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1) If C be the centroid of the triangle having vertices $(3,-1),(1,3)$ and $(2,4)$. Let P be the point of intersection of the lines $x+3y-1=0$ and $3x-y+1=0$,then the line passing through the point:
   a) $(-9,-7)$
   b) $(-9,-6)$
   c) $(7,6)$
   d) $(9,7)$

2) The product $2^{\frac{1}{4}} \times 4^{\frac{1}{16}} \times 8^{\frac{1}{48}} \times 16^{\frac{1}{128}} \cdots \infty$ to is equal to
   a) $2^{\frac{1}{4}}$
   b) 2
   c) $2^{\frac{1}{2}}$
   d) 1

3) A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at the rate of $50cm^3/min$.When the thickness of ice is 5cm, then the rate($cm/min.$) at which the thickness of ice decreases, is:
   a) $\frac{5}{6\pi}$
   b) $\frac{1}{54\pi}$
   c) $\frac{1}{36\pi}$
   d) $\frac{1}{18\pi}$

4) Let f be any function continuous on $[a,b]$ and twice differentiable on (a,b). If for all $x \in (a,b), f' > 0$ and $f'' < 0$, then for any $c \in (a,b)$, $(f(c)-f(a))-(f(b)-f(c))$ is greater than:
   a) $(b-c)/(c-a)$
   b) 1
   c) $(c-a)/(b-c)$
   d) $(b+a)/(b-a)$

5) The value of $\cos^3\left(\frac{\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)$ is:
   a) $\frac{1}{4}$
   b) $\frac{1}{2\sqrt{2}}$
   c) $\frac{1}{2}$
   d) $\frac{1}{\sqrt{2}}$

6) The number of real roots of the equation, $e^{4x}+e^{3x}-4e^{2x}+e^x+1=0$ is
   a) 3
   b) 4
   c) 1
   d) 2

7) The value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x+\cos^8 x}\,dx$ is equal to
   a) 2
   b) 4
   c) 2^2
   d) π^2

8) If for some α and β in R,the intersection of the following three planes
   $x+4y-2z=1$
   $x+7y-5z=\beta$
   $x+5y+\alpha z=5$
   is a line in R^3, then $\alpha+\beta$ is equal to:
   a) 0
   b) 10
   c) -10
   d) 2

9) If e_1 and e_2 are the eccentricities of the ellipse, $\left(\frac{x^2}{18}\right)+\left(\frac{y^2}{4}\right)=1$ and the hyperbola, $\left(\frac{x^2}{9}\right)-\left(\frac{y^2}{4}\right)=1$ respectively and (e_1,e_2) is a point on the ellipse,$15x^2+3y^2=k$.Then k is equal to:
   a) 14
   b) 15
   c) 17
   d) 18

10) If f(x) $= \begin{cases} \frac{\sin(a+2)x+\sin x}{x} & ,x<0 \\ b & ,x=0 \\ \frac{(x+3x^{\frac{2}{3}}-x^{\frac{1}{3}})}{x^{\frac{4}{3}}} & ,x>0 \end{cases}$
    is continuous at $x=0$ then $a+2b$ is equal to:
    a) -2
    b) 1
    c) 0
    d) 1

11) If the matrices
    $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}$, then $B=adjA$ and $C=3A$, then

$\frac{|adj\,B|}{|C|}$ is equal to

a) 16
b) 2
c) 8
d) 72

12) A circle touches the y-axis at the point $(0, 4)$ and passes through the point $(2, 0)$. Which of the following lines is not a tangent to the circle?

a) $4x - 3y + 17 = 0$
b) $3x + 4y - 6 = 0$
c) $4x + 3y - 8 = 0$
d) $3x - 4y - 24 = 0$

13) Let Z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$.Then the value of $|z + 3i|$ is:

a) $\sqrt{10}$
b) $\frac{7}{2}$
c) $\frac{15}{4}$
d) $2\sqrt{3}$

14) If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $\frac{-\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to:

a) $\frac{\pi+1}{4}$
b) $\frac{\pi+2}{4}$
c) $\frac{1}{4}$
d) $\frac{\pi-1}{4}$

15) Negation of the statement: ' $\sqrt{5}$ is an integer or 5 is irrational' is:

a) $\sqrt{5}$ is irrational or 5 is an integer.
b) $\sqrt{5}$ is not an integer or 5 is not irrational
c) $\sqrt{5}$ is an integer and 5 is irrational
d) $\sqrt{5}$ is not an integer and 5 is not irrational