle 1 + a \rangle$ is a prime ideal.
b) The ideal $\langle a + b \rangle$ is a prime ideal.
c) The ideal $\langle 1 + ab \rangle$ is a prime ideal.
d) The ideal $\langle a \rangle$ is not necessarily a maximal ideal.

7) Let X be a compact Hausdorff topological space and let Y be a topological space. Let $f : X \to Y$ be a bijective continuous mapping. Which of the following is **TRUE**?
a) f is a closed map but not necessarily an open map.
b) f is an open map but not necessarily a closed map.
c) f is both an open map and a closed map.
d) f need not be an open map or a closed map.

8) Consider the linear programming problem:

$$Maximize x + \frac{3}{2}y$$

$$subject\ to\ 2x + 3y \leq 16,$$

$$x + 4y \leq 18,$$

$$x \geq 0,\ y \geq 0.$$

If S denotes the set of all solutions of the above problem, then

a) S is empty.
b) S is a singleton.
c) S is a line segment.
d) S has positive area.

9) Which of the following groups has a proper subgroup that is **NOT** cyclic?
a) $\mathbb{Z}_{15} \times \mathbb{Z}_{77}$
b) S_3
c) $(\mathbb{Z}, +)$
d) $(\mathbb{Q}, +)$

10) The value of the integral

$$\int_0^\infty \int_x^\infty \left(\frac{1}{y}\right) e^{-y/2}\, dy\, dx$$

is ___________.

11) Suppose the random variable U has uniform distribution on $[0, 1]$ and $X = -2\log U$. The density of X is

a) $f(x) = \begin{cases} e^{-x} & if\ x > 0 \\ 0 & otherwise \end{cases}$

b) $f(x) = \begin{cases} 2e^{-2x} & if/, x > 0 \\ 0 & otherwise \end{cases}$

c) $f(x) = \begin{cases} \frac{1}{2}e^{-x/2} & if\ x > 0 \\ 0 & otherwise \end{cases}$

d) $f(x) = \begin{cases} \frac{1}{2} & if\ x \in [0, 2] \\ 0 & otherwise \end{cases}$

12) Let f be an entire function on $\mathbb{C}$ such that $|f(z)| \leq 100 \log |z|$ for each z with $|z| \geq 2$. If $f(i) = 2i$, then $f(1)$
a) must be 2
b) must be $2i$
c) must be i
d) cannot be determined from the given data

13) The number of group homomorphisms from $\mathbb{Z}_3$ to $\mathbb{Z}_9$ is ___________.