Quadratic Equations and Inequations(Inequalities)

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I. D.MCQs with One or More than One Correct

1) Number of integral divisors of the form $4n+2(n \geq 0)$ of the integer 240 is (1984-2 Marks)
   a) a positive integer
   b) divisible by n
   c) equal to n+$\frac{1}{n}$
   d) never equal to n
2) If $3^X = 4^x - 1$,then $x =$ (JEE Adv. 2013)
   a) $\frac{2\log_3 2}{2\log_3 2-1}$
   b) $\frac{2}{2-\log_2 3}$
   c) $\frac{1}{1-\log_4 3}$
   d) $\frac{2\log_2 3}{2\log_2 3-1}$
3) Let S be the set of all non-zero real numbers α such that quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality$|x_1 - x_2| < 1$. Which of the following intervals is(are) α subset of S? (JEE Adv. 2015)
   a) $(-\frac{1}{2}, -\frac{1}{\sqrt{5}})$
   b) $(-\frac{1}{\sqrt{5}}, 0)$
   c) $(0, \frac{1}{\sqrt{5}})$
   d) $(\frac{1}{\sqrt{5}}, \frac{1}{2})$

II. E.Subjective Problems

4) solve for $x : 4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$ (1978)
5) If $(m,n) = \frac{(1-x^m)(1-x^{m-1}).........(1-x^{m-n+1})}{(1-x)(1-x^2)...........(1-x^n)}$ Where m and n are positive integers $(n \leq m)$,Show that $(m,n+1) = (m-1,n+1) + x^{m-n-1}(m-1,n)$.(1978)
6) Solve for x: $\sqrt{x+1} - \sqrt{x-1} = 1$. (1978)
7) Solve the following equation for x:
   $2\log_x a + \log_{ax} a + 3\log_{a^2 x} a = 0$,$a > 0$ (1978)
8) Show that the square of $\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$ is a rational number. (1978)
9) Sketch the solution set of the following system of inequalities:
   $x^2+y^2-2x \geq 0; 3x-y-12 \leq 0; y-x \leq 0; y \geq 0$. (1978)
10) Find all integers x for which
    $(5x-1) < (x+1)^2 < (7x-3)$. (1978)
11) If α,β are the roots of $x^2+px+q=0$ and γ,δ are the roots of $x^2+rx+s=0$,evaluate $(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)$ in terms of p,q,r,and s. Deduce the condition that the equations have a common root. (1979)
12) Given $n^4 < 10^n$ for a fixed positive integer $n \geq 2$,
    prove that $(n+1)^4 < 10^{n+1}$. (1980)
13) Let $y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$
    Find all the real values of x for which y takes real values. (1980)
14) For what values of m,does the system of equations (1980)
    $3x + my = m$
    $2x - 5y = 20$
    has solution satisfying the condition x¿0,y¿0. (1980)
15) find the solution set of the system
    $x + 2y + z = 1;$
    $2x - 3y - w = 2;$
    $x \geq 0; y \geq 0; z \geq 0; w \geq 0.$ (1980)