JEE-MAINS-29/01/2023-shift-1

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1 Section A.MCQs

16) Fifteen football players of club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is

a) $\frac{5}{24}$
b) $\frac{2}{15}$
c) $\frac{1}{6}$
d) $\frac{5}{36}$

17) Let
$f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2\left(1 - \sin^2(2\theta)\right)$
and $S = \left(\theta \in (0, \pi) : f'(\theta) = -\frac{\sqrt{3}}{2}\right)$. If $4\beta = \sum_{\theta \in S} \theta$, then $f(\beta)$ is equal to

a) $\frac{11}{8}$
b) $\frac{5}{4}$
c) $\frac{9}{8}$
d) $\frac{3}{2}$

18) If p, q and r are the three propositions, then which of the following combinations of the truth values of p, q and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false ?

a) $p = T, q = F, r = T$
b) $p = T, q = T, r = F$
c) $p = F, q = T, r = F$
d) $p = T, q = F, r = F$

19) There rotten apples are mixed accidently withseven good apples and four apples are drawn oneby one without replacement. Let the random variable X denote the number of rotten apples. If $\mu$and σ^2 represent mean and variance of X,respectively, $10\left(\mu^2 + \sigma^2\right)$ then is equal to

a) 20
b) 250
c) 25
d) 30

20) Let $y = f(x)$ be the solution of the differential equation $y(x+1)\,dx - x^2 dy = 0, y(1) = e$. The $\lim_{x \to 0^+} f(x)$ is equal to

a) 0
b) $\frac{1}{e}$
c) e^2
d) $\frac{1}{e^2}$

2 Section B.Numerical

21) Let the co-ordinats of one vertex of ΔABC be $A(0,2,\alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. For $\alpha \in Z$, if the area of ΔABC is 21 sq.units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to _____.

22) Let the equation of the plane p containing the line $x + 10 = \frac{8-y}{2} = z$ be $ax + by + 3z = 2(a+b)$ and the distance of the plane P from the point $(1,27,7)$ be c. Then $a^2 + b^2 + c^2$ ii equal to _____.

23) Suppose f is a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in N$ and $f(1) = \frac{1}{5}$. If $\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then m is equal to _____.

24) Let $a_1, a_2, a_3, \ldots$ be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to _____.

25) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar vectors. Let the position vectors of four points $A, B, CandD$ be $\vec{a} - \vec{b} + \vec{c}, \lambda\vec{a} - 3\vec{b} + 4\vec{c}, -\vec{a} + 2\vec{b} - 3\vec{c}$ and $2\vec{a} - 4\vec{b} + 6\vec{c}$ respectively. If $\vec{AB}, \vec{AC}$ and $\vec{AD}$ are coplanar, then λ is :

26) If all the six digit numbers $X_1X_2X_3X_4X_5X_6$ with $0 < X_1 < X_2 < X_3 < X_4 < X_5 < X_6$ are arranged in the increasing order, then the sum of the digits in the 72^{th} number is _____.

27) Let $f: R \to R$ be a differentiable function that satisfies the relation $f(x+y) = f(x) + f(y) - 1$, $\forall x, y \in R$. If $f'(0) = 2$, then $|f(-2)|$ is equal to _____.

28) If the co-efficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to _____.

29) Let the coefficients of three consecutive terms in the binomial expansion of $(1+2x)^n$ be in the ratio 2: 5: 8. Then the codfficient of the term, which is in the middle of these terms, is _____.

30) Five digit numbers are formed using the digits 1, 2, 3, 5,7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is _____.