

- 1) The possible set of eigenvalues of a 4×4 skew-symmetric orthogonal real matrix is
- a) $\{\pm i\}$ b) $\{\pm i, \pm 1\}$ c) $\{\pm 1\}$ d) $\{0, \pm i\}$
- 2) The coefficient of $(z - \pi)^2$ in the Taylor series expansion of $f(z) = \begin{cases} \frac{\sin z}{z - \pi} & \text{if } z \neq \pi \\ -1 & \text{if } z = \pi \end{cases}$ around π is
- a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $\frac{1}{6}$ d) $-\frac{1}{6}$
- 3) Consider \mathbb{R}^2 with the usual topology. Which of the following statements are TRUE for all $A, B \subseteq \mathbb{R}^2$?
- P: $A \cup B = \overline{A \cup B}$.
 Q: $A \cap B = \overline{A \cap B}$.
 R: $(A \cup B)^o = A^o \cup B^o$.
 S: $(A \cap B)^o = A^o \cap B^o$.
- a) P and R only b) P and S only c) Q and R only d) Q and S only
- 4) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function with $f(1) = 5$ and $f(3) = 11$. If $g(x) = \int_1^3 f(x+t) dt$ then $g'(0)$ is equal to _____.
- 5) Let P be a 2×2 complex matrix such that $\text{trace}(P) = 1$ and $\det(P) = -6$. Then, $\text{trace}(P^4 - P^3)$ is _____.
- 6) Suppose that R is a unique factorization domain and that $a, b \in R$ are distinct irreducible elements. Which of the following statements is **TRUE**?
- a) The ideal $\langle 1 + a \rangle$ is a prime ideal.
 b) The ideal $\langle a + b \rangle$ is a prime ideal.
 c) The ideal $\langle 1 + ab \rangle$ is a prime ideal.
 d) The ideal $\langle a \rangle$ is not necessarily a maximal ideal.
- 7) Let X be a compact Hausdorff topological space and let Y be a topological space. Let $f : X \rightarrow Y$ be a bijective continuous mapping. Which of the following is **TRUE**?
- a) f is a closed map but not necessarily an open map.
 b) f is an open map but not necessarily a closed map.
 c) f is both an open map and a closed map.
 d) f need not be an open map or a closed map.

8) Consider the linear programming problem:

$$\begin{aligned} & \text{Maximize } x + \frac{3}{2}y \\ & \text{subject to } 2x + 3y \leq 16, \\ & \quad x + 4y \leq 18, \\ & \quad x \geq 0, y \geq 0. \end{aligned}$$

If S denotes the set of all solutions of the above problem, then

- a) S is empty. c) S is a line segment.
 b) S is a singleton. d) S has positive area.

9) Which of the following groups has a proper subgroup that is **NOT** cyclic?

- a) $\mathbb{Z}_{15} \times \mathbb{Z}_{77}$
 b) S_3
 c) $(\mathbb{Z}, +)$
 d) $(\mathbb{Q}, +)$

10) The value of the integral

$$\int_0^\infty \int_x^\infty \left(\frac{1}{y}\right) e^{-y/2} dy dx$$

is _____.

11) Suppose the random variable U has uniform distribution on $[0, 1]$ and $X = -2 \log U$.

The density of X is

- a) $f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$
 b) $f(x) = \begin{cases} 2e^{-2x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$
 c) $f(x) = \begin{cases} \frac{1}{2}e^{-x/2} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$
 d) $f(x) = \begin{cases} \frac{1}{2} & \text{if } x \in [0, 2] \\ 0 & \text{otherwise} \end{cases}$

12) Let f be an entire function on \mathbb{C} such that $|f(z)| \leq 100 \log |z|$ for each z with $|z| \geq 2$.

If $f(i) = 2i$, then $f(1)$

- a) must be 2
 b) must be $2i$
 c) must be i
 d) cannot be determined from the given data

13) The number of group homomorphisms from \mathbb{Z}_3 to \mathbb{Z}_9 is _____.