JEE-MAINS-25/02/2021-shift-1

AI24BTECH11024-Pappuri Prahladha

1) The coefficients a,b and c of the quadratic equation,$ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is:
   a) $\frac{1}{54}$
   b) $\frac{1}{72}$
   c) $\frac{1}{36}$
   d) $\frac{5}{216}$
2) Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$.Then the value of $\sin^4\alpha + \cos^4\alpha$ is:
   a) $\frac{3}{4}$
   b) $\frac{1}{2}$
   c) $\frac{5}{8}$
   d) $\frac{3}{8}$
3) The value of the integral
   $\int \frac{\sin\theta\cdot\sin 2\theta\left(\sin^6\theta - \sin^4\theta + \sin^2\theta\right)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1-\cos 2\theta}\, d\theta$
   is
   a) $\frac{1}{18\left[9-2\sin^6\theta-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\left(\frac{3}{2}\right)}} + c$
   b) $\frac{1}{18\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\left(\frac{3}{2}\right)}} + c$
   c) $\frac{1}{18\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\left(\frac{3}{2}\right)}} + c$
   d) $\frac{1}{18\left[9-2\cos^6\theta-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\left(\frac{3}{2}\right)}} + c$
4) A man is observing, from the top of a tower, a boat speeding point A, with uniform speed. At that point, the angle of depression of the boat with the man's eye is 30° (Ignore man's height).After sailing for 20 seconds towards the base of the tower(which is at the level of water),the boat has reached point B, where the angle of depression is 45°.Then the time taken (in seconds) by the boat from B to reach the base of the tower is:
   a) $10\left(\sqrt{3}-1\right)$
   b) $10\sqrt{3}$
   c) 10
   d) $10\left(\sqrt{3}+1\right)$
5) If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty}\cos^{2n}\theta$, $y = \sum_{n=0}^{\infty}\sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty}\cos^{2n}\theta\cdot\sin^{2n}\phi$ then:
   a) $xyz = 4$
   b) $xy - z = (x + y)z$
   c) $xy + yz + zx = z$
   d) $xy + z = (x + y)z$

6) The equation of the line through the point $(0,1,2)$ and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is:
   a) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$
   b) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$
   c) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$
   d) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$
7) The statement $A \to (B \to A)$ is equivalent to:
   a) $A \to (A \wedge B)$
   b) $A \to (A \vee B)$
   c) $A \to (A \to B)$
   d) $A \to (A \leftrightarrow B)$
8) The integer k, for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in R is:
   a) 3
   b) 2
   c) 4
   d) 0
9) A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$.which of the following points does NOT lie on it?
   a) $(0,3)$
   b) $(-6,0)$
   c) $(4,5)$
   d) $(5,4)$
10) Let $f, g\colon N \to N$ such that $f(n+1) = f(n) + f(1)$ for all $n \in N$ and g be any arbitrary function. Which of the following statements is NOT true?
   a) f is one-one
   b) If fog is one-one, then g is one-one
   c) If g is onto, then fog is one-one
   d) If f is onto, then $f(n) = n$ for all $n \in N$
11) Let the lines $(2-i)z = (2+i)\bar{z}$ and $(2+i)z + (i-2)\bar{z} - 4i = 0$, $\left(here\ i^2 = -1\right)$ be normal to this circle C, then its radius is:
   a) $\frac{3}{\sqrt{2}}$
   b) $3\sqrt{2}$
   c) $\frac{3}{2\sqrt{2}}$
   d) $\frac{1}{2\sqrt{2}}$
12) All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in
   a) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$
   b) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$
   c) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$
   d) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

13) The image of the point $(3, 5)$ in the line $x - y + 1 = 0$, lies on:

a) $(x-2)^2 + (y-4)^2 = 4$
b) $(x-4)^2 + (y+2)^2 = 16$
c) $(x-4)^2 + (y-4)^2 = 8$
d) $(x-2)^2 + (y-2)^2 = 12$

14) If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4, x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to:

a) $(-5, 8)$
b) $(5, 8)$
c) $(5, -8)$
d) $(-5, -8)$

15) If the curves $\frac{x^2}{a} + \frac{y^2}{b}$ and $\frac{x^2}{c} + \frac{y^2}{d}$ intersect each other at an angle of $90°$, then which of the following relations is true?

a) $a + b = c + d$
b) $a - b = c - d$
c) $ab = \frac{(c+d)}{(a+b)}$
d) $a - c = b + d$